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Résumé: La fin du monopole des agences publiques de placement dans la plupart des pays de l’OCDE permet aux agences de travail temporaire de développer leur activité de recrutement en emploi permanent. Dans le cadre d’un modèle d’appariement, cet article montre, dans un premier temps, que la concurrence sur le marché des placements facilite le retour à l’emploi des chômeurs. Cette amélioration du marché du travail se réalise sous certaines conditions : efficacité significativement plus élevée de l’agence privée, faible niveau du salaire de réservation. Cet article illustre, dans un second temps, le désintérêt des agences privées envers les travailleurs peu qualifiés plus nombreux et plus difficiles à placer. L’introduction d’une subvention au placement d’un travailleur peu qualifié corrige cet effet et permet de réduire le taux de chômage.

Mots-clé: Chômage, Modèle d’appariement, Services Publics de l’Emploi, Agences privées.

Classification J.E.L: J60, J63, J64, J68

Abstract: This paper deals with the consequences of the existence of private employment agencies on the labor market. Using a matching framework, we study the conditions of existence of private employment agencies and the consequences of competition on the market for job placement. We show that the private employment agencies enter in the labour market only if they are much more efficient than the private agency. Moreover, the level of the unemployment benefits is a disincentive to manage workers for the private agency. Because of a high fallback position for the worker, it is costly for the private employment agency to manage a worker having some low probabilities to exit from unemployment. If these conditions are satisfied, the existence of private employment agencies improve the labor market through shorter unemployment spells and a lower unemployment rate. Moreover, an improvement in the matching process is an incentive for the firms to post vacancies at the private agencies. Nevertheless, the workers managed by the private agencies receive some lower wages than the other workers because of the payment by the firm for recruiting a worker. Finally, we show that private employment agencies have a natural disincentive to manage unskilled workers. But, the introduction of a subsidy to match an unskilled worker with a vacancy seems to be a sufficient incentive to make the private employment agencies managing unskilled workers.

Keywords: Unemployment, Matching model, Public employment agencies, Private employment agencies

J.E.L Classification: J60, J63, J64, J68
1 Introduction

A lack of transparency causes problems in matching labor supply and demand. The fact that the information required to make the transactions on the labor market is costly, makes it reasonable to have institutions that facilitate a more efficient exchange. Examples of such information channels are newspapers advertisements, informal contacts. The role of placement services can be regarded as an other intermediary in the labor market. They take the role of match-makers who narrow down the set of employers and workers and reduce uncertainty on both side of the labor market.

The coexistence of high unemployment and vacancies raises questions about the efficiency of the public employment services. Empirical studies report that job seekers using public employment agencies have longer unemployment spells than those using other methods (see Carpenter and Wieglosz, 1987, Holzer, 1988, Blau and Robbins, 1990, for the US, Osberg, 1993, for Canada, Böheim and Taylor, 2002, for United Kingdom, Addison and Portugal, 2002 for Portugal). Since the early 1990s the tasks and the organisation of job placement have been undergoing comprehensive changes in all OECD countries. Private agencies are starting to play a more important role in the area of job placement, which has been dominated by public services for decades. Placement services can now be offered either by public or by private agencies. Until 2005, France had a moderate monopoly system where private employment services were banned. However there were ways to get around these restrictions. Due to the recent law of social cohesion, France can ratify the ILO-convention concerning private employment agencies. Particularly, temporary work agencies are allowed to develop their activities to open permanent jobs too. From now on, each private agency can propose, within the law, a service of placement to unemployed and a service of recruitment to the firms.

The Public Employment Services (PES) are set up by government or regional authorities, whereas private employment services are established as a result of private initiative. PES provide their services free of charge, while private employment services usually charge a fee and operate with the purpose of making profit. There is a wide set of types of private employment services. Headhunters provide an example of an agency service targeted at the labor market for high-skilled workers. Temporary work agencies are an important category of private services. They do not place job-seekers permanently but only for a limited period of time. Although the use of public placement services is basically voluntary, there is an obligation for insured unemployed workers to report to the employment service at regular intervals. In contrast there has never been an obligation on the part of the employers to register job vacancies with the employment services.

In general, the market share of the PES is higher than those of private employment services, and the two categories of job placement agencies complement each other. Indeed, they serve different types of unemployed workers. Private employment services typically take care of the placement of already employed and highly qualified job seekers. However, there is a potential competition between public and private employment services for the placement of low skilled unemployed workers.

The aim of this paper is to analyze the efficiency of a labor market where there is a competition on the "market" for job placement (private and public employment agencies). In the economic literature dealing with employment agencies, little attention has been devoted to providing a model in which firms can explicitly choose between alternative employment agencies.
(public or private) to fill their vacancies. The analyses of Pissarides (1979) and Yavas (1994) consider the role of an intermediary on the labor market causing a triangular relation between the agency, the firm and the worker. Yavas set up a formal framework for analyzing the efficiency of a labor market with placement agencies. The essential hypothesis is that an agency can ensure a better match between unemployed workers and vacancies than individual job search can. However, this analysis does not consider the simultaneous use of several job search methods. In addition, the complexity of the analysis does not make it possible to determine the remuneration of the employment agency. The article of Pissarides (1979) primarily aims at modelling the role of the public employment agencies. The remuneration of this intermediary is exogenous. Job seekers and firms with vacancies search the other side of the market through the employment agency and on the labor market. The policy implication is that the agency should try to encourage private search activity rather than offer incentives to firms to register more of their vacancies. Therefore, in the contributions of Pissarides (1979) and Yavas (1994), there is no competition between two types of employment agencies. Instead these two papers contributes to explain the role of an intermediary on labor market efficiency. More recently, Neugart and Storrie (2002) consider the role of temporary work agencies as matching intermediaries on the labor market. They show that the growth of these temporary work agencies does not necessarily crowd out permanent jobs.

In a more applied framework, our model, focused on two matching processes, allows us to consider explicitly the interaction between, on the one hand, employers and public/private employment agencies and, on the other hand, between job seekers and public/private employment agencies. More precisely the aim of our paper is to illustrate the trade-off between opening a vacancy in a public employment agency or in private ones. In the public employment agency, firms face a high number of registered job seekers. But according to empirical studies quoted above, the matching process through this channel is less efficient. For a given number of job seekers and vacant jobs, the higher the number of matches, the more efficient the matching process. In the private employment agency, firms face few registered job seekers and have to pay a fee for recruiting one of them. But the matching process is more efficient.

In this paper, we extend the model of Diamond (1981), Mortensen (1982) and Pissarides (2000) by assuming two matching channels. Private employment agencies are an additional way of search for the unemployed workers. Firms have choice to post their vacancies in public employment agencies without cost or in private ones. In the latter case, if the vacancy is filled, the firm has to pay a fee to the private agency. Unemployed workers need to register to the public employment agency in order to receive their unemployment benefits. They can also be contacted by private agencies to access a job. Public or private employment agencies can not charge any fees from job seekers. Because of the "no fees to staff" principle, private employment agencies may not request any fees from job seekers (Walwei, 2001). The main contributions of our paper are the followings. First, competition on the "market" for job placement makes the return to employment easier for job seekers. This improvement of the labor market is possible under some conditions: higher efficiency of the private employment agencies, lower fall-back position of the unemployed workers and the fees charged by the private agency should be reasonable. Second, we show that private employment agencies are more interested in the highest skilled workers. A subsidy for the placement of unskilled workers is a good motivation for private agencies to place them into an appropriate job. This is in line with the French subcontracting situation where the public employment agency pay private ones in return for the placement of unemployed workers.
in long lasting jobs.

The paper is organized as follows. The model is presented in section 2. We present the comparative static results in a case where wages are exogenous. This section sums up in a very simple way the effect of an additional job search channel. In section 3, wages are endogenized and we conduct a quantitative analysis. We show that the presence of additional employment agencies influences the decision of the firms to open vacancies and affects the outflow from unemployment. The introduction of an efficient private employment agency rise employment but to a lesser degree than with exogenous wages. In section 4, we discuss the efficiency of the competition between types of placement agencies when the fees charged by the private agencies are endogenous. We illustrate how the matching of a vacancy with a job seeker affiliated with a private agency influence the "price of placement" perceived by the private agency. In section 5, we consider two types of workers. We analyze the behavior of the firms and the private agencies in this context. Concluding comments are provided in section 6.

2 The model

2.1 A matching model with several employment agencies

We assume that the workers can be in 4 different states: unemployed workers using exclusively public employment agency (U), unemployed workers using public and private employment agencies (A), employed workers through public agency (G) and employed workers through private agencies (P). It will be assumed that there is no unemployed workers using exclusively private agencies, because they may be registered to the public agency in order to receive unemployment benefits. The size of the labor force is constant and is denoted: \( u + a + g + p = 1 \) where \( g \) and \( p \) are respectively the employment rate in the states \( G \) and \( P \). The unemployment rate in this model is also determined by \( u + a \); where \( u \) denotes the unemployed workers using exclusively the public agency and \( a \) the unemployed workers using the both network.

The figure 1 sums up the equilibrium flows on the labor market. To simplify, let’s say that \( U \) is the pool of unemployed people using the only public unemployment agency, and \( A \) is the pool of unemployed people using the both network.

