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Quantitative selection of hedge funds using data envelopment analysis

Huyen Nguyen-Thi-Thanh*[†]

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Abstract

Previous studies have documented that Data Envelopment Analysis (DEA) could be a good tool to evaluate fund performance, especially the performance of hedge funds as it can incorporate multiple risk-return attributes characterizing hedge fund's non normal return distribution in a unique performance score. The main purpose of this paper is to generalize this framework and to extend the use of DEA to the context of hedge fund selection when investors must face multi-dimensional constraints, each one associated to a relative importance level. Unlike previous studies which used DEA in an empirical framework, this research puts emphasis on methodological issues. I showed that DEA can be a good tailor-made decision-making tool to assist investors in selecting funds that correspond the most to their financial, risk-aversion, diversification and investment horizon constraints.

JEL CLASSIFICATION: G2, G11, G15

KEYWORDS: hedge funds, data envelopment analysis, performance measures, Sharpe ratio, fund selection.

*Ph.D candidate at Laboratoire d'Economie d'Orléans (LEO), Université d'Orléans, Rue de Blois, B.P. 6739, 45 067 Orléans Cedex 2. E-mail: Huyen.Nguyen.Thi.Thanh@univ-orleans.fr.

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1 Introduction

The highly successful performance of the so-called hedge funds over the past two decades, notably during the long bull equity market of the 1990s, has made them quickly well-known to financial communities as well as to the public. After several years of outstanding growth (60% in 2000, 40% in 2001), the net inflow of money into this industry is still recorded at 15% in 2004 to end at \$1 trillion, and a growth rate of 10-15% is estimated for 2005¹. While hedge funds still manage only a fraction of the \$8 trillion invested by mutual funds, their assets have ballooned from only about \$150 billion a decade ago. The main reason behind this magic success is that hedge funds are believed to be able to generate superior returns regardless of market environment, and thus offer investors a means to enhance returns, reduce risk exposure and diversify portfolios invested in traditional assets. This argument seems to be particularly attractive in the difficult and highly volatile environment that has prevailed in equity and monetary markets since the early 2000s.

This increasing multiplication of hedge fund industry makes the investment choice quite challenging for investors. With over 8,000 hedge funds now available, investors need efficient tools to select the best funds in which to invest. In general, the selection of funds is assumed to be essentially based on funds' past performance, that is the ratio of returns adjusted to risks. According to traditional financial theories, investors make their investment choices by considering simultaneously the returns approximated by the mathematical mean return and the risks measured either by the return dispersion (represented by variance or standard deviation) or by the correlation with the market (defined by beta). Many of them though validated in mutual fund and pension fund contexts might not be really adequate for hedge funds. In fact, hedge fund returns are much different from those of mutual fund and pension fund "buy-and-hold" portfolios in several ways². On the one hand, they are usually asymmetric and kurtotic, a characteristic largely imputed to the intensive use of short sales, leverage, derivative instruments and to the free call-option like incentive fee structure, all specific to only the hedge fund industry. On the other hand, their short-term movements across diverse asset categories and the market neutral absolute investment objective of hedge fund managers make it really delicate to identify market factors necessary to the use of multi-factorial models³. Recent techniques enlarge the evaluation dimension to the

¹According to a Morgan & Stanley's report.

²Unlike other kinds of investment funds, hedge funds are loosely regulated, and in many cases, are largely exempted from legal obligations as the case of offshore hedge funds. Hedge fund managers thus have a broad flexibility in determining the proportion of securities they hold, the type of positions (long or short) they take and the leverage level they make. As a consequence, they are free to make very short-term movements across diverse asset categories involving frequent use of short sales, leverage and derivatives to attempt to time the market.

³Betas and R^2 obtained from market models are usually very low or insignificant.

skewness or/and the kurtosis, and even other higher moments in order to take into account the non normality of return distributions (see, for example, Sortino & Price (1994); Leland (1999); Stutzer (2000); Keating & Shadwick (2002); Gregoriou & Gueyie (2003); etc.).

Yet, there are suggestions that actual selection criteria, in fact, may be more complicated and differ significantly from theoretical formulations since there are more attributes to consider, each one being associated with a priority level. Apart from risk-return characteristics, investors are also concerned about funds' performance over various time-horizon, about lock-up period, incentive fees, manager's reputation, manager's seniority and perhaps other qualitative criteria. In addition, although investors share the same selection criteria, importance level that each investor attach to each criterion is not necessarily the same because each investor has his own budget, diversification constraints, investment horizon and consequently different priorities. Even when fund selection is made solely on the basis of their risk and return, given the wide variety of risk and return measures without no measure absolutely dominant, an investor may want to consider several of them at the same time, with or without particular interest *a priori* to one (some) among them.

The need to simultaneously consider several criteria while incorporating selector own preferences is particularly important for institutional investors such as pension funds, mutual funds or endowment funds whose clients do not usually share the same financial objective, risk aversion, investment horizon, etc. From such a multi-objective decision-making perspective, the Data Envelopment Analysis approach (hereafter, DEA) seems particularly appealing as it provides the possibility of incorporating many criteria at the same time, together with the control over the importance level paid to each criterion by means of a tailor-made optimizing system. Unlike other performance measures, DEA does not provide a complete ranking of funds. Instead, it only shares them out between efficient (dominant) and inefficient (dominated) sets. The efficiency is simply relative to other funds in the same category and thus can be changed once the considered sample is modified. Nevertheless, relative evaluation is a well-established concept in economic literature (Holmstrom 1982). Moreover, the relative property of fund evaluation is quite valuable because in the investment industry, funds are often rated relatively to others in the same category and investors are only interested in top-performing ones. A broad literature documented that the investment fund market is a tournament and the managers compete against each other in the same category to attract investors (see Brown et al. (1996); Agarwal et al. (2003); Kristiansen (2005)). In addition, DEA is computationally simple and conceptually intuitive. These characteristics make DEA a powerful assistant tool in decision making and explain the increasing enlargement of its application fields including engineering (to evaluate engines), public administration (to evaluate hospitals, administration offices, education

establishments, etc.), commerce (to evaluate supermarkets) and finance (to evaluate bank branches, institutions of micro-finance, stocks, etc.). Recently, it has been used to assess empirically the relative performance of mutual funds and hedge funds⁴.

The main purpose of this research is to study how the DEA method can be adapted to the context of hedge fund selection. Unlike previous works that used DEA to evaluate empirically the performance of the hedge fund industry, I rather put emphasis on methodological aspects of fund selection when investors must face multiple constraints, each one associated to a different important level. I showed that DEA can be a good tailor-made decision-making tool to assist investors in selecting funds that correspond the most to their risk-aversion, financial, diversification and investment horizon constraints. To the best of my knowledge, this study is the first to deal with this question.

The remainder of the paper is organized as follows. After a succinct review of studies related to this research in section 2, section 3 introduces basic concepts of the DEA method. Section 4 discusses the use of the DEA framework to select hedge funds, particularly the choice of inputs, outputs, DEA models as well as formulating additional mathematical constraints to incorporate personal preferences towards inputs and outputs. Section 5 provides some numerical illustrations from a sample including 38 hedge funds. Section 6 summarizes and concludes the paper.

2 Related literature

This research emanated from two main streams of literature. The first one concerns the application of DEA into making a selection when decision-makers have multiple objectives. The second is about the use of DEA to evaluate empirically the performance of investment funds.

With respect to the first literature, I refer especially to two important projects on siting problems: Thompson et al. (1986) and Tone (1999). The former involves identifying feasible sites among six candidate sites for location of a very high-energy physics lab in Texas. A comparative site analysis was made by applying a DEA model under constant returns-to-scale setting, incorporating project cost, user time delay, and environmental impact data. The second project pertains to a huge long-range project with an initial budget more than 12 trillion yen (about 10 billion US dollars) of the Japanese government to transfer the political functions of Tokyo to a new capital. The selection criteria include distance from Tokyo,

⁴A brief review of these studies is presented further.

safety indexes regarding earthquakes and volcanoes, access to an international airport, ease of land acquisition, landscape, water supply, matters with historical associations, etc.⁵. A common interesting point of these two siting analyses is that the evaluators, basing on prior expert knowledge about the relative importance of chosen criteria, fixed lower and upper bounds to the weights associated to each criterion in the mathematical optimization⁶.

The second literature which motivated my work is related to studies using DEA to evaluate empirically the performance of mutual funds, ethical funds and more recently hedge funds (see Table 1 for a summary). Studies on mutual funds include Murthi et al. (1997), McMullen & Strong (1998), Choi & Murthi (2001)⁷, Basso & Funari (2001), Tarim & Karan (2001) and Sengupta (2003). The common point of these work lies in supposing that fund performance is a combination of multiple fund attributes such as mean returns (outputs), risk (total or systematic) and expenses⁸, and sometimes even fund size, turnover speed and minimum initial investment (inputs). Employing essentially basic DEA models like CCR (Charnes et al. 1978) or BCC (Banker et al. 1984), they sought to compare the efficiency of funds within a category or between several different categories of funds.

In the same vein, Basso & Funari (2003) found that DEA is particularly adapted to assess the performance of ethical mutual funds. They suggested including in the outputs an indicator measuring funds' ethical level fulfillments. As argued by the authors, "the solidarity and social responsibility features that characterize the ethical mutual funds satisfy the fulfillment of humanitarian aims, but may lower the investment profitability". Hence, we should not disregard the ethical component when evaluating ethical mutual funds.

