Communication impacting financial markets

Jørgen Vitting Andersen, Ioannis Vrontos, Petros Dellaportas, Serge Galam

– PANTHÉON - SORBONNE –
UNIVERSITÉ PARIS 1

Documents de Travail du
Centre d'Economie de la Sorbonne

CES Working Papers

Communication impacting financial markets

Jørgen Vitting ANDERSEN, Ioannis VRONTOS
Petros DELLAPORTAS, Serge GALAM

2014.29

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://centredeconomiesorbonne.univ-paris1.fr/
ISSN : 1955-611X

Communication impacting financial markets

Jørgen Vitting Andersen[1], Ioannis Vrontos[2], Petros Dellaportas[2] and Serge Galam[3]

[1] *CNRS, Centre d´Economie de la Sorbonne,*

Université Paris 1 Panthéon-Sorbonne,

Maison des Sciences Economiques,

106-112 Boulevard de lHôpital 75647 Paris Cedex 13, France

*

[2]*Department of Statistics*

Athens University of Economics and Business

Patission 76,104 34 Athens, Greece

and

[3]*Cevipof - Center for Political Reseach,*

Sciences Po and CNRS,

98 rue de l'Université, 75007 Paris, France

Background: Since the attribution of the Nobel prize in 2002 to Kahneman for prospect theory, behavioral finance has become an increasingly important subfield of finance. However the main parts of behavioral finance, prospect theory included, understand financial markets through *individual* investment behavior. Behavioral finance thereby ignores any interaction between participants.

Methodology: We introduce a socio-financial model that studies the impact of communication on the pricing in financial markets. Considering the simplest possible case where each market participant has either a positive (bullish) or negative (bearish) sentiment with respect to the market, we model the evolution of the sentiment in the population due to communication in subgroups of different sizes. Nonlinear feedback effects between the market performance and changes in sentiments are taking into account by assuming that the market performance is dependent on changes in sentiments (e.g. a large sudden positive change in bullishness would lead to more buying). The market performance in turn has an impact on the sentiment through the transition probabilities to change an opinion in a group of a given size. The idea is that if for example the market has observed a recent downturn, it will be easier for even a bearish minority to convince a bullish majority to change opinion compared to the case where the meeting takes place in a bullish upturn of the market.

Conclusions: Within the framework of our proposed model, financial markets stylized facts such as volatility clustering and extreme events may be perceived as arising due to abrupt sentiment changes via ongoing communication of the market participants. The model introduces a new volatility measure which is apt of capturing volatility clustering and from maximum likelihood analysis we are able to apply the model to real data and give additional long term insight into where a market is heading.

INTRODUCTION

The quote "In the short run, the market is a voting machine, but in the long run it is a weighing machine" is attributed to Benjamin Graham[2]. Graham himself used the quote to argue that investors should use the so-called fundamental value investment approach and concentrate on analyzing accurately the worth of a given financial asset. That is, he suggested ignoring the short run "voting machine" part, and instead concentrate on the long run, where somehow the "weighing machine"of the market would ensure to end up with a pricing of the true worth of an asset. The interesting part of Graham's quote is the allusion to the decision making of people and its impact on the markets. Graham however does not refer to how the decision making process actually takes places, and this will be our focus in the following.

In situations of uncertainty people often consult others to get more information and thereby a better understanding of the situation. This is in particular true with respect to financial markets, where market participants consult the media or other colleagues to get an idea of the origin behind price movements, or to assess which impact a given information could have on the markets. For example, copying the behavior of others is one of the predominant mechanisms in decisions to purchase[3]. Social households, i.e. those that interact with their neighbors, have also been shown to have an impact in the level of stock market investments. In [4] it was shown how the stock-market participation rates were higher in places that had a higher sociability. However the sole knowledge of the role of imitation is not sufficient to describe in an operative frame the price dynamics of financial markets since indeed the rules governing opinion dynamics and group decisions still needs to be understood and identified in more detail. In [5] interpersonal communication were found to be extremely important in investors decisions. Questionnaire surveys of institutional and individual investors were found to reveal a strong influence by word-of-mouth communications. In [6] it was shown that among fund managers in Germany information exchange with other financial and industry experts was the second most important factor influencing their investment decisions, complemented by conversations with their colleagues and reports from media. More recently impact of large scale human network structes and its flow of information were shown to have an impact on the asset pricing[7]. Communication and opinion formation among market participants therefore seems to be an important ingredient in the price formation of any financial market, the question however is how to quantify a general framework of such a scenario at the micro level? In the following we will introduce a socio-financial[1] model that in a quantitative manner does exactly this.

ANALYSIS

Insert Figure 1 about here

Consider a population of market participants, shown schematically as circles in Fig. 1A. We will proceed as in the so-called Galam model of opinion formation[8–10] and for simplicity imagine that people have just two different opinions on the market, which we can characterize as either "'bullish"' (black circles) or "bearish" (white circles). Letting $B(t)$ denote the proportion of bullishness in a population at time t, the proportion of bearishness is then $1 - B(t)$.

