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Abstract:

Increasing internally generated goodwill (IGG) is another way of depicting the rising gap between market and accounting values sometimes referred as the "book-to-market black box". Existing methods propose to value internally generated goodwill as the present value of abnormal earnings (e.g., residual income models) or to measure it indirectly through the excess of the enterprise value over the fair value of assets in a business combination. The critical drawback of these approaches is that they do not explain how the goodwill is created. In other words they do not enter into the "black box". We propose an alternative valuation method based on the recognition that using an asset in combination with other assets leads to an interaction affecting firm value. In this context IGG emerges from an inadequate theory of aggregation of assets. Using Choquet's capacities, which are non-additive aggregation operators, allows solving this adequacy issue as IGG arises as a consequence of specific synergies between assets. Our model is tested on the U.S. High Technology sector and benchmarked against the residual income model. To the extent of the accuracy to forecast enterprise value, our model performs better than the standard residual income model.

Key words:

Internally generated goodwill – Going concern goodwill – Synergy – Choquet integral – Residual income model

1. Introduction

In a context where value drivers are shifting from tangible to intangibles resources, valuation and recognition of intangible assets represent a challenge for financial reporting. Damodaran (2006) highlights the complexity associated with intangible valuation, whereas Lev (2003) regrets the only partial recognition of intangibles in financial reporting. The decrease in book-to-market ratios which has been observed over the years and originates from the rising gap between market and accounting values is leading to the "*book-to-market black box*" (Lev and Sougiannis 1999). One explanation for the decrease in this ratio is the non-accounting recognition of intangibles which, in turn, increases the internally generated goodwill (IGG in the rest of the paper) created by companies. In this paper, we try to explain the parts of this "*black box*" represented by the IGG. We propose a methodology to explain and illustrate the creation of the IGG and to value this IGG by using a non-additive aggregation method that allows for the measurement of the value of the structure, i.e., the interaction value of a set of assets.

The IGG (also called "going-concern goodwill") represents "the ability [of a company] as a stand-alone business to earn a higher rate of return on an organized collection of net assets than would be expected if those assets had to be acquired separately […]" (Johnson and Petrone 1998, p. 295). This definition is consistent with the definition formally given in the residual income model as developed by Ohlson (1995), where the unrecorded goodwill equals the expected present value of abnormal earnings.

In general, accounting standards do not recognize this IGG, which cannot be capitalized. Its economic nature is completely independent from a business combination. However in accounting, "goodwill is the part of the enterprise value that does not appear in financial statements but that emerges only when acquired" through a business combination, for instance (Zanoni 2009, p. 1).[1]

Two methodologies have been proposed in the literature to value the IGG. (1) The direct method, which consists of computing the present value of expected abnormal earnings relying on the residual income formula as expressed in Ohlson (1991, 1995) and implemented by Dechow et al. (1999) for instance. (2) The indirect method, which consists of subtracting a

[1] The concept of "enterprise value" is traditionally defined as market value + total debt – cash, which corresponds to an "equity side" logic in the context of a theoretical takeover. In this paper, we implement an "assets side" approach which represents an economic value of the assets of the firm. We define it as the fair value of total assets plus the internally generated goodwill. We do not restate cash because our research does not focus on a theoretical takeover price and also because cash may exhibit interactions with other classes of assets.

company's fair value of assets[2] from its enterprise value in the case of a business combination. The latter approach has been achievable since the introduction of the purchase method by FAS 141 (FASB 2001, 2007), which requires the acquirer to estimate and disclose the fair value of assets and liabilities acquired in a business combination.

The main drawback of these two approaches is that they focus on output flows and do not explain how the IGG is created. The nature of the IGG remains to a large extent a mystery. These approaches renounce exploring the "*black box*" and prefer to focus on the economic consequences of IGG i.e., abnormal earnings or excess enterprise value over fair value of assets.

We propose to enter into the "*black box*" to explain the IGG creation, by relaxing the additive postulate underlying financial valuation. Literature suggests that the fair value of a set of assets has no reason to equal the sum of the fair value of each asset i.e., fair value measure is not additive (McKeown 1971, Ijiri 1975). Goodwill emerges from an "*inadequate theory of aggregation of assets*" (Miller 1973, p. 280). Our approach consists of proposing an aggregation method that permits to recognize that using an asset in combination with other assets leads to an interaction affecting firm value i.e., a structured set of assets can possibly increase the overall value of a company. Hence, the IGG stems from synergies[3] between assets organized in a specific system. Using the Choquet's capacities (Choquet 1953) which are non-additive measures allows the modeling of the value of different combinations of assets. We can compute and explain the value of the IGG as a consequence of specific synergies between assets.

Our model is tested on the U.S. High Technology sector, which allows to identify several synergies and inhibitions between classes of assets in that industry leading to an overall excess value i.e., an IGG. This excess value is explained by specific interactions between classes of assets affecting the overall productivity. Additionally, two robustness checks are implemented in order to ascertain the validity of our results: first, out-of-sample predictions of enterprise values are computed and then compared to the actual enterprise values; second the model is tested against the residual income model used as a benchmark. To the extent of the accuracy to forecast enterprise value of the High Technology companies, our model performs better than the standard residual income model as forecasting errors produced by our approach

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[2] Assuming company's fair value of assets is known (see below).

[3] The term "synergy" is often used to refer to the synergistic effect that result of the combination of the acquirer and the target company (Ma and Hopkins 1988, p. 77, Johnson and Petrone 1998, Henning et al. 2000). In this paper, we only use the term "synergy" in relation to the interaction between assets, which exists inside a given company and which is not related to a business combination.

are smaller. A clear improvement is provided by relaxing the additive postulate in aggregation to the extent of enterprise value predictions.[4]

The remainder of this paper is organized as follows. Section 2 provides background on the IGG and existing valuation methods as well as our research question. Section 3 describes the research design, demonstrates how Choquet's capacities[5] allow solving the aggregation issue and highlights the similarities and differences between the residual income model and our approach. Section 4 describes the data and sample used to implement our model. Section 5 exposes the empirical results and presents a comparison of the performance of our valuation method with the residual income model in predicting enterprise value. Section 6 concludes this study.

2. The Nature of Goodwill

2.1. Internally Generated Goodwill

As mentioned above, IGG is generally not recorded in the accounting system and exists independently from any business combination. However, it becomes part of the recorded accounting goodwill when a company is acquired. In this context, the reporting requirements concerning the acquisition provide useful data. In an acquisition, the price paid by the acquirer often exceeds the book value of net identifiable assets of the acquiree. Using a bottom-up perspective (Johnson and Petrone 1998), i.e., starting from the book value, and in accordance with Henning et al. (2000, p. 376) we describe below the other resources that were acquired and have certainly some value to the acquirer.

1. Excess of the fair values over the book values of the acquiree's recognized net assets and fair values of the other net assets not recognized by the acquiree (Johnson and Petrone 1998), corresponding to what is often called "revaluations" in practice.
2. Fair value of the "going concern" element of the acquiree's existing business, also called "going-concern goodwill" (Johnson and Petrone 1998, Henning et al. 2000) or "internally generated goodwill" (Ma and Hopkins 1988, p. 77).
3. Fair value of synergies for combining the acquirer's and acquiree's businesses and net assets. This element is often called "combination goodwill" (Johnson and Petrone 1998, p. 296) or "synergy goodwill" (Henning et al. 2000).[6]

[4] Similar tests are conducted by Barth et al. (2005) and Nekrasov and Shroff (2009) on equity value predictions.
[5] Choquet's capacities, which are developed later in the article (see section 3), are commonly used in the field of expected utilities without additivity (see, e.g., Gilboa 1988, Schmeidler 1989).
[6] See a preceding footnote on the use of the term "synergy" in this paper. This "combination goodwill" relates to the synergistic effect of combining two companies.

4. Overvaluation of the consideration paid by the acquirer and overpayment (or underpayment) by the acquirer (Johnson and Petrone 1998). This last component is referred to as "residual goodwill" by Henning et al. (2000).

The main focus of the present article is on component ❷ of Figure 1 which might be thought as a pre-existing goodwill that was internally generated by the acquiree. The going concern element of goodwill reflects the ability of an established business to earn on a stand-alone basis a rate of return on a collection of assets and liabilities different from that which could be expected if the net assets were acquired separately (Arnold et al. 1994, p. 19, Johnson and Petrone 1998, p. 296).

Ma and Hopkins (1988, p. 77) explain that "the use of an asset in combination with other assets is often assumed to lead to an interaction affecting favourably the productivity of the other assets as well as its own productivity. This is the so-called synergy from asset interaction, which results in superior earnings".

Figure 1 summarizes the decomposition of the purchase price paid by the acquirer. The addition of the components ❷, ❸ and ❹ represents the total goodwill, also called purchased goodwill or accounting goodwill (component Ⓕ in Figure 1). Figure 1 also presents the computation of the main concepts used in this article (purchase price, fair value of assets, fair value of liabilities, enterprise value and IGG) and cites our data sources.

Insert Figure 1 About Here

2.2. Goodwill, Enterprise Value and Purchase Price Allocation

How can we get or compute the value of the IGG? As can be seen from Figure 1, several elements are necessary to get this value. If we are not in the context of an acquisition (business combination), only the book value of assets and liabilities (components Ⓑ and Ⓑ') is available from the published annual report. The fair value of equity as a stand-alone entity (component Ⓔ) can be assimilated to the market value of the firm (Johnson and Petrone 1998, p. 296). This value is available if the company is listed on a stock market.

In order to obtain the other components of Figure 1, we need to assume that the studied company (the acquiree) has been acquired. Consequently, the price paid (component Ⓐ), or "purchase price" (Henning et al. 2000), becomes available in the annual report of the acquirer. Beginning with the publication of the Financial Accounting Standard 141 (FASB 2001), U.S. acquiring entities have been required to allocate the cost of an acquired entity to the assets acquired and liabilities assumed based on their estimated fair values at date of acquisition and to disclose this "purchase price allocation" (PPA in the rest of this article) in the notes. This obligation has been maintained in the revised version of FAS 141 (FASB 2007) and is now included in the Accounting Standards Codification (ASC) as paragraphs 805-10-50 and 805-30-50. This latter paragraph namely states that the "acquirer shall disclose (…) the acquisition-date fair value of the total consideration transferred and the acquisition-date fair value of each major class of consideration, such as the following:

1. Cash
2. Other tangible or intangible assets, including a business or subsidiary of the acquirer
3. Liabilities incurred, for example, a liability for contingent consideration
4. Equity interests of the acquirer, including the number of instruments or interests issued or issuable and the method of determining the fair value of those instruments or interests" (§ 805-30-50-1). We provide in Appendix 1 an excerpt of an illustration given by the FASB in paragraphs 805-10-55-37 to 41.

The purchase price allocation allows to identify the fair values of identifiable assets and liabilities, including, especially, intangible assets.

2.3. Research Question

The preceding paragraph and Figure 1 describe a computation method of the IGG but does not explain the nature of the IGG. The role of synergy between assets in the formation of the IGG raises some concern about the additivity of fair values.

