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Abstract:

In 1932, Jan Tinbergen proposed an explanation of the Great Depression based on a specific treatment of unstable processes and multiple equilibria. While he outlined a possible mechanism based on a specific treatment of firms' interactions, this first explanation was later abandoned in favor of more dynamic mechanisms, but he did not cast aside the idea of multiple equilibria and instability. After his involvement in the early meetings of the econometric society, he started working on different dynamic models that would account for this instability. In 1934, Tinbergen built a model to generate new types of economic movements that did not return to an equilibrium. This led him in 1936 to consider the possibility of having two equilibria, one stable, one unstable, with damped or self-sustained cycles around the high equilibrium and a collapse once the economy reaches the low equilibrium. Tinbergen saw these models, with reference to Fisher's 1933 classic *Econometrica* paper, as a way to interpret the potential of a crisis to trigger the collapse of the economy. At the end of the day, it turns out that Tinbergen managed to open a new avenue of research which, strikingly, remains almost totally ignored in most history of macroeconomics and econometrics.

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Introduction

It has taken some time for economists to acknowledge the uniqueness of the 1929 crisis. Reading back on the works of Joseph Schumpeter,[2] Alvin Hansen (in the early years of the Depression), and many others, the crisis was often seen as a bump in the road, a somewhat painful moment on the road to renewed riches. Change had to come from somewhere else in the approach of the economy.

The late 1910s and 1920s were marked by the development of new empirical techniques which led economists of different backgrounds to bring to light multiple types of cycles.[3] It also shaped the view that the depression was an inevitable follower to the boom, and vice versa so that most of the authors which attempted to explain it favoured the idea that there existed mechanisms responsible for the oscillation of the world around a state of stationary equilibrium.

However, in the early 1930s Irving Fisher was the most adamant to put that vision into question.[4] As one of the godfathers of the Econometric Society created in 1931, he shared his views at one of their first meetings and published them subsequently in his famous "Debt-deflation" article[5] (Fisher, 1933), where he described the possibility of collapse away from equilibrium with several striking metaphors; among them, the idea that the economic boat can sway and roll only while it is erect, but once capsized cannot go back to its old equilibrium. Maybe those ideas were less conducive to water displays and intricate machinery, but Fisher left to others the task of exploring his 49 creeds of 1933, seemingly to little avail at first.[6]

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[2] For instance, regarding the 1825 crisis Schumpeter wrote in (1934) that: "Although the masses did not suffer in silence and their riots, if nothing else, should have been sufficient motive for public action, things were allowed to take their course. And business did recover. Losses had to be taken, South American investments to be written off, prices had to find their levels, what was unable to adapt itself had to be eliminated. All this took time. But as soon as it was done, an upward swing set in by itself." (7) For a more nuanced approach of Schumpeter's interpretation of the crisis, see Muriel Dal Pont Legrand and Harald Hagemann (2017).

[3] See in particular chapter 2 of Morgan (1990), who describes the development of business cycle analysis in the early 20th century, at Harvard (Persons), Columbia (Mitchell) and in the sprawling institutes of business cycle research. In Moscow, the institute created by Kondratieff favored his research on long-term cycles, while Joseph Kitchin developed at the same time the forty months cycle. Another avenue of research on the business cycle was that of Henry L. Moore and the periodogram analysis (see chapter 1 of Morgan [1990]), but the aim was still to decompose the business cycle into different waves.

[4] See Dimand (2019) on why Fisher thought that the business cycle was a "myth".

[5] Fisher's 1933 article was the outcome of his presidential address at the econometric society.

[6] Because Fisher's prestige in the United States had been tainted by his mistakes on the real state of the economy just before the crisis, and their cost for those following him, one has in fact to look toward Europe to find someone willing to express Fisher's ideas in a new, more formal way (see Dimand 2019).

We argue in this paper that Jan Tinbergen, one of the most active members of the young econometric society, was the first to try to meet Fisher's challenge. Tinbergen was perhaps the most concerned to account for the specificity of the 1929 crisis as evidenced by the models he built and presented, often in German, Dutch or French rather than English, between 1932 and 1936.[7] He ultimately proposed the idea that a disturbance such as the 1929 stock crash could carry the economy over the hill, into an abrupt and bottomless precipice.[8]

Tinbergen clearly thought of his models - with reference to Fisher but not only - as a way to interpret the potential of the crisis to throw the economy overboard. He first presented clear ideas on instability as early as 1932 in an article written in Dutch for the Board of the Association for Economics and Statistics (Tinbergen, 1932). In this article, Tinbergen also explored a line of research based on multiple equilibria derived from Walrasian theory[9] and a problem of coordination between firms, which explained for him the necessity for the state to intervene in the economy. In his somewhat cryptic presentation of the mechanisms preventing the economy to reach a "good" equilibrium, one can see him struggling with the idea that firms may face a demand constraint.

Following this article and the early econometric society meetings, Tinbergen was decisively pushed to develop his own macrodynamic models, which allowed him to explore other mechanisms and to express his position on how to account for the instability of the economy. In a 1934 German article (Tinbergen, 1934b), he developed clear ideas on what makes the dynamic nature of the economy. These ideas were partly developed in a 1934 survey for *Econometrica* on economic theory (Tinbergen, 1934a), and more importantly in a wide-ranging survey of business cycle theories the following year (Tinbergen, 1935). This new dynamic framework gave a renewed importance to economic policy, which aimed to mitigate shocks and fluctuations.

This early dynamic work culminated the following year with a simple yet powerful model that managed to encapsulate both the idea of sustained fluctuations around an equilibrium and the idea of collapse within the same equation (Tinbergen, 1936). From the 1932 article, Tinbergen kept the idea of multiple equilibria, which explained this possibility of global instability. Published somewhat anonymously in a *Festschrift* for the

[7] The following citations are all based on our own translations.

[8] The impact of Tinbergen on the early development of the Econometric Society and on other econometricians is discussed by historians of economics such as Erwin Dekker (2020), Mary Morgan (2019) or Marcel Boumans (1993). See also the 1987 interview with Morgan and Magnus (Tinbergen et al., 1987), where Tinbergen also comes back to his work in the interwar, the departure from an empirical analysis of the business cycle and the early construction of economic models. A biography of Tinbergen is currently being written by Dekker. Strikingly, the treatment of instability by Tinbergen has remained unaddressed.