Let \( S_i \) denote the number of job seekers in state \( i = G, P, A \) and \( V_i \) the number of vacancies in state \( i \). The number of job matches per unit of time is given by: \( M_i = Q_i V_i^\phi S_i^{1-\phi} \) where \( Q_i \) is the matching efficiency, \( \phi \) is the elasticity of the matching function with respect to vacancies.

The probability of filling a vacancy is then \( \frac{m_i}{M_i} = m_i(\theta_i) \), where \( \theta_i = \frac{V_i}{S_i} \) is the tightness of the labor market in sector \( i \). The bigger tightness \( \theta_i \), the smaller the probability to fill a vacancy.

Symmetrically, the probability of finding a job for a job seeker is \( \theta_i m(\theta_i) = \frac{M_i}{S_i} \), the larger \( \theta_i \), the bigger the probability of finding a job.

The number of job seekers concerned the matching function is then, in each state \( i: S_u = u, S_p = a, S_g = u + a \). The private unemployment agencies search people to manage in the pool of unemployed workers using the only public agency network. The vacancies of firms using the private network are filled by unemployed using the private unemployment agencies services. The vacancies posted in the public unemployment agencies can be filled by the whole unemployed people \( (u + a) \).

An unemployed becomes employed by the public agency network with probability \( \theta_g m_g(\theta_g) \). She can be registered in a private agency with the rate \( \theta_a m_a(\theta_a) \). The private agency determines
the optimal number of unemployed workers to manage, depending of the expected gains. The change of state between $U$ and $A$ can be interpreted as a search process between unemployed workers and private agencies looking for workers to manage and place in some firms of their network. This process takes time and is captured by the matching function $M_a$. $\theta_a$ is the tightness of the job placement market. A worker who is registered in a private agency obtains has some job offers at rate $\theta_p m_p(\theta_p)$. We assume the workers accept the first job they are offered.

We denote the job destruction rate by $q$. When a worker lose her job, she enters the pool of unemployed $U$. This hypothesis is realistic, since in the most OECD countries, unemployed workers have to register at unemployment agency to get unemployment benefits. We suppose that an unemployed worker registered by a private agency faces a risk of having her file deleted. Such shocks occur at rate $q_a$.

The steady-state flows equilibrium is determined by:

\[
\theta_g m_g(\theta_g) u + \theta_g m_g(\theta_g) a = q g 
\]

\[
q g + q p + q_a a = \theta_g m_g(\theta_g) u + \theta_a m_a(\theta_a) u 
\]

\[
\theta_a m_a(\theta_a) u = q_a a + \theta_g m_g(\theta_g) a + \theta_p m_p(\theta_p) a 
\]

\[
\theta_p m_p(\theta_p) a = q p 
\]

2.2 The behavior of firms

Firms choose the optimal way to post vacancies : in private or public agency. Vacancies posted in the public agency are matched with pool of unemployed workers of measure $u + a$, when the
vacancies posted at the private agency are matched with pool of unemployed worker of measure $a$. Then the probability to fill a vacancy by the public network is \textit{ceteris paribus}, stronger than in the private agency. But the efficiency of the matching function in the private agency is bigger.

In counterpart, the firms have to pay the private unemployment agency if the job is filled by its network. Let us explain the difference in efficiency of the matching process between the two types of employment agencies. The public agency face a high number of unemployed workers because of the compulsory registration for receiving unemployment benefits. As a result, the management of all these files is more difficult. Furthermore, the more efficient matching process through the private agency can be explained by a reputation effect. When a firm wants to hire a worker, she does not know anything about the quality of the worker. The employment agency reduces this uncertainty. In order to keep up her reputation, the agency is encouraged to propose the right worker to the firm. By the way, a survey of Hasluck and Purcell (1987) reports that 70% of british interviewed firms appreciate the quality and the selectivity of the workers that the private agencies propose to them. In other words, a private employment agency is more concerned by a reputation effect than an individual worker because the agency anticipates the possible expected gains following the placement of a worker$^4$.

The probability to fill a vacancy via the public agency is $m_g(\theta_g) = Q_g \left( \frac{V_g}{S_g} \right)^{\phi-1}$, while the probability to fill a vacancy via the private agency is $m_p(\theta_p) = Q_p \left( \frac{V_p}{S_p} \right)^{\phi-1}$. The fact $S_g = u + a > S_p = a$ may incitate firms to declare vacancies in the public agency, because of a bigger pool of job seekers. But, the scale parameters $Q_p > Q_g$ may incitate firms to declare their vacancies in the private agency. This captures the efficiency trade-off faced by firms.

It is assumed that the private agency receives payment (from the firm) only when a vacancy is filled via the private agency. In equilibrium, firms are indifferent between posting vacancies in the private agency or posting vacancies in the public agency

The expected profit for a firm using the public network.

Let $\Pi_{v,g}$ be the expected profit from a vacancy posted in the public unemployment agency, and $\Pi_v$ the expected profit from a job filled through the public agency. When the job is vacant and posted in a public agency, it costs $h_g$ per unit time. It is filled at endogenous rate $m_g(\theta_g)$. The net return of filling the job is $\Pi_g - \Pi_{v,g}$. Each filled job yields a net return $y - p$, where $y$ is the is real output and $w_g$ is the cost of labor for a worker. At rate $q_p$, the job is destroyed and becomes vacant. Then the firm chooses to post the vacant job in the public or in the private unemployment agency according to the respective expected profits. This assumption is not very important because of the free entry condition, implying a zero profit on any vacancy. Indeed, firms will open vacancies until the expected profit from a vacancy falls to zero.

$$r \Pi_{v,g} = -h_g + m_g(\theta_g) [\Pi_g - \Pi_{v,g}]$$
$$r \Pi_g = y - w + q \{ \max [\Pi_{v,g}, \Pi_{v,p}, 0] - \Pi_g \}$$

The expected profit for a firm using the private network

$^4$Let us remark that private agencies with a good reputation will be attracted by the "best" unemployed workers. Unskilled workers or long term unemployed workers who are "hard-to-place" will stay in the network of the public agency. We come back to this point later in the paper.
Firms can choose to post the vacant job in the private unemployment agency. Posting a vacant job in the public agency implies no cost but $h_g$. But the services of the private unemployment agency is costly. If the vacancy is filled, the private agency is paid a price $\Omega > 0$ by the firm. This amount is paid once, when the job is filled.

Let $\Pi_{v,p}$ denote the expected profit on a vacant job posted in the private agency and $\Pi_p$ denote the expected profit of a job filled by the way of the private agency. Then,

$$r\Pi_{v,p} = -h_p + m_p(\theta_p) [ -\Omega + \Pi_p - \Pi_{v,p}]$$  
$$r\Pi_p = y - w + q \{ \max\{\Pi_{v,g}, \Pi_{v,p}, 0\} - \Pi_p \}$$  

A vacancy posted in the private agency costs $h_p$ each unit of time, and the job can be filled at rate $m_p(\theta_p)$. The firm pays the agency for the placement service at price $\Omega$. Then, it is assumed that the productivity is independent of the way the vacancy was filled and is $y$. The cost of labor for a worker is $w_p$.

In equilibrium, the expected profit from jobs posted at public agency must equal the expected profit from job posted at private agency:

$$\Pi_{v,g} = \Pi_{v,p}$$

### 2.3 The behavior of the private unemployment agency

The private employment agency works as other firms. It meets, between the $u$ unemployed people, some workers they could manage and match. It posts some vacant "files" and searches between the unemployed to fill these files. If there's a match between an unemployed and the file, the file is "filled". This state is denoted $P$. The agency succeeds in placing the worker in a job at rate $\theta_p m_p(\theta_p)$. Upon which, it receives a payment $\Omega$.

Let $\Pi_{v,a}$ denote the expected profit from file in the private agency and let $\Pi_{p,a}$ denote the expected profit from a managed unemployed worker:

$$r\Pi_{v,a} = -h_a + m_a(\theta_a) [P_{p,a} - P_{v,a}]$$  
$$r\Pi_{p,a} = -f + \theta_p m_p(\theta_p) [ \max\{P_{v,a}, 0\} + \Omega - P_{p,a}] + \theta_g m_g(\theta_g) [ \max\{P_{v,a}, 0\} - P_{p,a}]$$  

Similarly to ordinary firms, the search for people workers to manage is costly, the flow-cost is denoted by $h_a$. It can meet some workers at rate $m_a(\theta_a)$ It is assumed that managing a worker is costly (appointments, searching for the most efficient match, etc.) ; this cost is denoted by $f$. The agency succeeds in matching a managed unemployed worker at rate $\theta_p m_p(\theta_p)$, upon which the file of the worker is deleted and a new one can be opened. A placement brings back $\Omega$. We assume that an unemployed worker managed by the private agency continues to receive offers from the public agency. The agent can then be hired by a firm using the public agency network and then give up the private agency at rate $\theta_g m_g(\theta_g)$. In such a case, the file of the worker at

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5 These costs take into account the expenses for the search of an employee and particularly the fees paid to the private agency. Implicitly, we assume that the cost of a vacancy posted in the public agency is lower than in a private agency, because of the fees charge by the firms of the private network.

6 Indeed, because of the obligation to be registered in the public agency to perceive the unemployment benefits, unemployed workers remain in contact with it.
the private agency is deleted, and the private agency decides whether or not to create a new vacant file. Finally, files in private agency face a risk of being exogenously deleted. Exogenous deletions occur at rate $q_a$.