The application of DEA in evaluating hedge funds emerged from the work of Gregoriou (2003) and has been supported by Gregoriou et al. (2005)⁹ and Kooli et al. (2005). One common point among these studies is considering only risk-return performance without referring to fees. They approximated outputs (what investors seek to maximize) by right-hand-side values of return distribution and inputs (what investors seek to minimize) by left-hand-side values. Hence, the inputs include (1) lower mean monthly semi-skewness, (2) lower mean monthly semi-variance, and (3) mean monthly lower return; the outputs include (1) upper mean monthly semi-skewness, (2) upper mean monthly semi-variance,

⁵I didn't have access to documents related to this project. All the information mentioned is extracted from Cooper et al. (2000), p.169.

⁶A detailed example is presented in Appendix 1.

⁷Choi & Murthi (2001) is in fact the extended version of their seminal work (Murthi et al. 1997). In the later version, the constant returns to scale assumption is replaced by the variable returns to scale.

⁸The concept of expenses differs from study to study. It might include transaction costs and administration fees (totalled in expense ratio) and loads (subscription or/and redemption costs).

⁹Gregoriou et al. (2005) is an extended version of Gregoriou (2003) and more complete while employing the same DEA methodology with Gregoriou (2003). Therefore, I will refer only to Gregoriou et al. (2005).

Table 1: Studies on investment fund performance within the DEA framework

Studies	Funds	DEA Models	Inputs	Outputs
Murthi et al. (1997)	MF	CCR	standard-deviation, turnover, loads	ratio, mean (gross) returns
McMullen & Strong (1998)	MF	CCR with restricted weights	standard-deviation, expense ratio, loads	one-year, three-year and five-year mean returns
Basso & Funari (2001)	MF	CCR	beta, lower semi-variance, loads	mean return, d_j
Tarim & Karan (2001)	MF	CCR with restricted weights	standard-deviation, expense ratio, loads	mean returns
Choi & Murthi (2001)	MF	CCR et BCC	standard-deviation, turnover, loads	ratio, mean returns
Sengupta (2003)	MF	BCC	beta, expense ratio, turnover, load	mean returns, skewness
Gregoriou (2003)	HF	BCC, cross-efficiency, super-efficiency	lower mean, lower semi-variance, lower semi-skewness	upper mean, upper semi-variance, upper semi-skewness
Gregoriou et al. (2005)	HF	BCC, cross-efficiency, super-efficiency	lower mean, lower semi-variance, lower semi-skewness	upper mean, upper semi-variance, upper semi-skewness
Kooli et al. (2005)	HF	super-efficiency	lower mean, lower semi-variance, lower semi-skewness	upper mean, upper semi-variance, upper semi-skewness

d_j = number of non dominated subperiods for fund j / (total number of subperiods). The lower and upper mean, semi-variance and semi-skewness are calculated with respect to the distribution mean, except for Kooli et al. (2005) where these parameters are computed with respect to risk-free return (one-month certificate of deposit secondary market).

and (3) mean monthly upper return¹⁰. An another similarity is that they put emphasis on an absolute ranking of funds by employing modified DEA techniques: super-efficiency (Andersen & Petersen 1993) and cross-efficiency (Sexton et al. 1986). After comparing DEA ranking results with those of Sharpe and modified Sharpe ratios by means of the Spearman correlation coefficient, they concluded that there was little consistency between these measures. In particular, Kooli et al. (2005) compared fund ranking issued from DEA technique with that of stochastic dominance and found a very low correlation between them, which they considered as a weak power of DEA¹¹.

3 DEA's basic concept

3.1 Technical efficiency

First initiated by Charnes, Cooper & Rhodes (1978) to assess the performance of educational organizations in the program "Follow Through", DEA can be roughly defined as a mathematical programming technique to measure the relative *technical efficiency* of similar Decision-Making Units (hereafter DMU) which use multiple resources (inputs) to produce multiple products or services (outputs). According to Fried, Lovell & Schmidt (1993, p.9-10), "productive efficiency has two components. The purely technical, or physical, component refers to the ability to avoid waste by producing as much output as input usage allows, or by using as little input as output production allows.... The allocative, or price, or economic, component refers to the ability to combine inputs and outputs in optimal proportions in light of prevailing prices.". Consequently, technical efficiency measurement is based solely on quantity information on the inputs and the outputs whereas the economic efficiency necessitates the recourse to information on prices as well as on economic behavioral objective of producers (cost minimization, profit maximization or revenue maximization). Conceptually, in DEA, the efficiency of each DMU under evaluation is determined by the distance from the point representing this DMU to the efficient frontier (production frontier in the case of technical efficiency; cost, revenue or profit frontier in the case of cost, revenue or profit efficiency respectively.). In Figure 1, the isoquant $L(y)$ represents the various combinations of the two inputs that a perfectly efficient firm like Q or Q' might use to

¹⁰The returns used are net returns already subtracted by 30-day US T-bill rate and defined as the value-added of funds.

¹¹In DEA, each Decision Making Unit (DMU) can freely choose its own weighting system associated to inputs and outputs to make it as efficient as possible in comparison to the others. In other words, there exists no common rule about weight limits for all DMUs. As a result, the original DEA model is not appropriate to rank funds. Instead, it provides a dichotomy of efficient and inefficient DMUs. Although the super-efficiency and cross-efficiency techniques are designed to classify completely DMUs, I am sceptical about their adequacy, especially the super-efficiency, just because of the above mentioned reason.

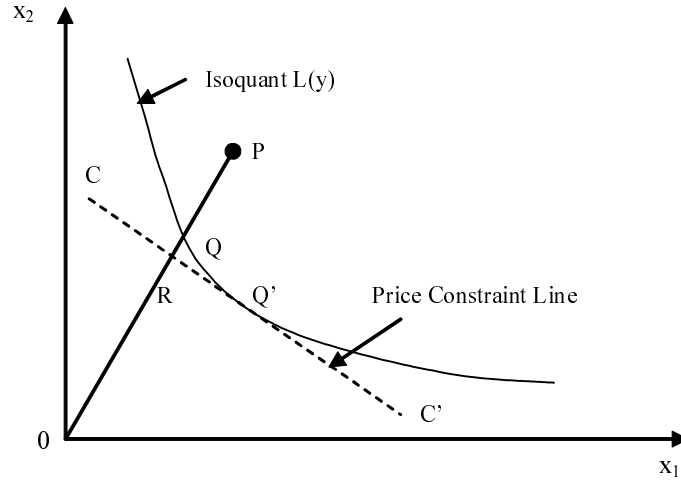


Figure 1: Technical efficiency versus economic (cost) efficiency with two inputs
Source: Farrell (1957)

produce an unit of output; CC' with its slope equal to the ratio of the prices of the two inputs represents the price constraints that all the firms must face. According to Farrell (1957), OQ/OP is defined as the technical efficiency, OR/OQ is the price (cost) efficiency and OR/OP is the overall efficiency of the firm P ¹². In DEA, the production frontier against which the (technical) efficiency¹³ of each DMU is derived is empirically constructed from observed DMUs, and thus without any assumption on the functional relation between inputs and outputs¹⁴; that is, it is formed by a set of best practices (the most efficient DMUs) and the other DMUs are enveloped by this frontier, which explains the origin of the name "Data Envelopment Analysis" of this method.

3.2 The model

3.2.1 The primal program

Consider n DMUs under evaluation that use m inputs (X) to produce s outputs (Y) with X and Y are semipositive¹⁵. The efficiency score h_k attributed to the DMU k is the solution of the following optimizing system:

¹²For more details, see Farrell (1957).

¹³For the shake of brevity, hereafter I will refer only to "efficiency" instead of "technical efficiency".

¹⁴In econometric methods, the efficient frontier is estimated by supposing a particular form of the production function (e.g., Cobb-Douglas, translog, etc.).

¹⁵The semipositivity signifies that all data are nonnegative but at least one component of every input and output vector is positive.

$$\max \quad h_k = \frac{\sum_{r=1}^s u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \quad (1)$$

$$\text{subject to:} \quad \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j = 1, 2, \dots, n \quad (2)$$

$$u_r, v_i \geq \varepsilon \text{ with } r = 1, 2, \dots, s; i = 1, 2, \dots, m \quad (3)$$

where: k is the DMU under evaluation, y_{rj} is the amount of output r of the DMU_j ; x_{ij} is the amount of input i of the DMU_j ; u_r and v_i are the weights assigned respectively to output r and input i ; ε is an infinitesimal positive number, imposed to assure that no input or output is being ignored during the optimization.

Mathematically, the model's objective is to seek for the most favorable (positive) weight system associated to each input and each output which maximizes the weighted sum of the outputs over the weighted sum of the inputs, provided that the maximum value of this ratio as well as this ratio of other DMUs do not exceed 1 (constraint (2)). Given that the production frontier contains efficient DMUs and envelopes inefficient ones, and that the efficiency level of each DMU is, by definition, the distance from its position to the production frontier, it is natural to fixe the maximal value of the objective function to unity¹⁶. Thus efficient DMUs will obtain a score of 1 and inefficient DMUs a score smaller than 1.