Figure 1A represents the opinions of the participants at the beginning of a given day. During the day people meet in random subgroups of different sizes, as illustrated by the different boxes in Fig. 1B, to update their view of the market. Take, for example, the leftmost box in Fig. 1B with six persons, two bullish, four bearish, who we can imagine are sitting around a table, or having a conference call, discussing the latest market developments. The outcome of the discussions for the different groups are illustrated in Fig. 1C. For simplicity we have illustrated the case where a majority opinion in a given subgroup manages to polarize the opinion of the group by changing the opinion of those who had an opinion belonging to the minority. If we take the afore mentioned group of six persons we can see that after discussing, because of the majority polarizing rule, they have all become bearish. More realistically, we will in the following instead assume that is a certain *probability* for a majority opinion to prevail, and that even under certain conditions a minority could persuade a part of the majority to change their opinion.

For a given group of size k with j agents having a bullish opinion and $k - j$ a bearish opinion, we let $m_{k,j}$ denote the transition probability for all (k) members to adopt the bullish opinion as a result of their meeting. After one update taking into account communications in all groups of size k with j bullish agents, the new probability of finding an agent with a bullish view in the population can therefore be written:

$$B(t+1) = m_{k,j}(t) C_j^k B(t)^j [1 - B(t)]^{k-j} \tag{1}$$

where

$$C_j^k \equiv \frac{k!}{j!(k-j)!} \tag{2}$$

are the binomial coefficients. Notice that the transition probabilities $m_{k,j}$ depend on time, since we assume that they change as the market performance changes (this point will be explained further below).

Taking the sum over different groups of different sizes and different composition of bullishness within each group(see Fig. 1B) one obtains a general term, $B(t+1)$, for the bullishness in a population at time $t+1$ due to the outcome of meetings of groups with different sizes and different composition of bullishness:

$$B(t+1) = \sum_{k=1}^{L} a_k \sum_{j=0}^{k} m_{k,j}(t) C_j^k B(t)^j [1 - B(t)]^{k-j} \quad (3)$$

$$\sum_{k=1}^{L} a_k = 1 \quad ; \quad a_k \equiv \frac{1}{L} \quad (4)$$

With L denoting the size of the largest group and a_k denoting the weight of the group of size k. The link between communication and its impact on the markets can then be taken into account by assuming that the price return $r(t)$ changes whenever there is a change in the bullishness. The idea is that the bullishness itself is not the relevant factor determining how prices will change. Those feeling bullish would naturally already hold long positions on the market. Rather, when people change their opinion, say becoming more negative about the market, or less bullish, this will increase their tendency to sell. Assuming the return to be proportional to the percentage change in bullishness, $RB(t)$, as well as economic news, $\eta(t)$, the return $r(t)$ is given by:

$$r(t) = \frac{RB(t)}{\lambda} + \eta(t), \quad \lambda > 0 \quad (5)$$

with $RB(t) = \frac{B(t)-B(t-1)}{B(t-1)}$ the change or "return" of the bullishness. The variable $\eta(t) = r(t) - \frac{RB(t)}{\lambda}$ is assumed to be Gaussian distributed with a mean $\mu \equiv 0$ and a standard deviation that varies as a function of time depending on changes in sentiment. We will assume that the market will react to fundamental economic news represented by η but that the amplitude of the reaction depends on changes in the sentiment $RB(t)$:

$$\sigma(t) = \sigma_0 \exp\left(\frac{|RB(t)|}{\beta}\right), \quad \sigma_0 > 0, \quad \beta > 0 \quad (6)$$

The influence of the financial market on decision-making can now be included in a natural way by letting the strength of persuasion depend on how the market has performed since the last meeting of the market participants. The idea is that, if for example the market had a dramatic downturn at the close yesterday, then in meetings the next morning, those with a bearish view will be more likely to convince even a bullish majority of their point of view. In the formal description below, this is taken into account by letting the transition probabilities for a change of opinion, i.e., the probabilities of transitions like Fig. 1.B $\rightarrow$ Fig. 1.C, depend on the market return over the last period:

$$m_{k,j}(t) = m_{k,j}(t-1) \exp\left(\frac{r(t)}{\alpha}\right); m_{k,j}(t=0) \equiv j/k, \quad \alpha > 0 \quad (7)$$

where α defines the scale for which a given return $r(t)$ impacts the transition probabilities. The condition $m_{k,j}(t = 0) \equiv j/k$ describes the initially unbiased case where in average no market participant changes opinion.

RESULTS

As a first demonstration of the properties of the model, Eqs. (3-7), Figure 2a shows an illustration of the link between the bullishness of the market participants obtained through communication (thin solid line) and market prices (thick solid line). One observes a clear almost continuous decrease in bullishness, whereas the market itself first decreases but then regains what it lost at the end of the time period, essentially ending up unchanged compared to its initial level. This illustrates the competition between persistence through memory effects in the bullishness (1-7) and the randomness of the return via the $\eta(t)$-term in (5). The decaying rate of the return of the bullishness over the last part of the time period and the simultaneous rise of the prices illustrates indeed the complex and highly nonlinear relationship between returns and sentiments. The model is able to reproduce the most important of the so-called stylized facts seen in real financial market data[11]. Figure 2b illustrates clustering of the volatility of the return of the time series in Fig. 2a. Furthermore the returns show fat-tailed behavior in the returns (Fig. 2c), with fat tail exponents similar to what is found in real markets[12]. Finally let us also mention that there are no arbitrage possibilities within the framework of the model seen by the zero autocorrelations of returns (thin line, Fig. 2d), whereas long memory effects are seen in the volatility, seen via the slowly decaying autocorrelations of volatility (thick line, Fig. 2d).

