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Cédric Clastres
Haikel Khalfallah

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AN ANALYTICAL APPROACH FOR ELASTICITY OF DEMAND ACTIVATION WITH DEMAND RESPONSE MECHANISMS

Cédric Clastres[1] and Haikel Khalfallah[2]

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Abstract

The aim of this work is to demonstrate analytically under what conditions activating elasticity of demand of consumers could be beneficial for the social welfare. It has added to the literature on analyzing the use of price signals in eliciting demand response by an analytical approach. We develop so an analytical Nash model to quantify the effect of implementing demand response, via price signals, on social welfare and energy exchanges. A prior result show that the trade-off between producing locally and exporting energy depends on the opportunity cost of the energy and the global efficiency of the generation technology. Results are moreover impacted by the degree of integration between the countries.

The novelty of this research is the demonstration of the existence of an optimal region of price signal for which demand response leads to increase the social welfare. This optimality region is negatively correlated to the degree of competitiveness of the generation technologies and to the market size of the system. We particularly notice that the value of un-served energy or energy reduction the producers could lose from such demand response program would limit the effectiveness of its implementation. This constraint is strengthened when energy exchanges between countries are limited. Finally, we demonstrate that when we only consider the impact in term of consumers' surplus, more aggressive DR could be adopted. The intensity of DR program is however negatively correlated to the degree of the elasticity of demand.

1. Introduction

The deployment of smart grids is seen as a major change in the electricity markets. One of the main considerations, presented as a necessary condition to benefit from this deployment, is to pilot demand or to make it reactive to prices or markets' constraints (Chao, 2010). Indeed, beside all improvements for networks management, consumers' behavior will impact the uses and investment planning in the midstream (networks) and the upstream (generation) (Strbac, 2008). The idea is to replace some of deep investments in infrastructures by a decrease in consumption. This decrease could be made by the consumer or by another entity (aggregators, suppliers, distributors) on behalf of consumers. Experiments have shown that investing in smart grids technology could be cheaper than deeper investments in infrastructures. For example, the smart grids project implemented in Orkney, UK, has an estimated cost of building new power lines of £30 million to integrate 28 MW of wind energy whereas a smart grids technology investment was of £0.5 million (Kema, 2012).

The main feature of the demand of electricity is its inelasticity (Stoft, 2002). Currently, most of the consumers of electricity are captive in the short run. Industrials customers could be more volatile but mainly in the long run (Lijesen, 2007; Patrick and Wolak, 1997). However, with the implementation of smart grids and demand response programs, the aim is to make short run elasticity appears in some uses. Thus, introducing price

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[1] Associate Professor, UPMF, +33 4 56 52 85 78, cedric.clastres@upmf-grenoble.fr
[2] Associate Professor, UPMF, +33 4 56 52 85 80, haikel.khalfallah@upmf-grenoble.fr

signals to consumers will always drop consumption[3]. The literature generally shows that demand is elastic when dynamic tariff is introduced. However, the value of the elasticity varies as regard to several factors, for instance the period of consumption, the equipments of households, the degree of deployment of SG technologies or the price differential between periods of consumption (Di Cosmo et al., 2014). Boisvert et al. (2004) demonstrated that there exists a significant elasticity of substitution between off-peak and peak periods. An increase in peak prices induces an increase in the off-peak demand; this is the load-shifting effect. More recently, Faruqui and Sergici (2010) have analyzed several American pilots in demand response. They conclude that households could react to dynamic pricing, mainly by reducing their peak-load demand. Thus, several papers have studied the impact of dynamic pricing on consumers and markets design to remunerate curtailments. In a perfect competitive market, Chao (2011) has demonstrated that a remuneration of curtailment equals to the difference between retail rate and real time price should be optimal for the welfare. With an applied model, he notes that consumers could increase their surplus with the implementation of a demand response program. As their payments stay constants, their satisfaction increases because of revenues of the demand response. Obviously, an increase in tariffs impacts negatively the consumers' surplus. However, this decrease in surplus could be offset by revenues from demand response. Thus, consumers could improve their welfare. Orans et al. (2010) show that a three-part tariff, including a Time of Use, a fixed fee and a remuneration of the demand response is an efficient tool to give incentives to modify consumers' behavior. They note the significance to design a tariff with a lower impact on the consumers' payments (with the demand response program).

We defer from these papers using analytical equilibrium in an interconnected market to study in which cases an increase in tariffs would reduce consumption without worsening social welfare. We do not introduce a remuneration for the curtailment, but previous researches show that it should improve the satisfaction of consumers. The more load-shedding is valued on the market or the more the shift of demand is, the more consumer could conserve part of their surplus from reduced bills and demand response valorization.

We use a deterministic optimization model with supply functions (Ventosa et al., 2005). As in others papers (Stoft, 2002), we assume perfect competitive markets. Each producer makes offers at its marginal costs. Woo (1990) shows that it is optimal to set a two-part tariff based on marginal costs when utilities could manage the demand of consumers with an ex-ante subscription. De Jonghe et al. (2011) studied the demand response and have shown with a computable model that demand response has an impact on investments and on renewable integration. To our knowledge, there is a few works that tries to model analytically the implementation of demand response in a situation of interconnected electricity markets. Vespucci et al. (2013) is the closer model of ours. They have studied an electricity market divided into interconnected zones. Their analysis focuses on the transformation of their optimization problem into a linear one, as electric supply and demand functions are discontinuous. Then, they show with applied data that a dominant firm has always incentives to use market power to achieve higher profit targets. Our model differs from their regarding two points. First, we compute theoretical equilibriums in a context of two interconnected markets with several technologies in each market. Secondly, we introduce an analysis of the demand response linked to the degree of the elasticity of consumer demand.

In our paper, a first analysis is made on how generation technologies structures could affect countries' merit orders and potential trades between them. The results show that the trade-off between producing locally and exporting energy depends on the opportunity cost of the energy and the global efficiency of the generation technologies. These results are impacted by the degree of integration between the countries. When interconnection capacity is limited, we demonstrate that interconnection prices would impact negatively the

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[3] This result could be overcome by the extend of the rebound effect (Muratori et al., 2014).

trades between the countries. Thus, incentives for exports are reduced when transport capacities become more expensive. The generation costs differential between the interconnected systems lessens this effect.

A second analysis, which constitutes the novelty of this research, demonstrated that there exists an optimal region of price signal for which demand response leads to increase the social welfare. We use computed equilibria of the first analysis to show that this optimality region is negatively correlated to the degree of competitiveness of the generation technologies and the market size of the system. The value of un-served energy or energy reduction the producers could lose from such consumption reduction would limit the implementation of such program. This constraint is moreover strengthened when energy exchanges between countries are limited. However, this constraint is alleviated if the considered system is cost-inefficient as well as weakly connected with its neighbors. Such surprising result is also explained by the weight of the value of lost energy inefficient producers will cope with when intensive DR is put in place.

The paper is organized as follows. Section 2 presents the modeling assumptions. In section 3, we show the analytical equilibriums as regarding two scenarios about the degree of interconnection between the markets. In section 4, we focus on the impact of a demand response program implementation through a modification in supply and demand functions. Section 5 concludes.

2. Modeling Framework

The following sub-sections discuss the main assumptions of this analysis regarding generation technologies characteristics and the calculation of demand and supply functions in the modeled markets.

2.1. Generation technologies and trades between countries

We assume that there are two interconnected countries[4] ($n=1,2$)[5]. The two system operators, after collecting their generation technologies' plans, balance the total offer and the total demand by considering possible exchanges between the countries. This exchange is limited by an interconnection capacity $Cap_{n,m}$, which has a price $P_{n,m}$ when the capacity is saturated. Each country is characterized by the presence of t generation technologies. We assume perfect competition in the two markets. So, we disregard the strategic behaviors of producers by admitting that the merit order of a given system is the result of the aggregation of the marginal costs of the generation technologies which are available to produce[6]. Each technology is characterized by a quadratic variable cost function[7], described as follows:

$$CV_{t,n}(x_{t,n}) = a_{t,n}.x_{t,n} + \frac{1}{2}.b_{t,n}.x_{t,n}^2 \tag{1}$$

$a_{t,n}$ and $b_{t,n}$ are parameters and $x_{t,n}$ is the quantity produced by technology t in a given country n.

By aggregating marginal costs of the technologies available in country n, we obtain the following inverse supply function:

$$S_n(X_n) = a_n + b_n X_n \tag{2}$$

a_n and b_n are aggregate parameters and $X_n = \sum_{t=1}^{2}(x_{t,n} + x_{t,n,m})$ is the total quantity produced in a given country. See *appendix 1* for more details about the construction of the supply function.

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[4] We note that the model could be generalized to analyze more than two interconnected systems.
[5] Index m is also used.
[6] Our model is a one shot game with a linear demand. Thus, all capacities are offered on the market through the aggregate supply function to serve the demand.
[7] As we assume a perfect competitive market, fixed costs of generation are disregarded since they do not impact our results.

For sake of simplicity, we rank the technologies from the less costly to the more costly. A linear relationship between the variable costs function of the technologies is made as follow:

$a_{t,n} = \alpha^{t-1}.a$ and $b_{t,n} = \alpha^{t-1}.b$

Where α is a ranking parameter of the technologies[8] and; a and b corresponds to the parameters of the less costly technology.

2.2. Supply and Demand

In a first scenario, we suppose that electricity demand in a given country is inelastic ($\overline{D}_n$). In the second scenario, we suppose, as detailed in the fourth section, that implementing a Demand Response mechanism will allow to modify consumers' behavior and then their demand becomes elastic. Inverse demand function in the last case is given by:

$$D_n^{-1}(X_n) = c_n - d_n.X_n \tag{3}$$

Where c_n and d_n are the parameters of the inverse demand function.

Market equilibrium is found at the intersection between demand and supply functions. While consumers are passive players, their demand function depends just on prices and their elasticity. The supply function is the result of a minimization program where system operator determines the aggregated supply function to be minimized to satisfy local demand. As in Vespucci et al. (2013), this minimization includes local generation technologies and potential import from the neighbor country (see section 3 below).

The final consumption in a given country Q_n will take this form:

$$Q_n = \begin{cases} \overline{D}_n & \text{if Demand is inelastic} \\ D_n^{-1}(Q_n) & \text{if Demand is elastic} \end{cases} \tag{4}$$

3. The Model

Each country n's system operator minimizes the total variable cost of the generation that satisfies its local demand and possible export to the other country. The objective function to be minimized (5) and the specific constraints (6-8) are defined as follows:

$$Min_{x_{t,n},x_{t,n,m}} \sum_t CV_{t,n}(X_{t,n}) \tag{5}$$

Subject to,

$$x_{t,n} + x_{t,n,m} \leq K_{t,n} \qquad (r_{t,n}) \tag{6}$$

$$\sum_t x_{t,n,m} \leq Cap_{n,m} \qquad (P_{n,m}) \tag{7}$$

$$\sum_t x_{t,n} + \sum_t x_{t,m,n} = Q_n \qquad (f_n) \tag{8}$$

Where,

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[8] We suppose that α is slightly >1 to keep a rational homogeneity in the generation technologies and to avoid exponential disparity between them.

$x_{t,n}$	Local generation of technology t in country n
$x_{t,n,m}$	Quantity produced by technology t in country n and exported to country m
$X_{t,n} = x_{t,n} + x_{t,n,m}$	Total generation of technology t in country n
$K_{t,n}$	Installed available capacity of technology t in country n
$Cap_{n,m}$	Interconnection capacity between country n and country m
Q_n	Final consumption in country n
$r_{t,n}$, $P_{n,m}$ and f_n	Dual variables of the constraints (6), (7) and (8) respectively

Energy generation in a given country must match with three constraints. Firstly, the quantity generated by each technology t, which includes local generation, $x_{t,n}$, and possible export to neighbor country, $x_{t,n,m}$, cannot exceed its installed available capacity (equation 6). Secondly, the export energy from country n to country m is limited by the capacity of the interconnection line, $Cap_{n,m}$ in equation 7. Finally equation 8 shows that each system must ensure its market balancing by equaling local consumption, Q_n, to total generation addressed to the system.

By solving the above constrained program, the Nash-equilibrium consists of determining the optimal generation quantities, $x^*_{t,n}$ and $x^*_{t,n,m}$, that minimize simultaneously generation variable costs in both countries. As demonstrated in *appendix 2*, the solution is unique since the cost functions are strictly convex and continuously differentiable. The solution is found by regrouping together all first order conditions, so a Mixed Complementarity Problem[9] (MCP) is formed.