Conceptually, each DMU is free to choose its own combination of inputs and outputs so that it is as desirable as possible relatively to other DMUs in the same category. Obviously, this combination must also be technically "feasible" for others, that is the efficiency level of any other DMU using this combination should not exceed the maximum attainable bounded by the production curve (the constraint (2) is also applied to $j = 1, \dots, n$ with $j \neq k$). The idea is under this set of weights, if one DMU can not attain an efficiency rating of 100% then it can never be attained from any other set. It should be noted that in practice, more constraints on weight systems can be imposed to take into account specific preferences of decision-makers. This point will be illustrated further in section 5.

Given the lack of common criteria in the evaluation process, it is important to keep in mind that basic DEA models do only provide a dichotomic classification, not a complete

¹⁶Mathematically, the maximal value of the objective function can be given any other number without changing the relative efficiency of the DMUs. The choice of unity is to assure the coherence between mathematical calculations and efficiency definitions.

ranking; The DMUs are simply divided into two sets: one includes efficient DMUs and the other inefficient ones.

According to Charnes & Cooper (1962, 1973) and Charnes et al. (1978), the fractional problem (1-3) can be conveniently converted into an equivalent linear programming problem by normalizing the denominator to one ($\sum_{i=1}^m v_i x_{ik} = 1$). We obtain now the input-oriented constant returns to scale developed by Charnes et al. (1978) - the so-called CCR model¹⁷:

$$\max \quad h_k = \sum_{r=1}^s u_r y_{rk} \quad (8)$$

$$\text{subject to:} \quad \sum_{i=1}^m v_i x_{ik} = 1 \quad (9)$$

$$\sum_{r=1}^s u_r y_{rj} \leq \sum_{i=1}^m v_i x_{ij}, \quad j = 1, 2, \dots, n \quad (10)$$

$$u_r, v_i \geq \varepsilon, \quad r = 1, 2, \dots, s; \quad i = 1, 2, \dots, m \quad (11)$$

In order to obtain the efficiency scores of n DMUs, the equation system (8-11) must be run n times with each time the DMU under evaluation changes.

3.2.2 The dual program

According to the linear programming theory, the system (8-11) has a dual equivalent formulated as follows:

¹⁷The output-oriented model is obtained by setting the numerator of (1) equal to one and minimizing the denominator as follows:

$$\min \quad h_k = \sum_{i=1}^m v_i x_{ik} \quad (4)$$

$$\text{subject to:} \quad \sum_{r=1}^s u_r y_{rk} = 1 \quad (5)$$

$$\sum_{r=1}^s u_r y_{rj} \leq \sum_{i=1}^m v_i x_{ij}, \quad j = 1, 2, \dots, n \quad (6)$$

$$u_r, v_i \geq \varepsilon; \quad r = 1, 2, \dots, s; \quad i = 1, 2, \dots, m \quad (7)$$

Under the constant returns to scale assumption, the relative efficiency of DMUs obtained by the two models is equivalent.

$$\min \quad \theta \quad (12)$$

$$\text{subject to:} \quad \theta x_{ik} \geq \sum_{j=1}^n \lambda_j x_{ij}, \quad i = 1, \dots, m \quad (13)$$

$$y_{rk} \leq \sum_{j=1}^n \lambda_j y_{rj}, \quad r = 1, \dots, s \quad (14)$$

$$\lambda_j \geq 0, \theta \text{ unconstrained in sign} \quad (15)$$

with θ and λ are dual variables. Note that θ can not, by construction, exceed unity¹⁸.

In economic terms, we are looking for a feasible activity - a virtual DMU which is a linear combination of the best practice set - that guarantees the output level y_k of the DMU_k in all components while using only a proportion of the DMU_k 's inputs (θx_{ik}). Hence, θ is defined as a measure of the efficiency level of the DMU_k . Graphically, in the input-output plan (see Figure 2), θ (input-oriented or input contraction) of the DMU A is the ratio DC/DA ¹⁹; The virtual DMU which serves as benchmark to measure the efficiency of A is C .

In practice, the dual program is usually preferred to the primal because the resolution of the former is computationally much more convenient due to a considerable reduction of constraints, from $n + s + m + 1$ (in the primal) to $s + m$ (in the dual), especially when dealing with a large sample.

4 Hedge fund selection under the DEA framework

4.1 Selector's preferences and the choice of inputs and outputs

In general, the selection of funds to invest in is essentially based on funds' past performance, that is the ratio of returns adjusted to risks. According to traditional financial theories, investors make their investment choices by considering simultaneously the returns approximated by the mathematical mean return and the risks measured either by the return dispersion (represented by variance or standard deviation) or by the correlation with the market (defined by beta), one risk measure at a time. Recent studies enlarge the evaluation dimension to the skewness or/and the kurtosis, and even other higher moments in order

¹⁸We can easily see that $\theta = 1, \lambda_k = 1, \lambda_j = 0 (j \neq k)$ is a feasible solution to (12-15). Hence, the optimal value of θ can not be greater than 1. Besides, the constraint 13 implies that θ must be positive, since X is required, by construction, to be semipositive.

¹⁹The CCR output-oriented efficiency score (output augmentation) of the DMU A is FE/FA .

to take into account the non normality of return distributions (see, for example, Sortino & Price (1994); Leland (1999); Stutzer (2000); Keating & Shadwick (2002); Gregoriou & Gueyie (2003); etc.).

Yet, there are suggestions that actual individual choices, in fact, may be more complicated and differ significantly from theoretical formulations since there are more attributes to consider, each one being associated with a priority level. While some investors might be more concerned with central tendencies (mean, variance), others may care more about extreme values (skewness, kurtosis). Let us consider the positive preference of individuals for skewness first invoked by Arditti (1967). It implies that individuals will be willing to accept a lower expected value from his investment in portfolio A, than in portfolio B if both portfolios have the same variance, and if portfolio A has greater positive skewness and all higher moments are the same. In other words, individuals attach more importance to the skewness than to the mean of returns. Similarly, McMullen & Strong (1998), Morey & Morey (1999) and Powers & McMullen (2000) documented that investors are also concerned about fund's performances over various time-horizons (1 year, 3 years, 5 years and sometimes 10 years) because they provides much more informative insight into fund's perspective than the performance over only one horizon. It is of a particular interest when investors have different investment horizons and risk-return parameters vary greatly from horizon to horizon (Nguyen-Thi-Thanh 2004). Basso & Funari (2003) argued that some categories of investors may also include ethical criteria (categorical variable) in decision-making process in order to satisfy their ethical need while others are interested in transaction costs and administration fees incurred by funds (Murthi et al. (1997); McMullen & Strong (1998); Choi & Murthi (2001); Sengupta (2003)). In the case of gross returns (before all fees pre-cited), the justification for including fees in fund evaluation is direct. But even when returns are net of all fees, fee consideration can still provide additional information about the manager's performance. As claimed by Choi & Murthi (2001), although having the same variance, a fund with 15% of gross returns and 5% of fees is not really comparable to another fund which yields 12% of gross returns at 2% of fees. Given the higher gross historical performance of the first fund in comparison with the second, one might hope that the first will have more chance to achieve a better performance in the future than the second, and rightly so. In addition, investors undoubtedly care about fund manager profile such as reputation, seniority, education level, etc., those documented as having substantial effects on fund performance.

In the context of the hedge fund industry, along with these attributes, it is also important for a potential investor to consider (sometimes simultaneously) other characteristics specific to only hedge funds such as high minimum investment, long lock-up period as well as

incentive fees required by fund managers. Although recently a lot of registered hedge funds, especially funds of funds, have lowered significantly the minimum investment (some lower than \$50,000) to enlarge their clientele to the public, this number is quite modest compared to an universe of up to 8,000 funds. Regarding the lock-up period, it is fixed, on average, about one year but varies greatly from funds to funds - from three months to as long as five years. These requirements form a veritable barrier to the portfolio diversifying task of both institutional and individual investors. Another feature that makes hedge fund selection more complex is the incentive fee scheme. Unlike other types of investment funds, hedge fund managers require incentive fees²⁰ in addition to administration fees which is in percentage of assets under management (about 1%, sometimes 6%). As the hedge fund industry is loosely regulated and in many cases exempted from many investment regulations applied to other kinds of funds, fee fixation is at will of managers. Consequently, incentive fees are very dispersed, ranging from as low as 0% to 50% (Ackermann et al. 1999).

Although recent performance measures make it possible to deal with higher moments than the mean and variance, they do not satisfy the need of investors to consider other attributes than the performance like those mentioned above. Even when fund selection is made solely on the basis of their risk and return, given the wide variety of risk and return measures without no measure absolutely dominant, an investor may want to consider several of them at the same time, with or without particular interest *a priori* to one (some) among them. Very often, selection criteria differ across investors. In the case where selection criteria are common, importance level that each investor attach to each criteria is not necessarily the same because each investor has his own budget and diversification constraints and consequently different priorities. This flexibility is particularly important for institutional investors such as pension funds, mutual funds or endowment funds whose clients do not usually share the same financial objective, risk aversion, investment horizon, etc. In this regard, DEA is a good tool to assist investors in multi-criteria problem of selecting the most appropriate funds to invest in. The merit of this technique lies in the possibility for fund selectors to construct a personalized tool that incorporates their own preference structure regarding fund selection criteria. These preferences are introduced both by the choice of parameters to include in the inputs and the outputs and by setting additional constraints on *absolute* or on *virtual* weights²¹ in the optimizing program. For instance, investors without

²⁰In the incentive fee scheme, the managers earn a pre-defined percentage of the net returns (on average 20%) above a certain threshold named "hurdle rate" (often fixed about 5%). Besides, hedge fund managers ought to recover their past loss (if any) before charging incentive fees, a mechanism known under the name "high water mark".

