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Abstract

We look at the optimal inter-temporal management of the fuel reservoir of nuclear units in a liberalized electricity market. We use the assumption that nuclear fuel works as a "reservoir" of energy due to the periodical shutdown of nuclear units to reload their fuel. In the medium-term, how a producer sets the nuclear fuel of the reservoir to respond to the variations of seasonal demand in order to maximize its production value on a multi-annual basis? The dynamic nature of the nuclear fuel reservoir highlighted the discontinuity of the price which complicates the resolution of the optimal inter-temporal production problem and even leads to a lack of solutions. Theoretically, at the optimum, nuclear is used to serve baseload and thermal follows demand's variations. Numerically, both nuclear and thermal units operate in load-following mode. Solutions characterized by a constant nuclear production do not exist which shows that the significant share of nuclear in the energy mix does not permit to produce at a constant rate unless further investments in thermal capacity are done. Inter-temporal optimization shows the role of nuclear for ensuring the equilibrium between supply and demand.

Key words: Electricity production, nuclear fuel reservoir, inter-temporal optimization, thermal production, merit order price, discontinuity problem.

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1 Introduction

The novelty of our research consists of the characteristic of the "reservoir" of nuclear fuel which results from the discontinuous reloading of nuclear reactors. Every 12 or 18 months, nuclear reactors reload their fuel and then a period of production named "campaign" begins. In the medium-term, a producer sets its production during a campaign of nuclear fuel reservoir to respond to the seasonal variations of demand and to maximize its profit. The important size of the French[2] nuclear set is the main reason why nuclear does not serve only as a baseload[3] generation technology but it has to be flexible[4] to participate significantly in the modulation of supply between winter (season of high demand) and summer (season of low demand) and therefore ensure the stability of the electricity grid (Pouret and Nuttall (2007), Bruynooghe et al. (2010)). This can be seen in the monitoring report of the Regulatory Commission of Energy which provides us with an illustration of the management of the French nuclear set (Regulatory Commission of Energy (2007)). In a market based electricity industry, the objective should be the maximization of the value of electricity production. Consequently, the question of the optimal management of the nuclear fuel reservoir during a campaign of production arises.

A number of technico-economical constraints regarding the nuclear production need to be considered which makes instantly our model complex. Firstly, we look at the constraints imposed by the flexible management of nuclear units (minimum/maximum production constraints). Generally, a nuclear unit can vary its capacity level between the nominal capacity and the technical minimum. In the case of an EPR, load follow enables planned variations in energy demand to be followed and can be activated between 25% of nominal capacity (technical minimum) and 100% of nominal capacity (technical maximum) (NEA/AEN (2011)). Secondly, nuclear fuel constraints result from the inter-temporal management of the nuclear fuel stock during the production period given the periodical interruptions of nuclear production to reload reactors with fuel. Thirdly, we have to take into account the constraints imposed by the generation capacity of nuclear and thermal units (nuclear, coal, gas). Finally, the equality between supply and demand every month is an essential constraint that all nuclear producers need to take into consideration because the very large proportion of nuclear in the national energy mix makes this constraint heavily dependent on nuclear generation.

The consideration of competition is another aspect that complicates our model since France in contrast to other countries (e.g. UK) has not completely opened till now its electricity market to competition. Actually, the French nuclear operator (EDF) is essentially public and dominates the electricity market which shows that there is not sufficient place for competition in the electricity market (Chevalier (2004)). Consequently, the optimization of the management

[2]Several countries with a significant share in nuclear generated electricity, Slovakia (51.8%) and Belgium (51.1%), partly operate in load-following mode.

[3]Baseload plants are the production facilities used to meet some or all of a given region's continuous energy demand, and produce energy at a constant rate, usually at a low cost relative to other production facilities available to the system.

[4]From a technical point of view, nuclear reactors of modern design (the third generation and its evolution III+) are capable of a flexible operation (Nuttall and Pouret (2007), Guesdon et al. (1985)). In fact, this flexibility is primarily due to the new types of fuels which affect the constraints that determine the speed of increase and decrease of production. This type of constraint (called ramping rate constraints) binds the change of operation level of a unit between two successive periods. In principle, all nuclear reactors might reasonably be regarded as having some capacity to follow load. In practice, however, the ability to meet grid needs efficiently and safely is restricted to a certain set of design types (for technical engineering, safety and licensing reasons). The new reactor EPR, which is an evolution of the pressurized water reactor (PWR), is an example of a III+ generation nuclear reactor which is designed to accommodate load-following operation (AREVA (2005), Goldberg and Rosner (2012)).

of the nuclear fuel reservoir of flexible nuclear plants in a purely competitive electricity system is an entirely new question in the public literature. Creating a benchmark in order to respond to this question could give insights into the optimal production behaviour of nuclear producers for policy and industry.

In the second chapter of the Ph.D. thesis of Lykidi (2014), we analyzed the optimal management of the nuclear fuel reservoir on a short-term (monthly) time horizon of operation in a competitive setting. Following, this partially "myopic" (short-sighted) approach of optimization of the nuclear production, we pass to the "non-myopic" case being the inter-temporal optimal management of the nuclear fuel reservoir. The optimal short-term production problem is an essential step in order to build a complex model based on the operation of flexible nuclear units in a competitive market dominated by the nuclear generation and the characteristic of the nuclear fuel reservoir, and then, to cope with a large number of mathematical, technical and computational difficulties in order to find an optimal production trajectory. Its existence permits to move to a more complex optimal production problem based on the inter-temporal optimization of the nuclear production in an identical framework. In addition, the analysis of the outcome of the optimal short-term production problem helped to discover the potential advantages and disadvantages of such approach before we proceed with a full optimization of the nuclear fuel stock. Here, the nuclear managers aim to optimize the operation of the reservoir on a time horizon which consists of several campaigns (typically 36 months).

In section 2, we start with the modelling of the optimal inter-temporal production problem in a framework of perfect competition. This framework also keeps modelling and assumptions identical to those made in the case of a short-term optimization of the operation of the nuclear fuel reservoir examined in the second chapter of the Ph.D. thesis of Lykidi (2014). Then, we introduce the modelling of the optimal inter-temporal production behaviour. It consists of the maximization of the inter-temporal profit under the optimization constraints mentioned above. In section 3, we study the optimization of the inter-temporal production. However, we meet a mathematical-economical-technical difficulty which is that of the discontinuity of the merit order price. We treat this difficulty and then, we provide a new property that fully characterizes the optimal solutions of the optimal inter-temporal production problem in the particular case that production constraints are not binding. In section 4, we propose a simple numerical model which analyzes the optimal production decisions resulting from the optimal inter-temporal production problem. In the same section, we study an economical feature of producers' optimal behaviour: the symmetry of an equilibrium of the optimal inter-temporal production problem. We apply Scilab[5] to run our simulations by using some basic data. To end, the numerical results of the optimal inter-temporal production problem are contrasted with the numerical results of the optimal short-term production problem in order to compare these two approaches. Section 5 concludes.

2 Model: Perfect competitive case

In this section, we present briefly our general deterministic multi-period model of perfect competition[6]. Producers operate both with nuclear and thermal plants and there exists perfect

[5]Scilab is an open source, cross-platform numerical computational package and a high-level, numerically oriented programming language. It can be used for numerical optimization, and modelling, simulation of dynamical systems, statistical analysis etc.

[6]Let us remember that in presence of perfect competition firms are price-takers: they treat price as a parameter and not as a choice variable. Price taking firms guarantee that when firms maximize their profits (by choosing the quantity they wish to produce and the technology of generation to produce it with) the market

information among them. Our modelling aims to determine the inter-temporal optimal management of the nuclear fuel reservoir in that competitive framework. We focus on the medium-term horizon in order to take into consideration the seasonal variations of demand during a campaign of production. Our optimization constraints contain nuclear fuel storage constraints, production constraints coming from the flexible operation of nuclear units and the generation capacity as well as constraints resulting from the equality between supply and demand. These constraints are decisive for the determination of the optimal solutions in this wholesale electricity market.

In our model, a number of assumptions have been made mainly in order to avoid complicate our model and because it is not possible to have access to detailed data. In the second chapter of the Ph.D. thesis of Lykidi (2014), we provide analytically the reasons which impose and/or motivate our hypothesis and the way that they may influence our modelling supposing that we did not effectuate them. In particular, importations and exportations as well as the production resulting from hydro-storage units[7] and renewables[8] are not considered within our model. Another point is that we calculate the profit without taking into account a discount rate or a mark-up rate. Note that mathematical proofs and numerical data can be found in the third chapter and the annexes of the Ph.D. thesis (Lykidi (2014)).

2.1 Modelling the generating units

We study a competitive electricity market with $N \geqslant 2$ producers who manage both nuclear and thermal generating units. A producer $n = 1, \cdots, N$ can operate with all types of nuclear generating units. Moreover, each producer disposes of a certain amount of thermal capacity.

2.1.1 Concept of type

Among the nuclear generating units, we distinguish several essential intrinsic characteristics:

- available nuclear capacity,
- minimum capacity when in use,
- month of their fuel reloading.

In our model, the minimum capacity is proportional to the available capacity, and this proportion is the same for all "physical" nuclear reactors. Therefore, for each "physical" nuclear reactor, we will focus on the month of fuel reloading, which permits us to define twelve "types" of nuclear units. Each type indexed by $j = 1, \cdots, 12$ corresponds to a different month of reloading of the nuclear unit. Then, a unit which belongs to the type of unit $j = 1$ (respectively $j = 2, \cdots, j = 12$) shuts down in the month of January (respectively February, $\cdots$, December).

price will be equal to marginal cost.

[7]There exists an extensive literature that studies the optimal management of hydro-reservoirs in mixed hydro-thermal competitive markets and where one can see several modellings of the optimal production problem and notice the increased level of difficulty from a theoretical and numerical point of view (Arellano (2004), Bushnell (2003)).

[8]The electricity production coming from renewable energy plants is variable or intermittent because of the stochastic nature of weather patterns which means that it should be a stochastic endogenous variable in our model. Its consideration would impose a radically different modelling, a stochastic modelling, whose nature is not consistent with the deterministic character of our model.

A nuclear plant[9] may contain several "physical" nuclear reactors, which (for operational reasons) do not reload on the same month. The characteristic "type" for the nuclear case is not related to the plant but to the reactor. Each producer $n = 1, \cdots, N$ owns a precise number of "physical" nuclear reactors that are grouped according to the month of reloading (independently of the locations) in order to constitute units. Therefore, it can hold a certain level of capacity from each type of nuclear unit.

The modelling regarding the thermal units is the same except that the minimum capacity is equal to zero and that there is no month of reloading. There is a unique type of thermal units.

2.2 Modelling the production costs

We recollect the modelling of the production costs. The cost functions of both nuclear and thermal production are common to all producers. The nuclear cost function is made of a fixed part determined by the cost of investment, the fixed cost of exploitation and taxes and a variable part which corresponds to the variable cost of exploitation and fuel cost. We assume that the cost function of the nuclear production is affine and defined as

$$C_{n,j}^{nuc}(q_{njt}^{nuc}) = a_{nuc}^{n,j} + b_{nuc} q_{njt}^{nuc}.$$

The thermal cost function is also made of a fixed part which corresponds to the cost of investment, the fixed cost of exploitation and taxes and a variable part covering the variable cost of exploitation, the fuel cost, the cost of CO_2 as well as the taxes on the gas fuel. We assume that the thermal production has a quadratic cost function $C_n^{th}(.)$ which is the following:

$$C_n^{th}(q_{nt}^{th}) = a_{th}^{n} + b_{th} q_{nt}^{th} + c_{th}^{n} {q_{nt}^{th}}^2.$$

The nuclear and thermal cost functions are monotone increasing and convex functions of q_{njt}^{nuc} and q_{nt}^{th} respectively. We choose a quadratic cost function and thus, an increasing marginal cost for the thermal production because: (i) the thermal production results from different fossil fuel generation technologies (e.g. coal, gas -combined cycle or not-, fuel oil), (ii) the high fixed costs of thermal production need to be recovered, (iii) we want to keep our model simple by choosing the simplest cost function for thermal (DGEMP & DIDEME (2003, 2008), MIT (2003, 2009), Cour des Comptes (2012)).