Insert Figure 2 about here

Having verified that the model is indeed able to describe the main characteristics of real market data, the next question concerns how to find the optimal parameters describing real market data. Using inference based on maximum likelihood (see appendix) we have first conducted a series of simulations in order to retrieve the parameter values used to generate the simulated data (Figure 2), namely $\lambda = 1.1$, $\sigma_0 = 0.01$, $\beta = 0.001$ and $a = 400$.

In order to avoid the positivity restrictions for the parameters, $\lambda > 0$, $\sigma_0 > 0$, $\beta > 0$ and $\alpha > 0$ we use the logarithmic transformation, so that $\lambda^* = \ln(\lambda)$, $\sigma_0^* = \ln\left(\sigma_0^2\right)$, $\beta^* = \ln(\beta)$ and $\alpha^* = \ln(\alpha)$. Thus the parameter vector to estimate is $\theta^* = (\lambda^*,\ \sigma_0^*,\ \beta^*,\ \alpha^*)'$. The technique of reparameterizing the model parameters is very useful in order to ensure that a numerical optimization algorithm always provides parameter values within certain specified boundaries. An attractive feature of the method of maximum likelihood estimation is its invariance to one-to-one transformations of the parameters of the log-likelihood. That is, the maximum likelihood solution is invariant under transformation of parameters. Finally, due to the highly non-linear nature of the proposed model, maximization of the conditional log-likelihood with respect to the model parameters is achieved by using numerical optimization algorithms.

Insert Table 1 about here

Different starting values of the model parameters were used to ensure that the maximization/minimization algorithm converges to the true simulated parameters. We present one example of optimization using $T = 5000$ data points (see Table 1) noting that other values of T gave similar results. The first column of Table 1 shows the 'true' parameter values $\theta = (\lambda,\, \sigma_0,\, \beta,\, \alpha)'$ used to simulate the data, while column 2 presents the 'true' transformed parameter values $\theta^* = (\lambda^*, \sigma_0^*, \beta^*, \alpha^*)'$. The maximum likelihood estimates $\widehat{\theta}^*$ of the transformed parameters θ^* and their standard errors are presented in column 3, while column 4 shows the corresponding estimated parameter values $\widehat{\theta}$. Finally, in the last column we present the value of the gradient of the log-likelihood function evaluated at the parameter estimates $\widehat{\theta}^*$. Different starting values for the model parameters have been used in the maximum likelihood estimation procedure (see Panel A-C). For example, the starting values used in Panel C are the 'true' parameter values used in the simulation experiments. The results presented in Table 1 indicate that the maximum likelihood inferential procedure provide estimates that are close to the 'true' parameter values, taking into account the estimates and the corresponding standard errors. The result provide evidence of convergence of the maximization/minimization algorithm since the estimates found by using different starting values are very similar, and the value of the gradient of the log-likelihood evaluated at the parameter estimates is near zero.

Insert Figure 3 about here

Next, we examine whether the proposed algorithm can estimate adequately the proportion of bullishness across time and the time-varying conditional volatilities. To this end, in Figure 3, we present the simulated prices $P(t)$, and the corresponding returns $r(t)$, as well as the simulated bullishness proportions $B(t)$ across time, for $T = 5000$ data points. We also present the estimated bullishness proportions $\widehat{B}(t)$ and the estimated conditional volatilities $\widehat{\sigma}(t)$ which are based on the parameter estimates of the model. Looking at Figure 3(b) and Figure 3(d) which illustrate the 'true' and the estimated bullishness proportions, respectively, we can observe that these proportions are almost identical. Thus, the proposed optimization procedure manages to correctly identify and detect the proportions of a bullish view on the simulated prices. Finally, looking at Figure 3(c) and Figure 3(e), which illustrate the simulated returns and the estimated conditional volatilities, respectively, we can see that periods of lower or higher deviation in the return series can be detected by the models volatility estimates. Therefore, the proposed model is able to capture the volatility clustering phenomenon of the return series.

Application to the FTSE-20 Athens stock index: We present an empirical application of the proposed socio-financial model to the FTSE-20 Athens stock exchange index. The idea is to consider a very volatile market to study abrupt and large changes in market performance and map the corresponding evolution in sentiments. The data consists of 1247 daily prices over the 7/11/2008-6/11/2013 period. We compute and analyse the returns of the FTSE-20 index.

