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Collusive Market-Sharing and Corruption in Procurement[1]

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ABSTRACT

This paper investigates links between corruption and collusion in procurement. A first-price multiple-object auction is administered by an agent who has legal discretion to allow for a readjustment of (all) submitted offers before the official opening. The agent may be corrupt, i.e. willing to "sell" his decision in exchange for a bribe. Our main result shows that the corrupt agent's incentives to extract rents are closely linked with that of a cartel of bidders. First, collusive bidding conveys value to the agent's decision power. Second, self-interested abuse of discretion to extract rents (corruption) provides a mechanism to enforce collusion. A second result is that package bidding can facilitate collusion. We also find that with corruption, collusion is more likely in auctions where firms are small relative to the market. Our main message to auction designers, competition authorities and criminal courts is that risks of collusion and of corruption must be addressed simultaneously. Some other policy implications for the design of tender procedures are discussed.

Keywords: auction, corruption, collusion.

JEL: D 44, H 57, K 42.

1 Introduction

Recent advances in auction theory allow obtaining deep insights into the design of complicated selling schemes in different environments (Krishna (2002), Milgrom (2004)). Yet, major problems of real-life auction mechanisms received only limited attention in the theoretical literature. Klemperer (2002) argues that collusion between bidders should be a major concern for auction designers. In this paper, we investigate how a collusive agreement can be sustained in the presence of a corrupt auctioneer. The main motivation for the paper is a mounting body of evidence that collusion and corruption often go hand in hand in public procurement.[2]

In France, practitioners and investigators in courts of accounts, competition authorities, and in the judiciary have long been aware of close links between collusion and corruption in public tenders. The "spectacular" testimony of J. C. Mery provides suggestive evidence of such links (*Le Monde*, September 22 and 23, 2000).[3] A recent judgment in the "Les Yvelinnes" case (Cour d'Appel de Versaille, January 2002) provides a vivid illustration as well. Detailed evidence revealed the ways in which corrupt politicians and procurement officials use to initiate and arbitrate collusion in the allocation of maintenance and construction contracts. Finally, according to a judge investigating a major corruption case in Paris, it is a rare exception that a large stake collusion in public procurement in France goes without corruption.[4]

Besides empirical evidence, there are theoretical issues motivating the study of links between collusion and corruption. First, any cartel must solve a series of problems including agreeing how to share the spoils, securing enforcement, and deterring entry (see McAfee and McMillan, 1992). A corrupt auctioneer can contribute to solving some of these problems, e.g. by providing means of retaliation to secure enforcement or creating barriers to entry. Second, corrupt auctioneers might seek to extract rents. Certain provisions in auction rules may provide them with ample opportunities to support collusion in order to create rents that they can appropriate.

[2] Although our focus in this paper is on public procurement, the theory could be applied to private procurement as well.

[3] J. C. Mery, a City Hall official, left a video tape as he died. On the tape he desribes how under ten years (1985-94) he organized and arbitrated collusion in the allocation of construction and maintenance contracts for the Paris City Hall. In exchange, firms were paying bribes used to finance political parties. The contracts in question were on average very profitable: they generated up to 30 percent profit in an industry that averages 5 percent. Mr. Mery also claimed that he had always managed to allocate the contracts to the lowest price bidder. Both these features suggest that the firms were not competing with each other, but were instead implementing some kind of market sharing agreement.

[4] The case concerns the procurement of a 4.3 billion euros construction market (see *Le Monde*, January 26, 2000).

We show that in a one-shot first-price multiple-object auction corruption can induce collusive market-sharing. The reason is that the auctioneer, who acts as an agent for the public interest, often has discretion to let firms simultaneously readjust their bids. If the auctioneer is honest, this provision does not create any inefficiency. If the auctioneer is corrupt, collusion becomes sustainable. The basic intuition is that a defection from collusive bidding creates an opportunity for the auctioneer to extract rents by abusing his right to let firms readjust their offers. When he exploits this opportunity, the auctioneer effectively makes defection less profitable.

In practice, formal procedures in procurement often include various provisions that allow the auctioneer to intervene during the tendering process, e.g., when a new information becomes available, to correct an undue informational advantage or to clear a tender document from an ambiguity. Upon such an intervention, bidders are allowed to readjust submitted bids; the submission deadline might be extended. The World Bank guidelines 'Procurements under IBRD loans and IDA credits' specify that "*Additional information, clarification, correction of errors or modification in bidding documents* shall be sent to each recipient of the original bidding documents in sufficient time before the deadline. *If necessary the deadline shall be extended.*" article 2.18.[5]

The second result of our paper is that with corruption the gains from a more flexible bidding procedure may be outweighed by an increased risk of collusion: package bidding can facilitate collusion. With bids on individual tasks only, the enforcement power of corruption is much more limited. This result is in contrast with the recent emphasis on advantages of package bidding.[6] Finally, our analysis predicts that collusive market-sharing is more likely to occur in auctions where firms are small relatively to the market. This is because the corrupt auctioneer's self-interest to deter defection implies an unusual role for the cartel's threat equilibrium: the *larger* is the "threat payoffs", the easier it is to deter defection.

A central message to auction designers and procurement agencies emerges from the analysis: risks of collusion and risks of corruption must be addressed simultaneously. This approach is recommended when dealing with the fine details of the procedural design regulating the auctioneer's role as well as when dealing with more profound auction rules such as the choice of the "bidding language".

In our theoretical model, we consider a sealed-bid multiple-object first-price auction with pack-

[5] To secure fair treatment, when the deadline is extended, those who have already submitted are allowed to submit a new bid.

[6] See, e.g., Cybernomics (2000), Ausubel and Milgrom (2002), and Ausubel, Cramton and Milgrom (2005).

age bidding. There is an inefficient public firm (the "government") and n private firms. Following Bernheim and Whinston (1986), we assume symmetric information among bidders. The auctioneer has some discretion with regard to the procedure. On the basis of a private signal, he decides whether or not to extend the deadline for submission so that the participating firms can readjust their offers. In the absence of corruption, any equilibrium is characterized by price competition between private firms. We then assume that before the official opening, the auctioneer can disclose the submitted offers to some bidders, and invite them to compete in bribes for the "right to decide" on the deadline.

We show that the effect of corruption is to impose a cost on defection from collusive bidding. The defector must outbid (in bribes) a firm whose collusive bid he displaced in order to avoid an extension of the deadline, which would trigger non-cooperative bidding. When the bribe needed to outbid any displaced bidder is sufficiently high, defection is deterred. Essentially, the sustainability of collusion is due to the opportunities to observe current action (submitted offers) and to react to them.[7] A contribution of this paper is to show that a combination of the corrupt auctioneer's self-interest and a common form of discretion provides these opportunities.

In the package auction, defection from a collusive bidding profile implies that a displaced bidder earns zero payoff: his stake in the "right to decide" is equal to his competitive payoff. In contrast, in an auction with single-item bidding , a defection from a collusive market-sharing scheme typically would not reduce to zero any other bidders' payoffs, i.e. no firm is fully displaced. No firm may therefore be willing to pay much to revert to a low-payoff competitive equilibrium. As a result, corruption may not suffice to deter defection, and collusion fails. Our results are consistent with early conjectures that package bidding may facilitate collusion (e.g., CRA 1998). To the best of our knowledge, a more formal argument was made in connection to the second-price auction only.[8]

While there exists a significant body of theoretical literature on collusion in auctions initiated by Graham and Marshall (1987) and McAfee and McMillan (1992), corruption in auctions has become a focus of economists' interest only recently. The rapidly growing literature distinguishes between two types of corruption. The first type of corruption, often referred to as "favoritism" in the economic literature, corresponds to cases when an auctioneer biases competition by offering a preferential treatment to some firm (e.g. Laffont and Tirole, 1993, Burguet and Che, 2003 and 2004, Celentani and Ganunza, 2002). The second type of corruption targets competition per se:

[7] If both those opportunities are present in the auction procedure, collusion can be sustained without corruption. Such situation might arise in a sequential auction where the auction ends when no new bid is submitted.