2.3 Notations and constraints

- T: the time horizon of our model. Its length is chosen to be equal to 36 months[10] beginning by the month of January in order to obtain a sufficiently long time horizon to follow up the evolution of the value of the optimal solutions and at the same time to be consistent with the absence of the discount rate. The complexity of our model leads to compromise refinement of the model and computational capacity by choosing a reasoning in months[11] rather than weeks.
- $T_{campaign}$: the time horizon of the campaign. A French nuclear producer has two main options regarding the scheduling of fuel reloading (Source: EDF (2008), CEA (2008)):

[9] A nuclear power plant is a thermal power station in which the heat source arises from nuclear reactions. A nuclear unit is the set that consists of two parts: the reactor which produces heat to boil water and make steam and the electricity generation system in which one associates: the turbine and the generator. The steam drives the turbine which turns the shaft of the generator to produce electricity (Source: SFEN).

[10] The time horizon of the model is a multiplicative of twelve, being expressed in months. Therefore it could be modified.

[11] This reasoning is also met in articles which study the optimal management of hydro-reservoirs in mixed hydro-thermal electricity systems (e.g. Arellano (2004), Bushnell (1998)).

- per third (1/3) of fuel reservoir (representing a reloading of reactor's core per third of its full capacity) that corresponds to 18 months of campaign and 396 days equivalent to full capacity for a unit of 1300 MW,
- per quarter (1/4) of fuel reservoir (representing a reloading of reactor's core per quarter of its full capacity) that corresponds to 12 months of campaign and 258 days equivalent to full capacity for a unit of 1500 MW.

Both options[12] of fuel reloading result from the operational schema of EDF (Electricité de France) that is strategically chosen in order to optimize the allocation of the shutdowns of nuclear reactors for reloading (EDF (2008, 2010)). So, the scheduling of fuel reloading is entirely exogenous within our model (Bertel and Naudet (2004), CEA (2008)). Our goal is to determine the optimal allocation of the nuclear fuel stored in the reservoir during the different campaigns of production for a reloading pattern provided by the French nuclear operator via the model ORION. We retain a duration of campaign equivalent to 12 months to get a cyclic model with a periodicity of one year. The one year period can be then decomposed into 11 months being the period of production and 1 month corresponding to the month of reloading of the fuel. We do not choose a campaign of 18 months because it is not in accordance with the "good" seasonal allocation of shutdowns of the nuclear units which consists of avoiding shutdowns in high demand periods (winter) and concentrating them as much as possible in low demand periods (between May and September). In fact, if the nuclear producer reloads fuel in summer when the demand is low the date of the next reloading will be then in winter when the demand is high. The case of having both a campaign of 12 and of 18 months is excluded in order to avoid complicate our model and because the choice of normative duration of the campaign can not be changed for a given nuclear reactor. The Nuclear Safety Authority (NSA)[13] has to give the authorization for any changes on the choice of duration of the campaign. Additional to that the optimal allocation of the shutdowns of all 58 nuclear reactors for reloading is decided in advance according to safety rules imposed by NSA.

- D_t: the level of demand observed in month $t = 1, \cdots, T$. The demand for electricity being an exogenous variable is assumed perfectly inelastic mainly because in the short-term to medium-term, we may consider that price variations can not be observed by consumers in real time and consumers habits and prior investments in electrical devices can not change immediately. If we include a price elasticity of demand in our model, it would have a random value since there are no particular elements that enable to assess its value.
- Q_t^{hyd}: the hydro-production coming from the run-of-river[14] hydro plants in month $t = 1, \cdots, T$. We assume that the monthly run-of-river hydro production is constant over the total time horizon of our model given: (i) the non-availability of the data with regard to the seasonal variations of hydro production because of precipitation and snow melting, (ii) its low volatility caused by a relatively low standard deviation which leads to a steady evolution of its monthly value near to the mean over a year. It is calculated by the mean of the yearly production. In this way, we deduce a significant part of the base load demand in order to have a more accurate picture of the demand served by the nuclear

[12] In the case of a unit of 900 MW, the scheduling of fuel reloading is the following: (*i*) 1/3 of fuel reservoir that corresponds to 18 months of campaign and 385 days equivalent to full capacity, (*ii*) 1/4 of fuel reservoir that corresponds to 12 months of campaign and 280 days equivalent to full capacity.

[13] The Nuclear Safety Authority (NSA) is tasked, on behalf of the state, with regulating nuclear safety in order to protect workers, the public and the environment in France.

[14] The run-of-river hydro plants have little or no capacity for energy storage, hence they can not co-ordinate the output of electricity generation to match consumer demand. Consequently, they serve as baseload power plants.

and thermal units. The intermittency that determines the base load production of the renewable energy plants makes our model more complex and additionally to this it is not coherent with the deterministic character of our model which is why we do not consider it.

- q^{nuc}_{njt}: the level of the nuclear production during the month $t = 1, \cdots, T$ for the unit j of producer n.
- $Q^{n,j,nuc}_{max}$: the maximum nuclear production that can be realized by the unit j of producer n during a month. The nuclear capacity is an exogenous variable.
- $Q^{n,j,nuc}_{min}$: the minimum nuclear production that can be realized by the unit j of producer n during a month.
- q^{th}_{nt}: the level of the thermal production during the month $t = 1, \cdots, T$ for the producer n.
- $Q^{n,th}_{max}$: the maximum thermal production during a month for the producer n. It corresponds to the nominal thermal capacity of producer n. A producer may use the thermal resources to produce electricity until it reaches the level of demand of the corresponding month respecting at the same time the constraint (5). The thermal capacity is an exogenous variable.
- $Q^{n,th}_{min}$: the minimum thermal production during a month for the producer n. There is no minimum for thermal production $Q^{n,th}_{min} = 0$.

The minimum and maximum production constraints have the form:

$$\begin{cases} Q^{n,j,nuc}_{min} \leqslant q^{nuc}_{njt} \leqslant Q^{n,j,nuc}_{max}, & \text{if no reload during month } t \text{ for unit } j \\ q^{nuc}_{njt} = 0, & \text{if unit } j \text{ reloads during month } t \end{cases} \tag{1}$$

$$0 \leqslant q^{th}_{nt} \leqslant Q^{n,th}_{max} \tag{2}$$

- $S^{n,j}_{reload}$: the nuclear fuel stock of reloading available to the unit j of producer n. This stock will be expressed thanks to the conversion between the quantity of energy and the corresponding number of days of operation at full capacity rather than expressing it in kilograms of uranium or number of nuclear fuel rods. In our model, the number of days of operation equivalent to full capacity is constant for all j, n and inferior than 11 months which permits and obliges at the same time to modulate the nuclear production. The nuclear fuel stock of reloading $S^{n,j}_{reload}$ is equal to the corresponding capacity of the units of type j of producer n (Capacity^{n,j,nuc}) multiplied by the number of hours equivalent to full capacity during a campaign. More precisely, one has:
  $$S^{n,j}_{reload} = 1\times \text{ Capacity}^{n,j,nuc} \times \text{Number of days equivalent to full capacity}\times 24$$
  which corresponds to the nuclear fuel stock of reloading over a campaign of production.
- $S^{n,j}_t$: the quantity of fuel stored in the nuclear reservoir and available to the unit j of producer n at the beginning of the month $t = 1, \cdots, T$. Evidently, we have $S^{n,j}_t \geqslant 0$. If t is the month during which the producer n reloads the fuel of the reactor then, the stock at the beginning of the following month (beginning of the campaign) is equal to $S^{n,j}_{reload}$. A producer has a quantity of nuclear fuel stock equal to zero at the end of a campaign (beginning of the month of reloading) which means that it spends all its nuclear fuel stock of reloading $S^{n,j}_{reload}$ during the campaign. The reasons that lead us to this ascertainment mainly concern the implicit costs that result from not consuming the totality of the nuclear fuel stock during a campaign. Moreover, a producer has to finish the period T at least with the same quantity of nuclear fuel as the initial one ($S^{n,j}_{T+1} \geqslant S^{n,j}_1$). The consideration of this constraint is motivated by some arguments analytically exposed in the second chapter (avoid to "over-consume" the nuclear fuel stock to reach the maximum nuclear production level because of induced negative effects, assure that each new cycle of simulations of 36 months starts with the same quantity of nuclear fuel ($S^{n,j}_1$)).

The nuclear fuel constraints for the nuclear unit j of producer n are defined as follows:

j=1	**j ∈ {2, ⋯, 11}**	**j=12**
$\sum_{t=2}^{12} q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=1}^{j-1} q_{njt}^{nuc} = S_1^{n,j}$	$\sum_{t=1}^{11} q_{n12t}^{nuc} = S_{reload}^{n,12}$
$\sum_{t=14}^{24} q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=j+1}^{j+12-1} q_{njt}^{nuc} = S_{reload}^{n,j}$	$\sum_{t=13}^{23} q_{n12t}^{nuc} = S_{reload}^{n,12}$
$\sum_{t=26}^{T} q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=j+12+1}^{j+2\cdot 12-1} q_{njt}^{nuc} = S_{reload}^{n,j}$	$\sum_{t=25}^{T-1} q_{n12t}^{nuc} = S_{reload}^{n,12}$
	$\sum_{t=j+2\cdot 12+1}^{T} q_{njt}^{nuc} = S_{reload}^{n,j} - S_1^{n,j}$	

Table 1

We can see that the nuclear units of type $\{2, \cdots, 11\}$ have two additional constraints than the nuclear units of type 1 and 12. This is because there exist, at the beginning and end of the game, campaigns that we will qualify as incomplete.

2.4 Number of optimization variables and of optimization constraints

The entire number of optimization variables is equal to $N \cdot (J \cdot T + T) = N \cdot (12 \cdot 36 + 36) = N \cdot 468$ within our model. The number of constraints resulting from the equality between supply and demand is $T = 36$. Moreover, the number of nuclear fuel constraints is $N \cdot ((2 \cdot K + 1) \cdot (J - 2) + (2 \cdot K) \cdot 2) = N \cdot ((2 \cdot 3 + 1) \cdot (12 - 2) + (2 \cdot 3) \cdot 2) = N \cdot 82$, where K represents the number of campaigns within our model. Finally, the number of minimum and maximum nuclear and thermal production constraints is equal to $N \cdot (J \cdot T + T) = N \cdot (12 \cdot 36 + 36) = N \cdot 468$. Consequently, the total number of optimization constraints is equal to $N \cdot 550 + 36$. Even in the case of a unique producer ($N = 1$), the number of variables (468) and of optimization constraints (586) is quite large which results in computational difficulties. This is due to the fact that the level of difficulty of the numerical program to compute a solution of an optimization problem is increasing with respect to the size of the model (number of optimization variables, number of optimization constraints).

