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Energy and Capital in a New-Keynesian Framework

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Abstract

The economic implications of oil price shocks have been extensively studied since the oil price shocks of the 1970s'. Despite this huge literature, no dynamic stochastic general equilibrium model is available that captures two well-known stylized facts: 1) the stagflationary impact of an oil price shock, together with 2) two possible reactions of real wages: either a decrease (as in the US) or an increase (as in Japan). We construct a New-Keynesian DSGE model, which takes the case of an oil-importing economy where oil cannot be stored and where fossil fuels are used in two different ways: One part of the imported energy is used as an additional input factor next to capital and labor in the intermediate production of manufactured goods, the remaining part of imported energy is consumed by households in addition to their consumption of the final good. Oil prices, capital prices and nominal government spendings are exogenous random processes. We show that, without capital accumulation, the stagflationary effect is accounted for in general, and provide conditions under which a rise (resp. a decline) of real wages follows the oil price shock.

Keywords: New-Keynesian model, DSGE, oil, capital accumulation, stagflation.

1 Introduction

The two episodes of low growth, high unemployment, low real wages and high inflation that characterized most industrialized economies in the mid and late 1970s' are usually viewed as the paradigmactic consequences of large price shocks that affect various countries simultaneously. Despite the huge literature devoted to the implications of oil prices, to the best of our knowledge, no dynamic general equilibrium model is available that captures two well-known stylized facts related to this scheme : 1) the stagflationary impact of an oil price shock, together with 2) two possible reactions of real wages: either a decrease (as in the US and in most industrialized countries) or an increase (as in Japan, rather the exception).

On the other hand, the events of the past decade seem to call into question the relevance of oil price changes as a significant source of economic fluctuations. Since the late 1990s' indeed, the global economy has experienced two oil shocks of sign and magnitude

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comparable to those of the 1970s' but, in contrast with the latter episodes, GDP growth and inflation have remained relatively stable in much of the industrialized world (cf. e.g., [Ś08], Blanchard and Gali, 2009 [BG08] ; Kilian, 2007 [Kil08]). In [BG08], the suggested reasons why the effects of oil price shocks have recently weakened are the decrease in real wage rigidities, smaller oil share in production, and improvements in the credibility of monetary policy. However, the model used for this purpose exhibits somewhat disturbing properties that seem at odds with otherwise well-known empirical facts: in particular, as we show in this paper, the simulation of the DSGE model introduced by [BG08] leads to an increase, not a decrease, of the real GDP after an oil shock —a side-product that was probably not intended by its authors. As shown by [Hon10], during the 1970s', Japan is the unique industrialized country for which such a counterintuitive macroeconomic impulse response is compatible with a structural VAR analysis. Unfortunately, however, the model introduced by [Hon10] succeeds in capturing an increase in real GDP after an oil shock but fails in reproducing the empirically observed increase of Japanese real wages.

This raises the following body of questions. Can we alter the dynamic stochastic general equilibrium (DSGE) models considered in the literature so as to account simultaneously for:

- the stagflationary impact of an oil shock ;
- the fall in real wages ;
- conditions under which the Japanese exception occurs (i.e., real wages increase) ;
- the reduction of the sensitivity of industrialized countries to such a shock in the 1990s'?

This paper provides a positive answer to this question. Following Blanchard and Galí [BG08], we develop a DSGE model by introducing capital accumulation and energy into an otherwise standard New Keynesian model. The economy is populated by a representative household, a continuum of intermediate good firms $j \in [0, 1]$ who are involved in a monopolistic competition, a representative final good producing firm without market power, a government, and a Central Bank. All energy is imported: it is consumed by households and used as input in the production function of intermediate goods producing firms, along with labor and capital.

Output is produced with inputs of capital, energy, and labor. Energy is imported from abroad at an exogenous world price, $p_{E,t}$, and energy imports are paid for with exports of output, with trade being balanced at every date. The household works, invests in government bonds and capital, pays taxes and consumes both energy and the final good. Each intermediate goods firm uses capital, labor and energy to produce the intermediate good and sells it to the final good producing firm. The representative final good firm uses intermediate goods to produce the consumption good and sells it to the household and to the government. The government finances its public expenditures by raising taxes. A central bank sets the short-run nominal interest rate according to some Taylor-like rule. With no capital accumulation and zero public expenditures, our model reduces to the one first introduced by [BG08] with two changes, in the monetary policy and in the definition of the GDP deflator.

While the first departure is taken only to facilitate numerical simulations, and is therefore anecdotic, the second is not. In [BG08], indeed, the CPI is defined as $P_{c,t}$, the core CPI, as $P_{q,t}$ and the GDP deflator, as $P_{y,t}$. The three indices are related by the following equations:

$$P_{q,t} := P_{y,t}^{1-\alpha_e} P_{e,t}^{\alpha_e}; \tag{1}$$

$$P_{c,t} := P_{q,t}^{1-x} P_{e,t}^{x}; \tag{2}$$

and

$$P_{y,t} := P_{q,t}^{\beta} P_{e,t}^{1-\beta} \tag{3}$$

where $P_{e,t}/P_{q,t}$ is the (exogenous) real price of energy at time t, $\alpha, x \in (0,1)$, but — and this turns out to be crucial—, $\beta > 1$.

These conventions have the paradoxical consequence that, when the energy price experiences an upward shock, then the GDP deflator decreases (everything else being kept fixed). This turns out to be responsible for the rise of the GDP, Y_t, after the shock. Both phenomena — a decrease of the GDP deflator and an increase of the GDP— are at odds with empirical evidence from the 1970s".

[Hon10] succeeds in fixing this issue by identifying the three price indices just alluded to: $P_{c,t} \equiv P_{q,t} \equiv P_{y,t}$. By doing so, he gets both an increase of the GDP deflator after a shock, together with an increase of the GDP (which is peculiar to the Japanese economy). However, this success is impaired by 1) a basic flaw in the model, where it is postulated that

$$Y_t = Q_t, \tag{4}$$

that is, where the GDP is identified with the output produced by firms, while the household's budget constraint implies:

$$P_{y,t} Y_t = P_{y,t} Q_t - P_{E,t} E_t. \tag{5}$$

2) Beyond this inconsistency, as acknowledged by [Hon10], his approach leads to a decrease of real wages as a response to a price shock, which again contrasts with empirical evidence. 3) [Hon10] leaves open the question of the conditions under which the response to an energy price shock may be an output decline (resp. growth).

In this paper, we fix these various problems by adopting a price convention similar to (but weaker than) the one used by [Hon10] —namely, $P_{c,t} = P_{y,t}$—, and by keeping (1) and (2), and (5). This enables us to get an increase of the GDP deflator and real wages after a shock, together with either an decline or an increase of output. Moreover, we show that the introduction of capital accumulation does not impair the first 3 stylized facts just recalled. Quite on the contrary, capital even amplifies the response of the economy to a shock. On the other hand, in our model, wages are perfectly flexible and the central bank's monetary policy is perfectly credible. However, the share of oil in production can be adjusted and we show that a reduction of this share suffices to imply a significant reduction of the effect of a shock on macroeconomic performance. This is the way the last fourth phenomenon is accounted for in this paper.

The next section describes the model. Section 3 provides our main findings.

2 Model

Let us describe carefully how capital accumulation and imported energy enter into the production process.

2.1 Household

The representative infinitely-lived household works, invests in government bonds and capital, pays taxes and consumes both energy and the final good.

The household has an instantaneous utility function

$$u(C_t, L_t) = log(C_t) - \frac{L_t^{1+\phi}}{1+\phi},$$

where C_t is the consumption at time t, L_t is the labor (e.g. hours worked) and ϕ is the inverse of the Frish elasticity.

Labor: at each time, the household is endowed with 1 unit of labor. Let W_t denote the nominal wage.

Capital: The dynamics of capital accumulation follows

$$I_t := K_{t+1} - (1-\delta)K_t,$$

where $\delta \in (0, 1)$ is the depreciation rate. If, at date t, the household lends K_{t+1} units of capital to the production sector, and if the price of capital is $P_{k,t+1}$ at time $t+1$, then the household will receive $r^k_{t+1} P_{k,t+1} K_{t+1}$ units of account, where r^k_{t+1} is the rental rate of capital.

Bond: i_t is the nominal short-run interest rate which is set by the central bank.

Tax: at time t, T_t denotes the tax paid by the household.

Dividend: Being the shareholder of the firms, the household receives the global dividend $D_t := \int_0^1 D_t(j)dj$, i.e., the sum of dividends of all intermediate good firms.

Given a (perfectly anticipated) sequence of energy prices $(P_{e,t})_{t\geq 0}$, of intermediate good i prices $(P_{q,t}(i))_{t\geq 0}$, final good prices $(P_{q,t})_{t\geq 0}$, nominal wages $(W_t)_{t\geq 0}$, taxes $(T_t)_{t\geq 0}$, capital prices $(P_{k,t})_{t\geq 0}$, capital rental rates $(r^k_t)_{t\geq 0}$ and nominal short-run interest rates $(i_t)_{t\geq 0}$, the household chooses a consumption and labor plan consisting of a sequence of final good consumption $(C_{q,t})_{t\geq 0}$, energy consumption $(C_{e,t})_{t\geq 0}$, capital lending $(K_t)_{t\geq 0}$, bond lending/borrowing[1] on the monetary market $(B_t)_{t\geq 0}$, and labor supply $(L_t)_{t\geq 0}$ in order to maximize its discounted utility. Thus, the problem of the household is

$$\begin{aligned} \max \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[u(C_t, L_t)\big], \quad 0 < \beta < 1, & (6) \\ \text{subject to :} \quad & P_{e,t}C_{e,t} + P_{q,t}C_{q,t} + P_{k,t}(K_{t+1} - (1-\delta)K_t) + B_t & \\ \leq \quad & (1+i_{t-1})B_{t-1} + W_t L_t + D_t + r^k_t P_{k,t} K_t + T_t, & \end{aligned}$$

where the consumption flow is defined as:

$$C_t := \Theta_x C_{e,t}^x C_{q,t}^{1-x}, \tag{7}$$

[1]Whenever $B_t \geq 0$ (resp. ≤ 0), the household is lending (resp. borrowing) money to (resp. from) the government.

with $x \in (0,1)$ being the share of oil in consumption, and $\Theta_x := x^{-x}(1-x)^{-(1-x)}$. We stress that (7) implies that part of the imported energy will be consumed by the household. In order to ensure that this programme has a solution, we impose the following transversality condition (no Ponzi scheme):

$$\lim_{k\to\infty} \mathbb{E}_t \left(\frac{B_{t+k}}{\prod_{s=0}^{t+k-1} (1+i_{s-1})} \right) \geq 0, \quad \forall t. \tag{8}$$

Optimal allocation of expenditures among different domestic goods yields:

$$\begin{aligned} P_{q,t}C_{q,t} &= (1-x)P_{c,t}C_t && (9)\\ P_{e,t}C_{e,t} &= xP_{c,t}C_t && (10)\\ \text{CPI index:} \quad P_{c,t} &= P_{e,t}^x P_{q,t}^{1-x} && (11) \end{aligned}$$

