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Self-esteem achievement through work and socio-demographic disparities in the labor market

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Self-esteem achievement through work, and socio-demographic disparities in the labor market

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Abstract

We develop a model in which agents choose whether to achieve self-esteem through work. When they do, they develop an intrinsic motivation to effort. Depending on the characteristics of the job to be filled, an employer may try, or not, to encourage this intrinsic motivation by an adequately designed contract. Although equally productive, assuming that agents from distinct socio-demographic groups differ in their propensity to achieve self-esteem through work, this may lead to unequal access to employment. We analyze the consequences of this model on labor market outcomes. The model can give an account of many important traits of socio-demographic disparities in the labor market (notably of vertical occupational segregation).

Keywords: Employment relation, self-esteem, intrinsic motivation, (seeming) hiring discrimination, occupational segregation, socio-demographic earnings gaps.


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1 Introduction

People are in search of self-esteem: some of their actions respond to the need to have an enhanced self-image. Here is the core result of social psychology that Akerlof and Kranton (2000) take up (and widely document) to motivate the introduction of identity into economic analysis.\(^1\) They show that taking this motivation into account allows a better understanding of some behaviors embedded into the social context, without departing from the individualistic paradigm. Employment relations are good example of the kind of social situation the understanding of which can be improved by such an approach.\(^2\) Indeed, it is quite sensible to deem that the exchange of labor for wages should not be reduced to a purely economic transaction. From a working person’s point of view, a job can embody much more than a simple source of income: it can be a significant channel for self-esteem.\(^3\) Our point is that this mere fact may shed light on some aspects of labor market outcomes.

More precisely, taking the need for self-esteem as a motivation for a working person’s behavior, this paper studies its consequences on the issue of socio-demographic disparities in the labor market. By these terms, we mean all phenomena reflecting differentiated individual experiences in the labor market, depending on non-productive features such as gender, race, or age. In particular, our point is about socio-demographic occupational segregation, and the gaps in average pay between socio-demographic groups.

In the following, we basically look at a Principal-Agent model in which we introduce self-esteem motives through identity building. Let us display the main characteristics of our approach in more detail. Our analysis of the employment relation comes within the framework of a standard Principal-Agent model with limited liability. We successively consider cases with complete information about effort (jobs whose monitoring is costless), and with moral hazard (jobs whose monitoring is not cost-effective). Indeed, it will be seen that moral hazard appreciably affects the conclusions of our analysis regarding labor market outcomes. Following Akerlof and Kranton (2000), we tackle issues of self-esteem through identity building. Let us recall the broad outlines of their modeling. Self-esteem derives from the assertion of an identity. Each agent

\(^1\)It is worth noting that reference to identity concerns is not such a recent trend in the economic literature. McCrate (1988) recalls Sen’s and Hirschman’s observation that people have tastes not just about external objects or other people, but also about themselves: in other words, about their identities. Identity is what these authors have called a "metapreference" or "value." McCrate insists that we do struggle regularly with ourselves over who we are and who we want to be: we have second order preferences, for instance, concerning such fundamental issues as manhood or womanhood.

\(^2\)For some accounts about the limits of standard analyses of employment relations, see Bewley (1999).

\(^3\)For a review of the sociopsychological experiments supporting this assertion, see Haslam (2001).
declares himself as belonging to some abstract social category. Possible categories are associated with different *ideal attributes* and *prescribed behaviors*. Exhibiting individual traits close to the ideal attributes associated with one’s category facilitates a sense of belonging (and hence access to self-esteem); following corresponding behavioral prescriptions affirms one’s self-image i.e. increases self-esteem, while violating them evokes anxiety and discomfort in oneself.

What are the trade-offs that feed our results? In our analysis, beyond their decision to expend effort, agents choose between achieving self-esteem through their job or through other activities outside the workplace. In terms of identity, they choose between a *workplace identity* and an *out-of-the-workplace identity*.\(^4\) When holding the workplace identity, agents have an *intrinsic motivation* to make an effort at work to the extent that it conditions their self-esteem (workplace identity involves an effort prescription). Employers have an obvious interest in this choice: an intrinsic motivation to make an effort may allow them to reduce the required extrinsic incentives. The identity decision of an agent is assumed to depend on the characteristics of the job offered by the principal but also on pay. Hence, the principal can influence the agent’s choice by offering wage amounts which meet the standards of the workplace identity (social status concern).

Yet, as suggested above, other factors condition an individual’s decision to achieve self-esteem through work: the distance from their personal traits to ideal attributes. Exhibiting particular non-productive traits may make the holding of the workplace identity more or less easy (comfortable). As a consequence, when choosing to encourage the workplace identity, the principal will target agents who exhibit traits that most easily fit into the workplace identity:\(^5\) a selective hiring\(^6\) will occur on this criterion.

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\(^4\)Among the four facts documented by Akerlof and Kranton (2000), we then mostly focus on two: 1) that people have identity-based payoffs derived from their own actions; 2) that some people may *choose* their identity. This latter point is carefully documented in their paper. Yet further a reference deserve attention. Surveying the findings of the Social Identity Theory, Ashforth and Mael (1989) mention studies asserting that an individual (consciously or not) identifies with a social category to enhance self-esteem. In her analysis of the domestic sexual division of labor, McCrate (1988) focuses on individuals’ choice of identity. She states that: "women [...] choose to *learn to prefer* mothering over auto mechanics [because] the expected payoff is higher."

\(^5\)Those whose characteristics are the closest to ideal attributes defining the workplace identity.

\(^6\)In the perspective of our model which put forward the role of work motivation, we will refer to the socio-demographic differences in hiring experiences as *selective hiring* (on socio-demographic criteria) rather than hiring discrimination. Indeed, from an economist’s point of view, hiring discrimination occurs when two individual with similar productive features do not have an equal chance to get a job as a result of differing socio-demographic belonging. Our approach suggests that such belonging can affect the productive features of workers: employer’s preference for some socio-demographic group over another is not, stricto sensu, discriminatory. We will thus refer to selective hiring (understood, on socio-demographic criteria).
Our main focus is therefore on issues of socio-demographic disparities in the labor market. However, to the extent that it is a key element of our contribution, we start with analysing how agents’ concerns about self-esteem affect the profitability of effort. Because job characteristics matter, the option for the principal to encourage the workplace identity may or not lead to some gains in the profitability of effort (compared with the standard case). We give a condition on job’s characteristics such that these gains are feasible for the employer. As regards selective hiring, this first result leads to conditions of their occurrence: these conditions involve in particular the degree of demands of the job under consideration. This is a first step towards a full and intuitive characterization of the set of jobs for which hiring might be selective. Once this characterization is available, it becomes possible to draw some conclusions about the earnings disparity between social groups. As far as jobs whose monitoring is costless are concerned, we show that the share of jobs for which hiring is selective is an increasing function of the wage standard under consideration. We then investigate the impact of moral hazard over previous results. While the set of jobs for which effort is induced obviously shrinks, one observes a stronger propensity from the principal to encourage the workplace identity. This has appreciable implications over the set of jobs for which hiring is selective as well as over the properties of the model regarding socio-demographic earnings disparities. The relation between the proportion of jobs for which hiring is selective, and the wage standard under consideration is no longer necessarily monotonic: under some circumstances, selection may be less likely in better paid jobs.

Akerlof and Kranton (2000) have already tackled the problem of occupational segregation stressing gender association with different types of work. This approach focuses on identity externality: a woman performing a "man’s job" provokes anxiety in her male co-workers. In the remaining, we do not assume this kind of externality, and develop arguments that go beyond gender association with different jobs. Akerlof and Kranton (2003) apply their model of identity to the analysis of work incentives. They consider workers who think of themselves either as part of the firm or as outsiders. When identifying with the firm, employees experience a loss in utility when not following its interests. So their main focus is on organizations’ ability to motivate their employees through identification. Our approach differs from theirs in two respects. First, we assume the organization is not able to change agents’ identity except through a change in its compensation schedule: aspects of corporate culture are not considered. Second, contrary to their rather radical approach to the identities available to workers (insider identity or outsider identity) which departs from strict individualism, we take up identities picked out
by contemporary psychologists which preserve the integrity of employees’ preferences.\footnote{In our approach, employees do not identify with the firm.}

We think of our contribution in two parts. The first is to provide a model of how self-esteem, as a motive for behavior, affects the employment relation: it leads us to focus on the role of job characteristics in the optimal designing of contracts. The second is to provide an alternative (or complementary) explanation to phenomena which challenge the dominant theories: seeming hiring discrimination and unequal earnings between socio-demographic groups. It is generally admitted that mainstream theories of discrimination do not do well in explaining lasting earnings disparities in the labor market. As Arrow (1998) states, if, as involved by most taste-based theories of discrimination, prejudiced employers make lower profits, competition should drive them out of the market. As regards statistical discrimination, it is often argued that, in the absence of real gaps in productivity between socio-demographic groups, recourse to such observables as race or sex in hiring decisions should disappear.\footnote{See Cain (1986).} In our model, employers fully observes workers’ productivity, and selective hiring goes with gains in profitability (therefore, our explanation should be competition-proof). As a theory of selective hiring, our model leads to a special kind of occupational segregation which provides a potential explanation of disparities between average earnings of different socio-demographic groups. Hence, it is consistent with the central evidence - see Blau and Kahn (2000), Holzer (1998) - that pervasive differences in occupational patterns are primarily responsible for persistent differences in earnings.\footnote{A more detailed discussion of our contribution is provided in section 6.}

This paper is organized as follows. Section 2 contains evidence gathered by psychologists about self-esteem achievement through work, and an informal exposition of our hypotheses regarding preferences. Section 3 displays our model of employment relations. Section 4 is devoted to the exposition of our results under complete information about effort, and Section 5 to the impact of moral hazard on these results. Section 6 provides a discussion of our contribution with regard to issues of hiring discrimination and unequal average earnings between socio-demographic groups, and concludes.

## 2 Psychological backgrounds

Identities are defined by a number of prototypical features abstracted from individuals.\footnote{For some references about the ideal-typical method, see Ashforth and Mael (1989).} From extensive analyses of typical ways of behaving and feeling in the working life, social psychology has gathered a sum of information, and reconstituted a set of identities which develop in the
workplace.\textsuperscript{11} In this section, among documented facts, we stress those that seem the most relevant from an economic perspective, and organize them to fit into the framework proposed by Akerlof and Kranton (2000). This leads us to define two identities: the workplace identity and the out-of-the-workplace identity.

2.1 Typical attitudes and feelings in the workplace

Industrial psychologists draw attention to workers who easily assert themselves within the organization.\textsuperscript{12} Such individuals are found to carry weight in the work group’s decisions. Their initial training is generally highly regarded, and the competences they claim recognized.\textsuperscript{13} Focusing on the topic of professional training, observers describe a particular zeal from this kind of worker for participation in training sessions that improve their mastery of the organization’s activities. These workers easily declare that their job is an important part of their life. Typical profiles are: professional workmen, employees whose promotion is based upon seniority, technical experts, executives or managers. In contrast, observers draw attention to individuals who hardly differentiate themselves in the work group: the latter generally have poor personal access to power, and little autonomy in the execution of job tasks. Psychologists stress that these individuals give their job a purely practical value insofar it allows them to benefit from material rewards.\textsuperscript{14} One can list typical profiles: young workers whose skills are considered as inappropriate, women (particularly mothers) employed in jobs considered as unimportant, recent immigrants or socially disadvantaged persons. More generally, all situations involving strong commitments outside the organization may predispose to such attitudes towards work.

Behind these observations lies Kanter (1977)’s argument that workers with few opportunities to advance at work tend to seek satisfaction outside work as a way of achieving a sense of efficacy and worth. Conversely, workers who have many opportunities in the job tend to consider work more central to their lives.

\textsuperscript{11}For a survey, see Haslam (2001).
\textsuperscript{12}See, for instance, the detailed observations of Sainsaulieu (1977).
\textsuperscript{13}See Dubar (1992).
\textsuperscript{14}Ashforth and Mael (1989) mention investigations in the field drawing the conclusion that "people working at menial jobs in a bank often distanced themselves from their implied identity (e.g., This is only a stopgap job; I’m trying to save enough to start my own business)."
2.2 Workplace or out-of-the-workplace identity: behavioral prescriptions and ideal attributes

Akerlof and Kranton (2000) refer to the fact that women can choose either to be a career woman or a housewife. On the bases of previous accounts, we would like to extend this perspective by defining two identities: the workplace and the out-of-the-workplace identity. Although clear-cut, Gecas and Seff (1990) show that this distinction was relevant (they regard work and home as two meaningful contexts of self-evaluation) and fruitful.