2.5 The modelling of the optimal inter-temporal production behaviour

The optimal inter-temporal production problem that producer n resolves is the following:

$$\max_{((q_{njt}^{nuc})_{j=1}^{J}, (q_{nt}^{th}))_{t=1}^{T}} \sum_{t=1}^{T} (p_t \cdot (\sum_{j=1}^{J} q_{njt}^{nuc} + q_{nt}^{th}) - \sum_{j=1}^{J} C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th})) \tag{3}$$

subject to the nuclear fuel storage constraints provided in table 1 and the minimum/maximum production constraints (4) and (5)

$$\begin{cases} Q_{min}^{n,j,nuc} \leqslant q_{njt}^{nuc} \leqslant Q_{max}^{n,j,nuc}, & \text{if no reload during month } t \text{ for unit } j \\ q_{njt}^{nuc} = 0, & \text{if unit } j \text{ reloads during month } t \end{cases} \tag{4}$$

$$\tag{5}$$

The price p_t is given by the marginal cost of the last technology of the merit order used to equilibrate supply and demand (perfect competition). It is calculated independently of n and is discontinuous on production vectors whose thermal component q_{nt}^{th} is equal to zero.

2.6 The notion of equilibrium

Let us now give a definition of equilibrium of the optimal inter-temporal production problem with respect to a system of prices $p \in \mathbb{R}_+^T$

Definition 2.1 The production vector $(\overline{q}_n)_{n=1}^N = (((\overline{q}_{1jt}^{nuc})_{j=1}^J, \overline{q}_{1t}^{th})_{t=1}^T, \cdots, ((\overline{q}_{Njt}^{nuc})_{j=1}^J, \overline{q}_{Nt}^{th})_{t=1}^T)$ is an equilibrium with respect to a system of prices $p \in \mathbb{R}_+^T$ if:

(i) for all n, $\overline{q}_n$ is a feasible production vector: (a) it respects the nuclear fuel constraints, for all j and (b) it respects the minimum/maximum production constraints, for all j, t.

(ii) for all n, it maximizes the inter-temporal profit of producer n on the set of feasible solutions.

(iii) the price, at each month t, is determined by the marginal cost of the marginal technology. It is called the merit order price associated with the production vector $(\overline{q}_n)_{n=1}^N$.

(iv) at each date t, it respects the equality between supply and demand

$$\sum_{n=1}^{N}(\sum_{j=1}^{J}\overline{q}_{njt}^{nuc} + \overline{q}_{nt}^{th}) = D_t - Q_t^{hyd}. \tag{6}$$

3 Optimization of the inter-temporal production

In this section, we show that under some assumptions, the inter-temporal profit decreases at production vectors characterized by thermal components equal to zero as a result of the discontinuity (decrease) of price at these production vectors. Then, we proceed by giving a property that characterizes the "interior" optimal solutions of problem (3) (see page 8).

3.1 The decrease of inter-temporal profit in the absence of thermal production

We define the set of feasible solutions of the optimal production problem (3) as

$$C = \left\{ q \in M \ s.t. \begin{array}{ll} Q_{min}^{n,j,nuc} \leqslant q_{njt}^{nuc} \leqslant Q_{max}^{n,j,nuc}, & \text{for all } n,j,t \\ 0 \leqslant q_{nt}^{th} \leqslant Q_{max}^{n,th}, & \text{for all } n,t \end{array} \right\}$$

where M is defined by all the production vectors of the form $q = ((q_{nj1}^{nuc})_{j=1}^J, \cdots, (q_{njT}^{nuc})_{j=1}^J, q_{n1}^{th}, \cdots, q_{nT}^{th})_{n=1}^N$ that respect the nuclear fuel constraints for all n as well as the supply-demand equilibrium constraint for all t. The set M is affine and the set C is compact (closed and bounded) and convex.

Moreover, we define F as the relative interior[15] of C ($F = ri(C)$). It has the following form

$$F = \left\{ q \in M \ s.t. \begin{array}{ll} Q_{min}^{n,j,nuc} < q_{njt}^{nuc} < Q_{max}^{n,j,nuc}, & \text{for all } n,j,t \\ 0 < q_{nt}^{th} < Q_{max}^{n,th}, & \text{for all } n,t \end{array} \right\}$$

Notice that if unit j reloads during month t then $q_{njt}^{nuc} = 0$ and thus, the strict inequality constraints that determine the nuclear production q_{njt}^{nuc} in the set F are no more valid.

[15]It is important to emphasize that the usual interior of C is empty since M is an affine set that is not equal to $\mathbb{R}^n$. Consequently, we focus on a classical generalization called relative interior (for the notion of the relative interior of a set cf. for example Florenzano and Le Van (2001), Boyd and Vandenberghe (2004), Pugh (2002)).

Let us focus on the set F^{th} defined as

$$F^{th} = \left\{ q \in M \ s.t. \begin{array}{ll} Q^{n,j,nuc}_{min} \leqslant q^{nuc}_{njt} \leqslant Q^{n,j,nuc}_{max}, & \text{for all } n, j, t \\ 0 < q^{th}_{nt} \leqslant Q^{n,th}_{max}, & \text{for all } n, t \end{array} \right\}$$

Remark 3.1 *F^{th} contains F and is contained in C and C is contained in M ($F \subset F^{th} \subset C \subset M$).*

We now proceed with Proposition 3.1 which we will use to prove the decrease of the inter-temporal profit at production vectors with zero levels of thermal production.

Proposition 3.1 *If F^{th} is a non-empty set, then the closure of F^{th} is C ($\overline{F^{th}} = C$).*

Proof
A proof of this proposition is given in the Ph.D. thesis on page 163 (Lykidi (2014)).

□

From a geometrical point of view one deduces from Proposition 3.1 that all the points of the set C and consequently those which belong to $C \setminus F^{th}$ and hence contain thermal components equal to zero can be approached by points that belong to F^{th}. This result is fundamental in order to show in the next proposition the discontinuity and more specifically the decrease of the inter-temporal profit at these particular points which results from a decrease of price (discontinuous problems have been analyzed in an economic framework (cf. for example Bich and Laraki (2011)).

Proposition 3.2 *For all $n \in \{1, \cdots, N\}$, if F^{th} is a non-empty set, $b_{nuc} < b_{th}$ and $\overline{q} \in C \setminus F^{th}$, there exists a sequence $(q_r)_{r \in \mathrm{N}} \in F^{th}$ with $\lim_{r\to\infty} q_r = \overline{q}$ such that $\lim_{r\to\infty} \pi^n(q_r) > \pi^n(\overline{q})$.*

Proof
A proof of this proposition is provided in the Ph.D. thesis on pages 163 − 165 (Lykidi (2014)).

□

In view of our data, $b_{nuc} < b_{th}$ holds, thus according to Proposition 3.2, the inter-temporal profit decreases for all production vectors that belong to the subset $C \setminus F^{th}$ of C and hence they are characterized by zero levels of thermal production. This leads all producers to search for a solution that maximizes the inter-temporal profit on the set F^{th} in which the thermal production is strictly positive and the price is provided by the thermal marginal cost.

The following corollary shows the relation between the optimal inter-temporal production problem on C and the optimal inter-temporal production problem on F^{th}.

Corollary 3.1 *The inter-temporal profit maximization problem determined on C is equivalent to the inter-temporal profit maximization problem determined on F^{th} (same set of solutions and same value[16]).*

Proof
This corollary is an obvious consequence of Proposition 3.2.

□

It should be noticed that the value of both optimization problems exists (in the real line) because the objective function (profit function) is polynomial and the set C together with the

[16] The value of an optimization problem is defined as the upper bound of the set $\{f(x)|x \in C\}$, where f is the objective function and C is the set of feasible solutions. The value always exists even if the set of solutions is empty. When the set of solutions is non-empty, the value of an optimization problem is the common value $f(\overline{x})$ for any solution $\overline{x}$.

set F^{th} are bounded. We also remark that if the inter-temporal profit maximization problem is determined on C which is a compact set, the objective function is not continuous in view of Proposition 3.2. If the inter-temporal profit maximization problem is determined on F^{th}, the objective function is continuous according to Proposition 3.2 while F^{th} is not a compact set. Therefore, we can not conclude on the existence of solutions of this problem (cf. for example Varian (1992)). If a solution of the inter-temporal profit maximization problem on F^{th} exists, then it is an equilibrium since all the conditions in order to be an equilibrium are satisfied (see Corollary 3.1 and Definition 2.1 on page 9).

3.2 Case of an "interior" optimal solution.

In view of Proposition 3.2 on page 10, we search for a solution that maximizes the inter-temporal profit within the set F^{th}. The next proposition gives a property when in addition the solution of the optimal inter-temporal production problem belongs to the set F.

Proposition 3.3 *For all $n \in \{1, \cdots, N\}$, if an equilibrium $(((\overline{q}^{nuc}_{njt})^J_{j=1}, \overline{q}^{th}_{nt})^T_{t=1})^N_{n=1} \in F$ exists such that the inter-temporal profit of the producer n is maximum on C and $((\overline{q}^{nuc}_{nt})^T_{t=1})^N_{n=1}$ is the corresponding monthly nuclear production vector then $\overline{q}^{nuc}_{n1} = \overline{q}^{nuc}_{n2} = \cdots = \overline{q}^{nuc}_{nT}$.*

Proof
A proof of this proposition is provided in the Ph.D. thesis on pages 167 − 170 (Lykidi (2014)). □

Since F is not a compact set, the inter-temporal profit maximization problem may not have a solution on F. Consequently, the existence of a solution of the problem (3) on F takes the form of an assumption in Proposition 3.3. Moreover, Proposition 3.3 implies that each producer holds a strictly positive level of nuclear capacity from all types of nuclear units (i.e. Capacity$^{n,j,nuc} > 0$ for all n, j).

3.2.1 Economic interpretation of the Lagrange multipliers of the optimal inter-temporal production problem

Following the proof of Proposition 3.3, we can interpret economically the Lagrange multipliers of the optimal inter-temporal production problem (3) presented within this proof. We recall that μ^n_t is the Lagrange multiplier associated with the supply-demand equilibrium constraint at each month t for the producer n and $\lambda^{n,k}_j$ is the Lagrange multiplier for the nuclear fuel constraint of the unit j during the campaign k for the producer n. In view of the equality between supply and demand and since $\overline{q} \in F$, we deduce from equation (3.7) (respectively (3.8)) appeared on page 169 of the proof that the sign of the multiplier μ^n_1 (respectively μ^n_2) is strictly positive for all n. By symmetry, the Lagrange multiplier μ^n_t is strictly positive ($\mu^n_t > 0$) for all n, t. Consequently, in view of equations (3.9) and (3.10) on page 170 of the proof, the multiplier $\lambda^{n,1}_3$ (respectively $\lambda^{n,k}_j$) has a strictly negative sign for all n. Indeed, if an additional unit of nuclear fuel became available for the unit j of producer n during the campaign k, the thermal production of producer n and consequently the total thermal production obtained during this campaign would decrease which would lead to a lower market price and thus to a lower revenue for the producer n. At the same time the nuclear production cost of producer n would increase while its thermal production cost would decrease. However, the first effect that concerns the decrease of the revenue is the most important. Consequently, the "additional" profit resulting from an additional nuclear fuel unit and thus the value of the multiplier $\lambda^{n,k}_j$ should be negative. The multiplier $\lambda^{n,k}_j$ indicates the "marginal value of nuclear fuel stock", i.e.

the additional profit $|\lambda_j^{n,k}|$ unit j of producer n would get if the nuclear fuel stock decreased by one unit during the campaign k.