²¹*Absolute* weights indicate simply normal multipliers, in this case u and v , which are opposed to *virtual* weights denoting the product of absolute weights times input/output quantities. For a discussion about weight restrictions, refer to Allen et al. (1997), Sarrico & Dyson (2004).

fee or fund lock-up constraints might look only at fund performance. Consequently, only return and risk parameters intervene in decision-making process with returns in the outputs and risks in the inputs. If investors want to consider simultaneously several return or/and risk measures and care more about one or some indicators than the others, they can set constraints that the multipliers associated to the former are greater than the multiplier associated to the latter. Based on his preferences or his prior beliefs about the importance of each input and output, one can also limit the definition field of these multipliers to a certain interval. This point will be illustrated later by further numerical examples.

Once selection criteria have been specified, the next step is to choose an appropriate setting to construct the reference production frontier or the efficiency envelope. This issue is addressed in the next section.

4.2 The choice of the efficiency envelope

The assumptions on underlying production technology are of particular importance in evaluating the relative productive efficiency of manufacturing DMUs. The original DEA model - the so-called CCR model, as presented in Section 2, assumes a constant returns to scale (CRS) technology. This assumption may be acceptable in some cases but is of little realism regarding economic theory of production. To mitigate this strong assumption, Banker et al. (1984)(hereafter, BCC) introduced a method to measure relative efficiency in a variable returns to scale (VRS) environment by making slight modifications in the optimizing program proposed by CCR (1978). The BCC primal problem is formulated as:

$$\max \quad h_k = \sum_{r=1}^s u_r y_{rk} - u_o \quad (16)$$

$$\text{subject to} \quad : \quad \sum_{i=1}^m v_i x_{ik} = 1 \quad (17)$$

$$\sum_{r=1}^s u_r y_{rj} - u_o \leq \sum_{i=1}^m v_i x_{ij}, \quad j = 1, 2, \dots, n \quad (18)$$

$$u_r, v_i \geq \varepsilon; \quad r = 1, 2, \dots, s; \quad i = 1, 2, \dots, m \quad (19)$$

The VRS is reflected in the value of u_o . If $u_o = 0$, we are in a CRS environment. $u_o > 0$ characterizes a decreasing return to scale while $u_o < 0$ characterizes an increasing return to scale.

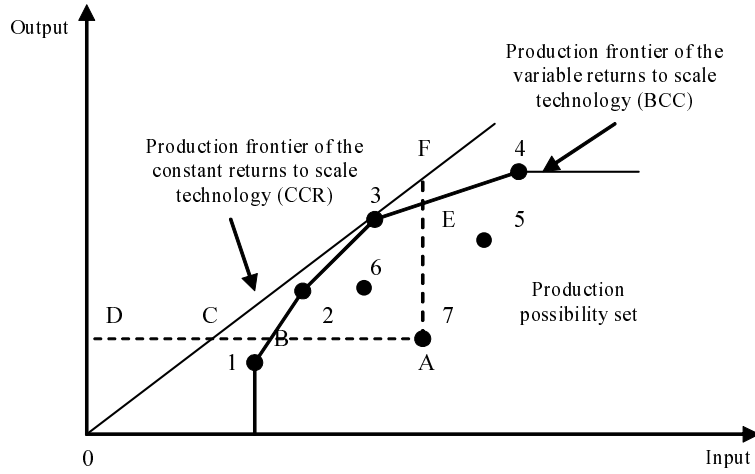


Figure 2: Production frontiers and efficiency measurement of the CCR and BCC models

The primal system (16-19) is equivalent to the following dual:

$$\min \quad \theta \quad (20)$$

$$\text{subject to:} \quad \theta x_{ik} \geq \sum_{j=1}^n \lambda_j x_{ij}, \quad i = 1, \dots, m \quad (21)$$

$$y_{rk} \leq \sum_{j=1}^n \lambda_j y_{rj}, \quad r = 1, \dots, s \quad (22)$$

$$\sum_{j=1}^n \lambda_j = 1 \quad (23)$$

$$\lambda_j \geq 0, \quad \theta \text{ unconstrained in sign} \quad (24)$$

The constraint (23) implies that the referenced DMU of the analyzed DMU is a *convex* linear combination of efficient DMUs while this is not the case in CCR program. Figure 2 illustrates how this additional constraint modifies the efficiency frontier. Graphically, the frontier is no longer a line but a piece-wise linear curve. This is because in CCR model, only the most efficient DMUs with *efficient scale sizes* (for their given inputs and outputs mixes) - like the DMU 3 - can lie on the efficiency frontier, those forming the BCC efficiency frontier (as the DMUs 1, 2 and 4) do not necessarily operate at the most efficient scales. Consequently, the BCC model measures the *pure* technical efficiency of DMUs at the given scale of operation (DB/DA) while the CCR model estimates the *overall* technical efficiency (DC/DA). The difference between these two values approximates the scale efficiency (DC/DB). As a result, there are more efficient DMUs under BCC setting than under CCR one.

Put in the production context, the BCC variable returns to scale model is obviously more realistic than the CCR constant returns to scale one. However, when DMUs to be evaluated are not manufacturing ones, where the inputs and outputs are selection criteria and not physical elements, justifying for using CCR or BCC models is not easy. Thompson et al. (1986) used CCR model to choose a site for locating a high-energy physics lab in Texas without having recourse to BCC model. In contrast, Powers & McMullen (2000) applied the BCC model to select efficient large market securities. The question that which model is more appropriate to this context, to the best of my knowledge, is still unanswered.

4.3 Negative inputs and outputs

DEA models as designed to measure the efficiency of production DMUs require that inputs and outputs are semipositive. In production economics, negative inputs and outputs make no sense. However, in fund selection context, it is likely that we sometimes have negative values like mean, skewness of returns, or returns of some lower quartiles, etc. This problem can be easily solved without any modification of the efficiency envelope in several ways.

When negative values are only present in some outputs (inputs), input-oriented (output-oriented) DEA models are required so that optimizing systems like (1-3) and (4-7) remain always soluble without modifying the original efficient set. Note that under the CRS setting, efficiency scores of input-oriented and output-oriented are equivalent while they are slightly different under the VRS. An other alternative consists of increasing the output y_{rj} (input x_{ij}) of all other DMUs in the sample by a value of φ_r (ψ_i):

$$\begin{aligned} \hat{x}_{ij} &= x_{ij} + \psi_i \text{ with } j = 1, \dots, n \\ \text{OR} \quad \hat{y}_{rj} &= y_{rj} + \varphi_r \text{ ; } j = 1, \dots, n \end{aligned}$$

such that the transformed data is all positive. According to Gregoriou & Zhu (2005), the efficiency frontier remains the same if x_{ij} and y_{rj} are replaced by \hat{x}_{ij} and \hat{y}_{rj} . In this case, either input-oriented models or output-oriented models can be employed.

However, when both input and output sets have negative data, only the solution of transforming data is possible. In a similar manner, the choice of input or output-oriented models is at discretion of selectors.

5 Hedge fund selection: Illustrative applications

5.1 Data

To illustrate the use of DEA in selecting hedge funds, I used a sample of 38 hedge funds belonging to the category Equity Hedge²². Data includes 60 monthly returns covering the period of January 2000 to December 2004. Table 2 reports some descriptive statistics of these funds.