If an agent defines himself as a workplace identity holder, we will assume that he derives self-esteem from: the adherence of his actions to a prescription of effort (he must be zealous); the scope associated with his job; the social recognition this job brings him.\textsuperscript{15} The first two points deal with prescriptions defining the workplace identity. Thereby, an agent will comfortably claim this identity (and enjoy self-esteem through his job) when exerting a not too low level of effort at work, otherwise, he will feel some discomfort in himself.\textsuperscript{16} The scope of a job refers to the autonomy, self-direction, and personal access to power that come with this job.\textsuperscript{17}

The third point makes explicit the link between self-esteem and social status as emphasized by social psychologists.\textsuperscript{18} This justifies our assumption that an agent holding the workplace identity is susceptible to social recognition as revealed by good pay: if his wage is too far below some exogenous standard, the agent will feel discomfort as he will see it as a drop in social status.\textsuperscript{19}

This assumption is already current in the economic literature with different underlying justifications.\textsuperscript{20} As for the ideal attributes defining the workplace identity, following the insights of

\textsuperscript{15}In support of this assumption, Gecas and Seff (1990) found that when work was a central aspect of men’s self-concept, occupational variables (occupational prestige, control at work) were more strongly related to self-esteem than when they were not; similarly, when home was important, home variables (control and satisfaction at home) were strongly related to self-esteem.

\textsuperscript{16}Lobel and St. Clair (1992) show that individuals with salient career identities were willing to expend extra effort at work. Less specifically, they provide evidence on how identity salience motivates attitudes and behavior in support of an identity.

\textsuperscript{17}For some references about the "motivational" properties of the scope associated with a job, see Dodd and Gangster (1996) who give the main conclusions of the Job Characteristics Approach. For the link between scope at work and self-esteem, see Gecas and Seff (1990). Falk and Kosfeld (2004) provide some behavioral findings.

\textsuperscript{18}See Rosenberg and Pealin (1978) as a seminal reference or, again, Gecas and Seff (1990) who explore the link between social class and self-esteem.


\textsuperscript{20}As a seminal reference, see Akerlof and Yellen (1990). For some behavioral evidence supporting the relevance of relative payoff concerns see Clark and Oswald (1996).
social psychology, we can assume that they involve: education (experiencing self-esteem at work may require having an educational background which is seen as appropriate); age (it is harder to experience self-esteem through work when too young as one may be viewed as inexperienced or, when too old, to be out-of-date); gender (through stereotypes\textsuperscript{21}); strong out-of-the-organization commitments. Akerlof and Kranton (2000) suggest adding race to this list.

The self-esteem associated with the out-of-the-workplace identity is assumed not to depend on any features of one’s working life.\textsuperscript{22} There exists a huge variety of fields in which one can achieve self-esteem as well as a large variability in the amounts different individuals may experience. This heterogeneity will not be taken into account in the sequel, and we will take the self-esteem of an agent holding the out-of-the-workplace identity as a fixed exogenous. As far as ideal attributes are concerned, one will have to keep in mind that, in our dichotomic approach to social identities, the out-of-the-workplace identity is defined relative to the workplace identity. Hence, ideal attributes associated with the workplace identity can be regarded as negative, in terms of self-esteem, when considering the out-of-the-workplace identity and vice versa.

This was a statement of the evidence at the root of our analysis. We now formally state the corresponding assumptions.

3 Identity building, and the employment relation

In this section, we display the framework of our analysis.

3.1 Effort and production

Let us consider an agent (he) characterized by an exogenous parameter $\theta \in \{0, 1\}$ (for instance his gender or the color of his skin), and identifying with $c \in C$.\textsuperscript{23} He can exert an effort $e \in \{0, 1\}$. Exerting effort $e$ implies a disutility\textsuperscript{24} equal to $\psi(e)$ with normalisation $\psi(0) = 0$ and $\psi(1) = \psi > 0$. The utility of the agent is assumed to be separable between: the utility he derives from his wage, the disutility of his effort, and his neutral self-esteem, that is the personal gratification he derives from his job for a neutral 0 transfer - which is actually the reservation transfer. If he receives a transfer $w$ from the principal (she) and experiences the

\textsuperscript{21}Akerlof and Kranton (2000) focus on these stereotypes. Dubar (1992) asserts that: "the workplace identity is marked by male stereotypes just as the out-of-the-workplace identity is marked by female stereotypes".

\textsuperscript{22}See the findings of Gecas & Seff (1990) already mentioned.

\textsuperscript{23}The identity held by the agent is an endogenous of our model.

\textsuperscript{24}In the sequel, we will always take it as characterizing the job rather than as a subjective parameter.
neutral self-esteem $I_c(e; \theta)$, his global utility is given by

$$U_c(w, e; \theta) = u_c(w) - \psi(e) + I_c(e; \theta)$$

where $u_c(.)$ is an increasing function such that $u_c(0) = 0$. We clarify in what follows how self-esteem concerns may influence the utility derived from a given wage.

Production is stochastic, and the effort of the agent affects the production level as follows: the stochastic production level $\tilde{q}$ can only take two values $\{\underline{q}, \overline{q}\}$ with $\overline{q} - \underline{q} = \Delta q > 0$. We will denote $q = (\underline{q}, \overline{q})$. The stochastic influence of effort on production is characterized by the probabilities $\Pr(\tilde{q} = \overline{q} | e = 0) = \pi_0$ and $\Pr(\tilde{q} = \overline{q} | e = 1) = \pi_1$ such that $\pi_1 > \pi_0$. We will denote $\pi = (\pi_0, \pi_1)$, and $\Delta \pi = \pi_1 - \pi_0$.

### 3.2 Self-esteem and identity in the workplace

**Two identities.** The agent has the choice between two identities: $C = \{A, B\}$. Identity $A$ corresponds to the workplace identity while identity $B$ corresponds to the out-of-the-workplace identity. An agent considering himself as an $A$ extracts his self-esteem from: (a) the appropriateness of his trait $\theta$ to the ideal attribute defining $A$ (that we fix to 1), (b) the extent of his scope within the organization $\phi \in \mathbb{R}^+$,\(^{25}\) (c) the fact of complying his effort $e$ to the prescription defining category $A$ (that we also fix to 1), (d) the appropriateness of his wage to the exogenous standard $w_A$ prevailing among $A$ agents. As we said above, this latter assumption aims to capture the idea that social status - which we suppose to be revealed (at least partially) through the amount of $w$ - fuels self-esteem.\(^{26}\)

An agent whose identity is $B$ extracts his self-esteem from activities outside the organization. As a consequence, we will consider this level $I_B > 0$ as exogenous.

**The form of the agent’s preferences according to his identity.** Assuming the agent is risk-neutral, the material utility derived from a transfer $w$ will simply amount to $w$. This material utility is obviously a component of $u_c(w)$ whatever $c \in \{A, B\}$. However, it may not encompass the whole utility derived from a transfer $w$. Indeed, taking into account self-esteem concerns, we assume

\(^{25}\)In fact $\phi$ can include any characteristics of a job entering positively in the identity $A$ holders’ utility but not in that of $B$ holders.

\(^{26}\)For individuals holding the workplace identity, $w_A$ is what they proudly consider as the worth of their productive contribution. They experience the case $w < w_A$ as a negative public signal.
\[
\begin{align*}
\quad u_c (w) + I_c(e; \theta) &= \begin{cases} 
  w + \phi - \gamma_w (w_A - w) - \gamma_e (1 - e) - \gamma_\theta (1 - \theta) & \text{if } c = A \\
  w + I_B & \text{if } c = B
\end{cases}
\end{align*}
\]

where \( \gamma_w, \gamma_e, \) and \( \gamma_\theta \) are positive parameters. As a consequence, for all \( w > 0 \) : \( u_A (w) = (1 + \gamma_w) w > u_B (w) = w \) while

\[
I_A(e; \theta) = \phi - \gamma_w w_A - \gamma_e (1 - e) - \gamma_\theta (1 - \theta)
\]

which involves a perfect substitutability between the various ways to fit into the workplace identity.

What if the agent is an outsider? The reservation wage is fixed to 0 so that an outsider’s only source of utility consists in his self-esteem. It amounts to \( I_B > 0 \) for an identity \( B \) holder. The self-esteem of an outsider holding identity \( A \) amounts to \( -\gamma_w w_A - \gamma_e - \gamma_\theta (1 - \theta) < 0 \). Indeed, the agent is then deprived of the main factor making identity \( A \): a job.

We will denote \( \gamma = (\gamma_w, \gamma_e, \gamma_\theta) \) and refer to \( (I_B, w_A, \gamma) \) as an agent’s self-esteem concerns. Although it enters agents’ utility, \( \phi \) and \( \psi \) must be understood as objective measures characterizing a job rather than an agent. \( \phi \) stands for the scope attached to the job while \( \psi \) measures how demanding this job is. In the remaining sections, we will refer to the pair \( (\phi, \psi) \) as some job characteristics.

### 3.3 The contracting game

**Timing of decisions and information.** The timing of the contracting game is the following: 1) the agent and the principal learn the agent’s trait \( \theta \in \{0, 1\} \); 2) the principal offers a contract; 3) the agent accepts or refuses the contract, chooses his identity, and exerts an effort or not; 4) the outcome \( \tilde{q} \) is realized; 5) the contract is executed.

With moral hazard, the agent’s level of effort is not directly observable by the principal (a fortiori non-verifiable). The principal can only offer a contract based on verifiable variables. We assume identities are non-verifiable. Hence, with moral hazard, contracts are functions \( w(\tilde{q}, \theta) \) linking an agent with trait \( \theta \)’s compensation to the random output \( \tilde{q} \). With two possible outcomes \( \tilde{q} \) and \( \tilde{q}^* \), the contract can be defined, whatever \( \theta \), by a pair of transfers \( (w(\theta), \psi (\theta)) \).\(^{27}\)

\(^{27}\)Under complete information, since \( e \) is verifiable, it can be included into a contract enforced by a benevolent court of law. We will denote \( w_e (\theta) \) and \( \psi_e (\theta), e \in \{0, 1\} \), the transfers under complete information.
Principal’s set of actions, and payoffs under limited liability. The risk-neutral (with respect to transfers) principal’s expected utility is written as

\[ V_e = \pi_e \left( S(\tau) - w \right) + (1 - \pi_e) \left( S(q) - w \right) \quad \text{with } e \in \{0, 1\} \]

where \( S(.) \) is assumed to be a strictly increasing function. We denote \( \Delta S = S(\tau) - S(q) \). In the sequel, when talking about job technology, we will refer to the triplet \((\pi, q, S(.)\)) characterizing this job. If the principal does not induce the participation of the agent, we assume that she gets 0.

Note that the principal only pays attention to the identity adopted by the agent in as far as it may modify the expected transfer: she tries to encourage the identity that will make its holder exert the desired level of effort for the least (expected) cost. This aspect differentiates our approach from taste-based theories of discrimination.

The assumption that the agent’s liability is limited is written: \( \bar{w} \geq 0 \) and \( \underline{w} \geq 0 \).\(^{28}\) In the remaining, we will denote \( \mathbf{w} = (\underline{w}, \bar{w}) \).

Agent’s set of actions. Let \( a \) denote the agent’s answer to the contract \( \mathbf{w} \) offered by the principal: \( a \in \{\text{in, out}\} \), \( a = \text{out} \) meaning remaining an outsider, \( a = \text{in} \) meaning taking the offer and becoming an insider.\(^{29}\) An action of the agent is a vector \((a, c, e) \in A \) where\(^{30}\)

\[ A = \{(\text{out}, B, 0), (\text{out}, A, 0), (\text{in}, B, 0), (\text{in}, A, 0), (\text{in}, B, 1), (\text{in}, A, 1)\} \]

Given the agent’s payoff, it is straightforward to observe that strategy \((\text{out}, A, 0)\) is strictly dominated by \((\text{out}, B, 0)\) whatever \( \mathbf{w} \): an outsider will always hold identity \( B \) obtaining a utility \( I_B > 0 \).

Principal’s problem with moral hazard. Assuming that it is a best choice for the principal to induce effort \( e = 1 \), with obvious writings, her problem is written as

\[ \max_{\mathbf{w}} \pi_1 \left( S(\tau) - \bar{w} \right) + (1 - \pi_1) \left( S(q) - \underline{w} \right) \]

subject to

\[
\begin{aligned}
&EU_A(\mathbf{w}, 1; \theta) \geq EU_A(\mathbf{w}, 0; \theta) \quad (IC_A) \\
&EU_A(\mathbf{w}, 1; \theta) \geq EU_B(\mathbf{w}, 0; \theta) \quad (IC_{A/B}) \\
&EU_A(\mathbf{w}, 1; \theta) \geq I_B \quad (PC_A)
\end{aligned}
\]

\(^{28}\)Under complete information, limited liability states that \( \forall e \in \{0, 1\}, \bar{w}_e \geq 0 \) and \( \underline{w}_e \geq 0 \).

\(^{29}\)Do not confuse the "out-of-the-workplace" identity with the fact of being an outsider nor the "workplace" identity with the situation of being an insider.

\(^{30}\)For example, \((a, c, e) = (\text{in}, B, 0)\) stands for "accepting the contract, becoming a \( B \) without exerting effort".

