Collapse Theories as Beable Theories

Guido Bacciagaluppi

Collapse theories as beable theories*

Guido Bacciagaluppi

Department of Philosophy, University of Aberdeen
and
Institut d'Histoire et de Philosophie des Sciences et des Techniques
(CNRS, Paris 1, ENS), Paris†

Abstract

I discuss the interpretation of spontaneous collapse theories, with particular reference to Bell's suggestion that the stochastic jumps in the evolution of the wave function should be considered as local beables of the theory. I develop this analogy in some detail for the case of non-relativistic GRW-type theories, using a generalisation of Bell's notion of beables to POV measures. In the context of CSL-type theories,

*I am very grateful to Décio Krause for this excellent opportunity to complete a project I had been involved with for some time. Indeed, while Sections 4 and 5 of this paper are new and were written at the Centre for Time, Department of Philosophy, University of Sydney, Sections 1–3 were written during my tenure of an Alexander von Humboldt Fellowship at the University and the IGPP of Freiburg, and were presented at the Twelfth U.K. Conference on the Philosophy of Physics, Leeds, September 2003. Since then, I have become aware that ideas on the analogy between collapse theories and beable theories have been discussed independently both before and since (Kent 1989, Dowker and Henson 2004, Dowker and Herbauts 2004, 2005, Tumulka 2006, Allori *et al.* 2008). Gambetta and Wiseman (2004) have also independently generalised beable theories to the case of POVMs, similarly to what done in Section 3.2. I am extremely grateful to Lajos Diósi for pointing out the serious mistake of my original conjecture that the approach to GRW-type theories described in Section 3 could be extended to CSL. The end of that section has been revised accordingly, and I discuss a different approach to CSL in Section 4. Finally, I wish to thank Fay Dowker, Wayne Myrvold and Alberto Rimini for discussions on these topics in the early phases of my work on this paper, and Owen Maroney, Max Schlosshauer, Ward Struyve and Hans Westman for more recent discussions.

†Address for correspondence: Department of Philosophy, University of Aberdeen, Old Brewery, High Street, Aberdeen AB24 3UB, Scotland, U.K. (e-mail: guidob@univ-paris1.fr).

this strategy appears to fail, and I discuss instead Ghirardi and co-workers' mass-density interpretation and its relation to Schrödinger's original charge-density interpretation. This discussion is extended to relativistic CSL-type theories. A few remarks on Everett's interpretation conclude the paper.

Keywords: quantum mechanics, collapse theories, beable theories, Bell, Schrödinger.

1 Introduction

Collapse theories are variants of quantum mechanics in which the Schrödinger evolution is modified in order to reproduce the phenomenology of the "reduction of the wave packet" or "collapse of the wave function". Setting aside attempts based on non-linear modifications of the Schrödinger equation (summarised e. g. in Doebner and Goldin 1996), collapse theories are generally stochastic. We shall divide them in three classes:

- more or less *ad hoc* theories in which the wave function is assumed to collapse upon measurement onto the eigenstates of some observable (such theories can perhaps be attributed to Dirac and/or von Neumann);
- spontaneous collapse theories of the Ghirardi-Rimini-Weber (GRW) type, in which the Schrödinger equation is supplemented by certain random discontinuous transformations, so-called "hits" (Ghirardi, Rimini and Weber 1986);
- continuous spontaneous localisation (CSL) and related theories, in which the wave function obeys a stochastic differential equation of a certain type (Pearle 1989).

Stochastic theories for the evolution of the quantum state have been proposed also in other contexts (e. g. Percival 1998) — but we shall focus specifically on these "collapse" theories, in particular GRW and CSL.[1]

The *prima facie* interpretation of these theories is of course in terms of a "wave ontology", whatever the details of this ontology may be, i. e. it will

[1] For an excellent introduction and source of references for these collapse theories, see also the entry in the *Stanford Encyclopedia of Philosophy* (Ghirardi 2002).

take the (universal) wave function as a complete and correct description of an individual quantum system.

In this paper I wish to discuss an alternative approach to the interpretation of collapse theories, inspired by a remark by Bell to the effect that the "beables" in the GRW theory should be the "hits" (or "jumps") themselves:

> However, the GRW jumps (which are part of the wavefunction, not something else) are well localized in ordinary space. Indeed each is centred on a particular sp[a]cetime point $(\mathbf{x}, t)$. So we can propose these events as the basis of the "local beables" of the theory. These are the mathematical counterparts in the theory to real events at definite places and times in the real world [...] . A piece of matter then is a galaxy of such events. (Bell 1987a, p. 205)

This is by all means a rather cryptic remark, and comparing the GRW theory and the notion of "hits" with Bell's (1984) own construal of "beable theories", as we shall do in Section 2, seems scarcely enlightening. As shown in Section 3, however, this alternative interpretation can be easily constructed for the case of GRW-type theories (whether or not it corresponds exactly to Bell's original intentions[2]). However, as we shall also see, it cannot be extended to CSL.

In the context of CSL, Ghirardi, Grassi and Benatti (1995) have proposed a different interpretation based on a Schrödinger-type mass density. I discuss this in Section 4 and argue that, much in keeping with Schrödinger's own ideas, as expressed in particular at the 1927 Solvay conference (Schrödinger 1928), this can be seen not as a separate interpretation, but as a return to a "wave only" interpretation. I also discuss this approach in the relativistic setting.

I believe that the present discussion may further serve to show that differences between different approaches to quantum mechanics (collapse, beable, or perhaps even Everett, as suggested in Section 5) are somewhat subtler than one might expect!

[2]See also the remarks in Section 5.

2 Collapse theories vs beable theories (in Bell's sense)

In order to understand Bell's remark and see what is initially puzzling about it, we must first present the GRW theory in more detail, as well as Bell's own "beable theories".

The original version of the GRW theory (Ghirardi, Rimini and Weber 1986) consists in the following stochastic evolution for the wave function, formulated in position representation. For one particle, the Schrödinger equation is supplemented at random times (with average frequency $1/\tau$) by a transformation known as a "hit", consisting of a multiplication with a normalised (in modulus-squared norm) three-dimensional Gaussian $\alpha_\lambda(q-x)$, with fixed width λ, centred at some random position x, together with a renormalisation of the resulting wave function:

$$\psi(q) \mapsto \frac{1}{\sqrt{\int |\alpha_\lambda(q-x)\psi(q)|^2 dq}} \alpha_\lambda(q-x)\psi(q) \ . \tag{1}$$

The probability density in x for the hits is given by

$$\int |\alpha_\lambda(q-x)\psi(q)|^2 dq \ , \tag{2}$$

which indeed integrates to 1 if $\alpha_\lambda(q)$ is normalised. The suggested values of the two parameters are $\tau = 10^{16}$s and $\lambda = 10^{-5}$cm.

For N particles there will be N independent such three-dimensional hitting processes supplementing the Schrödinger equation, which greatly increases the frequency of the hits, and in typical measurement situations (due to the entangled form of the wave function) even a single hit will induce localisation of the wave function on a macroscopic scale.

Indeed, the theory is specifically intended to provide an approach to quantum mechanics that makes it universally applicable (in particular both to the microscopic and the macroscopic domains, as well as to their interaction), and that makes it interpretable in an observer-independent manner.

An example of interpretation of the theory in terms of a wave ontology is given by the standard eigenstate-eigenvalue link, i. e. the assignment of a property P (identified with a subspace or a projection in Hilbert space) to the system if and only if the quantum probability of P is 1 (if the system is in a pure state, this is equivalent to saying the state of the system is an eigenstate of P). One can argue that due to the macroscopic localisation properties of the dynamics, the quantum state with such a wave-based ontology directly corresponds to visible reality, and that there is no need for "many worlds" or for additional "hidden variables" or "beables".[3]

In this sense, the theory is *about the wave function.* The wave function is what there is, the ontology of the theory, or at least the visible, manifest part of the ontology, while the hits are part of what determines the evolution of the wave function.

By contrast, Bell's (1984) beable theories are no-collapse theories in which the wave function "guides" the evolution of some preferred observable ("beable") much like in de Broglie-Bohm theory (de Broglie 1928, Bohm 1952) it guides the evolution of particle positions.

Formally, beable theories are constructed as follows. Bell takes a (discrete) projection-valued (PV) measure $X \mapsto \sum_{i \in X} P_i$ (a quantum observable — the more traditional identification with a self-adjoint operator follows via the spectral theorem) and assumes that it always has a determinate value (i or P_i), even if $|\psi(t)\rangle$ is not one of its eigenstates (therefore the designation as a *beable*).