We now proceed by showing that a constant monthly nuclear production constitutes a sufficient condition for optimality on C. Let us state the following proposition:

Proposition 3.4 *For all $n \in \{1, \cdots, N\}$, if $(((\overline{q}_{njt}^{nuc})_{j=1}^{J}, \overline{q}_{nt}^{th})_{t=1}^{T})_{n=1}^{N}$ is a production vector belonging to F^{th} such that $\overline{q}_{n1}^{nuc} = \overline{q}_{n2}^{nuc} = \cdots = \overline{q}_{nT}^{nuc}$, where $(\overline{q}_{nt}^{nuc})_{t=1}^{T}$ is the corresponding monthly nuclear production vector of producer n then $(((\overline{q}_{njt}^{nuc})_{j=1}^{J}, \overline{q}_{nt}^{th})_{t=1}^{T})_{n=1}^{N}$ is a solution of the inter-temporal profit maximization problem on C.*

Proof

A proof of this proposition is provided in the Ph.D. thesis on pages $171-173$ (Lykidi (2014)). □

Remark 3.2 *We can prove that the strict concavity of the profit function π^n with regard to the thermal production q_n^{th} implies the unicity of solutions with respect to the thermal component for all n (Lykidi (2014)). However, if we consider the other variables which do not impact the profit, the profit function π^n is concave with regard to q_n for all n, which does not entail automatically the unicity of the entire solution.*

3.2.2 Economic analysis of Proposition 3.3 and of Proposition 3.4

In view of Propositions 3.3 and 3.4 (pages 11, 12), we conclude that in the absence of binding productions constraints, the solutions of the optimal inter-temporal production problem are fully characterized by a constant nuclear production. Consequently in such situations, from a theoretical point of view, each producer maximizes its inter-temporal profit by using its nuclear units in order to produce at a constant rate while it operates its thermal units to follow-up load so that the global equilibrium between supply and demand is satisfied each month. Hence, at the optimum, the nuclear production does not follow the seasonal variations of demand which means that nuclear units operate only at baseload on a monthly basis. This implies that the amplitude of demand has to be inferior than the thermal capacity Q_{max}^{th} so that imbalances between supply and demand are avoided every month. Note that the level of the nuclear production of each producer being constant could never reach its maximum value given the definition of the nuclear fuel stock of reloading $S_{reload}^{n,j}$ on page 7.

4 Numerical modelling

In this section, we proceed with an analysis of the nuclear and thermal production decisions as well as the storage decisions obtained by the optimal inter-temporal production problem, within a simple numerical model solved with Scilab.

4.1 Equivalence of equilibrium between an economy with N producers and an economy with one aggregate producer

From a mathematical point of view, the complexity of the optimal inter-temporal production problem makes "necessary" to decrease the number of optimization variables and operational constraints. Indeed, even in the simplified case of one aggregate producer (for $N = 1$), we obtain 586 operational constraints and 468 optimization variables given the time period of our model ($T = 36$), the number of campaigns ($K = 3$) and the different types of nuclear units ($J = 12$). We show in the next proposition that the equilibrium of the original economy with $N \geqslant 2$ producers is "equivalent" to the equilibrium of an alternate economy with one aggregate producer ($N = 1$). Thanks to this mathematical proposition, we simplify the resolution of our optimization problem by determining an equilibrium of the economy with a unique producer operating with the aggregate nuclear and thermal capacity instead of the economy with N producers. This approach independent of the number of producers (reduction to a unique producer) makes coherent the numerical modelling of the optimal short-term production problem and the optimal inter-temporal production problem.

Proposition 4.1 *Let us consider an economy E with several producers and let $\widetilde{E}$ be the auxiliary economy with a unique producer obtained by the aggregation of the N producers of E.*

(α) *If $(q_n)_{n=1}^N$ is an equilibrium of E, then its aggregation defined by $q = \sum_{n=1}^N q_n$ is an equilibrium of $\widetilde{E}$ with respect to the same prices.*

(β) *Conversely, if q is an equilibrium of $\widetilde{E}$, it can then be decentralized as an equilibrium $(q_n)_{n=1}^N$ of E for the same prices.*

Proof

A simple proof of this proposition is provided in the Ph.D. thesis on pages $177 - 182$ (Lykidi (2014)). Note that in part (β), there is no unicity in the process of decentralization.

□

4.1.1 Economic consequences of Proposition 4.1

According to Proposition 4.1 on page 13, we can say that the equilibrium q of the optimal inter-temporal production problem of the auxiliary economy $\widetilde{E}$ is equivalent to the equilibrium $(q_n)_{n=1}^N$ of the optimal inter-temporal production problem of the decentralized economy E. In view of this proposition, we consider from now on an economy with a unique producer ($N = 1$) operating with the total nuclear and thermal capacity to meet the monthly demand. For example, the exogenous variable Q_{max}^{th} will now represent the aggregate maximum thermal production. From an economical perspective, the equivalence of equilibria of the optimal inter-temporal production problem between the economies E and $\widetilde{E}$ implies that the optimal production behavior in the case of the decentralization of the nuclear generation set is "neutral" with respect to the optimal production behavior observed in the case of the centralized management by an aggregate producer within our model. These results can be also found in the case of the optimal short-term production problem.

4.2 Symmetry of equilibrium of the optimal inter-temporal production problem

In this section, we introduce the notion of symmetrisability within our model in order to provide an economical property of producer's optimal behaviour: under the assumption that each producer disposes of the same level of nuclear and thermal capacity, we show that an equilibrium of the inter-temporal profit maximization problem (3) is "almost" symmetric. More precisely, we prove that the thermal component of the equilibrium is symmetric while the nuclear component of the equilibrium is "symmetrisable", i.e., it can be symmetrised. The interest of showing this property lies in the notion of symmetrisability and its mathematical-economical implications. We will show through a simple example that despite the symmetry of nuclear capacities, the nuclear component of the equilibrium is potentially asymmetric which leads to asymmetric equilibriums of the inter-temporal profit maximization problem. Therefore, considering the feature of symmetrisability, we can move from an asymmetric equilibrium to a symmetric equilibrium by focusing on the symmetric nuclear component of the equilibrium. From a mathematical point of view, this property permits to concentrate only on the symmetric solution and thus, it may be used to decrease the number of the optimization variables of the optimal inter-temporal production problem (3) which simplifies its numerical resolution. In addition, it provides us with an appealing economical feature regarding the profit resulting from a symmetrisable equilibrium.

4.2.1 Existence of an asymmetric equilibrium of the optimal inter-temporal production problem: An example

It should be noticed that the nuclear component of the equilibrium $(q^{nuc}_{1jt}, q^{nuc}_{2jt}, \cdots, q^{nuc}_{Njt})$ is potentially asymmetric. In order to understand this asymmetry, let us give an example in the case of two producers ($N = 2$). Let $(\widehat{q}^{nuc}_{1jt}, \widehat{q}^{nuc}_{2jt})$ be a symmetric equilibrium such that the price is the same during the period 1 and the period 2 (i.e. $p_1 = p_2$). This occurs in particular, if nuclear is the marginal technology in periods 1 and 2. Then, any feasible production realized by the unit 3 of producer 1 (respectively 2) in periods 1, 2 means a solution of the following system:

$$\begin{cases} q_{131} + q_{231} = \widehat{q}_{131} + \widehat{q}_{231} = D_1 \text{ - } Q^{hyd}_{Tot,1}, & \text{supply - demand equilibrium constraint} \\ & \text{in month 1} \\ q_{132} + q_{232} = \widehat{q}_{132} + \widehat{q}_{232} = D_2 \text{ - } Q^{hyd}_{Tot,2}, & \text{supply - demand equilibrium constraint} \\ & \text{in month 2} \\ q_{131} + q_{132} = \widehat{q}_{131} + \widehat{q}_{132} = S^3_1, & \text{nuclear fuel constraint for unit 3 of producer 1} \\ q_{231} + q_{232} = \widehat{q}_{231} + \widehat{q}_{232} = S^3_1, & \text{nuclear fuel constraint for unit 3 of producer 2} \end{cases}$$

and remains unchanged during the remaining periods ($q^{nuc}_{njt} = \widehat{q}^{nuc}_{njt}$, for all $n \in \{1, 2\}$, for all $j \neq 3$, and for $t \geqslant 3$) will be still an equilibrium. Consequently, there exists at least one asymmetric equilibrium of the nuclear inter-temporal profit maximization problem of producer n , where for example unit 3 produces more for producer 1 than for producer 2 in period 1 (and the opposite in period 2 in order to compensate). This is related to the absence of unicity in part (β) of Proposition 4.1.

4.2.2 The notion of symmetrisability

In view of the assumption that the nuclear and thermal capacities are symmetric among producers, if $(((q^{nuc}_{1jt})^J_{j=1}, q^{th}_{1t})^T_{t=1}, \cdots,\ ((q^{nuc}_{Njt})^J_{j=1}, q^{th}_{Nt})^T_{t=1})$ is an equilibrium of this problem, we show that the thermal component of the equilibrium is symmetric while the nuclear component of the equilibrium is symmetrisable.

We define a symmetrisable equilibrium as follows:

Definition 4.1 Let $(q_1, q_2, \cdots, q_N)$ be an equilibrium. This equilibrium is called symmetrisable if there exists a symmetric allocation $(\tilde{q}_1, \tilde{q}_2, \cdots, \tilde{q}_N)$, which is an equilibrium and "leads" to the same prices as the initial allocation $(q_n)^N_{n=1}$.

4.2.3 An economical property resulting from the notion of symmetrisability of an equilibrium

The notion of symmetrisability of an equilibrium provides us with an interesting economical feature: the profit of a symmetrisable equilibrium $(\pi_1, \pi_2, \cdots, \pi_N)$ is symmetric. This means that the production levels of a symmetrisable equilibrium are equivalently profitable for all producers. This arises from the fact that the price induced by the symmetrisable equilibrium is equal to the price induced by the symmetric equilibrium. Consequently, the profit $(\pi_1, \pi_2, \cdots, \pi_N)$ coming from a symmetrisable equilibrium is equal to the profit $(\tilde{\pi}_1, \tilde{\pi}_2, \cdots, \tilde{\pi}_N)$ resulting from the deduced symmetric equilibrium. For a symmetric equilibrium, the value of profit is equal among the different producers at the equilibrium state given that the price p_t as well as the production level are identical for all players and the production cost is symmetric for both technologies.

4.2.4 Symmetry of the thermal component and symmetrisability of the nuclear component of equilibrium

We proceed here with Proposition 4.2 which shows the symmetry of the thermal component and the symmetrisability of the nuclear component of equilibrium of the optimal inter-temporal production problem.

Proposition 4.2 *Let* $(((q^{nuc}_{1jt})^J_{j=1}, q^{th}_{1t})^T_{t=1}, \cdots, ((q^{nuc}_{Njt})^J_{j=1}, q^{th}_{Nt})^T_{t=1})$ *be an equilibrium of the optimal inter-temporal production problem* (3)*. If the nuclear and thermal capacity are symmetric among producers then the thermal component of the equilibrium is symmetric while the nuclear component of the equilibrium is symmetrisable.*

Proof

A proof of this proposition is provided in the Ph.D. thesis on pages $186-189$ (Lykidi (2014)). The proof is based on the separability[17] of the optimal inter-temporal production problem (3) with respect to the couple $(q^{nuc}_{njt}, q^{th}_{nt})$ and on the strict concavity of the thermal optimal production problem producer n. On the other side, the nuclear optimal production problem of producer n is not separable with respect to t, hence there is no symmetry of equilibrium.