[9] Tinbergen mentioned several times that he came up with the idea of multiple equilibria after discussions with a younger colleague, J.G. Koopmans (not to be mistaken with Tjalling Koopmans), who worked on Walrasian theory at the same time (see Koopmans, 1932).

Berlin Business Cycle Institute and its director, this article has never been translated and has surely been eclipsed by the publication the same year of the *General Theory*, and Tinbergen's own ulterior work for the League of Nations. It does not however take anything away from this model and its ability to describe the vision explained by Fisher: a sufficiently large shock could capsize the economic boat, and throw it into the abyss. Thus we can show that Tinbergen managed to combine two visions: one of repeating cycles around a stationary equilibrium, developed by the econometricians, and one of global instability and mythical business cycle, developed by Fisher.

The paper is organized in three sections explaining how Tinbergen progressively took up the theme of instability (section 1), transferred it into a macrodynamic model in two steps (sections 2 and 3), and eventually managed to provide a framework conveying at the same time Fisher's insights and the dynamic conceptions just worked out by his fellow econometricians (section 4).

I. Escaping bad equilibria with economic policy

Tinbergen's 1932 paper was the outcome of a talk he gave for the Association for Economics and Statistics, "to explain the position of socialism with respect to the question of the regulation of production by private individuals."[10] The issues he touched upon were however much wider in their scope, and Tinbergen's approach very original and personal.

To the question "is popular prosperity so great under the system of free competition?" Tinbergen answered with mixed feelings. He underlined the recurrence of "breakdowns in economic life," which he thought should be avoided to obtain a smoother distribution over time of the "joys and burdens of life" (Tinbergen, 1932: 50 ff.). The fluctuations in economic life, source of so much ailments, prevented a full use of resources and led to a waste of technical and natural resources. The suppression of the uncertainty brought on by the cyclical character of economic life would have the added benefit of suppressing calculations from firms about the decisions of other firms, a mechanism he tried to encapsulate in a diagrammatic representation of the economy as we will see. His remarks on the underutilization of productive capacities were illustrated by statistics on the available capacity of industrial enterprises in the United States and in Germany, which showed that even at the top of the boom, the economy is not using its full capacities.

[10] See Knoester and Wellink (1993: 19-20) on this address and the rest of the paper as well as their book on the wider context of the talk, one of five given by Tinbergen in front of the Association, that became the Netherland Economic Association in 1950 and received the "Royal" title in 1987.

More importantly, Tinbergen noted that in June 1932, the end of the disruption that had begun in 1929 was nowhere in sight;[11] this led him to discuss the possibility that the economic process was unstable and to reflect on the economic policy needed:

> The question has been posed whether we are not dealing here with a movement process that no longer returns automatically to an equilibrium state, or whether we are not dealing here with unstable states. (*ibid*.: 56)

This led to an entire section on "stable and unstable economic processes".[12] Tinbergen argued that although previous crises were usually followed by a return to equilibrium, there were examples of unstable processes that did not lead to an automatic recovery, such as the inflation that plagued European countries in the early 1920s. This begged the question: "are there among the economic-dynamical processes, which theory teaches us, such that show an ever-increasing removal of the equilibrium state?" (*ibid*.: 58). Although such mechanisms were not discussed in the theoretical literature, Tinbergen was confident that he could find dynamic models accounting for instance for exponential movements away from the equilibrium.

The discussion was introduced with the hog cycle model,[13] in which case he noted that fluctuations usually led back to the equilibrium, but that if the elasticity of production became too great it could lead to fluctuations of increasing amplitude. He also referred to his own model of the "shipbuilding" cycle, published in 1931, where he had observed that the solutions of his model could yield fluctuations of increasing amplitude, depending on the values of certain parameters like the sensitivity of the supply of ships to the price of freight, and that "increasingly violent fluctuations" could arise. Although he was concerned with the rather restricted shipbuilding cycle in this article, Tinbergen already viewed in 1931 the relevance of such mechanisms for the whole economy:

> The basic problem of any theory on endogenous trade cycles may be expressed in the following question: how can an economic system show fluctuations which are not the effect of exogenous, oscillating forces, that is to say fluctuations due to some "inner" cause? (Tinbergen, [1931] 1959: 1).

And he had concluded his examination of the shipbuilding model by underlining the importance of finding a good dynamic mechanism to approach the problem of stability at the aggregate level:

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[11] "The crisis which broke out in 1929 led to a disruption of production which did not occur in any previous crisis … And still nothing can be noticed of an end of this disruptive process" (Tinbergen, 1932: 56).

[12] Tinbergen uses here and later the term "labile" which also exists in French and English but whose meaning is best translated as "unstable".

[13] Tinbergen emphasized the importance of this model developed by Arthur Hanau during his 1987 interview with Morgan and Magnus (Tinbergen et al., 1987).

> It seems to me that this result is of vital importance to the theory of economic dynamics. … Moreover, it gives us a clue to a method of judging the stability of an economic system in general. (Tinbergen, [1931] 1959: 12).

He also referred to his work on market level price dynamics that had been presented in 1931, during the first European meeting of the Econometric Society in Lausanne (published in the first issue of *Econometrica* in 1933). There, he started again from a simple cobweb model of supply and demand, that he complexified first by introducing "speculation" (demand depends on the rate of change of prices). This transformed his model into a mixed difference-differential equations, but Tinbergen did not stop there: he introduced a "hyperbolic demand", transforming his difference equation into a nonlinear equation, a dependence of demand on the activity of the industrial sector, a durable consumption good, and finally introduced a second market producing a capital good. Thus Tinbergen had already started to build up toward a more intricate model of the economy and its interdependencies. In all those cases, he was chiefly interested in the periodicity of the oscillations and always used hypotheses on the parameters that would make those oscillations self-sustained.[14]

However, the shipbuilding cycle as well as the cobweb mechanism remained focused only on one market and not the whole economy. Tinbergen pointed out in 1932 that if those models helped to identify new economic processes, they remained of little help to provide an appropriate framework to make sense of the working of the whole economy and the factors responsible for the ongoing crisis, in large part because they did not explain satisfactorily a mechanism of "removal from the equilibrium state".