He then defines a stochastic process for the beable of choice. The process should be such that

$$p_i(t) = \langle \psi(t)|P_i|\psi(t)\rangle \tag{3}$$

is an asymptotic distribution of the process, in particular is preserved in time (is "equivariant", is a "time-dependent equilibrium" distribution). Among the possible probability currents $j_{ji}(t)$ for such a process, defined by

$$\dot{p}_j(t) = \sum_i j_{ji}(t) \ , \tag{4}$$

Bell chooses

$$j_{ji}(t) := 2\hbar \mathrm{Im}\langle \psi(t)|P_j H P_i|\psi(t)\rangle \ , \tag{5}$$

[3]It is sometimes argued that this is not true, due to a supposed difficulty of interpreting "tails" of the wave function. See Ghirardi (2002) for discussion. The mass-density interpretation by Ghirardi, Grassi and Benatti (1995) that we discuss below was partly motivated as a response to this criticism. I plan to deal with the "tails" problem in separate work with Max Schlosshauer.

and, given this current, as infinitesimal parameters $t_{ji}(t)$ of the process he chooses

$$t_{ji}(t) := \left[\frac{j_{ji}(t)}{p_i(t)}\right]^+ \tag{6}$$

(where $[.]^+$ denotes the positive part).[4]

Examples of beable theories based on continuous PV measures of course also exist, although I do not know of any fully general classification as the above. In the case of position, the paradigm examples are de Broglie-Bohm pilot-wave theory and the stochastic variant thereof equipped with Nelson's diffusion dynamics (cf. Nelson 1966).[5]

Also a theory of this type is intended to provide an approach to quantum mechanics that makes it universally applicable and interpretable in an observer-independent manner. To this end, at least standardly, the beable is interpreted as providing the ontology of the theory, or the visible part of the ontology.

As in the case of the "wave ontology" for GRW, there may be some ambiguity about how to understand a "beable ontology". One could identify the values of P_i with subspaces of Hilbert space. On the other hand, they obey a classical logic, so that this identification seems irrelevant. It also breaks down in the case of de Broglie-Bohm theory. It thus appears more natural to interpret the values of P_i simply as elements i of a relevant "state space" or "configuration space". (This will be the interpretation most suited to our purposes.)

In order to provide an acceptable ontology, the beable of course needs to be chosen appropriately: in the non-relativistic particle theory one usually chooses particle positions (such as in de Broglie-Bohm theory). Bell's aim was to construct a de Broglie-Bohm-like theory for fields, and his choice of

[4] From well-known results one can canonically reconstruct a stochastic process from such infinitesimal parameters, at least if they are bounded (Feller 1940). Note that the parameters (6) become singular at the nodes of $p_i(t)$. A proof of the existence and uniqueness of solutions also for this case was first given by Tumulka and Georgii (2005).

[5] The most general deterministic and diffusion-type dynamics for beable theories based on position have been discussed by Deotto and Ghirardi (1998) and by Bacciagaluppi (1999), respectively; see also Peruzzi and Rimini (1996). It is further known that if one chooses an appropriate limiting procedure, it is possible to recover de Broglie-Bohm dynamics and Nelson dynamics as continuum limits of discrete beable dynamics (in the latter case the choice of infinitesimal parameters differs from (6)); see Sudbery (1987) and Vink (1993).

beable was fermion number density on a lattice.

A plausible criterion for beable choice is that the beable should be a decohering variable, i. e. that on a suitable scale there should be no interference between wave components characterised by different values of P_i. If this is the case, then on this scale the transition probabilities for the beable ("visible") dynamics constructed using Bell's choice of current and infinitesimal parameters will reduce to the empirical quantum transition probabilities (which further reduce approximately to 0 and 1 in appropriate cases); see e.g. Bub (1997, pp. 161–163).[6]

It thus appears that in a beable theory it is the beable that provides the ontology, or the visible part thereof, while the wave function determines its evolution. In these respects such a theory seems to be the *exact opposite* of the GRW theory, and it thus seems surprising that the GRW hits could be understood as beables.

3 Collapse theories as beable theories (in Bell's sense)

We now proceed to show how GRW hits can indeed be understood as beables in a sense very close to that of Bell. We first take a step back, however, to a "toy" case modelled on collapse upon measurement.

3.1 Toy example

In the traditional (more restrictive) view, quantum observables are identified with projection-valued (PV) measures. Measurement-type collapse for these observables is defined only if their spectrum is discrete,[7] and is described as follows. For some discrete PV measure $\sum_{i\in X} P_i$, the state undergoes upon measurement the random transformation

$$\psi \mapsto \frac{1}{\sqrt{\langle\psi|P_i|\psi\rangle}} P_i\psi \tag{7}$$

[6] For further discussion of this point, see e.g. Bacciagaluppi (2003).

[7] This point is already discussed by von Neumann in his book (von Neumann 1932, Sections III.3, IV.3, V.1 and VI.3).

with probability $\langle\psi|P_i|\psi\rangle$. Probabilities are normalised to 1 because the PV measure is normalised to $\sum_i P_i = \mathbf{1}$.

Let us now define a toy spontaneous collapse theory by stipulating that the above transformation happens *spontaneously* at random times (with some given average frequency).

The theory as it stands is still a theory about the quantum state $|\psi\rangle$, and we can take the standard eigenstate-eigenvalue link as describing the "wave ontology" of the theory, i. e. the system has a property P at time t if and only if $\langle\psi(t)|P|\psi(t)\rangle = 1$.

While the "hits" P_i are clearly part of what determines the evolution of the state $|\psi\rangle$, this interpretation of the theory has implications for the *values* of P_i. Indeed, at the discrete times of collapse, the system discontinuously acquires one of the properties P_i. Thus, the standard wave ontology implies that at these discrete times the given observable has determinate values.

The wave ontology of course contains more than these values of P_i: at times between collapses in general *other* observables will have values, and even at collapse times — unless all P_i are one-dimensional — the value of P_i does *not* fix the state $|\psi(t)\rangle$.

If, however, we wish to construct an interpretation of the theory in terms of a *beable* P_i, we are free to ignore the additional ontology provided by the quantum state. This would lead to a description of the system in terms of values of P_i at certain discrete times. (We are free to "add" ontology to fill the temporal gaps, postulating that P_i has a value at all times, say, constant between "collapses".)

One can go even further. While it is natural to keep the *collapsing* wave function in the formalism in order to describe the evolution of the P_i, one could also consider the probabilities for whole stochastic trajectories $P_{i_0}, \ldots, P_{i_n}$ as given by the generalised Born rule

$$\langle\psi(t_0)|P_{i_0}U^*_{t_1t_0}P_{i_1}\ldots U^*_{t_nt_{n-1}}P_{i_n}U_{t_nt_{n-1}}\ldots P_{i_1}U_{t_1t_0}P_{i_0}|\psi(t_0)\rangle \ , \tag{8}$$

and argue that this describes a generally "non-Markovian" process determined by $|\psi(t_0)\rangle$ and the unitary evolution U (i. e. information equivalent to the uncollapsed $|\psi(t)\rangle$). In this case one can imagine a *non-collapsing* wave function as guiding the P_i (admittedly in a rather contrived way).[8]

[8] In the standard beable theory, it is not *a priori* necessary to take the initial distribution

We see that in this toy example we can turn a collapse theory into a theory about the beable P_i, with the quantum state (whether collapsing or non-collapsing) determining the evolution of the beable. In this sense, the difference between the standard beable theory for P_i, as sketched in the previous section, and the transformed collapse theory, as sketched in this, becomes purely *quantitative*. Further, in the special case in which the beable should happen to be also a decohering variable, the two theories will at least approximately coincide both qualitatively (in terms of what they are about) and quantitatively (in terms of the values for the probabilities), although the *form* of the laws will be different in the two theories.

3.2 GRW

We now show how the GRW theory can be reinterpreted as (or transformed into) a beable theory by analogy to the toy example. To this end, we point out two simple but apparently not widely known facts, both involving positive-operator-valued (POV) measures $\sum_{i \in X} E_i$.[9]

The first is that beable theories in Bell's sense can be formulated also based on POV measures, i. e. also POV measures can be used as beables.[10] As in Section 2, we sketch this in full generality only for the discrete case, but again examples based on continuous POV measures are also known.