The discussion on the additivity of fair values is not new. Ijiri (1975, p. 93) already mentioned that "the fair value measure is not additive. In the case of historical cost, the historical cost of resources *A* and *B* together is by definition the sum of the historical cost of *A* and the historical cost of *B*. Generally, we do not have this additivity in fair value. (…) The fair value of an entity is (…) known to be quite different from the sum of the fair values of the resources it owns". Ijiri (1975, p. 93) adds that "this lack of additivity means that the resources must be evaluated more than once to make a proper assessment because the values depend upon other resources that are bought or sold with the resources to be evaluated" and refers to McKeown (1971) for a further discussion on this additivity issue.

Ijiri (1975, p. 93) also reminds that "goodwill presents a serious aggregation problem because the value of the whole is not necessarily equal to the sum of the values of its parts". This author (1975, p. 93) adds that this has been one of the oldest issues in accounting as discussed by Yang (1927), Canning (1929) and Paton and Littleton (1940). This subject has also been discussed by Devine (1962), Gynther (1969) and Miller (1973).

Our main research question is the following: what is the nature of the IGG? We hypothesize that the IGG mainly arises from synergy between assets. The measurement of this synergy requires to abandon the additivity postulate in order to highlight the over-additive or under-additive effect of the combination of assets. In this case, does a model based on non-additivity perform better than a traditional additive model (e.g., Ohlson 1995)?

3. Research Design

3.1. Dropping the Additive Postulate

3.1.1. The Measurement Process. The object of measurement is to convey information about some other objects like the economic reality. Let P be the set of principals p observed in reality, let S be the set of surrogates s that intend to represent the principal. Then the function $m: P \rightarrow S$ is called measurement process. From a normative standpoint, a desirable property of the function m is that it preserves the structure of the principals that is represented in the surrogates.[7]

3.1.2. Unsuitability of the Classic Additivity Concept. Dropping the additive postulate is necessary to solve the aggregation issue as mentioned by Miller (1973). The financial accounting system relies on the additive postulate of values. Methods of evaluating a set of assets assume, for the sake of convenience, that the value of a set of N assets is equal to the sum of the values of its N components i.e., the overall value of the set equals the sum of the individual value of each asset.

This standard financial arithmetic uses the mathematical notion of "measure". This concept is generally defined for a finite set of elements X, as the mapping m, defined on the set $P(X)$, the power set of X, i.e., the set of all sub-sets of X, taking values in $[0,+\infty]$, satisfying the following properties:

(i) non-negativity: $m(A) \geq 0$ for all $A \in X$;

(ii) null empty set: $m(\emptyset) = 0$;

(iii) additivity: for all sets A disjoint from B, $m(A \cup B) = m(A) + m(B)$.

The third condition of this definition is really constraining. It is one of the main underlying assumptions of standard aggregation operators such as the Riemann (1857) or the Lebesgue (1918, 1928) integrals. The notion of "measure" is widely used in financial

[7] Ijiri calls such a measure *consistent* or *perfect* (1975, p. 42).

accounting. This approach of valuation constrains the vision of the organization in a very specific way, where no relation between financial resources exists. For instance in the case of a firm using three assets A, B, and C, the measurement process in financial accounting is represented by the left side of Figure 2.

Insert Figure 2 About Here

In the numerical representation, it is implicitly assumed that assets A, B, and C do not exhibit any interrelationships with each other. As a consequence, this particular setting hypothesizes that the sum of each fair value of asset is equal to the fair value of the sum of all assets. This mapping does not preserve the structure of the economic reality (which is organized) in the numerical representation because it does not represent any interactions between assets.

3.1.3 The Enterprise as a Structured Set of Assets: Relaxing the Additive Postulate. The additivity property, based on the hypothesis of the interchangeability of the monetary value of the different elements, seems intuitively justified. However, this method of calculation proves particularly irrelevant in the case of the structured and finalised set of assets which makes up an organization (Casta and Bry 2003). Indeed, the optimal combination of assets (for example: brands, distribution networks, production capacities, etc.) is a question of know-how on the part of managers and appears as a major characteristic in the creation of intangible assets like goodwill. This is why an element of a set may be of variable importance depending on the position it occupies in the structure. Moreover, its interaction with the other elements may be at the origin of value creation such that the overall value of a set of assets may exceed the sum of the individual value of the assets.

To achieve this goal, it is necessary to develop a methodology allowing modeling interactions between the assets of a firm and measuring the intensity of the relation between every sub-sets of assets.

Let's consider a company having a structured set X of three assets A, B, and C. We can then graphically represent the set $P(X)$, the power set of X, in order to analyze the potential interrelationships (synergy, inhibition, neutrality) which might occur amongst these three assets (see right side of Figure 2).

As the real structure of the economic reality is unknown, the most general form is presented in the numerical representation which exhibits every possible interaction between the three assets A, B, and C. At each node of the lattice, it is then possible to take into account the asset interrelationships of two or three components, through the non-additive measure μ (defined below) The function μ captures the nature of the relation between two or three assets.

In other words this function must offer special attributes allowing modelling (1) neutrality, (2) synergy, and (3) inhibition between assets.

To achieve this purpose of non-additive firm valuation, Casta and Bry (1998, 2003) suggest to use the Choquet's capacities (Choquet 1953) as the non-additive measure μ. The Choquet's capacities are a generalization of the measure concept. Two types of application based on the Choquet's capacities have been developed in the literature. Choquet's capacities are used as a representation of the degree of uncertainty of an event (e.g., non-additive expected utilities), or to model the strength of the coalition of elements (e.g., criteria, attributes, players, voters, etc.) (see, for a review, Grabisch and Labreuche 2010). This last type of application is used in multicriteria decision helping because it enables the recognition of the importance of interaction of criteria in a classification.

Formally, Choquet's capacity on a finite set of elements X is any set function $\mu: P(X) \rightarrow R^{+}$ respecting the following properties (Grabisch 2003):

(i) null empty set: $\mu(\emptyset) = 0$;

(ii) monotonicity: $\forall A \subset B, \mu(A) \leq \mu(B)$.

The second property: monotonicity, states that adding a new element to a combination cannot decrease its importance (Marichal 2002). It is a less constraining property than the additivity presented above. Following from the monotonicity property, for two disjoint sets A and B, Choquet's capacities can, depending on the modelling requirement, behave in the following manner:

- Additive: $\mu(A \cup B) = \mu(A) + \mu(B)$ (neutrality between assets)
- Over additive: $\mu(A \cup B) > \mu(A) + \mu(B)$ (synergy between assets)
- Under additive: $\mu(A \cup B) < \mu(A) + \mu(B)$ (inhibition between assets).

The definition of Choquet's capacities requires the measures of all subsets of X to be specified, that is to say $2^n - 1$ capacities to be estimated. They offer the possibility to identify all the interactions between a set of assets.

3.2. Non-Additive Aggregation and Company Valuation

Aggregation over (or under) additive seems to be a relevant approach to reckon the synergies effect between assets which is the source of IGG. However, using the mathematical concept of non-additive aggregation for firm valuation requires a few definitions of the mathematical tools that will be implemented.

3.2.1. The Concept of Non-Additive Aggregation. Choquet integral stems directly from the capacities presented above. It is a generalization of the integral to non-additive measures.

According to Grabisch et al. (2009), Choquet integral generalizes the Lebesgue integral (Lebesgue 1918, 1928) in the sense that the Choquet integral equals the Lebesgue integral when the capacities are additive (Marichal 2002).

Definition 1: *The Lebesgue integral $L_{(f)}$ of a measurable function f defined on a measure space (Ω, X, m) where Ω, is a set, X is a σ-algebra*[8] *of subset of R^+ and m is the Lebesgue's measure, is defined for a simple function*[9] *taking no more than finitely increasing distinct values $a_1 < a_2 < \ldots < a_n$ as:*

$$L_{(f)} = \int_\Omega f dm = \sum_{i=0}^{n-1} (a_{i+1} - a_i) m(A_{i+1}) \qquad (1)$$

Where $A_{i+1} = \{x \in A : f(x) < a_{i+1}\}$

Definition 2: *Let μ be a set of capacities on (Ω, X) and a function $f : \Omega \to [0,+\infty]$, when Ω is a finite set $\Omega = \{w_1, \ldots, w_n\}$, such that $w_1 < w_2 < \ldots < w_n$, the Choquet integral of f with regards to μ can be written as follows:*

$$C_{(f)} = \int_\Omega f \, d\mu = \sum_{i=0}^{n-1} [f(w_{i+1}) - f(w_i)] \mu(A_{i+1}) \qquad (2)$$

where $A_i = \{w_i, \ldots, w_n\}$ *and* $f(w_0) = 0$ *by convention.*

In the finite case presented above[10], it is easy to see that the Choquet integral is a generalization of the classical Lebesgue integral. Besides, if the capacity μ is additive, then the Choquet integral reduces to a Lebesgue integral (Marichal 2002). The Choquet integral extends the Lebesgue integral to possibly non-additive measures. As a result of monotonicity, it is increasing with respect to the measure and to the integrand. Hence, Choquet integral can be used as an aggregation operator. This integral allows non-additive aggregation of a set of assets where interactions between every sub-sets of assets create (or destroy) value.

3.2.2. Company Valuation with a Non-Additive Approach. To present in details the valuation method of a company using the Choquet integral, a graphic illustration is developed below. A graphic approach of the Choquet integral is also proposed by Gayant (1998) and Grabisch (2000).

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[8] In the definition of measures, a *σ-algebra* is the collection of sets over which a measure is defined.

[9] A *function f that takes no more than finitely distinct values $a_1, a_2, \ldots, a_n$ can be written as a simple function:* $f(x) = \sum a_n X_{A_n}(x)$ *where* $A_n = \{x \in A : f(x) = a_n\}$. Representing a set of assets by a simple function enables computing the value of a company as the area under the curve, as represented in section 3.2.3.

[10] For the definitions of the Lebesgue and Choquet integrals in the continuous case, see Appendix 2.

Graphic illustration of the additive approach. Let's assume a company having three assets A, B, and C ranked by value increasing order. The fair market values of these three assets are 100, 150, and 250 respectively; they can be represented by an *increasing simple function f.* The hatching area represents the global fair value of the assets of the firm according to the additive approach (see Figure 3).

Insert Figure 3 About Here

The standard valuation approach requires computing the area below the curve. The Lebesgue integral (equation (1)) of this *simple function* of assets is the following:

$$\begin{aligned} V_L &= \sum_{i=0}^{n-1} [f(x_{i+1}) - f(x_i)] m(A_{i+1}) \\ &= (100-0) * m(A,B,C) + (150-100) * m(B,C) + (250-150) * m(C) \\ &= (100-0) * 3 + (150-100) * 2 + (250-150) * 1 = 500 \end{aligned}$$

where V_L represents the global valuation of assets based on the Lebesgue integral, where $A_{i+1} = \{x \in A : f(x) < a_{i+1}\}$, and $m(A_i)$ is the Lebesgue measure of A_i, representing the length of the intervals.

The common Riemann integral of that function gives the same result:

$$V_R = \sum_{i=1}^{3} f(x_i) = 100 + 150 + 250 = 500$$

where V_R represents the global valuation of assets based on the Riemann integral, and $x_i = \{A, B, C\}$.