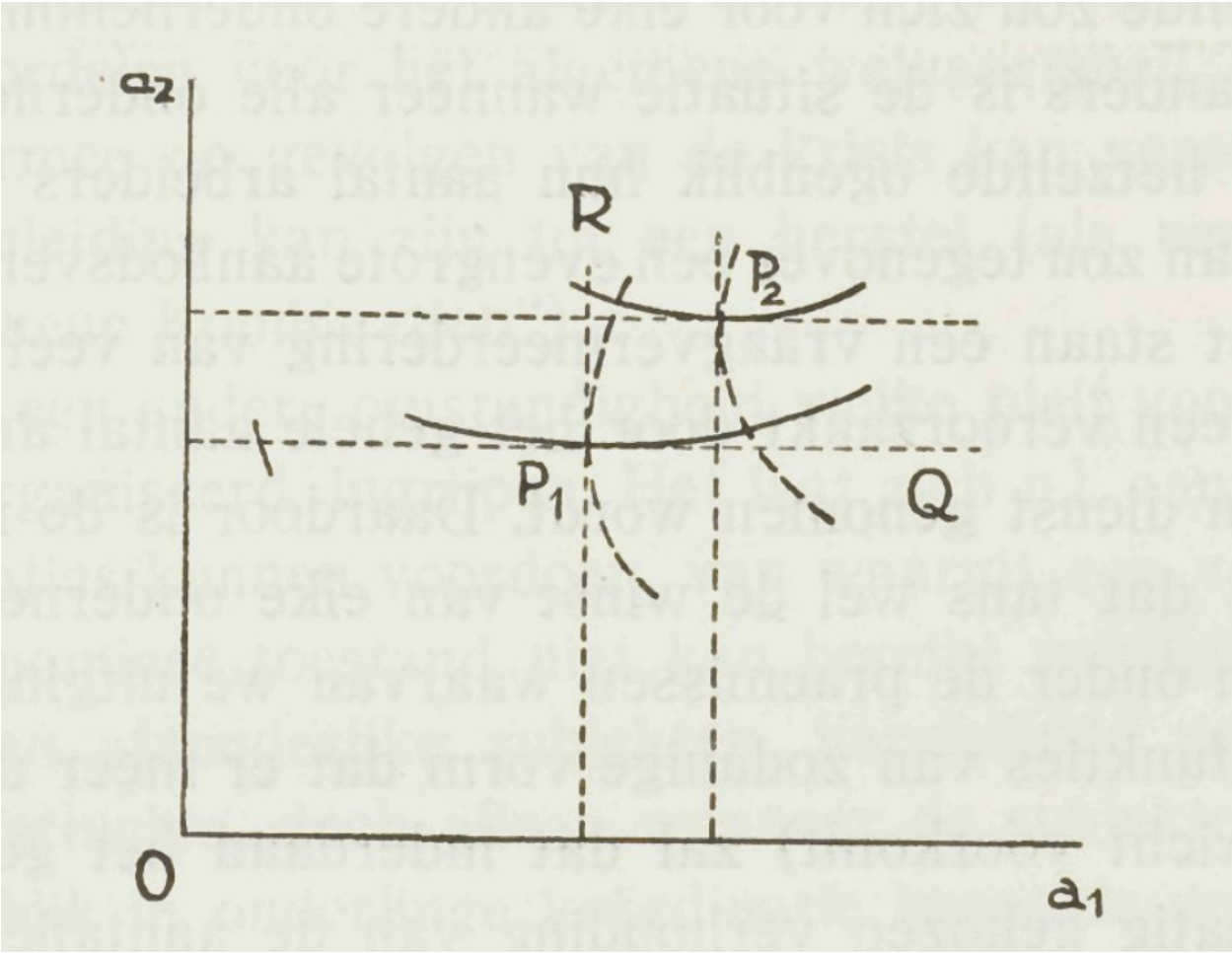
In 1932, he claimed that the solution to account for aggregate phenomena may come from the Walras-Cassel model of the economy.[15] According to him, this approach left ample room for multiple equilibria and instability to appear, and he described a possible mechanism based on a schematic representation of the interactions between two firms.

In this model with two sectors and two firms, the economy gets stuck in a "bad" equilibrium with high unemployment. Plotting the "profit lines" of each enterprise as a

[14] This was precisely the approach that Frisch reproached to Kalecki in his 1933 model. Tinbergen had prepared the ground for Kalecki's model, both through this approach of endogenous fluctuations, and his 1931 shipbuilding model, which is used by Kalecki to solve his own model. Following Boumans' (2004) idea that Kalecki's model was like a recipe, this shows that the ingredient coming from Tinbergen was more than a mathematical mould. The type of solutions favored by Tinbergen had a definitive impact on Kalecki.

[15] "Further investigation of the equilibrium equations (of, for example, Cassel or Walras) which determine the size of production, the level of prices, etc. in a stationary society, shows that there is not one equilibrium (in the sense of economic science), but that in general different equilibria are possible with the same technical and psychological data. It is quite possible, and even most likely, that with some form of utility functions and technical coefficients, two equilibria at least exist, both stable, one for instance with significant unemployment and the other without" (*ibid*.: 60). Rodenburg (2010: 10) mentions Tinbergen's argument in the context of his debate with the Dutch economist Goudriaan on dynamics and equilibrium.

function of their employment levels a_1 and a_2, that is, all the combinations of employment for which the profits of enterprise 1 and 2 are constant, Tinbergen provided the following diagram (*ibid*.: 62).



Tinbergen underlines that an increase of employment from only one firm will correspond to a displacement along a vertical or horizontal line. Because the curves meet at their minimum point, this means that a firm increasing its employment will always see its profits go down if the other firm does not increase employment as well. Tinbergen argues that the configuration where both firms have their profit curves crossing at their minimum points will thus correspond to stable equilibria, because there will be no incentive to deviate alone from such a situation.[16] In the picture above, there are two such equilibria: $P_1$and P_2. We can readily see that in the second of these equilibria,

[16] "Presence of two stable equilibrium points means, as can be easily ascertained, that there are two points P1 and P2, in which the profit line of 1 has a horizontal tangent and that of 2 has a vertical tangent. This nevertheless means that any change in the number of workers in one's own enterprise results in a reduction in profit: the condition for a stable equilibrium. In the figure, the action of separate companies always means a movement along vertical lines (for company 2) or horizontal lines (company 1)." (*ibid*.: 62).

there is more aggregate employment than in the first; this begs the question of how to move the economy from the "bad", low equilibrium, to the "better" equilibrium (*ibid*.: 61).