Insert Table 2 about here

Insert Figure 4 about here

To estimate the model parameters we apply the maximum likelihood method as mentioned above. Table 2 presents the estimates $\widehat{\theta}^*$ of the transformed parameters θ^* and their standard errors (columns 1 and 2, respectively), the corresponding estimated parameter values $\widehat{\theta}$ (column 3), and the value of the gradient of the log-likelihood function evaluated at the parameter estimates $\widehat{\theta}^*$ (last column). Different starting values for the model parameters been used in the maximum likelihood estimation procedure (see Panel A-C). These results provide evidence of convergence of the maximization algorithm since the estimates found by using different starting values are very similar, and the value of the gradient of the log-likelihood evaluated at the parameter estimates is near zero.

Next, we estimate the proportion of bullishness across time and the time-varying conditional volatilities. In Figure 4, we present the FTSE-20 prices $P(t)$, the corresponding returns $r(t)$, the estimated bullishness proportions $\widehat{B}(t)$ and the estimated conditional volatilities $\widehat{\sigma}(t)$, which are based on the parameter estimates of the socio-financial model. Comparing the FTSE-20 price evolution Figure 4(a) and the estimated bullishness proportions Figure 4c, one observes a steady decline in the bullishness proportions even after a relatively long (two years: from mid-2011 to mid-2013) and stable price evolution. We consequently suggest to consider this as indicating a 'bad signal" for the following FTSE-20 price levels. Finally, comparing the observed volatility of the FTSE-20, Figure 4(b), and the estimated volatility from the model, Figure 4(d), one notices that periods of high volatility of the market is indeed detected by volatility estimates of the socio-financial model. Therefore our proposed model can be seen as introducing an alternative measure to capture volatility clustering phenomena observed in return series of financial market data.

DISCUSSION

The main parts of traditional finance as well as behavioral finance, prospect theory[13] included, understand pricing in financial markets through *individual* investment behavior, thereby ignoring any interaction between market participants. We have on the contrary proposed a socio-financial model that focus on interaction between market participants via communication. We have shown how communication can impact the pricing in financial markets. Opinion formation of the market participants is modeled by communication that takes place in subgroups of different sizes. Nonlinear feedback effects were introduced to describe how market performance can change sentiments and vice versa. Our socio-financial model was shown to be able to reproduce the main stylized facts of financial market data. By estimating the model parameters via maximum likelihood method we are able to study extreme behavior of markets swings and sentiments illustrated by a case of the FTSE-20 Athens stock index over the last five years. Recent price behavior of the FTSE-20 index seems to indicate a reversal of the bearish trend whereas the sentiment measure obtained from our method seems to indicate that the market still has room for further downturns. Finally our model introduces a new volatility measure apt of capturing volatility clustering seen in the real financial market data

APPENDIX

In the following we present an inferential method adopted to estimate the parameters of the model (3-7) based on the maximization of the conditional likelihood. Below we describe the calculation of the likelihood function for the model for a sample of T observations $r = (r(1), r(2), ..., r(T))$ under the assumption of a Normal distribution for the error process $\eta(t)$.

Consider the probability distribution of $r(1)$, the first observation in the sample. Since $\eta(t)$ is assumed Gaussian, the density of the first observation, conditional on $B(0)$ and $m_{k,j}(0) = j/k$ (corresponding to the neutral case), takes the form

$$f\left[r(1)|B(0), m_{k,j}(0), \theta\right] = \frac{1}{\sqrt{2\pi}\sigma(1)} \exp\left\{-\frac{1}{2\sigma(1)^2}\left[r(1) - \frac{1}{\lambda}RB(1)\right]^2\right\},$$

where $\theta = (\lambda, \sigma_0, \beta, \alpha)'$ denotes the parameter vector to be estimated. Next, conditioning on $r(1)$, the density of the second observation $r(2)$ is

$$f\left[r(2)|r(1), B(0), m_{k,j}(0), \theta\right] = \frac{1}{\sqrt{2\pi}\sigma(2)} \exp\left\{-\frac{1}{2\sigma(2)^2}\left[r(2) - \frac{1}{\lambda}RB(2)\right]^2\right\}.$$

Proceeding in this fashion, the conditional density of the $t - th$ observation can be calculated as

$$f\left[r(t)|r(t-1), ..., r(1), B(0), m_{k,j}(0), \theta\right] = \frac{1}{\sqrt{2\pi}\sigma(t)} \exp\left\{-\frac{1}{2\sigma(t)^2}\left[r(t) - \frac{1}{\lambda}RB(t)\right]^2\right\}.$$

Therefore, the likelihood of the complete sample can be written as

$$f\left[r|B(0), m_{k,j}(0), \theta\right] = (2\pi)^{-\frac{T}{2}} \prod_{t=1}^{T} \sigma(t)^{-1} \cdot \exp\left\{-\frac{1}{2}\sum_{t=1}^{T} \frac{1}{\sigma(t)^2}\left[r(t) - \frac{1}{\lambda}RB(t)\right]^2\right\}.$$

The log-likelihood function, denoted $L_T\left(r|\theta\right)$, can be written as

$$L_T\left(r|\theta\right) = -\frac{T}{2}\ln\left(2\pi\right) - \sum_{t=1}^{T} \ln\sigma(t) - \frac{1}{2}\sum_{t=1}^{T} \frac{1}{\sigma(t)^2}\left[r(t) - \frac{1}{\lambda}RB(t)\right]^2.$$

Clearly, the value of θ that maximizes the conditional likelihood is identical to the value that maximizes the conditional log-likelihood.