In the discrete case, one can construct beable theories for any given POV measure by straightforwardly applying Bell's construction of Section 2. Instead of (3), the asymptotic (equivariant) distribution is

$$p_i(t) = \langle \psi(t) | E_i | \psi(t) \rangle \ . \tag{9}$$

One can take any current satisfying (4) for this distribution, e. g.

$$j_{ji}(t) := 2\hbar \mathrm{Im} \langle \psi(t) | E_j H E_i | \psi(t) \rangle \ , \tag{10}$$

equal to (3). To extend the analogy also to this point, one could perhaps use (8) only to calculate conditional probabilities, and refrain from identifying the initial distribution at t_0 with $\langle \psi(t_0) | P_{i_0} | \psi(t_0) \rangle$.

[9] Here, the E_i are so-called effects, i.e. positive operators with spectrum in the interval $[0, 1]$, and $\sum_i E_i = \mathbf{1}$. As a general reference for the use of POV measures, see Busch, Grabowski and Lahti (1995).

[10] As mentioned in footnote 1, Gambetta and Wiseman (2004) have also independently given a generalisation of beable theories to the case of POVM beables. Their construction of the relevant stochastic process is based on the Naimark dilation theorem.

and define the infinitesimal parameters for instance again using (6).

An example of a beable theory based on a continuous POV measure is Garcìa de Polavieja's (1996) family of phase-space pilot-wave theories, based on POV measures of coherent states in phase space, i. e. joint position-momentum observables.[11]

The second of the little-known facts is that the GRW theory already has the form of a spontaneous measurement-type collapse as in the toy case, with the difference that it is a measurement of a continuous POV-observable rather than a discrete PV-observable.

Consider the special case of a (discrete or continuous) POV measure of the form $X \mapsto \int_X E_x dx$, with E_x having spectrum in the interval $[0,1]$ and with the normalisation $\int E_x dx = \mathbf{1}$. A class of measurements of this POVM (via "pure operations", the only ones we shall need), is given by the following transformation:

$$|\psi\rangle \mapsto \frac{1}{\sqrt{\langle\psi|A_x^* A_x|\psi\rangle}} A_x|\psi\rangle \ , \tag{11}$$

with probability density $\langle\psi|A_x^* A_x|\psi\rangle$, where the A_x are arbitrary (i. e. not necessarily self-adjoint) operators satisfying $A_x^* A_x = E_x$. Comparing (11) to (1), we see that a GRW hit is just a special case of (11), with the corresponding $A_x = \alpha_\lambda(Q - x)$, i. e. given by the (self-adjoint) multiplication operator with the Gaussian $\alpha_\lambda(q - x)$. Thus the GRW dynamics is indeed a spontaneous measurement-type collapse, namely for a specific "unsharp realisation" of position. More general "GRW-type" spontaneous collapse theories can be further defined by setting $A_x = \alpha_\beta(A - x)$, with arbitrary width β, arbitrary frequency $\frac{1}{\sigma}$ and an arbitrary (self-adjoint) operator A, which may itself have discrete or continuous spectrum.

The eigenstate-eigenvalue link no longer gives us values for the POV measure at collapse times, but we postulate, by analogy to the toy example, that at collapse times the beable being spontaneously measured has a value, namely the value x corresponding to the centre of the Gaussian instantiating the hit. If we also (optionally) postulate that the values of all beables (one for each particle) remain constant between hits, we have a well-defined prescription for the stochastic evolution of the configuration of the system at all times.

[11] One of the interesting features of this theory is the fact that the POV measure used is informationally complete, i. e. the (equivariant) distribution of values not only is determined by the quantum state but completely determines it in turn.

The GRW theory is thus transformed into a position-based beable theory, with a non-standard dynamics, which is not deterministic, not diffusion-type and obviously not allowing for reinterference effects! It also has quite different commitments as to what the visible ontology is supposed to be: no longer the wave but the particle positions, just as in de Broglie-Bohm theory.

Note that for this reason, the answer to the question of whether the theory is a good candidate for a fundamental approach to quantum mechanics is not automatically the same as in the case of the GRW theory. And in fact, the following suggests that it is not. Macroscopic localisation in the GRW theory is obtained throught he fact that a hit associated with a single particle can and will collapse the wave function of the whole system to one that is localised with respect to a macroscopic number of spatial variables. In the beable version, the wave function also collapses (or effectively collapses) with the first hit, but only the particle associated with this particular hit will be localised where the hit has taken place. Since the other particles are guided (or effectively guided) by the now localised wave function, they will also jump to more or less the same location, given sufficient time. The time it takes for a macroscopic number of particles to congregate around the location of the first hit, however, is of the order of magnitude of the average jump frequency times the macroscopic number of particles. That is, the macroscopic localisation features of the theory are much less satisfactory than in the case of GRW, where the visible ontology is given directly by the wave function.[12]

This problem of course is only a problem if we stick to the original GRW theory (i. e. if we want to implement literally Bell's suggestion that the GRW hits ought to be the beables). One could for instance take instead a GRW-type theory with A some kind of product observable taking values in the configuration space $\mathbf{R}^{3N}$ of a system of N particles. A "hit" would then correspond to a jump of all particles simultaneously. We shall, however, not labour this point, as the approach will anyway fail to generalise to the context of CSL.

[12]Another related difficulty, or at least an unintuitive consequence of the theory, is that isolated microscopic systems will hardly evolve at all (one jump every 10^{16} seconds!).

3.3 CSL

In this subsection, we shall first briefly recall the formalism of CSL. We shall then recall a well-known but perhaps not widely known result about CSL being a certain infinite-frequency limit of a family of GRW-type processes. Unfortunately, the limit for the corresponding beable theories (constructed as in the previous subsection) is singular, so that it appears that CSL cannot be reinterpreted as a beable theory in Bell's sense.

Following Ghirardi, Pearle and Rimini (1990), let us first define a general CSL-type process as given by a stochastic Itô equation for the quantum state of the form

$$d|\psi\rangle = \left[(-\frac{i}{\hbar}H - \frac{1}{2}\gamma \mathbf{A}_\psi^2)dt + \mathbf{A}_\psi \cdot d\mathbf{B} \right] |\psi\rangle \ , \tag{12}$$

where $\mathbf{A}_\psi = \mathbf{A} - \langle\psi|\mathbf{A}|\psi\rangle$ and $\mathbf{A} = (A_i)_{i\in I}$ is a (discrete or continuous) family of not necessarily commuting self-adjoint operators, and $d\mathbf{B} = (dB_i)_{i\in I}$ is a corresponding family of Wiener processes satisfying

$$\overline{dB_i} = 0, \qquad \overline{dB_i dB_j} = \delta_{ij}\gamma dt \tag{13}$$

(where δ_{ij} is to be interpreted as a Kronecker or Dirac delta depending on the index set I).

Specifically, the original CSL theory (Pearle 1989) is obtained by taking as $(A_i)_{i\in I}$ the family $(N^{(k)}(x))_{k\in K, x\in\mathbf{R}^3}$, with k labelling particle type and

$$N^{(k)}(x) := \int dq \alpha_\lambda(q-x) \sum_s a_k^*(q,s) a_k(q,s) \ , \tag{14}$$

where $a_k(q,s)$ and $a_k^*(q,s)$ are the creation and annihilation operators for a particle of type k at point q with spin component s, and $\alpha_\lambda(q)$ is the three-dimensional Gaussian with fixed width λ as in the GRW theory. That is, the $N^{(k)}(x)$ are number density operators locally averaged in a small region around x, with the width of the average specified by the Gaussian. The parameter γ in (12) and (13) is chosen as $\gamma = \frac{1}{\tau}(4\pi/\alpha)^{3/2}$, with τ and α as in the GRW theory.

A slightly different version of the theory (Ghirardi, Pearle and Rimini 1990) uses locally averaged mass density operators $(M(x))_{x\in\mathbf{R}^3}$ instead of the number density operators (14):

$$M(x) := \sum_k m_k N^{(k)}(x) \ , \tag{15}$$

where m_k is the mass of a particle of type k, and uses $\gamma = \frac{1}{m_0^2 \tau}(4\pi/\alpha)^{3/2}$, where m_0 is the mass of the proton.