Graphic illustration of the Choquet's non-additive approach. Now we want to value the same firm by taking into account the interactions between the assets. The Choquet's capacities can play that role. In the next paragraph, a learning method for capacities estimation will be presented and implemented. But first, let's assume that the capacities (i.e., each μ) are known for this set of assets.

The Choquet's capacities of the set of assets A, B, and C are the following in this simplified example:

- $\mu(A) = \mu(B) = \mu(C) = 1$;
- $\mu(A,B) = 2$; i.e., neutrality between assets A and B (because $\mu(A,B) = \mu(A) + \mu(B)$);
- $\mu(A,C) = 2$; i.e., neutrality between assets A and C (because $\mu(A,C) = \mu(A) + \mu(C)$);
- $\mu(B,C) = 1.5$**;** i.e., **25% inhibition** between assets B and C (because $\mu(B,C) < \mu(B) + \mu(C)$, (i.e., μ(B,C)/[μ(B)+μ(C)] = 1.5/2 = 0.75, hence 25% of inhibition).

- $\mu(A,B,C) = 4$; i.e., **33% synergy** between assets A, B, and C (because $\mu(A,B,C) > \mu(A) + \mu(B) + \mu(C)$, i.e., μ(A,B,C)/[μ(A)+μ(B)+μ(C)] = 4/3 = 1.33, hence 33% of synergy).

It is worth noting that for a company having three assets with a given ranking of fair values, only three Choquet's capacities will be used to compute the overall value of the company, because only three kinds of interactions are possible as we will see below.[11]

Comparing the traditional additive approach with the non-additive method, this second method can be seen as an area expansion (in the case of synergy), or contraction (in the case of inhibition) of the area associated with every differentials of fair value below the curve. To represent graphically the value of this structured set of assets we can proceed in two steps: the first step will only take into account the interaction value between assets A, B, and C (i.e., the synergy of 33%) whereas the second step will take into account both the synergy between assets A, B, and C and the inhibition between assets B and C (i.e., inhibition of 25%).

In the first step, we only take into account the synergy effect between assets A, B and C together (i.e., 33% of synergy). This corresponds to a 33% expansion of the area associated with the differential of value of the three assets (bold hatched area below the curve). As a result the rest of the initial curve is translated upwards (see Figure 4A).

Insert Figure 4A About Here

Note that the μ coefficient between assets B and C in the first step doesn't exhibit any interactions, because $\mu(B,C) = \mu(B) + \mu(C) = 2$, as we consider only interaction between assets A, B and C. Levels of translation are obtained as follows:

100 * 1.33 = 133 (as a result of synergies between assets A, B, and C);

133 + 50 = 183 (simple upwards translation of 33);

183 + 100 = 283 (simple upwards translation of 33).

The second step consists in taking both the synergy effect between assets A, B, and C (i.e., 33%) and the inhibition between assets B and C (25% of inhibition). This corresponds to a 25 % contraction of the area associated with the differential of value between assets B and C (doted area below the curve). As a result the remainder of the curve in Figure 4.A is translated downwards (see Figure 4B).

Insert Figure 4B About Here

[11] If we consider another company with another ranking of assets, three other interaction coefficients may be used. Consequently, considering many companies and every possibility, with three assets, seven capacities might be computed (see, later, in section 5.2).

Note that the μ coefficient between assets B and C displays an inhibition, because $\mu(B,C) = 1.5 < \mu(B) + \mu(C)$. Additionally synergies between assets A, B, and C are also measured, as in the previous example. Levels of translation are obtained as follows:

100 * 1.33 = 133 (as a result of synergies between A, B, and C);

133 + (50 * 0.75) = 133 + 37.5 = 170.5 (as a result of the first translation and the inhibition between assets B and C);

283 – (50 * 0.75) = 270.5 (simple downwards translation).

Formally, the overall value of the company calculated with a non-additive aggregation operator is equal to the hatching area (bold and thin) below the new curve, i.e., the value of the Choquet integral. The capacities weight the differential of fair values related to each combination of assets. The value of the Choquet integral (equation (2)) relative to the capacities for this set of assets is:

$$
\begin{aligned}
V_C &= \sum_{i=0}^{n-1} [f(x_{i+1}) - f(x_i)]\mu(A_i) \\
&= (100-0) * \mu(A,B,C) + (150-100) * \mu(B,C) + (250-150) * \mu(C) \\
&= (100-0) * 4 + (150-100) * 1.5 + (250-150) * 1
\end{aligned}
$$

where V_C represents the valuation based on the Choquet integral.

$$V_C = 400 + 75 + 100 = 575$$

In this second illustration, we recognize both the synergies between the assets A, B, and C, and the inhibition between assets B and C. This second valuation leads to a new value of 575 for this company (compared to only 500 with the additive approach).

3.3. Choquet's Capacities Learning Method

As explained by Casta and Bry (1998), modelling through Choquet integral presupposes the construction of a measure which is relevant to the semantic of the problem. Since the measure is not *a priori* decomposable, it becomes necessary to define the value of $2^n - 1$ coefficients $\mu(A)$ where $A \in P(X)$. Similar to Grabisch (2003) we suggest an indirect econometric method based on a regression model to estimate the coefficients. Moreover, in cases where the structure of the interaction can be defined approximately, it is possible to reduce the combinatory part of the problem by restricting the analysis of the synergy to the interior of the useful subsets (see Casta and Bry 1998). Determining Choquet's capacities (that is to say $2^n - 1$ coefficients) brings us back to a problem for which many methods have been elaborated (Grabisch et al. 2008). We propose a specific method of indirect estimation on a learning sample made up of companies for which the firm's overall evaluation and the individual value of each element of the set of assets are known.

Let us consider I companies described by their overall value V and a set X of J real variables x^j representing the individual value of each element in the assets. Let f_i be the function assigning to every variable x^j its value for company i $f_i : x^j \rightarrow x_i^j$. We are trying to determine a set of Choquet's capacities μ in order to come as close as possible to the following relationship:

$$\forall i : C_{(f_i)} = EV_i \qquad (3)$$

where EV_i is the Enterprise Value for firm i.

Let A be a subset of variables and $g_A(f_i)$ be the variable called *generator relative to A* and defined for company i as:

$$i \rightarrow g_A(f_i) = \int 1_{(A=\{x/f(x)>y\})} dy \qquad (4)$$

Then the Choquet and Lebesgue integrals can be written as in equation (5) and (6)[12]:

$$L_{(f)} = \sum_{A \in P(X)} g_A(f) * m(A) \qquad (5)$$

$$C_{(f)} = \sum_{A \in P(X)} g_A(f) * \mu(A) \qquad (6)$$

Thus, according to equations (3) and (6) we can write the following *econometric* model:

$$\forall i\ EV_i = \sum_{A \in P(X)} \mu(A) * g_A(f_i) + \varepsilon_i \qquad (7)$$

where $\mu(A)$ is now a parameter and ε_i is a residual which must be globally minimized in the adjustment. It is possible to model this residual as a random variable or, more simply, to restrict oneself to an empirical minimization of the ordinary least squares type. The model given below is linear with $2^J - 1$ parameters: the $\mu(A)$ for all the subsets A of variables x^j. The dependent variable is the enterprise value EV. The explanatory variables are the generators corresponding to the subsets of X. A classical multiple regression provides the estimation of these parameters, that is to say the required set of capacities. In practice, we shall consider the discrete case with a regular subdivision of the values:

$$y_0 = 0,\ y_1 = dy,\ \ldots,\ y_n = n*dy$$

where dy is the difference of the value of assets $i+1$ and i. For each group A of variables x^j, we compute the corresponding generator as:

[12] Proof: see Appendix 3.

$$g_A(f_i) = dy * \sum_{h=0}^{n} 1_{(A=\{x/x_i > y_h\})} \qquad (8)$$

The following principle will be used to interpret the measure thus obtained for A ∩ B ≠ Ø:

$\mu(A \cup B) > \mu(A) + \mu(B)$ => Synergy between *A* and *B*,

$\mu(A \cup B) < \mu(A) + \mu(B)$ => Inhibition between *A* and *B*.

It should be noted that the suggested model is linear with respect to the generators, but obviously non-linear in the variables x^j.

3.4. A Benchmark: The Residual Income Model

The objective of this section and the two following ones is to compare the residual income model, which will be used as a benchmark in the empirical section, and the synergy model. Using the well-known dividend discount valuation model, and clean surplus relation, the residual income model (e.g., Ohlson 1995) states the following relation between equity market value, book value and expected abnormal earnings:

$$MV_t = BV_t + \sum_{\tau=1}^{+\infty} R^{-\tau} E_t[x^a_{t+\tau}] = BV_t + GW_t \qquad (9)$$

Where:

MV_t: Market value at time t;

BV_t: Book value at time t;

R: 1 + cost of equity capital;

x^a_t: abnormal earnings in t defined as $x_t - (R-1) * BV_{t-1}$

x_t: reported earnings in *t;*

$E_t[.]$: the expectation operator in date t.

The market value of equity equals the firm's book value plus the present value of expected abnormal earnings. In this model the unrecorded goodwill appears as expressed in equation (9).[13]

To obtain the enterprise value (EV_t), one can add the market value of debt on both sides of equation (9) leading to equation (10):

$$MV_t + D_t = EV_t = (BV_t + D_t) + \sum_{\tau=1}^{+\infty} R^{-\tau} E_t[x^a_{t+\tau}] = TA_t + GW_t \qquad (10)$$

[13] In a context of unbiased accounting (assets are recorded at fair value), the unrecorded goodwill equals the IGG.

where TA_t are total assets value.

As we will demonstrate in the next section, the non-additive approach allows similar relations, though putting emphasis on interactions between assets instead of expectations of abnormal earnings as the source for IGG.

3.5. The Synergy Model Based on Non-Additivity

The synergy model based on non-additivity ("synergy model" in the rest of the article) states that the enterprise value can be computed via the Choquet integral of the assets of the firm using the appropriate set of capacities: For $A \in P(X)$, X being a set of assets, $\mu(A)$ the Choquet's capacities of the set A, and $g_A(f)$ the generator relative to A, we have the relation (11):

$$EV_t = C_{(f)} = \sum_{A \in P(X)} \mu(A) * g_A(f) \qquad (11)$$

We can also write this equation adding and subtracting a Lebesgue integral in the right hand side leading to equation (12) developed in equation (13):

$$EV_t = L_{(f)} + [C_{(f)} - L_{(f)}] \qquad (12)$$

$$EV_t = \sum_{A \in P(X)} m(A) * g_A(f) + \sum_{A \in P(X)} [\mu(A) - m(A)] * g_A(f) \qquad (13)$$

The first term represents the additive value of assets, whereas the second one provides the value of the combination of assets. It can be positive (synergies generate value) or negative (inhibitions destroy value). This equation allows for the differentiation between two components of enterprise value:

$$EV_t = L_{(f)} + [C_{(f)} - L_{(f)}] = TA_t + IV_t \qquad (14)$$

In equation (14), the first term represents the **additive value of total assets** (TA_t), and the second term the **value of interactions between assets** (IV_t). Hence, the same relations as in the Ohlson model can be expressed:

$$EV_t = MV_t + D_t = TA_t + IV_t \qquad (15)$$

$$MV_t = TA_t - D_t + IV_t \qquad (16)$$

$$MV_t = BV_t + IV_t \qquad (17)$$

The fair equity market value of the firm's equals its equity book value plus the value of interactions generated by combination of assets at time t. In the above equation (17), the goodwill emerges formally in a comparable manner as in the residual income formula. Although the difference is essential: the value of the goodwill is directly generated by interaction between assets and not by expected discounted abnormal earnings. Table 1 summarizes the similarities and differences between the residual income and synergy models.