### 2.4 The labor market equilibrium

A description of the labor market equilibrium requires us to pin down the labor market tightness $\theta_g, \theta_p, \theta_a$. At the free-entry equilibrium, the value of a vacancy is zero, employers are indifferent between posting vacancies in either one of the agency types. Independence of the way it is posted (public/private), and consequently in either one of the types is zero, wherever it is posted. Employers are then indifferent where to post their vacancies (public or private agency).

Since $\Pi_{v,g} = 0$, equation (5) can then be written as $\Pi_g = \frac{h_g}{m_g(\theta_g)}$, an upon substitution into (6), we have the labor demand via the public agency:

$$\frac{h_g}{m_g(\theta_g)} = \frac{y - w}{r + q}$$

(11)

This labor demand is such that the expected costs of a vacancy (left term) equals the expected profit of a filled job (right term of the equation). Because the probability of filling a vacancy decreases with the tightness indicator, the relation (11) defines an increasing relation between the wage $w_g$ and the tightness indicator $\theta_g$ of the labor market $G$.

Likewise the free-entry condition, $\Pi_{v,p} = 0$ implies that the labor demand addressed to the private agency is given as:

$$\frac{h_p}{m_p(\theta_p)} = \frac{y - w}{r + q} - \Omega$$

(12)

Equation (12) defines a decreasing relation between $\theta_p$ and $w_p$.

Since the free-entry assumption ($\Pi_{v,a} = 0$) and equations (9) and (10), we obtain the following expression for the managing demand from the private employment agency:

$$\frac{h_a}{m_a(\theta_a)} = \frac{-f + \theta_p m_p(\theta_p) \Omega}{r + \theta_p m_p(\theta_p) + \theta_g m_g(\theta_g) + q_a}$$

(13)

The existence of the private agency is then conditional on its placement efficiency. For given values of $\theta_p$ and $\theta_a$, relation (13) has a solution if an only if the following condition is satisfied:

$$\theta_p m_p(\theta_p) \Omega > f$$

(14)

Indeed, it follows from equation (13) that $\theta_a$, the indicator of tightness of the placement market, is positive if and only if the net output of matching a managed worker with a vacancy is positive. In other words, the net remuneration of the agency for placing an unemployed worker must outweigh the average cost of placing a managed unemployed.

From relations (11) and (12) we see that:

$$\Omega = \frac{h_g}{m_g(\theta_g)} - \frac{h_p}{m_p(\theta_p)}$$

(15)

$$\Omega = h\left(\frac{1}{m_g(\theta_g)} - \frac{1}{m_p(\theta_p)}\right)$$

(16)
It follows that firms are willing to pay a private agency if the agency generates faster matching, i.e., the average vacancy spell duration of a vacancy posted at the private agency (given by the ratio $1/m_p(\theta_p)$) is shorter than the average vacancy spells of a job posted in the public agency.

### 2.5 The comparative statics at the equilibrium

Finally, when wages are negotiated, the labor market equilibrium is defined by equations (11), (12) and (13). Table 1 presents the comparative statics properties of the labor market equilibrium. We present shortly the impact of a subset of parameters to illustrate how the model works. The calculus of this comparative statics are detailed in Appendices. A quantitative analysis is made in further section.

<table>
<thead>
<tr>
<th>$h_g$</th>
<th>$h_p$</th>
<th>$h_a$</th>
<th>$q$</th>
<th>$w$</th>
<th>$y$</th>
<th>$\Omega$</th>
<th>$Q_p$</th>
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<tr>
<td>$\theta_p$</td>
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<td>$\theta_a$</td>
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</tbody>
</table>

Table 1: Comparative statics in steady state equilibrium

**The cost of posting a vacancy**

An increase in $h_g$ leads to an increase in the average cost of posting a vacancy at the public employment agency. The labor demand through the public agency decreases. That implies an increase in the unemployment rate. The private agency has less difficulties to find unemployed workers to manage. The private employment agency looks for more unemployed workers to manage, leading to an increase in $\theta_a$. The probability to be affiliated with a private agency for an unemployed worker increases.

An increase in $h_p$ leads to an increase in the cost of posting a vacancy at the private agency. Firms react by posting less vacancies in the private agency. That leads to a decrease in $\theta_p$, *ceteris paribus*. The probability to find a job through the private employment agency decreases. In turn, the expected profit from managing an unemployed worker for the private agency falls (following equation (10)). The private agency lower the number of job seekers to manage, implying a fall in $\theta_a$. Consequently, the unemployed workers have a lower probability to be affiliated with a private agency.

An increase in $h_a$ has no influence on tightness indicators $\theta_g$ and $\theta_p$. Because of the increase in the search costs for the private agency, it manages less workers, leading to a decrease in the tightness indicator on the market of placement $\theta_a$.

**The level of wages**

An increase in wage decrease the expected profit on a filled job (relations (6) et (8)). Firms react by posting less vacancies. Two opposite effects take place.

On the one hand, posting less vacancies, the probability of exiting from unemployment decreases. That leads to an increase in the unemployment rate. It is easier for the private agency to find workers to manage. Because of a decrease in the average search cost to manage workers, the private agency has an incentive to search some workers to manage: $\theta_a$ increases.
On the other hand, firms, posting less vacancies at the private agency, contributes to lower the tightness of the labor market of the private network: $\theta_p$. That means the private agency has more difficulties to match its managed workers with a vacancy. The expected profit of managing an unemployed worker decreases. This effect is a disincentive of managing unemployed workers, and leads to a decrease in $\theta_a$.

*The job destruction rate*

Equations (11), (12) and (13) show that an increase in the job destruction rate $q$ is equivalent to an increase in the wage. The effects on $\theta_a$ are not obvious.

*Job productivity*

With an increase in $y$, the expected profit from a filled job increases, creating an incentive to post vacancies. Because of more vacancies, the probability of exiting from unemployment increases. The unemployment rate falls. It becomes harder to find unemployed workers to manage: the average cost of posting a vacancy at the private agency raises, leading to a decreases in $\theta_a$. But, because of the more vacancies, especially in the private agency, the unemployment exit rate through the private agency increases. The private agency has less difficulties to match workers with vacancies, increasing the expected utility from having a managed unemployed worker. This mechanism is an incentive to look for more unemployed workers to manage. Finally, the effect on $\theta_a$ in ambiguous.

*The efficiency of the private employment agency*

A higher efficiency in the matching process of the private agency increases the probability to fill vacancies through the private agency, leading to a decrease in the average cost of vacancies. *Ceteris paribus*, firms post more vacancies at the private agency and $\theta_p$ raises. The private agency match workers and vacancies faster ($\theta_p m(\theta_p)$ increases). That leads to an increase in the expected utility from managing an unemployed worker; the private agency looks for more workers to manage and $\theta_a$ increases. Finally, a higher efficiency of the private matching function increases the probability of exiting from unemployment through the private agency (increase in $\theta_p m_p(\theta_p)$). That leads to a higher expected profit of managing a worker, and an incentive to look from job seekers to manage.

*The price of placement*

The effect the price of placement on the tightness indicator of the market of placement is ambiguous. On the one hand, when $\Omega$ increases the private employment agency has a higher expected profit from managing a worker. It looks for more unemployment workers to manage, leading to a higher $\theta_a$. On the other hand, the increase in the price of placement $\Omega$ leads to an increase in the expected cost of posting a vacancy at the private agency. Firms reduce the vacancies posted at the private agency. Formally, $\theta_p$ decreases *ceteris paribus*. That leads to a decrease in the exit rate from unemployment through the private agency $\theta_p m_p(\theta_p)$). In other words, the private agency has more difficulties to match workers with vacancies. The expected profit from managing a worker is lower. This effect is a disincentive to manage workers, leading to a decrease in the tightness indicator of the market of placement $\theta_a$. Finally, the effect of $\Omega$ on
\( \theta_a \) is not determined. The effect due to the increase in the expected profit of the private agency increases \( \theta_g \); but the indirect effect on the expected cost of posting a vacancy at the private agency is a disincentive of using the private agency, decreasing the expected profit of the private agency.

It is interesting to note that the introduction of a private employment agency induces some network and congestion effects, affecting the exit rate from unemployment and the vacancies opening decisions. The consideration of a further search method influences also the wage settings.

3 The effects of the introduction of a private employment agency

We assume that wages are negotiated. The introduction of a private employment agency is an additional search method, it will modify wages settings and then the level of employment. We present in this section to analyse the consequences of a competition on the placement labor market. In a first time the behavior of workers is presented (3.1), and in a second time, we analyse the wage bargaining process (3.2). We end the section by a quantitative (simulated) analysis of the model (3.3).

3.1 The behavior of workers

Let \( V_g \) denote the expected utility of an employee getting her job through the public agency (state \( G \)), \( V_p \) denote the expected utility of an employee getting her job through the private agency (state \( P \)), \( V_u \) the expected utility of an unemployed using only the PSE (state \( U \)) and \( V_a \) the expected utility of an unemployed worker using both the public AND private unemployment agencies (state \( A \))^7.

\[
rv_g = w_g + q[V_u - V_g]
\]  

(17)

A worker employed in state \( G \), is paid \( w_g \) per unit of time. She looses her job at rate \( q \) and then becomes unemployed (state \( U \)).