ACKNOWLEDGMENTS

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7-SSH/2007-2013) under grant agreement n 320270 SYRTO.

* Electronic address: `Jorgen-Vitting.Andersen@univ_paris1.fr`

[1] J. Vitting Andersen and A. Nowak, "An Introduction to Socio-Finance", Springer, Berlin, 2013.

[2] Benjamin Graham (1894-1976) was an influencial American econmist mentor to several investors of whom the most known is probably Warren Buffet.

[3] S. Bikhchandani, D. Hirshleifer, I. Welch, "Learning from the behavior of others: conformity, fads, and informational cascades", J. Econ. Perspect. 12(3), 151170 (1998).

[4] Hong, H., J. D. Kubik, and J. C. Stein, "Social interaction and stock-market participation", Journal of Finance, 49, 137-163 (2004).

[5] R. J. Shiller and J. Pound,"Survey evidence on diffusion of interest and information among investors", Journal of Economic Behavior and Organization 12, 47-66 (1989)

[6] T. Arnswald,"Investment behaviour of German equity fundmanagers an exploratory analysis of survey data", Deutsche Bundesbank WorkingPaper 08/01 (2001).

[7] H. N. Ozsoylev and J. Walden, "Asset pricing in large information networks", Journal of Economic Theory, 146, 6, 2252-2280 (2011).

[8] S. Galam, "Local dynamics vs. social mechanisms: a unifying frame". Europhys. Lett. 70, 705711 (2005).

[9] S. Galam, "Sociophysics: A Physicists Modeling of Psycho-political Phenomena", Springer, New York, 2012.

[10] Y. Biondi, P. Giannoccolo and S. Galam, "Formation of share market prices under heterogeneous beliefs and common knowledge", Physica A 391, 5532-5545 (2012).

[11] R. Cont, "Empirical properties of asset return s: stylized facts and statistical issues." Quant. Financ. 1(2), 223235 (2001).

[12] V. Plerou, P. Gopikrishnan, L. Amaral, M. Meyer, H.E. Stanley, "Scaling of the distribution of fluctuations of financial market indicies." Phys. Rev. E 60, 65196529 (1999);

G. Xavier, P.Gopikrishnan, V. Plerou, H.E. Stanley,"A theory of power law distributions in financial market fluctuations." Nature 423, 267230 (2003).

[13] D. Kahneman, A. Tversky, "Prospect theory: an analysis of decision-making under risk". Econometrica 47(2), 263292 (1979);

D. Kahneman, A. Tversky (eds.), "Choices, Values, and Frames"' (Cambridge University Press, Cambridge, 2000).

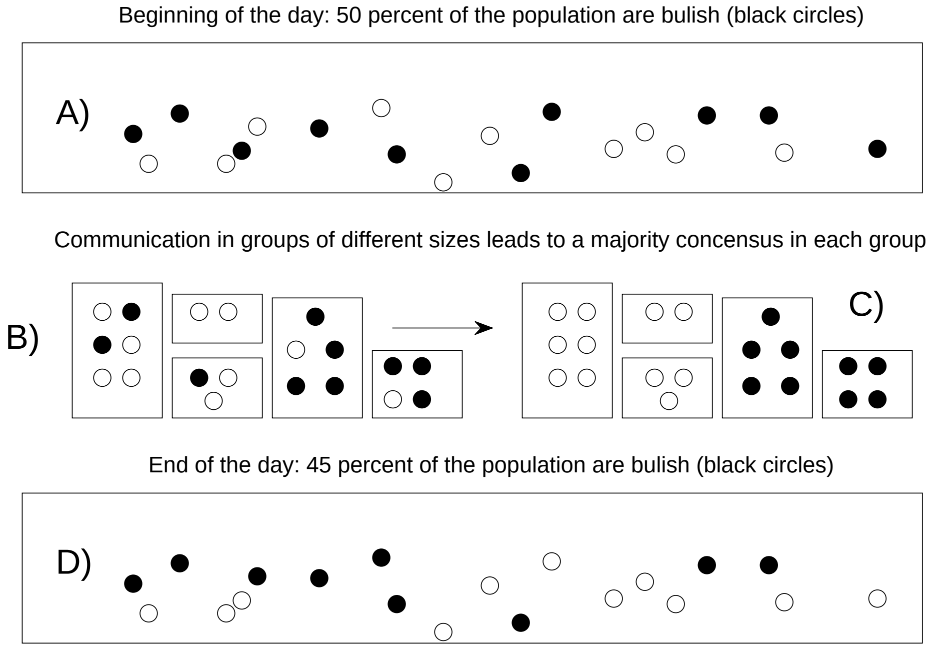

FIG. 1: Changing the "bullishness" in a population via communications in subgroups. **a):** At the beginning of a given day t a certain percentage $B(t)$ of bullishness. **b):** During the day communication takes place in random subgroups of different sizes. **c):** illustrates the extreme case of complete polarization $m_{k,j} = \pm 1$ created by a majority rule in opinion. In general $m_{k,j} \simeq j/k$ corresponds to the neutral case where in average the opinion remains unchanged within a subgroup of size k. **d):** due to the communication in different subgroups the "bullishness" at the end of the day is different from the beginning of the day.