□

In conclusion, through the property of "symmetrisability", it is possible to derive a symmetric equilibrium from an asymmetric equilibrium and thus to focus on the symmetric equilibrium which shows the usefulness of this property for complex computational optimization problems like ours.

4.3 Data

The data used in our numerical model comes from the French electricity market and it is collected by different entities and for different years because of the difficulty of collection. In particular, consumption data comes from the French Transmission & System Operator (named RTE) for the year 2007, the annual generation capacity of hydro (run-of-river) is given by the French nuclear operator (EDF) while the annual nuclear and thermal (coal and gas) generation capacity is given by RTE for the year 2009, the nuclear fuel stock of reloading has been provided by EDF for the same year. The fixed and variable costs of nuclear, coal and gas generation come from the official report "Reference Costs of Electricity Production" issued by the ministry

[17] For the notion of separability cf. for example Boyd and Vandenberghe (2004).

of industry (General Direction for Energy and Raw Materials (DGEMP) & Directorate for Demand and Energy Markets (DIDEME)) and they are calculated for the year 2007. In view of the specificities of the nuclear generation technology and of its production cost, we give a brief analysis with respect to the impact of the discount rate on the calculation of the nuclear cost, the economic consequences of a load-following mode of operation of nuclear reactors on the nuclear cost as well as the principal points of differentiation between nuclear and thermal production cost (Bertel and Naudet (2004)) in the Ph.D. thesis of Lykidi (2014). Then, we present explicitly some specific data assumptions considered for our numerical modelling regarding: (i) the value of the exchange rate, of the discount rate, of the cost of CO_2 per ton and the price of coal and gas (ii) the computation of the coefficients of the thermal production cost, (iii) the simulation of the capacity for each type of nuclear unit and of the initial value of the nuclear fuel stock (S_1^j), (iv) the calculation of the number of days equivalent to full capacity, (v) the technical minimum and maximum for an EPR reactor in order to determine the minimum and maximum nuclear production constraints. Lastly, we mention a couple of economical results which can be inferred within our data base and which are totally explicated in the second chapter: (i) the average nuclear cost calculated here (37.25 euros per MWh) is close to the range of nuclear electricity prices (37.5 - 38.8 euros per MWh) estimated for the NOME[18] law (Commission for Energy Regulation (CRE) estimated this range of prices in 2010 (before Fukushima accident in 2011 (Les Echos (20/04/2011)) in order to propose to EDF a fair price for selling nuclear capacity to alternative producers (Le Monde (01/02/2011))), (ii) the total monthly thermal production cost is never covered except if other generation technologies with higher marginal costs are called to meet demand (e.g. hydro-storage units).

4.4 Simulation results

In this section, we treat the problem of discontinuity of the merit order price which induces a discontinuity (decrease) of the inter-temporal profit in order to resolve numerically the optimal inter-temporal production problem within our data set. The proof of several mathematical propositions presented here can be found in the Annex B of the Ph.D. thesis of Lykidi (2014).

4.4.1 "Regularization" of the optimal inter-temporal production problem

From both a theoretical and a numerical point of view, we "regularize" the merit order price in order to deal with the problem of discontinuity and solve the optimal inter-temporal production problem. To treat the problem of discontinuity numerically, we introduce an alternative model in which the price is given by the thermal marginal cost $(mc^{th}(0) = b_{th})$ instead of (b_{nuc}) during periods when nuclear is the marginal technology. Hence, the price p_t during the month t will be

$$p_t = \begin{cases} mc^{th}(q_t^{th}), & \text{if } q_t^{th} > 0 \\ mc^{th}(0), & \text{if } q_t^{th} = 0 \end{cases} = \begin{cases} b_{th} + 2c_{th}q_t^{th}, & \text{if } q_t^{th} > 0 \\ b_{th}, & \text{if } q_t^{th} = 0 \end{cases} \tag{7}$$

Considering this "regularization" of the merit order price, the profit, being now a continuous function, is maximized on the entire set of feasible solutions C within our numerical model. This constitutes a continuous optimization problem called the "regularized" problem. Nevertheless,

[18]The "Nouvelle Organisation du Marché de l'Electricité" (NOME) law indicates the findings of the report of the Commission Champsaur which suggests access to nuclear electricity of the French nuclear operator (EDF) for all producers (Champsaur (2009)). Specifically, the NOME law forces EDF to sell at a competitive price to alternative producers of electricity and gas (GDF Suez, E.ON, ENEL, Poweo, Direct Energy, etc.) a quarter of its nuclear production until 2025. This price should include the total cost of the operating nuclear plants.

the "regularized" problem (continuous problem) is different from the economical problem (3) described in subsection 2.5 (discontinuous problem) with regard to the objective function i.e. the profit. More precisely, the profit considered in this problem is greater than the profit of the economical problem since the value of b_{th} (26.24 Euro/MWh) is greater than the value of b_{nuc} (5.01 Euro/MWh). However, we prove that the value of the "regularized" problem is the same with the value of the economical problem which means that the value of profit at the optimum is identical for both optimization problems (see Annex B, Proposition B.1). Hence, we deduce that the "regularized" problem is a "good" approximation of our economical problem (Boyd and Vandenberghe (2004)).

Theoretically, in Annex B, Proposition B.7 proves that a solution of the "regularized" problem which is not in the set F^{th} implies the emptiness of the set of solutions of the economical problem. From a numerical perspective, the solution of the "regularized" problem which is presented in this section is not in the set F^{th} because the thermal production is not always used to ensure the equality between supply and demand. This means that the set of solutions of the economical problem is empty which shows the importance of emphasizing the numerical solution resulting from the "regularized" problem. This numerical solution is only an "approximate" solution of our economical problem.

It should be mentioned that the discontinuity of price observed at production vectors with zero levels of thermal production poses an "economical problem". More precisely, a producer, who covers the monthly levels of demand during summer (low demand season) by running only its nuclear units, is penalized since its nuclear production is evaluated at a low price (b_{nuc}). This price does not allow the amortization of the important fixed costs of nuclear. Hence, by realizing an infinitesimal nuclear capacity withholding, thermal becomes the marginal technology leading prices to a higher level (almost equal to b_{th}) and valorizing nuclear production which justifies our "regularization" of the merit order price.

4.4.2 General results

According to the economic analysis of Proposition 3.3 and Proposition 3.4 on page 12, the thermal production is adjusted on the seasonal variations of demand, while the nuclear production remains constant during the entire time horizon of the model. However, in view of our data, a constant nuclear production is not a feasible solution in the case of a large nuclear set because the deduced thermal production violates both minimum and maximum[19] thermal production constraints during some months (see Figure 2). In particular, the thermal production does not meet demand during high demand months (winter). As one can see in figure[20] 1 and in figure 3, the amplitude of demand exceeds the maximum level of thermal production Q^{th}_{max}. Consequently, given that the nuclear production is constant, the thermal production can not balance supply and demand each month. For this reason, the nuclear production has to be flexible and follow the variations of demand. In conclusion, Proposition 3.3 says that the inter-temporal profit maximization problem has no solutions on F within our numerical model.

[19] The maximum thermal production during a month is represented by the white blue dotted line and corresponds to the nominal thermal capacity (including coal, gas, fuel, etc.) of the French set.

[20] The amounts of monthly demand D_t are obtained for the period January 2007 - December 2009. In particular, the values of monthly demand during the period January 2007 - December 2007 come from our historical data. Then, we reproduce these values by applying a positive rate of 1% per year on the monthly demand for the years that follow (2008 and 2009) to take into account the increasing trend of demand from one year to another. We did a rescaling on this data to take into account the diversity on the length of the months. Note also that the monthly demand in 2007 results from the aggregation of the hourly demand found within our historical data.

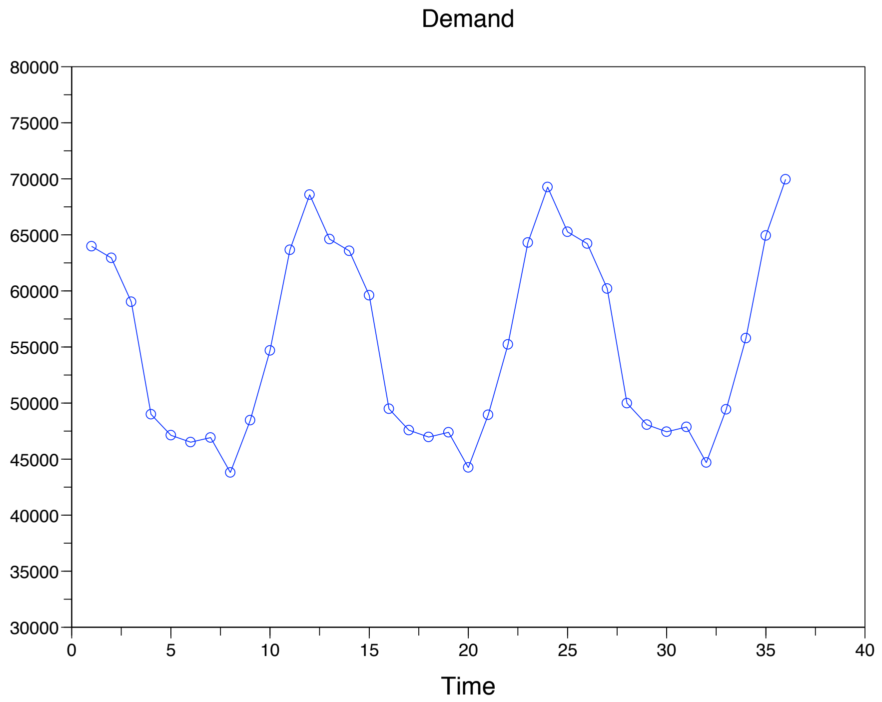

Figure 1: Simulated demand (in MW)

We also observe (both graphically and with a numerical test) that the thermal and nuclear production increase (and respectively decrease) simultaneously during almost the entire time

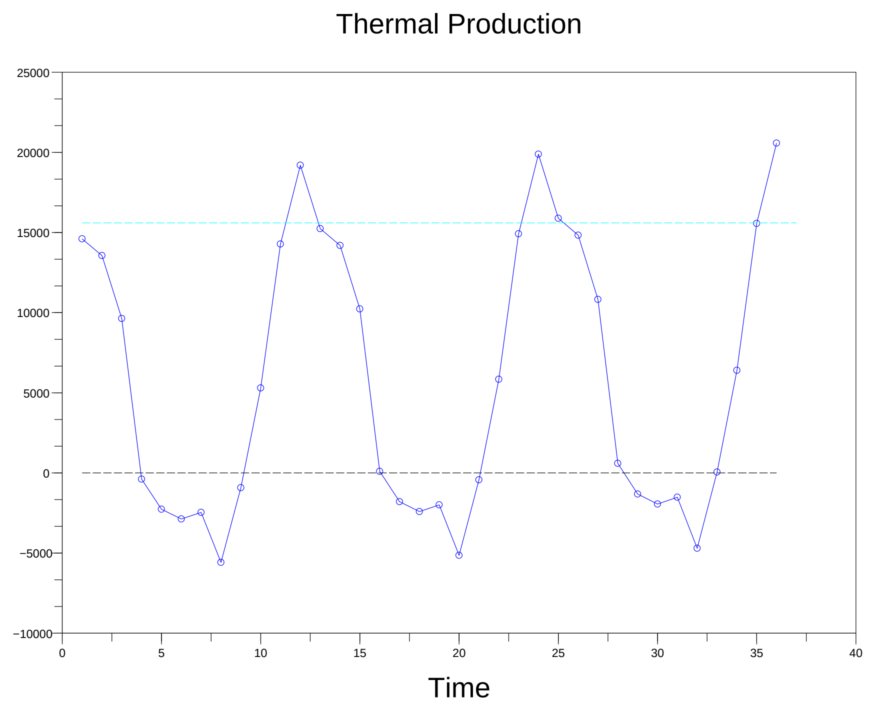

Figure 2: Simulated thermal production resulting from a constant nuclear production (in MW)