[8] In a single auction context, collusion can be sustained in weakly dominated strategies

the abuse of discretion makes collusion sustainable (Compte *et al*, 2005). They show that in a first-price single object auction conducted by a corrupt auctioneer, collusion may obtain in equilibrium. The auctioneer provides *one* firm with an *illegal* opportunity to secretly resubmit a bid in exchange for a bribe. A key feature in the model is that the competition in bribes for the opportunity to secretly resubmit a bid is imperfect. In contrast, our results do not rely on any imperfection in the bribe competition. Also, our focus is on the role of *legal* provisions in procurement: (i) procedures that imply effective discretion to give *all* bidders a chance to readjust their offer, and (ii) rules pertaining to the formulation of bids in a multiple object context.[9]

The paper proceeds as follows. In section 2, we describe our model. Section 3 starts with benchmark results and then proceeds with our investigation of collusion when the auctioneer is corrupt. Section 4 contains a discussion of central assumptions and some policy recommendations.

2 The Model

There is a large project denoted Ω to be procured. The project is divided into k different tasks indexed with superscript $j : \omega^j$. We denote $S \subseteq \Omega$ a subset of tasks or a package. There exists $2^k - 1$ possible combinations (packages) of tasks. The packages are indexed with a superscript h. The government can implement the project at a cost of 1 per task.[10] We refer to $\overline{p}(S) = |S|$, where $|S|$ denotes the number of tasks in package S, as the reservation price.

There are n private firms indexed $i = 1, ..n$. They have private costs for implementing tasks, $c_i : \mathbb{N} \rightarrow \mathbb{R},\ c_i\left(S^h\right) = c_i\left(\left|S^h\right|\right)$.[11] Let $\Delta c_i(x)$, where $x = |S|$, denote the cost increment imputable to the last task in S. Firms' cost function are characterized by $\Delta c_i(x) < 1$ for $x < m_i$, and $\Delta c_i(x) > 1,\ x > m_i$, for some $m_i < \infty,\ i = 1, ..., n$.[12] Following Bernheim and Whinston (1986), we assume symmetric information among firms: the firms' costs for all packages are known to all firms, while the auctioneer knows the distribution of the firms' costs only.

The auction procedure views each task as *unique*. A package is defined as a set of identified tasks, rather than a quantity of tasks. Thus, an offer made by firm i is a collection of bids $\left(S^h, p_i^h\right)$ where

[9] A larger share of the corruption cases in e.g. France do pertain to situations where the market is made out of a number of contracts. In the 'Les Yvelinnes' case mentioned above, 88 construction/maintenance contracts were simultaneously allocated, and 9 firms were involved, together with civil servants and politicians.

[10] This assumption is similar to the free disposal assumption in standard auctions.

[11] This particular cost structure is not critical to any of qualitative results in the paper, and greatly simplifies presentation.

[12] We do not need to impose more structure on the cost function.

p_i^h is the minimum price firm i requires for delivering S^h, and is denoted by $B_i = \left\{ \left(S^h, p_i^h \right) \right\}_{h \leq 2^k - 1}$. Bids belonging to one offer are mutually exclusive (such bids are called XOR bids).

We consider a first-price sealed-bid auction with package bidding. Such auction has some specific features. In an offer, each price bid applies to a *bundled* set of tasks, e.g., it might be a bid of \$100 on $S = \{\omega_1, \omega_2\}$. Such a price bid does *not* imply any bid on packages $\{\omega_1\}$ and $\{\omega_2\}$. Thus, an offer that addresses all possible packages must include $2^k - 1$ distinct bids. Typically, package auction rules include no obligation to bid on all packages. In particular, a firm making a bid on a package does not necessarily submit a bid on the subsets of tasks included in that package. This is *the* critical feature that distinguishes our setting from a multiple-unit auction (with interchangeable tasks). In a multiple-unit auction, bidders submit a supply function.[13] We will see that the option "*not* to bid seriously on all packages" plays an important role in maintaining collusion.[14]

The role of the auctioneer, i.e. the government agent[15] who administers the procedure is to publicly open the envelopes and select the cost-minimizing collection of packages among submitted bids under the constraint that all tasks are awarded. By convention, the public firm submits a bid on each task at price equal to 1. In the case of a tie with the government, the auctioneer must select the private firm. In the case of a tie between private firms, the auctioneer randomizes with equal probability. The auctioneer pays the winning firms according to their bid.

Let S_i^* denote i's package in the winning collection of packages. We assume that there is no externality, so that the firm i's payoff depends solely on S_i^*:

$$v_i = p_i \left(S_i^* \right) - c_i \left(S_i^* \right).$$

Discretionary power

The auctioneer has discretion to decide whether or not to *simultaneously offer to all firms* an opportunity to readjust their offers, prior to the official opening. We refer to this decision as "extending the deadline" or "overturning bids" interchangeably. The decision to overturn the bids is motivated by alleging a defect in the procedure. We assume that firms and the auctioneer who

[13] Even if individual tasks are interchangeable, we could use the package auction 'bidding language'. Bidders would make bids on bundles corresponding to exact quantities (e.g., s tasks in exchange for \$100 or nothing). However, this is not a standard model of a multiple-unit auction. Thus, we assume that tasks are unique. This constrains the agent's choice, but it is not critical to our main argument.

[14] Bernheim and Whinston (1986, p.6) observe that "It appears that inefficient allocations arise from the failure of bidders to make serious bid on every alternatives."

[15] The ultimate principal is tax-payers. The agent is a player who has been delegated the power to administer the allocation procedure (the auctioneer).

conducts the auction, but not the government, share information about the relevance of the alleged defect for competition. In our analysis, we focus on those decisions to extend the deadline which are motivated by the auctioneer's self-interest. In the real life, it is of course possible that the deadline is extended for good reasons.

The auctioneer may be either honest or corrupt. If the auctioneer is honest, his incentives are perfectly aligned with that of the government, his principal. If he is corrupt, he may abuse discretion to extract rents. In this case, his payoff is equal to the total amount of bribes he receives. We assume that when the auctioneer is indifferent between abusing his discretion and not abusing, he chooses not to.

The time line of events in the auction game without corruption (alternatively when the auctioneer is honest) is as follows:

Timing

$\tau = 0$: The project $\Omega = \left\{\omega^j\right\}_{j=1}^k$ is announced, and bidders learn the costs for all packages and for all firms.

$\tau = 1$: Each firm submits its offer, a collection of prices and associated packages, in a sealed envelope.

$\tau = 2$: The auctioneer selects from among the submitted offers (including the public firm's offer), the cost minimizing collection of bids under the constraint that all tasks must be allocated. The packages from the winning collection are awarded to (one of) the firm(s) that made a lowest-cost bid. Winners are paid according to their bids.

3 Analysis

3.1 Benchmark

The case when the auctioneer is honest serves as a benchmark. In a single-object first-price auction with symmetric information, the problem of efficient allocation entails no subtleties whatsoever: in equilibrium, the contract goes to a firm that has the lowest costs.[16] The equilibrium price corresponds to the second-lowest cost. In contrast, a multiple-object auction with package bidding

[16] Suppose that it was not so. Since the equilibrium price p^e cannot be lower than the lowest cost, any firm with lower cost than the winning firm would be better off by offering $p^e - \varepsilon$ for some $\varepsilon > 0$ - a contradiction.

may have multiple equilibria, some of which are inefficient[17].