Resolving the model as demonstrated in *appendix 3* implies the following results:

3.1. Trade-off between local production and export

Proposition 1: *The trade-off between producing locally or exporting energy would depend on the opportunity cost of energy and the global efficiency of the production technology. Interconnection prices in both senses would limit the exchanges between countries. This impact is however weighted by the generation costs differential between interconnected markets.*

Optimal levels of production are defined as follows:

$$x^*{}_{t,n} = x^*{}_{t,n,m} + \frac{f_m - f_n}{\alpha^{t-1}.b} \tag{9a}$$

With,

$$f_m - f_n = \frac{b.(1-\alpha)(Q_n - Q_m)}{1 - \alpha^t} \tag{10}$$

Proof. *See appendix 3*

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[9] It solves directly the necessary conditions of the Nash equilibrium. Writing the first order optimality conditions simultaneously for all decision makers results in a MCP problem that we solve with Kuhn and Tucker's conditions.

First order conditions of the model show that the dual variables f_m and f_n are negative. They could be interpreted as the cost supported by the country to balance its national electricity market. Their difference is positive when local consumption is higher than foreign country consumption: $Q_n \geq Q_m$. In this case, the production that satisfies local consumption $x^*_{t,n}$ is far higher than the exported quantity $x^*_{t,n,m}$. Indeed, when the absolute value of dual price f_n is higher than the one of the other country, the opportunity cost of selling energy in the local market is higher than exporting. So, producers of country n will have to produce more in their local country before exporting to country m. Moreover, they have incentives to do so because the higher valuation of their production in market n. A contrario, when the opportunity cost of exporting energy from country m to country n is less costly than producing locally, producers of country m will favor the export. The analysis of the demand level in each country must be linked with the analysis of DR program made in the following sections of the paper. A DR program would modify these equilibria. Reducing the consumption in the countries must distress the system, making some cheaper generation available for export. Empirical studies have shown this result. Indeed, a DR program in a country reduces its needs of generation. Thanks to the DR program, efficient countries must increase their exports, improving their revenues and the efficiency of the overall system (Bergaentzlé et al, 2014). Moreover, by reorganizing equations 9a and 10 we can observe that the efficiency of the technology t compared to the other technologies would influence the trade-off between producing locally and exporting. Let's assume that $\omega_t = \frac{\alpha^{t-1}}{\alpha - 1}$ is the global efficiency factor of technology t. This term ω_t also give information on price levels in the two markets that balance supply and demand. So, according to this term, producers in country n must export capacities when their technologies are more efficient rather than competitors' ones in country m. Equation 9b shows that the more the technology t is efficient i.e. ω_t decreases, the more it will be preferred for balancing the system of the country having higher energy consumption:

$$x^*_{t,n} - x^*_{t,n,m} = \frac{Q_n - Q_m}{\omega_t} + \frac{(1-\alpha).\Sigma_n P_{n,m}}{\alpha^{t-1}.(1-\alpha^t).b} . \Sigma_t \frac{1}{\alpha_m^{t-1}} \tag{9b}$$

When interconnection capacities are limited, i.e. $P_{n,m} \geq 0$, local production is more preferred to serve local demand. Indeed, high values of $P_{n,m}$ reduces the incentives for country n to export and $P_{m,n}$ has the same effect for country m. Thus, the higher $P_{m,n}$ is, the lower producers of country m are incited to export, the higher $x^*_{t,n}$ is. The influence of interconnection capacity price on generation decisions is weighted by the efficiency factor of the importer country's technologies defined by: $\Sigma_t \frac{1}{\alpha_m^{t-1}}$. The more the importer country has expensive technologies, i.e. α_m^{t-1} increases, the more this factor decreases. Thus, to minimize total systems costs, it would be beneficial to rely more on export even if there exist significant interconnection prices, leading to an increase in $x^*_{t,n,m}$. Moreover, producers in country n could valorize their production at a greater marginal costs or energy price in country m. Exchanges between the countries would still significant when potential importer country holds relatively more expensive technologies.

3.2. Technology efficiency and markets balancing

Proposition 2: *Efficient technologies would be called first for balancing the markets. Other technologies would satisfy residual demands with respect to their global and relative efficiencies. Exchanges are however strongly correlated to interconnection prices and cost intensity of importer country.*

Each efficient technologies t would offer all its installed capacity to the interconnected markets. Optimal quantities for local production and for export are given as follow[10]:

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[10] N.B : under the condition that $x^*_{t,n}$ and $x^*_{t,n,m}$ are non negative, with $x^*_{t,n} + x^*_{t,n,m} < K$. Otherwise, if $x^*_{t,n} + x^*_{t,n,m} \geq K_{t,n}$, the equilibrium is $x^*_{t,n} + x^*_{t,n,m} = K_{t,n}$ and others technologies serve the residual demand.

$$x^*{}_{t,n} = \frac{K_{t,n}}{2} + \frac{Q_n - Q_m}{2.\omega_t} + \frac{\sum_n P_{n,m}.\Sigma t \frac{1}{\alpha_{t,m}}}{b.\omega_t} \tag{11}$$

$$x^*{}_{t,n,m} = \frac{K_{t,n}}{2} - \frac{Q_n - Q_m}{2.\omega_t} - \frac{\sum_n P_{n,m}.\Sigma t \frac{1}{\alpha_{t,m}}}{b.\omega_t} \tag{12}$$

However, inefficient technologies $\bar{t}$ would just deserve residual demand in the interconnected markets, with quantities lower than their total installed capacities. Their optimal production levels are given by[11]:

$$x^*{}_{\bar{t},n} = \frac{Res_n - \phi_{\bar{t}}}{\omega_{\bar{t}}} + \frac{P_{m,n}}{b.\omega_{\bar{t}}} . \Sigma t \frac{1}{\alpha_{t,m}} \tag{13}$$

$$x^*{}_{\bar{t},n,m} = \frac{Res_m - \phi_{\bar{t}}}{\omega_{\bar{t}}} - \frac{P_{n,m}}{b.\omega_{\bar{t}}} . \Sigma t \frac{1}{\alpha_{t,m}} \tag{14}$$

Where,

$$Res_n = Q_n - \sum_t x^*{}_{t,n} - \sum_t x^*{}_{t,m,n}$$

$$Res_m = Q_m - \sum_t x^*{}_{t,m} - \sum_t x^*{}_{t,n,m}$$

$$\omega_t = \frac{\alpha^{t-1}}{\alpha - 1} \text{ with } t \text{ is the index of an efficient technology}$$

$$\omega_{\bar{t}} = \frac{\alpha^{\bar{t}-1}}{\alpha - 1} \text{ with } \bar{t} \text{ is the index of an inefficient technology}$$

$$\phi_{\bar{t}} = \sum_{\bar{\bar{t}},n} \frac{a_{\bar{t},n} - a_{\bar{\bar{t}},n}}{b_1} \text{ with } \bar{\bar{t}} \text{ the index of the other inefficient technologies besides } \bar{t}$$

Proof. *See appendix 4*

Thus, the call to produce of a given technology would depend on its efficiency compared to other technologies. When its efficiency is higher than a certain threshold as discussed in *appendix 5*, all its available capacity will be used to balance the systems and minimize the total cost of energy supply. The other technologies will either satisfy the efficiency conditions and hence they will be called at full capacity or just serve the residual demand, verifying then the solution of equation 13 and 14.

Supposing that interconnection capacities are unlimited, i.e. $P_{n,m} = 0$. When technologies are efficient, the share of capacity between local production and export is mainly directed by energy consumption differential, $Q_n - Q_m$, as shown in figure 1, which return us back to the analysis of equations 9a, 9b and 10 above and more precisely on the impact of price differential, $f_m - f_n$. Again, the more the local consumption is higher than the one in neighbor country, the more local price is higher, the more generation is kept to satisfy local needs, and *vice versa*. We can observe from figure 1 that generation is symmetrically shared between local generation and export, with a constant slope that depends on the global efficiency factor of the technology, $\frac{1}{2.\omega_t}$; the total cannot exceed the available generation capacity, $K_{t,n}$. We could conclude that if technology t of country n is efficient with respect to other technology, its production increases to serve both the local demand and the export to neighbor country m until its full capacity $K_{t,n}$ is reached.

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[11] N.B : under the condition that $x^*{}_{\bar{t},n}$ and $x^*{}_{\bar{t},n,m}$ are non-negative and their sum does not exceed the technology capacity $K_{\bar{t},n}$. Otherwise, inefficient technologies produce at full capacity and a risk of unbalanced market appear with no equilibrium. The cost of the energy is then the Value of Loss Load. As mentioned in our article, DR could reduce this risk.

We can also see from figure 1 that generation sharing is impacted by the global efficiency factor of the technology, ω_t. An increase in α^{t-1}, involving that the technology tends to be more costly, leads to an increase in ω_t, meaning that the technology becomes less efficient. In this case, the incurred inefficiency will be supported by the system facing low demand. For instance, if $Q_n - Q_m > 0$, local generation from efficient technology in country *n* decreases, and the residue will be covered from import, and *vice versa*. This behavior should stop as soon as the global inefficiency factor of efficient technology reach a limit beyond it the technology becomes absolutely inefficient and hence, will be shared given the solutions in equation 13 and 14.
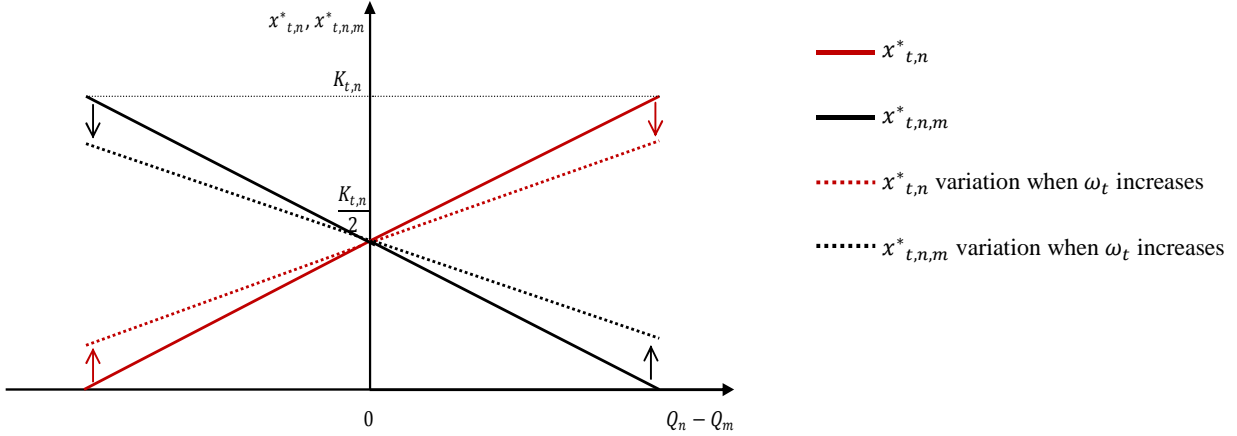

Figure 1. *Efficient technologies sharing given demand differential and technology's global efficiency with unlimited interconnection capacities*

Interconnection capacity prices in both senses of exchange would however reduce the incentives to export into the neighbor country. We can see from figure 2 a shifting to the left of the point for which local generation equals exported quantity. Moreover, for the same level of demand differential and whatever its sign, local generation increases in spite of export, compared to the situation with no constraint on inter-countries exchanges. The variation corresponds to the sum of interconnections capacity prices weighted by a factor of cost structure characteristics of the importer country, $\frac{\Sigma_t \frac{1}{\alpha_{t,m}}}{b.\omega_t}$. This means that when restraining interconnection capacity, systems with efficient technologies are less incentivized to externalize their costs advantage. Moreover, we can see from the figure that local generation will reach more quickly its generation capacity, when the country's needs are higher than in the other country, while exporting all the generation capacity will be slowly reached in the opposite case. This result goes toward our previous analyzing underlying the disincentive interconnection capacity prices will have on energy exchanges.
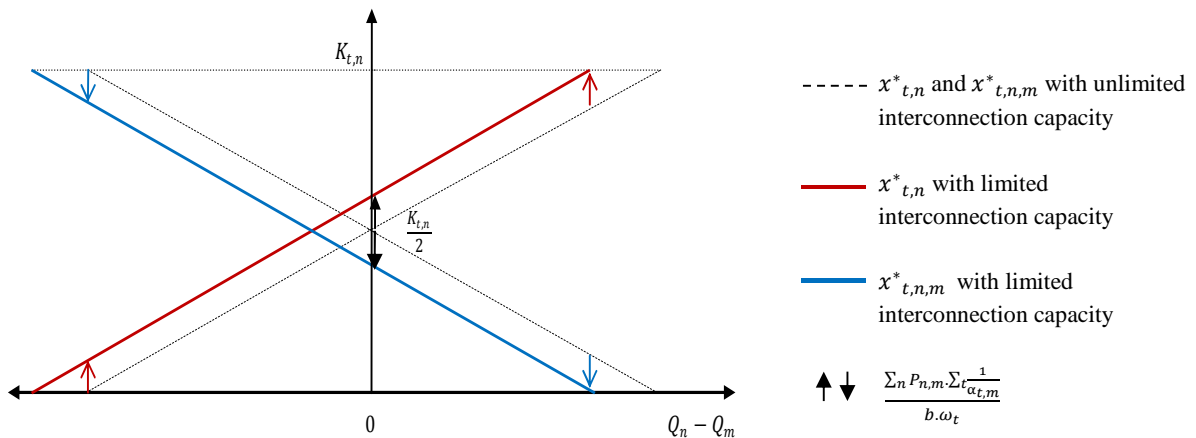