A useful way of getting an intuitive grasp of what the CSL formalism yields is as a limit of GRW-type processes, in the following sense (Ghirardi, Pearle and Rimini 1990). Take a GRW-type process, defined by a spontaneous collapse (with average frequency $\frac{1}{\sigma}$) of the form (11), with $A_x = \alpha_\beta(A - x)$, with an arbitrary (self-adjoint) operator A. Quite generally, this process corresponds to a measurement of an unsharp realisation of A, which disregarding the Hamiltonian evolution would asymptotically drive the state $|\psi\rangle$ to some eigenstate of A (that is, if A has discrete spectrum). For fixed A, we can now vary the parameters β and σ, in particular we can take the limit $\beta \to \infty$, $\frac{1}{\sigma} \to \infty$, with $\beta\sigma = \gamma$ (constant). Since all of these processes are measurements of unsharp realisations of the same operator A, they all share the same asymptotic behaviour, and one should intuitively expect the same from the limit. The limit is well-defined, as discussed by Diósi (1988) for the case of $A = Q$ and by Nicrosini and Rimini (1990) in the general case, and in fact corresponds precisely to the process (12) for the case of a single A in the family $(A_i)_{i \in I}$.

In order to obtain the full (12), one has to consider several such GRW-type processes, each with A given by a different A_i, happening independently and simultaneously, and all being taken to the limit. The different A_i could be for instance the positions Q_i of the different particles $i = 1, 2, \ldots N$. If one chooses them to be the different $N^{(k)}(x)$ or the different $M(x)$, one recovers the CSL dynamics for the quantum state. Therefore the CSL dynamics can be understood as a suitable infinite frequency limit of GRW-type processes for local number density or for local mass density (one for every particle type and every spatial point or, respectively, one for every spatial point), and at least intuitively drives the system towards simultaneous eigenstates of these operators.[13]

One might hope similarly to understand CSL as a limiting theory also when the GRW-type theories are seen as beable theories. For that to work, how-

[13]For a more rigorous discussion of this point, see Ghirardi, Pearle and Rimini (1990). Note that the localisation properties of the CSL dynamics are not governed by the width β of the Gaussian in the definition (11) of the GRW-type process (since this represents width in number or mass density and not in space, and anyway it goes to infinity), but by the width λ of the Gaussian used in defining the local averages (14) or (15). For further details of how the theory works and what it does, see Ghirardi (2002), Ghirardi, Grassi and Benatti (1995) and Ghirardi, Pearle and Rimini (1990).

ever, there must be a well-defined limit also at the level of the corresponding beable dynamics. Diósi (1988) discusses both the limit of the process for the wave function and the limit of the process for the beable (in the case of position). His analysis shows that, in the limit, the variance of the position beable goes to infinity (cf. Diósi 1988, eq. (3.7b)). Therefore, the behaviour of the beable dynamics in the passage from GRW-type theories to CSL is very singular, and this picture of CSL as a beable theory turns out not to be viable.[14]

4 Collapse theories as beable theories (in Schrödinger's sense)

4.1 Schrödinger's charge-density interpretation[15]

In 1926, as is well known, Schrödinger published a series of papers in which he developed his theory of wave mechanics. From the start, Schrödinger thought that the wave function, the fundamental object in the theory, could be used to give an anschaulich description at least of the stationary states of the atom, and believed that by combining two stationary wave functions one might find a mechanism for the emission of radiation with the characteristic Bohr frequency $\nu_{ij} = \frac{E_i - E_j}{h}$. While far from being the end of the story, this is the basis for the semi-classical radiation theory still used today in applications.

Schrödinger arrived at a workable proposal through his investigation of the relation between wave mechanics and matrix mechanics (which already contained expressions for radiation intensities), and proposed that the expression for charge density should be proportional to $|\psi|^2$ for the first time in print in an addendum in proof to his third paper on quantisation (Schrödinger 1926a, footnote on p. 476). At the same time, he explained his proposal in detail in a letter to Lorentz of 6 June 1926 (Przibram 1967, p. 56). The fourth paper on quantisation (Schrödinger 1926b) then elaborated on this idea, giving an anschaulich picture of the interpretation as the superposi-

[14] I am extremely thankful to Lajos Diósi for pointing out this important fact to me (private communication, Bielefeld, February 2004).

[15] For further background details, see the discussion in Bacciagaluppi and Valentini (2009, Chapter 4, esp. Section 4.4). Instead, the more speculative of the suggestions below are peculiar to this paper.

tion of all classical configurations of a system weighted by $|\psi|^2$, a picture described by Schrödinger shortly thereafter as that of the system as a kind of "mollusc".[16]

Possibly the most detailed articulation of the charge-density interpretation, however, was given by Schrödinger in his report at the 1927 Solvay conference in Brussels and in the ensuing discussion (Schrödinger 1928, see pp. 451–454 and 466–471).

There, Schrödinger presents his idea in the following terms (his emphasis in this and subsequent quotations):

> What does the ψ-function mean now, that is, *how does the system described by it really look like in three dimensions*? [...] I myself have so far found useful the following perhaps somewhat naive but quite concrete idea. The classical system of material points does not really exist, instead there exists something that continuously fills the entire space and of which one would obtain a "snapshot" if one dragged the classical system, with the camera shutter open, through *all* its configurations, the representative point in q-space spending in each volume element $d\tau$ a time that is proportional to the *instantaneous* value of $\psi\psi^*$. [...] Otherwise stated: the real system is a superposition of the classical one in all its possible states, using $\psi\psi^*$ as "weight function". (Schrödinger 1928, p. 451)

Schrödinger then goes on to define a "charge density" based on the configuration-space wave, the contribution from each single particle being calculated by integrating out all the other particles.

It is clear that Schrödinger was very well aware that such a charge density as he had defined does *not* generally behave like a (classical) charge density. Indeed, as Schrödinger explains, these charges are not affected by other charges or by external fields according to the classical force laws, because the evolution of ψ is already determined completely by the Schrödinger equation. Instead, they may be usefully considered to be classical sources of the electromagnetic field, yielding insights into Bohr's frequency condition

[16]Schrödinger to Lorentz, 23 June 1927, Archive for the History of Quantum Physics, microfilm LTZ-13 (original with Schrödinger's corrections) and microfilm 41, section 9 (carbon copy) (in German).

and the selection rules (pp. 452–453). Nevertheless, Schrödinger emphasises that this is only an approximation: first, radiation reaction is left out of the picture, and, second, the action of the charges on some other systems should in principle also be described by the Schrödinger equation (or some relativistic generalisation thereof), but then there is again no place for a classical electromagnetic field to mediate the interaction (pp. 453–454).[17]

Historically, Schrödinger ran into problems with this interpretation, partly due to the limitations of the semi-classical picture (which attracted criticism in the discussion in Brussels), but more importantly due to his difficulties with the spreading of wave functions and, later on, to the realisation that the Schrödinger equation leads inevitably to Schrödinger-cat situations. If one realises, however, that classical behaviour and localisation are essential only as emergent features on a macroscopic scale, then collapse theories seem ideally suited for an interpretation along the lines of Schrödinger's views, and indeed a Schrödinger-type beable interpretation of collapse theories has been proposed by Ghirardi and co-workers, as will be presently discussed.

An important point to note, however, is that Schrödinger presents this interpretation as a description of how the configuration-space wave *appears* in ordinary space and time. In the discussion (p. 469), he is explicit that the wave equation in configuration space will presumably remain the central tool in the theory, but that he wishes to grasp better its "physical meaning". Here, Schrödinger — the champion of the wave picture over the particle picture — is not saying that the fundamental ontology of the theory is given by a charge density beable, nor by a dual ontology of wave function together with charge density. He is saying (or so I suggest) that the ontology of the theory is given by the wave function, and that the charge density he has defined is a proposal for capturing the three-dimensional manifestation (and hence the "physical" meaning[18]) of the fundamental configuration-space ontology. Whether or not this is the most accurate historical reading, I suggest, as we shall discuss more fully in the next subsection, that this position should be taken as the most natural to adopt in this context.

[17]Note that, although formally a "classical" field, Schrödinger's charge density is indeed merely a quantity that behaves approximately in certain respects and emergently in a certain "semi-classical" regime as a classical charge density would. If we apply the term "charge density" to denote this quantity, we must be conscious of its change of meaning. (This is quite analogous to the case of the term "mass" in special relativity, which denotes a quantity that behaves like Newtonian mass only approximately and in a certain regime.)

[18]For further remarks on "physical" meaning, see the next subsection.