If we denote by R and Q the two other points where the profit lines intersect, we can see that the profits of both firms will only increase together in the $P_1\ Q\ P_2\ R$ quadrilateral. But in order for the economy to move in this direction, it is necessary that both firms increase employment at the same time, so that none has the incentive to reduce employment to go back to a higher level of profits.[17] The bottom line is that a new stable equilibrium can be obtained for a specific ratio between $a_1$and a_2 that only an organized action may help to reach: "In other words, it has been shown that there are cases in which competition cannot get out of a certain (low) situation, while organized action will make it possible" (*ibid*.: 61).

Unfortunately, Tinbergen does not explicit the set of equations that he might have used to derive the two profit lines, making it thus difficult to identify the exact mechanisms at work in the move of the economy from one equilibrium to the other. He describes however some potential mechanisms explaining how such a situation could arise, mainly from the mismatch in the increase of demand and supply when firms act alone, while when they act together this mismatch is construed to have a positive effect on profits because of a potential reduction in costs.

As he points out, in the case of a firm acting alone from equilibrium P_1, "the increase in demand which would arise as a result would by no means outweigh the increase in the supply of the commodity produced by that company" (*ibid*.). But with no other information on the consumption of the workers of the two firms and on the way both firms decide to invest, the argument is a bit enigmatic. The possibility that demand may eventually adjust to supply and reach the "good" equilibrium would in fact be due to the evolution of cost: "Expansion of the number of workers [...] also lowers the unit cost of production" (*ibid*.: 62), which, for reasons also left unexplained, is assumed to have a positive effect on demand of both goods.

Whatever the limits of Tinbergen's argument, it is clear that the issue of the stability of an equilibrium point is well present to his mind. From that point of view, the response J. G. Koopmans published a few months later proves helpful to make clear some features of Tinbergen's model and how far, in fact, Tinbergen eventually departed from the Walrasian approach.

[17] "Only an organized action - changing the number of workers in both companies at the same time - creates a movement along other lines. Now, from the point P1, one can only reach the point P2 without going through 'regions of lower gain', if one stays within the hatched curvilinear quadrilateral. And again, this is only possible if one does not go along horizontal or vertical lines." (ibid.: 62).

As we have seen, Tinbergen associated the idea of multiple equilibria with a Walrasian system of equations. In a footnote, he admitted that he was puzzled to see that while the idea was mentioned in passing in the literature, it had “not been elaborated anywhere” except in “some unpublished studies of Mr. J. G. Koopmans” suggesting that his analysis derived from it (*ibid*.: 60). A couple of months later, Koopmans published a paper with the aim to clarify his position with respect to Tinbergen’s analysis. While he acknowledged in the first paragraph of his paper “the much appreciated help” which “as a non-mathematician” he received from Tinbergen (Koopmans, 1932: 679), he felt the need to underline the differences between his and Tinbergen’s analysis. In fact, Koopmans made clear how far Tinbergen’s analysis departed from his own approach and this for two main reasons.

First and foremost, Koopmans emphasized that neither in Walras’s nor in Cassel’s works, one can show that there might be an equilibrium let alone a "stable equilibrium” as far as a “significant degree of unemployment is assumed” (*ibid*.: 682).[18] For Koopmans, if this condition is not met anymore, one market equation at least will have to disappear, letting the system undetermined with the result that many potential solutions become possible. If now one assumes that wages are fixed, it becomes possible to define a set of equilibria, each parametrized by a given level of money wage. Koopmans does not show how any such equilibrium may be reached. However, Tinbergen does assume that wages are given, and the problem for him is to show that for a given level of money wage, there may exist multiple equilibria and that among these equilibria some are stable while others are not.

The question now is to understand what equilibrium among the whole set of equilibrium will be reached. It is precisely that question that Tinbergen tackled when he discussed the stability properties of his two equilibria. What Koopmans did not see is that Tinbergen was concerned by the possibility to define a stable equilibrium and hence to define new mechanisms allowing to identify which equilibrium among all the equilibria defined by a given level of money wage was likely to be reached.

The second issue raised by Koopmans concerns the possibility to rank the equilibria. As Koopmans argues, the equilibria displayed by Walrasian systems cannot be easily ordered. From the point of view of policy, this makes a huge difference between Tinbergen’s multiple equilibria approach and Koopmans’ conception of the economy, a world with no nominal rigidities where there is no way to choose among the different equilibria:

[18] “After all, one of the essential conditions for the existence of a state of equilibrium in the sense of these two authors [Walras and Cassel] is precisely that, at the prices applicable to that situation, the supply of all end products and production factors, including human labor, is fully absorbed by demand. The equations in which this condition is expressed are therefore an indispensable constituent of the whole system of "equilibrium equations" through which, in the thinking of the aforementioned authors, the economic phenomena are examined.” (ibid.: 682).

> such an appraisal judgment can no longer be justified with the same certainty when it comes to the comparison between two (or more) alternative equilibrium states … which, after all, is characterized precisely by the fact that in neither of them there is a surplus of unused production factors … The remarkable thing about a double or multiple economic equilibrium in this last sense … consists precisely in the fact *that the various alternative equilibrium states are in every respect, thus also viewed from a normative point of view, completely "equivalent"*, or in others words, that none of these equilibria can be given a "preference" over one or more of the others on the basis of any objective criterion. (*ibid*.: 685, original emphasis)

Thus Koopmans underlined that the different equilibria did not imply that some factors of production were underemployed, and they did not justify a socialist policy of planning in order to obtain a higher employment equilibrium. It was necessary to introduce another criterium from which to establish that an equilibrium was better or worse than the other.

It seems that this critique was acknowledged by Tinbergen, for he did not pursue this line of research afterward. Nevertheless, he did not abandon the idea that there could exist multiple equilibria in the economy, and the fact that some of these equilibria may be unstable, and he quickly tackled the question from the perspective of the dynamical models used at the same time by the econometricians.

II. Stable unstable processes in macrodynamic models

Following his 1932 paper, Tinbergen started to work more seriously on dynamic models that could account for unstable fluctuations, in line with the program he set out in this article. This shift bears similarities with the work of other econometricians, especially Frisch and Kalecki, who were both present at the Leyden meeting of the Econometric Society organized by Tinbergen in late September 1933.