11
OR

\[
\begin{align*}
EU_B(w, 1; \theta) & \geq EU_B(w, 0; \theta) \quad (IC_B) \\
EU_B(w, 1; \theta) & \geq EU_A(w, 0; \theta) \quad (IC_B/A) \\
EU_B(w, 1; \theta) & \geq I_B \quad (PC_B)
\end{align*}
\]

AND

\[w \geq 0 \quad (LL)\]

Among previous constraints, one will immediately recognize the standard moral incentive and participation constraints. The only supplement compared with the standard case comes from the necessity for the contract to meet a crossed incentive constraint. This latter constraint aims at preventing the agent from possibly changing his identity (and thereby his preferences) with the intention of exerting \(e = 0\). This requirement is particularly stringent when the principal is to maintain the workplace identity (A), and we will see that the corresponding constraint, denoted \((IC_{A/B})\), plays a crucial part in our results.

4 Profitability and selective hiring in jobs whose monitoring is costless (observable and verifiable effort)

This section is both a first step in our analysis, and a benchmark for the case with moral hazard. As a first step, it raises the question of the consequences of an agent’s caring about self-esteem over employment relations for jobs whose monitoring is costless.

Notation Let us denote \(\Delta I(\phi; \theta) = I_B - I_A(0; \theta) = I_B - \phi + \gamma w_A + \gamma e + \gamma \theta (1 - \theta) \leq 0\).

\(\Delta I\) is the relative (neutral) self-esteem of an identity B holder compared with that of an A exerting effort \(e = 0\). It is the relevant variable in all the results that follow. Indeed, as regards self-esteem concerns, \(\Delta I\) will capture the relative reservation utilities of the identities A and B facing the contract offered by the principal. The higher \(\Delta I\), the stronger A holder’s (relative) reservation, and the weaker B’s (relative) reservation.

In the sequel, as far as \(\Delta I\) is concerned, we will focus successively on the roles of \(\phi\) and \(\theta\).

4.1 Job characteristics, self-esteem concerns, and the profitability of effort

Optimal contracts. In the following claim we describe the equilibrium of the contracting game under complete information. We denote \(E_1 w^*_1\) the lowest expected transfer inducing \(e = 1\)

31This echoes our dichotomous approach to identity as far as working life is considered.
when effort is verifiable. It is useful to have in mind what prevails in the standard case: in the absence of a workplace identity, the lowest expected transfer ensuring effort \( e = 1 \) is \( \psi \).

**Claim 1** Let \((\phi, \psi)\) characterize a job (whose monitoring is costless) which the principal might like to be filled, and \((I_B, w_A, \gamma)\) an agent’s self-esteem concerns. Under complete information, with limited liability,

\[
E_1 w_1^* (\theta) = \begin{cases} 
\max \left\{ \frac{\psi - \gamma_e}{1 + \gamma_w}, 0 \right\} & \text{if } \Delta I (\phi; \theta) \leq 0 \\
\max \left\{ \frac{\psi - \gamma_e + \Delta I (\phi; \theta)}{1 + \gamma_w}, 0 \right\} & \text{if } 0 < \Delta I (\phi; \theta) \leq \gamma_w \psi + \gamma_e \\
\psi > 0 & \text{otherwise}
\end{cases}
\]

and effort \( e = 1 \) is induced if and only if \( E_1 w_1^* (\theta) \leq \Delta \pi \Delta S \). When effort is not induced by the principal \((e = 0)\), participation requires a transfer of 0, and she keeps inducing it if and only if \( E_0 S \geq 0 \). Otherwise, the job is left unfilled.

**Proof.** See the appendix.

Under complete information, the principal can punish the agent for exerting \( e = 0 \). However, the limited liability constraint prevents her from reducing transfers below 0. This implies that incentive constraints can be active, although effort is verifiable. To give an intuitive commentary on the previous claim, let us distinguish three types of jobs from the expression of the minimal transfers they require.

**Definitions** Given \((I_B, w_A, \gamma)\), an agent’s self-esteem concerns, a job will be said to be:

- **strongly fulfilling** if its characteristics \((\phi, \psi)\) are such that the crossed incentive constraint \((IC_{A/B})\) is relaxed in the optimum;
- **weakly fulfilling** if its characteristics \((\phi, \psi)\) are such that the crossed incentive constraint \((IC_{A/B})\) is binding in the optimum;
- **unfulfilling** if its characteristics \((\phi, \psi)\) are such that the crossed incentive constraint \((IC_{A/B})\) is violated in the optimum.

The more fulfilling a job, the lower the workplace identity \((A)\) relative reservation. We comment on the claim in terms of decreasing identity \(A\) relative reservation (decreasing \(\Delta I\)) starting from \(\Delta I > \gamma_w \psi + \gamma_e\). Jobs under consideration are then unfulfilling and it would require a relatively high compensation from the principal to induce the agent to develop an intrinsic motivation. Since these jobs are not that demanding, it is a best choice for her not to seek stimulating such added motivation i.e. to let the agent hold the out-of-the-workplace

\[32\text{ Assuming } \gamma_e < \psi, \text{ but also that it is profitable for the principal to induce effort } e = 1.\]
identity: the latter receives a full compensation for the "objective" disutility $\psi$ attached to the job. Such is no longer the case once the job becomes weakly fulfilling. Indeed, it is then demanding enough for it be profitable for the principal to stimulate intrinsic motivation. But this intrinsic motivation is paradoxically strongly dependent upon transfers: the self-esteem provided by the job mostly responds to the social status concerns it meets. When strongly fulfilling, beyond its compensation, the job is then appealing in itself, for the self-esteem its characteristics feed. Social status concerns are now dominated by "pure" intrinsic motivation responding to the (relatively) high scope the agent benefits from in his work.

**Motivation-based gains in profitability.** Here we would like to contrast the results of our model involving a workplace identity, with those of the standard model (in which agents can only hold identity $B$) in terms of *profitability*. It turns out that effort profitability is *not* necessarily improved by workplace self-esteem concerns. Recall that, in the standard model, effort $e = 1$ is induced if and only if $\psi \leq \Delta \pi \Delta S$.

**Implication 1** Self-esteem concerns extend the profitability of effort if and only if $\Delta I (\phi; \theta) < \gamma_w \Delta \pi \Delta S + \gamma_e$.

Figure 1 illustrates this implication.

These graphs give the threshold in the degree of demands over which it is no longer profitable for the principal to induce effort 1 (self-esteem concerns may extend effort profitability in the
sense that they may move this threshold to the right). Implication 1 says that employment relations profitability is constrained by the characteristics of the job which needs to be carried out. When the condition in implication 1 holds, \( e = 1 \) is induced for jobs whose "objective" disutility exceeds the expected added surplus which effort provides: that is what we mean when talking of extended profitability. When it does not, the principal renounces inducing \( e = 1 \) before it is profitable for her to encourage intrinsic motivation. The job under consideration is then definitely unfulfilling.

Our point was to show that beyond technologies, job characteristics and workers’ self-esteem concerns interplay in the determining of the profitability of employment relations. This comes from the potential stimulation of an intrinsic motivation. The question now is what if some agents are less sensitive than others to this stimulation?

4.2 Motivation-based gains in profitability and selective hiring in jobs whose monitoring is costless

Although we omitted its role in the previous step, motivation-based gains in profitability also depend on individual aspects through trait \( \theta \). Some individuals are better suited to the workplace identity than others (or, conversely, better suited to the out-of-the-workplace identity). As we have already stated: psycho-sociological analyses reveal that, for instance, being a woman, an old worker, having a depreciated qualification, etc. (within the framework of our model, having a \( \theta = 0 \)) predisposes to the out-of-the-workplace identity (identity \( B \)). In what follows, raising the question of selective hiring, we move gradually from the analysis of some particular employment relation to a model of labor market functioning that stresses job characteristics. We come to matters of earning disparities between socio-demographic groups through occupational segregation.

Suitability of agents to the workplace identity, and selective hiring. Here, it is assumed: that a principal faces a pool of agents only differentiated from each other by their trait \( \theta \in \{0, 1\} \) - \( (I_B, w_A, \gamma) \) is common to all the agents in the labour pool; that there is no shortage of workers of any trait. Technology \((\pi, q, S(.))\) is fixed so that we can focus on the role of job characteristics \((\phi, \psi)\) over selective hiring.

Because some individuals feel better suited to the out-of-the-workplace identity than to the workplace identity, they may be pushed aside by the principal: it all depends on the type of the available job. The next implication states conditions, for some particular job, that make it
prejudicial to exhibit trait $\theta = 0$. It also stresses the role of the degree of demands of jobs. Note that $\Delta I(\phi; 1) < \Delta I(\phi; 0)$.

**Implication 2** The relative ease with which agents hold identity $A$ or $B$ may or not, according to the job characteristics and technology, involve a selective hiring. More precisely,

- if $\Delta I(\phi; 1) \geq \gamma_e \Delta \pi \Delta S + \gamma_e$ or $\Delta I(\phi; 0) \leq 0$ then hiring is not selective whatever $\psi > 0$;

- if $\gamma_e \Delta \pi \Delta S + \gamma_e > \Delta I(\phi; 1)$ and $\Delta I(\phi; 0) > 0$: (i) hiring is selective for low and/or medium degrees of demand $\psi$; (ii) hiring stops being selective as degree of demands $\psi$ becomes high.

Hence, workers whose $\theta = 0$ may be crowded out by those whose $\theta = 1$ despite any apparent differences in terms of productivity. Some possible corresponding situations are depicted in figure 2.

Implication 2 provides a characterization of jobs for which hiring is selective. The underlying
argument is simple: it states that, according to job characteristics, agents exhibiting traits $\theta = 0$ or $\theta = 1$ can be perfect substitutes or not. Hiring is only selective if not and it has nothing to do with employer’s tastes as regards individual traits.

**Non-discrimination, and motivation-based gains in the profitability of effort.** Let us stress an important property of our model which figure 2 illustrates. Selection may be a requirement for the highest motivation-based gain in profitability.

**Implication 3** (i) for $\Delta I(\phi;0) \geq \gamma_e$, the highest motivation-based gain in profitability requires hiring selection; (ii) for $\gamma_e > \Delta I(\phi;0) > 0$, the highest motivation-based gain in profitability may require hiring selection or not, according to the job’s degree of demands; (iii) for $0 \geq \Delta I(\phi;0)$, the highest motivation-based gain in profitability does not involve any selection.

This latter implication highlights that, contrary to what holds for taste-based theories of discrimination, there could be an incompatibility between improving the profitability of effort, and avoiding socio-demographic selection. As a consequence, when fighting hiring discrimination, one should have in mind possible consequences in terms of profitability. In particular, quota policies are bound to be: ineffective as one seeks to reduce socio-demographic disparities (if firms are allowed to hire agents whose $\theta = 0$ in the type of job they want); source of loss in profitability (if the policy maker imposes the hiring of some agents whose $\theta = 0$ in jobs that are neither unfulfilling to $\theta = 1$ nor strongly fulfilling to $\theta = 0$).

We now turn to the analysis of some likely consequences of self-esteem concerns over the labor market as a whole.

**Self-esteem concerns and selective hiring in the labor market.** While agents (labor suppliers) are still assumed to be only differentiated from each other by $\theta$, we comprehend labor demand as segmented according to the characteristics of available jobs. For each technology $(\pi, q, S(,))$ and characteristics $(\psi, \phi)$, we assume there is a unique available job: employers are *monopsonists* on each segment of the labor market.$^{33}$

On this basis, it is trivial that when only the agent participation is required ($e = 0$) hiring will not be selective: indeed, in that case $E_0 w^*_0 (1) = E_0 w^*_0 (0) = 0$. We consider cases in which effort is induced in the next proposition.

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$^{33}$Beyond matters of simplicity, this assumption is made to neutralize the impact of competition over the distribution of workers between available jobs. Supporting the relevance of such an hypothesis, see Bhaskar, Manning, and To (2002).
Proposition 1 Consider a job for which it is profitable for the principal to induce effort \( e = 1 \). Then, hiring is selective if and only if this job is either weakly fulfilling to agents whose \( \theta = 1 \) or strongly fulfilling to them but not to those whose \( \theta = 0 \).

Proof. We show the contra-positive statement i.e. that hiring is not selective if and only if the job is either strongly fulfilling to agents whose \( \theta = 0 \) or unfulfilling to those whose \( \theta = 1 \). Consider a job for which hiring is not selective. It must be the case that the principal makes an equal profit when hiring an \( \theta = 1 \) or an \( \theta = 0 \). This is true when \( E_1 w_1 (0) = E_1 w_1 (1) \), that is, when the job in question is strongly fulfilling or unfulfilling both to an \( \theta = 1 \) and to an \( \theta = 0 \). Take a job which is strongly fulfilling (respectively unfulfilling) both to an \( \theta = 1 \) and to an \( \theta = 0 \). Then \( E_1 w_1 (0) = E_1 w_1 (1) = \psi - \frac{\gamma}{1+\gamma}w \) (respectively \( E_1 w_1 (0) = E_1 w_1 (1) = \psi \)) so that the principal makes an equal profit when hiring an \( \theta = 1 \) or an \( \theta = 0 \) and hiring is not selective.