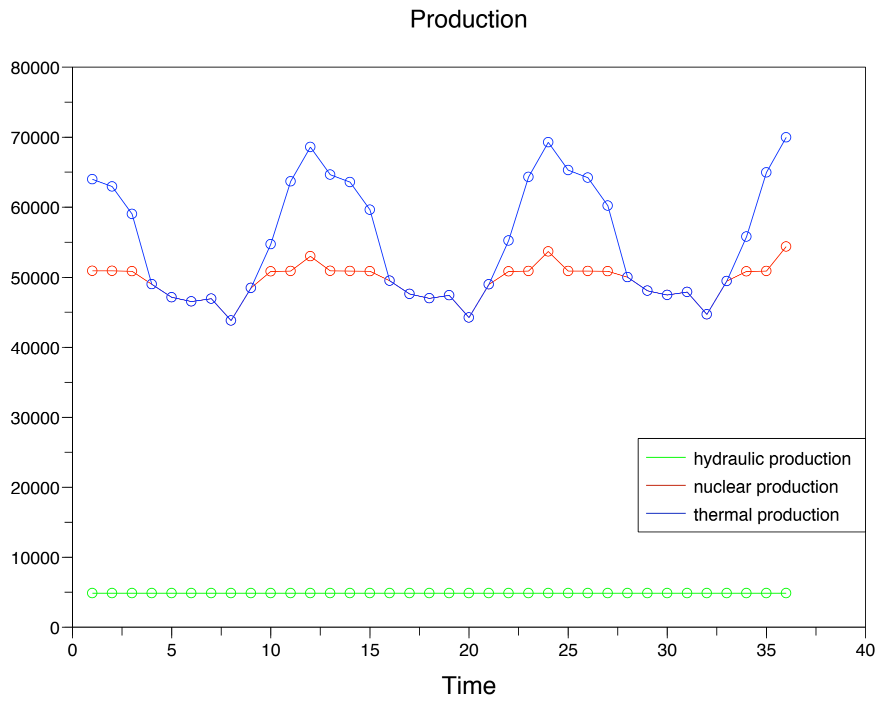

Figure 3: Simulated hydro(run-of-river)/nuclear/thermal production (in MW)

horizon of our model, which corresponds to the notion of "comonotonicity"[21] introduced by Yaari (1987) (see Figure 3). We also deduce, following a theoretical reasoning, that the nuclear production being comonotonic to the thermal production and obviously to itself is comonotonic to the total supply (the sum of nuclear and of thermal production). Consequently, if we take into account the equality between supply and demand, we deduce that the nuclear production is comonotonic to the demand. From a theoretical point of view, this result is not trivial and it is particular appealing: (i) demand is dynamic but is not periodic[22] because we assumed an augmentation of the demand by a rate of 1% per year (see Footnote 20 on page 17), (ii) there is a third variable which interferes between the demand and the nuclear production: the available nuclear capacity during the month t, $(\sum_{j=1}^{J} Q_{max}^{j,nuc}(t))$ considering that some unit is inactive during this month (month of reloading). Its evolution over time is periodic as it also appears in figure[23] 4. From (i) and (ii) results that despite the fact that the available nuclear capacity is periodic and thus, it repeats its values every year while the demand has an increasing tendency from one year to another, the nuclear production follows constantly the variations of demand i.e. they decrease and increase simultaneously. This shows why the comonotonicity between

[21]The vector $(X_t)_{t=1}^{T}$, by definition, is comonotonic to the vector $(Y_t)_{t=1}^{T}$ if $(X_{t'} - X_t)(Y_{t'} - Y_t) \geqslant 0$ holds for all t, t'. It forbids the opposite evolution between two dates for X and Y which means mathematically that it does not exist (t, t') such that $X_{t'} > X_t$ and $Y_{t'} < Y_t$.

[22]In mathematics, a periodic function is a function that repeats its values in regular intervals or periods. The most important examples are the trigonometric functions, which repeat over intervals of length 2π radians. Periodic functions are used throughout science to describe oscillations, waves and other phenomena that exhibits periodicity.

[23]The maximum nuclear production during the month t given that some unit is inactive during this month (month of reloading) is represented by the purple dotted line. This quantity is obviously below the nominal capacity of the French nuclear set represented by the crossed purple line. The minimum nuclear production during the month t given that some unit is inactive during this month (month of reloading) is represented by the purple line of asterisks.

the demand and the nuclear production is a non-obvious result of particular interest.

4.4.3 Analytical results

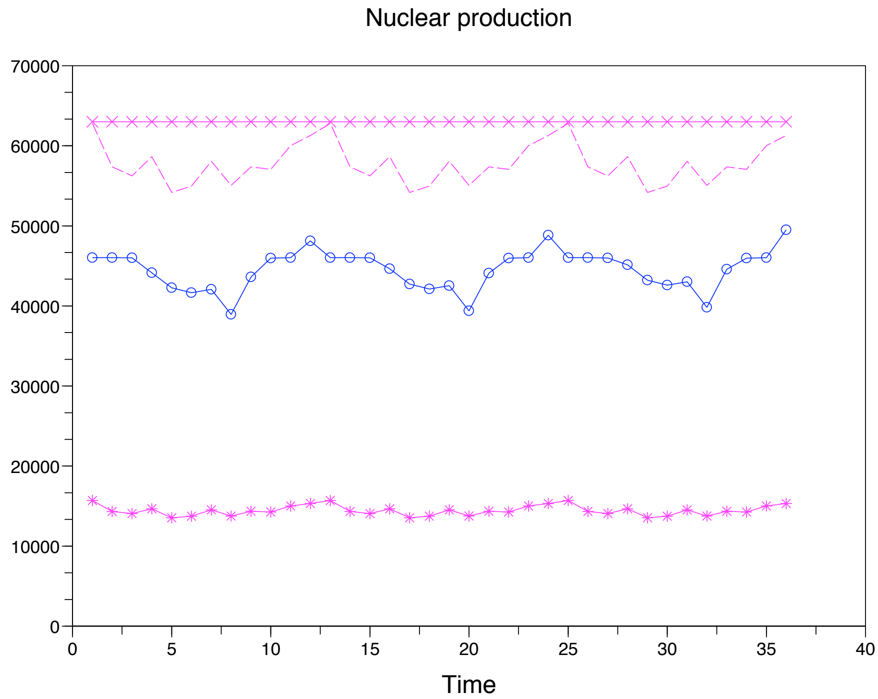

Figure 4: Simulated nuclear production (in MW)

In figure 4, we can see that the nuclear production follows the seasonal variations of demand (high production during winter – low production during summer) but it never reaches its maximum value[24] over the entire period T. The essentially periodic evolution of nuclear production leads to a periodic evolution for the nuclear fuel stock which oscillates around the "stock of reference"[25]. It increases during low demand seasons, exceeding the "stock of reference", while it decreases during high demand seasons falling below the "stock of reference" (see Figure 4, Figure 6).

From a theoretical thinking using the notion of comonotonicity like we did in the case of nuclear, the thermal production is comonotonic to the nuclear production which is itself comonotonic to the demand. Thus, we conclude that the thermal production is essentially comonotonic to the demand. In particular, it is increasing during winter (beginning from October) until it reaches its peak value in December. Afterwards, it decreases progressively until it takes its lowest value during summer.

The "regularized" price[26] is high during winter by taking its highest value in December and relatively low during summer (see Figure 7). From a theoretical point of view, this is explained by the fact that the "regularized" price is comonotonic to the thermal production since the price is determined by the thermal marginal cost, which is an increasing function of

[24]cf. Footnote 23.

[25] The "stock of reference" is represented by the blue dotted line which shows the value of stock at the beginning, being also the value of stock at the end.

[26]The red dotted line indicates the level of the "regularized" price when nuclear is the marginal technology.

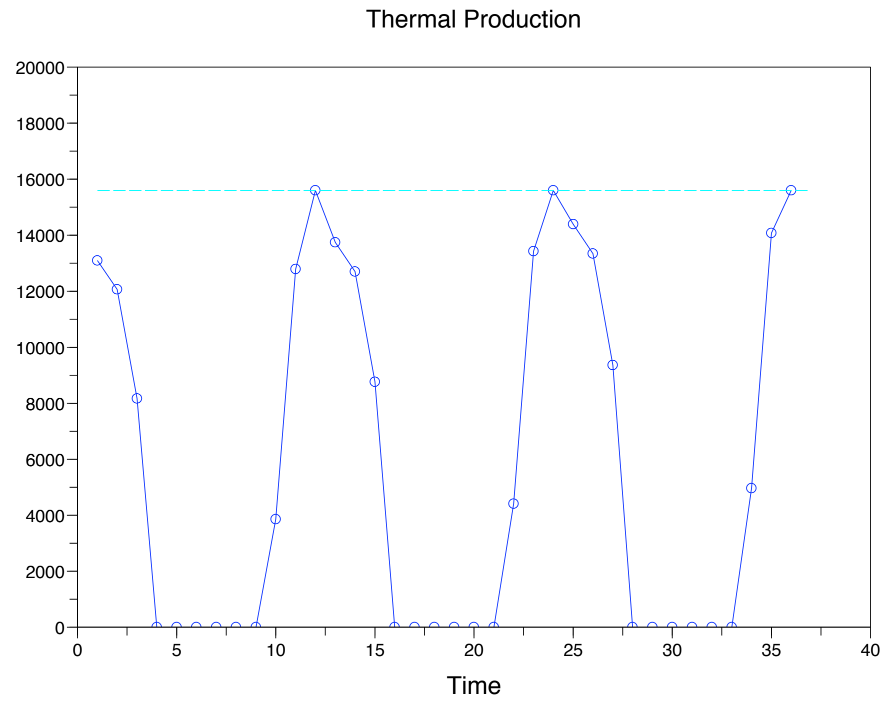

Figure 5: Simulated thermal production (in MW)

the thermal production. Additionally to this, in the previous paragraph, we showed that the thermal production is comonotonic to the demand. Hence, the price is essentially comonotonic

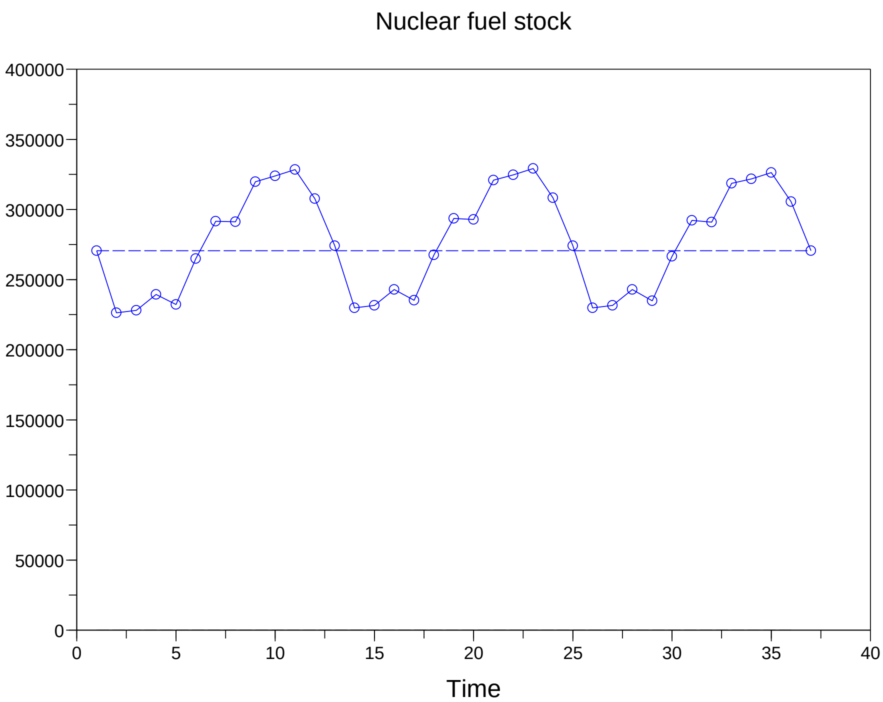

Figure 6: Simulated nuclear fuel stock (in MW)