Table 2: Descriptive statistics

Fund name	Min (%)	Max (%)	Mean (%)	SD (%)	SK	KU	S-W	K-S	J-B
1 IKOS Equity	-6.16	10.20	0.68	3.77	0.34	-0.35	0.98	0.11	1.43
2 Enterprise Lng/Shrt Eq	-7.89	7.69	0.16	3.33	-0.07	-0.25	0.99	0.06	0.20
3 Galleon Omni Tech. A	-12.51	19.66	-0.30	5.59	0.32	1.86	0.95**	0.12	9.72***
4 Primeo Select	-7.25	5.63	0.25	3.04	-0.27	-0.49	0.98	0.07	1.31
5 Gabelli Intl A	-11.37	11.95	0.10	4.65	-0.10	0.09	0.99	0.06	0.12
6 GAM Japan Hedge Open	-6.50	5.92	0.06	2.34	-0.36	0.29	0.98	0.08	1.50
7 Foyil Focused	-14.67	24.36	-0.11	6.23	0.91	3.37	0.95***	0.11	36.60***
8 Gruber & McBaine Cap	-22.96	33.89	-0.18	8.86	0.71	2.69	0.96**	0.06	23.09***
9 GAM Selection	-7.87	8.59	-0.01	3.92	-0.01	-0.43	0.98	0.07	0.46
10 SR Asia	-11.84	13.05	0.03	5.40	0.16	-0.12	0.99	0.08	0.29
11 AJR International	-8.19	17.11	1.08*	4.95	1.18	2.13	0.92***	0.16*	25.40***
12 Liberty Ermitage Selz	-13.49	9.11	0.01	4.35	-0.43	0.66	0.98	0.08	2.98
13 Momentum Stock Master	-6.77	7.23	-0.57	3.36	0.12	-0.75	0.98	0.07	1.53
14 Park Place Intl Columbia	-40.85	19.45	-0.73	9.02	-1.42	5.88	0.90***	0.15	106.6***
15 Permal US Opportunities	-12.04	14.17	0.23	5.53	-0.05	0.06	0.98	0.07	0.03
16 Wimbledon Class M	-5.76	6.58	0.24	2.83	0.13	-0.22	0.99	0.06	0.29
17 Wimbledon Class C	-7.10	6.27	0.33	3.25	-0.15	-0.49	0.98	0.06	0.83
18 Absolute Alpha Oppt.	-6.33	5.94	0.15	2.65	0.00	-0.33	0.99	0.08	0.28
19 IKOS Offshore Arbitrage	-7.21	8.18	0.25	3.43	0.17	-0.46	0.99	0.06	0.81
20 Sofaer Capital Global A	-9.90	14.45	0.12	4.54	0.25	0.71	0.99	0.05	1.91
21 Lansdowne Europe	-6.81	9.77	0.64*	2.75	0.48	2.08	0.96*	0.10	13.19***
22 Park Place Europe \$	-9.20	7.57	-0.23	3.95	-0.09	-0.38	0.99	0.05	0.44
23 Park Place Europe	-5.31	6.82	0.33	2.48	0.62	0.59	0.96**	0.15	4.66***
24 HSBC Selection Euro	-13.75	15.03	-0.81	5.72	-0.10	0.30	0.98	0.09	0.33
25 Park Place Galileo Intl	-9.78	19.59	0.09	5.46	1.35	3.85	0.90***	0.12	55.13***
26 Leonardo Capital	-16.34	25.90	1.34 *	5.31	1.35	10.02	0.72***	0.24***	269.3***
27 Key Europe	-1.24	5.45	0.52***	1.10	1.64	5.81	0.88 ***	0.17*	111.2 ***
28 TR Kingsway A	-2.37	15.86	0.74 **	2.39	4.36	27.27	0.63 ***	0.19**	2050***
29 SVM Highlander	-15.48	22.37	0.64	5.07	1.23	7.06	0.85 ***	0.16*	139.7***
30 SR Europe USD	-13.76	17.90	-0.35	5.29	0.50	1.86	0.97	0.09	11.22***
31 SR Europe EURO	-14.30	17.93	0.28	5.00	0.21	2.69	0.95**	0.10	18.58***
32 GAM Europe Hedge Open	-6.93	11.54	0.00	3.15	0.73	2.47	0.96**	0.08	20.64***
33 Lansdowne Europe Equity \$	-7.88	11.53	0.08	4.07	0.58	0.71	0.97	0.08	4.57
34 Aspect European Equity	-7.12	8.67	-0.07	4.12	0.20	-0.64	0.97	0.08	1.42
35 Zulauf Europe	-5.68	10.93	1.10***	2.97	0.38	1.30	0.96 *	0.12	5.66*
36 Zulauf Europe \$	-10.13	7.95	0.58	3.75	-0.29	-0.05	0.98	0.08	0.82
37 Sofaer Capital Europe	-6.48	11.01	0.38	2.60	0.71	4.25	0.93 ***	0.10	50.14***
38 Sofaer Capital Europe \$	-9.93	12.48	-0.13	4.10	0.04	0.71	0.98	0.07	1.26

SD = Standard deviation, SK = Skewness, KU = Kurtosis excess relatively to the normal distribution, S-W = Shapiro-Wilk, K-S = Kolmogorov-Smirnov, J-B = Jarque-Bera are normality tests on return distributions. ***: Rejection of the normality assumption at the 99% confidence level, **: Rejection of the normality assumption at the 95% confidence level, *: Rejection of the normality assumption at the 90% confidence level.

²²These 38 funds are extracted from a database provided by the company Standard & Poor's. Equity Hedge covers several different strategies whose investments are focused on the equity markets. Its two large categories are Global Macro and Relative Value.

As we can see, return distributions of many funds show highly positive (negative) skewness signifying higher probability of extreme positive (negative) values relatively to the normal distribution. Besides, many of them possess high kurtosis excess, which indicates more returns close to the central value but also more regular large positive or negative returns than a normal distribution. The normality assumption of return distributions is tested by means of three tests: Shapiro-Wilk, Kolmogorov-Smirnov, and Jarque-Bera. Results according to the Shapiro-Wilk and Jarque-Bera tests are quite similar although they are much different from those provided by the Kolmogorov-Smirnov test. This divergence is likely due to the sample's limit size as the Kolmogorov-Smirnov test is more appropriate to large samples. According to the Shapiro-Wilk test, documented as the most reliable for small samples, the normality assumption is rejected in 14 out of 38 cases at the confidence level of 95%. These findings imply much higher gain or risk of these funds than approximated under normality assumption, and highlight the importance of incorporating moments of order higher than the mean and variance when appraising their gain and risk profiles.

5.2 Methodology

Due to unavailable data on other characteristics of funds (lock-up period, manager profile, minimum investment, incentive fees, etc.) in the sample used, illustrations are limited to considering their return and risk profiles. This framework is plausible by assuming a category of investors who are only concerned about funds' performance without facing any other constraints. This future performance is approximated by historical risk and return parameters which are unique selection criteria. Since the distribution of hedge fund returns is documented as usually non gaussian, i.e. asymmetric and having fat tails, it is important to incorporate these features into the selection of inputs and outputs. Several settings are likely.

I first considered the case where investors have a positive preference for odd moments and a negative preference for event moments. In this spirit, outputs can be mean and skewness of returns where inputs can be standard deviation and kurtosis of returns.

Alternatively, as suggested by Gregoriou et al. (2005) and Kooli et al. (2005), it seems more clever to reason in terms of partial variations. In reality, investors are likely to be averse only to variations under the Minimum Accepted Return (MAR)²³, which are called lower variations, and appreciate those above this value, which are called upper variations. Thus,

²³The determination of the Minimum Accepted Return is purely subjective and specific to each investor. It can be a risk-free rate or any rate required by investors.

input and output parameters can also be determined in the following manner. The inputs include lower mean, lower semi-standard deviation, lower semi-skewness and lower semi-kurtosis. The outputs contain upper mean, upper semi-standard deviation, upper semi-skewness and upper semi-kurtosis. The method of computing these inputs and outputs is reported in Appendix 2. Besides, as discussed earlier, it is also possible that some categories of investors care more about extreme values than central tendencies. This preference can be taken into account by adding four more constraints on weight vectors into the optimization system:

$$\begin{aligned}
y_{3j}u_3 &\geq y_{1j}u_1 & ; & & x_{3j}v_3 &\geq x_{1j}v_1 \\
y_{3j}u_3 &\geq y_{2j}u_2 & ; & & x_{3j}v_3 &\geq x_{2j}v_2 \\
y_{4j}u_4 &\geq y_{1j}u_1 & ; & & x_{4j}v_4 &\geq x_{1j}v_1 \\
y_{4j}u_4 &\geq y_{2j}u_2 & ; & & x_{4j}v_4 &\geq x_{2j}v_2
\end{aligned}$$

where $y_{1j}, y_{2j}, y_{3j}, y_{4j}$ are the amount of upper mean, upper standard deviation, upper skewness and upper kurtosis of the fund j under consideration; $x_{1j}, x_{2j}, x_{3j}, x_{4j}$ are the amount of its lower mean, lower standard deviation, lower skewness and lower kurtosis. $u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4$ are the weights associated respectively to these outputs and inputs. Conceptually, these additional constraints require that the contribution of the upper and lower skewness and kurtosis to the performance score of the fund j must be greater than the contribution of the upper and lower mean and standard deviation.

Otherwise, if investors care uniquely about extreme events rather than ordinary ones, i.e. distribution tails rather than central values, representing inputs and outputs respectively by the first (lower) quantiles (what investors want to minimize) and the last (upper) quantiles (what investors want to maximize) can be a viable alternative. It is noteworthy that from a statistical viewpoint, the use of quantile-based metrics is especially useful and robust when historical returns are not long enough or returns are skewed and contain outliers²⁴. In this spirit, I also considered a scenario where inputs are the first 5%, 10%, 15% and then 20% of the return distribution (lower quartiles) and outputs are determined in a similar manner with corresponding upper quantiles - 95%, 90%, 85% and 80%.

In order to illustrate another case where an investor needs to reconcile funds' performance over several horizons (from a long time in the past to a more recent period) while still regarding non normal characteristics of returns, I modeled inputs by the modified Value-At-Risk (MVAR) (Favre & Galeano 2002) representing the loss limits and outputs by mean

²⁴The author thanks Sessi TOKPAVI (LEO) for pointing out this issue.

returns over three horizons: 1 year, 3 years and 5 years. The merit of the MVAR is that it takes the asymmetric and kurtotic features of returns into account. It is equal to:

$$MVAR = W \left[\mu - \left\{ z_c + \frac{1}{6} (z_c^2 - 1) S + \frac{1}{24} (z_c^3 - 3z_c) K - \frac{1}{36} (2z_c^3 - 5z_c) S^2 \right\} \sigma \right] \quad (25)$$

with W is the amount of portfolio at risk, μ is return mean, σ is the standard deviation of returns, S is skewness, K is excess kurtosis, z_c is the critical value for probability $(1 - \alpha)$ ($z_c = -1.96$ for a 95% probability).

