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A Dynamic Analysis of Tied Aid

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Abstract: In this paper we examine the impact of tied aid on capital accumulation and welfare in the presence of a quota on imports. Using a simulation model we establish that tied aid can lower the relative domestic price of the manufactured good and therefore reduce the stock of capital. In the presence of a strong production externality from capital accumulation and high tying ratio, tied aid may immiserize the recipient country. [71 words]

Key words: Tied aid, quotas, capital, welfare
JEL classification: F11, F35

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1. Introduction

Ever since the famous debate between Keynes (1929) and Ohlin (1929), movements in the terms of trade and its impact on welfare have been a central issue in the study of the transfer problem. Keynes claimed that aid can alter the donor’s terms of trade and therefore welfare. Ohlin questioned this line of reasoning. If aid can result in the terms of trade to favour the donor country, a paradoxical result, donor enrichment and recipient impoverishment, given by Leontief (1936) may arise. However, assuming market stability, Samuelson (1952) showed that the welfare of the donor decreases while the welfare of the recipient increases, regardless of the direction of shift in the terms of trade (which may be induced, for example, by a transfer). Later research nevertheless established that the transfer paradox can still happen in a stable economy with either more than two countries or in the presence of distortions. Subsequently, the research on international transfers has been focused to find specific distortions that can cause transfer paradox.

Bhagwati, et al. (1985) classify distortions into exogenous and (transfer-induced) endogenous distortions. The literature related to exogenous distortions includes Bhagwati (1971) on distortions and welfare, Brecher and Bhagwati (1982) on a production externality, Brecher (1974) and Beladi (1990) on unemployment, and Choi and Yu (1987) on variable returns to scale. In addition, exogenous distortions may arise from government policy interventions. For example, tariffs distort the importable sector of the economy. In this setting, as shown by Ohyama (1974), Jones (1985) and Yano and Nugent (1999), transfers can lower the recipient’s welfare because it aggravates the tariff distortion in the importable sector.

As far as endogenous distortions induced by transfers are concerned, there exist two lines of thought that have been considered: rent-seeking and tied aid. The former activity may be unproductive, as pointed out by Bhagwati and Srinivasan (1980), and therefore aid may be wasted, while the latter imposes a restriction on usage of aid, reducing the flexibility in the recipient country. For example, Kemp and Kojima (1985) and Schweinberger (1990) discuss the case of tying aid to consumption. This forced expenditure pattern can be immiserizing for
the recipient country. Recently, Lahiri and Raimondos (1995) consider aid tied to purchases of the importable good, which is under a quota restriction. Although aid in this form mitigates the trade distortion, the paradox of donor enrichment and recipient impoverishment may still arise. Chao and Yu (2001) extend their tied-aid model to a dynamic setting by focusing on the impact of tied aid on capital accumulation. Although tied aid brings a direct gain, it also causes a decline in the stock of capital. The welfare effect of tied aid depends on the direct gain of aid and the induced loss of capital decumulation; therefore the overall effect is ambiguous in a dynamic setting.

The purpose of this paper is to employ a simulation method to investigate the welfare implication of aid tied to a purchase of quota-restricted imports in a dynamic economy. To highlight the role played by capital accumulation, a production externality arising from the capital stock is explicitly incorporated in the production function. This externality was first considered by Arrow (1962) in a growth framework, in which knowledge creation is a side product of investment. Essentially, each firm’s knowledge is a public good so that firm can have free access to it. Romer (1986, 1989) extended this insight to model endogenous growth. Barro and Sala-i-Martin (2003) provide a comprehensive survey of this literature. We will show that accumulation of capital is positively related to the price of the quota-restricted foreign good. Aid tied for purchasing the foreign good can lower its relative price and hence capital accumulation in the economy, if the tying ratio is substantial. This suggests that to avoid the fall in the capital stock, tightening import quota (i.e., zero quota) is optimal when a large capital externality is present. Thus, tied aid for relaxing quota may immiserize the recipient country. On the contrary, the optimal level of import quota is large when the capital externality is low. In this case, tied aid that relaxes the quota restriction can be welfare improving if the initial level of quota is set too low.