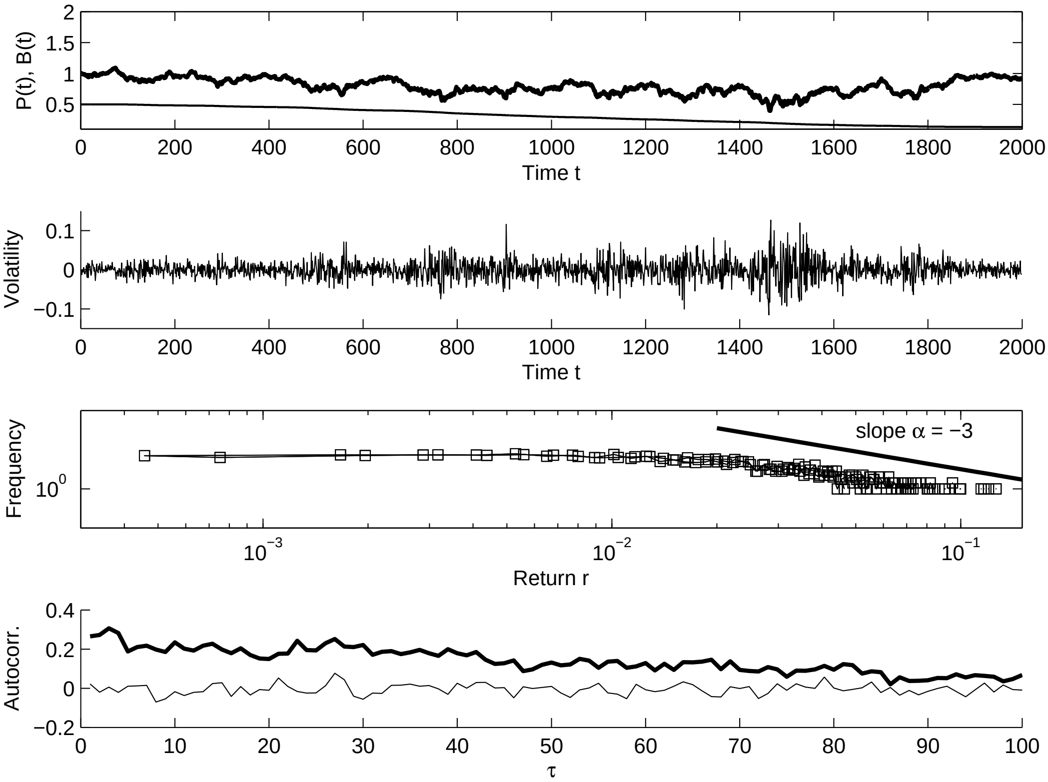

FIG. 2: Reproducing the "stylized facts". (a) Example of how a change in bullishness $B(t)$ in a population (thin line) can have an impact on prices $P(t)$ (thick solid line). (b) Volatility clustering as a function of time. (c) "Fat tailed" returns. (d) No arbitrage possibilities, i.e. zero autocorrelations of returns (thin solid line), and long time memory effects in volatility, i.e. nonzero autocorrelations of volatility versus time (thick solid line). Parameter values used: $\lambda = 1.1$, $\sigma_0 = 0.01$, $\beta = 0.001$, $L = 5$.