By solving the first order conditions we have the following inter-temporal optimality conditions:

$$\begin{aligned} 1 &= \beta \mathbb{E}_t \Big[(1+i_t) \frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \Big] && (12)\\ 1 &= \beta \mathbb{E}_t \Big[\frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \frac{P_{k,t+1}}{P_{k,t}} (r_{t+1}^k + 1 - \delta) \Big] && (13)\\ \frac{W_t}{P_{c,t}} &= C_t L_t^{\phi} && (14) \end{aligned}$$

Condition (12) is the standard Euler equation on consumption. Condition (13) is the familiar Fisher relationship between the capital rate of depreciation, the rental rate of capital and the subjective discount rate. The stochastic discount factors are defined as follows:

1. Stochastic discount factor from date t to date $t+1$

$$d_{t,t+1} := \frac{\beta u'_C(C_{t+1}, L_{t+1})}{u'_C(C_t, L_t)} \frac{P_{c,t}}{P_{c,t+1}}, i.e, \quad \frac{1}{1+r_t} = \mathbb{E}_t(d_{t,t+1}).$$

2. Stochastic discount factor from date t to date $t+k$

$$d_{t,t+k} := \prod_{s=t}^{t+k-1} \Delta_s^{s+1}, \text{ then, } \quad d_{t,t+k} := \frac{\beta^k u'_C(C_{t+k}, L_{t+k})}{u'_C(C_t, L_t)} \frac{P_{c,t}}{P_{c,t+k}}.$$

2.2 Final good firm

There is a continuum, $[0,1]$, of intermediate goods that serve in producing the consumption commodity. A representative final good producing firm maximizes its profit without market power. Her CES production function is given by

$$Q_t = \Big(\int_{[0,1]} Q_t(i)^{\frac{\epsilon-1}{\epsilon}} di \Big)^{\frac{\epsilon}{\epsilon-1}}, \tag{15}$$

where $\epsilon > 0$ is the elasticity of substitution among intermediate goods. For simplicity, no energy is needed to produce the final commodity out of the intermediate goods.

Given all the intermediate goods prices $(P_{q,t}(i))_{i\in[0,1]}$ and the final good price $P_{q,t}$, the final good firm chooses quantities of intermediate goods $(Y_t(i))_{i\in 0,1}$ in order to maximize her profit. Her problem is then

$$\max_{Y_t(\cdot)} P_{q,t}Y_t - \int_{[0,1]} P_{q,t}(i)Y_t(i)di \tag{16}$$

$$\text{subject to :} \qquad Y_t = \Big(\int_{[0,1]} Y_t(i)^{\frac{\epsilon-1}{\epsilon}}di\Big)^{\frac{\epsilon}{\epsilon-1}} \tag{17}$$

Therefore, the demand of good i is given by $Y_t(i) = \Big(\frac{P_{q,t}(i)}{P_{q,t}}\Big)^{-\epsilon}Y_t$. The production function of the final good firm exhibits constant return to scale, so that, at equilibrium, we the zero profit condition will hold. The price of the final good will therefore be:

$$P_{q,t} = \Big(\int_{[0,1]} P_{q,t}(i)^{1-\epsilon}di\Big)^{\frac{1}{1-\epsilon}}. \tag{18}$$

2.3 Intermediate goods firms

Each intermediate commodity is produced through a Cobb-Douglas technology involving energy:

$$\begin{aligned} Q_t(i) &= AE_t(i)^{\alpha_e}L_t(i)^{\alpha_\ell}K_t(i)^{\alpha_k} \\ &\quad \alpha_e, \alpha_\ell, \alpha_k \geq 0, \quad \alpha_e+\alpha_\ell+\alpha_k \leq 1 \end{aligned} \tag{19}$$

The strategy of firm i: Firm i takes prices $P_{e,t}$ and W_t, and demand $Q_t(i)$ as given, it chooses quantities of energy $E_t(i)$, labor $L_t(i)$, and capital $K_t(i)$ in order to minimize cost. And then it chooses price $P_{q,t}(i)$ so as to maximize its profit. Regarding the price formation process, two cases will be considered: flexible prices and staggered prices *à la* Calvo.

We suppose that $F(E_t(i), L_t(i), K_t(i)) = AE_t(i)^{\alpha_e}L_t(i)^{\alpha_\ell}K_t(i)^{\alpha_k}$. The cost minimization problem of firm i is

$$\text{minimize cost:} \qquad P_{e,t}E_t(i) + W_tL_t(i) + r_t^kP_{i,t}K_t(i) \tag{20}$$

$$\text{subject to} \qquad E_t(i), L_t(i), K_t(i) \geq 0, \tag{21}$$

$$F(E_t(i), L_t(i), K_t(i)) \geq Q_t(i) \tag{22}$$

The first order conditions of this problem give

$$\text{marginal cost} = \lambda(i) \quad := \quad \frac{W_t}{\alpha_\ell\dfrac{Q_t(i)}{L_t(i)}} = \frac{r_t^kP_{i,t}}{\alpha_k\dfrac{Q_t(i)}{K_t(i)}} = \frac{P_{e,t}}{\alpha_e\dfrac{Q_t(i)}{E_t(i)}}. \tag{23}$$

Then we have

$$\begin{aligned} cost(Q_t(i)) &:= P_{e,t}E_t(i) + W_tL_t(i) + r_t^kP_{i,t}K_t(i) \\ &= (\alpha_e + \alpha_\ell + \alpha_k)\lambda(i)Q_t(i). \end{aligned}$$

Denote $F_t := \Big(\dfrac{A\alpha_e^{\alpha_e}\alpha_\ell^{\alpha_\ell}\alpha_k^{\alpha_k}}{P_{e,t}^{\alpha_e}W_t^{\alpha_\ell}(r_t^kP_{i,t})^{\alpha_k}}\Big)^{\frac{-1}{\alpha_e+\alpha_\ell+\alpha_k}}$, then

$$\text{cost function:} \quad cost(Q_t(i)) = (\alpha_e + \alpha_\ell + \alpha_k)F_tQ_t(i)^{\frac{1}{\alpha_e+\alpha_\ell+\alpha_k}}, \tag{24}$$

$$\text{marginal cost:} \quad mc_t(i) := \lambda(i) = F_tQ_t(i)^{\frac{1}{\alpha_e+\alpha_\ell+\alpha_k}-1}. \tag{25}$$

2.3.1 Flexible prices

At each date t, firm i's profit maximization problem is

$$\max_{P_{q,t}(i)} P_{q,t}(i)Q_t(i) - cost(Q_t(i)) \tag{26}$$

$$\text{subject to} \quad Q_t(i) = \Big(\frac{P_{q,t}(i)}{P_{q,t}}\Big)^{-\epsilon}Q_t.$$

Note that this problem does not depend on i either. Consequently its solution $P_{q,t}(i)$ does not depend on i, i.e. $P_{q,t}(i) = P_{q,t}^*$ for every i. Combining with the fact that $P_{q,t} := \big(\int_{[0,1]} P_{q,t}(i)^{1-\theta}di\big)^{\frac{1}{1-\theta}}$, we have $P_{q,t}(i) = P_{q,t}$ for every i.

The first order condition for $P_{q,t}^*$ gives

$$P_{q,t}^* = \frac{\epsilon}{\epsilon - 1}mc_t^*, \tag{27}$$

where $mc_t^* := F_tQ_t^{\frac{1}{\alpha_e+\alpha_\ell+\alpha_k}-1}$.

In order to distinguish the two price formation settings, we denote by Q_t^* the output at date t with flexible prices. Let $\mu^p := \dfrac{\epsilon}{\epsilon-1}$ be the price markup.

2.3.2 Calvo price setting

Assumption 1. (Calvo price setting)
As in Calvo ([Cal83]), we suppose that a fraction, θ, of intermediate good firms cannot reset their prices at time t:

$$P_{q,t}(i) = P_{q,t-1}(i).$$

and a fraction $1-\theta$ set their prices optimally:

$$P_{q,t}(i) = P_{q,t}^o(i).$$

We will prove that all $P_{q,t}^o(i)$ do not depend on i, so we can write $P_{q,t}^o(i) = P_{q,t}^o$. for every i. Assumption 1 gives the following "Aggregate Price Relationship"

$$P_{q,t} = \Big(\theta P_{q,t-1}^{1-\epsilon} + (1-\theta)(P_{q,t}^o)^{1-\epsilon}\Big)^{\frac{1}{1-\epsilon}}. \tag{28}$$

At date t, denote $Q_{t,t+k}(i)$ be the output at date $t+k$ for firm i that last reset its price in period t. Firm i's problem is

$$\max_{P_{q,t}(i)} \mathbb{E}_t\Big[\sum_{k=0}^{\infty}\theta^k d_{t,t+k}\big[P_{q,t}(i)Q_{t,t+k}(i) - cost(Q_{t,t+k}(i))\big]\Big] \tag{29}$$

$$\text{subject to} \qquad Q_{t,t+k}(i) = \Big(\frac{P_{q,t}(i)}{P_{q,t+k}}\Big)^{-\epsilon} Q_{t+k}, \quad \forall k \geq 0.$$

Note that this problem does not depend on i, hence its solution $P_{q,t}(i)$ does not either, we write

$$P_{q,t}(i) = P^o_{q,t}.$$

From the first order condition for $P^o_{q,t}$ we have

$$\mathbb{E}_t \sum_{k=0}^{\infty}\theta^k d_{t,t+k} Q^o_{t,t+k}\Big[P^o_{q,t} - \mu^p mc^o_{t,t+k}\Big] = 0, \tag{30}$$

where $mc^o_{t,t+k} := F_{t+k}(Q^o_{t,t+k})^{\frac{1}{\alpha_e+\alpha_\ell+\alpha_k}-1}$, and $Q^o_{t,t+k} = \Big(\frac{P^o_{q,t}}{P_{q,t+k}}\Big)^{-\epsilon} Q_{t+k}$ for every $k \geq 0$.
Denote

$$A^o_t \;:=\; \mathbb{E}_t \sum_{k=0}^{\infty}\theta^k d_{t,t+k} Q^o_{t,t+k}, \tag{31}$$

$$B^o_t \;:=\; \mathbb{E}_t \sum_{k=0}^{\infty}\theta^k d_{t,t+k} Q^o_{t,t+k} mc^o_{t,t+k}. \tag{32}$$

We have

$$P^o_{q,t}A^o_t \;=\; \mu^p B^o_t, \tag{33}$$

$$A^o_t \;:=\; Q^o_{t,t} + \theta\mathbb{E}_t d_{t,t+1} A^o_{t+1}, \tag{34}$$

$$B^o_t \;:=\; Q^o_{t,t} mc^o_{t,t} + \theta\mathbb{E}_t d_{t,t+1} B^o_{t+1}. \tag{35}$$

2.4 Monetary policy

Let $\Pi_{q,t} := \dfrac{P_{q,t}}{P_{q,t-1}}$ be the core inflation.
We suppose that the Central Bank sets the nominal short-term interest rate by the following monetary policy

$$i_t = \frac{1}{\beta} ln(\Pi_{q,t})^{\phi_\pi} ln\Big(\frac{P_{y,t}Y_t}{\overline{P}_y\overline{Y}}\Big)^{\frac{1}{2}}, \tag{36}$$

where $\overline{Y}$ and $\overline{P}_y$ represent the steady state of Y_t and $P_{y,t}$. We will define these concepts further in the paper.