4.2 Ghirardi's mass-density interpretation

After Ghirardi, Rimini and Weber (1986) introduced their original GRW model, the question of how to best interpret collapse theories was first raised by Bell (1987a), with his suggestion of the hits as local beables. Such a question, however, started to generate a strong debate only after the so-called tails problem was raised by Shimony (1990), namely the problem of whether multiplication by Gaussians (which have infinitely extended tails) could really be taken to localise a macroscopic superposition. We refer the reader to Ghirardi (2002) for more details on this debate. Suffice it to say that Ghirardi himself and co-workers were eventually motivated by the tails problem to propose an interpretation based on mass density.

This interpretation was proposed in Ghirardi, Grassi and Benatti (1995), and it applies equally to both GRW and CSL theories. Ghirardi, Grassi and Benatti take as the candidate for the local beable in the theory the expectation value of the averaged mass density (15), and they postulate that this quantity corresponds to an objective property of the system if and only if its variance is much smaller than 1. While this condition is generally not fulfilled in standard quantum mechanics, it is in collapse theories for the bulk of the collapsed wave function (and not for the tails). The idea is clear: only where the variance of the averaged mass density is small, does the density emerge as a useful physical property, and in a collapse theory it supports classical behaviour in the macroscopic regime.[19] As a further justification, Ghirardi, Grassi and Benatti point out that differences in mass density provide a much more useful metric on macrostates than the usual Hilbert-space metric. Later, Ghirardi (2002, 2007) pointed out that taking the average of the mass density operator is inessential, and that one can base the interpretation just as well on the unaveraged mass density.

As presented in Ghirardi (2007), the mass-density interpretation has a dualist ontology. We now wish to expand on our comments above on Schrödinger's interpretation, and argue that also Ghirardi's (similarly defined) mass density should best be understood as the three-dimensional manifestation of a *wave-only* ontology.

[19]Unfortunately, this objectivity criterion (although very useful in other respects) rules out by *fiat* the only version of the tails problem that should be taken seriously, namely the Everettian one. As mentioned above, I hope to address the tails problem separately. See Section 5 for some further Everett-related remarks.

We have mentioned in Section 2 that the standard wave-only interpretation of the wave function is given by the so-called eigenstate-eigenvalue link (henceforth EEL): a quantum system possesses all and only those properties that are given probability 1 by the quantum state. Ghirardi's mass density, however, is an expectation value, so in general not a property in the sense of EEL. What attitude ought one to adopt towards expectation values? They are fixed by the quantum state, and include as a special case the (eigen)values associated with properties via EEL. Why not extend property assignments to include *all* expectation values? Indeed, the collection of all expectation values fixes in turn the quantum state, so that one could very well identify the state of a quantum system with the collection of all expectation values.[20] But if one does so, what makes the expectation value of the local mass density special?

This is where Ghirardi, Grassi and Benatti's objectivity criterion comes in. One normally restricts property status to those expectation values that are dispersion-free, because they have a certain dynamical stability (namely, they support repeatability of measurements). But expectation values of local density operators, in particular the mass density operator, also have interesting dynamical properties, at least in the context of GRW and CSL. Indeed, the expectation value for mass density acquires a relatively small dispersion in macroscopic states thanks to the collapse mechanism. That is, the expectation value of mass density can be justifiably be considered an (emergent) physical property of a quantum system *in the classical regime*.

Even classically, any function of the phase point of a system, say $3q^2p + 17\sqrt{q}p^3$, is a well-defined property of the system in an abstract sense. But only some of these functions play an interesting dynamical role (energy, angular momentum and so on), so that we are justified in calling them "physical" (or even "objective"). And some classical quantities only become useful in a suitable regime (temperature or pressure, when defined as averages).

Also Ghirardi, Grassi and Benatti's (1995) discussion of mass density providing a better metric on Hilbert space for distinguishing between macroscopic states can be seen in the same context, as an argument why mass density plays a useful role at the macroscopic level.

Thus, if we are looking for a local manifestation of the wave function, we are justified in saying that mass density is indeed such a local manifestation,

[20]This is precisely how von Neumann (1932) views the state for the purpose of deriving his no-hidden-variables theorem.

if only in the classical regime. With this proviso, which to a certain extent may have been clear already to Schrödinger (cf. his qualifications regarding the sense in which his charge density was indeed a charge density), we can indeed embrace what we have suggested was Schrödinger's view, that mass (or charge) density is nothing more or less than the three-dimensional appearance of the configuration-space wave function.

While a dualist position (wave *and* mass density beable) is perfectly justified in a no-collapse beable theory, where the beable is defined independently of the wave function, it seems less well motivated in the present case, in which the beable is functionally dependent on the wave function. Indeed, the beable appears to be ontologically completely derivative, so that a dualist interpretation collapses into a wave-only interpretation: I consider this *wave-only* interpretation to be the most natural reading of a mass-density interpretation.

The only other alternative is to deny not only property status to general expectation values, but to deny the eigenstate-eigenvalue link itself, i.e. abandon altogether the idea of the wave function as a compendium of the properties of the system, and postulate instead that the mass density beable provides the only ontology for quantum systems. As we understand it, it is this *beable-only* interpretation which is favoured by Allori *et al.* (2008). (And perhaps it is even Ghirardi's current position; cf. his remarks in Ghirardi 2002, Fall 2008 version.)

4.3 Relativistic local beables

We shall now discuss the generalisation of Schrödinger-type beables to the setting of relativistic collapse theories, in particular the proposal by Nicrosini and Rimini (2003). We shall discuss this proposal rather than the better-known one by Pearle (1999), because Nicrosini and Rimini's fits in particularly nicely with the discussion in Section 3.3. In a very good paper, Tumulka (2006) has discussed a relativistic generalisation of GRW-type theories where the local beables of the theory are hits (or "flashes", in Tumulka's terminology), thus beables in Bell's sense. We assume that this strategy is precluded in the CSL case for the same reasons as in the non-relativistic case.[21]

[21] Note that it is easy to generalise the toy case to a relativistic setting: the generalised Born rule (8) can be formulated in a relativistic setting, e.g. using the decoherent histories

When discussing relativistic collapse theories, it is useful to separate two questions. First, is the idea of collapse compatible at all with Lorentz invariance? Second, if it is, can one construct satisfactory Lorentz-invariant equations for a collapse theory?

The current consensus with regard to the second question seems to be that there is progress, as shown by the recent proposals by Pearle (1999) and by Nicrosini and Rimini (2003).[22] With regard to the first question, it appears very natural at first to doubt whether relativistic collapse is at all possible. Indeed, in a non-relativistic setting, if Alice and Bob perform spin measurements on an EPR pair in different directions, then the form of the state *between* the two measurements depends on the order in which they are performed; instead, the initial and final states do not depend on the time ordering of the measurements (see e.g. the discussion in Shimony 1986).

On the other hand, this turns out to be a problem only if one sticks to the idea that collapse is an instantaneous process along a single hyperplane, i.e. in a preferred frame. Indeed, there seem to be perhaps three ways to relax the assumption of a preferred collapse frame, as follows.

(a) One can try to define collapse as a process on the forward light cone (or on the backward light cone). This would make collapse manifestly Lorentz-invariant (if the corresponding equation can be defined Lorentz-invariantly). Indeed, a proposal for invariant collapse (along the backward light cone) has been made by Hellwig and Kraus (1970).[23]

formalism, where, in particular, it is *not* necessary to impose a separate decoherence condition in order to have the thus defined probabilities sum up to 1. (Note that if the histories decohere, the theory will be empirically indistinguishable from a corresponding toy beable theory.)

[22] However, Pearle (2005) now believes that one will obtain an overall more satisfactory theory by allowing violations of Lorentz invariance on a small scale.

[23] Aharonov and Albert (1984) claim to show that Hellwig and Kraus's approch, or indeed any covariant collapse approach, is inconsistent. While I believe forward or backward light cone approaches are not always fully covariant (only when one has a set of "first" or "last" collapses, respectively), I find Aharonov and Albert's two objections unconvincing. First, they construct a simple example (of a particle initially in two boxes, then collapsed to being in only one box) in which charge conservation is violated in every frame of reference (this will be true also with collapse along the forward light cone). However, why should we integrate charge along spacelike hyperplanes, if the collapse picks out as preferred certain piecewise lightlike surfaces? Second, they argue that Hellwig and Kraus's approach contradicts the predictions of standard quantum mechanics for certain observables measured using arrays of local apparatuses distributed along a hyperplane. Even if this is correct, it only means that it is an empirical question which approach is to be

(b) One can consider collapse to be a process that happens instantaneously along all hyperplanes, so that the quantum state itself becomes an intrinsically hyperplane-dependent quantity. This strategy has been for decades vigorously championed by Gordon Fleming (e.g. 1966, 1996).