Insert Table 1 About Here

3.6. Numerical Example

Assume a similar firm as in section 3.2., having total asset fair values of 500, book value of equity of 300, and market fair value of debts of 200. Let's also assume that the fair enterprise value of this company is 575. Finally, the fair market values of the three assets (A, B and C) of this firm, as in the section 3.2. are 100, 150, and 250 respectively.

Residual Income Model. This model, applied to this company, leads to the formula expressed in equation (10) (see above). Assuming that the market is efficient, the present value of expected abnormal earnings will equal the excess market enterprise value over the total fair assets value: 75. This is also the value of IGG. Thus we have the following relation:

$$EV_t = 300 + 200 + 75 = 575$$

Notice that without any reference to the market enterprise value, assumptions on the expected abnormal earnings dynamics should have been developed to compute their present value, whereas in the synergy model, only fair market values of assets and the appropriate Choquet's capacities are required.

Synergy Model. This model, applied to this company, will give the following results, using the Choquet integral (equation (2)) with the same set of capacities as in the example in section 3.2. As expressed in equation (14), we can split enterprise value between book value of assets and interaction value as an expression of Choquet and Lebesgue integrals.

Taking into account the value of the generators, the value of the total assets TA_t is:

$$TA_t = L_{(f)} = \sum_{A \in P(X)} m(A) * g_A(f) \qquad (18)$$

$$TA_t = m(A,B,C) * g_{A,B,C} + m(B,C) * g_{B,C} + m(C) * g_C$$

$$= [3] * 100 + [2] * 50 + [1] * 100 = 300 + 100 + 100 = 500$$

And the value of the interaction between assets IV_t is:

$$IV_t = \sum_{A \in P(X)} [\mu(A) - m(A)] * g_A(f) \qquad (19)$$

$$IV_t = [4 - 3] * 100 + [1.5 - 2] * 50 + [1 - 1] * 100 = 75$$

Hence, according to equation (14) we obtain the following expression:

$$EV_t = TA_t + IV_t = 500 + 75 = 575$$

This expression can be also rewritten:

$$EV_t = BV_t + D_t + IV_t = 300 + 200 + 75 = 575$$

As in the residual income model, the goodwill equals 75. However the key difference is that it is directly obtained as the value of interactions between assets through the Choquet's capacities and not as the consequences of these interactions on abnormal earning flows:

$$MV_t = EV_t - D_t = BV_t + IV_t = 300 + 75 \qquad (20)$$

4. Data and Sample

To compute the Choquet integral, fair values of assets and debts are required. These values are not generally observable for every company due to historical cost accounting (conservatism). However, since 2002, FAS 141 requires that fair values of identifiable tangible and intangible assets be estimated at the date of the acquisition through purchase price allocation in business combinations. Following Henning et al. (2000), we used business combination fair value estimates at the date of acquisition to obtain the fair value of firm's assets and liabilities.[14]

Furthermore, our valuation model is based on the concept of interactions between assets. As interaction between assets can vary from a sector to another, we decided to focus on a specific economic sector where one could make the assumption that the role of synergies between assets was important in the value creation process. We chose to study the High Technology sector (macro-industry: HT of Thomson One Banker), because the number of acquisition deals was the highest in this sector after the financial industry.[15]

We obtained our sample from the deals analysis database of Thomson One Banker for the period 2002-2009 with the following criteria:

- The deal has a value of at least $100 million;
- Both the acquiree and the acquirer are listed U.S. companies;
- The deal is completed;
- The target macro-industry is High Technology.

We obtained 180 business combinations satisfying these criteria from 2002 to 2009. Acquirers' 10-Q and 10-K reports, following the date of acquisition, available through the SEC EDGAR database were used to obtain the purchase price allocations of these business combinations. The purchase price is allocated to current, tangible, and identifiable intangible assets with various details from one company to another. The advantage of using assets' fair

[14] We acknowledge that Purchase Price Allocations and disclosures may be subject to managerial discretion and may constitute a bias estimate of assets' fair values (Shalev 2009). However, we believe that this methodology allows obtaining better estimates and recognition of fair value of assets and liabilities as compared to the use of book values.

[15] We do not consider the financial industry based on standard finance theory assuming that benefits from interactions between assets are already priced (a diversification effect).

values as estimated in purchase price allocations is that various intangible assets are identified only in business combinations, leading to a more extensive recognition of intangible assets. Due to insufficient and missing disclosure in 10-Q and 10-K reports, this process led to 101 firms for which fair values of assets and liabilities were available in the High Technology sector.

As explained in section 3.3., the Choquet's capacities learning method requires that the fair enterprise value be known in order to infer a set of capacities. The market values of equity of target firms were collected seven trading days prior to the acquisition announcement (Henning et al. 2000) using market value of equity available on Datastream.

To implement the model, we decided to regroup fair values of identified assets in three broad categories of assets. We provide in Appendix 4 the detailed content of these categories.

Fair values of liabilities (Debts) were also collected, allowing us to compute firms' enterprise values (being the addition of market value and debt). The excess of enterprise value over total fair values of assets leads to the IGG as indirectly measured. Table 2 presents the descriptive statistics of our sample.

Insert Table 2 About Here

The IGG represents the most important component of enterprise value in our sample with a mean of 33% of total enterprise value (median of nearly 33%). Current assets represent the second largest component of enterprise value with a mean of 31% (30%). As one could expect for the High Technology sector, the mean value of identified intangible assets represents a significant percentage of enterprise value as they account for 25% (24%). Finally, tangible assets do not account for more than 10% of enterprise values (5%).

In section 5, a residual income model is used as a benchmark to test the relative performance of the synergy model. To compute the appropriate variables, following Dechow et al. (1999), book value of equity, cost of capital, earnings, and analysts' estimates of earnings are requested to implement the model. Book values and earnings before extraordinary items were collected from Compustat annual, market values from Datastream, and earnings forecasts extracted from I/B/E/S. Costs of capital were obtained using a CAPM approach, using 5 years beta and the implied equity premium made available by Damodaran[16] for the U.S. market. Table 3 summarizes these panel data variables.

Insert Table 3 About Here

[16]http://pages.stern.nyu.edu/~adamodar/.

No analysts' coverage has been found for 16 firms, reducing the size of the sample for the model including that variable.

5. Empirical Results

5.1. Descriptive Statistics of Explanatory Variables: Generator Functions

Generator functions are computed according to equation (11) for each 101 companies. They merely represent a different approach of describing a set of assets for a company that is practical for the estimation of the capacities. As this equation isn't trivial, let's consider one company of our sample: DataDomain Inc, having the following set of assets (in millions $): Tangible Assets (40.46); Current Assets (81.73) and Intangible Assets (357.90). Figure 5 represents graphically this set of assets:

Insert Figure 5 About Here

Generators are computed in the following manner: for X the set of assets of DataDomain Inc. and $x \subseteq P(X)$, we have:

$$\{x / f_i(x) > 357.90\} = \{\varnothing\}$$
$$\{x / f_i(x) > 357.89\} = \{IA\}$$
$$\vdots$$
$$\{x / f_i(x) > 81.73\} = \{IA\}$$
$$\{x / f_i(x) > 81.72\} = \{CA, IA\}$$
$$\vdots$$
$$\{x / f_i(x) > 40.46\} = \{CA, IA\}$$
$$\{x / f_i(x) > 40.45\} = \{TA, CA, IA\}$$
$$\vdots$$
$$\{x / f_i(x) > 0.00\} = \{TA, CA, IA\}$$

Hence, taking dy = 1 in equation (8), we derive the value of the generators for DataDomain Inc. as:

$$g_{TA,CA,IA} = 1 * \sum_{h=0}^{40.46} 1_{(TA,CA,IA=\{x/f(x)>0\})} = 40.46$$

$$g_{CA,IA} = 1 * \sum_{h=40.46}^{81.73} 1_{(CA,IA=\{x/f(x)>40.46\})} = 81.73 - 40.46 = 41.27$$

$$g_{IA} = 1 * \sum_{h=81.73}^{357.90} 1_{(IA=\{x/f(x)>81.73\})} = 357.90 - 81.73 = 276.17$$

the other generators being equals to 0.

Table 4 reports descriptive statistics of generator functions, obtained in a similar manner for the total sample.

Insert Table 4 About Here

5.2. Choquet's Capacities Estimation

According to the learning procedure presented in section 3.3 and equation (7), the following model has been implemented on the sample in order to estimate the Choquet's capacities:

$$EV_i = \mu_1 g_{TAi} + \mu_2 g_{CAi} + \mu_3 g_{IAi} + \mu_4 g_{TA,CAi} + \mu_5 g_{CA,IAi} + \mu_6 g_{TA,IAi} + \mu_7 g_{TA,CA,IAi} + \varepsilon_i \qquad (21)$$

The estimation of the set of Choquet's capacities of the sample is reported in table 5.

Insert Table 5 About Here

Every Choquet's capacities are positive. Table 5 provides the values of every sub-set of assets in the structure. The adjusted R^2 has been reported although it has no relevance in a regression without an intercept. The standard errors and therefore p-values of the estimates for the capacities μ(TA,CA) and μ(CA,IA) are not reported as the constraint of monotonicity is binding for these capacities (see note below table 5). The estimated capacities are interpreted in the next section.

5.3. Results' Interpretations: Effect of Interactions Between Assets in the HT Sector

Given our estimated set of capacities, determining the importance of each class of assets in the general structure seems to be critical. The overall importance of a sub-set $i \in T$ is not solely determined by the quantity $\mu(i)$, but also by all $\mu(T)$ such that $i \in T$. Indeed, we may have $\mu(i) = 0$, suggesting that element i is not important, but it may happen that for many sub-sets $T \subseteq N$, $\mu(T \cup i)$ is much greater than $\mu(T)$, suggesting that i is actually an important element. Therefore the importance index or Shapley value (Shapley 1953) of any $i \in N$ is defined by:

$$\phi_i = \sum_{T \subseteq N \setminus i} \frac{(n-t-1)!t!}{n!} [\mu(T \cup i) - \mu(T)] \qquad (22)$$

where t is the cardinal of the subset T.

The quantity ϕ_i represents the weighted average value of the marginal contribution $\mu(T \cup i) - \mu(T)$ of element i alone in all combinations.[17] The higher the Shapley value, the more important the element is in the structure. Hence, if the Shapley value gives the overall importance of an individual asset, it provides no information about the interactions existing between assets. Yet, it seems critical given our research question to appraise the importance of interaction between assets. Using the rules mentioned in section 3.1 regarding neutrality,

[17] In game theory, it was originally an approach to allocate the gains obtained in a cooperative game fairly. It is worth noting that a property of the Shapley value is that they take values from 0 to 1 and that their sum equals 1 such that $\sum_{i=1}^{n} \phi_i = 1$.

synergy and inhibition, an important concept helping interpreting the capacities has been developed: the interaction index, initially suggested by Murofushi and Soneda (1993, see also Grabisch 1997, Marichal 2000).