Figure 2. *Efficient technologies sharing given demand differential and interconnection prices with limited interconnection capacity*

Varying now our results regarding the parameter $\sum_t \frac{1}{\alpha_{t,m}}$, which is an indicator on the cost level of importer country m, we can see from the figure 3 below that the more generation technologies of the neighbor country are cheaper, i.e. a decrease in $\alpha_{t,m}$ will increase $\sum_t \frac{1}{\alpha_{t,m}}$, the less export from country n to country m is, i.e. $x^*_{\bar{t},n,m}$ decreases, and hence the higher the local generation is i.e. $x^*_{\bar{t},n}$ increases. This result holds whatever the sign of demand differential. Because the neighbor country m is cheaper, local generation will prefer to import from country n, $x^*_{t,n,m}$ decreases, and because of the technology is efficient and has to generate at its maximum, the residue involved will be covered by increasing local generation, $x^*_{t,n,m}$ increases.
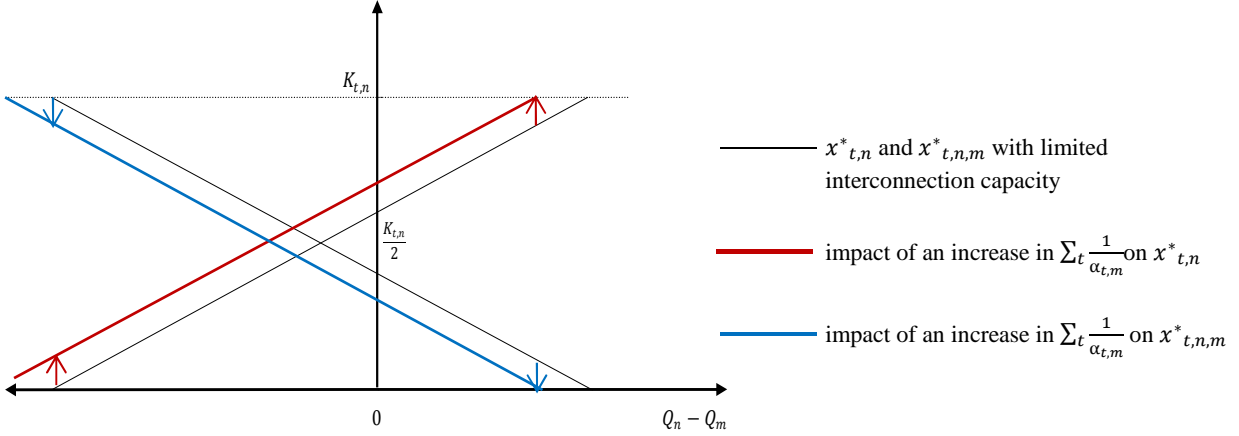

Figure 3. *Efficient technologies sharing given demand differential and cost structure of importer country*

Considering now inefficient technologies. Their call is increasingly dependent on the existence of residual demand, $Q_n - \sum_t x^*_{t,n,m}$ and $Q_m - \sum_t x^*_{t,n}$, as shown in equations 13 and 14, and on the global, $\omega_{\bar{t}}$, and relative, $\varphi_{\bar{t}}$, efficiencies of the technology compared to the other inefficient ones. Not surprisingly, when choosing between inefficient technologies, the less inefficient one are preferred, i.e. $x^*_{\bar{t},n}$ and $x^*_{\bar{t},n,m}$ decrease when $\omega_{\bar{t}}$ or $\varphi_{\bar{t}}$ increases.

Let's assume that interconnection capacities are unlimited. Figure 4 shows that until certain limits of residual demands in both countries, inefficient technologies would produce a quantity given the solutions in equation 13 and 14. However, beyond this limit, mainly because one of the residual demands becomes very high, *zone B* in figure 4, while the technology still inefficient, its total generation would correspond to its total available capacity and optimal generation will correspond to the one of efficient technologies, given the solutions in equation 11 and 12. This means that not only the cost structure of the technology which determines its optimal generation, the level of the specific demand addressed to them too.

However, the more the technology is costly, the more its generation quantity still in the inefficiency zone as shown in the figure 4 i.e. when $\omega_{\bar{t}}$ or $\varphi_{\bar{t}}$ increases, the inefficiency limit increases. The relative efficiency of the technology, which is defined by the parameter $\varphi_{\bar{t}}$ impacts the call to generate of such technology. This indicator is constituted by the parameters $a_{\bar{t},n}$ and b which measure respectively the cost of an infinitesimal quantity of energy and the evolution of the generation cost when the generation quantity increases. From this indicator and its impact on the solutions in equations 13 and 14, we can admit that the call of a technology $\bar{t}$ increases with its efficiency, but the increase is moderated by the trajectory b.
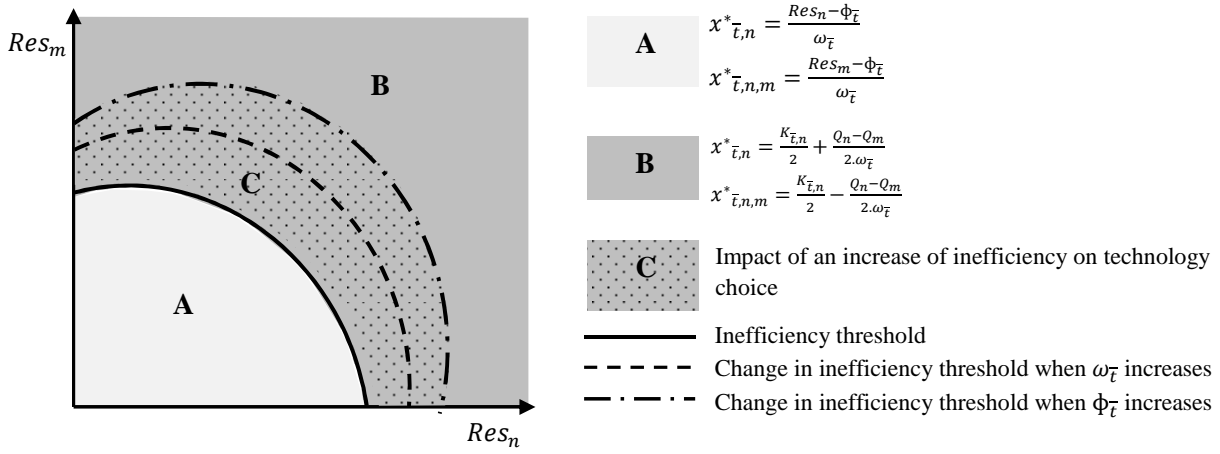
Figure 4. *Inefficient technologies sharing given residual demands and technology's efficiency factors*

When interconnection capacities are limited, local generation would be directly impacted by the interconnection capacity price in the inverse sense of export only, $P_{m,n}$. If technologies in country m are more efficient, balancing supply and demand in market n should be done with imports, except if imports are costly. Potential export is however directly impacted only by the capacity price in the sense of export, $P_{n,m}$. Indeed, when generation technologies are inefficient, systems operators would weakly rely on them to balance their systems; these technologies will just act as auxiliary technologies. It would be more beneficial to rely on import than calling the inefficient local technologies, upon the condition of course that imported technologies still less expensive. This would be ensured if the cost of transport is low enough. That's why our solutions in equations 18 and 19 show that any increase in the cost of transport, $P_{m,n}$ and $P_{n,m}$ respectively[12], would reduce the incentive for import and thus would impact positively local production, even if coming from inefficient ones. This result is shown in figure 5 where we observe a shifting to the left of $x^*_{\bar{t},n}$ function. For the same amount of Res_n ,local generation will increase. Exported quantity from inefficient technologies will however decrease. The variation is identical and expressed by $\frac{\sum_n P_{n,m} \cdot \sum_t \frac{1}{\alpha_{t,m}}}{b.\omega_t}$.

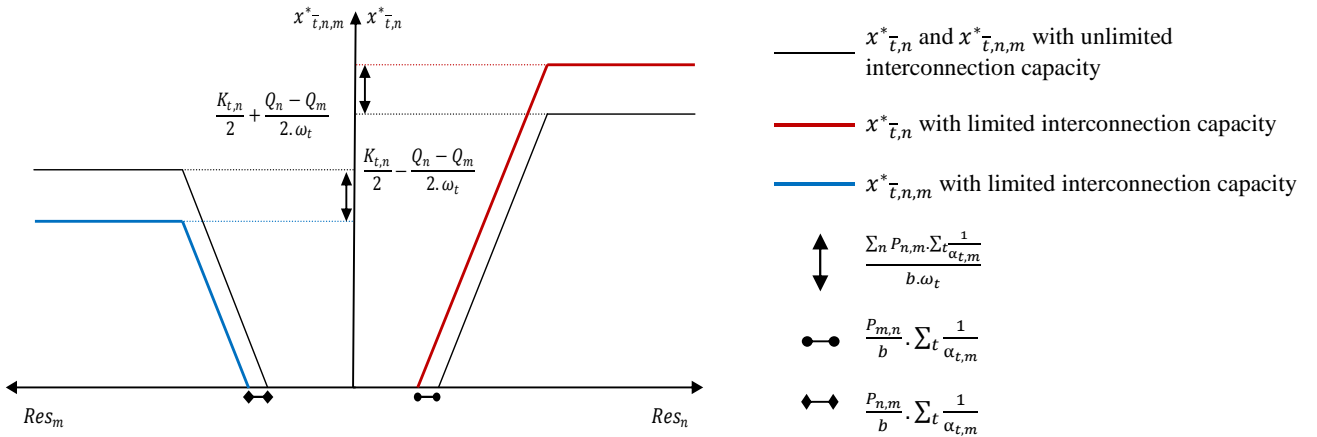**Figure 5.** *Inefficient technologies sharing given residual demands and with limited interconnection capacity*

In this section, we have seen that in an interconnected market, equilibria rely on technology efficiency, interconnection capacities, the opportunity cost of energy and the market sizes. Thus, introducing DR program

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[12] Assuming that only capacity prices could increase, other parameters are unchanged.

will impact these equilibria mainly because of the modification of the demand level and an increase in the availability of power plants.

4. Demand response and consumers' elasticity

We focus now on the effectiveness of the implementation of a Demand Response (DR) program. We can admit that a generalized application of DR would have several impacts on system equilibrium. The main objective of this program is to make consumers sensitive to prices in the short run[13]. They could actively participate in managing system security, instead of relying only on supply managements when the system is close to rationing.

Such mechanism would have a large cost of implementation. Consumers need to adapt and modify their behavior, going from being captive and inelastic to really elastic. It is worth noting that under DR programs, demand becomes elastic for a large range of consumers, independently of their own appliances. Obviously, as several pilots in US have shown it (Faruqui et al., 2009), main gains are in peak periods. Electric heating or air conditioning demand are the main consumption appliances that could be managed in DR programs. However, dynamic pricing gives incentives to reduce consumption to large part of consumers, regardless of the ownership of for example air conditioning (Di Cosmo et al., 2014). These effects are increased with information technologies, as in home display, that give continuous information to consumers on its consumption and tariffs.

Let's assume that at least two consequences will happen in the future because of a generalized application of DR. First, we can admit that the additional DR cost supported by the suppliers will lead to modify and increase their supply function. Indeed, investments in metering, energy boxes and sensors should be done to precisely assess energy consumption and to maximize the impact of DR programs. Firms should also invest in people to manage big data and to pilot DR program. All these investments should be recovered by the firms leading to increase their fixed and variable costs. Suppliers will undertake such investment after expecting their specific market share whereas the cost will be transferred to consumers by a tariff increase. This could be modeled by assuming that operational costs of the technology would increase by a convex function, $\frac{1}{2}\gamma_n \cdot x_{t,n}^2$ where γ can be interpreted as a DR intensity parameter. Thus, we assume a positive correlation between the intensity of suppliers' investments in smart grids system and the opportunity for consumers to manage their consumption with large variety of tools (In-Home-Display, Direct Load Control, Energy boxes, etc…). On the other hand, we can assume that consumers, being now price sensitive, will become demand elastic. Their inverse demand function is defined as in equation 3 in section 2 above. Graphically, the electricity price goes from point *A* to point *B*, as it is shown in the figure 7 below, when going from no DR scenario to a generalized DR scenario. Indeed, with the deployment of SG, informations are provided to consumers, allowing them to make decisions, to change their electricity consumption patterns and to adapt their demand. DR programs change the cost of electricity for consumers: they receive new price signals and, with new SG technologies, they could react to them. Consumers' reserve price with DR, c_n, is now higher than $\overline{P}_n$, the reserve price when all consumers are price inelastic. Indeed when consumers are price elastic, becoming able to pay higher prices for electricity when the system is rationed, will value much more the electricity scarcity which implies an increase in the value of lost energy (VoLL) [14].