The paper is organized as follows. Section 2 provides a discrete-time dynamic model of trade and foreign aid. Section 3 uses a simulation method to examine optimal quota structures under various capital externalities and its welfare implications of tied aid. Section 4 provides conclusions.
2. The model

Consider a two-country model and each country produces two goods: an agricultural good $X$ and a manufactured good $Y$. The home country exports good $X$ and imports good $Y$. There are no restrictions on the exports of good $X$ but a quota $Q$ is imposed by the home country on the imports of good $Y$. This quota restriction limits the exports of good $Y$ by the foreign country. It is also assumed that the home country receives an aid, denoted by $T$, from the foreign country. To increase the exports beyond the quota level, the foreign country requires the home country to use $\tau$ portion of the aid to purchase more good $Y$. Total supply of good $Y$ in the domestic economy therefore consists of domestic production, import quota and tied-aid purchase (i.e., $Y + Q + \tau T$). This form of tying aid to imports was first discussed in Lahiri and Raimondos (1995) in a static model. This paper extends their model to a dynamic analysis with capital accumulation.

2.1. Firms

The domestic firms produce goods $X$ and $Y$ by using labour ($L_i$) and capital ($K_i$). Total capital ($K_1$) is inherited from the past and it can be allocated freely between sectors at the beginning of the period, i.e., $K_1 = K_X + K_Y$. As expressed in Romer (1986, 1989), total capital serves as a proxy for knowledge in the economy, and it yields a positive externality to the production of the manufactured good $Y$. The production functions are specified in the Cobb-Douglas form:

$$X = A_x L_x^\alpha_1 K_x^\alpha_2,$$

$$Y = A_y K_{-i}^{\beta_1} L_{-i}^{\beta_2},$$

where $A_x > 0$, $A_y > 0$, $\alpha_i > 0$, $\beta_i > 0$ for $i = 1, 2$, and $\alpha_1 + \alpha_2 < 1$ and $\beta_1 + \beta_2 < 1$ are respectively the total shares of labour and capital income in production of goods $X$ and $Y$. It is assumed that $\beta_2 > \alpha_2$, i.e., the manufactured-good sector is capital intensive relative to the agricultural-good sector. It is noted that $\beta_2 \geq 0$, capturing the externalities from total capital to
the global productivity of factors in the manufactured-good sector. The presence of this externality also highlights the role of capital accumulation in the dynamic model.

Let good $X$ be the numeraire, and the domestic relative price of the manufactured good $Y$ is denoted by $p$. The goods and factor markets are assumed to be perfectly competitive. Given the relative price $p$, wage rate $w$ and the externality from $K_{-1}$, the production sectors choose $L_i$ and $K_i$ to maximize the returns on capital, i.e., $\text{Max } X + pY - w(L_X + L_Y)$, subject to (1) and (2). For a given state variable $K_{-1}$, the optimal conditions with respect to $L_i$ and $K_i$ are:

$$\alpha_1 A_X \left(\frac{K_X}{L_X}\right)^{\alpha_1} L_X^{\alpha_2 - \alpha_1 - 1} = \beta_1 p A_Y K_Y^{\beta_1} \left(\frac{K_Y}{L_Y}\right)^{\beta_2} L_Y^{\beta_1 - \beta_2 - 1} = w, \quad (3)$$

$$\alpha_2 A_X \left(\frac{L_X}{K_X}\right)^{\alpha_2} K_X^{\alpha_1 - \alpha_2 - 1} = \beta_2 p A_Y K_Y^{\beta_1} \left(\frac{L_Y}{K_Y}\right)^{\beta_2} K_Y^{\beta_1 - \beta_2 - 1} = r, \quad (4)$$