2.5 Government

The Government budget constraint is:

$$(1+i_{t-1})B_{t-1} + G_t = B_t + T_t, \tag{37}$$

where G_t is the nominal government spending which we take as exogenously given by

$$G_t = func(G_{t-1}, P_{q,t}Q_t, e_{g,t}).$$

We choose this function to be

$$\ln(G_t) = (1-\rho_g)(\ln(\omega \overline{P}_q \overline{Q})) + \rho_g \ln(G_{t-1}) + e_{g,t}$$

where ω represents the share of output that the government takes for its own spending.

2.6 Equilibrium

At equilibrium,

(i) Each economic agent solves its maximization problem;

(ii) All markets clear, i.e., the following equations hold:

$$\text{Capital:} \quad K_t = \int_{[0,1]} K_t(i)di, \tag{38}$$

$$\text{Labor:} \quad L_t = \int_{[0,1]} L_t(i)di, \tag{39}$$

$$\text{Energy:} \quad E_t = \int_{[0,1]} E_t(i)di, \tag{40}$$

$$\text{Resource constraint:} \quad P_{c,t}C_t + P_{k,t}I_t + G_t = P_{q,t}Q_t - P_{e,t}E_t. \tag{41}$$

(iii) And the government budget constraint is fulfilled:

$$(1+i_{t-1})B_{t-1} + G_t = B_t + T_t, \tag{42}$$

2.7 Dispersion of relative prices

We have (proof is given in Appendix A.6)

$$\Big(\int_{[0,1]} (\frac{P_{q,t}(i)}{P_{q,t}})^{\frac{-\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} di \Big)^{\alpha_e+\alpha_\ell+\alpha_k} Q_t = AE_t^{\alpha_e} L_t^{\alpha_\ell} K_t^{\alpha_k}. \tag{43}$$

Define $v_t := \int_{[0,1]} (\frac{P_{q,t}(i)}{P_{q,t}})^{\frac{-\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} di$. Then by Calvo setting, we get

$$v_t = \theta v_{t-1} \Pi_{q,t}^{\frac{\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} + (1-\theta)\Big(\frac{P^o_{q,t}}{P_{q,t}}\Big)^{\frac{-\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} \tag{44}$$

2.8 GDP and GDP deflator

In Blanchard and Galí [BG08] the following identification is used.

Definition 1. *At each date t, the value added (or GDP), Y_t, and GDP deflator, $P_{y,t}$, are defined implicitly as follows*

$$P_{q,t} = P_{y,t}^{1-\alpha_e} P_{e,t}^{\alpha_e}, \tag{45}$$

$$P_{y,t}Y_t = P_{q,t}Q_t - P_{e,t}E_t. \tag{46}$$

We adopt this definition of value added but, as we will see in the sensibility analysis, this definition of GDP deflator, $P_{y,t}$, is quite fragile. We propose another approach. Empirically there are not much differences between the CPI index and the GDP deflator, therefore, we suppose that

$$P_{y,t} := P_{c,t} \tag{47}$$

2.9 Real prices of oil and capital

We define real prices of oil and capital relative to the price of final good

$$S_t := \frac{P_{e,t}}{P_{q,t}} \tag{48}$$

$$S_{k,t} := \frac{P_{k,t}}{P_{q,t}}. \tag{49}$$

In order to complete our model, we suppose that both the real price of oil and the price of capital follow AR(1) processes

$$ln(S_t) = \rho_s ln(S_{t-1}) + e_{o,t} \tag{50}$$

$$ln(S_{k,t}) = ln(S_{k,t-1}) + e_{k,t}. \tag{51}$$

3 Simulations and results

We calculate the steady state ourselves and then by using the software Dynare we simulate it at the second order. This software gives us also the impulse response functions (IRFs) to the three different shocks included in our model. Using the algorithm of Kim, Kim, Shaumburg and Sims ([KKSS03]) implemented in Dynare by the command "pruning" we have that the model is robust to the size of the shocks, meaning that a bigger shock in any of the stochatic variables, will not change qualitatively the response of the endogenous variables of the model.

3.1 Impulse response function analysis

In a first step, we run our model with the standard parameters from the literature. Their values and sources are shown in Table 1. The time period represents a quarter.

3.1.1 IRF of a shock on the real price of oil

The impulse response functions (IRFs) of each variable is calculated as a percentage deviation from its steady state. As seen in Figure 1 a shock in the real price of oil provokes an increase in all prices. As expected the nominal price of oil rises much more than the other prices and the price of capital follows the same pattern as the price of domestic goods. Both inflation measures increase instantly but quickly recover their original levels. The quantity of energy used by the intermediate firms decreases and follows a similar pattern, in opposite direction, to the real price of oil. The same effect is observed in the consumption of energy by the households. This effect comes from the Cobb-Douglas specification assumed for the production function : the elasticity of substitution between factors of production is one, therefore the firms can perfectly substitute capital and labor to energy, when faced

Parameter	Value	Source
β	0.99	Classic
ϕ	1	Classic
ϵ	6	[BG08]
θ	0.75	[BG08]
δ	0.06	[FBQF12]
ω	0.2	[Sim11]
α_ℓ	0.7	[BG08]
α_k	0.2	
α_e	0.015	[BG08]
x	0.023	[BG08]
ρ_s	0.97	[BG08]
ρ_{sk}	0.85	
ρ_{eg}	0.85	[Sim11]
σ_{sk}	0.01	
σ_g	0.01	[Sim11]
$var(ln(S_t))$	0.16^2	[BG08]
σ_s	$((1-\rho_s^2)var(ln(S_t)))^{\frac{1}{2}} = 0.038$	[BG08]

Table 1: Basic Parametrization

with a positive oil price shock. Moreover, saying that the elasticity of production with respect to energy use is low, firms are further prone to substitute capital and labor to energy. The same applies to private consumption. In this line, labor demand increases slightly, which implies higher wages. Capital does not vary instantly, this last effect could be associated to the fact that the price of capital rises only slightly. However, after the shock, the capital decreases and this could be because the energy use increases, so the firms revert the substitution between energy, labor and capital. GDP goes down. Global consumption goes down, seeing the fall in GDP and the rise in prices. As in the case of the firms, the elasticity of substitution between domestics goods and energy is one so the decline in the energy consumption is bigger than the decline in domestic consumption.

3.1.2 IRF of a real price of capital shock

Figure 2 shows the IRFs to a shock on the real price of capital. This kind of shock could be associated for example to an increase in real estate prices. In this case, at the moment of the shock, the nominal price of capital increases at the same amount than the real one. Other prices remain at their steady state values, because the shock in the real price of oil is zero, and each of them follows the final good price , i.e they progressively increase. Both inflation measures have the same pattern as the previous case, yet with less accentuated reactions. The main differences between this shock and the last one, is that capital and employment react in an opposite way when the shock occurs, both types of private consumption increase in the same amount. The interest rate largely decreases as the rental rate of capital by a no-arbitrage relation. Also wages fall in contrast with the last shock. This last effect comes from the fact that the labor decreases.

3.1.3 IRF of government spending shock

Finally, Figure 3 shows the IRFs to a shock on the government spending. With the formulation of the government expenditure function that we have adopted, a shock on e_g of standard deviation equal to 1% produce an increase 0.8% in the Government spending. Here, all prices do the same and follow the reaction of the final consumption good price, falling when the shock occurs, but in any case less than a 0.01%. This shock does not produce to much inflation either. Energy and capital fall, but energy recovers sharply its level a few quarters later. Employment rises and so does domestic production also rises, along with GDP, consistent with higher government spending.

3.2 Sensitivity analysis

3.2.1 Sensitivity to price stickiness

Going back to the shock in the real price of oil, we now change some parameters in order to study the sensitivity of our model to some changes in elasticities, the stickiness of prices and the coefficients of the Taylor rule. Let us first make the prices more flexible, we put $\theta = 0.5$ instead of 0.75. Figure 4 shows the IRFs. If there are more firms that can reset their prices, then the latter will increase even more, which will produce also an increase in the price of capital, provoking then a bigger decrease in the quantity of capital, in investment and in the real return of capital. P_q rises more, hence the production of final goods will decrease more and then labor too. However, this increase is much muted than before and then the impact on wages will be slightly negative.

3.2.2 Sensitivity to the definition of GDP deflator

Let us now adopt the definition of the the GDP deflator give by Blanchard and Galí ([BG08]), $P_{y,t} = P_{q,t}^{\frac{1}{1-\alpha_e}} P_{e,t}^{\frac{-\alpha_e}{1-\alpha_e}}$. Figure 5 shows the comparison of the endogenous variables that substantially change using this both definitions. We observe that under the definition of Blanchard Galí the GDP deflator falls and the value added increases, which oppose with the empirical data, that shows that in every oil shock, a rise on the CPI index and on the GDP deflator is observed. Empirically an increase in the GDP could be observed in the last oil shock of 2000s but in this part we are using the parametrization that approaches more to the 1970s' environment, so this increase in GDP is suspicious. That is why we feel confortable with the definition that we have adopted.

Actually this effect is even verified in the model of Blanchard and Galí where capital does not exist and $B_t = 0$ (so $G_t = 0$). Figure 6 shows the IRFs where the only modification is on the Taylor Rule, using instead the one defined by Blanchard and Galí, the one we imply in our model. Theoretically, this effect on the GDP deflator is not surprising, it just come from its definition $P_{y,t} = P_{q,t}^{\frac{1}{1-\alpha_e}} P_{e,t}^{\frac{-\alpha_e}{1-\alpha_e}}$. When S_t rises, $P_{e,t}$ rises a lot more than $P_{q,t}$ and so $P_{y,t}$ falls. GDP then has rose, because its price responses negatively.

3.2.3 Sensitivity of the Taylor response to inflation coefficient

Let us now make a change on the coefficient of the Taylor rule ϕ_π, meaning that the central bank reacts more against the inflation. Figure 7 shows the IRFs in both cases. We observe that global consumption and final good consumption are a little bit more negative when the Central Bank fights more agains the inflation. As expected, the price of domestic goods and its inflation react less when the coefficient is bigger, causing also a reduction on the other prices in exception to the energy price. The reaction of the energy is the same in both cases, but when the coefficient is bigger, labor does not increase to much provoking a decrease in the wages. Capital falls a bit more, the interest rate increases less leading to a decrease in the rental rate of capital. The decrease in the input use provokes an even bigger decrease in output and value added, so contrary to what some other papers have found, a bigger reaction of the Central Bank against the inflation affects somehow worse these two variables.