During this meeting, Frisch and Kalecki presented their models of economic fluctuations both based on the same family of mixed difference-differential equations already used by Tinbergen in 1931. The latter presented a communication seeking to answer the question “Is the theory of harmonic oscillations useful in the study of business cycles?” Building on the ideas presented in Lausanne in 1931, he tried to find the mathematical translation of different types of economic mechanisms (speculation, the effect of profits and losses, short lags, the rate of current investments, stock surpluses, changes in employment…). From the discussion, it clearly emerged however that Tinbergen was behind Kalecki when it came to economic mechanisms, although he tried perhaps more than any other to test empirically those different mechanisms; the energy spent on empirical verification and the quest for a good mechanism may explain why he did not have a fully-fledged model at the meeting.

A few days after the meeting, Tinbergen gave his inaugural lecture at the Rotterdam University of Applied Sciences, and a couple of months later, he published an essay based on this talk, which dealt with “The influence of purchasing power regulation on the business cycle” (Tinbergen, 1934b). The novelty of his approach in this article does not lie so much in the equations, which built on those already presented in Lausanne, but on the outlook he has on the model, and particularly on the importance of economic policy. Tinbergen used Frisch’s distinction between impulse and propagation mechanisms, and underlined the importance of finding the economic variables responsible for a particular mechanism:

> Further, distinction may be made between exterior forces causing disequilibrium and a mechanism propagating it. The mechanism in the abstract sense may be separated from the economic entities playing a role in it. (*ibid.*: 431. From the translated abstract on the back cover of the issue)

He also abandoned the idea of using the walrasian system, coming to the conclusion that its static character destroyed much of the interesting behavior that stemmed precisely from dynamic mechanisms, and chose to illustrate his ideas with several models explaining how the business cycle could be influenced by different economic policies. In the same way as he did in Lausanne, he introduced gradual complexifications into his model, regarding flexible wages and profit rates, a delay of production, changes in productivity, speculation and a depreciation of capital goods.

The fact that the same equations could be used to model a simple market or the whole business cycle was completely endorsed by Tinbergen, who separated what he called the “bearers of the mechanism” from the mathematical mechanism itself (*ibid.*: 290). In this article, the problem is truly macroeconomic: “Our analysis of the economic drivers of the economy should be started with the remark that the degree of employment or the production volume is at the center of the economic problem. The most important thing is to explain the movement of these two closely related phenomena.” (*ibid.*: 292). The basic model developed is the same as in 1933 for the study of one market: the problem was to explain the price level from the relation between a global supply, dependent on delayed prices (period of production, something taken from the work of Aftalion), and global demand, which is “assumed to be simply the quotient of purchasing power and price that comes onto the market.” (*ibid.*: 294). The equilibrium relation allowing the modeler to find the evolution of prices is that supply is equal to demand:

$$A + ap(t - \theta) = \frac{K+k(t)}{P+p(t)}$$

In this model, prices and purchasing power are measured as deviation from their equilibrium values, such that when they stop moving, we have $A = \frac{K}{P}$. The four cases

he presented varied the purchasing power and the supply functions, and he was interested in the economic policies needed to tame the fluctuations in purchasing power in each case. We will only describe the first one, which is used again in 1936 to show the problems arising from nonlinearities as explained below.

In the first case, Tinbergen introduced the idea that purchasing power fluctuates as a function of employment,[19] itself dependent on the decisions of production during a past period θ and thus ultimately on prices:

$$K + k(t) = K + \epsilon \int_{t-\theta}^{t} p(\tau) d\tau$$

Where ϵ is a constant that determines the relationship between variations in employment and purchasing power. This generated a nonlinear integro-differential equation of the form:

$$[P + p(t)][A + ap(t - \theta)] = K + \epsilon \int_{t-\theta}^{t} p(\tau) d\tau$$

In the introduction of the article, Tinbergen had pointed out that he remained within the bounds of linear models, justifying it by arguing that he was interested only in small deviations from equilibrium (*ibid*.: 292). It allowed him to discard higher terms of his nonlinear equation and transform it into the linear equation:

$$Ap(t) + aPp(t - \theta) = \epsilon \int_{t-\theta}^{t} p(\tau) d\tau$$

It can easily be seen that this equation can be transformed into a mixed difference-differential equation after differentiating it with respect to time, and Tinbergen applied the methods he had already described in 1931 to study the behavior of these equations. In particular, he was only interested in the solution with the lowest frequency (longest period), discarding other terms of the general solution as economically uninteresting and negligible.

Tinbergen's solution allows him to separate his parameter space into different regions of stability and dynamic form. In particular, he remarked that small changes in ϵ, the sensibility of purchasing power to employment, could transform an oscillation into a monotonous movement away from the equilibrium:

[19] "The assumption that the purchasing power used per unit of time depends on the level of employment seems to be in good agreement with reality, since most types of income are closely related to employment." (Tinbergen, 1934b: 300).

> as soon as ϵ is more than twice the size of $\frac{A}{\theta}$, there are no fluctuations at all, but only monotonous, either upward or downward movements. These latter movements can be compared with periods of inflation such as the German one, or with the dream of eternal prosperity across the ocean if the movements are upward, or finally with the "Crisis in perpetuity" as seen by left-wing socialists and communists when it comes to downward movements. It is striking that these movements can occur even at such low values of epsilon; a rather small structural change is enough to make an automatically returning movement a non-returning one! (*ibid*.: 303)

Thus Tinbergen realized that if economic policy could change the parameters of the system, and in particular the relationship between employment and purchasing power, the resulting movement could be completely transformed.[20] The parallels that Tinbergen drew between an exponential solution of his model and the real world consequences of hyperinflation, unbounded growth or total collapse also underline that he did not see those solutions as mere epiphenomenon, but as indication that his model could describe a wide variety of economic situations.

This interpretation by Tinbergen of the solutions of his model is particularly interesting, because it was not widespread at the time; in his 1933-35 model, Kalecki discarded non-oscillatory solutions, because he was only interested in the business cycle, as were most econometricians then. Samuelson (1974: 10) blamed Frisch's concern with stable cycles for the failure of most econometricians to see the exponential solutions of multiplier accelerator models until Harrod and Domar came along. The passage quoted above shows in fact that Tinbergen was very much conscious of those solutions and seriously considered their economic meaning.