The idea is to consider two elements *i, j,* their individual importances $\mu(i)$, $\mu(j)$, as well as their joint importance $\mu(i, j)$. Accordingly to the way the elements interact, the following quantity $[\mu(i, j) - \mu(i) - \mu(j)]$ may be positive (synergy), null (neutrality) or negative (inhibition). In the same manner as for the Shapley value, the interaction index $I_{i,j}$ is the average of the quantities described above:[18]

$$I_{i,j} = \sum_{T \subseteq N \backslash i,j} \frac{(n-t-2)!t!}{(n-1)!} [\mu(T \cup i, j) - \mu(T \cup i) - \mu(T \cup j) + \mu(T)] \quad (23)$$

The definition has been extended by Grabisch (2003) to interactions between any number of elements:

$$I_A = \sum_{T \subseteq N \backslash A} \frac{(n-t-a)!t!}{(n-a+1)!} \sum_{B \subset A} (-1)^{a-b} [\mu(T \cup B)] \quad (24)$$

where *t*, *a*, and *b* are respectively the cardinals of subsets *T*, *A*, and *B*.

Table 6 presents a direct interpretation of the capacities (Panel A). For the reasons mentioned above it is completed by the Shapley values (Panel B) and interaction indexes (Panel C) for the capacities estimated for our sample.[19]

Insert Table 6 About Here

From Panel A, we can see that only one synergy appears at the dual combination level (between tangible assets [TA] and intangible assets [IA]. Synergies are also generated between the three categories of assets, between [tangible assets and current assets] and intangible assets and also between [current assets and intangible assets] and tangible assets. A synergy is also created between the three categories of assets, considered separately. The synergies outweigh inhibitions because their size, as measured by the product of the capacities (see Table 5) and the corresponding generators (summarized in Table 4), outweighs the size of inhibitions (measured in the same way). They generate on average an overall positive value (i.e., an IGG).[20]

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[18] The interaction index can be negative or positive, a negative value indicates an inhibition and a positive value a synergy. Also note that $I_{i,i} = \phi_i$.

[19] Computation details of the Shapley values and interaction index are provided in Appendix 5.

[20] The predicted enterprise value of a company is given by the product of the estimated capacities presented in table 4 and its specific set of generators which are summarized in table 5. This value is in excess of the fair values of the assets for 97% of the companies of the sample.

Panel B of Table 6 gives an idea of the relative importance of every individual class of assets given their marginal contribution in all possible combinations in the structure. It appears that, whereas tangible assets and current assets exhibit relatively the same importance, intangible assets are the most important class of assets, i.e., they generate on average more value in the different combinations than the two other classes. Given the intangible-intensive nature of the industry this result seems to be consistent with expectations.

Additionally, Panel C provides insight on the effect on firm value of interactions between any combinations of the three categories of assets. It reveals that combining tangible assets with current assets generate a limited amount of synergies ($I_{TA,CA}$ =0.055), whereas combining intangible assets with tangible assets generate much larger synergies ($I_{TA,IA}$ =0.408). The combination of the three classes of assets also generates an important synergy ($I_{TA,CA,IA}$ =0.299). However, Panel C provides evidence that inhibitions exist between current assets and intangible assets as the interaction index for this combination is negative ($I_{CA,IA}$ = -0.297).

5.4. Robustness Checks

5.4.1. Performance of the Model. Cohen et al. (2009) argue that asset-pricing models should be evaluated by their ability to provide close estimates to the current stock price. Hence, similarly to Barth et al. (2005), we decided to focus on the accuracy of the synergy model to predict out-of-sample enterprise value given a set of Choquet's capacities and the fair asset values of companies. The following jackknifing procedure was implemented to generate contemporaneous out-of-sample enterprise value predictions for each firm without using that firm's data to generate its predicted equity value.

(i) The model (21) is estimated on (N-1) firms, to generate a set of Choquet's capacities;

(ii) The enterprise value of the firm not used in the sample for the learning procedure is estimated on the basis of the fair values of the assets and the set of capacities estimated in step (i) with model (21);

(iii) We compare the enterprise value estimation to its actual value;

(iv) We repeat this procedure for the N firms of the sample.

The prediction error metric employed is the absolute percentage error (AE):

$$AE = abs(EV_{it} - \text{predicted } EV_{it})/EV_{it} \qquad (25)$$

The performance of the model, as tested by this procedure, is reported in table 7.

Insert Table 7 About Here

The synergy model misses the right enterprise value with a median value of 20%. The mean prediction error (25%) is slightly higher due to one bad prediction. Additionally 75% of enterprise values of the sample are predicted with an error of less than 39% of their actual values. These results are better than those obtained on out-of-sample predictions of equity values by Barth et al. (2005, p. 331-332) with a residual income model at the industry level and similar to those obtained by Nekrasov and Schroff (2009, p. 1997) at the industry level for their fundamental risk adjusted residual income model[21]. To confirm the superiority of the synergy model, we also run a comparative procedure in section 5.4.2.

5.4.2. Relative Performance of the Synergy Model Compared to the Residual Income Benchmark. In order to estimate the predictive power of the synergy valuation model, similarly to Barth et al. (2005) and Nekrasov and Schroff (2009), we decided to benchmark our model with the residual income valuation model, because of their accounting-based nature, and the similarity of the two models in terms of valuation of the IGG, although based on different method as explained in the section 3. Dechow et al. (1999) provide an empirical implementation of this class of model based on the Ohlson (1995) model. Using three assumptions: the dividend-discount model (26), the clean surplus relation (27), and the abnormal earnings dynamics (28 and 29), Dechow et al. (1999) derive equation (30):

$$P_t = \sum_{\tau=1}^{\infty} R^{-\tau} E_t\left[d_{t+\tau}\right] \qquad (26)$$

$$BV_t = BV_{t-1} + x_t - d_t \qquad (27)$$

$$x_{t+1}^a = \omega x_t^a + v_t + \varepsilon_{1,t+1} \qquad (28)$$

$$v_{t+1} = \gamma v_t + \varepsilon_{2,t+1} \qquad (29)$$

$$MV_t = \alpha_0 + \alpha_1 BV_t + \alpha_2 x_t^a + \alpha_3 v_t + \varepsilon_t \quad (30)$$

Where:

d_t: Dividend flow at time t;

R: 1 + cost of equity capital;

x_t: Earnings at time t;

x_t^a: Residual income at time t;

BV_t: Book value at time t;

v_t: Other information at time t;

ω: Auto-regressive coefficient of abnormal earnings dynamics, $\omega \in [0;1[$;

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[21] We compare our results to those obtained in these two papers because the authors provide out-of-sample prediction errors of different versions of the Ohlson model. However, our approach focuses on examining enterprise value that merely corresponds to the sum of equity value and total debt whereas Barth et al. (2005) and Nekrasov and Schroff (2009) examine solely equity value.

γ: Auto-regressive coefficient of other information dynamics, $\gamma \in [0;1[$;

To compute v_t, following Dechow et al. (1999), we use the difference between expectation of abnormal earnings for period t + 1 and the expectation of abnormal earnings based only on current period abnormal earnings:

$$v_t = E_t\left[x_{t+1}^a\right] - \omega x_t^a \qquad (31)$$

One can measure the period t conditional expectation of period t + 1 earnings using the median consensus analyst forecast of period t + 1 earnings, denoted f_t. We obtain:

$$E_t\left[x_{t+1}^a\right] = f_t^a = f_t - (R-1) * b_t \qquad (32)$$

Thus, we can measure the other information as:

$$v_t = f_t^a - \omega x_t^a \qquad (33)$$

Values for R, ω must be established. For the discount factor R, we used the CAPM model to determine the appropriate cost of capital for firms (Sharpe 1964). The classic formula was implemented:

$$k_{i,t} = r_{f,t} + \beta_i (r_{prem,t}) \qquad (34)$$

The U.S. annual equity market premium was provided by Damodaran and the risk free rate was proxied by the T-bond rate. Hence, unlike Dechow et al. (1999) who used a constant cost of capital of 12% for every firm, we obtained a specific cost of equity capital for each firm and each year.

We determined ω value as the first order autoregression coefficient for abnormal earnings, estimated using a pooled time-series cross-sectional regression from 1975 for earliest data to 2009 on our sample. The persistence coefficient was estimated at 0.425 (p-value of 0.000). Using the equations presented above, the remaining variables are then straightforward to obtain.

Equation (30) is estimated on the exact same sample as the synergy model. Due to missing value of analysts' forecasts, 16 firms of the initial sample were not used in the regression integrating the other information variable. We also run the model without this variable on the entire sample, to see the relative impact of the reduction of the sample compare to the new right hand-side variable.

Similarly to Barth et al. (2005), the accuracy of the Ohlson model has been tested and compared to the synergy model using the jackknifing procedure presented in section 5.4.1. Table 8 reports the performances of the Ohlson and synergy models in predicting enterprise values[22]. To be able to compare the models, the out-of-sample percentages of error of the two models have been computed with expression (25) on the exact same samples for which all data were available. To assess the statistical significance of differences in prediction errors, we compared means, median, and standard deviation for absolute percentage error (*AE*). For tests comparing means and variances we used standard paired t-test and F-test respectively, for medians, we used the Wilcoxon matched-pairs signed-rank test.

Insert Table 8 About Here

The synergy model clearly outperforms the Ohlson model, both in terms of central predictions (mean and median) and in term of dispersion of predictions. The differences are statistically significant both in terms of mean, median and standard deviations.

The predictive power of the Ohlson model integrating the other information variable slightly improves the model by 2% in mean. The loss in the sample size is outweighed by the predictive power of the other information variable. It also highly reduces the dispersion of the predictions (with a standard error dropping from 43% to 27%). Barth et al. (2005, p. 331-332) and Nekrasov and Schroff (2009, p. 1997) obtained similar results in terms of median industry prediction errors of equity values with a residual income model on a much larger sample[23]. Table 8 also clearly indicates that the predictive power of the synergy model is higher both in terms of central predictions of enterprise value as judged by mean and median errors and in terms of dispersion of errors.

6. Discussion, Limitations, Conclusion, Directions for Future Research

IGG emerges as interactions between assets generate synergies that create abnormal profitability for the company. Existing valuation methods propose to compute the present value of abnormal earnings (i.e., residual income models) or measure it indirectly by subtracting the fair value of assets identified in a business combination from the enterprise value. The drawback of these approaches is that they do not explain how goodwill is created. As a result a paradox emerges: the *internally* generated goodwill is evaluated with measures that focus on *external flows* (i.e., abnormal flows). This paradox is consistent with the aggregation issue identified by Miller (1973), indicating that goodwill emerges from an "*inappropriate theory of aggregation of assets*." Authors focus on external flows because

[22] As the residual income model predict equity values, the total value of debts is added to the estimated values of equity.
[23] For instance they obtained typically 35% of median prediction error in their models for industry specific prediction of equity.

additive measure is appropriate in that case, whereas focusing on assets is impossible as fair value of a set of assets is not additive. We aim at solving this paradox.