This proposition tells us that the way workers view a given job conditions their chance of being hired. Indeed, on this perception depends their capacity to develop intrinsic motivation to effort: that is what employers care about! These comments lead to figure 3 which displays, for a given technology \((\pi, q, S(.))\), the set of jobs for which hiring is selective in the space \((\phi, \psi) \subseteq \mathbb{R}^2_+\).

Each point in this space represents a particular job, described as a couple (scope, degree of demands). Our model suggests that all the jobs are not equally likely to give rise to motivation-based selection. Selective hiring should be scarce for jobs such as, for instance, cashier or menial bank clerk: tasks are such that, whatever \( \theta \in \{0, 1\} \), intrinsic motivation hardly balances the need for extrinsic rewards. These cases correspond to the bottom left area of figure 3. In contrast, reporters, doctors or soldiers often view their occupation as missions to be completed rather than just as a way of earning a living. They generally enjoy wide scope and give their job a particular importance in their personal fulfilment. According to our model, motivation-based selection should not arise in this kind of job because of the strong intrinsic motivation that comes with them: so strong that it does not really matter to exhibit an \( \theta = 0 \) or an \( \theta = 1 \). These cases echo the area to the right of the figure. All other situations between the last two sets of cases refer to jobs that are either weakly fulfilling to agents whose \( \theta = 0 \) or to those whose \( \theta = 1 \). For these jobs, extrinsic and intrinsic motivations compete and \( \theta \) makes a difference to

\[ I_B + \gamma w_A - \Delta \pi \Delta S > 0 \quad \text{and} \quad \gamma_h < \gamma_e < \Delta \pi \Delta S. \]  

The latter assumption about parameters is not crucial as the shape of the set of selective jobs is considered. As for the first, the opposite would have implied a vertical cut in the set of selective jobs: since it does not dramatically affect the content of our analysis, we do not consider this case graphically.
the principal: she targets agents who should develop the strongest intrinsic motivation.

So far, we have mostly adopted the principal’s perspective, stressing the profitability of effort. What has our model to say about earnings within each socio-demographic group?

4.3 The potential gap in average earnings

Here, we question the impact of the occupational segregation to which our analysis leads on the average earnings of socio-demographic groups whose \( \theta = 0 \) and \( \theta = 1 \). In the absence of any assumption about the distribution of jobs in the space \((\phi, \psi)\) we cannot address the question of earnings differences nor make any prediction. Nevertheless, we would like to put forward some properties our model exhibits. To do this we introduce a measure of potential hiring selection.

The potential share of discriminating jobs. Let \( \lambda(E_1w) \in [0,1] \) denote the potential share of discriminating jobs among those of wage standard \( E_1w > 0 \). This share is "potential" to the extent that it is built upon the assumption that jobs are uniformly distributed over a closed subset \([0, \hat{\phi}] \times [0, \hat{\psi}]\) of \(\mathbb{R}^2_+\) with \( \hat{\phi} > I_B + \gamma_ww_A + \gamma_e + \gamma_\theta \) and \( \hat{\psi} > (1 + \gamma_w)\Delta\pi\Delta S + \gamma_e \), so that all possible situations are encompassed. These strong assumptions respond to our will
Figure 4: Iso-pay curve and the set of jobs for which discrimination occurs.

to display the structural implications of our model regarding earnings disparities between socio-demographic groups.

Proposition 2 Consider the set of jobs whose monitoring is costless. Then

- λ is increasing in $E_1w$;
- $0 < \lambda (E_1w) \leq \min \left\{ \lambda (\Delta \pi \Delta S), \lambda \left( \frac{I_B}{\gamma_w} + w_A \right) \right\}$.

Proof. On the next figure, we draw the iso-pay curve corresponding to $E_1w^* = E_1w > 0$ (the bold dotted broken line).

For $0 < E_1w \leq \Delta \pi \Delta S$, our measure of potential selection is simply

$$
\lambda = \frac{X_1X_2 + X_2X_3}{X_0X_1 + X_1X_2 + X_2X_3 + X_3X_4}
$$

Hence, for $0 < E_1w \leq \Delta \pi \Delta S$, the potential share of discriminating jobs is written

$$
\lambda (E_1w) = \begin{cases} 
\frac{(\gamma_w E_1w + \gamma_e)\sqrt{2}\gamma_e}{(\gamma_w E_1w + \gamma_e)(\sqrt{2} - 1) + \gamma_e} & \text{if } E_1w \leq \frac{I_B}{\gamma_w} + w_A \\
\frac{(I_B + \gamma_w w_A + \gamma_e)\sqrt{2}\gamma_e}{(I_B + \gamma_w w_A + \gamma_e)(\sqrt{2} - 1) + \gamma_e} & \text{if } E_1w > \frac{I_B}{\gamma_w} + w_A 
\end{cases}
$$

which involves the previous result.

Earnings disparity. The latter proposition states that the higher the wage standard, the more (potentially) likely it is that a (randomly drawn) job will involve selection between $\theta = 0$
and θ = 1. Hence, our model leads to a possible explanation of the gap in average earnings between socio-demographic groups that the evidence displays. The argument would be the following: the proportion of agents whose θ = 1 should be higher in well paid jobs than in poorly paid ones - at least under the assumption that there are (at least) as many θ = 0 and θ = 1 in the two remaining sets of jobs. As a consequence, when comparing the average earnings between socio-demographic groups, it is likely that it will be higher among θ = 1 than among θ = 0. This corresponds to the fact that the set of jobs for which hiring is selective includes more demanding jobs than the set of jobs that are unfulfilling both to θ = 0 and θ = 1.

**Comparative statics.** Let us start with the analysis of a set of jobs with common expected added surplus ΔπΔS. For ΔπΔS < \( \frac{I_B}{\gamma w} + w_A \), all other things being equal, an increase in ΔπΔS implies an extended salary range with λ higher in the top earnings: it is bound to widen the gap in average earnings between socio-demographic groups. Once ΔπΔS is over \( \frac{I_B}{\gamma w} + w_A \), while still extending the salary range, the effects in terms of unequal average pay of a rise in ΔπΔS are no longer amplified by an increased λ for top earnings. Hence, \( \frac{I_B}{\gamma w} + w_A \) should be comprehended as a boundary limiting the increase of the weight of agents whose trait is θ = 1 in top earnings when computing average pay by socio-demographic groups.

What if \( I_B \) or/and \( w_A \) rise? As one considers jobs whose technologies were such that, initially, \( \Delta \pi \Delta S < \frac{I_B}{\gamma w} + w_A \), neither the salary range nor the weight of θ = 1 in top earnings are affected. Such is not the case when considering jobs whose associated initial expected added surplus was below \( \frac{I_B}{\gamma w} + w_A \). Then, for any given \( E_1 w \) initially higher than \( \frac{I_B}{\gamma w} + w_A \), λ is increased: the weight of θ = 1 among well-paid jobs is increased. Hence, on the whole economy scale, the potential gap in pay between socio-demographic groups is widened by a rise in \( I_B \) or \( w_A \).

Therefore, our argument is based on the relative concentration of well paid jobs in the set of jobs for which hiring is selective. Notice that it does not involve any competitive mechanisms: by designing a measure of "potential selective hiring" we focus on a force that is inherent in our model (involving agents’ preferences). Besides, this mechanism may not operate since effective selective hiring eventually depends on assumptions over the actual distribution of jobs in the space (φ, ψ).

In this section, while giving the implications of self-esteem concerns over employment relations for jobs whose monitoring is costless as well as potential implications over labor market outcomes, we brought to light some forces operating whatever the observability of effort: we

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35See the discussion below.
will see that most of the previous results hold when effort is not observable. Let us nevertheless turn to the problem with moral hazard, and question matters of discrimination and profitability for jobs whose monitoring is not cost-effective.

5 Profitability, and discrimination for jobs whose monitoring is not cost-effective (non-verifiable effort)

5.1 Self-esteem concerns, and optimal contracts with moral hazard

As a preamble, recall that, as holds under complete information, the contract \( w = 0 \) is necessary and sufficient to induce the participation of a non-zealous agent (agent exerting \( e = 0 \)) with moral hazard. In the next claim, we describe the equilibrium of the contracting game with moral hazard. It will be seen that \( \Delta I \), the relative reservation utility of identities \( A \) and \( B \), keeps playing a crucial role. We denote \( w^* \) the contract minimizing the expected transfer while inducing effort \( e = 1 \) with moral hazard, and \( E_1 w^* \) the corresponding expected transfer.

Claim 2 Let \( (\phi, \psi) \) characterize a job (whose monitoring is not cost-effective) which the principal might like to be carried out, and \( (I_B, w_A, \gamma) \) an agent’s self-esteem concerns. With moral hazard and limited liability, the contract minimizing expected transfer while inducing effort is written

\[
w^*_1 = \begin{cases} 
0, & \text{if } \Delta I(\phi; \theta) \leq \frac{\gamma_w}{1 + \gamma_w} \Delta \pi (\psi - \gamma_e) \\
\max \left\{ \frac{\psi - \gamma_e + \Delta I(\phi; \theta)}{(1 + \gamma_w) \Delta \pi}; 0 \right\}, & \text{if } \frac{\gamma_w}{1 + \gamma_w} \Delta \pi (\psi - \gamma_e) < \Delta I(\phi; \theta) \leq \gamma_w \frac{\Delta \pi}{1 + \gamma_w} \psi + \gamma_e \\
0, & \text{otherwise}
\end{cases}
\]

and effort \( e = 1 \) is induced if and only if \( E_1 w^* \leq \Delta \pi \Delta S \). When effort is not induced by the principal \( (e = 0) \), participation requires a transfer of 0, and she keeps inducing it if and only if \( E_0 S \geq 0 \). Otherwise, the job is left unfilled.

Proof. See the appendix. ■

With moral hazard, the principal can no longer punish a shirking agent: the contract is only contingent upon the realization of \( \tilde{q} \). Hence, inducing effort \( e = 1 \) requires making the gap between the expected payoffs for a zealous agent and a shirker as large as possible.

In the following, we will focus on the comparison with what we obtained for jobs whose monitoring is costless as well as with the standard case (absence of a workplace identity). To make clearer the connection to our previous results, let us make explicit the expected transfers...
corresponding to the contracts of the latter claim:

\[ E_1^w = \begin{cases} 
\max \left\{ \frac{\psi - \gamma_e}{\Delta \pi} \right\} & \text{if } \Delta I (\phi; \theta) \leq \frac{\gamma_e}{1 + \gamma_w} \frac{\pi_0}{\Delta \pi} (\psi - \gamma_e) \\
\max \left\{ \frac{\psi}{\Delta \pi} \frac{(1 + \gamma_w) \Delta \pi}{(1 + \gamma_w) \pi_1} \right\} & \text{if } \frac{\gamma_e}{1 + \gamma_w} \frac{\pi_0}{\Delta \pi} (\psi - \gamma_e) < \Delta I (\phi; \theta) \leq \gamma_e \frac{\pi_1}{\Delta \pi} (\psi - \gamma_e) \\
\frac{\pi_1}{\Delta \pi} & \text{otherwise}
\end{cases} \]

In this form, the connection to the standard case may seem clear. As one considers strongly fulfilling or unfulfilling jobs, the impact of the unobservability of effort is exactly what one usually obtains: from what agents get under complete information, required transfers rise by a factor \( \frac{\pi_1}{\Delta \pi} > 1 \) which corresponds to standard limited liability rent. This is not the case for weakly fulfilling jobs for which a factor \( \frac{(1 + \gamma_w) \Delta \pi}{(1 + \gamma_w) \pi_1} < 1 \) emerges that curbs the impact of the unobservability of effort. This difference echoes the fact that only for weakly fulfilling jobs (by definition) is the crossed incentive constraint binding: but (as we will see in detail in the sequel) the unobservability of effort induces a relative relaxing of (\( IC_A/B \)) compared to (\( IC_B/A \)) which curbs the increase of required expected transfer.

In fact, things are not that simple. Indeed, in the previous interpretation, we considered jobs that kept the same type under complete and incomplete information about effort: this may not be the case as we will see below.\(^\text{36}\)

As for the implications of the latter claim, the forces we described under complete information still operate. As a result, many differences from the previous analysis are only quantitative, leaving our generic results unchanged. One can check that this is true regarding implication 1 in particular. This results from the fact that moral hazard does not affect an agent’s self-esteem concerns. Hence, the wage threshold over which the agent prefers to hold the workplace identity is the same whether effort is observable or not.

But moral hazard also leads to qualitative differences from the case of jobs whose monitoring is costless. Since with moral hazard, matters of hiring discrimination involve both quantitative and qualitative differences, we postpone analyzing them. First, we would like to stress the qualitative differences from what we obtained for costlessly monitored jobs: they are induced by the fact that moral hazard may change the type of a job despite fixed characteristics.