3.2.4 Sensitivity to the environment change

Now we change the values of α_e and x to 0.012 and 0.017 respectively, in order to evaluate the decline of the energy shares argued by Blanchard and Galí. Figure 8 shows the IRFs in both cases. As expected from the definition of the model, a decrease in the parameters α_e and x will produce a decrease in the reactions of the endogenous variables face an oil shock.

3.2.5 Comparison to the model without capital

Finally we make a comparison between three different modelizations of the problem. A first model which includes energy and labor in the production function, a second which include energy, capital and labor, where the price of capital is exogenous and equal to P_q and a third the one of the paper. Figure 9 plot the three models.Here, the inclusion of capital in the model increases the reaction of the variables to an oil shock.

4 Conclusion

We have developed a DSGE model by incorporating capital accumulation and imported energy into the standard New-Keynesian framework.

Our first contribution is that the response of the real GDP to an oil shock is very sensitive to the definition of the GDP deflator. Second, the inclusion of the capital accumulation in the model amplify the negative response of output and GDP due to the persistent effect of capital accumulation in the dynamic of the model. Third, an increase in the Taylor response to inflation coefficient amplify the negative impact on output and GDP, moreover with active monetary policy ($\phi_\pi = 5$ instead of 1.2), the response of real price of oil on nominal wages turn out to be negative. Fourth, the reduction of oil share in consumption and in production certainly account for a muted impact on macroeconomic variables face to a real price of oil shock.

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A Appendix

A.1 First order conditions of households

The Lagrangian associated with the maximization problem of the household has the following from

$$\begin{aligned}\mathcal{L}_0 &= \sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\Big[u(C_t,L_t)+\lambda\big[P_{c,t}C_t+P_{k,t}I_t\\ &\qquad +B_t+T_t+(1+i_{t-1})B_{t-1}+W_tL_t+D_t+r_t^kP_{k,t}K_t\big]\Big]\end{aligned}$$

$$C_t: \quad u_C(C_t,L_t)=\lambda P_{c,t} \tag{52}$$

$$L_t: \quad u_L(C_t,L_t)=\lambda W_t \tag{53}$$

$$B_t: \quad \lambda=\beta\mathbb{E}_t\Big[(1+i_t)\lambda_{t+1}\Big] \tag{54}$$

$$K_{t+1}: \quad \lambda P_{k,t}=\beta\mathbb{E}_t\Big[\lambda_{t+1}\big(r_{t+1}^k+1-\delta\big)P_{k,t+1}\Big]. \tag{55}$$

A.2 First order condition for the Final good Producer

FOC for $Y_t(i)$

$$P_{q,t}\frac{\epsilon}{\epsilon-1}\Big(\int_{[0,1]}Y_t(i)^{\frac{\epsilon-1}{\epsilon}}di\Big)^{\frac{\epsilon}{\epsilon-1}-1}\frac{\epsilon-1}{\epsilon}Y_t(i)^{\frac{\epsilon-1}{\epsilon}-1}-P_{q,t}(i)=0. \tag{56}$$

A.3 First order condition for the Intermediate Firms

A.3.1 Cobb-Douglas production function

We suppose

$$\begin{aligned}Q_t(i) &= AE_t(i)^{\alpha_e}L_t(i)^{\alpha_\ell}K_t(i)^{\alpha_k} \qquad (57)\\ &\quad \alpha_e,\alpha_\ell,\alpha_k\geq 0, \quad \alpha_e+\alpha_\ell+\alpha_k\leq 1\end{aligned}$$

We have the following Lagrangian for the firms maximization problem

$$\mathcal{L}_0=P_{e,t}E_t(i)+W_tL_t(i)+r_t^kP_{k,t}K_t(i)-\lambda(i)\Big(AE_t(i)^{\alpha_e}L_t(i)^{\alpha_\ell}K_t(i)^{\alpha_k}-Q_t(i)\Big) \tag{58}$$

First-order conditions:

$$E_t(i): \quad P_{e,t}=\lambda(i)\alpha_eAE_t(i)^{\alpha_e-1}L_t(i)^{\alpha_\ell}K_t(i)^{\alpha_k} \tag{59}$$

$$L_t(i): \quad W_t=\lambda(i)\alpha_\ell AE_t(i)^{\alpha_e}L_t(i)^{\alpha_\ell-1}K_t(i)^{\alpha_k} \tag{60}$$

$$K_t(i): \quad r_t^kP_{k,t}=\lambda(i)\alpha_kAE_t(i)^{\alpha_e}L_t(i)^{\alpha_\ell}K_t(i)^{\alpha_k-1}. \tag{61}$$

We define

$$\text{marginal cost}=\frac{\frac{d(cost)}{d(worker)}}{\frac{d(output)}{d(worker)}}=\frac{\frac{d(cost)}{d(capital)}}{\frac{d(output)}{d(capital)}}=\frac{\frac{d(cost)}{d(energy)}}{\frac{d(output)}{d(energy)}}.$$

And so equation (23) follows.
On the other hand, we have

$$\begin{aligned}
Q_t(i) &= AE_t(i)^{\alpha_e}L_t(i)^{\alpha_\ell}K_t(i)^{\alpha_k} \\
&= A\big(\frac{\alpha_e\lambda(i)Q_t(i)}{P_{e,t}}\big)^{\alpha_e}\big(\frac{\alpha_\ell\lambda(i)Q_t(i)}{W_t}\big)^{\alpha_\ell}\big(\frac{\alpha_k\lambda(i)Q_t(i)}{r_t^kP_{k,t}}\big)^{\alpha_k} \\
&= \frac{A\alpha_e^{\alpha_e}\alpha_\ell^{\alpha_\ell}\alpha_k^{\alpha_k}}{P_{e,t}^{\alpha_e}W_t^{\alpha_\ell}(r_t^kP_{k,t})^{\alpha_k}}\big[\lambda(i)Q_t(i)\big]^{\alpha_e+\alpha_\ell+\alpha_k}.
\end{aligned}$$

If we denote $F_t := \Big(\frac{A\alpha_e^{\alpha_e}\alpha_\ell^{\alpha_\ell}\alpha_k^{\alpha_k}}{P_{e,t}^{\alpha_e}W_t^{\alpha_\ell}(r_t^kP_{k,t})^{\alpha_k}}\Big)^{\frac{-1}{\alpha_e+\alpha_\ell+\alpha_k}}$ then equations (24) and (25) follows.

A.3.2 Production function: general case

We have the Lagrangian

$$\mathcal{L}_0 = P_{e,t}E_t(i) + W_tL_t(i) + r_t^kP_{k,t}K_t(i) - \lambda(i)\Big(F(E_t(i),L_t(i),K_t(i)) - Q_t(i)\Big) \tag{62}$$

First-order conditions:

$$E_t(i): \quad P_{e,t} = \lambda(i)F_E(E_t(i),L_t(i),K_t(i)) \tag{63}$$

$$L_t(i): \quad W_t = \lambda(i)F_L(E_t(i),L_t(i),K_t(i)) \tag{64}$$

$$K_t(i): \quad r_t^kP_{k,t} = \lambda(i)F_K(E_t(i),L_t(i),K_t(i)). \tag{65}$$

We define

$$\text{marginal cost} = \frac{\frac{d(cost)}{d(worker)}}{\frac{d(output)}{d(worker)}} = \frac{\frac{d(cost)}{d(capital)}}{\frac{d(output)}{d(capital)}} = \frac{\frac{d(cost)}{d(energy)}}{\frac{d(output)}{d(energy)}}.$$

We have

$$\text{marginal cost} = \lambda(i) = \frac{W_t}{F_L(E_t(i),L_t(i),K_t(i))} \tag{66}$$

$$= \frac{r_t^kP_{k,t}}{F_K(E_t(i),L_t(i),K_t(i))} = \frac{P_{e,t}}{F_E(E_t(i),L_t(i),K_t(i))}. \tag{67}$$

We have

$$\begin{aligned}
cost(Q_t(i)) &= P_{e,t}E_t(i) + W_tL_t(i) + r_t^kP_{k,t}K_t(i) \\
&= \lambda(i)\Big(E(i)F_E(E_t(i),L_t(i),K_t(i)) + L_t(i)F_L(E_t(i),L_t(i),K_t(i)) \\
&\quad +K_t(i)F_K(E_t(i),L_t(i),K_t(i))\Big).
\end{aligned}$$

Assume that F is constant return to scale, i.e., for every $\lambda > 0$, we have

$$F(\lambda E, \lambda N, \lambda K) = \lambda F(E,N,K).$$

We obtain

1. $F(E,L,K) = EF_E(E,L,K) + LF_L(E,L,K) + KF_K(E,L,K)$.

2. Denote $x_t(i) := \dfrac{E_t(i)}{L_t(i)}, y_t(i) := \dfrac{K_t(i)}{L_t(i)}$. FOCs are equivalent to

$$\begin{aligned} \lambda(i)F_E(x_t(i),1,y_t(i)) &= P_{e,t} \quad (68)\\ \lambda(i)F_L(x_t(i),1,y_t(i)) &= W_t \quad (69)\\ \lambda(i)F_K(x_t(i),1,y_t(i)) &= r_t^k P_{k,t} \quad (70)\end{aligned}$$

This system has 3 equations, 3 variables $x_t(i), y_t(i), \lambda(i)$. In some crucial condition, we can compute $x_t(i), y_t(i), \lambda(i)$ as functions of $P_{e,t}, P_{k,t}, W_t, r_t^k$, hence $\lambda(i)$ does not depend on i.