The value function of an employee matched by private agency reads:

\[
rv_p = w_p + q[V_u - V_p]
\]  

(18)

Being employed in state \( P \), the worker earns a wage \( w_p \) per unit of time. Job destruction shocks occur at rate \( q \), then the worker becomes unemployed in state \( U \). It is the assumed that each unemployed is initially in state \( U \). This state is a condition to be registered as unemployed and to perceive unemployment benefits. This registration in the public employment agencies SPE is an obligation in most of the OECD countries. We assume that once the worker is hired, he loose her links with the private employment agency.

The expected utility of an unemployed using only the public unemployment agency services reads:

\[
rv_u = b + \theta_g m_g(\theta_g)[V_g - V_u] + \theta_a m_a(\theta_a)[V_a - V_u]
\]  

(19)

An unemployed worker in the public agency receives an amount \( b \) of unemployment benefits. At rate \( \theta_g m_g(\theta_g) \) she is matched with a firm through the public agency and becomes employed.

---

^7 We assume that agents are risk neutral. This assumption excludes the most important function of the unemployment benefits but allows to underline that allocations influence on placement is not the same than unemployment insurance.
in state $G$. The unemployed worker can also match with the private agency at rate $\theta_a m_a (\theta_a)$, after which she remain unemployed but access to the private network offers. His expected utility is denoted $V_a$ and is written:

$$r V_a = b + \theta_p m_p (\theta_p) [V_p - V_a] + \theta_g m_g (\theta_g) [V_g - V_a] + q_a [V_a - V_a]$$  \hspace{1cm} (20)$$

If registered at a private agency, the unemployed worker receives offers from the public agency and receives offers through the private network as well. At rate $\theta_g m_g (\theta_g)$, she is matched with a firm through the public agency network$^8$. The private agency can also match the unemployed worker: at rate $\theta_p m_p (\theta_p)$, he meets a firm from the private network. The intrinsic idea is obviously that the private agency will generate a better match than the public agency: in particular because the private agency limit the number of its own unemployed workers and because the public agency cannot and does not want to limit the number of unemployed workers registred there. In other words there are less competition between the unemployed workers in the private agency.

Comparison between equations (17), (18), (19) and (20) implies $V_u < V_a$. The expected utility of a worker using only the public agency is less than the expected utility of an unemployed worker using also the private agency. Indeed, the managed worker has a shorter spell of unemployment because of the usage of both networks. Consequently, an unemployed worker (state $U$) always has an interest to be affiliated with a private agency (state $A$).

### 3.2 Wages bargaining

We here assume that all wages are bargained. The price paid by a firm to hire a worker through private agencies inflicts on the wage settings. Consequently, the wage bargained by a worker only registered in the public agency is different from the wage bargained by an unemployed worker affiliated also with a private employment agency. In both cases, the wages are set through Nash bargaining.

*Bargaining in state $G$*

Consider a match between a worker and a firm with a vacancy posted in the public agency. In the one hand, if the parties can agree on a wage, the worker becomes employed and has expected utility $V_g$. In the other hand, if bargaining fails, the worker has the expected utility $V_u$. For the firm, the fallback option is $\Pi_{v,g}$. The wage is the result of the maximization of net gains of workers and firms in state $G$

$$\max_{w_g} [V_g - V_u]^{1-\beta} [\Pi_g - \Pi_{v,g}]^{\beta}$$  \hspace{1cm} (21)$$

where $\beta$ is the bargaining power of workers. With (17), we can write the expected utility of an employed worker in state $G$ as:

$$V_g = \frac{w_g + qV_u}{r + q}$$

$^8$We could imagine a system with different unemployment benefits according to the agency (public or private). Every unemployed workers would receive the offers of their network. Such a situation would characterize a private unemployment insurance system that would not be compulsory. As in Denmark, employment agencies would propose unemployment benefits to eligible workers and a service of placement. The public employment agencies would have to take care of non eligible unemployed workers.
Implying to:

\[ V_g - V_u = \frac{w_g - rV_u}{r + q} \]

Also, with (6), we can obtain, at free-entry equilibrium, an expression of firms expected profit if a vacancy if filled through the public agency:

\[ \Pi_g = \frac{y - w_g}{r + q} \]

The first order condition of the program (21) yields the following expression for the settled wage when the match was made through the public agency:

\[ w_g = \beta y + (1 - \beta) rV_u \] (22)

The worker obtains a share \( \beta \) of the surplus, and a share \( 1 - \beta \) of his fallback option. The bargained wage is then an increasing function of the productivity \( y \) and of his reservation utility \( rV_u \).

**Bargaining in state \( P \)**

If the bargaining is successful, the expected utility of the worker is \( V_p \) and the expected profit of firm is \( \Pi_p - \Omega \), since the firm must pay the private agency for generating be successful. The reservation utility for unemployed workers registered at the private agency is \( V_u \). Indeed, we consider that the private agency proposes to the managed workers some jobs they have to accept. If bargaining fails, we assume that the worker is excluded from the private agency. The worker can benefit of its services again at rate \( \theta_a(\theta_a) \). Firms profit, if negotiations fail is \( \Pi_{v,p} \). The wage \( w_p \) is then the result of the following maximization program:

\[ \max_{w_p} [V_p - V_u]^\beta [\Pi_p - \Omega - \Pi_{v,p}]^{1-\beta} \] (23)

With (18), we can write again the expected utility of a worker in state \( P \) de as follows:

\[ V_p = \frac{w_p + qV_u}{r + q} \]

\[ V_p - V_u = \frac{w_p - rV_u}{r + q} \]

Equation (8) can now be written:

\[ \Pi_p = \frac{y - w_p}{r + q} \]

The first order condition allows us defining the wage \( w_p \) to a worker who obtained her job through the private agency:

\[ w_p = \beta [y - (r + q) \Omega] + (1 - \beta) rV_u \] (24)

\[ w_p = w_g - \beta (r + q) \Omega \quad < w_g \text{ pour } \Omega > 0 \] (25)

Hence, the wage obtained by workers affiliated with the private agencies is lower than the wage obtained by workers placed by the public agency. This is a consequence of the payment for the
private agency services which is effectively split between the firm and the worker in the wage bargaining. We showed that the private agency must be efficient to come on the placement market (equation (14)). It permits to workers to have some smaller unemployment spells. In fine, it appears that these workers indirectly pay the cost of this private placement.

The improvement of the efficiency of the matching process has a positive influence on the labor market tightness of the private network ($\theta_p$ increases). That increases the exit rate from unemployment through the private agencies. The bargained wages changes with this new search possibility. The increase in bargaining wages could lower the positive effects of the additive efficient search method. The simulation exercises following confirm these intuitions.

3.3 A quantitative analysis

3.3.1 Calibration

The aim of the calibration done here is to depict a plausible situation of the labor market. We calibrate the model without private agency is calibrated to represent the french labor market situation.

The interest rate is fixed at 5%. We normalize the productivity of workers to 1. The replacement ratio $\rho$ is set at 55% of the wage. The job destruction rate is calibrated to represent some employment flows in France, and the unemployment exit rate is set at 0.15. The costs of vacancies posting ($h_g$) and the public network matching efficiency ($Q_g$) are set such that average unemployment spell is around 11 months and the unemployment rate is 11.9%. In France in 2004, the average unemployment spell was 332 days, so 11 months(source ANPE 2004). We assume that the elasticity of the matching function is 0.5 (cf Petrongolo and Pissarides (2001) for a survey of matching function estimations). The bargaining power of workers is set at 0.5, implying the Hosios condition is satisfied. Table 2 summarizes the parameters of a situation without private unemployment agency.

<table>
<thead>
<tr>
<th>y</th>
<th>$\rho$</th>
<th>$q$</th>
<th>$Q_g$</th>
<th>$h_g$</th>
<th>$\phi_i$</th>
<th>$\beta$</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.55</td>
<td>.15</td>
<td>1</td>
<td>3</td>
<td>.5</td>
<td>.5</td>
<td>.05</td>
</tr>
</tbody>
</table>

Table 2: Calibration of the benchmark situation

To introduce the influence of the private employment agency, we propose the following calibration. The private agency has to be sufficiently efficient to enter the market (see the equations (13) and (14)). We set the scale parameter of the matching function 1.5. We assume that the private unemployment agencies have some destruction rates higher than other destruction rates ($q_a = .20$)$^9$. The costs of vacancies posted in the private agency are higher than those in the public agency because of the incurrend when using the private agency. We set the managing costs for the private agency at $f = .1$, lower than the costs of blank files ($h_a = .2$). Indeed, it is easier for the private agency to meet people already in its files than to meet some unknow people. Finally, we have to determine the price of the placement, $\Omega$. In Germany,$^{10}$ where the

$^9$The results of the model are robust, whatever the level of $q_a$.

$^{10}$The private agencies can freely propose their services at the unemployed workers and firms since 1994.
private agency remuneration is a function of the worker wage, the placement price is about 12 to 15% of the worker wage. In our case, we set $\Omega$ to 1, representing about 10% of the wage. The introduction of a private unemployment agency with these parameters (Table 3) implies an unemployment rate at 11.7%, a little lower than the case without private agencies. With these characteristics, they register 2% of vacancies and realize 4% of hirings.