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[13] Electricity consumers are recognized to be elastic only in the long run, even if in the short run, part of them can be partially elastic too. We note also that we do not make any distinction between residential and industrial consumers which have different consumption profiles and would in practice react differently on intensive DR program implementation.

[14] In practice, the reserve price varies with the type of consumers, theirs elasticities as well as the duration of the outage. See Leautier (2000) for more details.
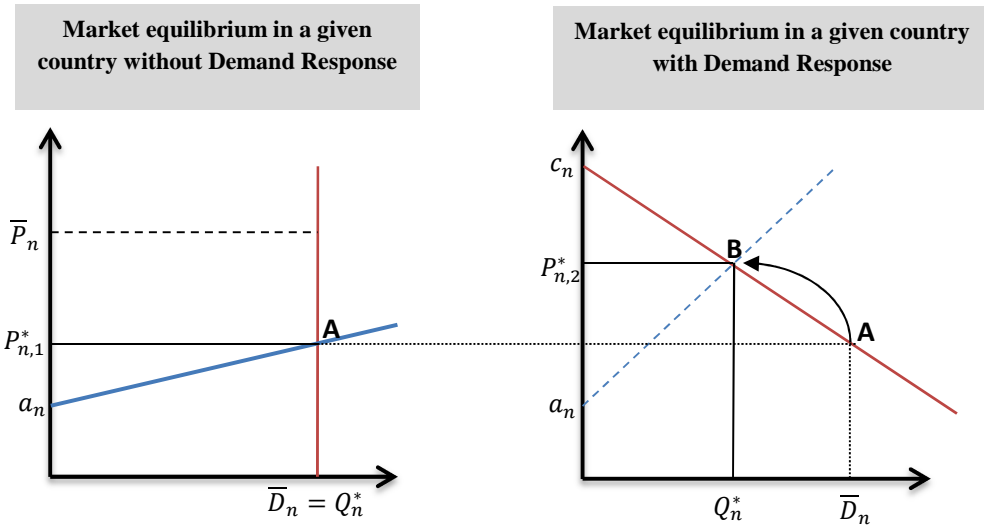
Figure 6. Impact of a demand response program on market equilibrium.

The impact of the DR programs on price bids has been studied in the literature. DR is a key to make smart grids to work because it allows efficient interactions between the segments of the electricity chain[15]. DR programs are separated into incentives and price based programs (Muratori et al., 2014). The first one remunerates consumers for their load-shedding whereas the second only gives a price signal to consumers : the cut in energy bills is then their only remuneration. However, lower forecasted gains for consumers in electricity bills lead the utilities to use the two kind of DR at the same time. For example, fees could be transferred from utilities' gains to consumers to reinforce load-shedding incentives. The literature has studied a variety of DR tools (Bergaentzlé et al., 2014, Faruqui and Sergici, 2010 and Horowitz and Lave, 2014), going from the simplest one that requires dividing the consumption periods in some price blocks where price increases with the short term system vulnerability (Time-Of-Use or Critical Peak Pricing), to the complex one were consumers react in real time to electricity prices (Real Time Pricing). All these tools imply an increase in electricity price in the periods of tensions and a low price otherwise. It is somewhere a realistic assumption to assume that an activation of a DR is translated by an increase in electricity price.

As prices increase, some consumers could be worse off considering this increase (Horowitz and Lave, 2014). As the main gains of DR program is the reduction of consumption in peak-load demand, incentives prices or direct load control are used to make the volume effect greater than the price effect, i,e to compensate the increase in prices by the decrease in electricity consumption. This should be done by minimizing the impact on consumers' utility[16], as consumers must have incentives to participate in DR program. Thus, social welfare and consumers' surplus are two economic indicators we have to study to gauge the efficiency of the DR programs.

In our model, we only study the impact of price based DR on the market equilibrium of two interconnected area, on consumers' surplus and on welfare. So, to look at the social efficiency of a DR measure, we first consider the maximum level of demand reduction the system operator can reach without any lose in social welfare. So, we define $Q_{n,min}$ as the minimum quantity of demand beyond it social welfare does not decrease.

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[15] For instance, generators of conventional or intermittent energies could easily manage variations of their productions and integrate renewable energies in the power system.
[16] One of the main field of suppliers'research is to convince consumers to participate in DR program. Thus, to be accepted by consumers, DR program must minimize the impact on consumers' modern conveniences and on electricity bills.

Then, we will introduce an analysis of consumers' surplus to show the existence of some designs of DR program that could not deteriorate this surplus. A sensitivity analysis regarding some key parameters and the level of demand elasticity is also done.

Without loss of generality, four extreme scenarios regarding markets integration and market efficiency are analyzed. In the first two scenarios, country n is more efficient and so less costly than country m. We look at welfare variation for two situations regarding the extent to which there is a sufficient interconnection capacity between the two countries. In the last two scenarios, we assume that country n is now more expensive than its neighbor[17].

The results of section 3 have emphasized the impact of the opportunity cost of energy and the technologies' cost efficiency on energy exchanges between countries, subject to interconnection capacity prices. The trade-off between costs and volumes efficiencies would drive the exchanges and the generation decisions. In what follow, we apply these theoretical predictions by considering the particular case of elastic demand with demand response. The results of proposition 1 and 2 will so serve as the analytical basis of the following efficiency study.

4.3. Demand Response, technology efficiency and volume efficiency

In this section, we analyze the efficiency of a DR program using social welfare as indicator of effectiveness. The introduction of $\gamma_n > 0$ leads consumers to react to prices. Thus, in this analysis, we show when system operator could implement DR programs and minimize the consumption without any deterioration in social welfare.

Let's define Δ_n as the welfare variation after implementing a DR program in the country n as follows:

$$\Delta_n = SW_{n,2} - SW_{n,1} \tag{15}$$

Where,

$SW_{n,1} = \overline{D}_n . (\overline{P}_n - P_{n,1}^*) + \overline{D}_n . \left(\frac{P_{n,1}^* - a_n}{2} \right)$ [18] : Social welfare before implementing DR program.

$SW_{n,2} = Q_n^* . \left(\frac{c_n - a_n}{2} \right)$: Social welfare at the equilibrium after implementing DR program.

When developing the above function, we obtain that $\Delta_n \geq 0$, i.e. DR is socially efficient, when the equilibrium quantity belongs to the optimal region below:

$$Q_n^* \geq Q_{n,min} = \frac{2.SW_{n,1}}{c_n - a_n} \tag{16}$$

Q_n^* is the equilibrium that is effectively reached according to supply and demand functions and the parameter of DR, γ_n. $Q_{n,min}$ is the minimum equilibrium quantity that could be reached with a DR program without a decrease in social welfare. This threshold exists for each equilibrium Q_n^*, i,e for all configurations of supply and demand functions. According to supply and demand parameters, some of these equilibriums will induce a positive variation of the social welfare.

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[17] In our modeling assumption, the two countries are differentiated only regarding their volume structures (demand and generation capacities) and their cost structures. By integrating these differences, the results of the fourth scenarios guarantee a certain level of exhaustiveness.

[18] When developing $SW_{n,1}$, we have $SW_{n,1} = \overline{D}_n . (\overline{P}_n - a_n) - \frac{b_n . \overline{D}_n^2}{2}$

Knowing also that at the intersection point B in the figure 6, $Q_n^* = \frac{c_n - a_n}{\gamma_n + b_n + d_n}$ [19], we obtain a negative correlation between the intensity of the DR program, γ_n, and the probability to cope with $\Delta_n \geq 0$ as $\lim_{\gamma_n \to +\infty} Q_n^* = 0$. It is a rational result since the more DR is intensive, the higher is the consumption reduction. Thus, as it is optimal for the DR program to reach $Q_n^* \in \left[Q_{n,min}, Q_{n,max} = \frac{c_n}{d_n} \right]$ to keep social welfare gains, an intensive DR should stronger reduce consumers' surplus as well as the profits of the firms which compete in marginal costs. So, there are no incentives to design a γ_n such as $Q_n^* < Q_{n,min}$ because of loses in social welfare when the demand function is elastic.

As Δ_n is an increasing function of Q_n, we could conclude that $Q_n^* = Q_{n,min}$ is the minimal condition the equilibrium must respect to keep gains from DR programs.

According to this prior result, we can see that the minimum quantity is linked to parameters of the supply function a_n and b_n [20] reflecting the supply marginal costs, and the reserve electricity prices before and after implementing the modification of their consumption behavior due to the DR program, $\overline{P}_n$ and c_n respectively. This threshold quantity is decreasing with the supply function parameters [21]. Thus, the less the country is efficient, the more it is possible to send a price signal to consumers to adapt and reduce their consumption without a decrease in the welfare. The intuition is that the SW_{n,1} is reduced because of inefficient supply costs. Thus, elasticity of demand and a higher reserve prices c_n could offset the DR increase in costs to keep gains in social welfare. In the same way, the more the value of unserved energy when consumers are elastic is, i.e. c_n increases, the more there are able to reduce their consumption level [22]. Finally, the higher the levels of initial demand and initial maximum price are, $\overline{P}_n$ or $\overline{D}_n$ increases, the less there is a margin for system operator to go through an intensive DS program and important consumption reduction because of a strong initial social welfare SW_{n,1}.

This prior analysis just served to have an indication on the main factors influencing the optimal consumption limit. We will study now in depth the extent to which this limit would depend on (i) the system cost efficiency and market sizes, (ii) systems integrations by considering the level of the price of the interconnection capacity between the countries, and (iii) the impact of the main parameters of the systems configurations on the efficiency of a DR program.

Proposition 5. *If the country is efficient, there exists an optimal region for which a demand response program does not reduce social welfare. This region is negatively correlated with the degree of competitiveness of the generation technologies and its market size.*

In this scenario, we assume that country *n* is enough more efficient than country *m*. This means that with or without price signal, only its generation technologies are called to produce to satisfy the local demand.

Integrating our results from ***Proposition 2*** with $P_{n,m} = 0$ into equation (16), the optimal minimum quantity is also equal to:

$$Q_{n,min} = \frac{\sum_t K_{t,n} - \alpha . Q_m}{2 - \alpha} = \frac{2.SW_{n,1}}{c_n - a_n} \tag{17}$$

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[19] This result holds by equaling inverse supply function to inverse demand function, i.e. $a_n + (\gamma_n + b_n)X_n = c_n - d_n . X_n$

[20] We remind that electricity price in the first situation is given by: $P_{n,1} = a_n + b_n \overline{D}_n$

[21] $\frac{dSW_{n,1}}{db_n} < 0$ and $\frac{dSW_{n,1}}{da_n} < 0$ and $\frac{dQ_{n,min}}{da_n} < 0$.

[22] $\frac{dQ_{n,min}}{dc_n} < 0$

The left term (from cost minimization program (5-8)) and the right term (from equation 15) are equals at the equilibrium. Any change in one parameter of one term will be accompanied by a proportional change in other parameter in the other term, guaranteeing that the above equality always holds and the equilibrium stills the one of efficient technologies. The link between the two terms is obtained through the parameter b_n as it is detailed next[23].

We can see from equation (17) that the optimal region limit depends on many parameters about the interconnected systems configurations and technologies specifications. We define three criteria to study the sensitivity of $Q_{n,min}$ to the main parameters. The first one is about the market size of the efficient country. It is impacted by $K_{t,n}$ and $\overline{D}_n$. Indeed the more these parameters are important, the more we can admit a large size of the system. The second one is about the consumption level of the inefficient country, i.e. Q_m. Technologies' cost efficiency level is the third indicator. Parameters like the supply function's parameters, a_n and b_n fall into this category.

Market size of the efficient country

When the capacities of country n increase, i.e. $\sum_t K_{t,n}$ increases, two impacts on $Q_{n,min}$ could happen. Knowing that country n is cheap, any increase in its technology capacity will be exported to the neighbor country. Such capacity addition can also offer more availability of cheapest technologies to country's n, leading to reduce the slope of the energy mix (the parameter b_n) and increasing then the initial social welfare $SW_{n,1}$. We observe so an identical increase in the left side and the right side of equation 17. The optimal region of a DR program, not reducing the social welfare, is reduced[24]. This reduction is due to the difficulty to implement a DR program without a reduction in social welfare because the supply costs to serve the captive demand are cheap. Thus, consumers' rationing is costly for the collectivity. Firms must implement lower DR program, with γ_n positively correlated to the elastic consumers' reserve price c_n. Indeed, we have seen in the previous equilibriums that an increase in generation capacities of the efficient country leads in general to increase the local generation as well as export. When available capacities are competitive, the system is far from rationing. Consumers do not need to be rationed and to adapt their consumptions, the generation being available and cheap. Indeed, the loss in term of surplus could be very high in such situation when generation is cheap while consumption must decrease because of the voluntarily increase in the price. There exists a kind of opportunity cost to ration the consumers without constraining the system. This is translated by a relaxation of the constraint 6 in the initial minimization program. Second, the optimal region could however still unchanged[25] if the increased capacity will be totally exported to the neighbor country, i.e. any increase in $\sum_t K_{t,n}$ is off-set by a proportional increase in Q_m in equation 17 above.