Bernheim and Whinston (1986) established a few key results applying to symmetric information first-price "menu auctions".[18] In particular, they show that any first-price menu auction has a Nash equilibrium in *profit-target* strategies,[19] which is coalition-proof and yields an efficient allocation. Formally, let $\pi = \{S_j(\pi)\}_{j=1}^{n}$ denote a feasible allocation of tasks. Strategy B_i is called an r_i-profit-target strategy if for any π, we have $B_i(S_i(\pi)) = c_i(S_i(\pi)) + r_i$. Firms simply bid their cost plus a mark-up r_i.

We start with a lemma. Recall that m_i denotes the number of tasks for which firm i has a cost advantage over the public firm.

Lemma 1 *For $\sum_{i=1}^{n} m_i \leq k$, there exists a Nash equilibrium of the first-price multiple-object package auction that yields a total expenditure equal to k. Otherwise, any Nash equilibrium yields a total expenditure strictly less than k.*

Lemma 1 simply states that when the market is large relative to the private firms' supply of tasks (priced at $p(\omega^j) = 1$), there exist equilibria where these firms do not compete with each other. Instead, they bid the reservation price corresponding to the public firm's price bid. When the market is small, i.e. $\sum_{i=1}^{n} m_i > k$, there exists no partition of the market such that private firms do not compete with each other. In any equilibrium the average price paid per task is less that 1. This simple result is the starting point for our investigation: can corruption help bidders avoid costly competition?

In what follows, we shall assume that $\sum_{i=1}^{n} m_i > k$, and that the non-cooperative outcome of the package auction is a profit-target strategy equilibrium. Among these equilibrium allocations, we denote π^* the task allocation(s) that maximizes the lowest payoff among firms. The corresponding profit-target strategy equilibrium will be used as the threat point in collusive schemes we investigate. The analysis focuses on the issue of existence of equilibria in a game extended with corruption where the bidders collude to share the market at the reserve price. Side transfers between firms are not allowed.

[17] Bernheim and Winston (1986) discuss the introduction of uncertainty and conclude that this does not eliminate the inferior equilibria.

[18] In a menu auction, the bidders put their bids on the whole allocation of tasks, while in the package auction they only bid on their own packages. This distinction is not relevant in our context.

[19] Bernheim and Winston (1986) use the term *truthful strategy*. We opted for a more descriptive term *profit-target strategy* introduced by Milgrom (2004).

3.2 Market-sharing and corruption

We now consider a situation where the auctioneer is corrupt. Our first objective is to exhibit a complementarity between the corrupt auctioneer's self-interest in extracting rents and the bidders' interest in avoiding competition. To this end, we extend the benchmark model with a corruption stage. Two cases are of interest. In the first case the auctioneer's discretion conveys no extortion power. We derive our central complementarity result. In the second case, the auctioneer can also credibly threaten to disrupt collusion which enables him to appropriate some of the collusive rents.

3.2.1 A complementarity

Informally, the bribing game may be described as follows. The auctioneer opens the envelopes and learns the content of all offers. If there is a deviator from the collusive agreement, the auctioneer discloses the currently winning collection of bids to the deviator and some other firm. Thereafter, he invites the defector and the other informed firm to compete in bribes. The firm that made the highest bribe offer is awarded the "right to decide" whether or not to overturn the bids. This bribing game is intuitively appealing. We shall see that corruption imposes a cost on the defector. It also keeps the detection risk low. At most two firms are involved in corruption, both with regards to the disclosure of secrete information and bribery. An additional appeal of this scheme is that it does not involve any sophisticated (and hazardous) updating of the auctioneer's beliefs (about firms' cost). Those beliefs play no role.

Formally, after the project has been announced and the firms learned the costs ($\tau = 0$ in *Timing*), the game has three stages:

(i) *First submission of offers:*

Each firm submits its offer B_i.

(ii) *Corruption game:*

a. The auctioneer learns the content of the offers and discloses the winning collection of bids to two firms (including the defector if any). Or he chooses not to disclose any information in which case the game moves to (iii).

b. The auctioneer invites the two informed firms to make bribe offers.

c. The auctioneer selects a winner who pays the proposed bribe and decides whether or not to overturn the offers. If the winner chooses to maintain the offers, the game proceeds to (iii).

d. If the winner decides to overturn the offers, all the firms are invited to resubmit.

(iii) *Selection:*

The auctioneer selects, from among the last submitted offers (including the public firm's offer), the cost minimizing collection of bids under the constraint that all tasks must be allocated. The packages from the winning collection are awarded to the firms that made a lowest bid. The winners are paid according to their bids.

The above timing defines a multiple-step game. We focus on subgame perfect equilibria characterized by collusion in the procurement auction[20].

Definition 1 *Offers* $\{B_i^c\}_{i=1,...,n}$ *form a market-sharing bidding profile in a first-price multiple object package auction if and only if* $B_i^c = \{(S_i^c, p_i^c), (S_i', p_i')\}$ *where* $S_i^c \cap S_j^c = \emptyset$ *and* $p_i^c = |S_i^c|$, $i \neq j$, $i, j = 1, ..., n$ *and* $p_i' > |S_i'|$ *for* $S_i' \neq S_i^c$.

Given any partition of the market $\pi^c = \{S_i^c\}_{i=1,...,n}$, the corresponding market-sharing strategy profile maximizes the cartel's payoff. A key feature of a market-sharing offer is that it contains a single "serious" bid, the one on the collusive package. Other bids are "non-serious": they exceed the reservation price, and therefore have no chance to be a part of the winning collection.

Let $\mathbf{B}^0 = \left\{B_i^0\right\}_{i=1,...,n}$ denote a profit-target bidding profile relative to π^* so that $v_i\left(\mathbf{B}^0\right)$, $i = 1, ...n$ are payoffs associated with the equilibrium $\mathbf{B}^0$ and let $\underline{v_i}\left(\mathbf{B}^0\right) = \min_{j\neq i, j=1,..n-1}\left\{v_j\left(\mathbf{B}^0\right)\right\}$, $\underline{v_i}\left(\mathbf{B}^0\right)$ is the lowest non-cooperative payoff among the $n - i$ firms. We assume that $\underline{v_i}\left(\mathbf{B}^0\right) > 0$ for all i.

Consider $\{B_i^c\}_{i=1,...,n}$, a market-sharing strategy profile with $v_i\left(\mathbf{B}^c\right) \geq v_i\left(\mathbf{B}^0\right)$, $i = 1, .., n$.

Proposition 1 *There exists a subgame perfect equilibrium of the first-price multiple object package auction in which firms play collusive strategies* $\{B_i^c\}_{i=1,...,n}$ *provided that*

$$COR_i : v_i\left(\mathbf{B}^c\right) \geq v_i\left(\widehat{B}_i, \mathbf{B}_{-i}^c\right) - \underline{v_i}\left(\mathbf{B}^0\right),\ i = 1, ..., n,$$

for any offer $\widehat{B}_i$.