<table>
<thead>
<tr>
<th>$Q_p$</th>
<th>$Q_a$</th>
<th>$q_a$</th>
<th>$h_p$</th>
<th>$h_a$</th>
<th>$f$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.3</td>
<td>.2</td>
<td>.5</td>
<td>.2</td>
<td>.1</td>
<td>.1</td>
</tr>
</tbody>
</table>

Table 3: Calibration of parameters concerning the private unemployment agency

3.3.2 The impact of an further search method

The liberalization of employment agencies started in 1990's in most countries; except in United Kingdom where competition always existed. In France, this liberalization started at the beginning 2005. The purpose of this paper is to evaluate the impact of this further search method on the French economy.

Impact of a higher efficiency of the job matching proposed by the private agency

We analyse here the effects of the efficiency of the private agency. We showed that the private agencies can play a role on the labor market only if they are sufficiently efficient\(^{11}\) (cf equation (14)). An increase in $Q_p$ signifies that private agencies are more efficient in matching between their managed workers and their client firms. The results of the simulations are represented in figure 2.

With the increase in $Q_p$, the average vacancy spell duration in private agencies falls. The firms react by opening more vacancies in the private agency: the proportion of vacancies posted at the private agency increasing from 2% to 15%. The tightness of the labor market of the private agency $\theta_p$ increases. Then the probability of exit from unemployment $\theta_p m_p(\theta_p)$ is higher.

This in turn implies that workers have a higher fallback option in the bargaining process leading to an increase in the bargained wage. Moreover, for the private agency, the expected profit of managing a worker increases, giving it an incentive to manage more workers. The workers have a higher probability of being contacted by the private agency. The workers benefit a higher fallback option, then they bargain a higher wage, also in jobs obtained through the public agency ($w_g$ increases). Note that the unemployment exit rate through the public agency decreases. Note this effect is small and does not balance the increase in the probability to be affiliated with a private agency. We also note that the bargained wages by workers managed by private agencies are about 1% lower than the wages obtained by other workers.

Finally, the increase in posted vacancies following the introduction of an efficient private unemployment agency contributes to decrease the unemployment rate. Nevertheless, this decreasing is balanced by the wages increasing. The unemployment rate decreases about 1.8 point. The figure 3 present the effect of an improvement of the efficiency of the private unemployment with rigid wages.

\(^{11}\)This calibration implies, knowing equation (14), that the efficiency parameter must be as $Q_p \geq 1.5$. 

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The unemployment rate decreases about 2.4 points. Moreover, note that the more efficient the private agency is to obtain workers to manage, the larger is the decrease in the unemployment rate: with $Q_a = 2$, the unemployment rate decreases by 3 points. The welfare is here measured with an utilitarian criterion. An increase in $Q_p$ implies a positive effect on welfare because of the decrease in the unemployment rate and the increase in wages.

**Impact of the price of placement**

Next, we study now the impact of the price asked by private agencies to match a vacancy to a worker on the behavior of firms and workers. The results are represented in figure 4. We vary the price between 10% and 55% of the wage.

An increase in $\Omega$ has two opposite effects. On the one hand, for private agency, the profit of matching a managed worker to a vacancy is increasing in the price of a match. On the other hand, from using the private agency is decreasing in the price of a match. These effects occur in two steps. In the first step, the increase in the price contribute to an increase in the private agencies gain from managing a worker. The private agency meets more people and creates more files, giving firms an incentive to post vacancies at private agencies. Indeed, because unemployed are in private agencies, firms fill their vacancies faster. This leads to a decrease in the unemployment rate and an increase in the welfare level of workers. For low values of the price of placement, this effect is dominant.
Figure 3: Impact of the efficiency of the private agency when wages are exogenous

In the second step, the higher the $\Omega$, the larger is the influence of the prices on the expected profit of a filled vacancy. The cost of using the private agency to fill a vacancy becomes too high; firms react by posting less vacancies in the private agencies. The tightness indicator $\theta_p$ decreases, reducing the probability exit from unemployment by the private agencies. This effect is larger than the incentive to manage workers because of a higher placement price. In turn, that implies an increase in the unemployment rate and a fall in welfare. Note that the consequences on the unemployment rate are not very large: the unemployment rate, initially set at 11.7%, decreases to 10.9% (for a placement price equal to 30% of wage). For a price equal to 55% of wage, the unemployment rate reaches 11.4%.

We can also note that workers coming from the private agency bargain lower wages than the workers matched by the public agency. The larger is $\Omega$, the larger is the wage gap between workers using the private agency network and the ones using the public agencies (from 1% to 5%). The intuition is that the increase in the placement price reduces the surplus of the job. Indeed, the employer considers the placement cost he has to pay if the bargaining succeeds and the job is filled. Consequently, for the same bargaining power, the worker exiting from unemployment by the private network has a lower wage and the employer a lower profit.

Effects of an increase in unemployment benefits

The effects of unemployment benefits are represented in the figure 5.

An increase in the unemployment benefits improve the situation of unemployed workers independent of whether they are affiliated with the private employment agency or not (both increase in $V_u$ and $V_a$). The unemployed workers bargain higher wages, $w_g$ and $w_p$, because of higher fallback position. When the replacement ratio increases, the situation of unemployed workers
Figure 4: Impact of the price of placement
Figure 5: Impact of an increase in the unemployment benefits
becomes closer to the situation of employees. The increase in wages makes the unemployment rate increase. The relative profit to take a job is lower with a bigger replacement ratio. With the increase in the wages, the profit of a job for a firm falls, because of a higher cost of labor. As a consequence, firms post less vacancies. The higher unemployment rate makes it easier to fill a posted vacancy, reducing firms’ incentive to use the costly and slightly more efficient private agencies. Because of a higher reservation wage and difficulties in finding a job for the unemployed, the private agencies have no incentive to manage workers: the number of unemployed affiliated with the private agency decreases with the level of unemployment benefits.

We showed that the existence of an efficient private employment agency can reduce the unemployment rate. Indeed, the existence of a further additional on the labor market can make increase exit from unemployment and reduce the vacancy spells. Nevertheless, the consideration of wage bargaining reduces the positive effects on the employment.

Notice that the more efficient private agencies are in matching workers, the less incentive the firm have to post vacancies in the public agencies. This has the feedback effect that the relative efficiency of private agencies increase further, since the public agencies now have a smaller pool un vacancies to fill. Hence, the competition, the competition on the market of the placement can increase the inefficiency of the public agency. Nevertheless, the existence of the private agency strongly depends of the situation of the labor market. Indeed, when unemployment is high, firms do not have difficulties in filling their vacancies, and they have less incentive to pay private agencies to fill their vacancies.

This study has assumed an exogenous price of matching an unemployed worker and a vacancy. The mechanism is the following: the higher the price of finding a job for an unemployed, the higher is the incentive to manage a worker. But the higher is the price, the less incentive to firms to use the private agency to fill their vacancies. We showed the effects of the price for finding a worker for a firm are not obvious on unemployment.

We now consider the case where prices are endogenous and are determined by a bargaining between firms and private agencies.

4 The effects of competition when the price is endogenous.

In this section, we consider the case where the price of matching a worker with a vacancy is determined by a bargaining between the firm having vacancies to fill and a private agency having a worker to find a job for. First, we present the remuneration system of private agencies (4.1), second, we study how this price system influence the level of unemployment. Especially, we will analyse the influence of the parameters of the model on the resulting price (4.2).

4.1 The remuneration of the private employment agency

We assume that the private agency is paid by the firm it finding a job for. The price of this service is assumed to be a bargaining between the private agency and the firm using the services of the agency. This bargaining protocol is Nash bargaining, i.e the price of finding a worker for a firm is the result of maximizing the product of net gains of the firm and the private agency.

If the bargaining process is successful, the firm fill its job with the worker the agency proposes to it. This service costs $\Omega$ to the firm, paid when the job is filled. The firm obtains then $\Pi_p - \Omega$ where $\Pi_p$ is still defined by the relation (8). The private agency earns the amount $\Omega$. If the
bargaining fails, the job stays vacant and the profit for the firm is $\Pi_{v,p}$, it is defined by (7). The private agency keeps the managed worker, and the profit is $\Pi_{p,a}$, still defined by (10). Formally, denoting $\gamma \in [0,1]$ the bargaining power of the firm on the price, the price is determined by the maximization of the following program:

$$\max_{\Omega} \left\{ [-\Omega + \Pi_p - \Pi_{v,p}]^\gamma [\Omega - \Pi_{p,a}]^{1-\gamma} \right\}$$

(26)

Using the free entry conditions, the surplus share is as follows:

$$\gamma [\Omega - \Pi_{p,a}] = (1 - \gamma) [-\Omega + \Pi_p] \frac{r + \theta_g m (\theta_g) + q_a}{r + \theta_g m (\theta_g) + \theta_p m (\theta_p) + q_a}$$

(27)

Then:

$$\Omega = \gamma \frac{-f}{r + \theta_g m (\theta_g) + q_a} + (1 - \gamma) \left( \frac{y - w_p}{r + q_a} \right)$$

(28)

This expression takes the standard form of the Nash bargaining process. The placement price is a linear combination of expected gain to have a worker to find a job for (for the private unemployment agency), and the expected gains of a job filled with the private agency.

We supposed here that the firm was paying the private agency only once. We could imagine a system of contributions paid each month. We show in the Appendices that these systems are strictly equivalent.