(c) One can take the quantum state to be defined relative to arbitrary spacelike hypersurfaces, as in the Tomonaga-Schwinger formalism. This strategy was first proposed by Aharonov and Albert (1984), and put in practice explicitly by Pearle (1990).

Admittedly, if one gives up the assumption of a preferred frame, the collapse mechanism in general no longer provides an explanation for the correlations in an EPR experiment.[24] However, it is not necessary to find such an explanation, if what one really requires of a collapse theory is to provide a microscopic account of collapse without the need for vague primitives such as “measurement”.

Pearle’s model, and its analysis by Ghirardi, Grassi and Pearle (1990) show that strategy (c) can be implemented explicitly. This possibility has begun to be recognised also in the philosophical literature, especially since the work of Myrvold (2002, 2003).[25] Still, the model suffers from technical difficulties (specifically, infinite energy production). Later models show progress also on such technical matters, for instance Pearle (1999) and Nicrosini and Rimini (2003). We shall now follow the latter in order to present this strategy in somewhat more detail.

The Tomonaga-Schwinger equation describes the evolution of an interaction-picture state vector $|\psi\rangle$ as a function of an arbitrary spacelike hypersurface σ, as follows:

$$\delta|\psi(\sigma)\rangle = -\frac{i}{\hbar}\mathcal{H}(\mathbf{x})|\psi(\sigma)\rangle\delta\sigma(\mathbf{x}) \ . \tag{16}$$

Here $\mathcal{H}(\mathbf{x})$ is a Lorentz scalar field and $\delta\sigma(\mathbf{x})$ is a small four-dimensional volume through which a segment of σ around the space-time point $\mathbf{x}$ is being

preferred, and does not show the inconsistency of the approach itself.

[24] With the obvious exception of collapse along the backwards light cone, if it should prove to be practicable (see the previous footnote). It is just as obvious that the correlations remain *un*explained in the case of collapse along the forward light cone. This is less obvious in the case of hyperplane- or hypersurface-dependence, but note that, e.g., the requirement that one have the same collapsed state regardless of how one arrives at it goes beyond the requirement of integrability of the equations, i.e. of the path-independence of the probabilities for the possible collapsed states.

[25] Let me publicly note my admiration for these two lovely papers.

"moved forward".

The relativistic CSL equation proposed by Nicrosini and Rimini has the form

$$\delta|\psi(\sigma)\rangle = \Big[(-\frac{i}{\hbar}\mathcal{H}(\mathbf{x}) - \frac{1}{2}\gamma S_{|\psi(\sigma)\rangle}(\mathbf{x})^2)\delta\sigma(\mathbf{x}) + S_{|\psi(\sigma)\rangle}(\mathbf{x})\delta\beta(\mathbf{x})\Big]|\psi(\sigma)\rangle \ , \quad (17)$$

where

$$S_{|\psi(\sigma)\rangle}(\mathbf{x}) = S(\mathbf{x}) - \langle\psi(\sigma)|S(\mathbf{x})|\psi(\sigma)\rangle \ , \quad (18)$$

with $S(\mathbf{x})$ an appropriate Lorentz scalar field (which Nicrosini and Rimini give explicitly, and which is related to mass density), and where, for any family of non-overlapping $\delta\sigma(\mathbf{x}_i)$, $\delta\beta(\mathbf{x}_i)$ is a Gaussian random variable with

$$\overline{\delta\beta(\mathbf{x}_i)} = 0, \qquad \overline{\delta\beta(\mathbf{x}_i)\delta\beta(\mathbf{x}_j)} = \delta_{ij}\delta\sigma(\mathbf{x}_i)\delta\sigma(\mathbf{x}_j) \ . \quad (19)$$

If (17) is integrable (which is not known[26]), i. e. if the evolution between any two hypersurfaces σ_1 and σ_2 is independent of the way σ_1 is moved forward to σ_2 (the relativistic analogue of Alice and Bob obtaining the same final state irrespective of the order of their measurements), then it is indeed a Lorentz-invariant collapse equation of the CSL-type.

The question we wish to discuss now is: can one define *local* mass density beables (in Schrödinger's sense) in such relativistic collapse approaches? The case of strategy (a) is straightforward. Indeed, the quantum state is covariant, so Schrödinger-like or Ghirardi-like mass density is always well-defined. Instead, in the case of strategies (b) and (c), since the state is hyperplane- or hypersurface-dependent, respectively, there is no unique (reduced) state defined in a neighbourhood of a point $\mathbf{x}$, and therefore no unique mass density either.

Ghirardi, Grassi and Pearle (1990) have proposed to define an objective local property at the spacetime point $\mathbf{x}$ (or in an appropriate neighbourhood) if and only if it is assigned probability close to 1 by the states on *all* hypersurfaces through $\mathbf{x}$.[27] They also show (at least schematically) that on the macroscopic scale, such a criterion yields the usual classical macroscopic behaviour. That is, since the trigger mechanism for collapse is sensitive to

[26] Note that Pearle's (1990) original model is integrable, as shown by Ghirardi, Grassi and Pearle (1990).

[27] The motivation for requiring the probability to be only close to 1 is again supplied by the desire to address the "tails" problem.

macroscopic superpositions, all the states defined on the different hypersurfaces will approximately coincide on the macroscopic scale (except for times shorter than the typical collapse rate).

If I am not mistaken, in order to make the argument fully general one must formulate it in terms of mass densities and of Ghirardi, Grassi and Benatti's (1995) objectivity criterion. If one does so, then, given the collapse mechanism, and except for times shorter than collapse times, it follows that macroscopic mass densities defined on all hypersurfaces through $\mathbf{x}$ will be objective and approximately coincide. This applies equally to strategies (b) and (c).

An alternative criterion for property ascription in the relativistic case has been proposed by Ghirardi (2000): a local property at $\mathbf{x}$ is objective iff it is given probability 1 by the state on the past light cone of $\mathbf{x}$. This form of the criterion has been championed also by Myrvold (2003). This now gives us an alternative way of defining Schrödinger-like mass densities also in the case of strategy (c), namely to take the mass density to be defined by (the limiting case of) the state on the past light cone. Provided this is well-defined (a question which is presumably related to, though separate from, that of integrability), it provides the theory with a unique local beable.

Also in this case, we wish to argue that a dualist ontology collapses to a wave-only ontology, because the arguments given in Section 4.2 apply equally well to EEL and other criteria of property ascription given (unique) non-relativistic quantum states as to the generalisation of EEL and other criteria of property ascription to the case of (unique) states on the past light cone.

As a closing remark, note that in this last case (and at least approximately, also in the case of properties common to all hypersurfaces or hyperplanes), the local mass density is exactly the same as the unique mass density in the case of collapse along the forward light cone of strategy (a). That is, the local mass density evolves as if the wave collapsed along the future light cone, so that these different approaches to relativistic collapse will presumably be experimentally indistinguishable and perhaps not so different after all.[28]

[28]This blurring of the different approaches is even more marked in the (no-collapse) Everett interpretation. Indeed, in this interpretation the differences in the approaches to "relativistic branching" are purely conventional. These approaches are the branching along arbitrary hypersurfaces favoured by Myrvold, and the branching along the forward light cone proposed by Bacciagaluppi (2002), while many-minds variants do not invoke

5 Schrödinger and Everett

We have argued above in Section 3.2 that there are some deep parallels between collapse theories of the GRW type and beable theories in Bell's sense. Thus, the distinction between these two approaches, which seemed very marked as sketched in Section 2, becomes somewhat blurred. We now wish to return, independently of collapse theories, to Schrödinger's own approach to interpreting quantum mechanics, and perhaps blur some further distinctions.

In August 1935, Schrödinger, describing to Einstein his "cat" experiment, had written explicitly: "I am long past the stage where I thought that one can consider the ψ-function as somehow a direct description of reality" (quoted in Fine 1986, p. 82). And yet, later on Schrödinger returned to emphasising the idea of waves and continuous wave-like processes as providing the best candidate for an interpretation of quantum mechanics.