As there exists no other preferences of the evaluator, no additional constraint on the weight vectors is needed.

After inputs and outputs corresponding to evaluator preferences are specified, the next step consists in running the foregoing inputs and outputs under the CCR model. For each model, the weights associated to each output and input are constrained to be equal or greater than 0.01 ($\varepsilon = 0.01$)²⁵ to assure that all criteria are considered in the optimization program.

5.3 Results

Table 3 displays detailed results on DEA score, absolute weights (u, v) and virtual weights (uy, vx) obtained under a CCR setting with mean and skewness as outputs, standard deviation and kurtosis as inputs. Funds with negative scores are those having simultaneously negative mean and negative skewness. Given the difference of unit between mean, standard deviation on the one hand and skewness, kurtosis on the other hand, virtual weights rather than absolute weights provide more informative identification of key factors (inputs and outputs) that make some funds (1, 11, 27, 28, 35) efficient relatively to others in the sample. By contrasting these results to the statistics of returns given in Table 2, we notice that fund 27 and fund 35 are considered as efficient because they have fairly high mean and small standard deviation in comparison with the others, which results quite important weights assigned to these elements. By contrast, the relative efficiency of fund 28 is due to the height weight associated to its positive skewness. In fact, this fund has the highest positive skewness among funds in the sample. It is important to keep in mind that not all efficient funds fit the selector's preferences as each efficient fund possesses its own weighting system which is a function of its relatively favorable inputs and outputs. An investor who is more

²⁵In fact, all calculations were already tested with 4 increasing values of ε : 0, 0.0001, 0.001 and 0.01. However, the performance scores changed very slightly while the relative rank between funds remains unchanged. Thus, ε is fixed to be equal to 0.01 to facilitate result presentation.

or less markowitzian should be in favor of fund 27 or fund 35 while those willing to sacrifice some mean in exchange for some extremely high returns (positive skewness) should choose fund 28.

Table 3: DEA detailed results using mean-skewness as outputs and standard deviation-kurtosis as inputs (moment-based)

Fund name	Score ^a	Absolute Weights (u, v) ^b				Virtual Weights (u*y, v*x)			
		Mean	SK	SD	KU	Mean	SK	SD	KU
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1 IKOS Equity	1	146.98	0.01	0.21	0.37	0.997	0.003	0.008	0.992
2 Enterprise Lng/Shrt Eq	0.23	141.60	0.01	0.21	0.36	0.228	-0.001	0.007	0.993
3 Galleon Omni Tech. A	0.29	0.01	0.89	0.01	0.21	0.000	0.286	0.001	0.999
4 Primeo Select	0.39	155.48	0.01	0.22	0.40	0.388	-0.003	0.007	0.993
5 Gabelli Intl A	0.13	125.88	0.01	0.01	0.32	0.127	-0.001	0.001	1.000
6 GAM Japan Hedge Open	0.07	118.94	0.01	0.18	0.30	0.077	-0.004	0.004	0.996
7 Foyil Focused	0.62	0.01	0.68	0.01	0.16	0.000	0.618	0.001	0.999
8 Gruber & McBaine Cap	0.54	0.01	0.76	0.01	0.18	0.000	0.537	0.001	0.999
9 GAM Selection	-0.00	0.01	0.01	24.87	0.01	0.000	0.000	0.974	0.026
10 SR Asia	0.24	0.01	1.51	0.01	0.35	0.000	0.240	0.001	1.000
11 AJR International	1	66.71	0.24	1.28	0.18	0.719	0.281	0.064	0.937
12 Liberty Ermitage Selz	-0.00	106.35	0.01	0.17	0.27	0.006	-0.004	0.007	0.993
13 Momentum Stock Master	0.22	0.01	1.92	0.01	0.44	0.000	0.222	0.000	1.000
14 Park Place Intl Columbia	-0.01	0.01	0.01	10.10	0.01	0.000	-0.014	0.911	0.089
15 Permal US Opportunities	0.29	127.28	0.01	0.01	0.33	0.294	-0.001	0.001	0.999
16 Wimbledon Class M	0.35	122.73	0.44	2.36	0.34	0.290	0.057	0.067	0.933
17 Wimbledon Class C	0.51	155.09	0.01	0.22	0.39	0.513	-0.002	0.007	0.993
18 Absolute Alpha Opportunistic	0.21	146.46	0.01	0.21	0.37	0.214	0.000	0.006	0.994
19 IKOS Offshore Arbitrage	0.41	126.42	0.56	0.01	0.39	0.318	0.094	0.000	1.000
20 Sofaer Capital Global A	0.30	0.01	1.17	0.01	0.27	0.000	0.297	0.001	1.000
21 Lansdowne Europe	0.68	68.22	0.51	15.01	0.12	0.438	0.246	0.412	0.588
22 Park Place Europe \$	-0.00	0.01	0.01	24.65	0.01	0.000	-0.001	0.974	0.026
23 Park Place Europe	0.79	0.01	1.29	6.88	0.23	0.000	0.793	0.171	0.829
24 HSBC Selection Euro Equity	- 0.00	0.01	0.01	16.90	0.01	0.000	-0.001	0.967	0.033
25 Park Place Galileo Intl	0.89	0.01	0.66	3.52	0.12	0.000	0.888	0.192	0.808
26 Leonardo Capital	0.70	29.62	0.22	6.52	0.05	0.397	0.298	0.346	0.654
27 Key Europe	1	190.54	0.01	83.05	0.01	0.984	0.016	0.912	0.088
28 TR Kingsway A	1	13.89	0.21	29.18	0.01	0.102	0.898	0.697	0.303
29 SVM Highlander	0.59	0.01	0.48	2.58	0.09	0.000	0.592	0.131	0.869
30 SR Europe \$	0.45	0.01	0.89	0.01	0.21	0.000	0.448	0.001	1.000
31 SR Europe	0.21	60.50	0.22	1.16	0.17	0.166	0.046	0.058	0.942
32 GAM Europe Hedge Open	0.64	0.01	0.87	4.65	0.16	0.000	0.639	0.146	0.854
33 Lansdowne Europe Equity \$	0.67	0.01	1.17	0.01	0.27	0.000	0.675	0.000	1.000
34 Aspect European Equity	0.36	0.01	1.84	0.01	0.42	0.000	0.361	0.000	1.000
35 Zulauf Europe	1	90.94	0.01	31.40	0.02	0.996	0.004	0.931	0.069
36 Zulauf Europe \$	0.77	131.96	0.01	0.20	0.34	0.771	-0.003	0.007	0.993
37 Sofaer Capital Europe	0.50	55.54	0.41	12.22	0.09	0.208	0.294	0.318	0.683
38 Sofaer Capital Europe \$	0.04	0.01	1.17	0.01	0.27	0.000	0.044	0.000	1.000

Note: SD = Standard deviation, SK = Skewness, KU = Kurtosis.

^aFunds with negative scores are those having simultaneously negative mean and negative skewness.

^bu and v are constrained to be equal or greater than 0.01. For further explanations, refer to p.20.

Results on DEA scores across various sets of inputs and outputs are summarized in Table 4. Note that funds with negative scores are those whose outputs are all negative. Several points are noteworthy. In general, efficiency results are rather sensitive to the choice of input and output parameters. Not only the number of dominant funds²⁶ varies across

²⁶The terms "efficient funds" and "dominant funds" denote funds having DEA scores equal to 1 and are

scenarios – 3 according to the (partial moment-based) weight-restricted model (column 3) to 5 according to the standard partial moment-based model (column 2) and the standard moment-based model (column 1) – but also dominant members differ across tested sets of inputs-outputs. Look at for example fund 25 that is considered as efficient only by the standard partial moment-based model (column 2).

Regarding the introduction of additional constraints on weight vectors in the partial moment-based model (p.19), I found that attaching more importance to tail values rather than central values generally deteriorates slightly efficiency scores but in some cases alters radically the subgroup to which funds belong, i.e. from efficient to inefficient subgroup (columns 2 and 3). For instance, funds 11 and 25 do not belong to the dominant group anymore once weight restriction constraints are added.

These findings highlight the importance of the choice of appropriate inputs and outputs as well as of a correct specification of additional constraints corresponding effectively to the selectors’ preferences. In all cases, results must be interpreted with caution.

Besides, despite this general disparity, I noticed a certain concordance in fund (dichotomous) classification for several funds as 11, 26, 27, 28, 35. Let us look at fund 28. Whatever set of inputs and outputs used, it is always considered as dominant. This feature can be considered as a sign of the robustness of fund 28’s relative performance.