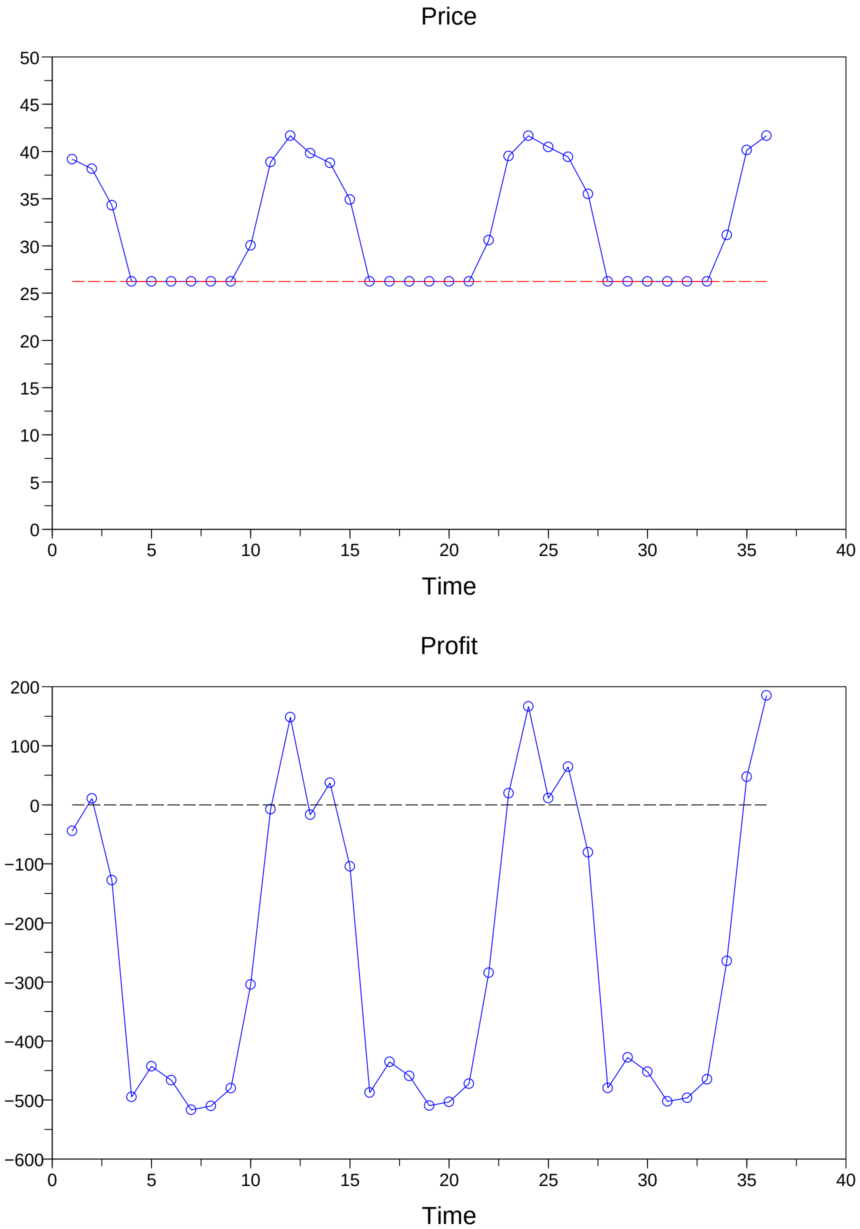

Figure 7: Simulated "regularized" price (in Euro/MWh)/Simulated "regularized" profit (omitting profit coming from hydro (run-of-river) generation) (in Euro (million))

to the demand and, therefore, it follows closely demand's seasonal variations.

The inter-temporal profit, being comonotonic to the price within our numerical model, is basically comonotonic to the demand which leads to high profits during winter and at the beginning of spring while lower profits are realized during summer and at the end of spring (see Figure 7). We may also observe that its value can break-up in a cyclical component and a linear trend which is slightly increasing. The reader should not pay attention to the exact value of profit as it is conditional on the too many approximations we made (e.g. discount rate, no mark-up rate, absence of profits coming from the hydro technology (run-of-river), etc.).

In view of the remark 3.2, the solution is unique with respect to the thermal component but bearing in mind the other variables which do not have an impact on the inter-temporal profit the entire solution is not necessarily unique.

Finally, we observe that a variation of the length of the time horizon T of the model does not lead to different behaviour patterns. The basically periodic evolution of the nuclear and thermal production is the same during the entire time horizon of the model (e.g. for $T = 72$, see Figure 8).

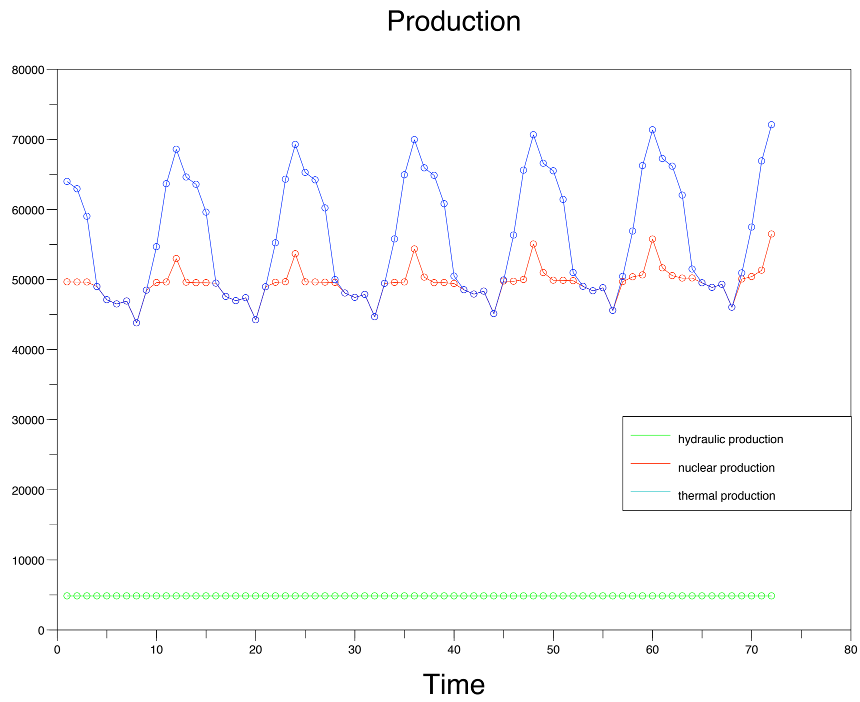

Figure 8: Simulated hydro(run-of-river)/nuclear/thermal production (in MW) (T=72)

4.4.4 The time period of marginality of nuclear and thermal generation technology

Simulation results show that the thermal production is marginal during months of high demand while the nuclear production is marginal during months of low demand meaning half part of the time (18 months over $T = 36$ months). In particular, nuclear stays marginal during almost the entire period of spring and summer (April - September), while thermal is marginal during autumn and winter (October - March) (see Figure 3, Figure 4, Figure 5). Hence, we see that a producer maximizes its inter-temporal profit by running only its nuclear units when the demand is low. The high duration of marginality of nuclear is first met in the theoretical optimal solutions in the particular case that the production constraints are not saturated. In view of our data, a constant nuclear production determines the price in the market during months of low demand and the length of its period of marginality is almost the same with that observed numerically. The time period of marginality of nuclear can be also explained by the fact that the price is "regularized" and consequently, the nuclear production is evaluated in a higher price than its own marginal cost (b_{nuc}) in our numerical model.

4.5 Optimal inter-temporal production problem VS Optimal short-term production problem

In the case of the optimal short-term management of the nuclear fuel reservoir, the optimization of production (nuclear, thermal) is realized over a management horizon equal to a month. In this paper, we carried on with the inter-temporal optimization based on the direct optimization of production over the entire time horizon T of the model (36 months). Each optimization problem has been treated within a simple numerical model in which the price is "regularized" (i.e. is determined by the thermal marginal cost when nuclear is the marginal technology). The symmetric approach regarding the "regularization" of these optimization problems makes pertinent their comparison.

The optimal short-term production problem provides us with a solution that can be seen as a "local" optimum, since it is optimal within a subset of feasible points. By contrast, the optimal inter-temporal production problem determines a global optimum which is the optimal solution among all the possible solutions. Consequently, from a theoretical point of view, the optimal inter-temporal profit has to be greater than the optimal short-term profit and it is what we also deduce from our numerical results. However, the variation of the profit when we move from a full optimization of production to a per month optimization (and vice versa) is of the order of 5% and is due to the slight variation of the variable cost of production since the initial losses (fixed costs) are identical for both problems (see Table 2, Table 3). Furthermore, the profit's value can be decomposed in a cyclical component and a linear trend that increases progressively from one year to another in both optimization problems.