The valuation of IGG through synergies between assets identified by the Choquet's capacities offers an interesting approach that solves this aggregation issue. This method is consistent with the fact that fair value of a set of assets is not additive, and goodwill comes as a result of positive interactions between assets. To assess the validity of this approach, the Ohlson Model was used as a benchmark. We compare the accuracy in predicting enterprise value and conclude that our synergy model outperform the Ohlson model.

Using a non-additive aggregation method based on Choquet's capacities opens an interesting field allowing modeling interactions between assets as well as the effect of the structure on firm value. However the methodology presented and implemented in this study suffers from a couple of limitations.

Choquet's capacities belong to a class of operators that allow modeling over (or under) additivity between elements in a context of firm valuation. Other operators take into account the way interactions of elements in a set influence the value of the whole (fuzzy measures, t-norms and t-conorms, Ordered Weighted Average, etc.).

Another limitation is that even in the case of a business combination from where our data were collected, some intangible assets may still not be identified. As we value IGG only by interactions between assets, we may overstate the role of synergies as compared to unidentified intangible assets.

Implementing Choquet's capacities requires specifying every interaction of sub-samples of a set of assets. That is to say $2^n - 1$ interaction coefficients. It can be complex to implement and interpret. However, it is possible to group the assets into major classes (assuming no interaction within each of these classes) and some methods exist to limit the order of interactions (i.e., using 2-additive capacities instead of k-additive capacities, see Miranda et al. (2005)).

The additivity assumption is a stabilized and implicit hypothesis that is often unintentionally accepted in financial accounting. It constitutes an "invisible" management instrument (Hatchuel and Weil 1995), bounding the representation of organizations in a specific view. By relaxing the additive postulate, this paper does not only aim at measuring IGG but also opens the debate of the role of additivity in management.

Appendix 1 Illustration of Disclosure Requirements

Paragraph 805-10-55-38, Accounting Standards Classification (FASB)

On June 30, 20X0, Acquirer acquired 15 percent of the outstanding common shares of Target. On June 30, 20X2, Acquirer acquired 60 percent of the outstanding common shares of Target. Target is a provider of data networking products and services in Canada and Mexico. As a result of the acquisition, Acquirer is expected to be the leading provider of data networking products and services in those markets. It also expects to reduce costs through economies of scale.

Paragraph 805-10-55-39, ASC

The goodwill of $2,500 arising from the acquisition consists largely of the synergies and economies of scale expected from combining the operations of Acquirer and Target. All of the goodwill was assigned to Acquirer's network segment.

Paragraph 805-10-55-41, ASC

At June 30, 20X2	$
Consideration	
Cash	5,000
Equity instruments	4,000
Contingent consideration arrangement	1,000
Fair value of total consideration transferred	10,000
Fair value of acquirer's equity interest in Target held before the business combination	2,000
	12,000
Acquisition-related costs (including in selling, general, and administrative expenses in Acquirer's income statement for the year ending December 31, 20X2)	1,250
Recognized amounts of identifiable assets acquired and liabilities assumed	
Financial assets	3,500
Inventory	1,000
Property, plant, and equipment	10,000
Identifiable intangible assets	3,300
Financial liabilities	(4,000)
Liability arising from a contingency	(1,000)
Total identifiable net assets	12,800
Noncontrolling interest in Target	(3,300)
Goodwill	2,500
	12,000

Appendix 2 Definition of the Lebesgue and Choquet Integrals in the Continuous Case

Definition 1: *The Lebesgue integral $L_{(f)}$ of a measurable function f defined on a measure space (Ω, X, m) where Ω, is a set, X is a σ-algebra of subset of R^+ and m is the Lebesgue's measure, is defined as:*

$$L_{(f)} = \int_\Omega f dm = \int_\Omega f(x) m(dx) \qquad \text{(A1)}$$

Definition 2: *Let μ be a set of capacities on (Ω, X) and a function $f : \Omega \to [0,+\infty]$ the Choquet integral of f with regards to μ is defined by:*

$$C_{(f)} = \int_\Omega f \ d\mu = \int_0^{+\infty} \mu(\{x : f(x) > t\}) dt \qquad \text{(A2)}$$

provided $\{x : f(x) > t\} \in X, \forall\, t \in R^+$.

Appendix 3 Proof of Equations (5) and (6)

Let $1_{(A=B)}$ being the indicator function which takes value 1 if $A = B$ and 0 otherwise, we can rewrite equations (1) and (3) as in equation (A1) and (A2):

$$L_{(f)} = \int (\sum_{A \in P(X)} 1_{(A=\{x/f(x)>y\})}) dy * m(A) \quad \text{(A1)}$$

$$C_{(f)} = \int (\sum_{A \in P(X)} 1_{(A=\{x/f(x)>y\})}) dy * \mu(A) \quad \text{(A2)}$$

The expression of the Lebesgue integral (A1) and the Choquet integral (A2) are equivalent to equation (A3) and (A4) respectively:

$$L_{(f)} = \sum_{A \in P(X)} (\int 1_{(A=\{x/f(x)>y\})} dy) * m(A) \quad \text{(A3)}$$

$$C_{(f)} = \sum_{A \in P(X)} (\int 1_{(A=\{x/f(x)>y\})} dy) * \mu(A) \quad \text{(A4)}$$

If we denote $g_A(f)$ as the value of the expression $\int 1_{(A=\{x/f(x)>y\})} dy$, Lebesgue integral may be expressed as stated in equation (A5) and Choquet integral as in equation (A6):

$$L_{(f)} = \sum_{A \in P(X)} g_A(f) * m(A) \quad \text{(A5)}$$

$$C_{(f)} = \sum_{A \in P(X)} g_A(f) * \mu(A) \quad \text{(A6)}$$

Appendix 4 Categories of Assets

The three categories of assets, current assets, tangibles assets, and intangible assets, contain the following items:

(i) Current assets:

- Account receivables;
- Cash or equivalent;
- Other current assets;
- Tax asset.

(ii) Tangible assets:

- Property, plant, equipment;
- Non-current assets.

(iii) Intangible assets:

- Completed technologies;
- Customer relationships;
- Trade names and trademarks.

Appendix 5 Computation of the Shapley Values and Interaction Index

The Shapley values are computed using equation (22). As the sum of the Shapley values equals 1 by definition, the value of the capacities must be normalized by $\mu(N)$. In our case $\mu(N) = \mu(TA,CA,IA) = 4.628$, so all the capacities of table 5 must are divided by 4.628. The following computations follow:

$\phi(TA) = 1/3 * (\mu(TA) - \mu(\emptyset)) + 1/6 * (\mu(TA,CA) - \mu(CA)) + 1/6 * (\mu(TA,IA) - \mu(IA)) + 1/3 * (\mu(TA,CA,IA) - \mu(CA,IA))$

$\phi(CA) = 1/3 * (\mu(CA) - \mu(\emptyset)) + 1/6 * (\mu(CA,IA) - \mu(IA)) + 1/6 * (\mu(TA,CA) - \mu(TA)) + 1/3 * (\mu(TA,CA,IA) - \mu(TA,IA))$

$\phi(IA) = 1/3 * (\mu(IA) - \mu(\emptyset)) + 1/6 * (\mu(TA,IA) - \mu(CA)) + 1/6 * (\mu(CA,IA) - \mu(CA)) + 1/3 * (\mu(TA,CA,IA) - \mu(TA,CA))$

The interaction index is computed on the normalized capacities using equation (23) in the case of two classes of assets and (24) in the case of three classes of assets:

$I(TA,CA) = 1/2 * (\mu(TA,CA) - \mu(TA) - \mu(CA) + \mu(\emptyset) + \mu(TA,CA,IA) - \mu(TA,IA) - \mu(CA,IA) + \mu(IA))$

$I(TA,IA) = 1/2 * (\mu(TA,IA) - \mu(TA) - \mu(IA) + \mu(\emptyset) + \mu(TA,CA,IA) - \mu(TA,CA) - \mu(CA,IA) + \mu(CA))$

$I(CA,IA) = 1/2 * (\mu(CA,IA) - \mu(CA) - \mu(IA) + \mu(\emptyset) + \mu(TA,CA,IA) - \mu(CA,IA) - \mu(TA,IA) + \mu(TA))$

$I(TA,CA,IA) = \mu(TA,CA,IA) - \mu(TA,CA) - \mu(TA,IA) - \mu(CA,IA) + \mu(TA) + \mu(CA) + \mu(IA) - \mu(\emptyset).$

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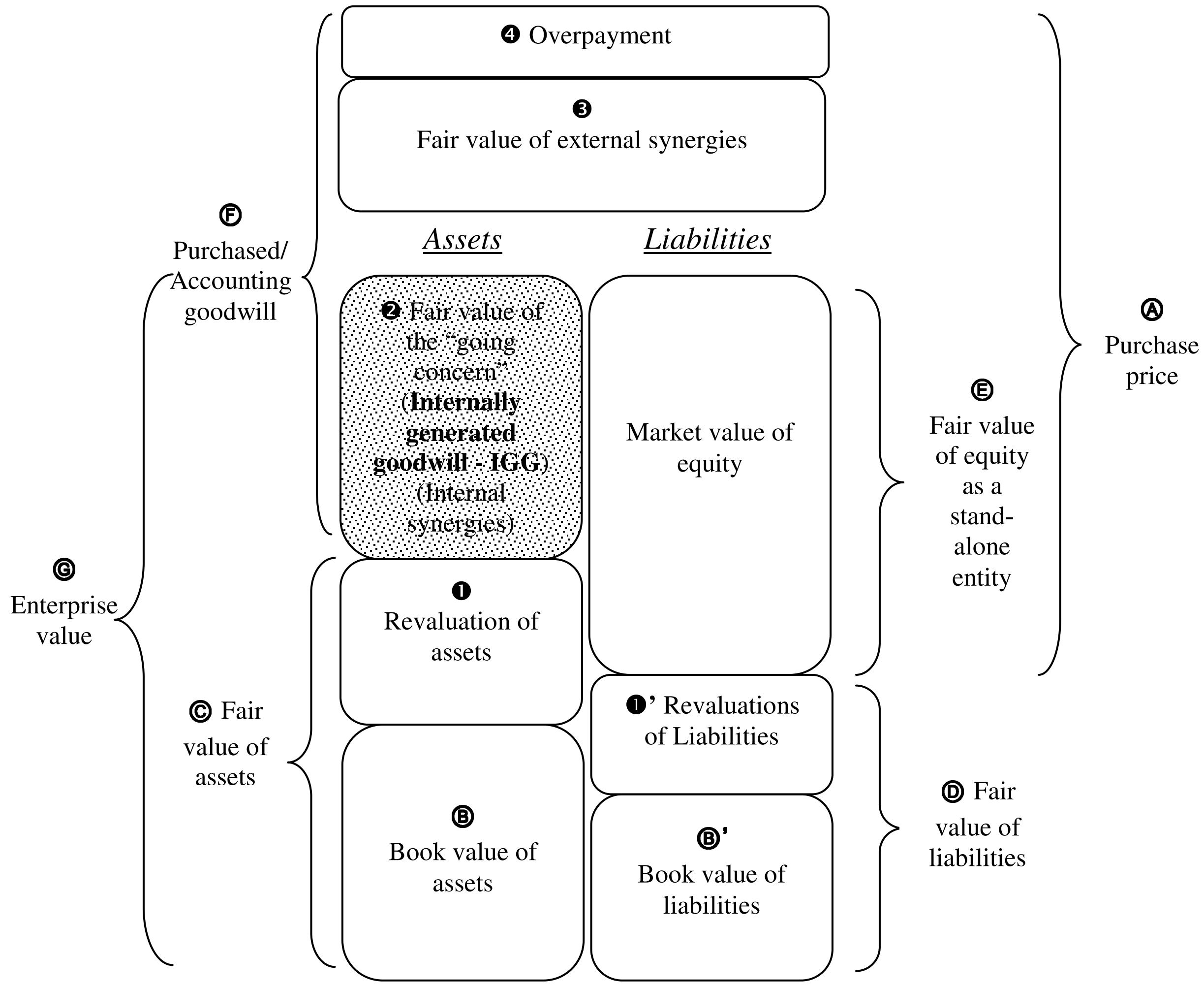
Figure 1 Purchase Price Decomposition