Indeed, this is the position he advances in his famous article "Are there quantum jumps?" (Schrödinger 1952) and in a closely related colloquium given in Dublin in July 1952, published posthumously by Michel Bitbol (Schrödinger 1995, Chap. 1). In the latter, Schrödinger is even more explicit about his aims than in the better-known paper:

> Let me say at the outset, that [...] I am opposing [the] basic views [of quantum mechanics] that have been shaped 25 years ago, when Max Born put forward his probability interpretation [...] The view I am opposing is so widely accepted, without ever being questioned, that I would have some difficulties in making you believe that I really consider it inadequate and wish to abandon it. It is, as I said, the probability view of quantum mechanics. (Schrödinger 1995, p. 19)

In both the paper and the colloquium, Schrödinger goes on to discuss prominent examples of discontinuous and particle-like behaviour in quantum mechanics and to explain them in purely continuous wave-theoretic terms.

It is evident that, despite the qualms in the years leading up to 1935, there is continuity between Schrödinger's thinking of the 1920s and of the 1950s.

---

global branching at all (see e.g. the reviews in Bacciagaluppi 2002 and 2003).

Indeed, many of the examples he gives in 1952 he had already described in papers from the 1920s, in particular his discussion of energy exchange in resonant systems (Section 3 in both paper and colloquium), which follows his well-known paper on the subject from just before the Solvay conference (Schrödinger 1927), and his discussion of the photoelectric effect (Section 5 in both paper and colloquium), which follows one given many years earlier in *Die Naturwissenschaften* (Schrödinger 1929).[29]

It is also rather clear that Schrödinger's criticism is directed as well against "quantum jumps" in their original 1920s incarnation as it is against any form of discontinuous evolution of the wave function, or even discontinuous guidance of beables, so that his approach of choice would presumably not have been to interpret a collapsing wave function in terms of charge or mass density, as we have described in Section 4, nor a beable theory in Bell's sense. It is less clear what Schrödinger's approach of choice actually is, but if one leaves historical scruples aside, it is not too difficult to imagine a position that is quite consistent both with Schrödinger's pronouncements in 1952 and with his main interpretive ideas in the 1920s, as follows.

Let us return to Schrödinger's charge-density interpretation (or rather, its mass-density variant) and apply it to a Schrödinger-cat situation. The straightforward application of Schrödinger's definition of mass density yields a weighted average of the mass densities corresponding to the live and the dead cat. This is clearly not a "classical" mass density distribution. It is yet another little-known fact, however,[30] that in this regime, because of decoherence, the two components of the mass density corresponding to the live and dead cat evolve *separately* as two classical mass density distributions would. One half of the total mass (that which is distributed as a live cat) could stand up and walk straight across the other half (that which is distributed as a dead cat), without noticing in the least that it is there. While this is again obviously not the behaviour of a classical mass density (note that it violates Ghirardi, Grassi and Benatti's "objectivity" criterion), it can in fact be described (as we just have) as corresponding to two mass density distributions moving in the same three-dimensional space, both oblivious to each other. If we renormalise the apparent mass, we get, as the three-dimensional manifestation of the wave function, two completely independent copies of the

[29] A slightly more detailed discussion of these earlier papers can be found in Bacciagaluppi and Valentini (2009, Section 4.6.3).

[30] As pointed out to me by Ward Struyve, the only other authors who appear to be aware of this, and indeed discuss it at some length, are Allori *et al.* (2008).

cat, one of which has died, while the other one perhaps continues undeterred to play with the phial of poison.

It is worth emphasising that the three-dimensional manifestation of the wave function is purely field-like, whether or not one has a Schrödinger-cat situation, so that any appearance of particulate or macroscopic objects has to be extracted from the field *a posteriori* in terms of stable patterns. In passing, we remark that Bell's original proposal of interpreting the GRW hits as local beables also relies on the extraction of patterns from the wave function, or, more specifically, from its evolution: recall, indeed, that Bell writes that the GRW hits "are part of the wavefunction, not something else".[31]

The picture of the two cats, while surely an overinterpretation of Schrödinger's own views, may resonate with modern Everettians.[32] In fact, if the above remarks on mass density as the three-dimensional manifestation of the Hilbert-space wave function are correct, then this picture just *is* the Everett interpretation. The difference to the usual presentations in terms of Hilbert-space components of the wave function that constitute "worlds", is merely that here the emphasis is on the three-dimensional manifestation of the different components of the wave function.

As remarked by Bitbol (Schrödinger 1995, p. 17), it also resonates with some of Schrödinger's own words (with the appropriate qualifications and warnings about overinterpretation):[33]

> Nearly every result [a quantum theorist] pronounces is about the probability of this *or* that or that... happening — with usually a great many alternatives. The idea that they be not alternatives but *all* really happen simultaneously seems lunatic to him, just *impossible*. He thinks that if the laws of nature took *this* form for, let us say, a quarter of an hour, we should find our surroundings rapidly turning into a quagmire, or sort of a featureless jelly or plasma, all contours becoming blurred, we ourselves probably becoming jelly fish. It is strange that he should believe this. (Schrödinger 1995, p. 19)

[31] We conclude from this that Bell would not have endorsed a dualist ontology, nor going as far as considering the POV beable as guided by a *non*-collapsing wave function.

[32] That is, with decoherence-based Everettians. See e.g. Bacciagaluppi (2003) and references therein, particularly Saunders (1993) and Wallace (2003).

[33] I wish to thank Antony Valentini for first pointing out to me the existence of such quasi-Everettian passages in Schrödinger's writings.

References

AHARONOV, Y., ALBERT, D. "Is the usual notion of time evolution adequate for quantum-mechanical systems? II. Relativistic considerations". Physical Review D, 29, pp. 228–234, 1984.

ALLORI, V., GOLDSTEIN, S., TUMULKA, R., ZANGHÌ, N. "On the common structure of Bohmian mechanics and the Ghirardi-Rimini-Weber theory". British Journal for the Philosophy of Science, 59, pp. 353–389, 2008.

BACCIAGALUPPI, G. "Nelsonian mechanics revisited". Foundations of Physics Letters, 12, pp. 1–16, 1999.

BACCIAGALUPPI, G. "Remarks on space-time and locality in Everett's interpretation". In Placek and Butterfield (2002), pp. 105–122.

BACCIAGALUPPI, G. "The role of decoherence in quantum mechanics". In Zalta (1995–), 2003. Winter 2003 version, http://plato.stanford.edu/archives/win2003/entries/qm-decoherence/ . Dynamic version, http://plato.stanford.edu/entries/qm-decoherence/ .

BACCIAGALUPPI, G., VALENTINI, A. Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference. Cambridge: CUP, 2009. (Page references to the draft at http://arxiv.org/abs/quant-ph/0609184 .)

BELL, J. S. "Beables for quantum field theory". CERN-TH, 4035/84, 1984. Reprinted in Bell (1987b), pp. 173–180.

BELL, J. S. "Are there quantum jumps?". In Schrödinger: Centenary of a Polymath. Cambridge: CUP, 1987a. Reprinted in Bell (1987b), pp. 201–212.

BELL, J. S. Speakable and Unspeakable in Quantum Mechanics. Cambridge: CUP, 1987b.

BOHM, D. "A suggested interpretation of the quantum theory in terms of 'hidden' variables, I and II". Physical Review, 85, pp. 166–179, 180–193, 1952.

DE BROGLIE, L. "La nouvelle dynamique des quanta". In Électrons et photons. Rapports et discussions du cinquième conseil de physique Solvay.

Paris: Gauthier-Villars, 1928. Translated in Bacciagaluppi and Valentini (2009), pp. 374–407.

BUB, J. Interpreting the Quantum World. Cambridge: CUP, 1997.

BUSCH, P., GRABOWSKI, M., LAHTI, P. Operational Quantum Physics. Lecture Notes in Physics, Vol. m31. Berlin: Springer, 1995. (2nd corrected printing, 1997.)

CLIFTON, R. K. (ed.). Perspectives on Quantum Reality. Dordrecht: Kluwer, 1996.

DEOTTO, E., GHIRARDI, G.C. "Bohmian mechanics revisited". Foundations of Physics, 28, pp. 1–30, 1998.

DIÓSI, L. "Contnuous quantum measurement and Itô formalism". Physics Letters A, 129, pp. 419–423, 1988.

DOEBNER, H.-D., GOLDIN, G. A. "Introducing nonlinear gauge transformations in a family of nonlinear Schrödinger equations". Physical Review A, 54, pp. 3764–3771, 1996.

DOWKER, F., HENSON, J. "A spontaneous collapse model on a lattice". Journal of Statistical Physics, 115, pp. 1327–1339, 2004.

DOWKER, F., HERBAUTS, I. "Simulating causal collapse models". Classical and Quantum Gravity, 21, pp. 2963–2980, 2004.