Therefore, with CES function, we have

$$cost(Q_t(i)) = \lambda(i)F(E_t(i), L_t(i), K_t(i)) = \lambda(i)Q_t(i) = \lambda Q_t(i).$$

Then in flexible price, we have (with CES function, and some other conditions)

$$P_{q,t} = \frac{\epsilon}{\epsilon-1}\lambda. \quad (71)$$

A.4 Calvo Price Setting

Lemma A.1. *We have the following "Aggregate Price Relationship"*

$$P_{q,t} = \left(\theta P_{q,t-1}^{1-\epsilon} + (1-\theta)(P_{q,t}^o)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \quad (72)$$

Proof. By definition we have

$$\begin{aligned} P_{q,t}^{1-\epsilon} &= \int_{[0,1]} P_{q,t}(i)^{1-\epsilon}di\\ &= \int_{\text{can not change price}} P_{q,t}(i)^{1-\epsilon}di + \int_{\text{set price optimally}} P_{q,t}(i)^{1-\epsilon}di\\ &= \int_{[0,1]} \theta P_{q,t-1}(i)^{1-\epsilon}di + \int_{[0,1]} (1-\theta)P_{q,t}(i)^{1-\epsilon}di\\ &= \theta P_{q,t-1}^{1-\epsilon} + (1-\theta)(P_{q,t}^o)^{1-\epsilon}.\end{aligned}$$

□

FOC for $P_{q,t}^o$:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k d_{t,t+k} Q_{t,t+k}^o \left[P_{q,t}^o - \mu^p mc_{t,t+k}^o\right] = 0, \quad (73)$$

where $mc_{t,t+k}^o := F_{t+k}(Q_{t,t+k}^o)^{\frac{1}{\alpha_e+\alpha_\ell+\alpha_k}-1}$, and $Q_{t,t+k}^o = \left(\dfrac{P_{q,t}^o}{P_{q,t+k}}\right)^{-\epsilon} Q_{t+k}$ for every $k \geq 0$.
Denote

$$A_t^o := \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k d_{t,t+k} Q_{t,t+k}^o, \quad (74)$$

$$B_t^o := \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k d_{t,t+k} Q_{t,t+k}^o mc_{t,t+k}^o. \quad (75)$$

We have

$$
\begin{aligned}
P^o_{q,t}A^o_t &= \mu^p B^o_t, &(76)\\
A^o_t &:= Q^o_{t,t} + \theta \mathbb{E}_t d_{t,t+1} A^o_{t+1}, &(77)\\
B^o_t &:= Q^o_{t,t} mc^o_{t,t} + \theta \mathbb{E}_t d_{t,t+1} B^o_{t+1}. &(78)
\end{aligned}
$$

Denote

$$
m^o_{t,t+k} := \frac{mc^o_{t,t+k}}{P_{q,t+k}}.
$$

Then we have

$$
\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k d_{t,t+k} Q^o_{t,t+k} \Big[P^o_{q,t} - \mu^p m^o_{t,t+k} P_{q,t+k} \Big] = 0, \tag{79}
$$

For each variable, we denote $\hat{x}_t := \frac{x_t - x}{x}$ where x is value of x_t at steady state. By log-linearizing the above equation, we get that

$$
\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \Big[\hat{P}^o_{q,t} - \hat{m}^o_{t,t+k} - \hat{P}_{q,t+k} \Big] = 0, \tag{80}
$$

A.5 Dispersion of relative prices

Lemma A.2. *We have*

$$
\Big(\int_{[0,1]} \big(\frac{P_{q,t}(i)}{P_{q,t}}\big)^{\frac{-\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} di \Big)^{\alpha_e+\alpha_\ell+\alpha_k} Q_t = A E_t^{\alpha_e} L_t^{\alpha_\ell} K_t^{\alpha_k}. \tag{81}
$$

Proof. We have

$$
\begin{aligned}
\Big(\frac{P_{q,t}(i)}{P_{q,t}}\Big)^{-\epsilon} Q_t &= Q_t(i) = A E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k}\\
&= A E_t(i)^{\alpha_e} \Big(\frac{P_{e,t}E_t(i)}{W_t} \frac{\alpha_\ell}{\alpha_e} \Big)^{\alpha_\ell} \frac{P_{e,t}E_t(i)}{r^k_t P_{k,t}} \frac{\alpha_k}{\alpha_e}^{\alpha_k}\\
&= A E_t(i)^{\alpha_e+\alpha_\ell+\alpha_k} \Big(\frac{P_{e,t}}{W_t} \frac{\alpha_\ell}{\alpha_e} \Big)^{\alpha_\ell} \Big(\frac{P_{e,t}}{r^k_t P_{k,t}} \frac{\alpha_k}{\alpha_e} \Big)^{\alpha_k}.
\end{aligned}
$$

Hence we get

$$
\Big(\frac{P_{q,t}(i)}{P_{q,t}}\Big)^{\frac{-\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} Q_t^{\frac{-\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} = E_t(i) \Big[A \Big(\frac{P_{e,t}}{W_t} \frac{\alpha_\ell}{\alpha_e} \Big)^{\alpha_\ell} \Big(\frac{P_{e,t}}{r^k_t P_{k,t}} \frac{\alpha_k}{\alpha_e} \Big)^{\alpha_k} \Big]^{\frac{1}{\alpha_e+\alpha_\ell+\alpha_k}}.
$$

By taking integral, then take power $\alpha ;= \alpha_e + \alpha_\ell + \alpha_k$, we have

$$
\Big(\int_{[0,1]} \big(\frac{P_{q,t}(i)}{P_{q,t}}\big)^{\frac{-\epsilon}{\alpha}} di \Big)^{\alpha} Q_t = E_t^{\alpha_e+\alpha_\ell+\alpha_k} A \Big(\frac{P_{e,t}}{W_t} \frac{\alpha_\ell}{\alpha_e} \Big)^{\alpha_\ell} \Big(\frac{P_{e,t}}{r^k_t P_{k,t}} \frac{\alpha_k}{\alpha_e} \Big)^{\alpha_k}. \tag{82}
$$

Recall that

$$
\frac{W_t L_t(i)}{\alpha_\ell} = \frac{r^k_t P_{k,t} K_t(i)}{\alpha_k} = \frac{P_{e,t} E_t(i)}{\alpha_e}.
$$

By taking integral, we have

$$\frac{W_t L_t}{\alpha_\ell} = \frac{r_t^k P_{k,t} K_t}{\alpha_k} = \frac{P_{e,t} E_t}{\alpha_e}.$$

Combining with (82), we obtaint the result. □

Define $v_t := \int_{[0,1]} (\frac{P_{q,t}(i)}{P_{q,t}})^{\frac{-\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} di$. Then by Calvo setting, we get that

Lemma A.3.

$$v_t = \theta v_{t-1} \Pi_{q,t}^{\frac{\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} + (1-\theta)\Big(\frac{P_{q,t}^o}{P_{q,t}}\Big)^{\frac{-\epsilon}{\alpha_e+\alpha_\ell+\alpha_k}} \tag{83}$$

Proof. See Lemma A.1 □

A.6 Simulations

A.6.1 Steady state

$$\begin{aligned}
\text{Static problem of Household:}\quad C &= \Theta_x C_e^x C_q^{1-x} && (84)\\
P_c &= P_e^x P_q^{1-x} && (85)\\
P_q C_q &= (1-x) P_c C && (86)\\
P_e C_e &= x P_c C. && (87)\\
1 &= \beta(r^k + 1 - \delta) && (88)\\
\text{Budget constraint:}\quad P_c C + \delta P_k K &= WN + r^k P_k K + \Pi, && (89)\\
\Pi &= P_q Q - P_e E - WL - r^k P_k K && (90)\\
\text{FOCs of household:}\quad W &= P_c C L^\phi && (91)\\
\text{Production function:}\quad Q &= A E^{\alpha_e} L^{\alpha_\ell} K^{\alpha_k} && (92)\\
\text{FOCs of firms :}\quad \frac{P_e E}{\alpha_e} &= \frac{WL}{\alpha_\ell} = \frac{r^k P_k K}{\alpha_k} && (93)\\
P_e E &= \frac{\alpha_e(\epsilon-1)}{\epsilon} P_q Q. && (94)
\end{aligned}$$

And we assume that

$$\begin{aligned}
S &= \frac{P_e}{P_q}, \text{where } S \text{ is exogenous} && (95)\\
S_k &= \frac{P_k}{P_q}, \text{where } S_k \text{ is exogenous} && (96)
\end{aligned}$$

Solution: we have to find $(C, C_e, C_q, P_c, P_e, P_q, P_k, r^k, W, Q, E, N, K)$. Without loss of generality, we can take $P_e = 1$. By using $P_c C + \delta P_k K + G = P_q Q - P_e E$, $E = \frac{\alpha_e(\epsilon-1)}{\epsilon} P_q Q$, and $\frac{P_e E}{\alpha_E} = \frac{r^k P_k K}{\alpha_k}$, we have

$$\begin{aligned}
P_c C &= P_q Q - P_e E - \delta P_k K - G\\
&= P_q Q\Big(1 - \omega - \frac{\alpha_e(\epsilon-1)}{\epsilon} - \frac{\alpha_e(\epsilon-1)}{\epsilon}\frac{\delta\alpha_k}{\alpha_e r^k}\Big) && (97)
\end{aligned}$$

Therefore, we can compute $\dfrac{P_qQ}{P_cC}$. The system of equations becomes

$$\begin{aligned}
P_e &= 1, P_c = P_q^{1-x}, && (98)\\
P_q &= 1/S, \quad P_k = P_qS_k \text{ with } S, S_k \text{ will be chosen} && (99)\\
r^k &= \frac{1}{\beta} - 1 + \delta && (100)\\
C &= \Big(1 - \omega - \frac{\alpha_e(\epsilon-1)}{\epsilon}(1 + \delta\frac{\alpha_k}{\alpha_e r^k})\Big)P_q^xQ && (101)\\
C_q &= (1-x)P_cC && (102)\\
C_e &= xP_cC && (103)\\
W &= P_cCL^\phi && (104)\\
Q &= AE^{\alpha_e}L^{\alpha_\ell}K^{\alpha_k} && (105)\\
\frac{E}{\alpha_e} &= \frac{WL}{\alpha_\ell} && (106)\\
E &= \frac{\alpha_e(\epsilon-1)}{\epsilon}P_qQ && (107)
\end{aligned}$$

By combining $W = P_cCL^\phi$ and $\dfrac{E}{\alpha_e} = \dfrac{WL}{\alpha_\ell}$, we can compute L

$$\begin{aligned}
L^{\phi+1} &= \frac{(\epsilon-1)\alpha_\ell}{\epsilon}\frac{P_qQ}{P_cC} && (108)\\
Q^{1-\alpha_e-\alpha_k} &= AL^{\alpha_\ell}\Big(\frac{\epsilon-1}{\epsilon}\alpha_\ell P_q\Big)^{\alpha_e}\big(\frac{\epsilon-1}{\epsilon}\frac{\alpha_kP_q}{r^kP_k}\big)^{\alpha_k} && (109)\\
E &= \frac{\alpha_e(\epsilon-1)}{\epsilon}P_qQ && (110)\\
W &= \frac{\alpha_\ell(\epsilon-1)}{L\epsilon}P_qQ. && (111)
\end{aligned}$$

$S = S_k = 1$, then we can calculate P_q, P_k and so on.