where $r$ denotes the domestic rental rate on capital. From equations (3) and (4), the factor price frontiers are described by

$$\left(\frac{w}{\alpha_1}\right)^{1 - \alpha_1} \left(\frac{r}{\alpha_2}\right)^{\alpha_2} L_X^{(1 - \alpha_1 - \alpha_2)} = A_X, \quad (5)$$

$$\left(\frac{w}{\beta_1}\right)^{1 - \beta_1} \left(\frac{r}{\beta_2}\right)^{\beta_2} L_Y^{(1 - \beta_1 - \beta_2)} = A_Y K_Y^{\beta_1} p, \quad (6)$$

Note that $L_X + L_Y = L$, which shows the endowment of labour. Due to the flexible wage rate, labour is fully utilized in the economy. We assume that there is no growth of the labour force in the economy.

2.2. Goods Market Equilibrium

Domestic households consume goods $X$ and $Y$, denoted by $C_X$ and $C_Y$. By the Walras’ law, we need to consider the market of good $Y$ only. In the dynamic model, the manufactured good $Y$ is used not only for consumption ($C_Y$) but also for capital investment ($K - K_{-1}$). The market equilibrium condition for good $Y$ requires the equality between its demand and supply:

$$C_Y + (K - K_{-1}) = Y + Q + \tau T. \quad (7)$$
As mentioned earlier, $T$ is the aid received in terms of good $Y$ and $\tau$ is the tying ratio for purchasing the quota-restricted good. Note that by letting the world price of good $Y$ be $p^*$, the value of the aid is $p^*T$ to the home economy. Thus, tied purchase of good $Y$ is $\tau T$ described in equation (7).

2.3. Households

Households set their consumption plan over time by maximizing their intertemporal utility subject to the budget constraint. The current utility function of households is chosen as:

$$U(C) = C^{(1-\lambda)}/(1 - \lambda),$$

where $C = \left[b^{1/(1+\sigma)}C_X^{\sigma/(1+\sigma)} + \tilde{b}^{-1/(1+\sigma)}C_Y^{\sigma/(1+\sigma)}\right]^{1/(1+\sigma)}$ represents their total consumption. It is noted that $\sigma$ is the elasticity of substitution between goods with $1 + \sigma \geq 0$.

Here, $b \in [0, 1]$ and $\tilde{b} = 1 - b$ capture the relative preferences for each good, and $\lambda > 0$ expresses the inverse of the intertemporal rate of substitution. The intertemporal utility of consumers is thus specified as:

$$W = \sum_{t=0}^\infty (1 - \rho)^t U(C),$$

where $0 < \rho < 1$ is the subjective discount rate of households.

As for revenues, domestic households receive factor income from production and aid from the foreign country. Since each firm operates under decreasing returns to scale with respect to its production factors, it earns a rent. Total rent, $R = X + PY - (wL + rK)$, is assumed to be distributed to households (who own the firms). We assume that when households make their saving/investment decision, they do not internalize the positive effect of the capital externality in each manufacturing firm. Additionally, there are quota rents accrued to the domestic government, which is assumed to be returned to domestic households in a lump-sum fashion. The budget constraint of the households is:

$$C_X + pC_Y + p(K - K_1) = wL + rK_1 + R + (p - p^*)(Q + \tau T) + p^*T,$$

where $p > p^*$ due to the quota restriction of the imports of good $Y$. Denoting $\mu$ as the Langrange multiplier, the optimality conditions of the households’ program for $C_X$, $C_Y$ and $K$ are:

$$b^{1/(1+\sigma)}C_Y^{-\sigma/(1+\sigma)}C_X^{\sigma/(1+\sigma)} = \mu,$$

(8)
Combining equations (8) and (9), the relationship between the good price and consumption is derived as: \( bC_Y / \bar{b}C_Y = p^{(1+\sigma)} \). This gives total consumption: \( C = (C_Y/b)(b + \bar{b} p^{-\sigma})^{(1+\sigma)/\sigma} \).