Proposition 1 establishes that, subject to the COR_i conditions, corruption makes collusive market-sharing sustainable. First, the corrupt auctioneer makes firms' actions observable by disclosing the current winning collection of bids. Second, he offers an opportunity to react to those

[20] The fact that the auctioneer has incomplete information about firms' cost has no implication for the outcome of the game, i.e., we could as well assume that the knows the costs. What is important is that the auctioneer learns all the information relevant to his decision as he opens the envelops.

actions by letting firms influence his decision on extending the deadline. When the firms play market-sharing strategies, defection of one bidder implies that some other bidder earns zero payoff because his single serious bid is being displaced. Therefore, a displaced bidder has incentives to bribe the auctioneer to extend the deadline, so he can readjust his offer and subsequently earn the no-collusion payoff. The deviator also has an incentive to pay a bribe to counter the displaced bidder, i.e., *to avoid* that bids are overturn and that firms readjust their offer to the competitive bidding profile. Conditions (COR) yield that for any bidder, the cost of outbidding in bribes any other bidder is so large that no profitable defection exists.[21]

We already argued that the proposed bribing game has an intuitive appeal in terms of punishing deviation from the collusive scheme. In the appendix, we show that in the subgame following a defection, a first-bribe competition is optimal, and it is optimal to confine bribe competition to just two firms, one of which is a displaced bidder. A sufficient condition for this mechanism to be an optimal rent-extracting mechanism is that the auctioneer's expected payoff is maximized when the auctioneer invites the defector rather than another complying firm to bid against any displaced bidder. In the Appendix, we show that when it is a common knowledge that $v_i(\mathbf{B}^c) \geq v_j(\mathbf{B}^0)$ for all $i, j = 1, ...n$, it is optimal for the auctioneer to select the defector to participate in the bribing competition. In an earlier version of the paper we show that a result similar to that in Proposition 1 obtains with an alternative (simple) bribing game.[22]

Remark 1 *The larger is the lowest non-cooperative equilibrium payoff, the larger is the set of allocations that can be sustained in a collusive equilibrium.*

This result follows immediately from the observation that raising $\underline{v_1}(\mathbf{B}^0) = \min_{i=1,...n} \{\underline{v_i}(\mathbf{B}^0)\}$ (weakly) relaxes the COR conditions. Remark 1 points to a feature of the equilibrium in Proposition 1 that may at first appear counter-intuitive. Indeed, firms' incentive to comply do not rely on the threat of ending up with their own non-cooperative payoff: deterrence obtains because the defector anticipates that he will have to pay a bribe *to avoid* the non-cooperative outcome.[23]. The highest

[21] By assumption a firm's cost for a package is a function of the number of tasks only. Hence, if a firm decides to defect it will displace the bidder who is cheapest to outbid in the bribe auction.

[22] We considered the following alternative formulation of the corruption game: the auctioneer discloses the offers to a displaced bidder (if any) and makes a take-it-or-leave-it offer: he offers to extend the deadline in exchange for a bribe equal to z. If the firm rejects the offer, the auctioneer proceeds to the official opening. It is easy to show that for $z \leq \underline{v_{1^*}}(\mathbf{B}^0) \min_{i=1,...n} \{\underline{v_{i^*}}(\mathbf{B}^0)\}$ collusion is an equilibrium of the game.

[23] The non-cooperative payoff might be obtained only in the subgame following a defection where the bribe exceeds the difference between the defection payoff and the non-cooperative payoff.

bribe that the displaced bidder is willing to pay to overturn the initial offers is equal to his non-cooperative payoff. Hence, the *larger* is the *lowest non-cooperative payoff*, the larger is the bribe needed to outbid the bidder who is the "cheapest to displace", and consequently the smaller is the set of collusive allocations that can be sustained. Corruption modifies the role played by the threat equilibrium in the cartel. The decision to "revert" to the non-cooperative equilibrium is an outcome of competition in bribes. What matters to incentives in collusion is the price that must be paid to control that decision.

Remark 1 provides some motivation for our selection of the threat equilibrium. We know from Bernheim and Whinston (1986) that a profit-target equilibrium always exist and although they are not the only stable equilibria, the profit-target equilibrium outcomes are the only stable outcomes. Milgrom (2004) shows (Theorem 8.7) that profit-target strategy equilibria of the first-price package auction correspond to competitive equilibria allocations that *maximize* public expenditures, which reflects the bidders' advantage of making the offers.

3.2.2 Equilibrium corruption

In Proposition 1, corruption is a necessary ingredient for sustaining a collusive ring. However, bribery always happens out of the equilibrium path. In equilibrium, no defection occurs as the firms correctly predict each other's behavior, and so the auctioneer's rents are equal to 0. As such, our theory fails to explain the occurrence of bribery in procurement. Still, our view is that in situations where both collusion and corruption are present, equilibrium bribes often are a "secondary" phenomenon that can be explained fairly easily once we pinned down the role of corruption in sustaining collusion. To show this, we note that the zero-equilibrium-bribe result hinges upon the assumption that extending the deadline is costly for the auctioneer.[24] As a consequence, in the subgame where no firm deviates, the auctioneer cannot extract any rent. The threat of extending the deadline is not credible.

Next, we consider a slight variation of the model which assumes that the auctioneer incurs no cost when extending the deadline. As a consequence, he can threaten to overturn the bids even in the case of a successful market-sharing. Let stage (ii) of the corruption game be modified as follows:

(a) The auctioneer learns the content of the offers and discloses the winning collection of bids

[24] This cost is normalized to zero. Still, it plays a role as we assume that when the auctioneer is indifferent, he chooses not to extend the deadline.

to two firms (including the defector if any). Or he discloses no information in which case the game moves to (iii).

(b) The auctioneer invites the two informed firms to make a bribe offer and the game moves to (c). Otherwise, he requests a "conventional" bribe from both[25]; the firms either pay or refuse to pay. The auctioneer decides whether to maintain the bids, and the game moves to (iii). Alternatively, the auctioneer overturns the bids, and all firms are invited to readjust their offers.

(c) The winner of the bribe auction pays his bribe bid and decides whether or not to keep the submitted bids unchanged. If the winner chooses to maintain the bids, the game proceeds to (iii). If the winner decides to extend the deadline, all firms are invited to readjust their offers.

Stage (iii), the selection stage, is as before.

As above, let $\mathbf{B}^0 = \left\{B_i^0\right\}_{i=1,...,n}$ denote the profit-target bidding profile relative to π^*, and let $\{B_i^c\}_{i=1,2}$ with $v_i\left(\mathbf{B}^c\right) - t \geq v_i\left(\mathbf{B}^0\right),\ i = 1, ..., n$ be a market-sharing strategy profile where $t \geq 0$ denotes the conventional bribe, which we assume to be a common knowledge.

Proposition 2 *There exists a subgame perfect equilibrium of the first-price multiple-object package auction in which firms play the collusive strategies $\{B_i^c\}_{i=1,...,n}$ provided that*

$$(COR_i^*) : v_i\left(\mathbf{B}^c\right) - t \geq v_i\left(\widehat{B}_i, \mathbf{B}_{-i}^c\right) - \underline{v_i}\left(\mathbf{B}^0\right),\ i \neq j,\ i,j = 1, ..., n.$$

for any offer $\widehat{B}_i$. In equilibrium, the auctioneer earns a total bribe income equal to $2t$.

In the equilibrium described in Proposition 2, collusion is always accompanied by bribery. A firm pays the conventional bribe to maintain collusive offers under the auctioneer's credible threat that he would overturn the offers, which would induce a readjustment to the competitive outcome. On the other hand, defection from the collusive strategy is deterred by the same mechanism as in Proposition 1.

It is possible to propose a number of extensions of the basic setup that yield results similar to that in Proposition 2. For instance, we might allow the auctioneer to alert a supervisory agency. Indeed, when practitioners talk about "silence money", they typically refer to bribes paid to the auctioneer when he refrains from reporting about non-serious bids that indicate collusion.