### 5.2 Fulfilling and unfulfilling jobs with moral hazard

Formally, the main differences come from the fact that, with moral hazard, the degree of demands \( \psi \) enters the condition that defines a job as strongly fulfilling: for \( \psi > \gamma_e \), a job can be strongly

\(^{36}\)The analysis of the impact of the unobservability of effort in terms of efficiency is available upon request.
fulfilling although $\Delta I > 0 \iff I_B > I_A (0; \theta)$. The recognition of one’s workplace identity through $E_0 > 0$ leaves an $A$ shirker relatively better off with moral hazard than under complete information about effort.

**Proposition 3** Moral hazard extends the class of fulfilling jobs.

**Proof.** Consider the technology $(\pi, q, S(\cdot))$ of a job whose characteristics are given by $(\phi, \psi)$, and an agent’s self-esteem concerns $(I_B, w_A, \gamma)$ such that $\Delta I = \gamma_w \psi + \gamma_e + \varepsilon$ with $0 < \varepsilon < \gamma_w \frac{\pi_0}{\Delta \pi} \psi$. Since $\Delta I > \gamma_w \psi + \gamma_e$, the job belongs to the class of unfulfilling jobs under complete information while since $\Delta I < \gamma_w \psi + \gamma_e + \gamma_w \frac{\pi_0}{\Delta \pi} \psi = \gamma_w \frac{\pi_1}{\Delta \pi} \psi + \gamma_e$ it belongs to the class of fulfilling jobs with moral hazard.

Furthermore, if a job is fulfilling under complete information then it is also fulfilling with moral hazard. Suppose it does not hold. Then, there would exist a technology $(\pi, q, S(\cdot))$, job characteristics $(\phi, \psi)$, and an agent’s self-esteem concerns $(I_B, w_A, \gamma)$ such that

$$\Delta I \leq \gamma_w \psi + \gamma_e$$ and $$\Delta I > \gamma_w \frac{\pi_1}{\Delta \pi} \psi + \gamma_e$$

which is impossible since $\pi_1 > \pi_0 \geq 0$.

The next figure illustrates the latter proposition.

Note that for $\Delta I \in [\gamma_w \psi + \gamma_e; \gamma_w \frac{\pi_1}{\Delta \pi} \psi + \gamma_e]$, an unfulfilling job under complete information becomes a fulfilling one with moral hazard. This is an important point for the remaining section.

Proposition 3 suggests that moral hazard tends to make employers "enrich" (in fulfillment capacity) the jobs they offer, that is to extend recourse to intrinsic motivation. What forces support this consequence of moral hazard? The idea is the following. Moral hazard allows the agent to benefit from a rent: whatever the identity that the principal finally encourages, she will

24
have to concede this rent. Therefore, we are dealing with better-paid jobs (for a given degree of demands) as moral hazard holds. Principals are then closer to the wage threshold making it profitable to induce intrinsic motivation (encourage the workplace identity).\textsuperscript{37} In fact, the extension of the class of fulfilling jobs is an echo of the shrinking of the class of jobs for which effort $e = 1$ is induced (through the limited liability rent).

5.3 Self-esteem concerns, and hiring discrimination for jobs whose monitoring is not cost effective

How does moral hazard change matters of hiring discrimination?

**Qualitative differences to the case with complete information about effort.** With figure 5, we illustrate the role of $\theta$ directly in the case $\frac{\pi_1 - \psi}{\pi_0} \gamma_w < \Delta \pi \Delta S$.\textsuperscript{38} The next three graphs reveal that conditions over $\Delta I(\phi; 0)$ and $\Delta I(\phi; 1)$ such that hiring is selective for some values of $\psi$ are exactly what we obtained under complete information.

But still, these graphs also complement the four configurations we analysed previously.

Let us first focus on what remains unchanged. As we were saying, implication 2 (condition of selective hiring for some $\psi$) and proposition 1 are still relevant for jobs whose monitoring is not cost-effective. This directly derives from the fact that implication 1 still holds with moral hazard (that moral hazard does not affect agents’ self-esteem concerns). Besides, the content of the implication 3 stressing the possible incompatibility between non-discrimination and profitability remains unaffected by the unobservability of effort.

The differences come from the fact that, for large enough $\psi$, the principal can no longer content herself with binding the crossed incentive constraint ($IC_{A/B}$): she meets the standard incentive constraint. In other words, as the degree of demands increases, the job turns from a weakly fulfilling into a strongly fulfilling one. The intuition follows. By considering more demanding jobs, we consider higher wage standards. We eventually exceed the wage threshold

\textsuperscript{37}To put it in more detail, we saw that the caring of identity $A$ holders about the meaning of their wage (social status) leads to a possible extra-valuation of a given wage (through parameter $\gamma_w$). To clarify the source of the latter result, this must be related to the fact that, with moral hazard, the expected transfer of a shirker is strictly positive - which was not the case under complete information. Hence, whereas the crossed incentive constraint ($IC_{A/B}$) corresponding increase is curbed by the extra-valuation of $E_{1w}$, ($IC_{B/A}$) corresponding increase is amplified by this extra-valuation (which plays over $E_{0w}$): ($IC_{B/A}$) becomes relatively more restrictive than ($IC_{A/B}$).

\textsuperscript{38}For $\frac{\pi_1 - \psi}{\pi_0} \gamma_w \geq \Delta \pi \Delta S$ graphical analysis only quantitatively differs from the corresponding under complete information.
resulting from moral hazard. As for the latter, two facts are illustrated: some jobs that were
strenuous of the extrinsic motivation to e
blunts the intrinsic motivation linked to the workplace identity, from which results the necessary
a relative weakening of the e
The set of jobs for which hiring is selective.

As far as our model properties are concerned, as the left and middle figures show, selective hiring may disappear although the principal keeps implementing action \((in, 1, A)\), as the degree of demands is increased. Indeed, as we noted above, the degree of demands \(\psi\) enters the condition that changes a weakly fulfilling job into a strongly fulfilling one: once the degree of demands is high enough so that the job is strongly fulfilling for agents whose \(\theta = 0\), selective hiring no longer occurs. As far as selective hiring is considered, this new mechanism leads to properties that depart from what we obtained for jobs whose monitoring is costless.

The set of jobs for which hiring is selective. In figure 6, as we did under complete information, we depict the set of jobs for which hiring is selective in the space \((\phi, \psi)\). The dotted polygon depicts the corresponding set when effort is observable.

This figure both illustrates the shrinking of the set of jobs for which effort \(e = 1\) is induced (the standard loss in efficiency), and the distortion of the set of jobs for which hiring is selective resulting from moral hazard. As for the latter, two facts are illustrated: some jobs that were
unfulfilling under complete information become weakly fulfilling to $\theta = 1$ (and enter the set of jobs for which hiring is selective) with moral hazard; some jobs that were weakly fulfilling under complete information become strongly fulfilling (in particular to agents whose $\theta = 0$) with moral hazard (and then exit the set of jobs for which hiring is selective). The intuition for the first fact is that of proposition 3: for a given degree of demands, the rent conceded by the principal to the agent involves higher pay; thus, when effort is induced, compensation is closer to $w_A$, and the workplace identity is encouraged for lower scope with moral hazard. As for the second fact, it echoes the same logic, to which is added the renewed need for extrinsic motivations as the workplace identity becomes more comfortable (high scope, and adequate pay).

Let us examine the consequences of moral hazard upon the potential gap in average earnings.

5.4 Moral hazard, and the potential gap in average earnings

Let $\lambda^{MH} (E_1 w) \in [0, 1]$ denote the potential\(^{39}\) share of discriminating jobs among those of wage standard $E_1 w$ which involve moral hazard.

**Proposition 4** Consider the set of jobs whose monitoring is not cost effective. Then,

\(^{39}\)The word "potential" involving the same set of restrictions as above.
require the arousing of intrinsic motivation which feeds selective hiring. For information: higher wages correspond to more demanding jobs; the latter are more likely to equal (in particular for a given expected transfer $E_1 w$ over $\left[0, \frac{I_0}{\gamma_w} + w_A\right]$; strictly decreasing in $E_1 w$ over $\left[\frac{I_0}{\gamma_w} + w_A, \Delta \pi \Delta S\right]$).

Proof. For $0 < E_1 w \leq \Delta \pi \Delta S$, the potential share of jobs for which hiring is selective is written:

$$\lambda^{MH} \left( E_1 w; \frac{\pi_1}{\pi_0} \right) = \begin{cases} 
\left( \gamma_w \left( \frac{1-\theta}{\phi} \right) E_1 w + \gamma_e \right) \sqrt{\pi_1 + \gamma_e} 
& \text{if } E_1 w \leq \frac{I_0}{\gamma_w} + w_A \\
\left( I_B + \gamma_w w_A + \gamma_e - \gamma_w \frac{\pi_0}{\pi_1} E_1 w \right) \sqrt{\pi_1 + \gamma_e} 
& \text{if } E_1 w > \frac{I_0}{\gamma_w} + w_A 
\end{cases}$$

which involves our claim. ■

Let us comment on the first item of the latter proposition. It states that, all other things being equal (in particular for a given expected transfer $E_1 w$), the potential share of discriminating jobs is lower with moral hazard than under complete information about effort. Indeed, with moral hazard, $E_1 w$ comprehends a (strictly positive) limited liability rent, which is not the case under complete information. Thus, a given $E_1 w > 0$ corresponds to less demanding jobs with moral hazard than under complete information. But selective hiring is all the more likely when more demanding jobs are considered so that $\lambda^{MH} < \lambda$.

Earnings disparity. As regards the class of jobs whose technology is such that $\Delta \pi \Delta S \leq \frac{I_0}{\gamma_w} + w_A$, $\lambda^{MH}$ is strictly increasing in $E_1 w$ which reinforces what we obtained under complete information: higher wages correspond to more demanding jobs; the latter are more likely to require the arousing of intrinsic motivation which feeds selective hiring. For $\frac{I_0}{\gamma_w} + w_A < \Delta \pi \Delta S$, $\lambda^{MH}$ rises in $E_1 w$ up to $\frac{I_0}{\gamma_w} + w_A$, it is then strictly decreasing in $E_1 w$. This results from the expansion of the class of jobs that are strongly fulfilling both to $\theta = 0$ and $\theta = 1$ as $E_1 w$ rise: for a given scope $\phi$, jobs which were weakly fulfilling to $\theta = 0$ for low levels of $E_1 w$ (of $\psi$) become strongly fulfilling for higher levels of $E_1 w$ (of $\psi$). Therefore, as we consider the class of well-paid jobs for which effort brings high expected benefits, the potential share of jobs for which hiring is selective may decrease. This implies that the over-representation of $\theta = 1$ in the better-paid jobs should be reduced, curbing unequal average earnings between groups. Hence, it is not within this class of jobs that we should witness the widest gap between socio-demographic groups.

Comparative statics. As for technological aspects, it is desirable to distinguish the stochastic productivity of effort $\Delta \pi$, from the non-stochastic productivity of effort $\Delta S$. Indeed, contrary

\[40\] Furthermore, $\lim_{\gamma_0 \to +\infty} \lambda^{MH} \left( E_1 w; \frac{1}{\gamma_0} \right) = \lambda (E_1 w)$. 

28
to what prevailed under complete information, the consequences of a change in the productivity of effort are not the same, whether it involves a change in $\Delta \pi$ or in $\Delta S$. The consequences of a change in the latter are broadly similar to those of a change in $\Delta \pi \Delta S$ under complete information: mainly a change in the extension of the salary range. With moral hazard, to the extent that a change in $\Delta \pi$ is also a change in $\frac{\Delta \pi}{\Delta S}$, it results in different effects. Previous expressions of $\lambda^{MH}$ imply that, whatever $E_1 w \in [0, \Delta \pi \Delta S]$, whatever the relative worth of $\Delta \pi \Delta S$ and $\frac{I_B}{\gamma w} + w_A$, a gain in $\pi_1$ (given $\pi_0$) increases $\lambda^{MH}$. Yet, this is not the only consequence of an increase in $\Delta \pi$.

The next figure depicts a numerical illustration. We draw the potential share of jobs for which hiring is selective for two technologies: the bold curve corresponds to a stochastic productivity of effort which is higher than that corresponding to the thin curve. The dotted curve represents the same measure under complete information.

Potential share of jobs for which discrimination occurs for two technologies under complete information or with moral hazard.

Numericals are such that $\Delta \pi \Delta S > \frac{I_B}{\gamma w} + w_A$. As mentioned above, we see that $\lambda^{MH}$ is higher for all wage standards below the initial $\Delta \pi \Delta S$, which suggests a widened average pay gap between $\theta = 0$ and $\theta = 1$. The ambiguity comes from the fact that the extended salary range goes with lower potential selection in top earnings.

We now provide a discussion of our results, relating them both to available theories and to available evidence about disparities in the labor market.