Substituting \( C \) into equation (8) and then using equation (10), the evolution of capital in terms of its rate of return is governed by

\[
1 + r_{x+1} = \frac{1}{1 - \rho} \left( \frac{C_X}{C_{X,+1}} \right)^{-\lambda} \left( \frac{(b + \bar{b} p^{-\sigma})^{(1+\sigma)/(1-\lambda) - 1}}{(b + \bar{b} p^{-\sigma})^{(1+\sigma)/(1-\lambda) - 1}} \right).
\]

In steady state, \( C_X = C_{X,+1} \) and \( p = p_{+1} \), we have \( 1 + r = 1/(1 - \rho) \). Hence, the rate of return on capital is approximately equal to the rate of time preference.

It is worthwhile to note the direct impact of aid on the domestic price of good \( Y \) in the steady state. Aid tied to purchase the importable good increases the supply of good \( Y \) by \( \tau \) in equation (7) for each unit of \( T \). On the demand side, tied aid raises revenue by \( p^* + \tau(p - p^*) \) expressed in the budget constraint, thereby increasing the demand for good \( Y \): \( \partial C_Y / \partial T = [p^* + \tau(p - p^*)](U_{XY} - pU_{XX})/\Delta \), where \( \Delta > 0 \). With the given specification of \( U \), we have: \( \partial C_Y / \partial T = [p^* + \tau(p - p^*)]C_Y/(C_X + pC_I) \). Consequently, the domestic price of good \( Y \) rises (falls) if the demand effect is larger (smaller) than the supply effect [i.e., \( \partial C_Y / \partial T > (<) \tau \) or \( p^* C_Y/C_X > (<) \tau/(1 - \tau) \)]. The change in the domestic price of good \( Y \) plays a crucial role in affecting capital accumulation in equation (7) and hence welfare in the economy as a whole.

3. Simulations

3.1. Reference steady state

On the basis of the above theoretical analysis, it is instructive to conduct simulations. At the outset, we calibrate the reference steady state. Initial values for sectoral outputs, goods price and total labour employment are chosen as: \( X = 0.5, Y = 1, p = 1 \) and \( L = 10 \), and parameters are specified as: \( \alpha_1 = 0.60, \alpha_2 = 0.10, \beta_1 = 0.40, \beta_2 = 0.40, \beta_3 = 0.50, \rho = 0.05, \sigma = \).
-0.50 and \( \lambda = 0.50 \). The initial amount of foreign aid is set as: \( T = 0.01 \), and the tying ratio is absent (i.e., \( \tau = 0 \)). We also assume that the quota on imports represents 20% of domestic output of the importable good \( (Q = 0.2Y) \), and that the world price of the importable good is equal to 90% of its domestic price \( (p^* = 0.9p) \).

Using equations (1) – (11) in the steady state, we compute the endogenous variables:

\[
C_X = 0.329, \quad C_Y = 1.2, \quad K = 8.55, \quad K_X = 0.95, \quad K_Y = 7.6, \quad L_X = 4.2857, \quad L_Y = 5.7143, \quad w = 0.07, \quad r = 0.0526, \quad U = 2.473 \text{ and } W = 49.4611.
\]

In addition, we compute the direct effect of aid on the demand for good \( X \):

\[
\frac{\partial C_Y}{\partial T} = 0.706.
\]

The eigenvalues in the neighbourhood of the reference steady state are equal to 0.4354 and 1.087. The local condition of existence and uniqueness are satisfied. As we will compare sums of discounted utilities when the convergence speed to the steady state may be slow, we simulated the model over 500 periods.

3.2. Optimal quota

We first look for the optimal quota by starting from the reference steady state and then by fixing the quota at different levels. For each new value of the quota, the economy will progressively adjust to this new quota and converge to a new steady state. We will compute the value of the sum of discounted utilities of households and look for the quota that maximizes this sum. It should be noted that, in the reference steady state, the quota is equal to 20% of the reference value of the output of good \( Y \).