[25] Evidence (e.g. coming from court cases or TI's Bribe Payers Surveys; www.transparency.org) suggests that bribes in public procurement have a conventional character. In France, it is common that bribes correspond from 2 to 3 percent of the value of a construction contract. In South East Asia, bribes can amount to up to 10 percent of the value of an international procurement contract.

3.3 Package bidding and collusion

This section aims at illustrating the role of package bidding in facilitating collusive market-sharing in procurement tenders. For this purpose, we compare the vulnerability to collusion of the package auction with that of a first-price multiple-object auction with single-item bidding. In view of the limited objective of this section, we make the comparison in a simpler environment: costs are assumed to be additive: $c_i\left(S^h\right) = \sum_{\omega^j \in S^h} c_i\left(\omega^j\right)$. Since costs are attached to specified tasks rather than a number of tasks, we denote $\Omega_i = \left\{\omega^j; c_i\left(\omega^j\right) \leq 1\right\}$ and assume that $\cup_{i=1}^n \Omega_i = \Omega$, so all tasks can be produced at a cost not exceeding the reservation price (corresponding to the public firm's production).

A single-item bid auction is defined as follows. Each firm (including the public firm) submits an offer, which is a collection of *non-exclusive* single-item bids, in a sealed envelope. The auctioneer selects the cost minimizing collection of bids under the constraint that all tasks must be allocated. The tie-breaking rules are the same as before: in a tie with the public firm, a private firm is selected. In a tie between private firms, the auctioneer randomizes.

Lemma 2 *In the additive cost environment, the Nash equilibrium of the first-price multiple object auction with single-item bidding is unique and yields an efficient allocation* $i^*\left(\omega^j\right) = \arg\min_{i=1,\ldots,n} c_i\left(\omega^j\right)$ *with* $p^*\left(\omega^j\right) = \min_{i \neq i^*,\ i=1,\ldots,n} c_i\left(\omega^j\right)$. *When* $\Omega_i \cap \Omega_j \neq \varnothing$ *for some* $i \neq j,\ \ i,j = 1,\ldots,n,\ \sum p^*\left(\omega^j\right) < k$.

The allocation of each task can be viewed as an independent first-price auction. When the set of tasks for which two firms have a cost advantage of the public firm overlap, the equilibrium price is below the reservation price.

Definition 2 *Offers* $\{B_i^c\}_{i=1}^n$ *form a market-sharing bidding profile in a first-price single-item auction if* $B_i^c = \left\{\left(\omega^j, p_i^j\right)_{j=1,..k}\right\}$ *;* $p_i^j = 1$ *for* $\omega^j \in S_i^c$ *and* $p^j > 1$ *for* $\omega^j \notin S_i^c$ *with* $S_i^c \cap S_k^c = \emptyset$, *for* $i = 1,\ldots,n$.

Despite the similarities with the market-sharing strategies of the first-price package auction, it turns out to be more difficult to sustain collusion with single-item bidding. To demonstrate that, we extend the single-item auction with a corruption game identical to the one introduced in Subsection 3.2.1. Let $\mathbf{B}^0$ denote the non-cooperative equilibrium strategy profile, and let $\underline{v_i}\left(\mathbf{B}^0\right)$ be defined as above. Let $\{B_i^c\}_{i=1,\ldots n},\ v_i\left(\mathbf{B}^c\right) \geq v_i\left(\mathbf{B}^0\right),\ i = 1,\ldots,n$ be a market-sharing strategy profile

Proposition 3 *There exists a subgame perfect equilibrium of the first-price multiple object single item auction in which the firms play the collusive strategies* $\{B_i^c\}_{i=1,...,n}$ *provided that*

$$(COR_i') : \; v_i\left(\mathbf{B}^c\right) \geq v_i\left(\widehat{B}_i, B_{-i}^c\right) - \max\left\{0, \underline{v_i}\left(\mathbf{B}^0\right) - v_j\left(\widehat{B}_i, B_{-i}^c\right)\right\}$$

$i \neq j, \; i, j = 1, ..., n$ *for any offer* $\widehat{B}_i$.

We immediately see that (COR') conditions are more restrictive than the corresponding (COR) conditions in Proposition 1: the second term in the bracket on the right-hand side includes the negative term $v_j\left(\widehat{B}_i, B_{-i}^c\right)$. This negative term is the main difference between the two auction formats with respect to their vulnerability to collusion. In the package auction, the market-sharing strategies are designed so as to give maximum incentives for a displaced bidder to bribe the auctioneer: in the absence of readjustments, he earns a payoff of zero: $v_j\left(B_i^c, \widehat{B}_{-i}\right) = 0$. Such strategies are not available in the auction with single-item bidding as tasks are not bundled into packages. When a defector (j)'s bid and the collusive bid of a bidder (i) overlap, the displaced bidder (i) generally wins some of the tasks belonging to his collusive "package". If bids are maintained, those tasks are paid at the high collusive price $v_i\left(\widehat{B}_j, B_{-j}^c\right) \geq 0$.. Therefore, he may not be willing to pay much in bribes to overturn the bids and readjust to the competitive equilibrium. This makes defection more profitable and reduces the set of equilibria with collusion compared with that of the package auction.[26]

The main lesson we derive from this exercise is that while package bidding is potentially efficiency improving as suggested by both theoretical and experimental works (e.g., Cybernomics 2000, Ausubel and Milgrom 2002 and Ausubel, Crampton and Milgrom 2005), the flexibility of the package auction may have a substantial flip side. Rules designed to improve allocation efficiency can be exploited to defeat competition. With package bidding, this happens as the larger set of bidding strategies allows bidders to select offers that imply a credible threat of retaliation in response to defection from a collusive agreement. The key issue revealed by our comparison with the single-item auction is that, with package bidding, firms may choose not to make serious sub-package bids. Our findings are consistent with remarks concerning the risks connected with flexibility of other rules in procurement. One example of such a situation was discussed in connection with the FCC package auction design. Plott and Salmon (2000) argue that the right to withdraw bids opens up gaming opportunities detrimental to competition.

[26] In a working paper version, we proposed a numerical example where collusion is sustainable with package bidding, but not with single-item bidding.

4 Discussion and Policy Recommendations

Assumptions Our main result, in Proposition 1, relies on several critical assumptions: (i) The auctioneer has some discretion to give all firms a chance to readjust their offer; (ii) the auctioneer knows the content of the offers; and (iii) the auctioneer's objective is to extract rents. We discuss them in turn.

(i) There is ample evidence of discretionary rules in procurement laws and guidelines that, in effect, give the auctioneer the right to let firms readjust their offers before the official opening. These rules are motivated by the consideration that the auctioneer may privately observe an ambiguity in some tender document or learn that some firm has an undue information advantage. One of the objectives of competitive public procurement procedures is to secure fair and fierce competition. The auctioneer is therefore expected to intervene to clear ambiguities (remove undue information advantage) and to offer firms an opportunity to readjust their offer when needed. In addition, it has been argued that such rules help combating favoritism.[27]

(ii) It might seem problematic to assume that the auctioneer knows the content of the offers so he can disclose it. Indeed, a main rule of public procurement auctions is that no one should have access to that information before the official opening. However, there is empirical evidence that procurement officials have been able to learn the offers before the official opening. One example is in the court case concerned with the construction of the High Speed line North in France (Cartier-Bresson, 1998).[28]

(iii) The assumption on self-interest is supported by widespread empirical evidence of corruption in procurement around the world (see e.g., Transparency International Global Report, 2002).