Since we are situated in the case where investors are assumed to be concerned about only funds’ return and risk, it can be interesting to contrast DEA results with fund rankings provided by the traditional Sharpe ratio (Sharpe 1966) and the recent modified Sharpe ratio (Gregoriou & Gueyie 2003). The formulas of these performance measures are given below:

$$Sharpe = \frac{\bar{r}_p - \bar{r}_f}{\sigma} \quad (26)$$

$$M\text{-}Sharpe = \frac{\bar{r}_p - \bar{r}_f}{MVAR} \quad (27)$$

where \bar{r}_p is the average return on fund p , \bar{r}_f is the average risk-free rate approximated here by the US 3-month T-bill rate, MVAR is described by equation (25). Note that the Sharpe ratio is based on the mean-variance paradigm while the modified Sharpe ratio takes into account skewness and kurtosis of returns.

Fund rankings according to these two ratios are reported in the columns 6 and 7 of Table 4. Several main observations can be drawn from these results. We can see easily that despite differences in the approach taken by the two measures, fund rankings are surprisingly quite

used interchangeably.

Table 4: DEA, Sharpe and modified Sharpe ranking results

	Fund name	Moment - based ^b	Partial moments ^a		Quantile - based ^c	Horizon - based ^d	Sharpe ranking	M-Sharpe ranking
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	IKOS Equity	1	0.91	0.87	0.79	0.40	7	7
2	Enterprise Lng/Shrt Eq	0.23	0.78	0.76	0.58	0.41	18	18
3	Galleon Omni Tech. A	0.29	0.92	0.91	0.35	-0.00	33	34
4	Primeo Select	0.39	0.76	0.74	0.60	0.16	14	15
5	Gabelli Intl A	0.13	0.80	0.78	0.52	0.08	21	20
6	GAM Japan Hedge Open	0.07	0.72	0.71	0.51	0.91	29	28
7	Foyil Focused	0.62	0.93	0.92	0.52	0.17	27	29
8	Gruber & McBaine Cap	0.54	1	1	0.57	0.11	25	26
9	GAM Selection	-0.00	0.83	0.80	0.50	0.21	28	27
10	SR Asia	0.24	0.86	0.82	0.51	0.11	23	23
11	AJR International	1	1	0.96	0.90	0.75	5	4
12	Liberty Ermitage Selz	0.00	0.75	0.73	0.46	0.23	26	25
13	Momentum Stock Master	0.22	0.78	0.76	0.46	-0.00	38	38
14	Park Place Intl Columbia	-0.01	0.74	0.72	0.44	1	34	31
15	Permal US Opportunities	0.29	0.89	0.87	0.51	0.08	17	17
16	Wimbledon Class M	0.35	0.78	0.76	0.63	0.22	16	16
17	Wimbledon Class C	0.51	0.81	0.72	0.66	0.61	12	12
18	Absolute Alpha Opportunistic	0.21	0.76	0.74	0.61	0.12	22	21
19	IKOS Offshore Arbitrage	0.41	0.82	0.80	0.65	0.16	15	14
20	Sofaer Capital Global A	0.30	0.86	0.82	0.52	1	19	19
21	Lansdowne Europe	0.68	0.80	0.78	0.76	0.63	6	6
22	Park Place Europe \$	-0.00	0.87	0.73	0.53	-0.00	36	35
23	Park Place Europe	0.79	0.91	0.74	0.71	0.19	11	11
24	HSBC Selection Euro Equity	-0.00	0.85	0.83	0.33	0.48	37	37
25	Park Place Galileo Intl	0.89	1	0.97	0.50	0.04	20	22
26	Leonardo Capital	0.70	0.89	0.89	1	1	4	5
27	Key Europe	1	1	1	1	0.87	2	2
28	TR Kingsway A	1	1	1	1	1	3	1
29	SVM Highlander	0.59	0.88	0.85	0.63	0.36	9	8
30	SR Europe \$	0.45	0.92	0.83	0.45	0.31	35	36
31	SR Europe	0.21	0.88	0.81	0.50	0.68	13	13
32	GAM Europe Hedge Open	0.64	0.84	0.84	0.43	0.15	30	32
33	Lansdowne Europe Equity \$	0.67	0.93	0.83	0.50	0.24	24	24
34	Aspect European Equity	0.36	0.89	0.83	0.58	-0.00	31	30
35	Zulauf Europe	1	0.85	0.83	1	0.94	1	3
36	Zulauf Europe \$	0.77	0.86	0.72	0.81	0.32	8	9
37	Sofaer Capital Europe	0.50	0.84	0.83	0.54	0.39	10	10
38	Sofaer Capital Europe \$	0.04	0.81	0.80	0.50	-0.00	32	33
	Number of dominant funds	5	5	3	4	4		
	Correlation (Sharpe,M-Sharpe)						0.995	

Note: All DEA efficiency scores are obtained from the standard CCR model.

^aThe partial moment-based set includes upper mean, upper standard deviation, upper skewness and upper kurtosis as outputs, lower mean, lower standard deviation, lower skewness and lower kurtosis as inputs (see Appendix 2). Standard denotes optimization without additional constraints on weights. WR implies optimization with additional constraints on virtual weights (see p.19).

^bThe moment-based set denotes the use of mean and skewness as outputs, standard deviation and kurtosis as inputs. Funds with negative efficiency score are those having simultaneously negative mean and negative skewness.

^cThe quantile-based set includes the first 5%, 10%, 15% and 20% of the return distribution as inputs, and corresponding upper quantiles 95%, 90%, 85% and 80% as outputs.

^dThe horizon-based set includes mean returns over 1 year, 3 years and 5 years as outputs and corresponding modified VARs as inputs. Funds with negative scores are those having simultaneously negative mean returns over three considered horizons.

similar, both in terms of correlation coefficient (0.995) and in terms of rank contrasting in couples from fund to fund. Does this strong similarity imply that the return distribution of all funds is quite close to the normal one? The answer according to the Shapiro-Wilk normality test is rather negative because the normality assumption is rejected in 14 among 38 cases at the confidence level of 95% (Table 2). However, finding explanations to such problem is beyond the scope of this paper.

Related to the connexion of DEA classifications with Sharpe and modified Sharpe rankings, the results show that most dominant funds (except for those classified by the horizon-based model) are generally among the seven funds the most highly ranked by Sharpe and modified Sharpe ratios. Nevertheless, funds 8 and 25 – considered as dominant under the standard partial moment-based model – are ranked only in the middle of the sample by the two performance measures. A closer examination of their return distributions shows that this contradiction is likely due to the much wider dispersal of returns and higher frequency of extreme positive values in these two distributions than in those of other funds. This finding implies that Sharpe and modified Sharpe ratios might not price properly good surprises, at least in the case of this sample. As a result, investors would have missed top-performing funds. Such a result supports the claim that DEA can be an efficient supplementary tool to assist investors in selecting correctly funds satisfying their preferences.

5.4 Efficiency stability analysis

Once efficient and inefficient funds are identified, it is also very important to examine the robustness of the DEA-efficient funds under possible unfavorable changes²⁷. Specifically, it is desirable to determine the range of worsening inputs and outputs (a decrease in outputs or/and an increase in inputs) within which an efficient fund remains efficient. The need for a sensitivity analysis of efficient funds can be justified in two different manners. At one level, results may be subject to erroneous data and we need to ensure that the obtained classification is tolerated within a certain extent. At another level, the knowledge about the stability region of efficient DMUs is valuable to assess their "what-if" robustness throughout various environment changes. In this sense, such kind of sensitivity can be a complementary tool to differentiate efficient funds and helps evaluators to choose the best ones.

For illustrative purpose, I applied the method developed by Seiford & Zhu (1998a), in

²⁷Stability analysis can be conducted on both efficient and inefficient DMUs. However, given that we are merely interested in efficient funds, only the first analysis is needed. Unfavorable changes may include DEA-model change, diminution or augmentation of the number of DMUs in the sample, data variations, etc. Here, I restricted my attention to the effect of data variations on efficient DMUs. For a review of the sensitivity analysis, refer to Cooper et al. (2001).

which the inputs and the outputs of the test DMU are worsened (decreased outputs and increased inputs) while those of other DMUs are improved (decreased inputs and/or increased outputs). Since this framework corresponds to the worst scenario, the mathematical optimization solution provides the largest stability region than any other methods. Besides, it is shown that this technique yields more exact robust results compared to those obtained by other methods (Zhu 2001). The mathematical program is presented in Appendix 3. Table 5 reports the results obtained under a CCR moment-based framework with mean and skewness as outputs, standard deviation and kurtosis as inputs. Let δ and τ denote the percentage variations of the inputs and outputs tested. As shown by Seiford & Zhu (1998a) and Zhu (2001), when there are simultaneous changes in inputs and outputs of all DMUs (including the test DMU), if $1 \leq \delta \leq \sqrt{1 + \Gamma_k}$ and $\sqrt{1 - \Gamma_k} \leq \tau \leq 1$, then DMU_k remains efficient. In Table 5, input increase (%) and output decrease (%) indicate the difference between the lower bound and the upper bound of δ and τ respectively. According to the results, fund 35 is the most stable among efficient funds as its inputs (outputs) can be increased (decreased) within the largest range without altering its efficiency score: an increase up to 10.78% of inputs along with a decrease up to 12.10% of outputs. By contrast, fund 1 is the less stable efficient fund because of its quite small range of unfavorable variations allowed: an increase of only 1.09% of inputs and a decrease of only 1.10% of outputs. Other things being equal, fund 35 is undoubtedly preferable to fund 1.