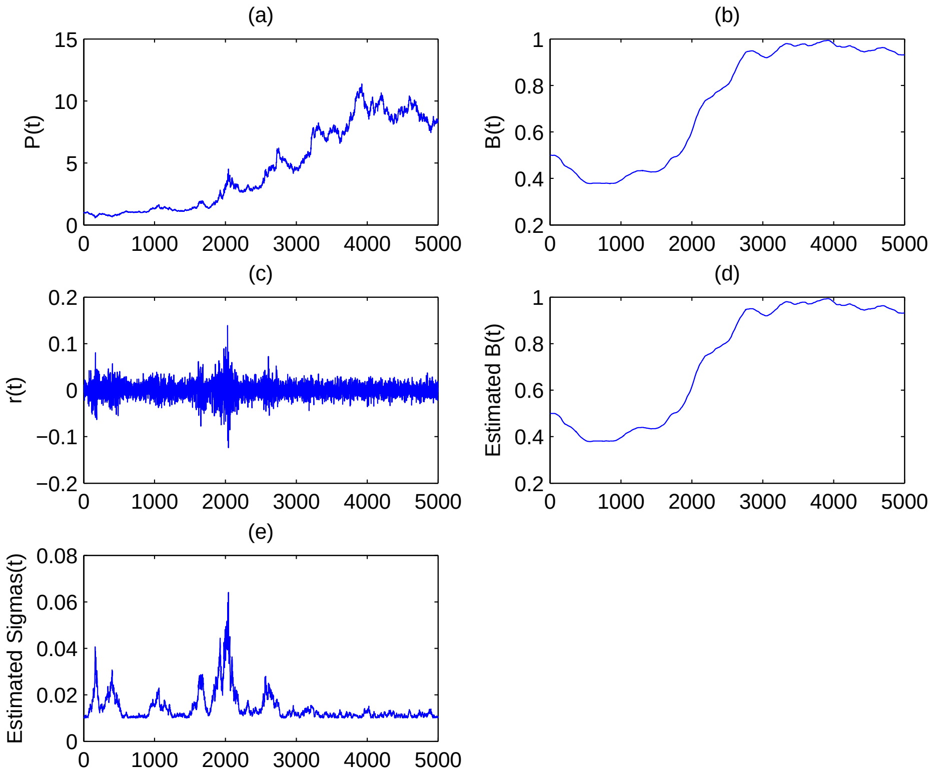

FIG. 3: Simulated price data ($T = 5000$). (a) Simulated prices $P(t)$, (b) Simulated bullishness proportions $B(t)$, (c) Simulated price returns $r(t)$, (d) Estimated bullishness proportions $\widehat{B}(t)$, (e) Estimated conditional volatilities.

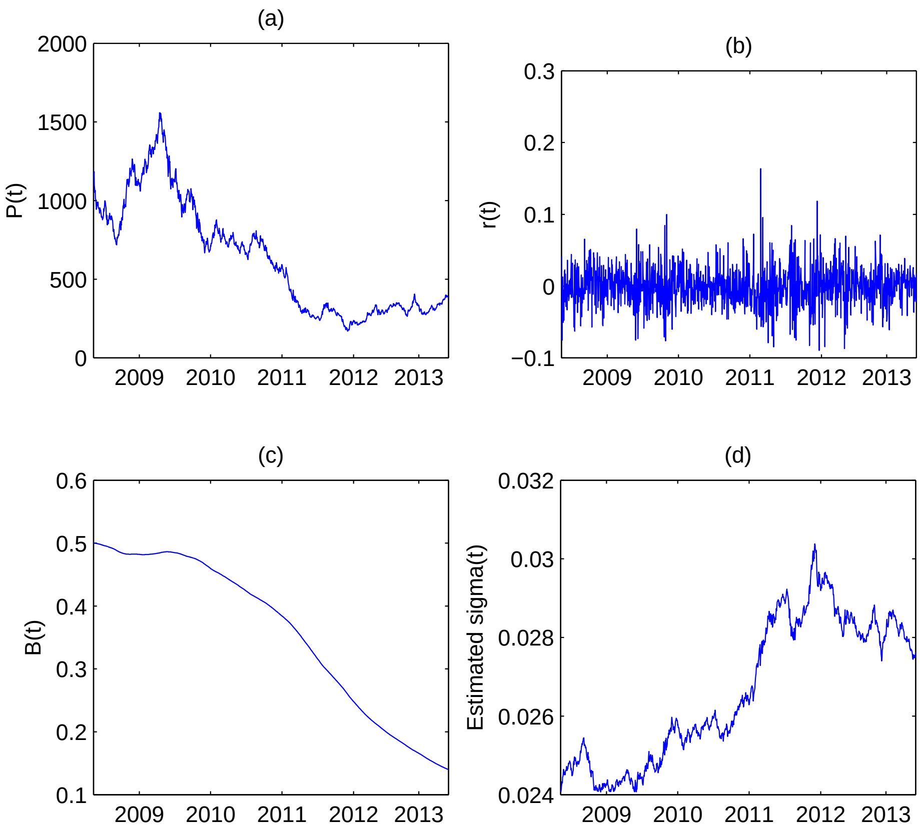

FIG. 4: FTSE-20 Athens stock index prices and returns, as well as the corresponding estimated conditional volatilities and bullishness proportions using optimal parameters estimated via the maximum likelihood method. (a) FTSE-20 price $P(t)$, (b) FTSE-20 returns $r(t)$, (c) Estimated bullishness proportions $\widehat{B}(t)$. (d) Estimated conditional volatilities $\widehat{\sigma}(t)$.

Table 1: Maximum likelihood estimation results of the simulation experiments ($T = 5000$).