---

41Self-esteem concerns are $(I_B, w_A, \gamma) = \left(\frac{1}{2}, 1, \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\right)$, the non-stochastic productivity of effort $\Delta S = 30$, and the technological shock consists in a move from $\pi = \left(\frac{1}{2}, \frac{3}{4}\right)$ to $\left(\frac{1}{2}, \frac{3}{4}\right)$. We further take $\phi = \frac{1}{2}$.

42The next section is a short version of Baguelin (2004) which provides a more comprehensive discussion.
6 Discussion and conclusion

Most available theoretical works addressing the problem of socio-demographic disparities in the labor market treat various aspects of these disparities in isolation. Yet empirical studies rather support global approaches.

6.1 Accounting for vertical occupational segregation

The major features of disparities between socio-demographic groups in the labor market are hiring discrimination and occupational segregation. The distribution of employment by occupation or sector is still very much gender-segmented. Similar evidence exists for racial differences. The interesting thing is that occupation segregation tends to be vertical: this is both a documented micro reality (see Neumark (1996) and Bertrand and Mullainathan (2003)) and a statistical fact. These findings make an indirect analysis of statistical wage disparities look particularly justified: the idea is that the most significant channel to explain average earnings disparities lies in vertical occupational segregation rather than in pure wage discrimination.

Occupational segregation might result from more severe employer discrimination in one occupation than in others. Although both statistical discrimination and taste-based theories can predict horizontal segregation, they can hardly say where it should arise. The story involving prejudiced co-workers is of particular interest as regards vertical occupational segregation. It explains the "glass ceiling" impeding women’s (or blacks’) occupational advancement by assuming that men (or whites) do not accept being ordered about by women (or blacks). But vertical

---


44 Occupational segregation is said to be horizontal when it involves a segregated distribution of socio-demographic groups among jobs that correspond to a given earnings standard. It is said to be vertical when jobs under consideration differ with respect to an earnings standard.

45 Neumark (1996) studies sex discrimination in restaurant hiring. He finds that in high-priced restaurants (where waitpersons’ earnings are higher), job applications from women have an estimated probability of receiving a job offer significantly lower than those from men. A key contribution of Neumark (1996) is to document micro evidence of vertical occupational segregation by gender. In a single industry (catering), he distinguishes two statuses: waitperson in high-priced restaurants, waitperson in low-priced restaurants. The interesting thing is that vertical occupational segregation arises, with a majority of men working in high-priced restaurants (which pay well), and a majority of women working in low-priced restaurant (which pay poorly). Neumark mentions studies which conduct comparable tests for racial discrimination: it turns out that discrimination against blacks exists in high-priced restaurants.

46 Bertrand and Mullainathan (2003) conduct a global study of racial discrimination in hiring. Manipulating the perception of race (in otherwise similar resumés) by using distinctively ethnic names, they show that "callback" rates are significantly lower for distinctively black-named applicants.
occupational segregation does not necessarily involve hierarchical aspects, as Neumark (1996) shows.

One can also account for occupational segregation without mobilizing hiring discrimination. A first possibility for this perspective states that group differences in pre-labor market human capital investment and in non-labor market activities may lead to differences in comparative advantage across occupations. This can account for both horizontal and vertical occupational segregation. Yet the nature of the gender and racial differing comparative advantage across occupations remains unspecified. Altonji and Blank (1999, p.3176) mention another possible explanation: that members of different groups select into different occupations, notably because social norms regarding appropriate occupations may differ between groups. What is more, preferences for the characteristics of occupations may differ between groups, particularly men and women. But again the very nature of these differing preferences are not specified. As for gender differences, Corcoran and Courant (1985) provide some hypotheses about how sex role socialization might affect labor market outcomes. They mention four ways through which socialization might affect occupational behavior. Among them are two human capital arguments: that socialization may lead women to be more fearful or more anxious, or less confident than men; that sex role socialization may directly affect workers’ skills and personality traits. But they also mention two "taste" explanations: that children may internalize traditional notions of sex roles, accept these cultural sex stereotypes as fact, and eventually choose occupations that conform to these stereotypes; that sex role socialization may affect the values men and women attach to different activities so that workers of both sexes tend to value "sex appropriate" activities. In fact, comparable arguments could be invoked as regards racial differences as suggested in Akerlof and Kranton (2000). We believe our argument consistently connects with these latter intuitions.

6.2 A motivation-based theory of selective hiring which generates statistical earnings disparities

In our analysis, agents decide whether to achieve self-esteem through their job or through other activities outside their working life. Certain individual traits restrict this choice since the comfortable holding of the workplace identity requires the agent to fit in with some ideal attributes. According to the field studies we mentioned above, the ideal attributes when one holds the workplace identity are to be a white middle-aged man with a considered-as-proper initial education, devoid of strong commitments outside one’s working life. As a consequence, all other things
being equal, agents exhibiting traits which match this portrait should choose the workplace identity (and hence, develop intrinsic motivation to effort) for lower wage amounts than others. If the offered job characteristics make it profitable to encourage the workplace identity, employers will hire the former first (at the expense of the others). It is noteworthy that in our model, selective hiring is not independent of technological or organizational aspects (there is no arbitrary behavior from employers): the characteristics of jobs under consideration determine how likely selective hiring is and, consequently, occupational segregation should reflect differences in job characteristics.

From the perspective of our model, the basic interpretation of Bertrand and Mullainathan’s (2003) findings would be that being black moves an individual’s traits further from the ideal attributes associated with the workplace identity. Assuming a particular concentration of jobs whose characteristics make them at most (or at least) weakly fulfilling to a black (or to a white), whites are expected to develop stronger intrinsic motivation so that it is rational of employers to favor their applications. Our interpretation of Neumark’s conclusions would suggest that catering occupations do not use the same job characteristics, whether one considers low-priced restaurants or high-priced ones. Working as a waitperson in the latter brings wider scope but is likely to be more demanding than in low-priced restaurants insofar as the quality of the meal service is then crucial (higher price often corresponds to higher demands for service quality): catering jobs in luxury restaurants are presumed to be at least weakly fulfilling to a man but at most weakly fulfilling to a woman. The higher capacity of men to develop intrinsic motivation as waiters in establishments where meal service is formal encourages managers to give them an advantage over women.

From the building of the set of jobs for which hiring is selective within the space (scope, degree of demands), we give some potential consequences of the particular occupational segregation we obtained, in terms of unequal earnings between socio-demographic groups. The gap in average earnings (favorable to agents who fit in) may be a consequence of the fact that the potential share of jobs for which hiring is selective increases as expected pay increases: selective hiring is more likely in the class of well-paid jobs than in the class of poorly paid ones. Why is it so? Because pay increases according to the degree of demands, and the more demanding a job, the stronger the propensity of employers to try arousing intrinsic motivations (i.e. the workplace identity): it is precisely on that ground that selection takes place in our analysis. All things considered, our explanation of earnings disparities (as a macro statistical fact) is very simple: women and blacks earn less than white men because they are relatively more concentrated into
less demanding occupations.

An important aspect of socio-demographic earnings disparities is that they are lasting.\textsuperscript{47} Hence the question: how lasting are the gaps in average earnings our model generates? To be long lasting, selection should increase profit or non-discrimination be costly. This is precisely the case in our model. We obtain an unambiguous increase in profits associated with selection when it takes place. Moreover, our argument for this result seems more cross-occupational than existing alternatives allowing higher profits to discriminating employers, which is consistent with Mullainathan and Bertrand’s (2003) findings showing that the amount of (seeming) discrimination looks uniform across occupations and industries. What matter from a motivation-based perspective are the job characteristics (whether or not these characteristics make it profitable for the employer to encourage the workplace identity). The class of jobs for which hiring is selective is likely to be uniformly distributed across industries and we see no reason supporting the assumption that such jobs should disappear in the long run.

6.3 Concluding remarks

Although motivational aspects are sometimes invoked in the literature about the gaps in earnings between socio-demographic groups, few theoretical works have shed light on the problem. Our analysis suggests that, for some jobs whose characteristics are specified, black or female workers\textsuperscript{48} could manifest lower motivation at work than white men as a consequence of diverging strategies of identity building. This is the core insight of the present analysis. We derived from this micro analysis consequences as regards labor market outcomes, suggesting that earnings gaps between socio-demographic groups could correspond to the fact that the share of jobs for which hiring is selective was increasing with the earnings standard considered.

As regards policy implications, we would argue that our model suggests two ways to homogenize the opportunities in the labor market. The first is to shape jobs so that they become unfulfilling to members of the majority group: this corresponds to an economy with a very high level of labor division, leaving individuals with little scope at work. Although hiring selection would then disappear, economic efficiency would be severely compromised since intrinsic motivations that individuals could develop in the workplace would never be encouraged. The alternative way is obviously the better. It advises shaping as many jobs as possible so that

\textsuperscript{47}See Arrow (1998).

\textsuperscript{48}We did not focus on this above, but our approach applies to the issues of age discrimination - Lobel and St. Clair (1992) report finding that age had a negative effect on merit increases, despite having no significant effect on effort.
they be strongly fulfilling to members of the minority group. This would lead both to a gain in fairness and to more profit.

Our approach could shed light on other empirical issues or, at least, feed non-standard perspectives. There exists a large literature studying the consequences of societies’ relationship to leisure, comparing "labor societies" to "leisure societies". The comparative statics about changes in $I_B$ yield possibly interesting intuitions (leisure societies being understood as ones with high average value of $I_B$): how do different levels of $I_B$ affect possible disparities in the labor market? What about the link between a collective taste for leisure and earnings? What impact in terms of efficiency? We provide a new route to the study of such questions.

Anyway, the would-be predictions of our model as regards labor market outcomes remain questionable since they assume a rigid monopsonic structure (our model focuses on the heterogeneity in the characterization of jobs). We believe that this assumption could be relaxed without radically amending the results we display above, but this remains to be shown: an important improvement would be to apply the insight of this paper to a more relevant framework (labor market in monopsonic competition). Additionally, other improvements would be required (notably the endogeneisation of the standard $w_A$) that we leave for future research.

References


A Appendix

The payoffs of the agent are

\[ U_{\text{out}}^B (w, 0; \theta) = I_B > 0 \]
\[ U_{\text{out}}^A (w, 0; \theta) = I_{\text{out}}^A (\theta) < 0 \]
\[ EU_B (w, 0; \theta) = E_0 w + I_B \]
\[ EU_A (w, 0; \theta) = (1 + \gamma w) E_0 w + I_A (0; \theta) \]
\[ EU_B (w, 1; \theta) = E_1 w - \psi + I_B \]
\[ EU_A (w, 1; \theta) = (1 + \gamma w) E_1 w - \psi + I_A (1; \theta) \]

Observing the contract offered by the principal, the agent selects a best reply in \( A \). Denote \( W_c (e; \theta) \) the set of contracts implementing \( (in, e) \) at least from an agent holding identity \( c \), given \( \theta \). Suppose first that \( \Delta \pi \Delta S < E_1 w \) so that the principal decides not to induce effort \( e = 1 \). The question of participation remains raised. The agent at least participates if \( w \in W_c (0; \theta) \) for \( c = A \) or \( c = B \). Since the level of effort is not at stake, the contract is simply contingent upon \( \tilde{q} \) i.e. it is a couple \((w, \bar{w})\), and \( W_c (0; \theta) \subset \mathbb{R}^2 \).

\[ w \in W_A (0; \theta) \iff EU_A (w, 0; \theta) \geq U_{\text{out}}^B (0; \theta) \iff (1 + \gamma w) E_0 w + I_A (0; \theta) \geq I_B. \]
\[ w \in W_B (0; \theta) \iff EU_B (w, 0; \theta) \geq U_{\text{out}}^B (0; \theta) \iff E_0 w + I_B \geq I_B \]

Claim 0 With limited liability, the contract transfering 0 to the agent whatever the realization of \( \tilde{q} \), induces his participation for a zero-effort. Furthermore

\[ c^* = \begin{cases} 
A & \text{if } \Delta I (\tilde{q}) \leq 0 \\
B & \text{otherwise}
\end{cases} \]

Proof. Since liability is limited, the principal chooses the contract \( w \) that solves

\[ \min_w E_0 w \quad \text{s.t. } w \in (W_A (0; \theta) \cup W_B (0; \theta)) \cap \mathbb{R}^2_+ \]

It is straightforward to see that for \( E_0 w = 0 \) an agent with identity \( B \) participates. When \( I_A (0; \theta) \geq I_B (\iff \Delta I \leq 0) \), self-esteem concerns lead the agent to hold identity \( A \) which involves a higher self-esteem than the \( B \).

Notice that the problem of inducing the agent participation arises in exactly similar terms under complete or incomplete information. Hence, in both cases, assuming that inducing the effort is too costly for the principal, participation will nonetheless be induced if and only if \( E_0 S \geq 0 \).
A.1 Optimal contracts under complete information

Suppose that the principal tries to induce $e = 1$. We successively define the sets of incentive feasible contracts inducing effort from agent with identity $A$ and $B$.