### 4.2 Competition and price bargaining

We now present a quantitative analysis of the consequences of competition on the "finding a job for" market when the price of finding a worker for a firm is bargained. The calibration is the same as the one presented in section 3.3. We assume that the firm have a bargaining power $\gamma = 0.5$. We discuss later the consequences of the level of the bargaining power $\gamma$. The unemployment rate obtained in this situation (endogenous price) is 11.3%, a little lower than in the exogeneous price case (11.7%). The level of the price $\Omega$ obtained with these characteristics of the labor market is about 18% of the wage.

The consequences of a higher efficiency of the private agency

The effects of an increase in $Q_p$ are close to those obtained in the case where the price was exogenous. We recall the mechanisms, explaining the modifications in the price of finding a worker for a firm. Figure 6 shows the effect of an improving the matching process when the price is bargained. The improving in the private unemployment agency efficiency incitate
the firms to post their vacancies in it. The ratio of vacancies posted in the private agency grows from 3.9% to 17%. Unemployed workers affiliated with the private agency have some higher chances to exit from unemployment: the exit rate by this network is four time higher. The expected utility of this kind of unemployed ($V_a$) increases. Because of a higher number of vacancies posted in the private agency, it can find a job for a managed work more easily, increasing its expected profit to have some unemployed workers to find a job for. Then the private agency manages more unemployed. The unemployed have a higher probability to be affiliated with a private agency, increasing their expected utility.\textsuperscript{13} These effects contribute to reduce the unemployment rate, falling from 11.3% to 9.5%. Because of a higher fall-back position, the workers bargain some higher wages. In particular, $w_p$, the wage bargained by the managed unemployed workers, increases. That implies a decrease in the expected profit on a filled job in sector $P$. The bargaining rule described by (26) implies then, at the implique alors, pour un pouvoir de négociation inchangé, une baisse du prix du placement qui passe de 18% à 15% du salaire pour $Q_p$ allant de 1.5 à 3.

\textit{The impact of firms bargaining power $\gamma$}

Figure 7 present the impact of an increase in the bargaining power of firms. The increase in

\textsuperscript{13}Indeed, $V_a > V_u$
firms bargaining power when the private agency matches it with a worker implies a decrease in the price the firm has to pay the agency. With this increase in the bargaining power and the induced decrease in the price of using private agencies, the firms are more likely to use private agencies for posting their vacancies. The share of jobs posted at the private agencies increase from 1% for $\gamma \to 0$ to 4% for $\gamma = 0.5$. This is a result of the fact that private agencies have less trouble matching workers and vacancies, leading private agencies to expand, thus increasing $\theta_a$. Unemployed workers affiliated only with the public agency now have a higher probability of being contacted by a private agency that wants to manage that worker, thereby increasing the expected utility of unemployed workers. This effect lead to an increase in the unemployment rate. The workers negotiate a higher wage because $w_p$ increases with $\gamma$ (the bargaining power of firms vis a vis agencies). The expected profit for a vacancy filled through the private agency is decreasing with $\gamma$. During the price bargaining the higher the bargaining power of the firm is, the lower is its expected profit on a filled vacancy.

There is a threshold below which private agencies are not making any profit. Obviously the lower the expected profit the less incentive do private agencies have to seek workers to manage. The contact rate of private agencies decrease, and the firms observe the matching probability by the private agencies falling, and consequently, they post less vacancies at the private agencies. This phenomenon explains the decrease in the share of workers managed by private agencies if the bargaining of firms becomes "too" high. This effect works in the direction of increasing the unemployment rate. Finally, the higher the share of workers managed by private agencies, the larger the decrease in the unemployment rate. Consequently, if the firms negotiate a "too" low price or if agencies suggest a "too" high price the albor market will not benefit from the presence of private actors.
How does the price of placement vary with the parameters of the model?

We are interested in the qualitative impact of a subset of parameters on the price of placements $\Omega$. The results are reported in figure 8.

- An increase in the bargaining power of workers pushes the bargained wage up. This diminishes the expected profit of a filled vacancy. At the time of negotiation of the price of placements the rent of the firm is lower implying a lower bargained price.

- The effect of an increase in unemployment benefits is exactly the same as the effect of an increase in bargaining power. Improving the expected utility of unemployed workers, the unemployment benefit improves the effective bargaining power of prospective employees during wage bargaining. The lower the fallback option of unemployed workers, the higher the share of unemployed workers managed by the private agency.

- An increase in the destruction rate $q$ reduces the average spell-duration of jobs. It induces a fall in the expected profit of firms, which then negotiating a lower price of placement with the private agencies.

- An increase in $h_p$ leads to an increase of the average cost of vacancies posted in the private agencies, and therefore leads to a decrease in the number of vacancies posted in the private
agencies. Ceteris paribus, since fewer firms post vacancies at the private agencies, those firms that do have a higher probability to fill their vacancies. Consequently, they accept to pay a higher price $\Omega$.

- Following the same logic, when $h_a$ increase, the cost for the private agency of managing a worker is higher, so the private agency reduce the number of workers it manages. Firms realize this, and post fewer vacancies at the private agencies. For the firms continuing to post their vacancies in the private agency, this implies that they face a higher probability of having their vacancies filled, so the price of a placement increases with $h_a$. The effect of $q_a$ are equivalent.

![Figure 8: Impact of parameters on the price of placement ($\Omega$)](image)

We have in this section considered the case where workers are homogenous. The price of the placement is correspondingly the price the firm is willing to pay to fill their vacancies quicker, and at the same time, it is the price that makes private agencies enter the market and accepting to fill the vacancies posted by firms. What happens if we consider to types of workers, say, skilled and unskilled workers? What is the price the firm is willing to pay to recruit an unskilled worker? And what is the price the private agency want in order to match an unskilled worker?
5 Opening the market for placements with two types of workers

A critique often raised towards private agencies is that they are only interested in highly employable workers rather than less employable workers (longterm unemployed workers, old unemployed workers and unskilled unemployed workers). This concern is raised because private firms, and therefore private agencies, are maximizing their profit.

In the previous section we modelled a labor market with both private and public agencies and we covered the case of homogenous workers. This assumption is obviously strong if we want to represent the behavior of firms, workers and employment agencies vis a vis the population they face. We will now assume that two types of workers exist: Unskilled workers indexed by $l$ and skilled workers indexed by $h$.

We start by characterizing the production process (5.1) and the labor market (5.2) in the presence of skilled and unskilled workers. We present a quantitative evaluation of this model. In particular, we consider a situation where the government gives a benefit to the private agencies to give them an incentive to manage both skilled and unskilled workers (5.3).

5.1 The production process

Each type of agent produces an intermediate good sold on a competitive market and transformed into a final good. We fix the price of the final good at unity to simplify the notation. Then the function of the final production is CES and denoted $Y$

\[
Y = F(Y_l, Y_h) = \left( \alpha Y_l^{\frac{\rho-1}{\rho}} + (1 - \alpha) Y_h^{\frac{\rho-1}{\rho}} \right)^{\rho/(\rho-1)}
\]

where $Y_i$ is the total production of intermediate good of type $i = l, h$. $\rho$ represents the elasticity of substitution between $Y_l$ and $Y_h$, and $\alpha$ represent the relative importance of $Y_l$ in the final production process.

Due to the assumption of a competitive output market, the price of the intermediate goods are given by$^{14}$:

\[
y_l = \alpha Y_l^{-1/\rho} Y^{1/\rho}
\]

\[
y_h = (1 - \alpha) Y_h^{-1/\rho} Y^{1/\rho}
\]

We exclude capital from the final production process assuming that the use of capital is isolated to the production of intermediate goods. It is assumed that the wages of skilled workers are higher than the wages of unskilled workers, and consequently $y_h > y_l$.

5.2 The labor market

We assume that there exist one labor market per skill-level. This segmentation of the labor market is justified by the lack of competition between the less skilled workers (blue collars) and the more skilled workers (white collars). That means we exclude the possibility of having ladder effects.

$^{14}$This specification of the production process has been explored in e.g. Acemoglu (2001).
The link between the two types of workers is the production process, and works through the relative labor demands.

The flow equations, and the behavior of workers and firms are identical to the equations of the previous sections, except they are now indexed by the skill level $i = l, h$.

On the firms side of the market, the expressions that represent their behavior now read:

$$r \Pi_{v,j}^i = -h_j^i + m_j^i \left( \theta_j^i \right) \left[ \Pi_j^i - \Omega_j^i - \Pi_{v,j}^i \right]$$

$$r \Pi_j^i = y_i - w_j^i + q_j^i \left[ \max \left\{ \Pi_{v,j}^i, \Pi_{v,-j}^i \right\} - \Pi_j^i \right]$$

(29)

$\forall i = l, h ; \forall j = g, p$ et où $\Omega_j^i = 0$ si $j = g$.

Note that the expected profit from a job now depends on the price $p_i$ which is the price of the intermediate good. This price is equal to the productivity, and is now endogenous.