Panel A: Starting values $\widehat{\lambda}^*$=-0.6931, $\widehat{\sigma}_0^*$=-6.4378, $\widehat{\beta}^*$=-3.5066, $\widehat{\alpha}^*$=5.7038									
True initial		True trans.		MLEs trans.		StdErr	MLEs		Gradient
λ	1.1	λ^*	0.095	$\widehat{\lambda}^*$	0.1747	0.8095	$\widehat{\lambda}$	1.1909	0.0004
σ_0	0.01	σ_0^*	-9.210	$\widehat{\sigma}_0^*$	-9.1891	0.0277	$\widehat{\sigma}_0$	0.0101	0.0087
β	0.001	β^*	-6.908	$\widehat{\beta}^*$	-6.8884	0.0417	$\widehat{\beta}$	0.0010	-0.0167
α	400	α^*	5.991	$\widehat{\alpha}^*$	5.9746	0.0382	$\widehat{\alpha}$	393.32	0.8311
Panel B: Starting values $\widehat{\lambda}^*$=-2.9957, $\widehat{\sigma}_0^*$=-13.8155, $\widehat{\beta}^*$=-6.9078, $\widehat{\alpha}^*$=6.2146									
True initial		True trans.		MLEs trans.		StdErr	MLEs		Gradient
λ	1.1	λ^*	0.095	$\widehat{\lambda}^*$	0.1737	0.8063	$\widehat{\lambda}$	1.1897	-0.0048
σ_0	0.01	σ_0^*	-9.210	$\widehat{\sigma}_0^*$	-9.1891	0.0277	$\widehat{\sigma}_0$	0.0101	0.0777
β	0.001	β^*	-6.908	$\widehat{\beta}^*$	-6.8883	0.0402	$\widehat{\beta}$	0.0010	-0.0461
α	400	α^*	5.991	$\widehat{\alpha}^*$	5.9746	0.0348	$\widehat{\alpha}$	393.29	-0.3053
Panel C: Starting values $\widehat{\lambda}^*$=0.0953, $\widehat{\sigma}_0^*$=-9.2103, $\widehat{\beta}^*$=-6.9078, $\widehat{\alpha}^*$=5.9915									
True initial		True trans.		MLEs trans.		StdErr	MLEs		Gradient
λ	1.1	λ^*	0.095	$\widehat{\lambda}^*$	0.1738	0.8100	$\widehat{\lambda}$	1.1898	-0.0007
σ_0	0.01	σ_0^*	-9.210	$\widehat{\sigma}_0^*$	-9.1891	0.0277	$\widehat{\sigma}_0$	0.0101	-0.0208
β	0.001	β^*	-6.908	$\widehat{\beta}^*$	-6.8884	0.0417	$\widehat{\beta}$	0.0010	-0.0219
α	400	α^*	5.991	$\widehat{\alpha}^*$	5.9746	0.0382	$\widehat{\alpha}$	393.32	0.0883

True initial: denotes the 'True' simulated parameter values θ, True trans.: denotes the 'True' transformed simulated parameter values θ^*, MLEs trans: denote the maximum likelihood estimates $\widehat{\theta}^*$ of the transformed parameters θ^*, StdErr: denotes the standard errors of the transformed parameters θ^*, MLEs: denote the maximum likelihood estimates $\widehat{\theta}$ of the parameters θ, Gradient: denotes the value of the gradient of the log-likelihood evaluated at the parameter estimates.

Table 2: Maximum likelihood estimation results of the FTSE-20 stock index return series.

Panel A: Starting values $\widehat{\lambda}^*$=-0.6931, $\widehat{\sigma}_0^*$=-6.4378, $\widehat{\beta}^*$=-3.5066, $\widehat{\alpha}^*$=5.7038					
	MLEs trans.	StdErr	MLEs		Gradient
$\widehat{\lambda}^*$	1.2825	3.7466	$\widehat{\lambda}$	3.6055	-0.0001
$\widehat{\sigma}_0^*$	-7.4537	0.0702	$\widehat{\sigma}_0$	0.0241	0.0001
$\widehat{\beta}^*$	-4.5640	2.9009	$\widehat{\beta}$	0.0104	-0.0000
$\widehat{\alpha}^*$	6.7058	2.9388	$\widehat{\alpha}$	817.16	-0.0004
Panel B: Starting values $\widehat{\lambda}^*$=-0.6931, $\widehat{\sigma}_0^*$=-9.2103, $\widehat{\beta}^*$=-4.6052, $\widehat{\alpha}^*$=6.6846					
	MLEs trans.	StdErr	MLEs		Gradient
$\widehat{\lambda}^*$	1.2869	3.7417	$\widehat{\lambda}$	3.6214	0.0002
$\widehat{\sigma}_0^*$	-7.4537	0.0702	$\widehat{\sigma}_0$	0.0241	-0.0001
$\widehat{\beta}^*$	-4.5615	2.8968	$\widehat{\beta}$	0.0104	-0.0001
$\widehat{\alpha}^*$	6.7033	2.9345	$\widehat{\alpha}$	815.12	-0.0001
Panel C: Starting values $\widehat{\lambda}^*$=0.0953, $\widehat{\sigma}_0^*$=-9.2103, $\widehat{\beta}^*$=-6.9078, $\widehat{\alpha}^*$=5.9915					
	MLEs trans.	StdErr	MLEs		Gradient
$\widehat{\lambda}^*$	1.2832	3.7284	$\widehat{\lambda}$	3.6082	-0.0001
$\widehat{\sigma}_0^*$	-7.4537	0.0702	$\widehat{\sigma}_0$	0.0241	-0.0007
$\widehat{\beta}^*$	-4.5641	2.8921	$\widehat{\beta}$	0.0104	0.0005
$\widehat{\alpha}^*$	6.7059	2.9298	$\widehat{\alpha}$	817.23	0.0003

MLEs trans: denote the maximum likelihood estimates $\widehat{\theta}^*$ of the transformed parameters θ^*, StdErr: denotes the standard errors of the transformed parameters θ^*, MLEs: denote the maximum likelihood estimates $\widehat{\theta}$ of the parameters θ, Gradient: denotes the value of the gradient of the log-likelihood evaluated at the parameter estimates.