The importance of exponential solutions for Tinbergen is confirmed by the fact that, when he proposed several numerical examples based on different values of ϵ, he computed two trajectories of damped cycles and one trajectory of "collapse" to a zero level of the endogenous variables (prices, purchasing power and supply), underlining with an exclamation mark that after only 6 periods, the purchasing power was theoretically below zero (*ibid*.: 304).

At the end of this section, he noted that those collapsing results were in part characteristic of the nonlinear form of the equation, that was not simplified for the numerical example, and argued that when compared to the linearized equation without the approximation: "you get somewhat different results, which, if I am correct, could be characterized as follows: an infinitely upward movement becomes impossible, an

[20] "small structural variations may completely change the type of the movement in converting for instance a cyclic movement in a one-sided one" (*ibid.*: 431. From the translated abstract on the back cover of the issue)

infinitely downward movement becomes possible even if it did not occur in the linear approximation." (*ibid*.: 305). Tinbergen proceeded to verify that intuition during the next couple of years, which led to the publication of a paper exploring the multiple equilibria generated by his nonlinear equation.

III. Multiple equilibria, the business cycle and complete collapse

Ach, mit offenen Armen stand ich gegen den Abgrund und atmete hinab! Hinab![21]
Goethe, *Die Leiden des jungen Werther*

Wenn nun schliesslich p(2) noch größer gewählt wird, so wird das zweite Gleichgewicht überschritten, und die Bewegung verliert sich sozusagen im Abgrund.[22]
Tinbergen (1936: 216), "Über den Wert Mathematischer Konjunkturtheorien"

Tinbergen referred to his 1936 article in his well-known survey on quantitative business cycle theories as part of a short section on "nonlinear equations" and multiple equilibria (Tinbergen, 1935: 295 ff.). With reference to the works of Vito Volterra and his Dutch student already mentioned, J.G. Koopmans, much of the section was devoted to the idea of saturation in economics (the fact that most economic variables have a maximum value, which introduces a *de facto* nonlinearity). Although the field of nonlinear dynamics had been pioneered by Volterra in the 1920s, the behaviour of nonlinear equations remained at that time unfamiliar to most economists.

Because the survey was the only publication in English of Tinbergen on this matter, and because Tinbergen had not chosen to put it at the heart of the discussion, this may have contributed to the neglect of his contribution in this field. Yet, and while his work is never referenced in modern accounts of the field which really gained momentum in the 1970s, one can agree that he showed "remarkable" results (Tinbergen, 1936: 212). In the survey, they were presented as a new exploration of possible trajectories:

> For instance, when the initial impulse is assumed to be increasing, the period increases, reaching infinity for a finite value of the initial disturbance. Parallel with this growing period, the form of the oscillations is changed. When the movement comes near the second, unstable equilibrium that exists in this case, it tends to move for a long time parallel to that equilibrium and so to prolong the part of the oscillatory movement situated between the two equilibria. When the original disturbance surpasses a certain limit, however, the movement breaks through the

[21] "Oh, with open arms I stood against the abyss and breathed down! Down!"
[22] "If finally p(2) is chosen to be even larger, the second equilibrium is exceeded and the movement is lost in the abyss, so to speak."

> unstable equilibrium and the movement does not come back. (Tinbergen 1935: 295)

The argument was eventually elaborated in Tinbergen's "Wagemann *Festschrift*" paper under the title: "On the value of mathematical business cycle theories".[23] On the basis of his 1934 model and with reference to the idea of multiple equilibria that he had introduced in 1932, Tinbergen was finally given a chance to clarify the properties of a nonlinear model. The first four sections summarized the results of his 1934 paper, with the introduction of some new variants into his linearized original scheme while the nonlinear complete system was discussed in the fifth section:

> Up to now we have limited ourselves to small disturbances of equilibrium. This made it possible to neglect quadratic terms … and therefore to obtain a more systematic solution to these equations. We want to drop this restriction now. Unfortunately, this also means that the possibility of a systematic solution disappears, and one has to do with less general methods. However, since the results, it seems to me, are extraordinarily remarkable, something should be said about them. (Tinbergen, 1936: 212)

The model was essentially the same as in 1934,[24] but it is explicitly the prices that Tinbergen wants to find, and the integral is replaced by its average such that

$$P + p(t) = \frac{K+k\int_{t-\theta}^{t} p(\tau)d\tau}{A+ap(t-\theta)} = \frac{K+k\theta p(t-\frac{\theta}{2})}{A+ap(t-\theta)}$$