Purchase price = Ⓐ = Ⓒ + Ⓕ - Ⓓ

Fair value of assets = Ⓒ = (❶ + Ⓑ)

Fair value of liabilities = Ⓓ = (❶' + Ⓑ')

Enterprise value = Ⓖ = Ⓔ + Ⓓ [Equity side] = ❷ + Ⓒ [Assets side]

Internally generated goodwill (IGG) = ❷ = Ⓖ - Ⓒ

Sources of data:

- Annual report (PPA): Ⓐ, Ⓒ, Ⓓ, Ⓕ
- Annual report (Balance sheet): Ⓑ, Ⓑ'
- Market capitalization: Ⓔ.

Figure 2 Numerical Representation of the Observed Reality According to the Accounting System (left) and the Non-Additive Approach (right)

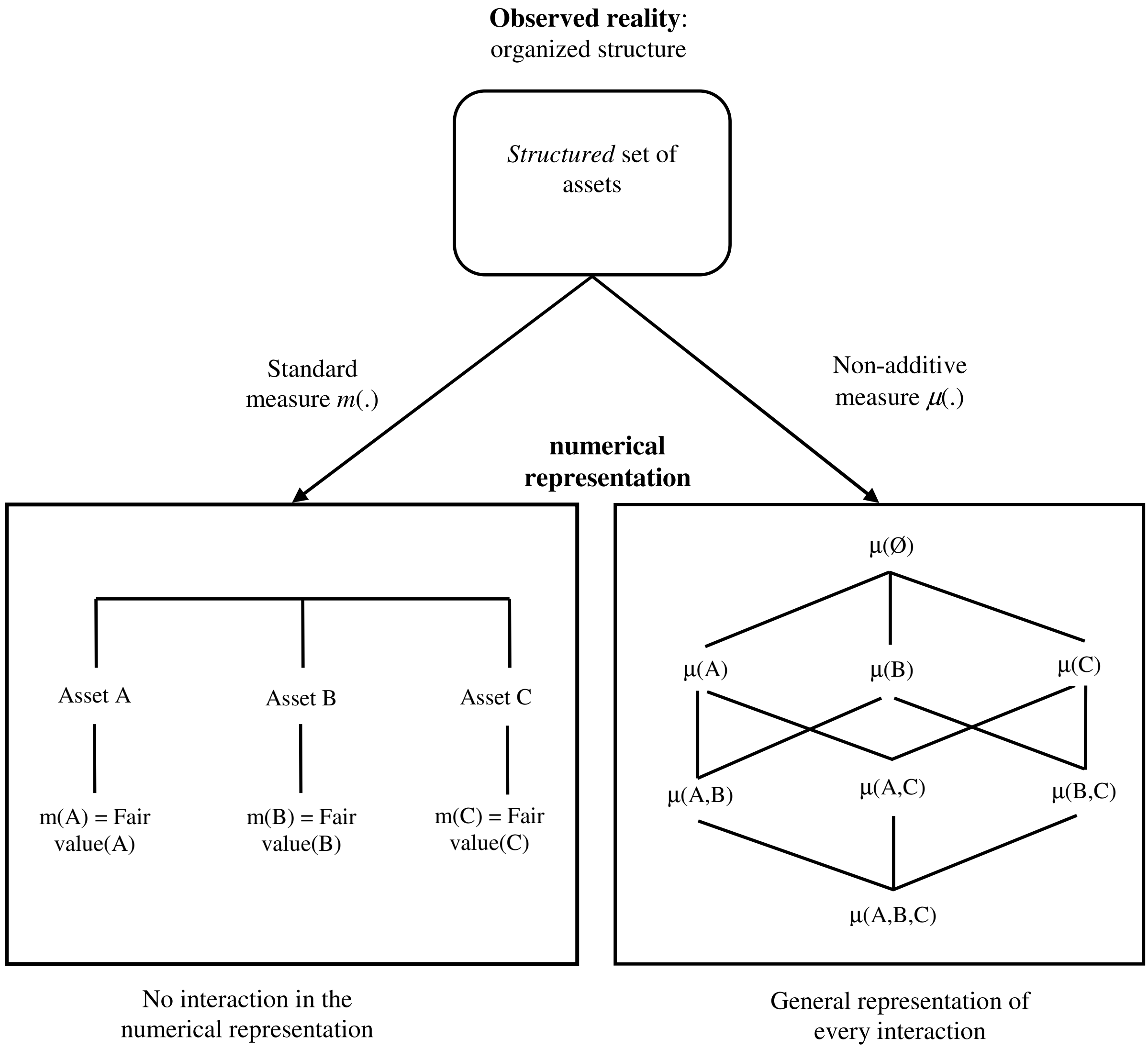


Figure 3 Additive Valuation of a Set of Assets

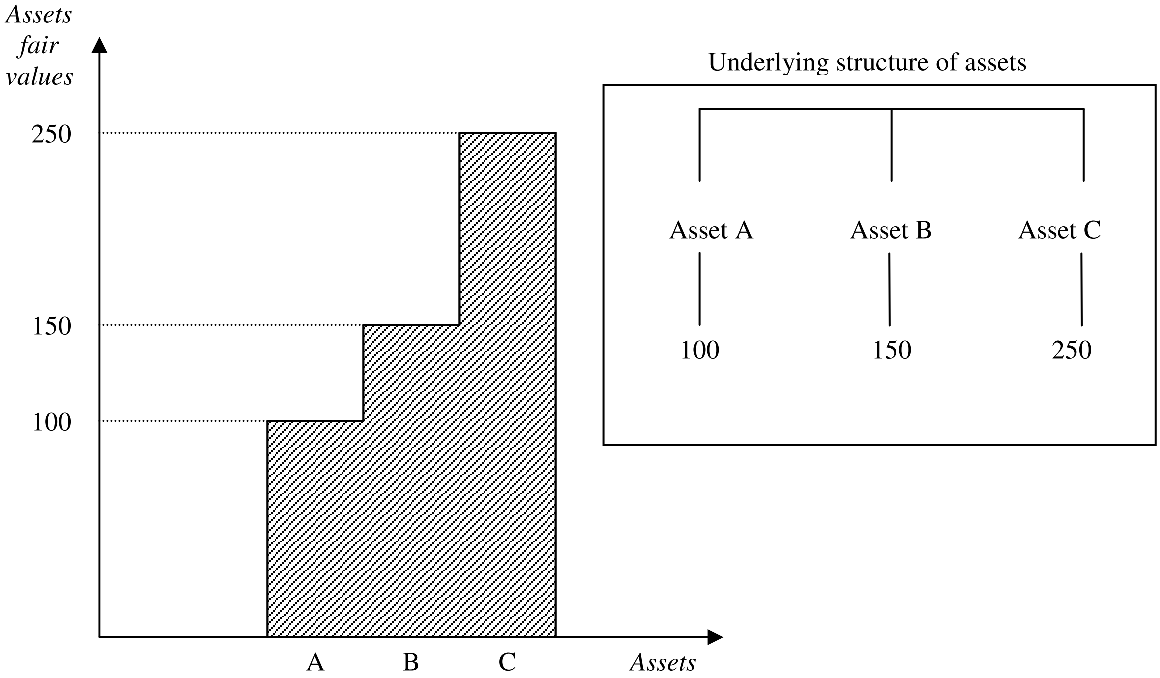


Figure 4A Valuation of a Set of Assets using Choquet integral for the Synergy between Three Assets Only

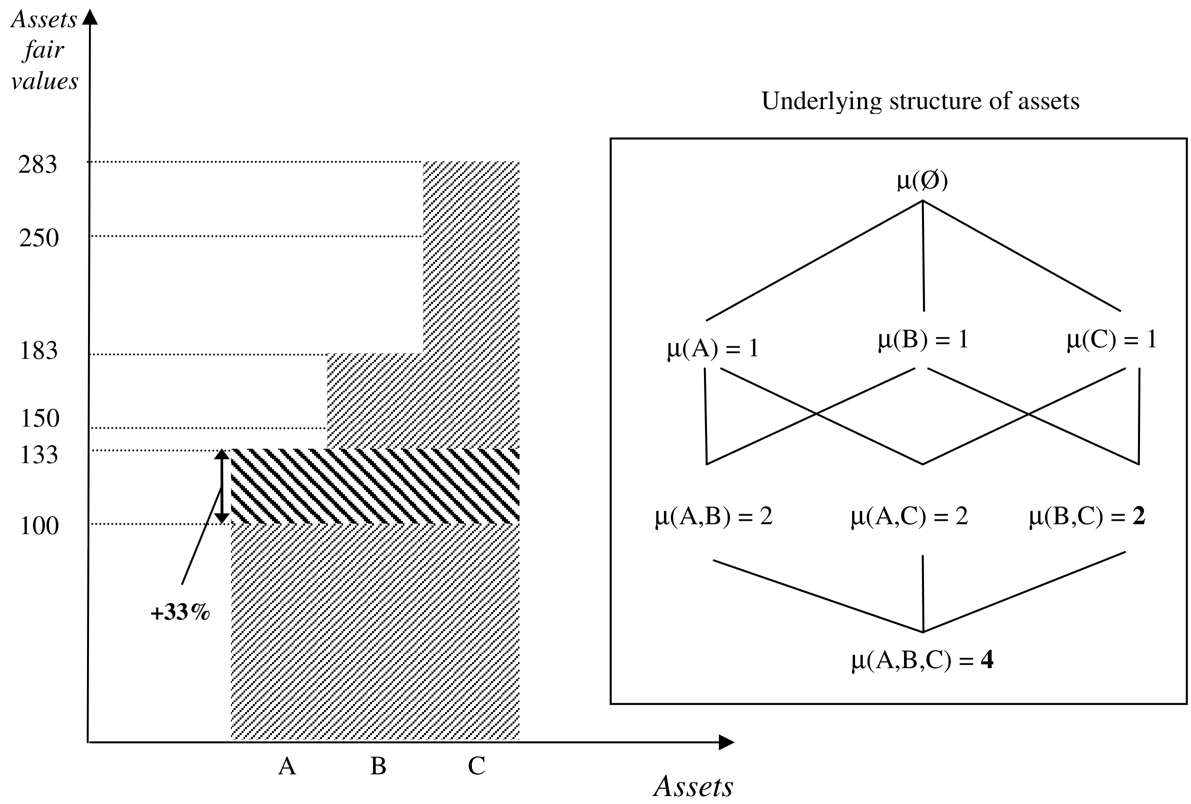


Figure 4B Valuation of a Set of Assets using Choquet integral for Synergy and Inhibition Effects