	Optimal inter-temporal production problem	Optimal short-term production problem
Total "regularized" profit (in Euro)	-9.147×10^9	-9.636×10^9
Total "regularized" nuclear profit (in Euro)	-5.957×10^9	-6.332×10^9
Total "regularized" thermal profit (in Euro)	-3.189×10^9	-3.304×10^9

Table 2

	Optimal inter-temporal production problem	Optimal short-term production problem
Total cost (in Euro)	5.261×10^{10}	5.250×10^{10}
Total variable cost (in Euro)	1.075×10^{10}	1.064×10^{10}
Total fixed cost (in Euro)	4.186×10^{10}	4.186×10^{10}

Table 3

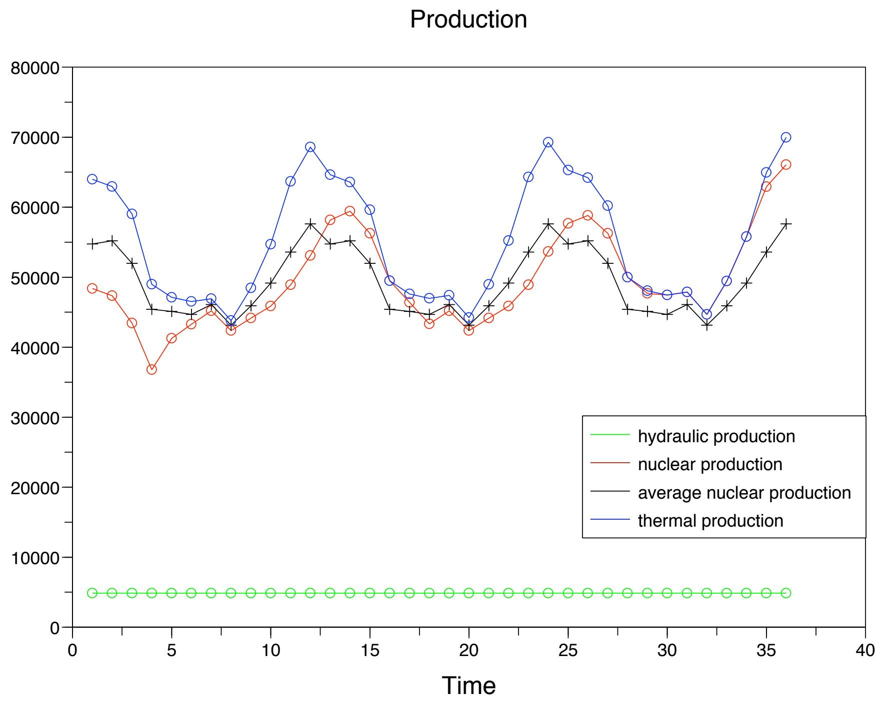

Figure 9: Simulated hydro(run-of-river)/nuclear/thermal production (in MW) resulting from the optimal short-term production problem

An important point of differentiation between the optimal inter-temporal production problem and the optimal short-term production problem consists of the length of the period of marginality of nuclear. In both numerical models, nuclear production is paid at price b_{th} which is significantly higher than its marginal cost b_{nuc}. This means that no penalty is caused by the exclusive use of nuclear production. However, we observe in figure 9 on page 25 that thermal is marginal during the majority of the months of period T in the case of the short-term optimization of production (the only exception is the last sub-period when nuclear units "overproduce"). The producer allocates an important amount of nuclear fuel stock during periods of high demand and thus the stock remaining in the reservoir is not sufficient to equilibrate supply and demand throughout low demand periods. In the optimal inter-temporal production problem, thermal is marginal only during months of high demand since the nuclear production is sufficient to cover the entire demand during periods of low demand (see Figure 3 on page 19). So, we deduce that when the producer maximizes its instantaneous profit, the price (respectively the profit) increases during low demand seasons. On the contrary, when the producer looks at the maximization of its inter-temporal profit, the price (respectively the profit) decreases during periods of low demand. As a conclusion, when the producer does not know how to reach an equilibrium of the optimal production problem and thus, it searches to determine a possible optimal production path on a short-term (monthly) basis, we obtain only a signal regarding the duration of nuclear's marginality. This signal does not reveal what happens when the producer having determined an equilibrium proceeds with the inter-temporal optimization of production which is a limit of the optimal short-term production problem.

A common point between the optimal short-term production problem and the optimal inter-temporal production problem concerns their solutions. More precisely, we get unicity of solutions with respect to the thermal component in both the optimal short-term production problem and the optimal inter-temporal production problem. We can also see that, in both optimal production problems, a prolongation of the time horizon of the model T does not result in different production behaviours (see Figure 8 on page 23 and Figure 10 on page 26).

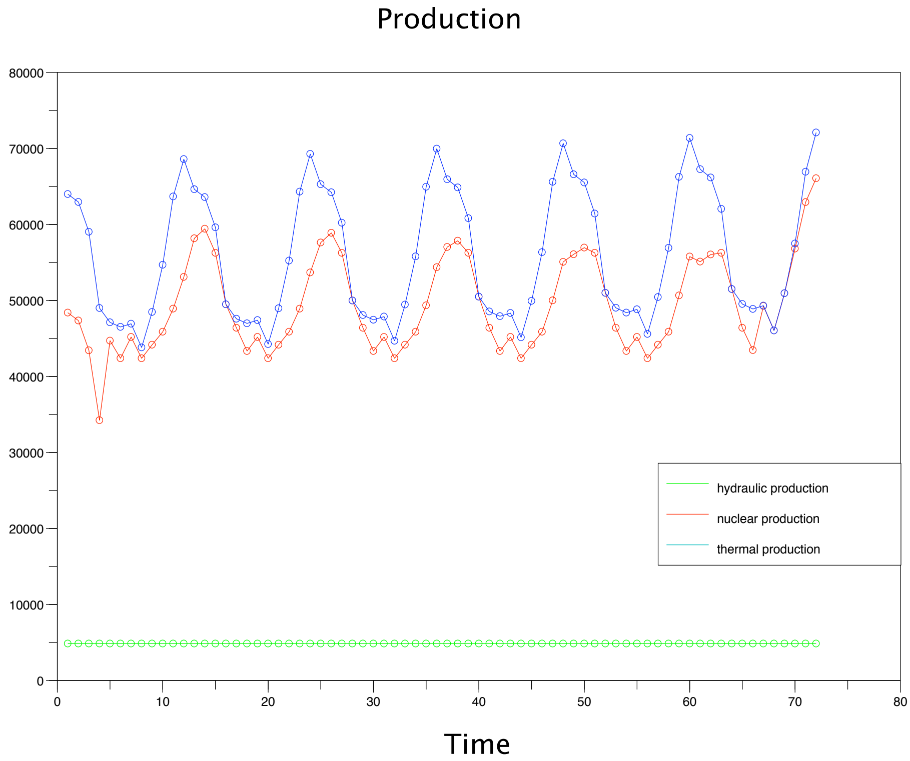

Figure 10: Simulated hydro(run-of-river)/nuclear/thermal production (in MW) resulting from the optimal short-term production problem (T=72)

5 Conclusion

In this paper, we examined the optimal inter-temporal management of the nuclear fuel reservoir of flexible nuclear plants in a competitive regime under production and storage constraints as well as constraints imposed by the equality between supply and demand each month. We focused on a medium-term horizon to take into account the seasonal variation of the demand between winter (high demand) and summer (low demand). The theoretical and numerical results obtained here are inherent in the assumptions made within our model.

First, we showed that, under some assumptions that hold within our model and for our data, the discontinuity (decrease) of price when nuclear becomes the marginal technology induces a discontinuity (decrease) of the inter-temporal profit of a producer. The dynamic nature of the nuclear fuel stock due to its modelling as a reservoir of energy make obvious the discontinuity of price which is the reason why it is difficult to find an equilibrium from a theoretical and numerical point of view. Mathematically, we simplified our optimal production problem by looking at a property of the "interior" solutions of the optimal inter-temporal production problem i.e. solutions that do not saturate the minimum and maximum production constraints. Economically, from a theoretical point of view, we proved that these solutions are fully characterized by a constant nuclear production given that the thermal capacity is sufficient in order to follow entirely the variations of demand which implies a baseload operation for nuclear plants.

From a numerical perspective, we "regularized" the price in order to deal with the problem of discontinuity of price. Then, the "regularized" price consisted of the sum of the nuclear marginal cost and an "opportunity" cost of using only nuclear capacities to meet demand instead of thermal capacities which results in a higher electricity price for consumers when nuclear is the marginal technology and in a higher market price for nuclear producers that helps to amortize their important fixed costs. This valorization of the nuclear production is met in the monitoring report of the French energy regulator (CRE) in 2007 (Regulatory Commission of Energy (2007)). Besides the economical consequences of the "regularization" of the price mentioned above, it also led to a satisfying situation which permitted to attain an equilibrium. The "regularization" of an economic phenomenon in order to reach an intellectually satisfying situation can be found for example in Balasko's work regarding the theory of general equilibrium (Balasko (1988)). The price "regularization" led to an alternative continuous problem (the "regularized problem") which has the same value with the economical problem (discontinuous problem) and permitted to obtain a "good" approximation of our economical problem. Proving that the set of solutions of the economical problem is empty, we showed the interest of focusing on the numerical solution resulting from the "regularized" problem which is only an approximative solution of our economical problem.

From a numerical perspective, we observed that solutions that are fully characterized by a constant nuclear production do not exist within our numerical model since the thermal capacity being inferior than demand's amplitude is not sufficient to cover the demand every month. This showed that, in France, where nuclear power is the principal electricity generation technology, nuclear can not be managed uniquely as a baseload generation technology at the optimum. It has also to operate at semi-base load following a part of the variable demand. Indeed, simulations showed that the nuclear production also adjusts to demand's seasonal variations to ensure the supply-demand equilibrium.

Theoretically and numerically, we found that in a decentralized market, producers optimum is obtained by making important investments in thermal capacity so that supply meets consumers needs for electricity (French case). Therefore, we conclude that the level of investments in order to build new capacities of a generation technology (e.g. nuclear) plays a major role in the determination of the production behaviour at equilibrium not only of the corresponding technology but also of the other generation technologies (e.g. thermal) that participate in the energy mix of a country.

The duration of marginality of nuclear is significant since the nuclear production covers the total monthly demand through the half of the time horizon T (18 months) and particularly during summer when the demand is low. A producer has to take into account the thermal generation capacity only during high demand periods to meet demand. The high duration of marginality of nuclear can be explained by: (i) the fact that an important time period of marginality of nuclear is originally observed in the theoretical solutions of the optimal inter-temporal production problem given our data, (ii) the "regularization" of the price which do not penalize the nuclear production within our numerical example.

Numerically, the results of the optimal inter-temporal production problem were contrasted with the results of the optimal short-term production problem. The producer uses practically all the time the thermal production to maximize its current monthly profit driving this way prices (respectively profits) in relatively high levels even during low demand seasons. This is not the case when producers maximize their inter-temporal profits since we noticed that, structurally, phenomena of marginality of nuclear may occur. More precisely, within our data set, the price in the market is determined by nuclear during periods of low demand from a theoretical and numerical perspective. Hence, despite the fact that the exclusive use of nuclear production is no more penalized in both problems, the length of the period of marginality of nuclear is more important in the optimal inter-temporal production problem than in the optimal short-term production problem where producers are short-sighted with respect to future gains. Nevertheless, the total profit resulting from the optimal inter-temporal production problem is higher than the total profit coming from the optimal short-term production problem since the solution of the first optimization problem constitutes a global optimum while the second problem gives a solution that can be thought as a "local" optimum.

In the real French electricity market, no behaviour coming from the resolution of the optimal inter-temporal production problem is fully observed. For example, as stated in the report of CRE in 2007, nuclear has been the marginal technology during periods of low demand. Nevertheless, the duration of marginality of nuclear indicated by the report was less important (a total of 1 or 2 months per year) than than the one resulted from the optimal inter-temporal production behaviour (the half part of the year) (Regulatory Commission of Energy (2007)). Furthermore, the nuclear units do not always produce at a constant rate as we deduced at the optimum from a theoretical point of view. These deviations from reality could be justified by the fact that the French nuclear operator (EDF) does not consider our assumptions that limit our model to a certain extent but help to overcome difficulties so that an equilibrium is found. Economically, the theoretical and numerical results obtained in our model may interest the system operator and provide insights for policy in the energy sector.

To conclude, in all cases, nuclear fuel modelled as a "reservoir" of energy follows the seasonal variations of demand in a competitive electricity market where nuclear capacity exceeds thermal capacity to a significant degree. But even if nuclear power does not possess the greatest part of the energy mix of a country (like France), it can be still operated at semi-base load following a part of demand's variations because, technically, modern nuclear reactors are capable of flexible operation. This could lead to a more significant use of nuclear in the electricity production of a country and therefore a higher share of nuclear power as a percentage of its national energy production especially since nuclear promotes: (*i*) reduction of CO_2 emissions, (*ii*) energy independence from fossil fuel generation technologies, (*iii*) large-scale deployment of intermittent electricity sources (renewable energy), (*iv*) economic competitiveness of a country's energy sector. All these factors play a very important role in the future of nuclear energy worldwide.

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