The analysis is performed in the context of a single auction. Yet, it is often argued that collusion in public tenders is enforced by repeated interaction and thus there is no enforcement role for corruption to play. There are both theoretical arguments and empirical evidence suggesting that corruption also can play an important role in a context where interaction between firms is repeated. A first argument is that there is often significant uncertainty and variation in the profitability of future contracts. This creates tensions in the cartel. This kind of problem is similar to the one encountered by a price cartel when the market is subject to demand shocks (see e.g., Green and

[27] Yet another rationale for such rules can be found in the seminal paper by Milgrom and Weber (1982). They show that the auctioneer can reduce his expenditure by improving bidders' information on a common value component.

[28] An SNCF (French Railroad) agent was convicted for having opened and disclosed the content of offers to members of a cartel. He also gave them two (?!) opportunities to readjust their offer.

Porter 1984, Athey, Bagwell, and Sanchirico 2004). In procurement, relying on corruption may turn out to be an optimal solution when uncertainty about the profitability of future contracts is large. A second argument appeals to the corrupt auctioneer's own interests: by helping avoid competition he contributes to creating rents that he can appropriate. This is best illustrated by the case of "Les Yvelinnes" mentioned in the Introduction. The details of the judgment clearly show that a corrupt politician and civil servants initiated and arbitrated the cartel. They selected the firms that were to participate. They divided the market among the firms. And they punished deviators.[29] The collusive scheme relied primarily on corruption despite repeated interaction between the firms. It appears that the same patterns are encountered in the earlier mentioned corruption/collusion case in the procurement of the Paris City Hall's construction projects. Nine of France's largest construction firms (including Bouygues, Dumez, SGE, SAEP etc..) are involved. Those firms have been interacting for decades in procurement tenders.

Policy recommendations The main insight from our analysis is that in multiple-object tenders corruption and collusion exhibit strategic complementarity.[30] In particular, seemingly innocuous details in the tendering procedures can be exploited to defeat competition. A second result is that package bidding, a feature of auction design aimed at enhancing efficiency, can facilitate collusion. A third result is that, in the presence of corruption, collusion is easier among firms that are not too close competitors. It should be noted that while the analysis has been performed in the context of public procurement, similar features are encountered in private procurement. In that context, we talk about fraud rather than corruption when referring to the procurement agent's abuse of his position in the firm.

Our most important message to auction designers in public procurement is that risks of collusion and risks of corruption must be addressed simultaneously. This message extends to control agencies. Often (e.g., in France) collusion issues are handled by a competition authority while corruption cases are addressed by criminal courts. This institutional separation makes it difficult to effectively unveil and prosecute many cases when collusion and corruption are closely entangled. The analysis

[29] This was made possible by communicating selectively an information that could be used to formulate a winning bid. In case a bidder deviated (tried to obtain more), the information could however be made worthless by the politician.

[30] We use the term strategic complemenetarity in a looser sense: Consider a much simplified version of our game formulated with zero/one decisions follows: playing corrupt (1) or not (0) and colluding (1) or not (0). We see that such a game would exhibit a more standard form of strategic complementarity.

entails that a close cooperation between competition authorities and criminal court is warranted. Alternatively, criminal investigators dealing with corruption in procurement should have a solid education in the economics of auction.

Our analysis suggests that the government should reduce the procurement agent's discretion and/or make him more accountable. More precisely this concerns features of the procedural design that in effect give the agent an opportunity to let firms readjust their offer. Yet, these features offer some valuable flexibility one often wishes to preserve. The policy should therefore target abuses rather than discretion as such. To this aim we believe that raising the level of competence of procurement agents is key. A highly competent agent can be made accountable for ambiguities and other defects in the bidding documents. On the other hand, strict accountability mitigates the agent's incentives to reveal private information about defects. Therefore, it might be counter-productive from the point of view of the fight against favoritism.

These conflicting arguments reveal a more general feature: namely, that measures aimed at combating favoritism can facilitate collusion and vice-versa[31], which provides an additional argument in support of our main conclusion that the issues of corruption and collusion must be addressed simultaneously. Our view is that strict accountability for the quality of the tendering documents is warranted. On the other hand, information about defaults in bidding documents favoring some firms could be generated in a mechanism that uses firms' private information. Indeed, the typical case is that firms would know (or suspect) that one of their competitor is being favored. The characterization of a mechanism that uses firms' private information to combat favoritism is on our future research agenda.

Another immediate recommendation is to limit the use of package bidding to situations where significant complementarities are expected. Where the patterns of complementarity are similar among firms, ex-ante bundling of objects may be preferable. However, when the patterns are different ex-ante bundling by the auctioneer generates a risk of favoritism: the auctioneer can bundle tasks to favor one firm. The analysis of the single-item bid auction reveals that it is because the firms fail to make serious sub-package bids that collusion is easier to sustain with package bidding. Hence, a recommendation is that a package bidding procedure requires that any package

[31] Another example concerns the secrecy of a reserve price or evaluation rule. The French Competition authority's contribution to the debate on the new procuremnt code included a fervent defence for secrecy of the evaluation rule - because it creates coordination problem for a cartel. When taking into account corruption, secrecy appears as a source of rents for the agent that can be realized either in favoritism or in support of the cartel.

bid also includes serious sub-package bids.[32] That may not always be feasible however as there may exist significant costs associated with the evaluation of the sub-packages.

Finally, the results in remark 1 suggest that procurement and control agencies should be particularly vigilant to the risk of collusion and corruption when competition between firms is not too fierce. This corresponds to situations when the market is characterized by a small number of medium size (relatively to the public market) firms of comparable efficiency. Interestingly, we see that when enforcement of the ring is secured by corruption the analysis suggests a quite different characterization of high risk markets compared to the one arising from standard analysis. The standard result yields that cet. par. the risks of collusion are highest when competition between firms is fierce.

[32] The IBM-Mars auction and the London bus auction used procedures with single items bids plus specified discounts for certain packages (see Milgrom (2004) p.315).

Appendix

Proof of lemma 1

i. Consider the case when $\sum_{i=1}^{n} m_i \leq k$. Then there exists a task allocation $\pi^* = \{S_i^*\}_{i=1}^n$; with $S_i^* \cap S_j^* = \varnothing$ and $|S_i^*| = m_i$ for all $i, j = 1, ..., n$ $i \neq j$, and such that $\cup_{i=1}^n S_i^* \subseteq \Omega$. Define $r_i^* = m_i - c_i(S_i^*)$ the profit target of firm i. The collection of offers $\{B_i^*\}_{i=1}^n$; $B_i^* = \left\{\left(S^h, p^h\right)\right\}_{h=1}^{2^k-1}$ where $p^h = c\left(S^h\right) + r_i^*$, constitutes a Nash equilibrium. It yields a total cost equal to k.

We first show that π^* is an allocation that minimizes the total cost of the project when firms play the above described profit target strategies. For any S^h such that $\left|S^h\right| > m_i$, $p^h = c\left(S^h\right) - c_i(S_i^*) + m_i > m^h$ because $\Delta c_i(x) > 1$ for $x > m_i$ so this bid is not competitive against the public firm. For S^h such that $\left|S^h\right| < m_i$, we also have $p^h = c\left(S^h\right) - c_i(S_i^*) + m_i > m^h$ since $c_i(m_i) - c\left(S^h\right) < m_i - m^h$ so even that bid is not competitive against the public firm. By construction, $p_i(S_i^*) = c(S_i^*) - c_i(S_i^*) + m_i = m_i$ so these are the only bids not dominated by those of the public firm. By assumption, in case of tie with the public firm the auctioneer selects the private firm.