Table 5: Results of efficiency stability analysis under the moment-based model

	Efficient funds	Γ^a	Input increase ^b (%)	Output decrease ^c (%)
1	IKOS Equity	0.022	1.09	1.10
11	AJR International	0.182	8.71	9.54
27	Key Europe	0.167	8.01	8.71
28	TR Kingsway A	0.101	4.92	5.18
35	Zulauf Europe	0.227	10.78	12.10

Note: The moment-based model denotes the use of mean and skewness as outputs, standard deviation and kurtosis as inputs.

^a Γ is the optimal value of the objective function issued from the model reported in Appendix 3.

^bInput increase (%) = $(\sqrt{1 + \Gamma} - 1) * 100$

^cOutput decrease (%) = $(1 - \sqrt{1 - \Gamma}) * 100$

When the sensitivity analysis program is infeasible (i.e. no solution is reached), which is not the case of my example, Seiford & Zhu (1998a) and Zhu (2001) showed that the test efficient fund can infinitely increase its corresponding inputs and decrease infinitely its outputs while maintaining its efficiency. In this situation, the fund in question is located at an extreme position in one input or output (Seiford & Zhu 1998b). Conceptually, it can be inferred that this fund has an input (output) excessively low (high) in comparison to

other funds. Consequently, this input (output) is attributed an excessively high weight in the evaluation process so that any unfavorable change can not affect its efficient status. However, should we always choose this extreme fund? It is important to keep in mind that extreme efficient funds are not necessarily the best ones to choose. In this case, investors should identify the extreme input or/and output in order to know if this input or/and output correspond(s) effectively to the criteria that they appreciate the most in comparison to other criteria.

6 Concluding remarks

Previous empirical studies have documented that DEA could be a good tool to evaluate fund performance, especially the performance of hedge funds as it can incorporate multiple risk-return attributes characterizing hedge fund's non normal return distribution in a unique performance score. In this paper, I showed that DEA is a particularly suitable tool to select hedge funds when investors must face multi-dimensional constraints (which is quite often the case), especially when each one is associated to different priority levels. Each investor can tailor his own DEA model to incorporate his selection criteria and his personal preferences for each criterium. In other words, the flexibility of DEA in terms of the number of selection criteria possibly included as well as of the control of weighting system (the relative importance attributed to each selection criteria) can help investors choose the most suitable funds that fit their financial, risk-aversion, diversification and investment horizon constraints.

This study is different from previous studies in several points. On the one hand, instead of applying DEA to evaluate empirically the performance of a large sample of hedge funds including several categories, I generalized the framework and extended it to the selection of hedge funds. On the other hand, I focused on methodological issues like the choice of inputs, outputs and the form of efficiency frontier. In this perspective, some numerical examples are then given on a sample of 38 hedge funds to illustrate the case where investors are concerned about fund returns and risks with various settings of alternative measures of return and risk. I found that results on DEA efficient funds are, in general, rather sensitive to the choice of input and output parameters. This finding highlights the importance of the choice of appropriate inputs and outputs as well as the specification of additional constraints corresponding effectively to the selectors' preferences. A comparison between DEA (dichotomic) classification and rankings provided by the traditional Sharpe and the modified Sharpe ratios indicates that they are sometimes radically inconsistent. Further

examinations of funds' return distributions suggest that these two performance measures might not properly price good surprises (extremely high positive returns). In this case, DEA is a good supplement to improve the precision of selection tasks.

In my illustrative applications, I also introduced the sensitivity analysis to appraise the robustness of efficient funds. By providing a range of unfavorable variations (stability region) in inputs or/and outputs within which an efficient fund remains efficient, such analysis can be a complementary tool to differentiate efficient funds so as to identify the best ones. This range can also be considered as a security zone within which any error in data does not affect classification results.

Like any other tools, DEA also has its pitfalls. One of the main weakness arises from the way the efficiency frontier is formed. Under the DEA non-parametric framework, the efficiency frontier is empirically determined from real observations. As a result, it is sensitive to noises, the approach lumps noise and inefficiency together and calls the combination inefficiency. The econometric approach, though attempts to distinguish the effects of noise from the effect of inefficiency, confounds the effects of misspecification of functional form with inefficiency. However, compared with the free distributional approach taken by DEA, which is particularly convenient to multi-criteria selection making when selectors have no information about the relation form between these criteria, I believe that this cost is quite small. Complementary tools as sensitivity analysis or statistical tests (if exist?) can be helpful to mitigate this problem.

Appendices

A Appendix 1: Principles of incorporating evaluator preference in siting analyses using DEA²⁸

Table 6: Weights on criteria of five evaluators

Evaluator	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Sum
Evaluator 1	1.67	3.33	1.67	3.33	10
Evaluator 2	2.11	3.16	1.58	3.16	10
Evaluator 3	2.50	1.88	1.88	3.75	10
Evaluator 4	2.00	2.00	2.00	4.00	10
Evaluator 5	2.40	1.90	1.90	3.80	10
Average	2.14	2.45	1.81	3.61	10

In order to take into account the various views which the evaluators show in the Table

²⁸Source: Cooper, Seiford & Tone (2000, p.171).

6, lower and upper bounds are established in the following way. Let the weight for Criterion i be u_i . The ratio u_2/u_1 takes the value $3.33/1.67=2$ for Evaluator 1, $3.16/2.11=1.5$ for Evaluator 2, $1.88/2.50=0.75$ for Evaluator 3, $2.00/2.00=1$ for Evaluator 4 and $1.90/2.40$ for Evaluator 5. Thus we have the range of the ratio u_2/u_1 as:

$$0.75 \leq u_2/u_1 \leq 2$$

In the same way we can find the range of u_j/u_i as shown in the table below.

Table 7: Upper bound and lower bound ratio

Ratio	Lower bound	Upper bound
u_2/u_1	0.75	2
u_3/u_1	0.74	1
u_4/u_1	1.5	2
u_3/u_2	0.5	1
u_4/u_2	1	2
u_4/u_3	2	2

B Appendix 2: Computing moment-based inputs and outputs

The inputs include lower mean M_L , lower semi-standard deviation SD_L , lower semi-skewness S_L and lower semi-kurtosis K_L . The outputs contain upper mean M_U , upper semi-standard deviation SD_U , upper semi-skewness S_U and upper semi-kurtosis K_U . They are computed using formulas given below.

$$M = \frac{1}{n} \sum_{i=1}^n r_i ; SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (r_i - \mu)^2} ; m_k = \frac{1}{n} \sum_{i=1}^n (r_i - \mu)^k$$

$$S = \sqrt[3]{\frac{n^2}{(n-1)(n-2)} \frac{m_3}{SD^3}} ; K = \sqrt[4]{\frac{n^2}{(n-1)(n-2)(n-3)} \frac{(n+1)m_4 - 3(n-1)m_2^2}{SD^4}}$$

where r_i is return at the month i , μ is the Minimum Accepted Return (MAR) approximated by the average return on the 3-month US T-bill over the 5-year studied period (2000-2004), n is the number of return observations. The skewness (S) and kurtosis (K) formulas are from Kendall & Stuart (1958) (used by RATS and give the same results as EXCEL). M_L , SD_L , S_L and K_L are obtained by using above formulas with all $r_i \leq MAR$. In a similar manner, M_U , SD_U , S_U and K_U are calculated with all $r_i \geq MAR$. Note that partial skewness and kurtosis are respectively put in cube and fourth roots so that they are in similar scale as the partial means and standard deviations.

C Appendix 3: Efficiency sensitivity analysis for simultaneous changes in all inputs and outputs

Let I and O denote the input and output subset in which we are interested, then the CCR sensitivity model is as follows:

$$\min \quad \Gamma_k \quad (28)$$

$$\text{subject to:} \quad \sum_{j=1, j \neq k}^n \lambda_j x_{ij} \leq (1 + \Gamma_k) x_{ik}, \quad i \in I \quad (29)$$

$$\sum_{j=1, j \neq k}^n \lambda_j x_{ij} \leq x_{ik}, \quad i \notin I \quad (30)$$

$$\sum_{j=1, j \neq k}^n \lambda_j y_{rj} \geq (1 - \Gamma_k) y_{rk}, \quad r \in O \quad (31)$$

$$\sum_{j=1, j \neq k}^n \lambda_j y_{rj} \geq y_{rk}, \quad r \notin O \quad (32)$$

$$\lambda_j (j \neq k) \geq 0, \quad \Gamma \text{ unrestricted.} \quad (33)$$

And the BCC sensitivity model is obtained by adding the constraint $\sum_{j=1, j \neq k}^n \lambda_j = 1$ into the problem (28)-(33).

Let δ and τ denote the percentage variation of the inputs and outputs tested. As demonstrated by Seiford & Zhu (1998a) and Zhu (2001), when there are simultaneous changes in inputs and outputs of all DMUs (including the test DMU), if $1 \leq \delta \leq \sqrt{1 + \Gamma_k}$ and $\sqrt{1 - \Gamma_k} \leq \tau \leq 1$, then DMU_k remains efficient, where Γ is the optimal value to the system(28)-(33). When such an optimizing program is infeasible, it is shown that the test DMU's efficiency will be always preserved however its inputs and outputs and those of the others vary.

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