Remark A.1. *We have to choose parameters such that there exists a steady state. Not difficult, but we have to pay attention.*

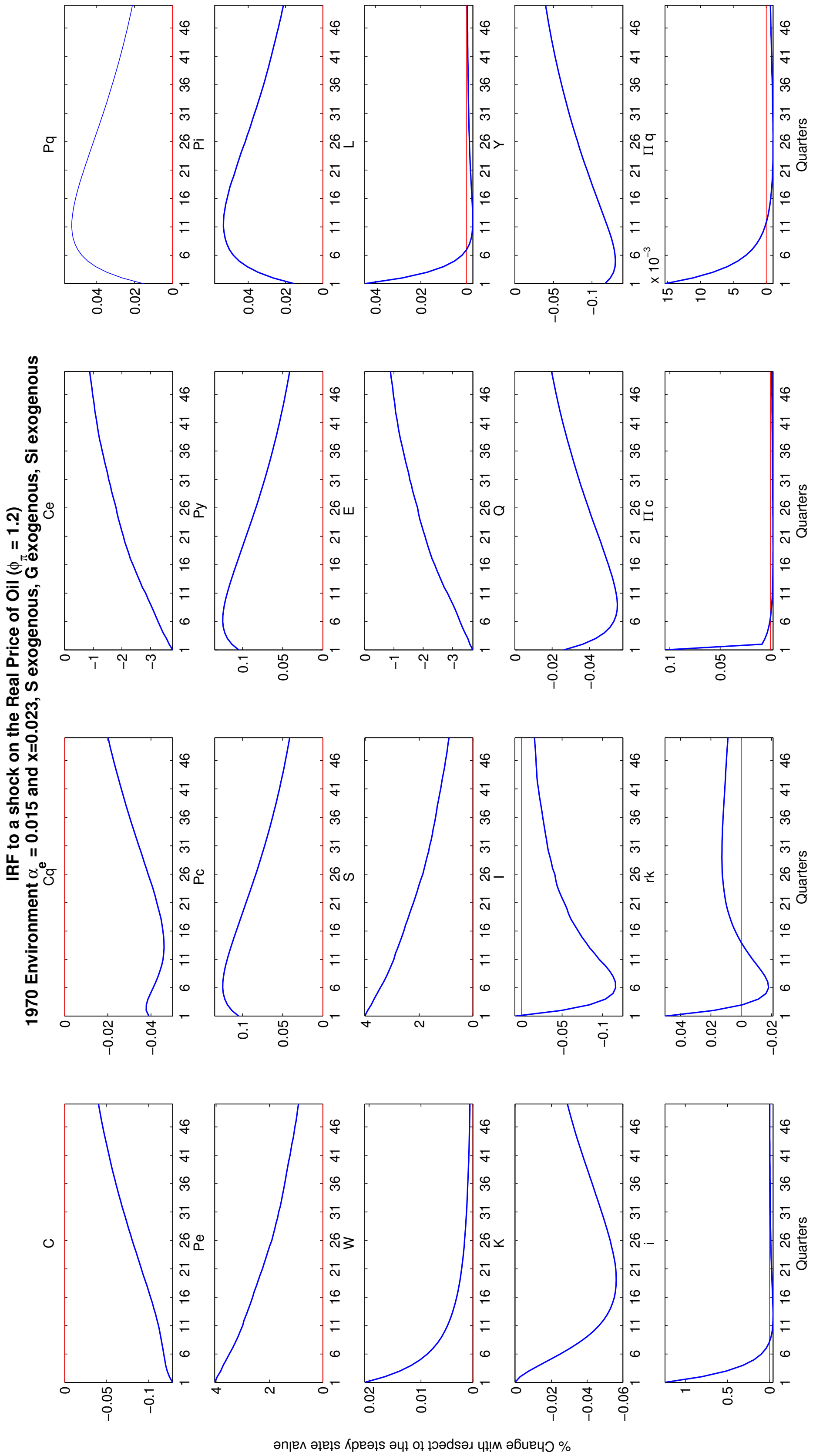

Figure 1: IRF to a shock on the Real Price of Oil

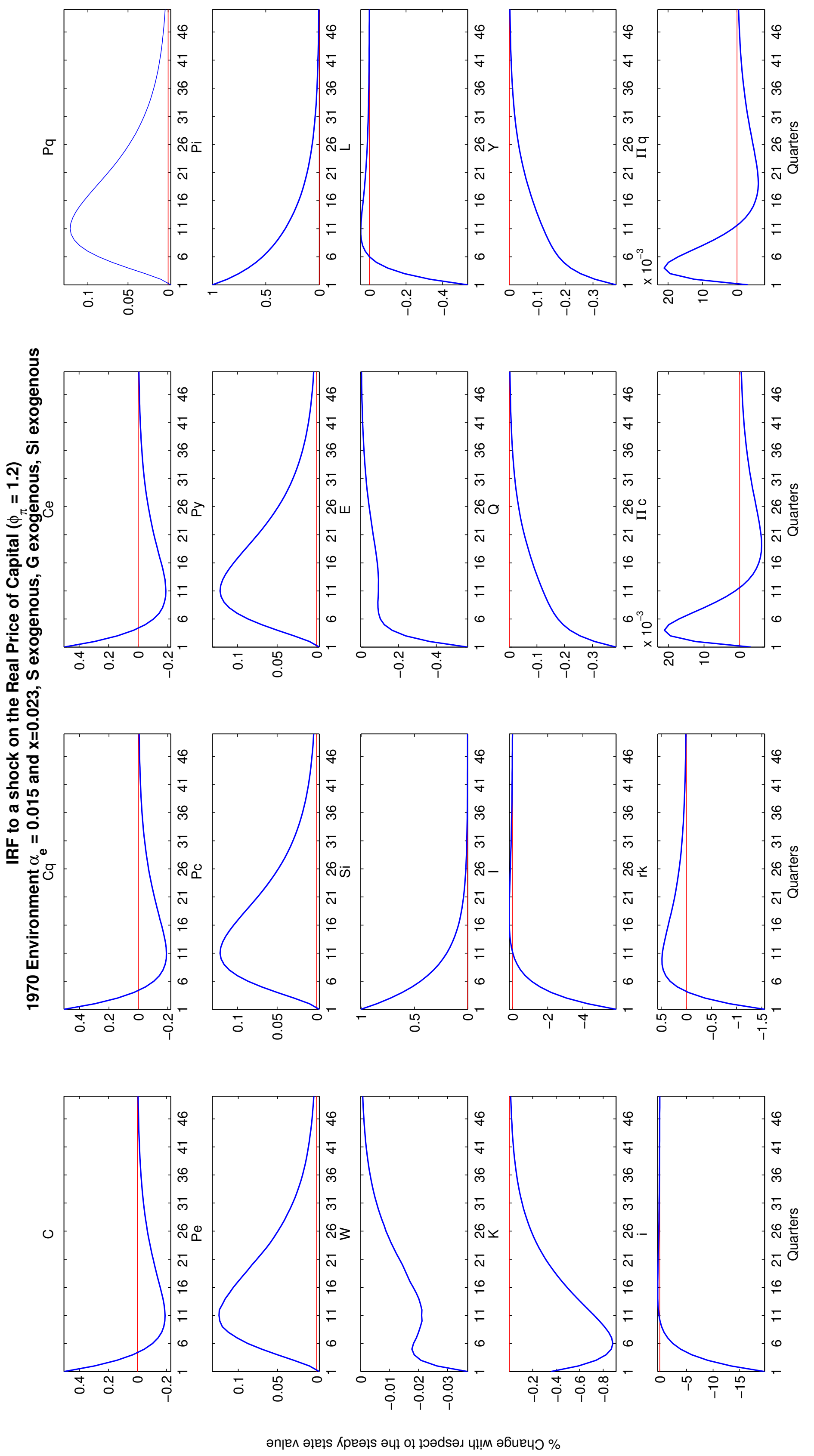

Figure 2: IRF to a shock on the Real Price of Capital