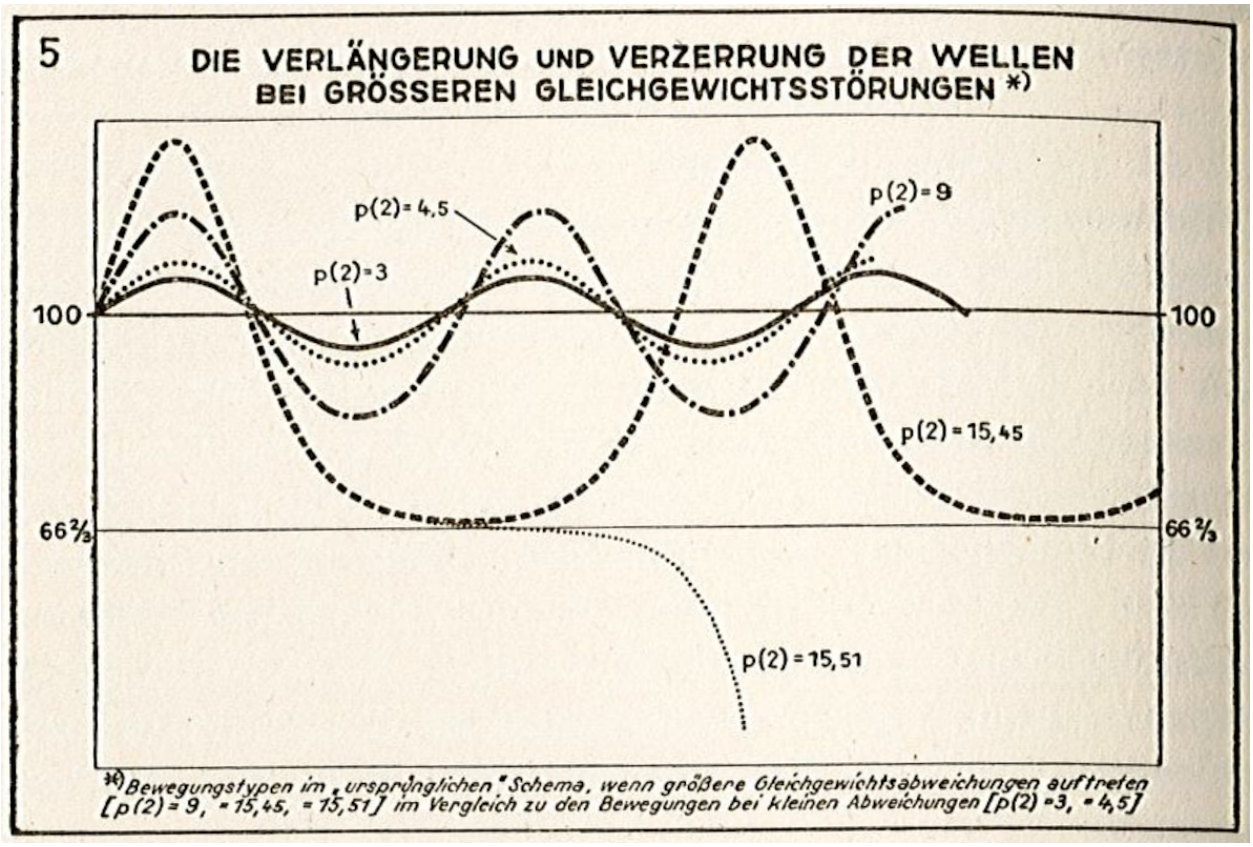
This transforms the equation into a nonlinear difference equation with two lags.[25] Because this equation is quadratic, there will be two equilibria, one stable and one unstable; Tinbergen notes that we already know the equilibrium when prices are equal to zero, and he assumes that the second will be equal to c. He expresses this equilibrium by replacing the values of p by this new equilibrium in the above equation, to obtain $P + c = \frac{k\theta-A}{a}$.

According to the values of the parameters, the second equilibrium will be above or below the first. By linearizing around the equilibrium in each situation, Tinbergen is able to show that it will always be the lowest equilibrium that will be unstable. One can also

[23] The "Wagemann *Festschrift*" was a book published to celebrate the ten-year anniversary of the Business Cycle Institute of Berlin, which was headed by Ernst Wagemann (see e.g. Tooze, 1999).
[24] Although the supply lag is not written in the 1936 article, computations and the fact that Tinbergen mentions it in the text show that this clearly is a misprint.
[25] This marked a first shift away from the mixed difference differential equations that were a dominant feature of his work and that inspired other early econometrician's work (in particular Frisch and Kalecki).

note that he still examines the case of undamped fluctuations, but this time he does not need to vary k (the ϵ of the 1934 model), to obtain a collapse of his model economy.

To compute a numerical solution of the model, one only needs two initial conditions for the two lags. To show very concretely the possible trajectories and behavior of his model, Tinbergen computes several trajectories generated by different initial impulses, that is, different values of the second initial condition $p(2)$ (while $p(1)$ is always assumed to start at equilibrium).[26] The following figure from the article summarizes his findings.[27]



One can clearly see in the figure that different impulses will yield a longer period, because when the economy is attracted by the second equilibrium during the depression phase, the force of this attraction will be a function of the initial shock. As the shock gets bigger, prices edge closer and closer to the symbolic line delimited by the unstable equilibrium, and at some point it will go overboard if the shock is higher than a certain value, much like a ball gathering enough momentum to go up a hill and over the top to a new part of the “dynamical landscape”, in that case a steep cliff - an abyss: “If

[26] Again, the paper has a misprint on the value of k, which is 500/6 and not 5/6.
[27] An application allowing the interested reader to reproduce those results is available here: https://economic-instability.com/wp-content/applications/tinbergen_1936/.

finally p(2) is chosen to be even larger, the second equilibrium is exceeded and the movement is lost in the abyss, so to speak." (*ibid*.: 216).

The fact that it is always the low equilibrium that is unstable is very interesting, because as Tinbergen pointed out in 1934, it means that a shock, whether it is positive or negative, cannot cause the economy to go on a growth path toward "eternal prosperity", but if large enough will in both cases cause the economy to sink into an abyss without return. Again, this means that a collapse can not only come from negative shocks of sufficient magnitude, but also from positive shocks that can throw the economy into such a situation that it eventually collapses after a period of boom!

While in 1934 the different possible dynamics of the system relied on a change of parameters, in 1936, Tinbergen realized that the nonlinear version of the model only needs to rely on the strength of a shock (initial conditions). Thus an oscillation produced by two different shocks can yield different periods of oscillations, or even a monotonous trajectory if the shock is sufficiently large. Linking this with Frisch's framework of propagation and impulses, this means that in the stream of shocks, a shock with a magnitude too important would potentially throw the economy in complete collapse, "into the abyss", a new part of the dynamical landscape.

IV. Meeting Fisher's (1933) challenge with multiple equilibria

The absence of fluctuations was for Tinbergen an important part of empirical findings that needed to be explained.[28] In that perspective, the notion of disturbance remains central, as for Frisch. But now, the issue was to determine both how a system once hit in one direction or the other, may swing back to normal and to include in the analysis the possibility that the economy may collapse in the absence of public intervention.

It is in that context that Tinbergen made a connection between his analysis and Fisher's 1933 *Econometrica* article.[29] Clearly, Tinbergen had in mind Fisher's critique of all business cycle theories which, in his opinion, remained confined in a vision that made it impossible to identify other trajectories than those characterized by the recurrence of ups and downs around a state of equilibrium.[30]

[28] With reference to Carl Snyder, Tinbergen admits that the fluctuations may not occur. But according to him "it is not necessary to always 'have' waves before you can start to say something; rather, it is interesting to talk about why clear cycles do not occur here and there" (Tinbergen 1936: 198).

[29] Tinbergen also mentioned an article of Wilhelm Röpke published in the Cassel festschrift "The secondary crisis and its overcoming" (Tinbergen, 1936: 204).