$Q_{n,min}$ is also positively correlated to $\overline{D}_n$ (the right term of equation 17)[26]. Indeed, such increase in the differential between the initial demand and targeted one would induce an important negative impact on producers, observing a great generation reduction as well as an increased value of their unused capacities. From other side, captive demand is high enough to allow the rationing a high number of users by a DR

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[23] This analysis procedure is followed for all the scenarios below.

[24] $\frac{d\left(\frac{\sum_t K_{t,n} - \alpha.Q_m}{2-\alpha}\right)}{dK_{t,n}} > 0$ if $\alpha.\frac{dQ_{n,min}}{dc_n} < 1$, meaning that only a part of capacity increase will be exported. At the same time, $\frac{dSW_{n,1}}{dK_{t,n}} > 0$ because of decrease in b_n.

[25] $\frac{d\left(\frac{\sum_t K_{t,n} - \alpha.Q_m}{2-\alpha}\right)}{dK_{t,n}} >= 0$ if $\alpha.\frac{dQ_{n,min}}{dc_n} = 1$, meaning that a high part of capacity increase will be exported. At the same time, $\frac{dSW_{n,1}}{dK_{t,n}} = 0$ because b_n is unchanged.

[26] $\frac{dSW_{n,1}}{d\overline{D}_n} > 0$ because $P_{n,1}^* = a_n + b_n\overline{D}_n < \overline{P}_n$.

program, without impacting negatively their welfare. Such increase in $\overline{D}_n$ and so, in the right term of equation 17 is accompanied by a proportional decrease in Q_m and then in the left term of equation 17. Indeed less export is envisaged when local system has more needs.

As a first conclusion, we can presume that intensive DR is constrained by the market size of the system because of the considerable impact of the value of non-served energy for producers. Not only consumers' rigidity would undermine the efficiency of DR but also the producers' side, regarding how much they will lose in term of profit..

Consumption level of the inefficient country

We can note that the demand in the neighbor country affects negatively $Q_{n,min}$[27]. Because interconnection capacity are unlimited and so are freely valued, an increase in Q_m will be covered by more export from country n, and then moving down the local generation in country n. Because export in now more preferred compared to local generation, country n will observe a reduction in $SW_{n,1}$, b_n being increased. These intermediate impacts lead to a reduction in $Q_{n,min}$. A reduction in Q_m will have however the opposite impact.

It is also to say that while the market size reduces the efficiency of DR, export opportunities given by the increase in foreign demand would alleviate such negative impact, reducing then the value of un-served energy caused by the DR program.

Cost efficiency of the efficient country

The second group of parameters regarding cost efficiency of the technologies is the specific parameters of the supply function (a_n and b_n). Again, when a_n or b_n decreases, meaning also that technologies are more efficient, we observe an increase in $Q_{n,min}$. Indeed, the initial social welfare is more important because the cost to serve the demand $\overline{D}_n$ with an efficient energy mix is now lower. Then a less intensive DR would be accepted by system actors, facing now a higher opportunity cost of avoiding the demand response.

Proposition 6. *When interconnection capacity is limited, the optimal region is reduced. System operators cannot introduce an intensive DR program because the generation reduction of its efficient producers, involved by the DR program implementation, cannot be totally evacuated by relying on export.*

Here, country n is still more efficient than country m. However, interconnection capacity between the two countries is limited in the both senses.

Integrating our results in ***Proposition 2*** with $P_{n,m} > 0$ into equation 16, the optimality region of the price signal is now given by:

$$Q_{n,min} = \frac{\sum_t K_{t,n} - \alpha.Q_m + \frac{P_{n,m}}{(\alpha-1).b}}{2-\alpha} = \frac{2.SW_{n,1}}{c_n - a_n} \tag{18}$$

All previous cost and market size parameters keep the same impact on $Q_{n,min}$ as previously. We still observe that the more the technologies that serve the demand are efficient, the more the rationing is costly and hence the more $Q_{n,min}$ increases. Idem, the more the size of the market is, the low the consumption reduction is.

Transmission capacity limit

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[27] $\frac{d\left(\frac{\sum_t K_{t,n} - \alpha.Q_m}{2-\alpha}\right)}{dQ_m} < 0$. At the same time, $\frac{dSW_{n,1}}{dQ_m} < 0$ since more export will induce higher b_n.

Due to the constraint on interconnection capacity, a capacity price effect associated to the use of interconnection appears and is represented by $\frac{P_{n,m}}{(\alpha-1).b}$. This effect leads to reduce the optimal region limit compared to the previous case, all things being equal. Indeed, implementing an intensive DS measure, by encouraging consumers to highly reduce their consumption, would involve a great reduction in local generation. Efficient producers, observing a huge reduction in their generation, can export their over-production to the neighbor less efficient country, lowering then the negative volume effect of such measure. When interconnection capacity is limited, export becomes more costly as well as less possible. Hence, the equilibrium for which efficient technologies are used at their maximum is much more difficult to reach. The cost to serve the rationed demand will significantly increase, leading the system operator to adopt a less intensive DR program in order to avoid system collapse. We can also understand this decrease in the optimal region by looking at the impact on the merit order of country n, a limitation on interconnection capacity will induce. The more interconnection capacity is limited, the less export to country m is reasonable. The reduced export can be seen as an excess of generation that has to be put in the local system. The impact is then twice. In the one hand, a direct impact would be the increase of local generation. On the other hand, the "returned" generation has a high probability to come from less costly technologies, where their supply in the local market will move down the merit order curve, because the decrease of b_n. Consequently, initial social welfare is increased, impacting then positively the quantity limit. This result shows that the impact of DR programs on interconnection capacities is ambiguous. On the one hand, DR programs, reducing the local demand, would distress the electric system. Thus, interconnection could be less used because of lower demand. DR program should be a substitute to network expansion. On the other side, there exist incentives to expand the interconnection capacity network to benefit from cheaper generations. Considering the opportunity cost of non-served energy and the capacity price, as the inefficient country has no incentive to implement intensive DR programs, we have shown that the second effect could be greater than the first one.

To sum up, let's focus on two indicators of the cost structure and the market size of the studied country. The first one includes, as said previously, parameters like supply function parameters, a_n and b_n. As it was demonstrated, the impact of these parameters on $Q_{n,min}$ is the same, the more technologies are cost efficient, the higher $Q_{n,min}$ is. We can so for simplicity focus only on parameter b_n, as a good indicator of the cost efficiency of the technology. Regarding market size indicator, it is also highlighted that the more initial captive demand, $\overline{D}_n$, is high, or the more generation capacity, $\sum_t K_{t,n}$, is high, the higher $Q_{n,min}$ is. For simplicity, we center our conclusion on the impact of $\overline{D}_n$. Figure 7 below shows the evolution of the optimal region regarding b_n and $\overline{D}_n$[28]. The curves are approximated based on our results from equations 17 and 18 above[29].

Two zones can be identified. The first zone A means that with low b_n and low $\overline{D}_n$, it is not possible to reach low demand reduction with intensive demand response. The fact that b_n is low enough, the rationing will lead to a high negative price effect on consumers. The low level of demand would not provide an important positive quantity effect of the measure. Consequently, we expect a high price effect of such measure, mainly for consumers, making a great loss of welfare if an intensive DR is implemented. In the opposite case when both parameters are high, the volume effect is now considerable compared to price effect, i.e. a first effect

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[28] We note that we disregard extreme situation, i.e. when country become inefficient compared to its neighbor. Such situation will be studied in the next sub-section.

[29] From equation 17 we can deduce the following relationship between $Q_{n,min}$ and $(b_n, \overline{D}_n)$: $Q_{n,min}(b_n) = Q_{n,min}(\overline{D}_n) = \frac{2\overline{D}_n.(\overline{P}_n-a_n)-b_n.\overline{D}_n^2}{c_n-a_n}$. Function of b_n, the function is strictly decreasing, whereas it is increasing and convex in $\overline{D}_n$. When the capacity price is non null (equation 18), $\frac{\frac{P_{n,m}}{\alpha.b}.\sum_t\frac{1}{\alpha_{t,m}}}{2-\alpha}$ is added to the last function. Function of b_n, it stills decreasing but with a concave form as shown in the figure 7. Function of $\overline{D}_n$, it is translated to the top.

comes from generation reduction and a second effect from the increased value of unused capacity. Moreover demand is initially high enough while the gain in term of price reduction is insignificant. Finally, a low demand reduction is also expected when b_n is low and $\overline{D}_n$ is high. While the positive volume effect for consumers would be off-set by the high negative price effect, producers would lose a high level of their surplus, as in the precedent case because of the important generation reduction and the increased value of unused capacity. The positive impact of price increase on producers is however moderate when we consider the increasing cost of the DR measure on their operational costs. Finally, when interconnection capacity is limited, the efficiency of DR in term of consumption reduction is moreover reduced (zone A in the right part of figure 7).

However, an aggressive demand response could only be put in place when b_n is high and $\overline{D}_n$ is low. This intensity is moderated by the extent to which the export is limited, i.e. a loss in the optimal region as shown in the right part of figure 7. We can conclude that to reach higher reduction of consumption without welfare loses, the volume effect of the DR measure on consumers and on producers has to be off-set by price effect, giving then an insignificant negative welfare impact on total surplus.
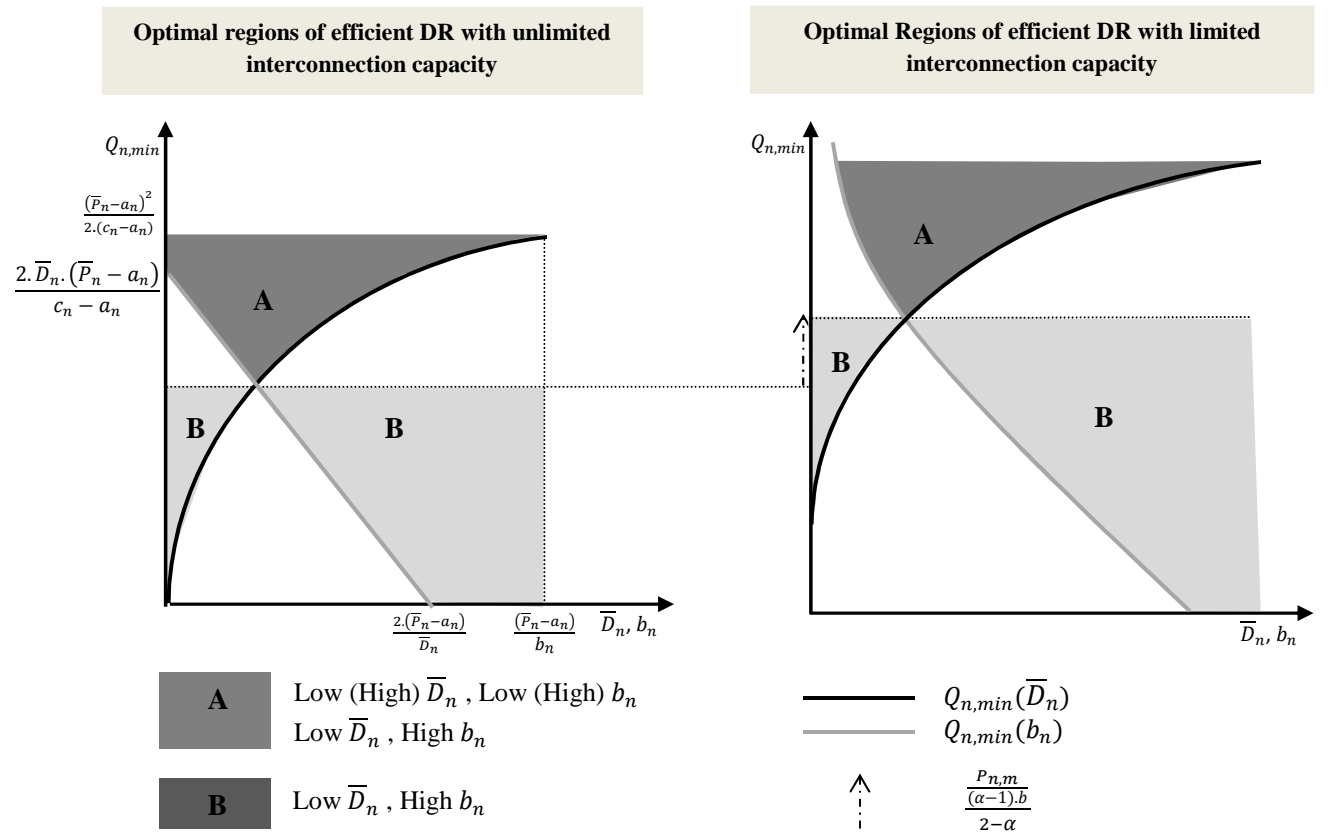

Figure 7. Optimal regions of efficient DR for efficient country

Proposition 7. *When country is inefficient, the higher the import is, the inefficient an intensive Demand Response is.*

In this scenario, we assume that country n is now enough less efficient than country m. This means that with or without price signals, it relies always on imports from country m to satisfy its local demand. We begin by admitting also that interconnection capacity is unlimited.