The labor used in the production of intermediate goods (the intermediate labor demand) satisfies:

- For a firm using the public agency services:

$$\frac{h_g^i}{m_g^i \left( \theta_g^i \right)} = \frac{y_i - w_g^i}{r + q_i}$$

(30)

$\forall i = l, h$

- For a firm using the private agency services:

$$\frac{h_p^i}{m_p^i \left( \theta_p^i \right)} = \frac{y_i - w_p^i}{r + q_i} - \Omega_p^i$$

(31)

$\forall i = l, h$

The price of a placement takes the exactly the same form as in the previous sections, i.e.:

$$\Omega_p^i = \gamma_i \frac{-f_i}{r + \theta_g^i m_g^i \left( \theta_g^i \right) + q_a^i} + (1 - \gamma_i) \left( \frac{y_i - w_p^i}{r + q_i} \right)$$

(32)

5.3 Results and Simulations

5.3.1 Calibration

The calibration we do here aims at representing a situation where jobs are only differentiated by the skill-level they require. We assume that the destruction rate, the replacement ratio, bargaining power, vacancy cost, matching functions and characteristics of the employment agencies are the same whatever the skill-level of workers. That is, we fix all the structural parameters, in order to remove any effect coming from these parameters in the results. The calibration is exactly the same as that carried out in the previous sections:
Concerning the production function we set the elasticity of substitution between skilled and unskilled workers to $\rho = 0.6$, using the estimation of Gianella (1999). The weight assigned to the intermediate good produced by unskilled workers in the production function ($\alpha$) is set to $\alpha = 0.4$.

The calibration results in a level of productivity (or the price of the intermediate goods) of unskilled and skilled workers, respectively, of $y_l = 0.7731$ and $y_h = 4.3186$.

The following table describes the "benchmark situation":

<table>
<thead>
<tr>
<th></th>
<th>Unskilled</th>
<th>Skilled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>13%</td>
<td>5.13%</td>
</tr>
<tr>
<td>Unemployment spell</td>
<td>11,8 months</td>
<td>2,8 months</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.7731</td>
<td>4.3186</td>
</tr>
<tr>
<td>Wage in the public network</td>
<td>0.7137</td>
<td>4.1729</td>
</tr>
<tr>
<td>Wage in the private network</td>
<td>0.6992</td>
<td>4.1268</td>
</tr>
<tr>
<td>Price of placement</td>
<td>0.1444</td>
<td>0.4609</td>
</tr>
<tr>
<td>Price of placement (% of the wage)</td>
<td>20.65%</td>
<td>11.17%</td>
</tr>
</tbody>
</table>

We note that even with the same characteristics of the public and private agencies, the price of placement for an unskilled worker is lower than the price of placement for a skilled worker. Because of the production structure is more favorable to skilled work, firms only have difficulties recruiting skilled labor. Hence, the firms have an incentive to use the private agencies services for the recruitment of skilled labor, because it is more efficient. The firms using unskilled workers know that they do not have difficulties in recruiting, because of higher unemployment rate. Therefore, the gain from using private agencies for such firms is low. The number of vacancies posted at the private agency by the firms using unskilled labor is lower, and hence, the private agencies have no incentive to manage unskilled workers.

Finally, a simple observation of the labor market structure, where all the private agencies have the same characteristics shows that the workers having the better characteristics have easier access to the private agency services.

### 5.3.2 Introduction of a subsidy to matching unskilled workers

The matching of an unskilled worker with a vacancy yields low gains to the private agencies. To make an incentive for these workers it is necessary to increase the gain received by the agencies.
The simulations confirm this intuition. Figure 9 shows the effect of a subsidy to the private agencies for placing unskilled workers. The subsidy gives the private agency an incentive to manage unskilled workers. Then the probability for an unskilled worker to be contacted by a private agency increases, and the price of the placement decrease with the level of the subsidy. Indeed, it may become negative. This
Figure 9: Impact of a subsidy for the placement of an unskilled workers $G_1$. 
result follows from (35):

\[ \Omega_l < 0 \]

\[ \iff \quad G_l > \frac{-f}{r + \theta_l^p m_g (\theta_l^p) + q_a} + \frac{(1 - \gamma) y_l - w_p^l}{\gamma (r + q)} \]  \tag{36}

Then, when the level of the subsidy \( G_l \) becomes higher than the sum of the expected profit to the private agency and the expected profit to the firm of matching an unskilled worker, the price becomes negative. That means that high levels of \( G_l \) gives an incentive to hire unskilled workers. Hiring an unskilled worker, the firm obtains a subsidy. In turn, the unemployment rate decreases from 13% to 11.89%. Finally, unskilled workers matched through private agencies are able to obtain higher wages through the negotiation with the employing firm. Since the bargained price of the placement is negative the wages bargained by the unskilled workers will be higher than the wages negotiated by the workers hired through the public agency. Indeed, the surplus coming from the match between a worker from a private agency and a firm increases: The firm obtains higher profits and the worker obtains higher wages.

The improvement of the labor market of unskilled workers propagates to the labor market of skilled workers. This effect is illustrated by figure 10. Indeed, because of better matching of unskilled workers the demand for unskilled labor increases, and the number of unskilled workers in jobs increases, thereby decreasing the productivity of unskilled labor, but increasing the global production. This leads to an increase in the demand for skilled labor. In this case, the interest of the private employment agency in skilled workers increases: It manages more skilled workers because of an higher expected profit for this type of workers. Then firms post more skilled vacancies at the private agency and the unemployment rate of skilled workers falls, so the labor market of skilled workers benefits from the subsidy as well.

Nevertheless, several obstacles might occur. First, the cost for the government can be prohibitively high. Indeed the initial price of the placement is 0.1444 for unskilled workers and 0.4609 for skilled workes. We simulated here subsidies ranging from 0 to 4.3 times the intial price for the placement of an unskilled worker. The decrease in the unemployment rate can raise 1.1 percentage point for the highest simulated subsidy. For the lower subsidies, the effect of the unemployment rate of course are lower. Finally, workers who do not come come into contact with a private agency face a risk of being marginalized in the labor market, because the firms post few vacancies in the public agencies.

6 Conclusion

In this paper we have studied the impact of a private employment agency, functioning on the principle of the temporary work agencies. The introduction of such an intermediary on the labor market makes it possible to reduce the unemployment rate. The more efficient the private agency is the more the unemployment rate decreases. The effect on the aggregate welfare is positive. Indeed, the presence of an efficient private agency encourages the firm to post there their vacancies there, which increases the chances of unemployed to find a job. The private employment agency play the role of an additional intermediary on the labor market. Empirical data has illustrated the fact that the use of several methods of research of employment increases
Figure 10: Aggregate product and the labor market of skilled workers.
the probability of exit from unemployment. We confirm this finding: the coexistence of a public and a private agency contributes to increase the exit rate from unemployment.

We show that the private employment agencies will be more present in the labor market as the level of unemployment benefits are lowered and that the bargaining power of the workers are lowered. We thus highlight a crowding-out effect of unemployment insurance on the private placement. A rise in the unemployment benefits improves the instantaneous gains of the unemployed but reduces their access to the private network of placement.

The behavior of the firms which open vacancies in private agencies determines also the efficiency of the competition on the market for job placement. We show in particular that if the price of the placement results from a process of bargaining, a too weak bargaining power of the firms does not encourage them to choose the most efficient agency because of the price to pay. Conversely, a strong bargaining power of the firms will discourage the employment agencies because of a lower expected profit. Finally, the presence of an inbalance during the bargaining can have a negative impact on unemployment. The government can have interest to fix itself the price of the placement to ensure the most effective system in term of keeping unemployment low.

Finally, we considered two types of workers: skilled workers and unskilled workers. Unskilled jobs are easier to fill than the skilled jobs because of a higher unemployment rate for the unskilled workers. The firms are not motivated to pay a private agency to recruit this type of workers. On the contrary, the skilled workers are fewer; while passing by a private agency, the firms expect to fill their skilled vacant jobs more quickly than by the public agency. The price they agree to pay is higher for skilled workers. In such a situation, private employment agencies can loose interest in the placement of less skilled unemployed workers because they are difficult to place in a job and source of less expected gains than the skilled unemployed ones. We show that the payment of a subsidy to the placement of a unskilled worker makes it possible to encourage the private employment agencies to enter the market of this type of workers. However, this system does not reduce the unemployment rate of unskilled workers significantly, because of the structural effect of the labor demand.

Considering one agency by skill constitutes obviously a limit of this model. Sub-contracting the placement of less skilled unemployed workers by proposing an increased remuneration with the difficulty of employability of the workers (as in France), can be an alternative solution. Then remains to define an optimal price of placement.
Appendix

A- Comparative statics when wages are exogenous

To study the properties of the comparative statics, we need to rewrite the relations (11), (12) and (13) in the following way:

\[
\begin{align*}
    m_g(\theta_g) &= \frac{h_g(r + q)}{y - w} \\
    m_p(\theta_p) &= \frac{h_p(r + q)}{y - w - (r + q) \Omega} \\
    m_a(\theta_a) &= \frac{h_a[r + q_a + \theta_g m_g(\theta_g) + \theta_p m_p(\theta_p)]}{-f + \theta_p m_p(\theta_p) \Omega} \\
\end{align*}
\]

- **Impact of** \(h_g\)

With (11), we have:

\[
\frac{d\theta_g}{dh_g} = \frac{r + q}{(y - w) m'_g(\theta_g)} < 0
\]

And (13) implies:

\[
\frac{d\theta_a}{dh_g} = \frac{h_a[m_g(\theta_g) + \theta_g m'_g(\theta_g)]}{[-f + \theta_p m_p(\theta_p) \Omega] m'_a(\theta_a)} \frac{d\theta_g}{dh_g}
\]

Recall that \(\theta_p m_p(\theta_p) \Omega > f\) according to (14), we obtain then:

\[
\frac{d\theta_a}{dh_g} > 0
\]

- **Impact of** \(h_p\)

Relation (12) implies:

\[
\frac{d\theta_p}{dh_p} = \frac{r + q}{[y - w - (r + q) \Omega] m'_p(\theta_p)} < 0
\]

Indeed, the condition \(y > w + (r + q) \Omega\) is necessarily satisfied. Conversely, if the firms expect a negative profit on a filled job, they will not post a vacancy in a private agency.