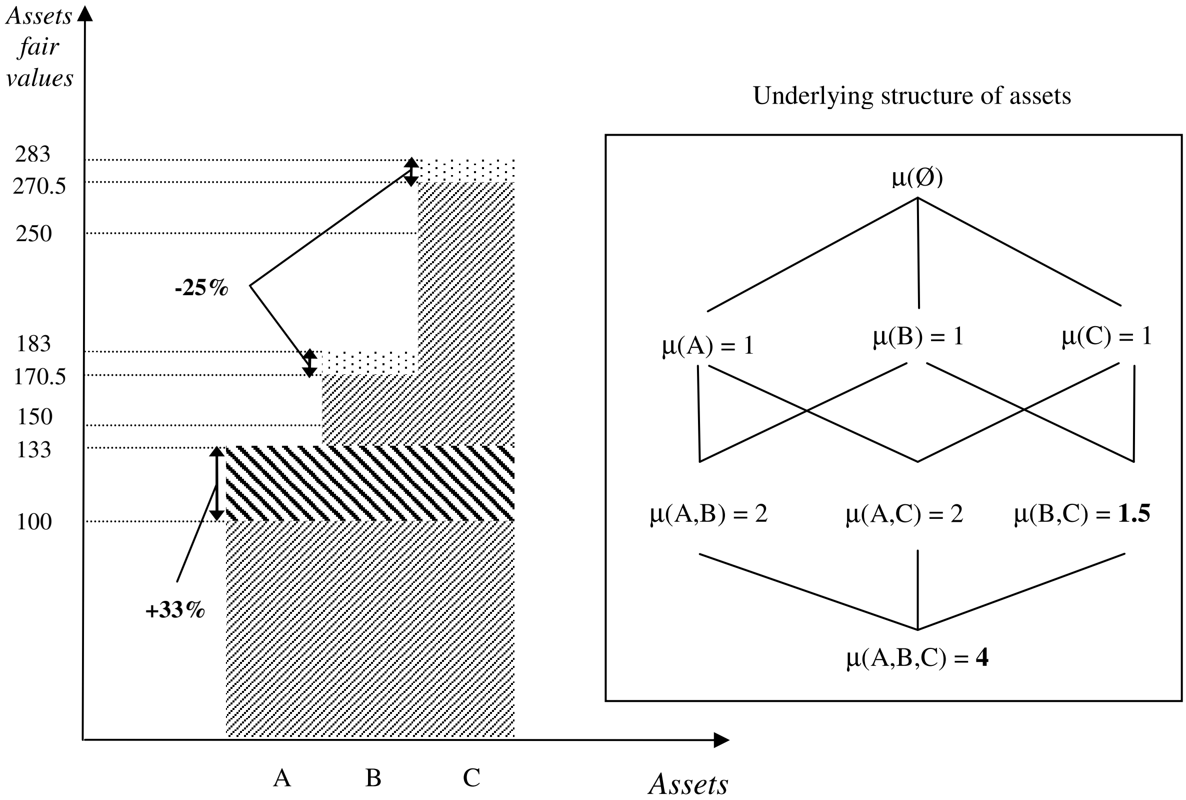


Figure 5 Graphical Representation of the Set of Assets of DataDomain Inc.

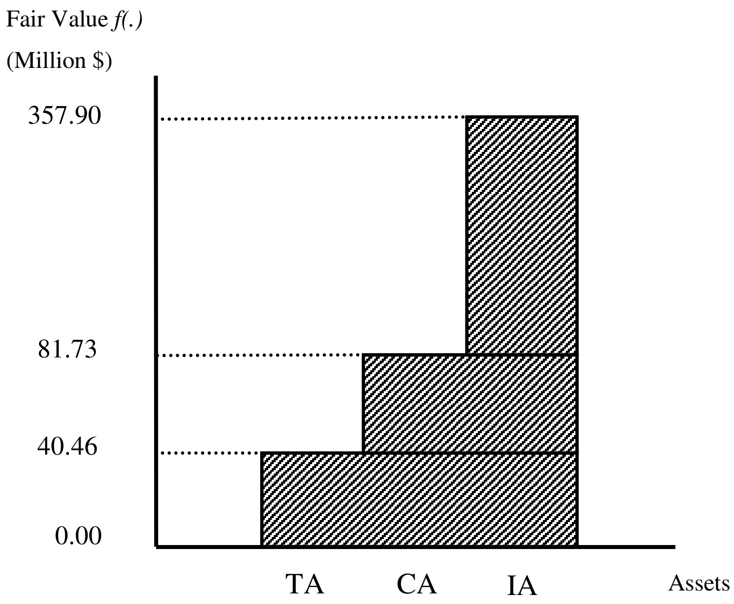


Table 1 Similarities and Differences between the Residual Income and Synergy Models

	Residual income model	Synergy model
Market value of equity	$BV_t + GW_t$	
Origin of goodwill	Output flows, abnormal earnings	Interactions between assets, positive synergies, value of the structure
Measure	Additive	Non-additive, Combinatorial
Expression of goodwill	$\sum_{\tau=1}^{\infty} R^{-\tau} E\left[x_{t+\tau}^{a}\right]$	$IV_t = C_{(f)} - L_{(f)}$

Table 2 Descriptive Statistics of the Sample (in million $ and % of Enterprise Value)

	N	Mean	St. dev.	1st quartile	Median	3rd quartile
Tangible assets (TA)	101	131.264	633.886	7.100	19.200	78.300
Tangible assets (% of EV)	101	9.9%	17.2%	2.3%	4.7%	9.3%
Current assets (CA)	101	350.675	748.147	52.212	97.000	251.569
Current assets (% of EV)	101	31.5%	15.9%	20.7%	29.8%	42.7%
Intangible assets (IA)	101	332.411	718.091	38.700	90.800	234.070
Intangible assets (% of EV)	101	25.3%	15.6%	12.8%	24.2%	34.1%
Internally generated goodwill	101	406.966	862.765	47.468	120.593	366.527
Internally generated goodwill (% of EV)	101	33.3%	24.5%	19.7%	32.9%	50.6%
Enterprise value (MV + D)	101	1,221.317	2,693.103	201.890	373.505	1,010.860

EV stands for Enterprise Value, MV for Market Value, D for Debts. Every asset is in estimated fair value at the date of acquisition.

Table 3 Book Value, Market Values, Earnings, Cost of Equity Capital and Earnings Forecasts

	N	Mean T (Panel data)	Mean	St. dev.	1st quartile	Median	3rd quartile
Book value	101	10.50	282.242	812.752	17.378	68.770	207.862
Market values	101	5.78	1,397.743	3,511.482	151.672	369.557	1,052.535
Earnings	101	10.47	2.287	195.007	-9.358	0.645	10.235
Cost of capital	101	5.78	0.146	0.069	0.093	0.129	0.186
Earnings forecasts	85	3.98	40.378	119.231	1.200	10.260	37.280

Table 4 Summary Statistics of the Generator Function

	N	Mean	St. dev.	1st quartile	Median	3rd quartile
g_{TA}	101	16.054	85.481	0	0	0.000
g_{CA}	101	84.017	201.088	0	21.889	59.223
g_{IA}	101	79.453	229.535	0	0	35.829
$g_{TA,CA}$	101	16.573	109.332	0	0	0
$g_{CA,IA}$	101	154.321	376.753	6.729	41.267	106.231
$g_{TA,IA}$	101	2.873	20.352	0	0	0
$g_{TA,CA,IA}$	101	95.764	455.145	6.075	17.238	54.872

Table 5 Choquet's Capacities Estimations

	Coefficient (μ)	Std. Error	p-value
g_{TA}	0.437	0.490	0.375
g_{CA}	2.040	0.491	0.000
g_{IA}	2.047	0.128	0.000
$g_{TA,CA}$*	2.040	na.	na.
$g_{CA,IA}$*	2.047	na.	na.
$g_{TA,IA}$	3.638	0.466	0.000
$g_{TA,CA,IA}$	4.628	0.125	0.000
N	101		
R-square	0.980		
Adj.R-square	0.979		

* Consistent with the properties of the Choquet capacities presented in section 3.5.3, the equation is estimated under the constraint of monotonicity (i.e. $\forall A \subset B, \mu(A) \leq \mu(B)$). These constraints are binding for μ(TA,CA) and μ(CA,IA). Therefore we have:

$$\{CA\} \subset \{TA, CA\}, \mu(CA) = \mu(TA, CA) = 2.040$$

$$\{IA\} \subset \{CA, IA\}, \mu(CA) = \mu(CA, IA) = 2.047$$

Table 6 Synergies and Inhibitions in the HT Sector

Panel A: Direct Interpretation

Additive value (1)		Interaction value (2)		Sign of interpretation
$\mu(TA) + \mu(CA)$	2.477	$\mu(TA,CA)$	2.040	(-)
$\mu(CA) + \mu(IA)$	4.087	$\mu(CA,IA)$	2.047	(-)
$\mu(TA) + \mu(IA)$	2.484	$\mu(TA,IA)$	3.638	(+)
$\mu(TA,CA) + \mu(IA)$	4.087	$\mu(TA,CA,IA)$	4.628	(+)
$\mu(CA,IA) + \mu(TA)$	2.484	$\mu(TA,CA,IA)$	4.628	(+)
$\mu(TA,IA) + \mu(CA)$	5.678	$\mu(TA,CA,IA)$	4.628	(-)
$\mu(TA) + \mu(CA) + \mu(IA)$	4.524	$\mu(TA,CA,IA)$	4.628	(+)

Panel B:

	Shapley value
ϕ(TA)	0.276
ϕ(CA)	0.273
ϕ(IA)	0.451

Panel C:

	Interaction index
I(TA,CA)	0.055
I(CA,IA)	-0.291
I(TA,IA)	0.408
I(TA,CA,IA)	0.299

Table 7 Out-of-Sample Prediction Errors in Percentage of Real Enterprise Value

N	Mean	St. dev.	1st quartile	Median	3rd quartile
101	25%	21%	6%	20%	39%

Table 8 Out-of-Sample Prediction Errors of the Ohlson and Synergy Models

Model	N	Mean	St. dev.	1st quartile	Median	3nd quartile
Ohlson (with v_t)	79	41%	27%	20%	37%	53%
Synergy	79	24%	22%	7%	19%	36%
Equality test p-value		0.000	0.045		0.000	
Ohlson (no v_t)	101	43%	43%	19%	34%	52%
Synergy	101	25%	21%	6%	20%	31%
Equality test p-value		0.000	0.000		0.000	