For packages larger than m_i, firm i's marginal cost is lower than its marginal revenue, which is equal to 1 per unit and above m_i the marginal cost is higher than the marginal revenue. Hence, $B_i^* = \left\{\left(S^h, p^h\right)\right\}_{h=1}^{2^k-1}$ where $p^h = c\left(S^h\right) + r_i^*$ is optimal for the firms.

ii. Assume now $\sum_{i=1}^{n} m_i > k$. Then any equilibrium task allocation $\pi^* = \{S_i^*\}_{i=1}^n$ must be characterized by $|S_i^*| = q_i < m_i$ for some i. Assume further that $p_j\left(S_j^*\right) = \left|S_j^*\right|$ $j = 1..., n$ (as before). Then firm i has an incentive deviate and offer $\widehat{B}_i$ with $\widehat{S}_i = S_i^* \cup \omega$ at price $\widehat{p} = |S_i^* \cup \omega| - \varepsilon$. The new bid must be part of the equilibrium allocation since it is cheaper than the competitors' bids which price the tasks at the reserve price. And, since $\Delta c_i(x) < 1$, for $x < m_i$, we have $v_i(B_i^*) < v_i\left(\widehat{B}_i\right)$ so the deviation is profitable. Hence, when $\sum_{i=1}^{n} m_i > k$ we must have $\sum_{i=1}^{n} p_i(S_i^*) < k$ in equilibrium.■

Proof of Proposition 1

Assume common knowledge of a standing partition π^c. The following strategies form a subgame perfect Nash equilibrium:

The auctioneer's strategy

i. In case of no defection from collusive bidding disclose no information and proceed with the official opening. In case of defection disclose the winning collection of bids to the defector and one of the displaced firms;

ii. Invite the two informed firms to bid in bribes and award the "right to decide" (hereafter rtd) to the highest bribe bidder.

Firm i's strategy, $i = 1, ..., n$

i. Submit the market–sharing offer corresponding to the standing partition;

ii. If the disclosed offers reveal no defection, wait for the official opening. If the offers reveal a defection,

iii. Submit a bribe bid $b_i^* = \min\left\{v_i\left(\mathbf{B}^0\right), V^{def}\right\}$ where V^{def} is defined below.

iv. If the initial offers are overturned, submit the non-cooperative Nash offer B_i^0.

We next show that these strategies are part of a subgame perfect equilibrium of the first-price package auction with corruption. We start with the corruption game. The selection stage is mechanic.

Corruption game

d. By construction B_i^0 is a best response to B_{-i}^0 so if initial offers are overturned, it is optimal for bidder $i = 1, ...n$ to readjust to B_i^0.

c. Let i and j be the two firms invited to bid in bribes and assume they submit b_i, b_j such that $b_i > b_j$. Clearly, the auctioneer maximizes his bribe income when awarding the rtd to i in exchange for b_i.

b. Several cases may present themselves. We may be in a subgame where the disclosed offers revealed no defection. In that case the bidders earn $v_i\left(\mathbf{B}^c\right)$ if the bids are maintained while they earn $v_i\left(\mathbf{B}^0\right)$ if the the bids are overturned. Since $v_i\left(\mathbf{B}^c\right) > v_i\left(\mathbf{B}^0\right)$ $i = 1, ...n$ all firms prefer the offers to be maintained. Since no extension is the outcome that the auctioneer chooses by default, the rtd has no value for the firms and it is indeed optimal to wait for the official opening.

Consider now a subgame where bidder j defected. When bidder j defects, he submits an offer B^{def}, $\left(S^{def}, p^{def}\right)$, with $v_j\left(B^{def}, \mathbf{B}_{-j}^c\right) > v_j\left(\mathbf{B}^c\right)$. Since $\sum_{i=1}^m m_i > k$, this implies that $S^{def} \cap S_i^c \neq \emptyset$, for some $i \neq j$: the defector displaces some bidder(s) $i's$ collusive bid. We first note that any non-displaced bidder including the defector prefers the bids not to be overturned $v_i\left(B^{def}, \mathbf{B}_{-j}^c\right) \geq v_i\left(\mathbf{B}^c\right) > v_i\left(\mathbf{B}^0\right)$. So if the second selected firm is not a displaced firm, there is no conflict of interest with the deviator and both submit zero bribes. In contrast firm i whose single serious bid has been displaced earns $v_i\left(B^{def}, \mathbf{B}_{-j}^c\right) = 0$ when the offers are maintained. If the deadline is extended, it readjusts and earns the non-cooperative payoff $v_i\left(\mathbf{B}^0\right) \geq \underline{v_i}\left(\mathbf{B}^0\right) > 0$. We now focus on the case where one of the informed firms is a displaced firm labelled 2 and the other the deviator labelled 1. Let V denote the value of the rtd: $V^1 = v_1\left(\widehat{B}_1, \mathbf{B}^c_{-1}\right) - v_1\left(\mathbf{B}^0\right)$ and $V^2 = v_2\left(\mathbf{B}^0\right)$. The outcome of the first-bribe auction is

i. For $V^1 \geq V^2,\ b_2 = V^2,\ b^* = b_2 = V^2$, where $b^* = b_1 = b_2$. By convention the highest value bidder wins He pays the second highest value. Bidder 1 decides not to extend. In case of tie in values (and bribes) the displaced bidder wins.

ii. For $V^1 < V^2,\ b_1 = V^1,\ b^* = b_2 = V^1$, the displaced bidder wins and decides to extend.

Since the auctioneer earns no bribe when he informs (about the winning collection) a non-displaced bidder as the second firm, while he earns at least $\min\left\{V^1, V^2\right\} > 0$ when informing a displaced bidder (in addition to the deviator), it is optimal for him to select a displaced bidder.

First submission stage

Given the equilibrium outcomes in the subgames following the first submission stage firm j knows that if it defects, it will compete in the bribe auction with a displaced bidder. By condition $(\mathrm{COR}_j) : v_j\left(\mathbf{B}^c\right) \geq v_j\left(\widehat{B}_j, \mathbf{B}^c_{-j}\right) - \underline{v_j}\left(\mathbf{B}^0\right)$, for all i so $b^* > V^{def}$ and there exists no profitable defection.

Hence, under conditions (COR_i) $i = 1, ..n$, the market-sharing bidding strategy profile $\mathbf{B}^c$ is part of a subgame perfect equilibrium of the auction game extended with corruption.■

Optimality of the bribing mechanism

We now assume that the auctioneer is free to choose any mechanism he wants to sell his rtd under the constraint that the defector be given a chance to bid in bribe. We first show that when only two firms can be involved in corruption, the first-price auction is optimal subject to self-enforceability. Next we show that restricting the number of bidder to two is optimal. Finally, we give a condition securing that inviting the deviator to bid in bribes is optimal.

i. In the subgame following defection, the defector and any of the displaced bidders have opposite interests with respect to the decision. Since the rtd has no value to the auctioneer, either imposing a reserve bribe strictly larger than zero or a bias in favor of one of the bidders is not ex-post incentive compatible for the auctioneer. Since the bidders know each other's value, the most the auctioneer can extract is the second highest value and a first bribe auction achieves that.

ii. The auctioneer could choose to invite three or more firms to bid in bribes. But there are only two outcomes "maintain the bids" and "overturn the bids". All displaced bidders share an interest for overturning the bids while all non-displaced bidders share an interest for maintaining them. It is easy to see that if two displaced bidders compete with the defector bidder, we have a free-riding problem. Both displaced bidders would prefer the other guy to pay for maintaining the bids. This is a standard problem which yields a mixed strategy equilibrium characterized by inefficiency. When both displaced bidders value the rtd more than the defector, they may still fail to win. So the auctioneer's expected revenue is lower than if only one displaced bidder was present. Under conditions COR$_i$ the displaced bidders do not have a value for the rtd lower than that of the defector.

iii. Ideally, the auctioneer would like to invite the highest value displaced bidder and let him compete with the highest value non-displaced bidder.