Integrating our results from **Proposition 2** into equation (16), the minimum quantity for efficient DR is equal to:

$$Q_{n,min} = \frac{\sum_t K_{t,m} - \alpha.Q_m + Res_n}{2-\alpha} = \frac{2.SW_{n,1}}{c_n - a_n} \qquad (19)$$

Parameters a_n and b_n (through $SW_{n,1}$) are playing the same role on $Q_{n,min}$. We still obtain that any decrease in the inefficiency of the cost of the local technologies is positively impacting $Q_{n,min}$. Idem, market size indicator of the inefficient country, $\overline{D}_n$, through $SW_{n,1}$, stills influence positively $Q_{n,min}$. The optimal region is however significantly influenced by the market size of the exporter country m, i.e. $K_{t,m}$, as we see below. As a preliminary interpretation, we can say that sincethe country relies massively on import, any local energy policy would be subject mainly to the (un)attractiveness of import, being structurally inefficient to go through independent measures.

Market size of exporter country

Compared to the scenario where country n is efficient, here generation capacity of exporter country, $K_{t,m}$, will influence $Q_{n,min}$ instead of $K_{t,n}$. Interconnection capacities do not constraint energy exchanges. Country n will intuitively import any less costly energy from its efficient neighbor country. An increase in $K_{t,m}$ will relax the neighbor system and then encourage more export to the inefficient system. Therefore, two situations are possible. Inefficient technologies of country n will be more replaced by less costly one, so an increase of $K_{t,m}$ will be accompanied by a decrease in the residual demand served by local producers, Res_n, the impact on $Q_{n,min}$ is then quite limited. Or the capacity expansion in the exporter country will be transferred to country n, and then satisfying more demand. $Q_{n,min}$ has to increase because Res_n will still unchanged. In this latter case, the rationing is more harmful for consumers. Indeed, when available capacities are competitive, consumers do not need to be rationed and to adapt their consumptions, the generation being cheap. The opportunity cost to ration the consumers without constraining the system is here significant. From other side, since cheapest technologies are more available in country's n, we obtain a reduction in the slope of the energy mix (the parameter b_n) and an increase in the initial social welfare $SW_{n,1}$. So, any capacity expansion in the exporter country that will be transferred to the inefficient country will make more probably restrictive the efficiency of DR program.

Consumption level of the exporter country

We can finally note that the demand in the neighbor country affects negatively $Q_{n,min}$. Again the increased demand in the exporter country would reduce the value of un-served energy of efficient producers caused by the DR program.

Proposition 8. *When interconnection capacities are constrained, the system operator of the inefficient system can exert more aggressive demand response program.*

Finally, considering that country n is enough less efficient than country m where also interconnection capacity between the two countries are limited in both senses.

Integrating our results in **Proposition 2** into equation (16), the optimality region of Demand Response is given by:

$$Q_{n,min} = \frac{\sum_t K_{t,m} - \alpha.Q_m - \frac{\alpha^2}{\alpha-1}.b.\sum_m P_{m,n} + Res_n}{2-\alpha} = \frac{2.SW_{n,1}}{c_n - a_n} \qquad (20)$$

Transmission capacity limit

Contrarily to the scenario with efficient country, when the country is holding expensive technologies, the more the import is costly, the more the DR could be intense. Indeed, when $P_{m,n}$ increases, it is less possible to rely on import to satisfy local demand of country n. Local inefficient generation will be used to balance the system. Given that the opportunity cost of reducing generation is much lower for inefficient technologies, i.e. local producers, than the efficient one, i.e. importers, the system operator can undertake more aggressive DR program without facing a social cost of unserved generation that would be caused if he relies only on efficient technologies. He can hence implement a price signal until an optimal level that does not reduce social welfare, while when import is not constrained, this level is lower because of the impact of the value of the unused energy of the efficient technologies.

To sum up, we will focus again on two criteria about technologies' cost level of the inefficient country and the market size of the exporter one since it significantly influence equilibrium in the under study market. We consider the parameter b_n as an indicator of the efficiency of the country. Second, we look at the impact of $K_{t,m}$ as an indicator about the market size the neighbor country to counterbalance the inefficiency of the considered country. Figure 8 below shows the evolution of the optimal region of DR regarding b_n and $K_{t,m}$. The curves' approximation is based on our results in equations 24 and 25 above and similarly to the curves in figure 7 above.

Here, three zones are identified. Zones *A* means that it is socially inefficient to target low demand reduction when either b_n and $K_{t,m}$ are low or only $K_{t,m}$ is high. When they are low this means that the system is less inefficient but it cannot rely so much on import. In the one hand consumers are less able to accept to be rationed, the generation being cheap. On the other side the System Operator have to rely on local producers to satisfy the local demand, an intensive demand reduction will then be socially suboptimal because of the high negative price effect of the DR measure on producers surplus, i.e. price differential from the initial situation to the targeted one is important. When $K_{t,m}$ is high, import could be abundant regardless the cost-intensity of the system. DR is also limited. While consumers could be less sensitive to demand rationing, the significant weight of cheap import will observe a significant decrease in their surplus, making the great demand reduction socially inefficient for all actors involved in such system. This result is very interesting and emphasizes the extent to which the development of interconnections between countries could condition energy policy strategies of the countries being highly dependent on energy import.

However, an aggressive demand response could be put in place only when b_n is high and $K_{t,m}$ is low. In this case, we have a very inefficient system as well as a low rate of import. In such case, consumers are more able to accept demand rationing because of the already excessive electricity price. On the other hand, as well as local producers and the low level of importers will not observe a significant decrease in their surplus, making then a great demand reduction socially acceptable regarding the actors involved in such system. It is to say that intensive DR could be put place when the system is already extremely inefficient and in the same time the system falls into a quite isolated situation. We finally observe that the intensity of DR program could be increased by the extent to which the export is limited, i.e. an increase in the optimal region as shown in the zone *C* in the right part of figure 10. The zone *C* indicates that more aggressive demand reduction could be targeted even if the degree of competitiveness of the technologies increases as long as import is more restricted.
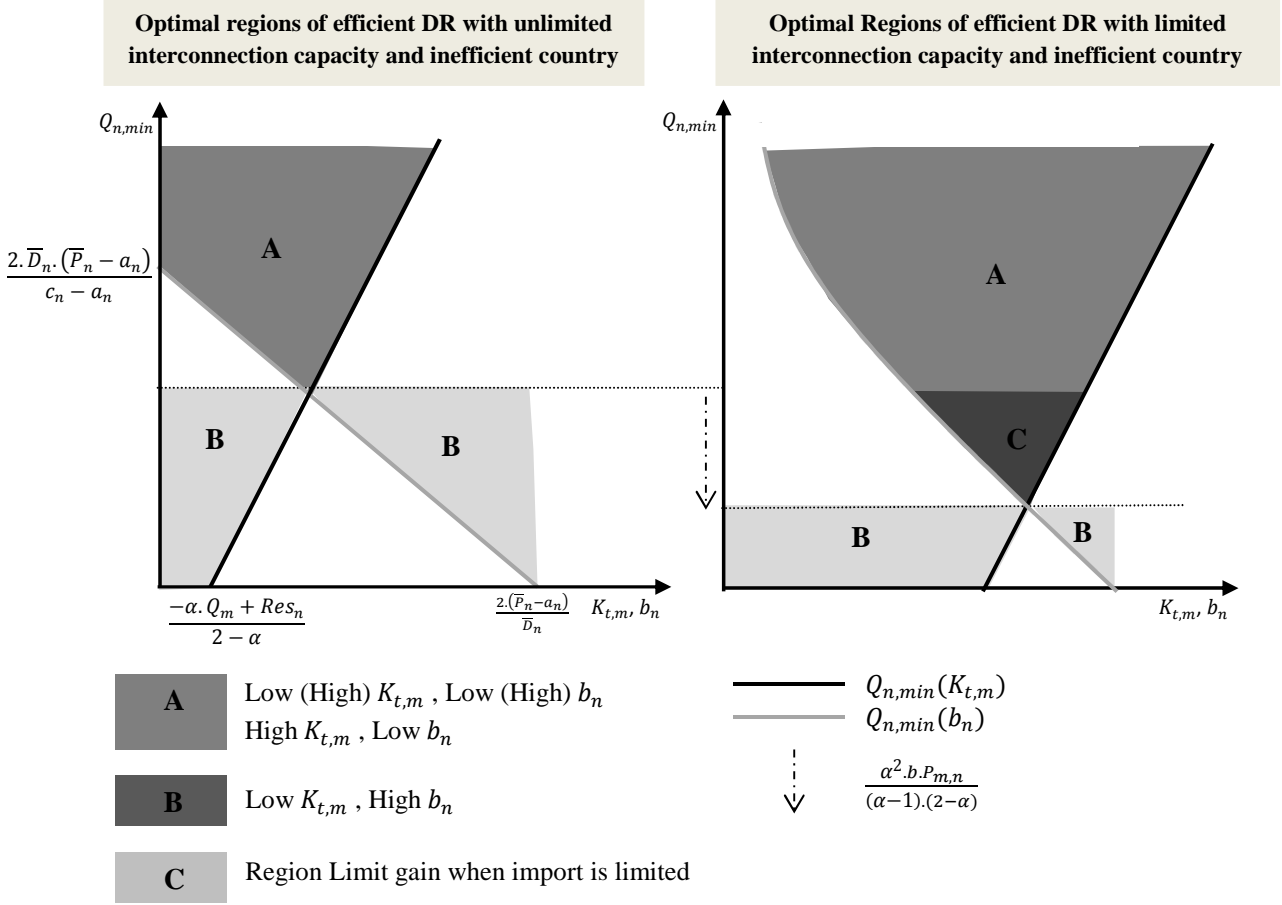
Figure 8. Optimal regions of efficient DR for inefficient country

4.4. Consumers' surplus analysis

We study now the extent to which the optimal region of DR programs would vary if consumers' surplus variation is considered instead of social welfare. Consumers are sensitive to financial losses that could occur with the adoption of a new technology (Park et al., 2014). In the case of a SG deployment with DR programs and dynamic pricing, these fears should be correlated with lower surplus, because of the risk of an increase in electricity bills[30]. This is one of the main risks that group consumers' fears together when DR program are used. Indeed, a DR mechanism has to act mainly on consumers profile; its efficiency would depend on their adaptation and their elasticity the DR program could succeed to activate. The impact on consumers' surplus could be an important social constraint that public authorities should seek to reach when trying to modify the equilibrium rational of their energy systems.

Proposition 8. *When the consumers' surplus variation is the used criteria instead of social welfare, more aggressive DR could be adopted, unless the system is cost-inefficient as well as the captive demand is low.*

Let's define Ω_n as the consumers' surplus variation after implementing a DR program:

$$\Omega_n = CS_{n,2} - CS_{n,1} \tag{21}$$

Where,

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[30] SG and DR pilots have shown that this risk exist but could be limited for consumers that react to signal from utilities or Distribution System Operators(Faruqui et al., 2007 ; Faruqui et al., 2010 ; Faruqui and Sergici, 2010).