[30] As Fisher noted in a provocative way about "cycle theory in general", whether it is based on "over- or under-production, over- or under-consumption, over- or under- spending, over- or under-saving, over- or under-investment, and over or under everything else" (Fisher, 1933: 339), existing theories can do no more than showing how the economy is cycling around its own equilibrium.

Fisher argued that the problem with that vision of the movements of the economy is that some crises like the Great Depression cannot be understood as a normal feature of the business cycle for the sheer fact that the economic system may well have no automatic tendency to reach its “perfect equilibrium”:

> There may be equilibrium which, though stable, is so delicately poised that, after departure from it beyond certain limits, instability ensues, just as, at first, a stick may bend under strain, ready all the time to bend back, until a certain point is reached, when it breaks. This simile probably applies when a debtor gets "broke," or when the breaking of many debtors constitutes a "crash," after which there is no coming back to the original equilibrium. To take another simile, such a disaster is somewhat like the “capsizing” of a ship which, under ordinary conditions, is always near stable equilibrium but which after being tipped beyond a certain angle, has no longer this tendency to return to equilibrium, but instead, a tendency to depart further from it. (Fisher 1933: 339)

So, moving away from “the cycle theory” compels one to adopt a view in accordance with the idea that the economic system may behave differently for large and for moderate displacement from equilibrium. Within some range of the equilibrium, the system may work well and shocks may trigger self-adjustment mechanisms. Outside of that range, these mechanisms may however become weaker so that the system may be increasingly subject to the self-perpetuating aggravating mechanism. This is because, depending on the size of the shocks, stabilizing effects may be too weak relative to destabilizing effects. In that case, and in the absence of countercyclical policy, the economy could hence slip into a bottomless depression similar to a stick which would have been bent too much or a ship which would have been tipped too far.

Very clearly, even though the mechanism to which he referred to was not similar to the one defined by Fisher,[31] Tinbergen came close to his vision but without breaking with the works of his fellows from the Econometric society. As he points out in the opening paragraph of his 1936 paper, macrodynamics as defined by Frisch is not confined to the study of the business cycle but may encompass, besides cyclical movements, all sorts of movements. With the possibility to have either damped or self-sustained cycles around the high equilibrium and a collapse once the economy is in the neighbourhood of the low equilibrium, Tinbergen was at the intersection between Frisch, Kalecki and

[31] Fisher argues that two dominant factors explain the possible transformation of business contraction into a deep depression: “over-indebtedness to start with and deflation following soon after.” When the level of debt is high and deflation is severe, stabilizing price level effects may well be dominated by destabilizing price change effects. In that perspective, Fisher does not deny that the central mechanism of most business cycle theories like over-investment, over speculation, etc, may play a role. Though, in his view, It is only in the presence of debt and deflation that great crises may occur: “if debt and deflation are absent, other disturbances are powerless to bring on crises comparable in severity to those of 1837, 1873, or 1929-33.” (Fisher 1933: 341)

Fisher. Depending on the degree of damping of the system, economic movements around the high equilibria are similar to those described by Frisch (when cycles are damped) or Kalecki (when cycles are undamped) while the one around the low equilibrium may account for Fisher's view.

The originality of this combination is all the more interesting given that other econometricians such as Frisch and Kalecki knew and mentioned Fisher's work, but did not want to or were unable to translate his vision into their models. In Ragnar Frisch's famous rocking horse model (Frisch, 1933), the propagation mechanism always tends back to its equilibrium, and is merely disturbed and pestered by external impulses. Frisch was ready to admit that Fisher's theory may help identify new mechanisms, implying that mixed differential and difference equations may provide a natural framework for his insights, although he did not propose himself an interpretation of Fisher's ideas. With the aim to explain business cycle and growth, Frisch resorted to a dynamic equation that could only give rise to damped oscillations and no trajectories close to the one defined by Fisher. It seems clear that if he was ready to base his analysis on Fisher's mechanisms, he did not go as far as to follow him on the road of instability and collapse.[32]

Concerning Kalecki, even if his model conveys the vision of a crazy world - where the capitalists are stuck in a running wheel and never stop running although they never move, a semblance of structural stability is preserved. The wheel stays attached and the ability of the system to reproduce itself is never questioned. Kalecki (1936) paid attention to the importance of debts and deflation and was sensitive to Fisher's arguments, but did not embrace his vision. In his opinion, there are always automatic factors that are likely to help an economy come out of a crisis. He considered the Great Depression to be one phase of the trade cycle that would, at least in theory, end up in an upswing. Sooner or later, the economy would move out even if he did not deny that this may take time and be accompanied with high unemployment.

Given the incredible richness of Tinbergen's model, one may wonder why Tinbergen did not pursue more of this avenue of research. Several explanations may be advanced: first, Tinbergen knew that nonlinear models were hardly tractable and had himself underlined the fact that he could give numerical example, and compute the position of equilibria, but it was difficult to say much else on the qualitative behavior of his system (of course this qualitative theory came much later and gave birth to a vast literature toward the end of the century). There was in general a steady evolution of Tinbergen

[32] See Carret (2020) on Frisch's model and its solutions. Vinci (1934), another early econometrician, attempted to capture Fisher's ideas but was not able to close his model. The visions developed by Frisch, Kalecki and Tinbergen are detailed in the forthcoming book (Assous and Carret, 2021), where the analysis is extended to other authors like Ludwig Hamburger, as well as Samuelson and Oskar Lange in the United States (see also Assous and Carret [2020] on the latter).

toward simpler equations, from nonlinear and mixed difference-differentials to recurrence relations with several lags, that made for easier testing and analysis. Thus the last system, although nonlinear, did not have differentials or integrals anymore but just two lags.[33]

One can also add that Tinbergen was very much aware that a linearisation around the equilibrium was often sufficient when working with difference or differential equations. In 1936, he went back to the full nonlinear system only to observe what would happen when a shock of a large magnitude would hit the system and cause it to fluctuate widely around the equilibrium. But in all his papers and his reflections upon the business cycle, the central concern was how to regulate it, how to mitigate the impact of shocks and the fluctuations of the propagation mechanism. Thus, in Tinbergen's framework, economic policy can be viewed as a "linearisation" mechanism that ensures the economy does not stray too far away from its equilibrium, lest it would be absorbed by another, dangerously unstable attractor.

[33] One can note that this was similar to Samuelson's famous 1939 accelerator multiplier model, a linear model with two lags (Samuelson, 1939).

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