We first note that the identity of the displaced bidder has no implication for the result in proposition 1 which must hold for the lowest non-cooperative payoff. The auctioneer's own incentive to select a displaced bidder with a high NE payoff only strengthens the deterrence power of corruption. In contrast the selection from among non-displaced bidders has a crucial implication for the result. There can be no equilibrium with collusion where it is ex-post optimal for the auctioneer to invite a complying firm rather than the defector to compete in bribe.

We know from above that $b^* = V^2$ when $V^1 \geq V^2$ i.e., $v_1\left(\widehat{B}_1, \mathbf{B}^c_{-1}\right) - v_1\left(\mathbf{B}^0\right) \geq V^2 = v_2\left(\mathbf{B}^0\right)$. By the COR conditions we know that $v_i\left(\mathbf{B}^c\right) \geq v_i\left(\widehat{B}_i, \mathbf{B}^c_{-i}\right) - \underline{v_i}\left(\mathbf{B}^0\right)$. So we have that $V^1 \geq V^2 \Rightarrow v_1\left(\mathbf{B}^c\right) \geq v_2\left(\mathbf{B}^0\right)$. Hence, if $v_i\left(\mathbf{B}^c\right) \geq v_j\left(\mathbf{B}^0\right)$ for all $i, j = 1, ...n$ it is indeed optimal for the auctioneer to invite the deviator to bid in bribes.■

Proof of proposition 2

Assume common knowledge of a standing partition π^c and of a conventional bribe t, and consider the following strategies:

The auctioneer's strategy

i. When no firm defects, he discloses the winning collection of bids to any two firms and asks them to pay the conventional bribe. He overturns the initial bids unless both firms pay t.

ii. In case of deviation from collusive bidding, he discloses the submitted offers to the deviator and a displaced bidder, and asks for bribe offers. He awards the rtd to the highest bribe bidder.

Firm i's strategy, $i = 1, ..n$

i. Submit the market-sharing offer corresponding to the standing partition;

ii. If the disclosed offers reveal no defection, pay the conventional bribe if the auctioneer asks for it. Otherwise don't pay. If the disclosed offers reveal that some firm defected,

iii. submit a bribe bid $b_i^* = \min\left\{v_i\left(\mathbf{B}^0\right), V^{def}\right\}$ where V^{def} is defined as in the proof of proposition 1 above.

iv. If initial bids are overturned, submit the non-cooperative Nash offer B_i^0.

We next show that these strategies constitute a subgame perfect equilibrium of the auction game extended with corruption.

Corruption game

The arguments at steps e,d, are similar to the ones in proposition 1 at steps d and c.

c. At step c we are in a subgame where the informed firms have been asked to pay a bribe. For the case where the disclosed bids revealed no defection, the firms face a coordination game like situation when deciding whether or not to pay. According to the auctioneer's equilibrium strategy if both firms pay the offers are maintained and the official auction outcome yields $v_i(\mathbf{B}^c)\ \ i = 1, ..., n$. While if one or both firms fail to pay, the offers are overturned which yields a payoff of $v_i\left(\mathbf{B}^0\right)\ \ i = 1, ..., n$. Since $v_i(\mathbf{B}^c) - t > v_i\left(\mathbf{B}^0\right)\ \ i = 1, ..., n$ both firms prefer to pay to maintain the bids. The firms' strategy entails paying if and only if the auctioneer asks for the conventional bribe t. This solves the coordination problem. In the equilibrium of this subgame both firms pay if and only if they are asked for t.

Consider now the subgame where a deviation has been uncovered but the auctioneer asks for the conventional bribe. The displaced bidder wants the offers to be overturned so he does not pay. Since the deviator who would like the bids to be maintained cannot affect the situation he does not pay either. No bribes are paid and the bids are overturned.

b. In the subgame where a firm deviated the auctioneer earns nothing when asking for the conventional bribe while he can earn at least $\min\left\{V^1, V^2\right\}$ (as defined in the proof of Proposition 1 above) when inviting the informed firms to make bribe offers so he chooses to do so. In the subgame where all firms submitted the market-sharing offers, he earns nothing when asking the firms to compete in bribes while he earns $2t$ when asking for the conventional bribe so it is optimal to do so.

a. For the same reasons as in proposition 1 in the subgame where a defection occurred the auctioneer discloses the offers to the deviator and a displaced bidder. In the subgame where all firms complied, it plays no role which firms he informs since we assume that $v_i(\mathbf{B}^c) - t \geq v_i\left(\mathbf{B}^0\right)$.

First submission stage

By the reasoning above firm i knows that if it defects, it will be invited to bid in bribe. By condition (COR_i^*) : $v_i(\mathbf{B}^c) - t \geq v_i\left(\widehat{B}_i, B_{-i}^c\right) - v_i\left(\mathbf{B}^0\right)$, $i = 1, ..., n$ and so there exists no profitable defection. The market-sharing bidding strategy profile $\mathbf{B}^c$ and the payment of the conventional bribe are part of a subgame perfect equilibrium of the auction game extended with corruption.■

Proof of lemma 2

Firm i's 'payoff-if-win S^h is given

$v_i\left(S^h\right)=\sum_{\omega^j\in S^h}\left[p_i\left(\omega^j\right)-c_i\left(\omega^j\right)\right],\ i=1,..n.$

This function is fully separable in the net income from each individual task. Hence, firms view bidding for each task as a separate auction. Each of these auctions are single-item first-price symmetric information auction (except for the public firm that always bid 1). The outcome is therefore $i^*\left(\omega^j\right)=\arg\min_{i=0,\ldots,n}c_i\left(\omega^j\right)$. The winner is paid the second lowest cost unless the winner is the public firm which is compensated with 1 par task. The equilibrium price is given $p^*\left(\omega^j\right)=\min_{i\neq i^*,i=1,\ldots,n}c_i\left(\omega^j\right)$ which is strictly less than 1 for $\Omega_i\cap\Omega_j\neq\varnothing$ since $c_{i(j)}\left(\omega_k\right)<1$ for $\omega_k\in\Omega_{i(j)}$. A firm's equilibrium offer is the collection of the unique single-item equilibrium bids.■

Proof of Proposition 3

The argument is identical to the one in the proof of proposition 1, except for the determination of displaced bidders' value of the rtd. In the subgame where firm j defected and submitted $S^{def}\neq S_j^c,\ p\left(S^{def}\right)\leq\left|S^{def}\right|$. By $\sum_{i=1}^n m_i>k$, we have $S^{def}\cap S_i^c\neq\emptyset$ for at least one firm i. So we have $S_i^{dis}=S_i^c\backslash\left(S^{def}\cap S_i^c\right)$ and $v_i\left(B^{def},\mathbf{B}_{-j}^c\right)=\sum_{\omega^j\in S^{dis}}\left(1-c_i\left(\omega^j\right)\right)\geq 0.$

In the subgame where defection occurred a displaced bidder i's value for the rtd is $V^{dis}=\max\left\{0,v_i\left(B^0\right)-v_i\left(B^{def},\mathbf{B}_{-j}^c\right)\right\}$.

By the same argument as in proposition 1, no bidder has any incentive to defect at the first submission stage when $v_i\left(\widehat{B}_i,B_j^c\right)-v_i\left(\mathbf{B}^c\right)\leq\max\left\{0,\underline{v}_i\left(B^0\right)-v_i\left(B^{def},B_i^c\right)\right\}$. The bribe needed to outbid the displaced bidder is so large that no defection is profitable.■

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