$CS_{n,1} = \overline{D}_n.\left(\overline{P}_n - P_n^*\right)$[31] : Consumers' Surplus after DR program

And;

$CS_{n,2} = Q_n^*.\left(\frac{c_n - P_n}{2}\right)$: Consumers' Surplus at the equilibrium before DR program

When developing the above function[32], we obtain that $\Omega_n \geq 0$, i.e. DR is efficient from consumers' side, when the equilibrium quantity belongs to the optimal region below:

$$Q_n^* \geq Q_{n,min} = \sqrt{\frac{2.CS_{n,1}}{d_n}} \qquad (22)$$

A consumption reduction without decreasing consumers' surplus could be reached until $Q_{n,min}$ in equation 22 above. We could verify that the intensity of the demand response is positively correlated to the elasticity of the demand function and negatively correlated to the captive demand prices. If we assume that public authority considers principally the impact in term of consumers' surplus when implanting DR measures, instead of social welfare, we can demonstrate that a higher consumption reduction could be obtained if the following condition is met[33]:

$$\sqrt{\frac{2.CS_{n,1}}{d_n}} \leq \frac{2.SW_{n,1}}{c_n - a_n} \qquad (23)$$

In what follow, we look at the additional consumption reduction under consumers' surplus constraint and regarding the level of two parameters about cost efficiency and market size of the system, b_n and $\overline{D}_n$ respectively[34]. The results shown in figure 9 below confirm our previous results regarding the impact of the weight of consumers' surplus in the social welfare variation, via the price and the volume effects of DR program. Two mains conclusions could come out from the figure. First, when market size of the system is high, i.e. $\overline{D}_n$ is high as shown is the right part of figure 9, and regardless the level of the cost efficiency, a higher consumption reduction could be reached if we only consider consumers' surplus impact (zone B in the right part of figure 9). This additional quantity reduction could also be obtained when demand is moderate but the system has to be highly inefficient in term of cost (zone B in the left part of the figure). Indeed, when demand is very high, negative price of an intensive DR program on consumers is off-set by the positive volume effect, while for producers, since positive price effect is insignificant (price variation is low because the additional cost of DR), the negative price effect will make them facing a high decrease in their own surplus variation. DR could not be then intensive if such surplus is considered. Likewise, if demand is not high enough, DR could be intensive only under consumers' surplus approach and if the system is expensive. In this case, consumers having already high bills would be more able to accept aggressive demand reduction, volume effect being higher than price effect.

The second conclusion is that, under consumers' surplus approach, we cannot expect a deep reduction in consumption when the system is already cheap while demand is also low (zone A in the figure). Again, negative price effect of the measure would be high enough for consumers and demand rationing would be less acceptable, energy being cheap as well as their consumption is already low.

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[31] When developing the function, we obtain: $CS_{n,1} = \overline{D}_n.\left(\overline{P}_n - a_n\right) - b_n.\overline{D}_n^2$

[32] $\Omega_n \geq 0$ if $Q_n.\left(\frac{c_n - P_n}{2}\right) \geq CS_{n,2}$. Knowing that $P_n = c_n - d_n.Q_n$, then $Q_{n,min} = \sqrt{\frac{2.CS_{n,1}}{d_n}}$.

[33] If $Q_{n,min}$ under consumers' surplus criteria is higher than $Q_{n,min}$ under social welfare criteria.

[34] A deep analysis of the impact of interconnection capacity price is disregarded here. Like previously, when the interconnection capacity is constrained, the optimal region is reduced when the system is efficient and increased in the other case. This holds under social welfare as well as consumers' surplus criteria.
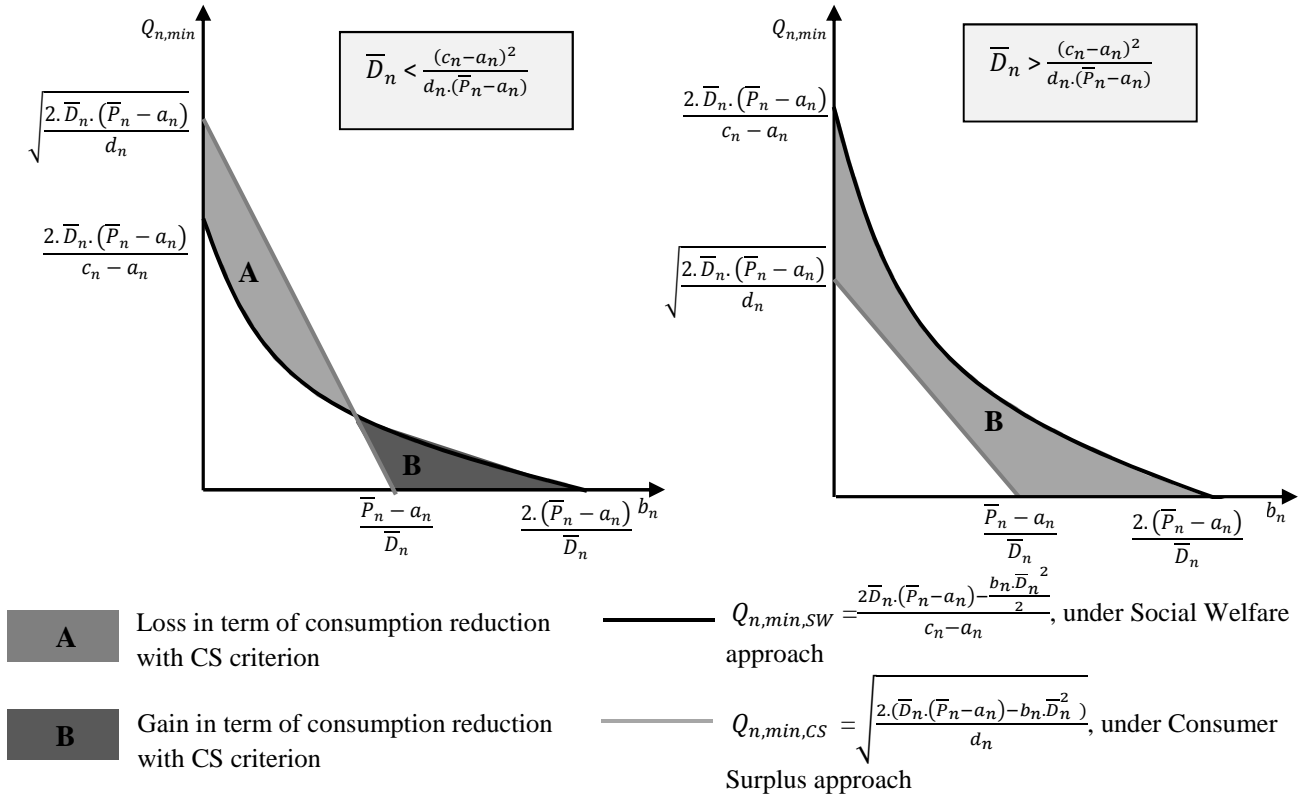
Figure 9. Maximal consumption reduction under consumers' surplus criterion and social welfare criterion

Legend for the figure:

| A | Loss in term of consumption reduction with CS criterion |

| B | Gain in term of consumption reduction with CS criterion |

$$Q_{n,min,SW} = \frac{2\overline{D}_n \cdot (\overline{P}_n - a_n) - \frac{b_n \cdot \overline{D}_n^2}{2}}{c_n - a_n}, \text{ under Social Welfare approach}$$

$$Q_{n,min,CS} = \sqrt{\frac{2 \cdot (\overline{D}_n \cdot (\overline{P}_n - a_n) - b_n \cdot \overline{D}_n^2)}{d_n}}, \text{ under Consumer Surplus approach}$$

4.5. Elasticity of demand

We end our analysis by studying the sensitivity of our results to the level of the elasticity of demand. We look at the consumption reduction a DR program could reach given the level of elasticity. Different situations are considered regarding the cost efficiency and the market size of the country as well as its degree of interconnection with the neighbor country.

Proposition 9. *Elasticity of demand is negatively correlated to the intensity of DR program. In general, less elasticity of demand is needed under a consumers' surplus approach. Finally, the more the system is efficient, the more consumers have to be elastic, to underpin the efficiency of the DR program.*

Let's define $\mathcal{E}_n$ as the level of the elasticity of demand in country n after DR being adopted:

$$\mathcal{E}_n = Q'(P_n) \cdot \frac{P_n}{Q_n} \tag{24}$$

A DR program is efficient when the level of the elasticity belongs to the following region[35]:

$$\mathcal{E}_n \geq 1 - \frac{c_n}{d_n \cdot Q_{n,min}} \tag{25}$$

Where,

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[35] From equation 24 and knowing that $P_n = c_n - d_n \cdot Q_n$, we find $\mathcal{E}_n = 1 - \frac{c_n}{d_n \cdot Q_n}$. Efficient DR is when $Q_n^* \geq Q_{n,min}$, so $\mathcal{E}_n \geq 1 - \frac{c_n}{d_n \cdot Q_{n,min}}$.

$$Q_{n,min} = \begin{cases} \dfrac{2.SW_{n,1}}{c_n - a_n} & \text{If Social Welfare criterion is used} \\[2ex] \sqrt{\dfrac{2.CS_{n,1}}{d_n}} & \text{If Consumers' Surplus criterion is used} \end{cases}$$

From the figure 10 below, we can make three important comments. First, the more the demand is elastic, the more DR could be aggressive, i.e. $Q_{n,min}$ decreases, whatever the efficiency of the system and under both criteria. This is an expected result since when consumers are increasingly elastic they are more able to accept a consumption reduction after a given price increase. Second, the impact of the elasticity of demand is less positive for consumption reduction if social welfare is considered instead of consumers' welfare, i.e. the shift to the left from the black curve to the gray one in the figures, except when the system is efficient in term of costs and has higher market size. Not surprisingly, for a given level of quantity reduction, we would require higher elasticity of demand under welfare approach than consumers' surplus approach. This supplement of elasticity is needed to compensate the loss of producers' surplus induced by the DR program. Finally, we observe again that when the interconnection capacity is limited, higher elasticity of demand is needed to reach a given demand reduction when the system is cost-efficient and inversely and low elasticity of demand is required when the system is cost-inefficient, i.e. the discontinued curves in the figures. When the system is efficient and export is less possible, as it was discussed in section 4.1, the cost to serve the rationed demand becomes more expensive. The un-exported quantity as an excess of generation should be injected in the system. The impact is twice. First impact is an increase in the merit order function. Second impact is an increase in the local generation in order to keep balanced a system having efficient and cheap technologies. A higher quantity is now needed to be produced for a price probably higher because the un-exported generation could be more expensive. Even consumers' surplus will be negatively impacted, energy being more expensive.

The opposite phenomenon is observed when the system is inefficient and relying massively on import. Under consumers' surplus approach, before implementing DR program, consumers' surplus is low due to the lack of efficient generation. With DR program, energy is now more expensive than the situation where import is free. To cover the increased loss in their surplus because of the increased negative price effect, they would accept higher consumption reduction and profiting from more positive volume effect. That's why we observe an increase in the quantity reduction under consumers' surplus approach too. This means that one would not require a high elasticity of demand to get intensive DR when the system is inefficient.
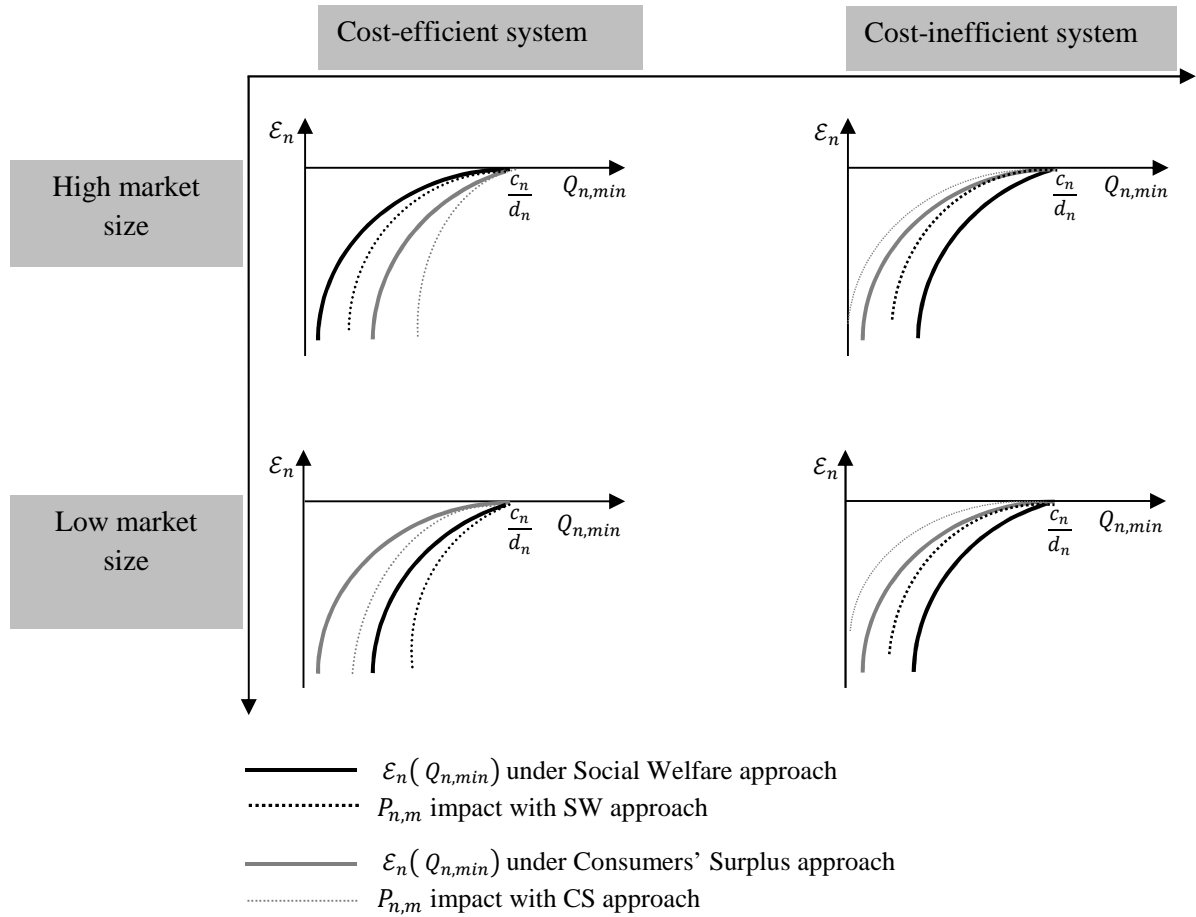
Figure 10. Elasticity of demand and optimal consumption reduction

5. Conclusion

The deployment of smart grids, seen as a major change in the electricity markets, would rely, among other transformations, on making demand reactive to prices or markets' constraints. This work aims to demonstrate analytically under what conditions activating consumers' elasticity of demand could be beneficial for social welfare. It has added to the literature on analyzing the use of price signals in eliciting demand response by an analytical approach. The developed model has quantified the effect of implementing demand response, via price signals, on social welfare and energy exchanges. A first analysis was made on how generation technologies structures could affect countries' merit orders and potential exchange between them. The results show that the trade-off between producing locally and exporting energy depends on the opportunity cost of the energy and the global efficiency of the generation technology. Technologies are differentiated regarding their efficiency levels where efficient ones would be called first for balancing the markets and inefficient ones would just satisfy the residual demands. These results are impacted by the degree of integration between the countries. When interconnection capacity is limited, we demonstrated that interconnection prices in both senses of the exchange would impact negatively the exchanges between the countries; an impact which is however weighted by the generation costs differential between the interconnected systems. Incentives for exports are also reduced when transport capacities become more expensive.