1. Consider the case when the externality coefficient \( \beta_3 \) is 0.5. Then the optimal quota is zero (i.e., \( Q^o = 0 \)), suggesting no imports of good \( Y \). When the level of quota is tightened to this level the domestic price of the importable good \( Y \) immediately rises and then it decreases at a slow rate and converges to a value lower than its initial value \( (p = 0.974) \). This reduces the consumption of the importable good but raises total capital over time until \( K = 11.5051 \). Consequently, current utility falls as a function of tightening the quota, and it then smoothly increases and converges to a value higher than its reference value \( (U = 2.7196) \). The result of a zero optimal quota is due to the high degree of the capital
externality. It requires a high price of good $Y$ for accumulation of more capital. Zero quotas on imports serve this purpose. Under this condition, tied aid for importing good $Y$ may induce a negative impact on welfare of the economy.

2. As the capital externality gets smaller, the optimal quota becomes positive. We illustrate this point by considering the case that $\beta_1 = 0.10$.\(^8\) The domestic price of good $Y$ increases sharply when a lower quota is set. Then, it decreases at a slow rate and converges to a value higher than its initial value. Capital increases smoothly and converges to a higher value. Current utility decreases, then increases at a slow rate to a value higher than its reference value. The optimal quota is then equal to 17% of the reference value of the output of good $Y$ (i.e., $Q^o = 0.17Y$). It is lower than the reference quota ($Q = 0.2Y$). So, tied aid for loosening the quota restriction may mitigate the direct gain of the aid to the economy.

3. As for the case in which the capital externality is absent ($\beta_1 = 0$), the optimal quota becomes large and is around 0.24 time the output of good $Y$ in the reference state ($Q^o = 0.24Y$). This is the best approximation of free trade that we can reach under the assumption of a permanent change to a constant quota. Of course, free trade is the optimal trade policy in the absence of externality. If we relax the quota by increasing it from its reference value, capital will smoothly decrease over time and converge to a lower value because the importable good is more capital intensive than the exportable good. The domestic price of imported good immediately decreases by a big amount, then it increases at a slow rate and converge to a value lower than its initial value. Current utility increases at the time of the change of policy, then smoothly decreases and converges to value lower than its reference value.

In summary, the optimal level of quota is inversely related to the degree of capital externalities. In particular, no imports are optimal when the externality is high, whereas free
trade is the first-best policy for the small open economy without the capital externality. The latter result echoes the traditional wisdom on free trade for the small open economy.

3.3. Tied aid

In this section, we turn to the welfare effect of tied aid. For the following experiments, we raise the amount of $T$ from 0.01 to 0.02 and then examine its welfare implications under various capital externalities.

1. When the capital externality is large ($\beta_3 = 0.50$), we obtain the following changes in welfare for different values of the tying ratio in Table 1:

TABLE 1 ABOUT HERE

When aid increases without tie, capital will smoothly increase over time and converge to a higher value. The domestic price of imported good immediately increases, then it decreases at a slow rate and converge to a value lower than its initial value. Current utility decreases at the time of the change, then smoothly increases and converges to a value higher than its reference value.

These results are exactly opposite to those reached under a relaxation of the quota. Note that 0.7665 is the long run value of $\frac{\partial C_1/\partial T}{\partial T}$. When $\tau < 0.7665$, the effect of the increase of aid is stronger than the effect of the relaxation of the quota, and we get the same dynamics as when there is no tie to aid. When $\tau > 0.7665$, the effect of the relaxation of the quota dominates the effect of the increase in aid. So, the domestic prices of the importable good and fixed capital have dynamics opposite to the previous one. When $\tau = 0.7665$, fixed capital and the price of imports remain the same. Current utility immediately increases to its higher level.