We infer from that:

\[
\begin{align*}
    \frac{d\theta_a}{dh_p} &= -\frac{h_a[m_p(\theta_p) + \theta_p m'_p(\theta_p)]}{[-f + \theta_p m_p(\theta_p) \Omega] m'_a(\theta_a)} \left(f + \Omega [r + q_a + \theta_g m_g(\theta_g)]\right) \frac{d\theta_p}{dh_p} \\
    \frac{d\theta_a}{dh_p} &< 0
\end{align*}
\]

- **Impact of** \(h_a\)

35
\( \theta_g \) and \( \theta_p \) do not depend on \( h_a \). Consequently, only the relation (13) is modified:

\[
\frac{d\theta_a}{dh_a} = \frac{r + q_a + \theta_g m_g (\theta_g) + \theta_p m_p (\theta_p)}{[-f + \theta_p m_p (\theta_p) \Omega] m'_a (\theta_a)} < 0
\]

- **Effect of \( w \)**

\[
\begin{align*}
\frac{d\theta_g}{dw} &= \frac{h_g (r + q)}{(y - w)^2 m'_g (\theta_g)} < 0 \\
\frac{d\theta_p}{dw} &= \frac{h_p (r + q)}{(y - w - (r + q) \Omega)^2 m'_p (\theta_p)} < 0 \\
\frac{d\theta_a}{dw} &= \frac{h_a [m_g (\theta_g) + \theta_g m'_g (\theta_g)] d\theta_g}{[-f + \theta_p m_p (\theta_p) \Omega]^2 m'_a (\theta_a) dw}
\end{align*}
\]

Positive effect

\[- \frac{h_a [m_p (\theta_p) + \theta_p m'_p (\theta_p)]}{[-f + \theta_p m_p (\theta_p) \Omega]^2 m'_a (\theta_a)} (f + \Omega [r + q_a + \theta_g m_g (\theta_g)]) \frac{d\theta_p}{dw} \]

Negative effect

- **Effect of \( q \)**

\[
\begin{align*}
\frac{d\theta_g}{dq} &= \frac{h_g}{(y - w) m'_g (\theta_g)} < 0 \\
\frac{d\theta_p}{dq} &= \frac{h_p}{(y - w - (r + q) \Omega) m'_p (\theta_p)} < 0 \\
\frac{d\theta_a}{dq} &= \frac{h_a [m_g (\theta_g) + \theta_g m'_g (\theta_g)] d\theta_g}{[-f + \theta_p m_p (\theta_p) \Omega] m'_a (\theta_a) dq}
\end{align*}
\]

Positive effect

\[- \frac{h_a [m_p (\theta_p) + \theta_p m'_p (\theta_p)]}{[-f + \theta_p m_p (\theta_p) \Omega]^2 m'_a (\theta_a)} (f + \Omega [r + q_a + \theta_g m_g (\theta_g)]) \frac{d\theta_p}{dq} \]

Negative effect

- **Effect of \( y \)**
If the bargaining on a contribution rate decreases the average cost of a vacancy. In response, firms post more vacancies. This increases the probability of filling a vacancy and decreases the average cost of a vacancy. In response, firms post more vacancies.

\[
\frac{d\theta_g}{dy} = -\frac{h_g (r + q)}{[y - w]^2 m'_g(\theta_g)} > 0
\]

\[
\frac{d\theta_p}{dy} = -\frac{h_p (r + q)}{[y - w - (r + q) \Omega]^2 m'_p(\theta_p)} > 0
\]

\[
\frac{d\theta_a}{dy} = \frac{h_a [m_p(\theta_p) + \theta_g m_g(\theta_g)]}{[-f + \theta_p m_p(\theta_p) \Omega]^2 m'_a(\theta_a)} \frac{d\theta_g}{dy}
\]

\[
\frac{d\theta_p}{dQ_p} > 0
\]

\[
\frac{d\theta_a}{dQ_p} = \frac{h_a [m_p(\theta_p) + \theta_g m_g(\theta_g)]}{[-f + \theta_p m_p(\theta_p) \Omega]^2 m'_a(\theta_a)} \frac{d\theta_p}{dQ_p}
\]

\[
\frac{d\theta_a}{dQ_p} > 0
\]

\[
\frac{d\theta_p}{d\Omega} = \frac{h_p (r + q)^2}{[y - w - (r + q) \Omega]^2 m'_p(\theta_p)} < 0
\]

\[
\frac{d\theta_a}{d\Omega} = \frac{h_a [r + q_a + \theta_g m_g(\theta_g) + \theta_p m_p(\theta_p)]}{[-f + \theta_p m_p(\theta_p) \Omega]^2 m'_a(\theta_a)} \frac{d\theta_p}{d\Omega}
\]

\[
\frac{d\theta_a}{d\Omega} \begin{cases} > 0 & \text{Positive effect} \\ < 0 & \text{Negative effect} \end{cases}
\]

**Effect of \( Q_p \)**

An improvement of the efficiency of the private matching process \( Q_p \) generates an increase in the number of hirings in the economy. This increases the probability of filling a vacancy and decreases the average cost of a vacancy. In response, firms post more vacancies.

\[
\frac{d\theta_p}{dQ_p} > 0
\]

\[
\frac{d\theta_a}{dQ_p} = \frac{h_a [m_p(\theta_p) + \theta_g m_g(\theta_g)]}{[-f + \theta_p m_p(\theta_p) \Omega]^2 m'_a(\theta_a)} \frac{d\theta_p}{dQ_p}
\]

\[
\frac{d\theta_a}{dQ_p} > 0
\]

**Effect of \( \Omega \)**

\[
\frac{d\theta_p}{d\Omega} = \frac{h_p (r + q)^2}{[y - w - (r + q) \Omega]^2 m'_p(\theta_p)} < 0
\]

\[
\frac{d\theta_a}{d\Omega} = \frac{h_a [r + q_a + \theta_g m_g(\theta_g) + \theta_p m_p(\theta_p)]}{[-f + \theta_p m_p(\theta_p) \Omega]^2 m'_a(\theta_a)} \frac{d\theta_p}{d\Omega}
\]

\[
\frac{d\theta_a}{d\Omega} \begin{cases} > 0 & \text{Positive effect} \\ < 0 & \text{Negative effect} \end{cases}
\]

**B- Bargaining on a contribution rate**

If the firms have a contribution on wages to pay, the expected gains rewrites:

\[
r_{\Pi_p} = y - (1 + \sigma) w_p + q (Max (\Pi_{wp}, 0) - \Pi_p)
\]

\[
r_{\Pi_{wp}} = -h_p + m_p(\theta_p) [\Pi_p - \Pi_{wp}]
\]
The payoffs of the private agency are more modified because of the existence of a permanent income following the placement. Like previously, private agencies continue with searching unemployed workers. When they find a job seeker, the expected profit from a managed unemployed worker does not write in the same way:

\[ r\Pi_{p,a} = -f + \theta_p m_p (\theta_p) [\Pi_{e,a} - \Pi_{p,a}] + \theta_g m_g (\theta_g) [\Pi_{v,a} - \Pi_{p,a}] + q_a [\Pi_{v,a} - \Pi_{p,a}] \quad (37) \]

where \( \Pi_{e,a} \) is the profit expected from a placement of a worker in a firm. This expected profit writes:

\[ r\Pi_{e,a} = \sigma w_p + q (\text{Max} (\Pi_{v,a}, 0) - \Pi_{e,a}) \quad (38) \]

The expected profit (37) rewrites then:

\[ \Pi_{p,a} = \frac{-f + \theta_p m_p (\theta_p) \Pi_{e,a}}{r + \theta_g m_g (\theta_g) + \theta_p m_p (\theta_p) + q_a} \quad (39) \]

During the bargaining on a contribution rate, the level of contribution comes from the following program:

\[ \sigma = \arg \max \left\{ (\Pi_p - \Pi_v)^\gamma (\Pi_{ea} - \Pi_{pa})^{1-\gamma} \right\} \quad (40) \]

The sharing of the surplus is defined by:

\[ \gamma (\Pi_{ea} - \Pi_{pa}) = (1 - \gamma) (\Pi_p - \Pi_v) \left( \frac{r + \theta_g m (\theta_g) + q_a}{r + \theta_g m (\theta_g) + \theta_p m_p (\theta_p) + q_a} \right) \quad (41) \]

The price of the placement satisfies:

\[ \sigma w_p = \frac{-f (r + q)}{r + \theta_g m (\theta_g) + q_a} + (1 - \gamma) (y - w_p) \]

\[ \sigma w_p = (r + q) \Omega \]

To conclude, if firms and agencies bargain on a contribution rate, the final price of placement is exactly the same as if firms and agencies bargain on a unique price paid at the moment of the placement.
References