$\mathbf{w} \in \mathbf{W}_A (1; \theta) \subset \mathbb{R}^4$ if and only if

\[
EU_A (\mathbf{w}, 1; \theta) \geq EU_A (\mathbf{w}, 0; \theta) \iff (1 + \gamma_w) E_1 w_1 - \psi + I_A (1; \theta) \geq (1 + \gamma_w) E_0 w_0 + I_A (0; \theta)
\]

\[
EU_A (\mathbf{w}, 1; \theta) \geq EU_B (\mathbf{w}, 0; \theta) \iff (1 + \gamma_w) E_1 w_1 - \psi + I_A (1; \theta) \geq E_0 w_0 + I_B
\]

\[
EU_A (\mathbf{w}, 1; \theta) \geq U_B^{opt} (\mathbf{w}, 0; \theta) \iff (1 + \gamma_w) E_1 w_1 - \psi + I_A (1; \theta) \geq I_B
\]

$\mathbf{w} \in \mathbf{W}_B (1; \theta) \subset \mathbb{R}^4$ if and only if

\[
EU_B (\mathbf{w}, 1; \theta) \geq EU_B (\mathbf{w}, 0; \theta) \iff E_1 w_1 - \psi + I_B \geq E_0 w_0 + I_B
\]

\[
EU_B (\mathbf{w}, 1; \theta) \geq EU_A (\mathbf{w}, 0; \theta) \iff E_1 w_1 - \psi + I_B \geq (1 + \gamma_w) E_0 w_0 + I_A (0; \theta)
\]

\[
EU_B (\mathbf{w}, 1; \theta) \geq U_B^{opt} (\mathbf{w}, 0; \theta) \iff E_1 w_1 - \psi + I_B \geq I_B
\]

Since liability is limited, the principal chooses the contract $\mathbf{w}$ that solves

\[
\min_{\mathbf{w}} E_1 w_1 \\
\text{s.t. } \mathbf{w} \in (\mathbf{W}_A (1; \theta) \cup \mathbf{W}_B (1; \theta)) \cap \mathbb{R}^4
\]

Claim 1 Optimal transfers under complete information.

Proof. Notice first that, since both the agent and the principal are risk-neutral, only expected transfers matter i.e. we are looking for a couple of expected transfers $(E_0 w_0, E_1 w_1)$ solving the latter program. Since the contract can be contingented upon $e$, a first step for the principal is to make the outside options (options that involve $e = 0$) as unrewarding as possible. Limited liability constraints prevent her from pushing corresponding transfers below 0. Hence, the strongest possible punishment entails $E_0 w_0^* = 0$ so that

\[
\mathbf{w} \in \mathbf{W}_A (1; \theta) \iff (1 + \gamma_w) E_1 w_1 - \psi + I_A (1; \theta) \geq \max \{I_A (0; \theta), I_B\}
\]

and

\[
\mathbf{w} \in \mathbf{W}_B (1; \theta) \iff E_1 w_1 - \psi + I_B \geq \max \{I_B, I_A (0; \theta)\}
\]

The most demanding constraint is obviously binding in the optimum. Taking into account limited liability constraints, the lowest expected transfer inducing effort is written as

\[
E_1 w_1^* = \max \left\{ \min \left\{ \frac{\psi + \max \{I_A (0; \theta), I_B\} - I_A (1; \theta)}{1 + \gamma_w}; \psi + \max \{I_B, I_A (0; \theta)\} - I_B \right\} ; 0 \right\}
\]

Hence, if $I_A (0; \theta) \geq I_B (> 0)$ (that is $\Delta I \leq 0$), since $I_A (1; \theta) = I_A (0; \theta) + \gamma_e$,

\[
E_1 w_1^* = \max \left\{ \min \left\{ \frac{\psi - \gamma_e}{1 + \gamma_w}; \psi + I_A (0; \theta) - I_B \right\} ; 0 \right\} = \max \left\{ \frac{\psi - \gamma_e}{1 + \gamma_w}; 0 \right\}
\]

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while for \( I_A (0; \theta) < I_B \) that is \( \Delta I > 0 \), we get
\[
E_1 w_1^* = \max \left\{ \min \left\{ \frac{\psi + I_B - I_A (1; \theta)}{1 + \gamma_w}; \psi \right\}; 0 \right\} = \max \left\{ \frac{\psi + \Delta_I - \gamma_e}{1 + \gamma_w}; 0 \right\} \text{ if } \gamma_w \psi + \gamma_e > \Delta I
\]
The remaining of the proof derives from claim 0. ■

**A.2 Optimal contracts with moral hazard**

With moral hazard, the principal can no longer make transfers depending on \( e \): \( w_0 = w_1 = w \) and \( \overline{w}_0 = \overline{w}_1 = \overline{w} \). This affects the set of incentive feasible contracts in the following way:
\[
w \in W_{\text{in}}^A (1; \theta) \subset \mathbb{R}^2 \text{ if and only if }
\begin{align*}
(1 + \gamma_w) E_1 w - \psi + I_A (1; \theta) &\ge (1 + \gamma_w) E_0 w + I_A (0; \theta) \quad (IC_A) \\
(1 + \gamma_w) E_1 w - \psi + I_A (1; \theta) &\ge E_0 w + I_B \quad (IC_{A/B}) \\
(1 + \gamma_w) E_1 w - \psi + I_A (1; \theta) &\ge I_B \quad (PC_A)
\end{align*}
\]
\[
w \in W_{\text{in}}^B (1; \theta) \subset \mathbb{R}^2 \text{ if and only if }
\begin{align*}
E_1 w - \psi + I_B &\ge E_0 w + I_B \quad (IC_B) \\
E_1 w - \psi + I_B &\ge (1 + \gamma_w) E_0 w + I_A (0; \theta) \quad (IC_{B/A}) \\
E_1 w - \psi + I_B &\ge I_B \quad (PC_B)
\end{align*}
\]
and the problem writes
\[
\min_w E_1 w \\
\text{s.t. } w \in (W_{\text{in}}^A (1; \theta) \cup W_{\text{in}}^B (1; \theta)) \cap \mathbb{R}_+^2
\]
The solutions of this program can no more be reduced to a couple of expected transfers. As a consequence, it is more convenient to work with variables \( \underline{w} \) and \( \Delta w = \overline{w} - \underline{w} \). A reformulation of incentives feasible sets is then required that we propose in the remaining. We will solve this program in three steps: (1) assuming that the solution involves the arousing of identity \( A \); (2) assuming that the solution involves the arousing of identity \( B \); (3) on the ground of the previous steps, making explicit conditions such that one identity is actually encouraged in the optimum.

**A.2.1 The lowest expected transfer inducing \( e = 1 \) and identity \( A \)**
\[
w \in W_{\text{in}}^A (1; \theta) \cap \mathbb{R}_+^2 \text{ if and only if }
\begin{align*}
\Delta w &\ge \frac{\psi - \psi e}{(1 + \gamma_w) \Delta \pi} \quad (IC_A) \\
\overline{w} + (1 + \gamma_w) \pi_1 - \pi_0 \Delta w &\ge \frac{\psi + \Delta_I - \gamma_e}{\gamma_w} \quad (IC_{A/B}) \\
\overline{w} + \pi_1 \Delta w &\ge \frac{\psi + \Delta_I - \gamma_e}{1 + \gamma_w} \quad (PC_A) \\
w &\ge 0 \text{ and } w + \Delta w \ge 0 \quad (LLC)
\end{align*}
\]
and the problem writes

\[
\min_{(w, \Delta w)} \ w + \pi_1 \Delta w \text{ s.t. } (IC_A), (IC_{A/B}), (PC_A), (LLC)
\]

**Lemma 1** The contract solving the previous problem is such that \( \Delta w \geq 0 \).

**Proof.** We prove it by contradiction.

Suppose there exists an optimum such that \( \Delta w < 0 \) (and \( w > 0 \) since \( (LLC) \) is satisfied). In that case, \( (PC_A) \) would be relaxed. Indeed, if \( \psi + \Delta I - \gamma_e \geq 0 \), \( \Delta w < 0 \) implies

\[
\frac{w + \pi_1 \Delta w > w + \frac{(1 + \gamma_w) \pi_1 - \pi_0 \Delta w}{\gamma_w} \geq \frac{\psi + \Delta I - \gamma_e}{\gamma_w} \geq \frac{\psi + \Delta I - \gamma_e}{1 + \gamma_w}}{i.e. (IC_{A/B}) \Rightarrow (PC_A), \text{ while if } \psi + \Delta I - \gamma_e < 0, \Delta w < 0 \text{ implies}}
\]

\[
\frac{w + \pi_1 \Delta w > w + \Delta w \geq 0 \geq \frac{\psi + \Delta I - \gamma_e}{1 + \gamma_w}}{i.e. (LLC) \Rightarrow (PC_A). \text{ Hence, consider the variation } d\Delta w \in [0; -\Delta w] \text{ and } dw \text{ such that}}
\]

\[
dw = - \min \left\{ \min \left\{ \frac{(1 + \gamma_w) \pi_1 - \pi_0}{\gamma_w}; 1 \right\} d\Delta w; w \right\}
\]

If \( \min \left\{ \frac{(1 + \gamma_w) \pi_1 - \pi_0}{\gamma_w}; 1 \right\} d\Delta w \leq w \), one obtains

\[
d\left( w + \frac{(1 + \gamma_w) \pi_1 - \pi_0}{\gamma_w} \Delta w \right) = \left( \frac{(1 + \gamma_w) \pi_1 - \pi_0}{\gamma_w} - \min \left\{ \frac{(1 + \gamma_w) \pi_1 - \pi_0}{\gamma_w}; 1 \right\} \right) d\Delta w \geq 0
\]

and

\[
d\left( w + \Delta w \right) = \left( 1 - \min \left\{ \frac{(1 + \gamma_w) \pi_1 - \pi_0}{\gamma_w}; 1 \right\} \right) d\Delta w \geq 0
\]

so that the couple of variations \( (d\Delta w, dw) \) does not involve any violation of \( (IC_{A/B}) \) or \( (LLC) \) while it relaxes \( (IC_A) \). Nevertheless,

\[
d\left( w + \pi_1 \Delta w \right) = \left( \pi_1 - \min \left\{ \frac{(1 + \gamma_w) \pi_1 - \pi_0}{\gamma_w}; 1 \right\} \right) d\Delta w < 0
\]

that is, the expected transfer is reduced which contradicts our initial assumption.

If \( \min \left\{ \frac{(1 + \gamma_w) \pi_1 - \pi_0}{\gamma_w}; 1 \right\} d\Delta w > w \), the couple of variations \( (w, -w) \) leaves all the constraints non violated. However,

\[
d\left( w + \pi_1 \Delta w \right) = -w + \pi_1 w < 0
\]

which contradicts our initial assumption. ■

The previous lemma implies that the solution to our problem also solves

\[
\min_{(w, \Delta w)} \ w + \pi_1 \Delta w \text{ s.t. } (IC_A), (IC_{A/B}), (PC_A), w \geq 0 \text{ and } \Delta w \geq 0
\]
As a preamble to what follows, notice that for $\Delta w \geq 0$, since $\gamma_w > 0$ and $\pi_1 > \Delta \pi > 0$, if $(IC_{A/B})$ is satisfied then $(PC_A)$ is satisfied. Let $w_1^A = (\underline{w}_1^A, \overline{w}_1^A)$ denotes the contract implementing effort $e = 1$ that encourages $A$, and minimizes the expected transfer.