Aid increases welfare, but the relaxation of the quota decreases it. For $\tau < 2.25$, the first effect dominates. However, for $\tau > 2.25$, an increase in tied aid decreases consumers’ welfare. Of course, the more tied the aid, the lower the increase in welfare (or the stronger its decrease). The threshold on $\tau$, after which tied aid decreases the welfare of the recipient country, decreases with the magnitude of the externality. For instance for $\beta_3 = 1$, the threshold is equal to 1.501.
2. When the externality is in the intermediate range \( (\beta_3 = 0.10) \), the optimal quota is positive \( (Q^* = 0.17Y) \) but it is lower than the reference quota \( (Q = 0.2Y) \). So, we expect to find results similar to those of the previous case, but with a slower decrease of welfare when the tying ratio \( \tau \) increases. The following result in Table 2 confirms this conjecture.

**TABLE 2 ABOUT HERE**

3. Finally, in the absence of externalities \( (\beta_3 = 0) \), the free-trade level of imports is optimal. Tied aid for relaxing the initial quota mitigates the distortion on imports of good \( Y \). Thus, welfare of consumers increases with the larger tying ratios. Table 3 provides the result on the change in welfare:

**TABLE 3 ABOUT HERE**

In the case when there is no tie, or more generally when \( \tau < 0.703 \), capital will smoothly increase over time when aid increases and converge to a higher value. The domestic price of imported good immediately increases, and then it decreases at a slow rate and converges to a value higher than its initial value. Current utility decreases at the time of the change, then smoothly increases and converges to a value higher than its reference value.

From the above experiments, we can conclude that for a small open economy with a quota restriction on imports, tied aid for importing more manufactured good can lower its domestic price and hence reduce the capital stock in a dynamic model. However, tied aid unambiguously improves welfare since the direct gain from the aid always outweighs the loss of capital if any.

4. **Concluding Remarks**

Using a dynamic simulation model, we have examined the welfare effect of tied aid for a small open economy with a quota restriction on imports. The economy considered receives aid which is tied to purchase the importable good from the donor country. To
highlight the effect of capital accumulation, we have added an externality from total capital into the production of the manufacturing sector. Capital accumulation is positively related to the price of the capital-intensive importable good. Our simulation results show that if the tie of aid is substantial, aid can lower this good price and thus reduce capital accumulation. When the capital externality is large, the optimal quota is zero. In this case, tied aid for relaxing quota will lower capital accumulation if the tying ratio is large enough. However, this capital decumulation effect is not large enough to outweigh the direct gain from aid. It is only for very high values of the tying ratio that tied aid will immiserize the recipient country. Further, when the capital externality is low, the optimal quota is large. Tied aid that relaxes the quota is always welfare improving.
Footnotes

1. See Bhagwati, et al. (1983) for discussions.


3. This production externality arising from the capital stock is employed, for example, in Liu and Turnovsky (2005).


5. This can be derived by maximizing steady-state utility $U = U(C_X, C_Y)$, subject to the budget constraint: $C_X + pC_Y = I$, where $I = wL + rK + R + (p - \dot{p}')(Q + \tau T) + p'T$. Solving the optimal conditions, we obtain: $\partial C_Y/\partial I = (U_{XY} - pU_{XX})/\Delta$, where $\Delta = 2pU_{XY} - p^2U_{XX} - U_{YY} > 0$. Note that $\partial I/\partial T = \dot{p} + \tau(p - p^*)$.

6. The important assumptions for the calibration are the relative sizes of both sectors and the difference between the domestic and the foreign price of the importable good. The amounts of import quota and foreign aid are also crucial.

7. The model was simulated and its eigenvalues computed with the software Dynare, which was run under MATLAB. Dynare was developed by Michel Juillard, and can be downloaded from the website: http://www.cepremap.cnrs.fr/dynare.

8. Changing the value of parameter $\beta_3$ does not alter the reference steady state, which is the initial state of the economy just before the change in the value of the quota.

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References


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Table 2. Medium Externality ($\beta_1 = 0.1$)

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Table 3. No Externality ($\beta_t = 0$)

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