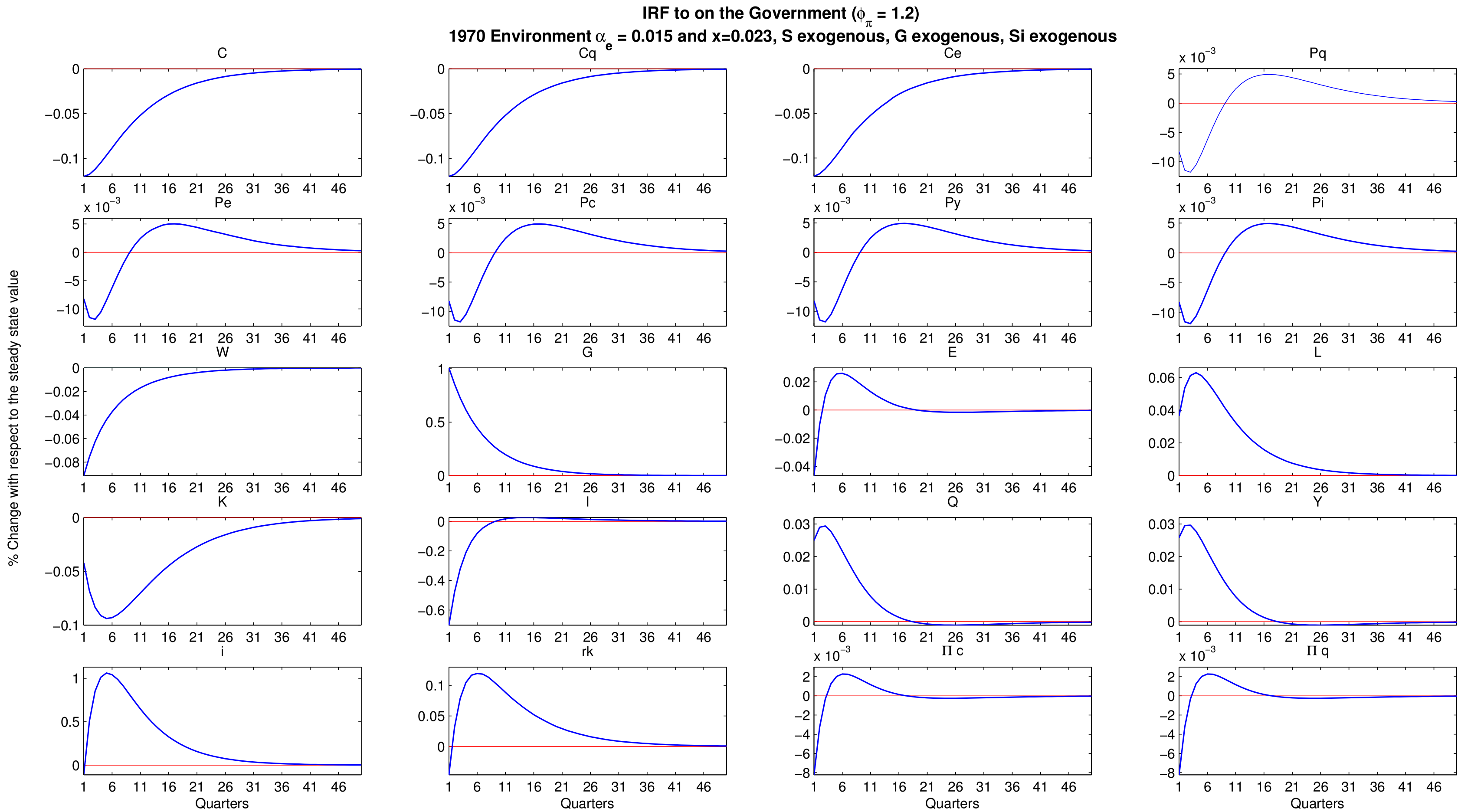

Figure 3: IRF to a shock on the Real Price of Government Spending

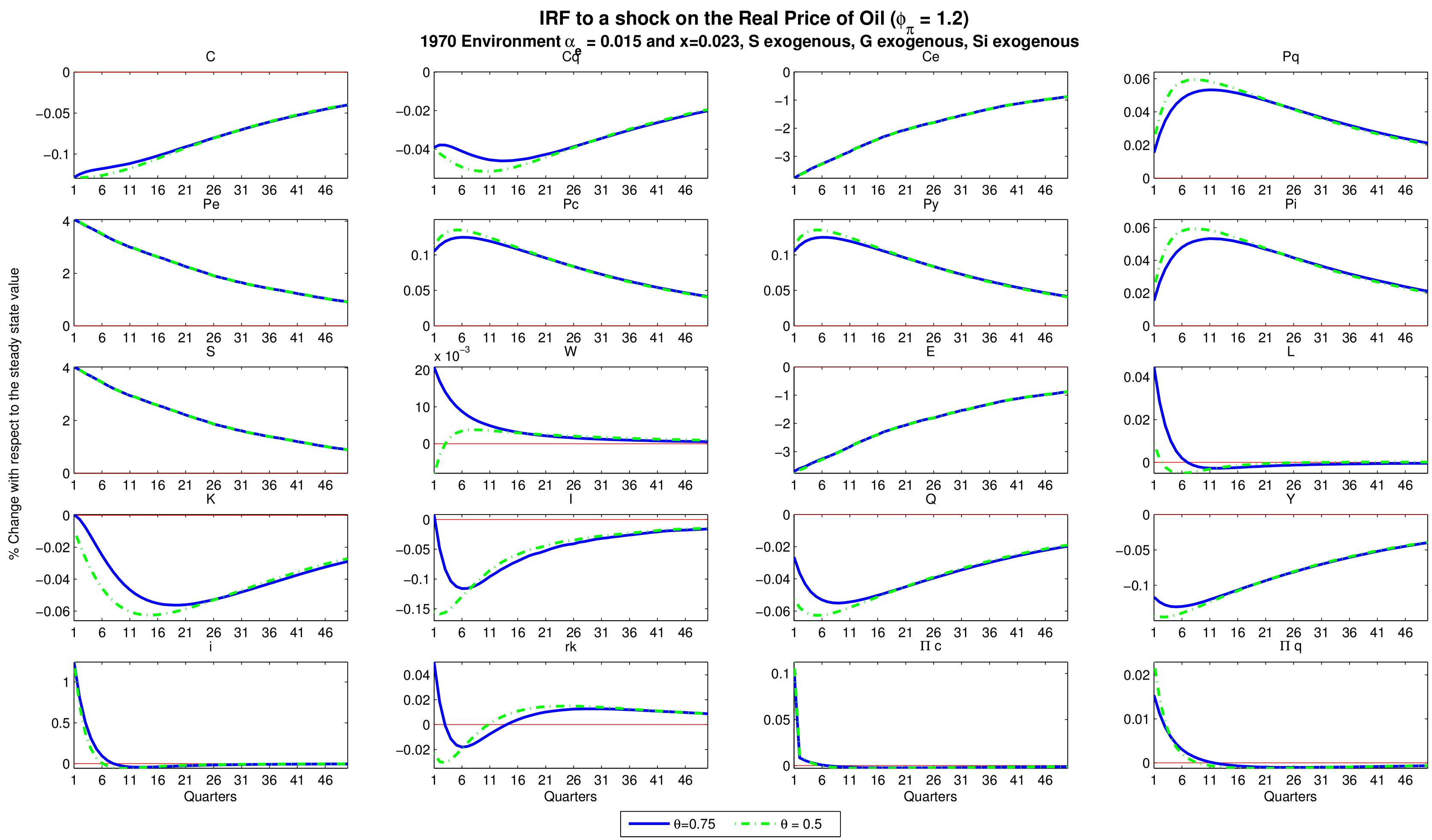

Figure 4: IRF to a shock on the Real Price of Oil

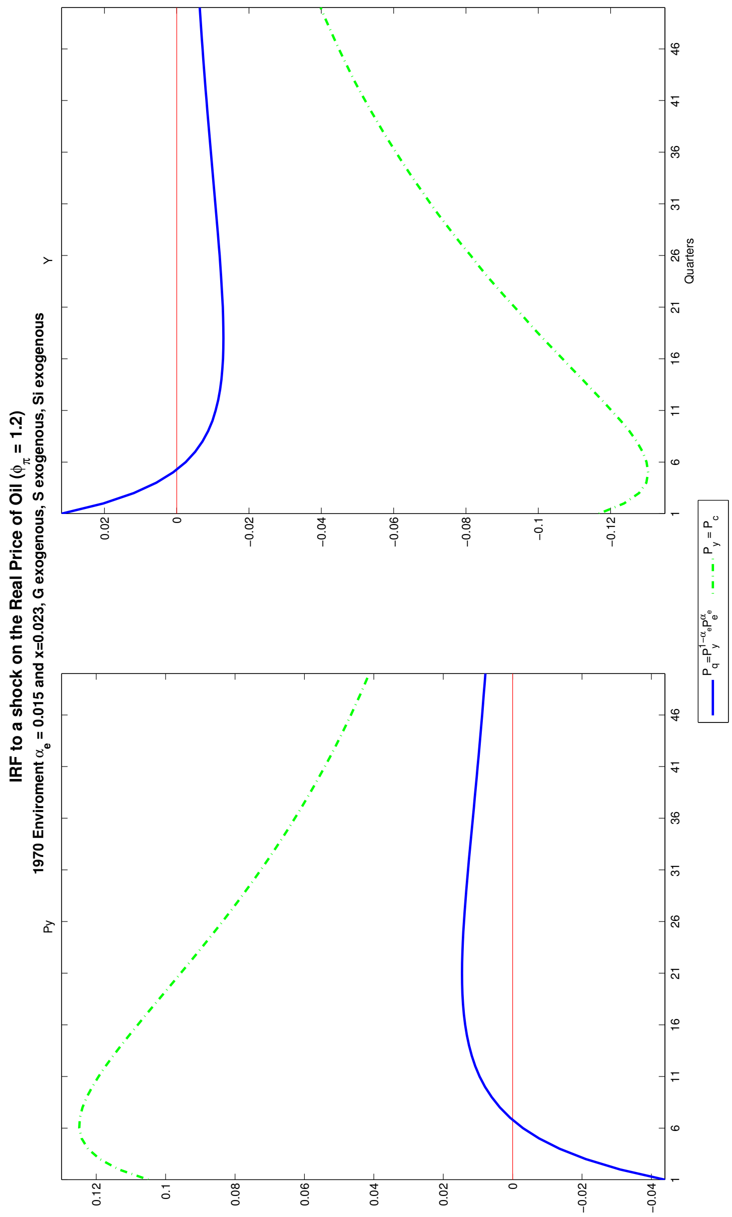

Figure 5: IRF to a shock on the Real Price of Oil

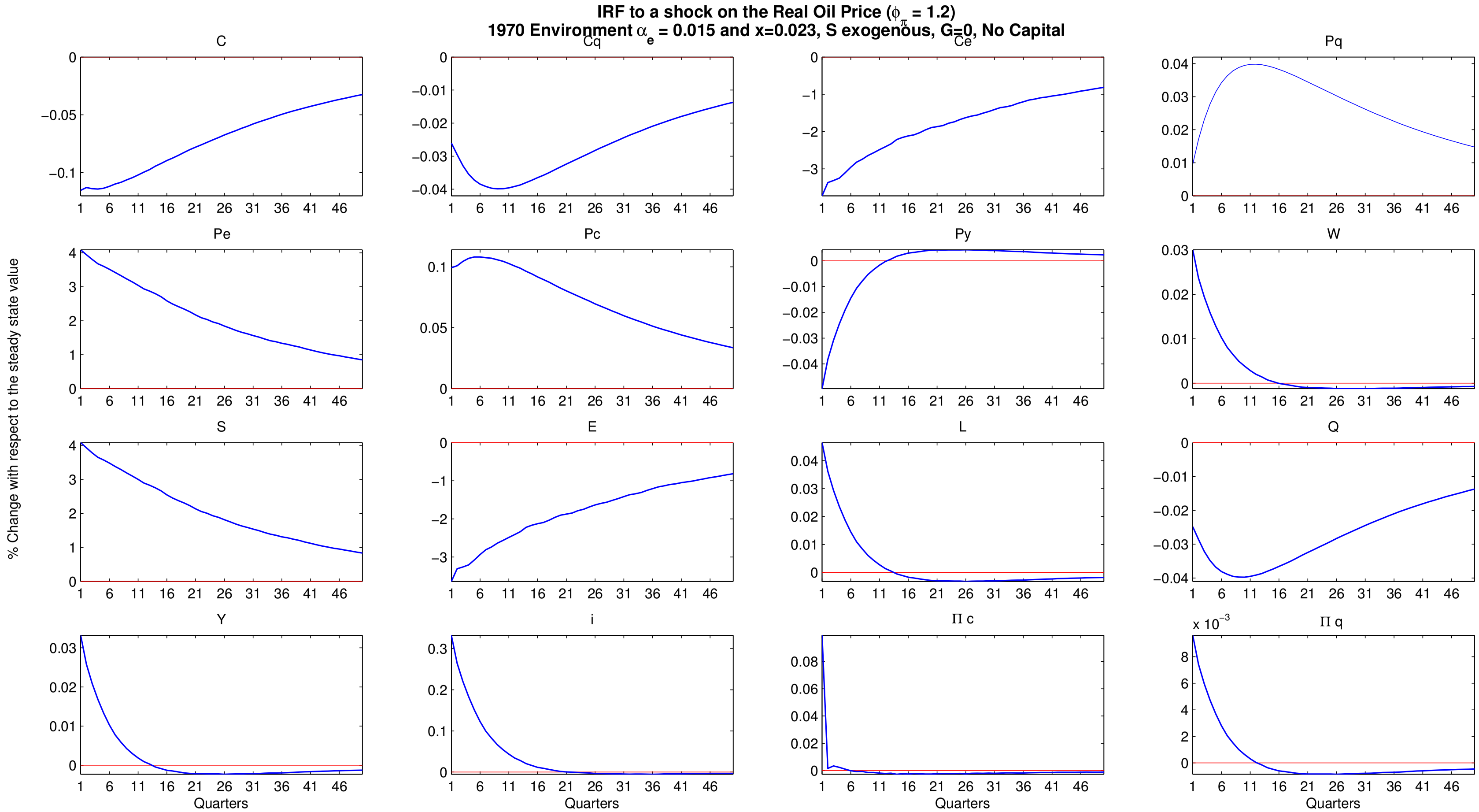