**Claim** With moral hazard and limited liability, the contract minimizing the expected transfer inducing identity $A$, and effort $e = 1$ entails:

$$w_1^A = \begin{cases} 
(0, \max \left\{ \frac{\psi - \gamma_e}{(1+\gamma_w)\Delta \pi}, 0 \right\}) & \text{if } \Delta I \leq \frac{\gamma_w \pi_0}{1+\gamma_w} (\psi - \gamma_e) \\
(0, \max \left\{ \frac{\psi - \gamma_e + \Delta I}{(1+\gamma_w)\pi_1 - \pi_0}, 0 \right\}) & \text{otherwise}
\end{cases}$$

**Proof.** The case $\gamma_e < \psi$.

First suppose that $\frac{\psi - \gamma_e}{(1+\gamma_w)\Delta \pi} \geq \frac{\psi - \gamma_e + \Delta I}{(1+\gamma_w)\pi_1 - \pi_0}$. We conjecture that $(LLC)$ and $(IC_A)$ are the only relevant constraints. Of course, since the principal is willing to minimize the payments made to the agent, both constraints must be binding. Hence, $\underline{w}_1^A = 0$ and $\overline{w}_1^A = \frac{\psi - \gamma_e}{(1+\gamma_w)\Delta \pi}$. We check that $(IC_{A/B})$ is satisfied since:

$$\frac{(1+\gamma_w)\pi_1 - \pi_0}{\gamma_w} \frac{\psi - \gamma_e}{(1+\gamma_w)\Delta \pi} \geq \frac{(1+\gamma_w)\pi_1 - \pi_0}{\gamma_w} \frac{\psi - \gamma_e + \Delta I}{(1+\gamma_w)\pi_1 - \pi_0} = \frac{\psi - \gamma_e + \Delta I}{\gamma_w}$$

For $\frac{\psi - \gamma_e}{(1+\gamma_w)\Delta \pi} < \frac{\psi - \gamma_e + \Delta I}{(1+\gamma_w)\pi_1 - \pi_0}$, we conjecture that $(LLC)$ and $(IC_{A/B})$ are the only relevant constraints. Both these constraints must be binding in the optimum so that $\underline{w}_1^A = 0$ and $\overline{w}_1^A = \frac{\psi - \gamma_e + \Delta I}{(1+\gamma_w)\pi_1 - \pi_0}$. Constraint $(PC_A)$ is then satisfied since:

$$\pi_1 \Delta w = \frac{\psi + \Delta I - \gamma_e}{1+\gamma_w} - \frac{\psi + \Delta I - \gamma_e}{1+\gamma_w}$$

In the case $\gamma_e \geq \psi$, $\Delta w \geq 0 \Rightarrow (IC_A)$. We minimize the expected transfer subject to $(IC_{A/B})$, $(LLC)$ and $\Delta w \geq 0$. It is then clear that, in the optimum, $\overline{w} = 0$, which leads to

$$\overline{w}_1^A = \Delta w = \max \left\{ \frac{\psi + \Delta I - \gamma_e}{(1+\gamma_w)\pi_1 - \pi_0}, 0 \right\}$$

We can now move on to the next step.

**A.2.2 The lowest expected transfers inducing $e = 1$ and identity $B$**

The limited liability condition $w \geq 0 \Rightarrow E_0 w \geq 0$ so that $(IC_B)$ implies $(PC_B)$. Hence, the set $W_B^{in}(1, \theta) \cap \mathbb{R}_d^2$ can be restricted to (and reformulated as) contracts $(\underline{w}, \Delta w)$ that satify

$$\Delta w \geq \frac{\psi}{\Delta \pi} (> 0) \quad (IC_B)$$

$$\underline{w} - \frac{\pi_1 - (1+\gamma_w)\pi_0}{\gamma_w} \Delta w \leq \frac{\Delta I - \psi}{\gamma_w} \quad (IC_{B/A})$$

$$\underline{w} \geq 0 \quad (LLC)$$
and the problem writes

\[
\min_{(w, \Delta w)} w + \pi_1 \Delta w \text{ s.t. } (IC_B), (IC_B/A), (LLC)
\]

As a preamble, we must state conditions guaranteeing \( W_B^{in} (1; \theta) \cap \mathbb{R}^2_+ \) non-emptiness.

**Lemma 2**

\[
W_B^{in} (1; \theta) \cap \mathbb{R}^2_+ \neq \emptyset \iff \left\{ \text{either } \gamma_w \frac{\pi_0}{\Delta \pi} \psi \leq \Delta I \text{ or } \pi_1 > (1 + \gamma_w) \pi_0 \right\}
\]

We denote \( C \) this condition.

**Proof.**

i) **Suppose \( \pi_1 > (1 + \gamma_w) \pi_0 \).**

If \(-\frac{\pi_1 - (1 + \gamma_w) \pi_0}{\gamma_w} \frac{\psi}{\Delta \pi} \leq \frac{\Delta I - \psi}{\pi_1 - (1 + \gamma_w) \pi_0} \) then \( 0, \frac{\psi}{\Delta \pi} \) obviously satisfies (LLC), (IC\(_B/A\)) and (IC\(_B\)).

Hence \( 0, \frac{\psi}{\Delta \pi} \) \( \in \) \( W_B^{in} (1; \theta) \cap \mathbb{R}^2_+ \neq \emptyset \).

ii) **Suppose \( \pi_1 \leq (1 + \gamma_w) \pi_0 \).**

Then, \( w \in W_B^{in} (1; \theta) \cap \mathbb{R}^2_+ \Rightarrow w + \frac{(1 + \gamma_w) \pi_0 - \pi_1}{\gamma_w} \Delta w \leq \frac{\Delta I - \psi}{\gamma_w} \). Furthermore, \( w \geq 0 \) and \( \Delta w \geq \frac{\psi}{\Delta \pi} \) imply

\[
w + \frac{(1 + \gamma_w) \pi_0 - \pi_1}{\gamma_w} \Delta w \geq \frac{(1 + \gamma_w) \pi_0 - \pi_1}{\gamma_w} \frac{\psi}{\Delta \pi}
\]

hence

\[
\frac{(1 + \gamma_w) \pi_0 - \pi_1}{\gamma_w} \frac{\psi}{\Delta \pi} \leq \frac{\Delta I - \psi}{\gamma_w} \Leftarrow \gamma_w \frac{\pi_0}{\Delta \pi} \psi \leq \Delta I
\]

If \( \gamma_w \frac{\pi_0}{\Delta \pi} \psi \leq \Delta I \) then \( 0, \frac{\psi}{\Delta \pi} \) \( \in \) \( W_B^{in} (1; \theta) \cap \mathbb{R}^2_+ \neq \emptyset \) \( . \)

Let \( W_1^B = (w_1^B, \pi_1^B) \) denotes the contract inducing effort that encourages identity \( B \), and minimizes the expected transfer.

**Claim** Assuming that \( C \) holds, with moral hazard and limited liability, the contract minimizing the expected transfer inducing identity \( B \), and effort \( e = 1 \) entails:

\[
W_1^B = \begin{cases} 
0, \frac{\psi - \Delta I}{\pi_1 - (1 + \gamma_w) \pi_0} \text{ if } \Delta I < \gamma_w \frac{\pi_0}{\Delta \pi} \psi \\
0, \frac{\psi}{\Delta \pi} \text{ otherwise }
\end{cases}
\]

**Proof.** We easily prove that \( w_1^B = 0 \). Indeed, if \( w_1^B \) was strictly positive then, by reducing it we could relax constraints \( (IC_B/A) \), and still reduce the expected transfer.
For $\gamma_w \frac{\pi\theta}{\Delta \pi} \psi \leq \Delta I \left( \Longrightarrow \frac{\psi - \Delta I}{\pi_1 - (1 + \gamma_w)\pi_0} \leq \frac{\psi}{\Delta \pi} \right)$, since $w^B_1 = 0$, $(ICB) \Rightarrow (IC_{B/A})$. Since in the optimum $(IC_B)$ is binding, $w^B_1 = \left(0, \frac{\psi}{\Delta \pi}\right)$.

For $\gamma_w \frac{\pi\theta}{\Delta \pi} \psi > \Delta I \left( \iff \frac{\psi - \Delta I}{\pi_1 - (1 + \gamma_w)\pi_0} > \frac{\psi}{\Delta \pi} \right)$, $W^B_1 (1; \theta) \cap \mathbb{R}_+^2 \neq \emptyset \iff \pi_1 > (1 + \gamma_w)\pi_0$ (see the lemma 2). If this latter condition holds, since $w^B_1 = 0$, $(IC_{B/A}) \Rightarrow (IC_B)$. Of course, in the optimum, $(IC_{B/A})$ is binding so that $w^B_1 = \left(0, \frac{\psi - \Delta I}{\pi_1 - (1 + \gamma_w)\pi_0}\right)$.

We can move on to our last step leading to optimal contract.

A.2.3 The principal’s choice

The principal encourages identity that minimizes expected transfer implementing effort $e = 1$. We denote $w^*_1 = (w^*_1, w^*_0)$ the contract inducing effort that minimizes the expected transfer. Whatever the encouraged identity, the wage in the bad state of nature ($\bar{q} = q$) is 0 - the limited liability constraint is binding. In the good state of nature, the principal encourages the identity that requires the least transfer

$$w^*_1 = \left\{ \begin{array}{ll}
\min \left\{ w_1^A, w_1^B \right\} & \text{whenever } W^B_1 (1; \theta) \cap \mathbb{R}_+^2 \neq \emptyset \\
w_1^A & \text{otherwise}
\end{array} \right.$$

Claim 2 Optimal transfers with moral hazard.

Proof. We have already shown that $w^A_1 = \max\left\{ \frac{\psi + \Delta I - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}, \frac{\psi - \gamma_e}{\Delta \pi}, 0 \right\}$.

- Suppose first that $(1 + \gamma_w)\pi_0 < \pi_1$ so that $W^B_1 (1, \theta) \cap \mathbb{R}_+^2 \neq \emptyset$.

For $\gamma_w \frac{\pi\theta}{\Delta \pi} \psi \leq \Delta I$, $w^B_1 = \frac{\psi}{\Delta \pi}$ and $\frac{\pi\theta}{\Delta \pi} \psi < (\psi - \gamma_e) < \Delta I$ so that $w^A_1 = \frac{\psi + \Delta I - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}$. Hence,

$$w^*_1 = \min\left\{ \frac{\psi + \Delta I - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}, \frac{\psi - \gamma_e}{\Delta \pi} \right\}.$$

If $(1 + \gamma_w)\pi_0 > \pi_1$ then $w^A_1 = \frac{\psi - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}$. Hence, $w^*_1 = \min\left\{ \frac{\psi - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}, \frac{\psi - \Delta I}{\pi_1 - (1 + \gamma_w)\pi_0} \right\}$.

For $\gamma_w \frac{\pi\theta}{\Delta \pi} \psi > \Delta I$, $w^B_1 = \frac{\psi - \Delta I}{\pi_1 - (1 + \gamma_w)\pi_0}$.

If $\frac{\gamma_w \frac{\pi\theta}{\Delta \pi} (\psi - \gamma_e)}{1 + \gamma_w} \geq \Delta I$ then $w^A_1 = \frac{\psi - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}$. Hence, $w^*_1 = \min\left\{ \frac{\psi - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}, \frac{\psi - \Delta I}{\pi_1 - (1 + \gamma_w)\pi_0} \right\}$.

If $\frac{\gamma_w \frac{\pi\theta}{\Delta \pi} (\psi - \gamma_e)}{1 + \gamma_w} \Delta I \iff \frac{\pi\theta}{\Delta \pi} (\psi - \gamma_e) \geq \frac{\Delta I}{1 + \gamma_w}$, then $w^A_1 = \frac{\psi + \Delta I - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}$. Hence, $w^*_1 = \min\left\{ \frac{\psi + \Delta I - \gamma_e}{(1 + \gamma_w)\pi_1 - \pi_0}, \frac{\psi - \Delta I}{\pi_1 - (1 + \gamma_w)\pi_0} \right\}$.

First: $(1 + \gamma_w)\pi_0 < \pi_1$ and $\gamma_e > 0 \Rightarrow \frac{\pi\theta}{\Delta \pi} - \frac{1}{\gamma_w} \gamma_e < 0$.

Second: $\gamma_w \frac{\pi\theta}{\Delta \pi} \psi > \Delta I \Rightarrow \frac{\pi\theta}{\Delta \pi} \psi > \frac{\Delta I}{\gamma_w}$.
third: $\gamma_w \frac{\pi_0}{\Delta \pi} \psi > \Delta I$ and $(1 + \gamma_w) \pi_0 < \pi_1 \Rightarrow \psi > \Delta I$.

So that $(2 \frac{\pi_0}{\Delta \pi} + 1) \psi > \left( \frac{1}{\gamma_w} + 1 \right) \Delta I + \left( \frac{\pi_0}{\Delta \pi} - \frac{1}{\gamma_w} \right) \gamma_e$, and $\overline{w}_1 = \frac{\psi + \Delta I - \gamma_e}{(1 + \gamma_w) \pi_1 - \pi_0}$.

- Suppose now that $(1 + \gamma_w) \pi_0 \leq \pi_1$ so that $W^1_{B_{1,\theta}} \cap \mathbb{R}_+^2$ can be empty.

For $\gamma_w \frac{\pi_0}{\Delta \pi} \psi < \Delta I$, $\overline{w}_1 = \frac{\psi}{\Delta \pi}$ and $\frac{\pi_0}{\Delta \pi} (\psi - \gamma_e) < \frac{1 + \gamma_w}{\gamma_w} \Delta I$ so that $\overline{w}_1 = \frac{\psi + \Delta I - \gamma_e}{(1 + \gamma_w) \pi_1 - \pi_0}$. Hence, $\overline{w}_1 = \min \left\{ \frac{\psi + \Delta I - \gamma_e}{(1 + \gamma_w) \pi_1 - \pi_0}, \frac{\psi}{\Delta \pi} \right\}$ a case we have already consider.

For $\gamma_w \frac{\pi_0}{\Delta \pi} \psi \geq \Delta I$, $W^1_{B_{1,\theta}} \cap \mathbb{R}_+^2 = \emptyset$. Hence $\overline{w}_1 = \overline{w}_1^A$.

- The remaining derives from claim 0.