DOWKER, F., HERBAUTS, I. "The status of the wave function in dynamical collapse models". Foundations of Physics Letters, 18, pp. 499–518, 2005.

FELLER, W. "On the integro-differential equations of purely discontinuous Markoff processes". Transactions of the American Mathematical Society, 48, pp. 488–515, 1940. "Errata". *Ibid.*, 58, p. 474, 1945.

FINE, A. The Shaky Game: Einstein, Realism, and the Quantum Theory. Chicago: University of Chicago Press, 1986.

FLEMING, G. "A manifestly covariant description of arbitrary dynamical variables in relativistic quantum mechanics". Journal of Mathematical Physics, 7, 1959–1981, 1966.

FLEMING, G. “Just how radical is hyperplane dependence?”. In Clifton (1996), pp 11–28.

GAMBETTA, J., WISEMAN, H. “Modal dynamics for positive operator measures”. Foundations of Physics, 34, pp. 419–448, 2004.

GARCÍA DE POLAVIEJA, G. “A causal quantum theory in phase space”, Physics Letters A, 220, pp. 303–314, 1996.

GHIRARDI, G.C. “Local measurements of nonlocal observables and the relativistic reduction process”. Foundations of Physics, 30, pp. 1337–1385, 2000.

GHIRARDI, G.C. “Collapse theories”. In Zalta (1995–), 2002. Spring 2002 version, http://plato.stanford.edu/archives/spr2003/entries/qm-collapse/ . Fall 2008 version,
http://plato.stanford.edu/archives/fall2008/entries/qm-collapse/ . Dynamic version, http://plato.stanford.edu/entries/qm-collapse/ .

GHIRARDI, G.C. “Some reflections inspired by my research activity in quantum mechanics”. Journal of Physics A: Mathematical and Theoretical, 40, 2891–2917, 2007.

GHIRARDI, G.C., GRASSI, R. “Outcome predictions and property attribution: the EPR argument reconsidered”. Studies in History and Philosophy of Science, 25, pp. 397–423, 1994.

GHIRARDI, G.C., GRASSI, R., BENATTI, F. “Describing the macroscopic world: closing the circle within the dynamical reduction program”. Foundations of Physics, 25, pp. 5–38, 1995.

GHIRARDI, G.C., GRASSI, R., PEARLE, P. “Relativistic dynamical reduction models: general framework and examples”. Foundations of Physics, 20, pp. 1271–1316, 1990.

GHIRARDI, G.C., PEARLE, P., RIMINI, A. “Markov processes in Hilbert space and continuous spontaneous localization of systems of identical particles”. Physical Review A, 42, pp. 78–89, 1990.

GHIRARDI, G.C., RIMINI, A., WEBER, T. “Unified dynamics for microscopic and macroscopic systems”. Physical Review D, 34, pp. 470–479, 1986.

HELLWIG, K.-E., KRAUS, K. “Formal description of measurements in local quantum field theory”. Physical Review D, 1, pp. 566–571, 1970.

KENT, A. “‘Quantum jumps’ and indistinguishability”. Modern Physics Letters A, 4, pp. 1839–1845, 1989.

MILLER, A. (ed.) Sixty-Two Years of Uncertainty. New York: Plenum, 1990.

MYRVOLD, W. “On peaceful coexistence: is the collapse postulate incompatible with relativity?”. Studies in the History and Philosophy of Modern Physics, 33, pp. 435–466, 2002.

MYRVOLD, W. “Relativistic quantum becoming”. British Journal for the Philosophy of Science, 54, pp. 475–500, 2003.

NELSON, E. “Derivation of the Schrödinger equation from Newtonian mechanics”. Physical Review, 150, pp. 1079–1085, 1966.

VON NEUMANN, J. Die mathematischen Grundlagen der Quantenmechanik. Berlin: Springer, 1932.

NICROSINI, O., RIMINI, A. “On the relationship between continuous and discontinuous stochastic processes in Hilbert space”. Foundations of Physics, 20, pp. 1317–1327, 1990.

NICROSINI, O., RIMINI, A. “Relativistic spontaneous localization: a proposal”. Foundations of Physics, 33, pp. 1061–1084, 2003.

PEARLE, P. “Combining stochastic dynamical state-vector reduction with spontaneous localization”. Physical Review A, 39, pp. 2277–2289, 1989.

PEARLE, P. “Toward a relativistic theory of statevector reduction’. In Miller (1990), pp. 193–214, 1990.

PEARLE, P. “Relativistic collapse model with tachyonic features”. Physical Review A, 59, pp. 80–101, 1999.

PEARLE, P. “Quasirelativistic quasilocal finite wave-function collapse model”. Physical Review A, 71, pp. 032101-1–032101-8, 2005.

PERCIVAL, I. Quantum State Diffusion. Cambridge: CUP, 1998.

PERUZZI, G., RIMINI, A. “Quantum measurement in a family of hidden-variable theories". Foundations of Physics Letters, 9, pp. 505–519, 1996.

PLACEK, T., BUTTERFIELD, J. (eds.) Non-locality and Modality. NATO Science Series, II. Mathematics, Physics and Chemistry, Vol. 64. Dordrecht: Kluwer, 2002.

PRZIBRAM, K. (ed.) Briefe zur Wellenmechanik. Wien: Springer, 1963. Translated as: Letters on Wave Mechanics. New York: Philosophical Library, 1967 (page references to this edition).

SAUNDERS, S. "Decoherence, relative states, and evolutionary adaptation". Foundations of Physics, 23, pp. 1553–1585, 1993.

SCHRÖDINGER, E. "Quantisierung als Eigenwertproblem (dritte Mitteilung: Störungstheorie, mit Anwendung auf den Starkeffekt der Balmerlinien)". Annalen der Physik, 80, pp. 437–490, 1926a.

SCHRÖDINGER, E. "Quantisierung als Eigenwertproblem (vierte Mitteilung)". Annalen der Physik, 81, pp. 109–139, 1926b.

SCHRÖDINGER, E. "Energieaustausch nach der Wellenmechanik". Annalen der Physik, 83, pp. 956–968, 1927.

SCHRÖDINGER, E. "Mécanique des ondes [Wellenmechanik]". In Électrons et photons. Rapports et discussions du cinquième conseil de physique Solvay. Paris: Gauthier-Villars, 1928. Translated (from the original German) in Bacciagaluppi and Valentini (2009), pp. 448–476.

SCHRÖDINGER, E. "Die Erfassung der Quantengesetze durch kontinuierliche Funktionen". Die Naturwissenschaften, 13, pp. 486–489, 1929.

SCHRÖDINGER, E. "Are there quantum jumps? Parts I and II". British Journal for the Philosophy of Science, 3, pp. 109–123 and 233–242, 1952.

SCHRÖDINGER, E. The Interpretation of Quantum Mechanics. Edited and with an introduction by M. Bitbol. Woodbridge, Connecticut: Ox Bow Press, 1995.

SHIMONY, A. "Events and processes in the quantum world". In R. Penrose and C. Isham (eds.) Quantum Concepts in Space and Time. Oxford: Clarendon Press, 1986. Reprinted in Shimony (1993), pp. 140–162.

SHIMONY, A. "Desiderata for a modified quantum dynamics". In A. Fine,

M. Forbes and L. Wessels (eds.) PSA 1990, Vol. 2. East Lansing, Michigan: PSA, 1990. Reprinted in Shimony (1993), pp. 55–67.

SHIMONY, A. Search for a Naturalistic World View, Vol. II. Cambridge: CUP, 1993.

SUDBERY, A. “Objective interpretations of quantum mechanics and the possibility of a deterministic limit”. Journal of Physics A: Mathematical and General, 20, pp. 1743–1750, 1987.

TUMULKA, R. “A relativistic version of the Ghirardi-Rimini-Weber model”. Journal of Statistical Physics, 125, pp. 825–844, 2006.

TUMULKA, R., GEORGII, H.-O. “Global existence of Bell’s time-inhomogeneous jump process for lattice quantum field theory”. Markov Processes and Related Fields, 11, pp. 1–18, 2005.

VINK, J. “Quantum mechanics in terms of discrete beables”. Physical Review A, 48, pp. 1808–1818, 1993.

WALLACE, D. “Everett and Structure”. Studies in History and Philosophy of Modern Physics, 34 B, pp. 87–105, 2003.

ZALTA, E. N. (ed.) The Stanford Encyclopedia of Philosophy. Stanford: Metaphysics Research Lab, CSLI, Stanford University.
http://plato.stanford.edu/ , 1995–.