The second analysis constitutes the novelty of this research. It demonstrated that there exists an optimal region of price signal for which demand response leads to increase the social welfare. This optimality region is negatively correlated with the degree of competitiveness of the generation technologies and the market size of the system. While the literature and policy recommendations have widely highlighted that the degree

consumers ability to adapt their behaviors is the main condition of the effectiveness of a demand response program, this work has demonstrated that the impact on producers' surplus has to be considered as a constraint to the implementation of DR. More particularly, the value of un-served energy or energy reduction the producers could lose from such consumption reduction would limit the implementation of such program. This constraint is moreover strengthened when energy exchanges between countries is limited. However, this constraint is alleviated if the considered system in cost-inefficient as well as weakly connected with its neighbors. Such surprising result is also explained by the weight of the value of lost energy inefficient producers will cope with when intensive DR is put in place.

Our intuitions was confirmed when the analysis has only considered the impact in term of consumers' surplus instead of social welfare. We demonstrated that under such condition, more aggressive DR could be adopted, the weight of producers welfare being removed. The analysis has finally demonstrated that the intensity of DR program is negatively correlated to the degree of the elasticity of demand. We do not need however higher elastic demand if consumers' surplus is the considered criteria.

This paper could be extended in several ways. First, strategic interaction between producers, which should affect merit orders functions and therefore market equilibrium, has to be introduced. Second, consumers' behaviours could be more robustly analysed by integrating their satisfaction of consuming energy with a kind of utility function. Consumers have also to be distinguished regarding their types and their consumption profiles.

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Appendix

Appendix 1:

Since we suppose that the merit order function is an aggregation of individual marginal costs of available technologies, let's define $P_{t,n}(x_{t,n}) = a_{t,n} + b_{t,n}.x_{t,n}$ as the individual inverse supply function of a given technology in a given country. Then, $x_{t,n}(P_{t,n}) = \frac{P_{t,n} - a_{t,n}}{b_{t,n}}$ is the individual supply function.

When the market is cleared, all market participants receive the same electricity price, therefore $P_{t,n} = P_n^*$ for all technologies producing in country *n*, i.e. local technologies as well exporters one are both considered. By aggregating so all individual supply functions, the aggregated function at the equilibrium will take this form (equation (2) in section 2):

$$P_n^* = a_n + b_n X_n$$

Replacing now X_n by the sum of the quantities of all generation technologies potentially producing in country *n* (by including import from the other country), and simplifying the equation above, we find the parameters of the aggregated inverse supply function as follows:

$$a_n = \frac{\sum_{t,\ddot{n}} a_{t,\ddot{n}}.b_{\hat{t},\ddot{n}} + \sum_{t,\ddot{n}} a_{t,\ddot{n}}.b_{\hat{t},\ddot{m}} + \sum_{t,\ddot{n}} a_{t,\ddot{n}}.b_{t,\ddot{m}}}{\sum_{t,\ddot{n}} b_{t,\ddot{n}}} \text{ and } b_n = \frac{\prod_{t,\ddot{n}} b_{t,\ddot{n}}}{\sum_{t,\ddot{n}} b_{t,\ddot{n}}}$$

Where, $\ddot{n}$ is the index of the countries selling electricity in country *n*, $\hat{t}$ is the index of the technology different from technology *t* and $\hat{m}$ is the index the other country selling electricity in country *n*.

Appendix 2:

Each country *n*'s system operator minimizes its total generation variable cost (5) under capacities constraints (6) and (7) and market balancing constraint (8). The decision variables are $x_{t,n}$ and $x_{t,n,m}$. We state the Cournot-based model as a Mixed Complementarity Problem (MCP) by determining first the first order optimally conditions associated to each country's program.

To calculate the optimality conditions of each program, we define the Lagrangien function of the corresponding optimization problem, Lag_n :

$$Lag_n = \sum_t \left[X_{t,n}.\left(a_{t,n} + \frac{1}{2}.b_{t,n}.X_{t,n}\right)\right] - r_{t,n}.(K_{t,n} - X_{t,n}) - P_{n,m}.\left(Cap_{n,m} - \sum_m x_{t,n,m}\right) - f_n.(L_n - \sum_t x_{t,n} - \sum_t \sum_m x_{t,m,n})$$

Then we calculate the gradient of the Lagrangien function with respect to each decision variable: $\frac{dLag_n}{dx_{t,n}}$ and $\frac{dLag_n}{dx_{t,n,m}}$.

Optimality conditions of each program are:

$$0 \leq \frac{dLag_n}{dx_{t,n}} \perp x_{t,n} \geq 0$$

$$0 \leq \frac{dLag_n}{dx_{t,n,m}} \perp x_{t,n,m} \geq 0$$

$$0 \leq (K_{t,n} - X_{t,n}) \perp r_{t,n} \geq 0$$

$$0 \leq (Cap_{n,m} - \sum_m x_{t,n,m}) \perp P_{n,m} \geq 0$$

$$L_n - \sum_t x_{t,n} - \sum_t \sum_m x_{t,m,n} = 0$$

$f_n \ free$

Grouping together the optimality constraints of the two programs lead to a MCP problem. Since the cost functions are convex and continuously differentiable, the KKT conditions are necessary and sufficient for the existence and the uniqueness of the solution (Bazaraa et al. (1993)). The solution x is unique and satisfies simultaneously the above constraints:

$$x = \begin{pmatrix} x_{t,n}^* & \forall t, n \\ x_{t,n,m}^* & \forall t, n \\ r_{t,n}^* & \forall t, n \\ P_{t,n}^* & \forall t, n \\ f_n^* & \forall n \end{pmatrix}$$

Appendix 3:

By developing the above program in appendix 3, we obtain the following equations:

$$\begin{cases} a_{t,n} + b_{t,n}.x_{t,n} = -r_{t,n} - f_n \\ a_{t,n} + b_{t,n}.x_{t,n,m} = -r_{t,n} - P_{n,m} - f_m \end{cases}$$

Subtracting the above equations and replacing $b_{t,n}$ by $\alpha^{t-1}.b$, we can verify that: $x_{t,n} = x_{t,n,m} + \frac{f_m - f_n + P_{n,m}}{\alpha^{t-1}.b_1}$.

Replacing $x_{t,n,m}$ by $x_{t,n} - \frac{f_m - f_n}{\alpha^{t-1}.b}$ as determined above. We find:

$$\begin{cases} \sum_t x_{t,n} + \sum_t x_{t,m} - \sum_t \frac{f_n - f_m + P_{n,m}}{b_{t,m}} = Q_n \\ \sum_t x_{t,m} + \sum_t x_{t,n} - \sum_t \frac{f_m - f_n + P_{m,n}}{b_{t,n}} = Q_m \end{cases}$$

We find that:

$$f_m - f_n = \frac{b_1.(L_n - L_m)}{\sum_{t,n} \frac{1}{\alpha^{t-1}}} + \frac{P_{m,n}.\sum_t \frac{1}{\alpha_{t,m}}}{\sum_{t,n} \frac{1}{\alpha^{t-1}}} - \frac{P_{n,m}.\sum_t \frac{1}{\alpha_{t,n}}}{\sum_{t,n} \frac{1}{\alpha^{t-1}}}$$

And,

$$x^*_{t,n} = x^*_{t,n,m} + \frac{Q_n - Q_m}{\omega_t} + \frac{\sum_n P_{n,m}}{\alpha^{t-1}.b_1} . \frac{\sum_t \frac{1}{\alpha_m^{t-1}}}{\sum_{t,n} \frac{1}{\alpha^{t-1}}}$$

Appendix 4:

System operators should prefer less costly technologies when balancing their systems. Let's assume first that $\sum_{t,n} K_{t,n} \geq \sum_n Q_n, K_{t,n} \leq \sum_n Q_n$.

Secondly, let's assume that technology t in country n is the most efficient. Therefore: $x_{t,n} + x_{t,n,m} = K_{t,n}$.

Hence, from equation 9b, $x^*_{t,n} = \frac{K_{t,n}}{2} + \frac{Q_n - Q_m}{2.\omega_t} + \frac{\sum_n P_{n,m} \cdot \sum_t \frac{1}{\alpha_{t,m}}}{b.\omega_t}$ and $x^*_{t,n,m} = \frac{K_{t,n}}{2} - \frac{Q_n - Q_m}{2.\omega_t} - \frac{\sum_n P_{n,m} \cdot \sum_t \frac{1}{\alpha_{t,m}}}{b.\omega_t}$.

For inefficient technology, we can admit that, if called to produce, total quantity put in both markets would not reach the technology capacity. Let's assume $\bar{t}$ the index of inefficient technology. In this case, $r_{\bar{t},n} = 0$, because the capacity constraint is not saturated. For $\forall \bar{t}$ and from $\frac{dLag_n}{dx_{\bar{t},n}} = 0$ and $\frac{dLag_n}{dx_{\bar{t},n,m}} = 0$, we find that:

$$x_{\bar{t},n} = \frac{a_{\bar{t},n} - a_{\bar{t},n}}{\alpha^{\bar{t}-1}.b_1} + \frac{\alpha^{\bar{t}-1}}{\alpha^{\bar{t}-1}} \cdot x_{\bar{t},n} , \forall \bar{t}$$

$$x_{\bar{t},n} = \frac{a_{\bar{t},n} - a_{\bar{t},n}}{\alpha^{\bar{t}-1}.b_1} + \frac{\alpha^{\bar{t}-1}}{\alpha^{\bar{t}-1}} \cdot x_{\bar{t},m,n} + \frac{P_{m,n}}{\alpha^{\bar{t}-1}.b_1} , \forall \bar{t}$$

Knowing that at the equilibrium $Q_n = \sum_t x_{t,n} + \sum_t \sum_m x_{t,m,n}$, and replacing $x_{\bar{t},n}$ and $x_{\bar{t},m,n}$ by their respective terms, we find that:

$$x^*_{\bar{t},n} = \frac{Q_n - \sum_t x^*_{t,n,m} - \Phi_{\bar{t}}}{\omega_{\bar{t}}} + \frac{P_{m,n}}{b.\omega_{\bar{t}}} \cdot \sum_t \frac{1}{\alpha_{t,m}}$$

$$x^*_{\bar{t},n,m} = \frac{Q_m - \sum_t x^*_{t,n} - \Phi_{\bar{t}}}{\omega_{\bar{t}}} - \frac{P_{n,m}}{b.\omega_{\bar{t}}} \cdot \sum_t \frac{1}{\alpha_{t,m}}$$

We note that if all technologies are inefficient, $\sum_t x^*_{t,n,m} = \sum_t x^*_{t,n} = 0$ in the above equations.

These solutions $x = \begin{pmatrix} x^*_{t,n} & \forall t,n \\ x^*_{t,n,m} & \forall t,n \\ x^*_{\bar{t},n} & \forall \bar{t},n \\ x^*_{\bar{t},n,m} & \forall \bar{t},n \end{pmatrix}$ hold of course only if $x \geq 0$ and the sum of local generation and

exported quantity for a given technology does not exceed the available capacity of the technology.