Figure 6: IRF to a shock on the Real Price of Oil

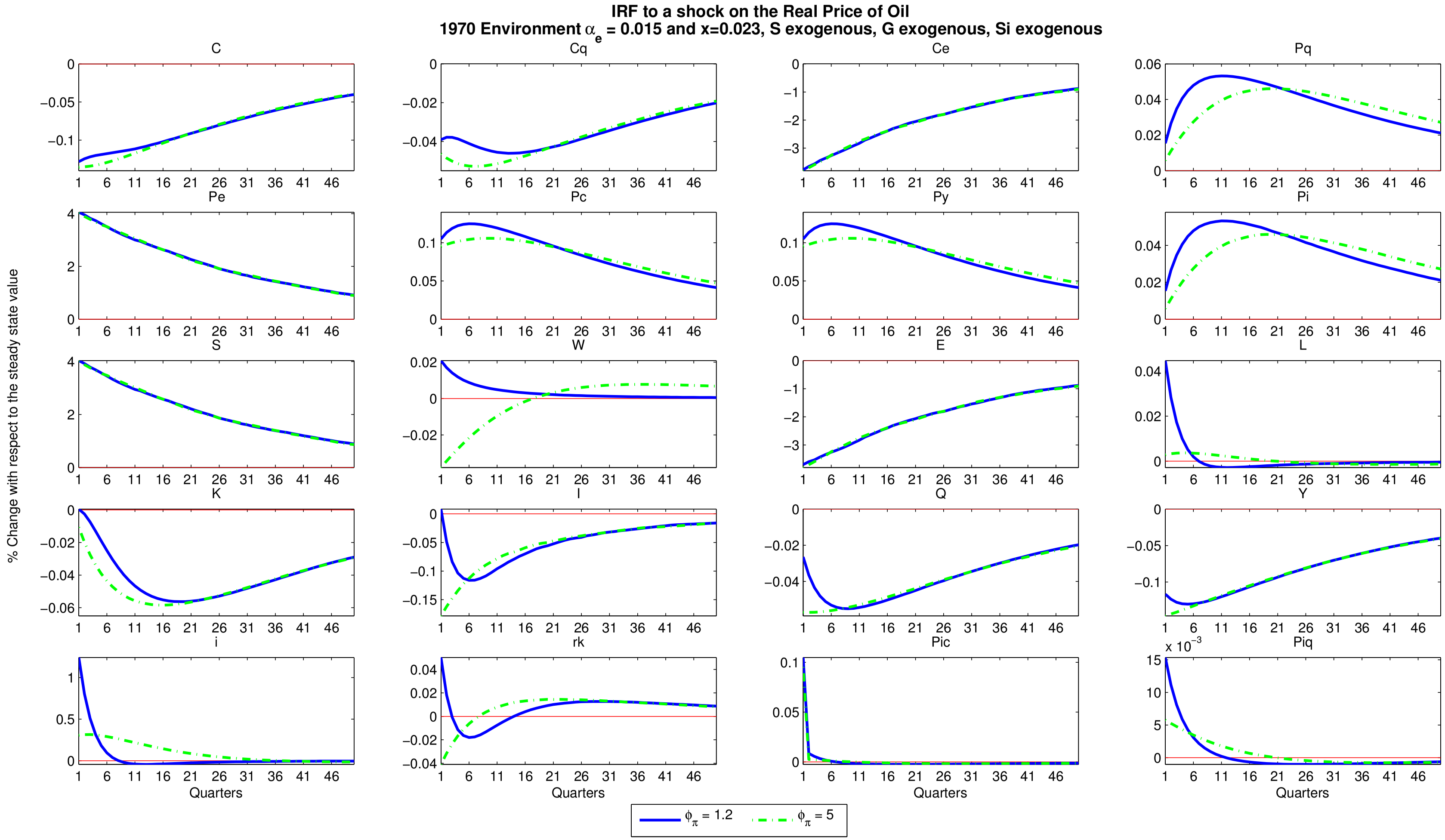

Figure 7: IRF to a shock on the Real Price of Oil

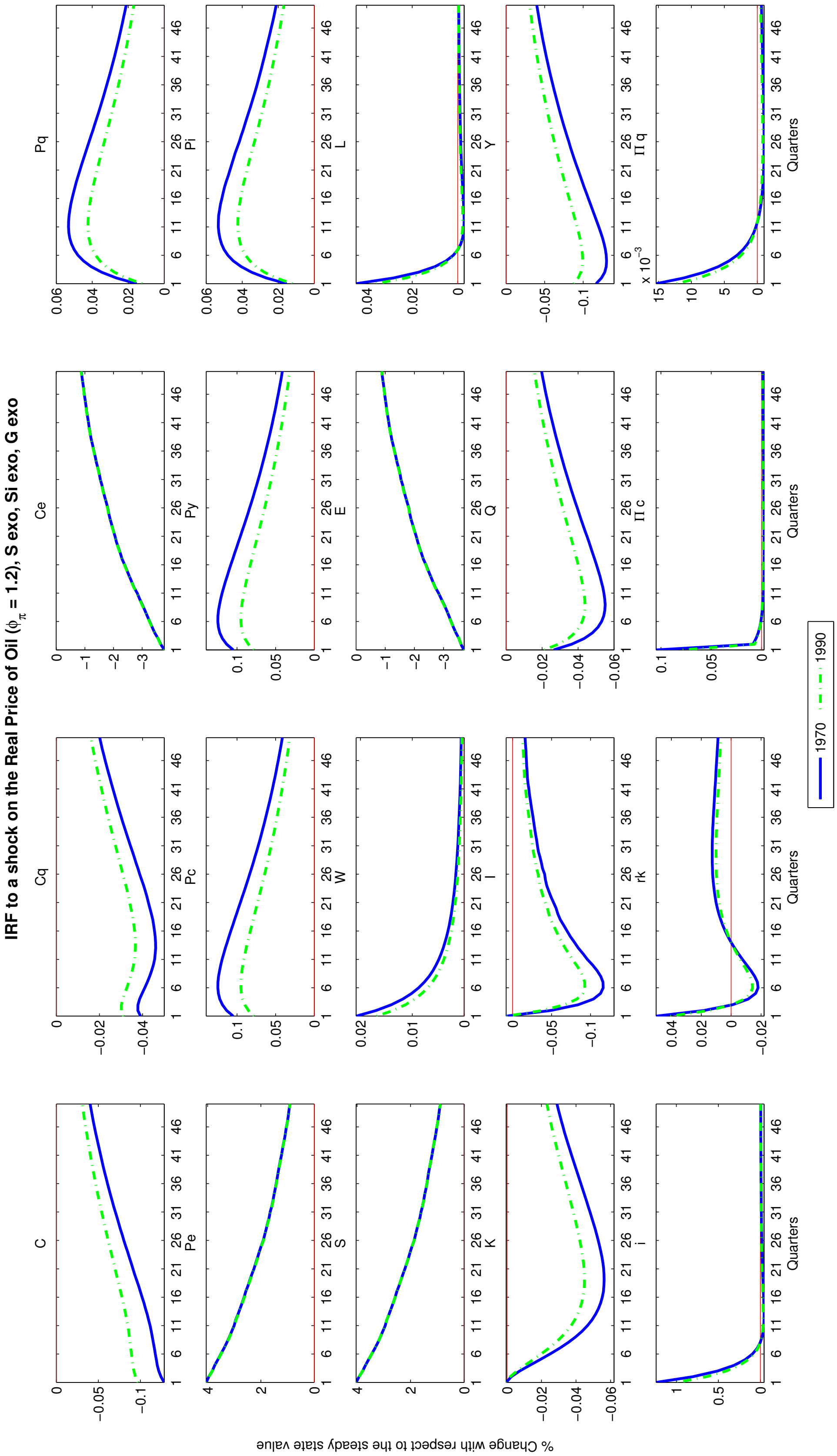

Figure 8: IRF to a shock on the Real Price of Oil

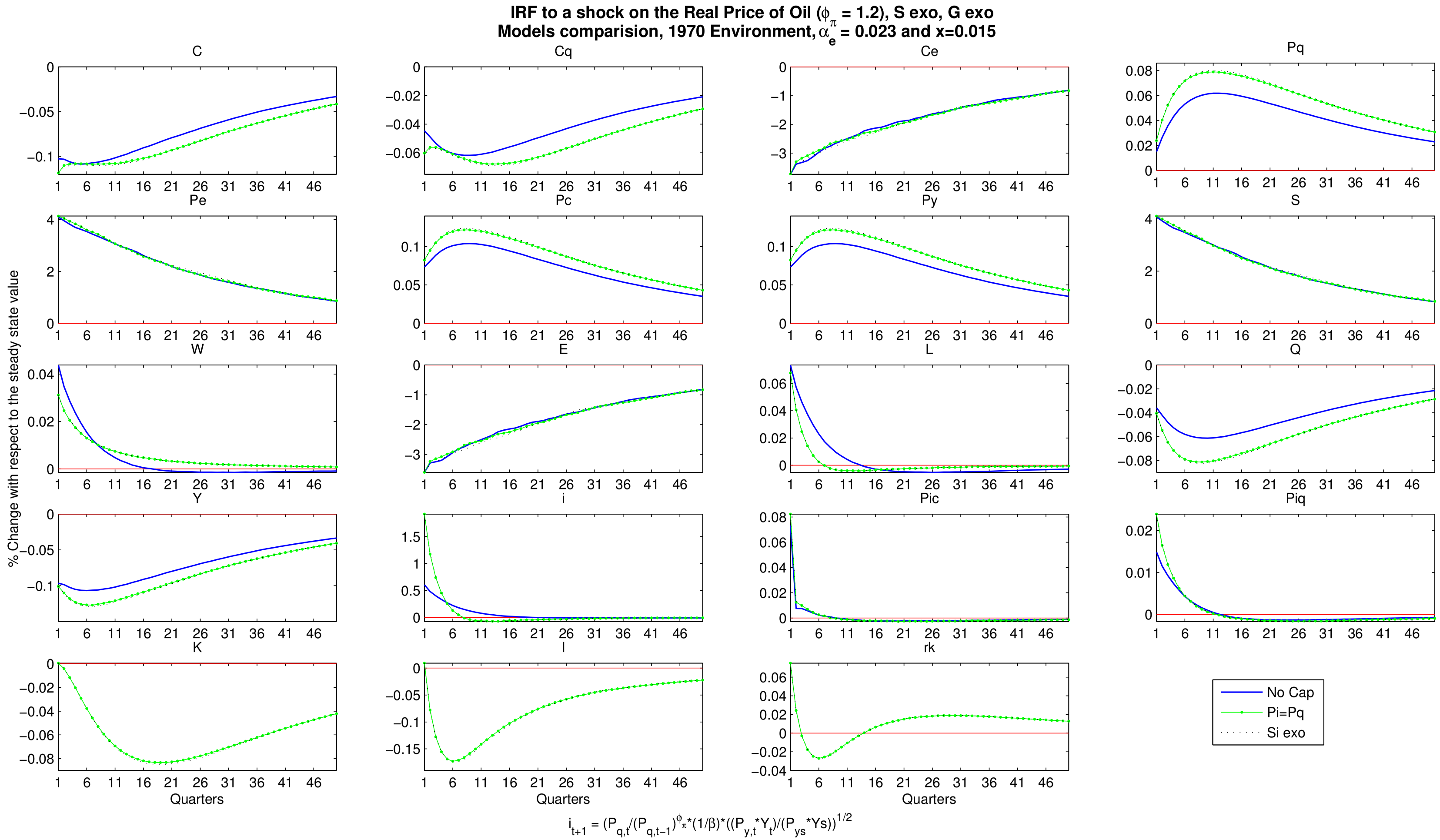

Figure 9: IRF to a shock on the Real Price of Oil