Prospect Theory and consumer behavior: Goals and Tradeoffs

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Résumé

Je montre qu'un consommateur averse à la perte qui doit répartir son budget entre deux biens préfère des allocations pour lesquelles la consommation est égale au point de référence pour au moins un des biens, ou des solutions en coin. L'intensité du phénomène dépend de la courbure de la fonction d'utilité. Ces résultats sont cohérents avec plusieurs faits stylisés qui ne peuvent pas être expliqués par la théorie standard du consommateur.

Mots-clés : Aversion pour la perte, théorie des perspectives.

Abstract

I show that a loss averse consumer who must share her budget between two goods prefer allocations for which consumption equals reference point for at least one good. The phenomenon intensity depends on the curvature of the utility curve. These results are consistent with several stylized facts which cannot be explained by the standard consumer theory

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1   Introduction

The standard consumer theory is based on the notion of utility maximization, through the search for the highest possible consumption, but in the reality, we set ourselves intermediary goals, which make decision-making and tradeoffs easier. These goals serve as reference points to assess our progress. It is therefore important to assess how these goals influence our behavior ; in particular, it turns out that agents behavior is often quite different depending on whether their consumption is above or below a given target.

These differences can be explained by the Prospect Theory. Prospect Theory was initially developped for risky choices (Kahneman and Tversky (1979)), then one of its components, namely loss aversion, was extended to certain choices (Tversky and Kahneman (1991)). But this extension was built mainly for labor supply (cf. *infra*) or for discrete choices (and more precisely binary choices, which are the closest to the traditional framework of risky choices), whereas the theory of the consumer behavior relies on continuous choices under a budgetary constraint.

Thus, Köszegi and Rabin (2006) deal with the issue of the reference point formation when there exists an initial uncertainty, which disappears before the final consumption decision. From this standpoint, their model is an intermediary step between risky and certain choice, not an analysis of certain choice *stricto sensu* ; in particular, their model continues to focus on binary choices, between buying and not buying for a single good, or between two lump goods.

In the next section, I study the optimal choice of a consumer who must share her budget between two goods and who is loss averse. I successively consider three functional forms for the utility function :

– linear,

– convex in losses and concave in gains,

– and finally concave everywhere.

In the third section, I discuss several behaviors which cannot be explained in the standard consumer theory, but which are consistent with loss aversion

2   Models

Prospect Theory endows the consumer with a utility function which is slightly concave in gains (when the consumption is higher than the reference point), and slightly convex in losses, with a stronger slope in losses than in gains. In practical applications, a tractable and often

used approximation is to consider an utility function which is piecewise linear, with a kink at the reference point.

On the contrary, standard consumer theory relies on utility functions which are everywhere concave, and which have no reference point (i.e. utility is equal to zero for a null consumption).

I will therefore proceed in three steps, and consider the effect of loss aversion in three cases :

– First, in the simplest case, namely a linear utility function ;
– Second, when the utility function is S-shaped as in the Prospect Theory ;
– Finally, when the utility is standard, that is concave, but has a reference point.

2.1 Linear utility

I begin with the case of a linear utility function with two goods, c_1 and c_2 :

$$U(c_1, c_2) = c_1 + \beta * c_2$$

$$s.t.\ p_1 c_1 + p_2 c_2 = W$$

with β the relative weight of the good 2 in terms of the good 1 in the utility function.

Figure 1 shows the graph of the utility function and the corresponding indifference curves. On a side note, for all figures in this article, I took $\beta = p_1 = p_2 = 1$ without loss of generality.

Figure 1: Linear Utility

(a)   (b)
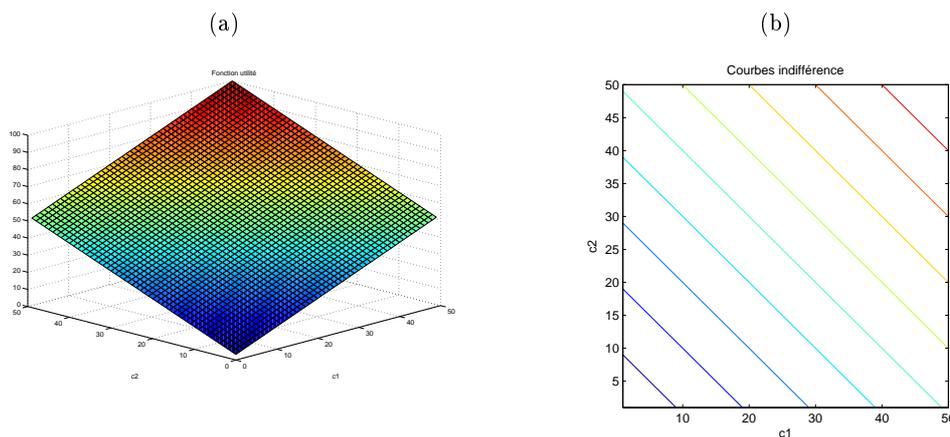
The solution of the consumer's utility maximization program is a corner solution if the slope of the budget constraint line is not equal to the slope of the indifference curves. If both slopes are equal, the consumer is indifferent between all possible allocations on her budget constraint.

Figure 2 shows an example of a possible budget line. The optimal solution is to allocate the whole budget to good 2.

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Figure 2: Linear Utility – Budget Line
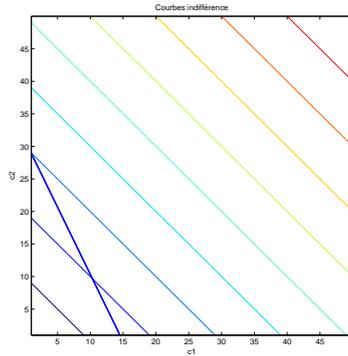
I now introduce loss aversion for each of the two goods. That is to say, for each good, the consumer has a reference point, at which utility is equal to zero.

The utility function is then

$$U(c_1, c_2) = u_1(c_1) + \beta * u_2(c_2)$$

$$s.t. \ p_1 c_1 + p_2 c_2 = W$$

with β the relative weight of good 2 with respect to the good 1, λ the loss aversion coefficient, W the consumer's budget, and

$$u_i(c_i) = c_i - r_i \quad \text{si } c_i \geq r_i$$

$$u_i(c_i) = -\lambda(r_i - c_i) \quad \text{si } c_i < r_i$$

where r_i is the reference point for good i, and c_i is the consumption of good i.

Figure 3 shows the utility function and the corresponding indifference curves, for a vector of reference points $(20, 20)$.

As the utility function for each good is stepwise linear, the indifference curves are also stepwise linear, and the Marginal Rate of Substitution is a step function (i.e. stepwise constant). We can notice that MRS are equal in quadrants South-West and North-East, and they are the inverse of each other in quadrants South-East and North-West.

Therefore, there always exists corner solutions, when the relative price is very low or very high. Formally, we have a corner solution when $\frac{p_2}{p_1} < \frac{\beta}{\lambda}$ or $\frac{p_2}{p_1} > \beta\lambda$. But when the relative price is between these two boundaries, there exists a new type of solutions, which we could call "interior corner solutions" : the indifference curves present kinks at the reference point for each

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Figure 3: Linear Utility with Loss Aversion

(b)
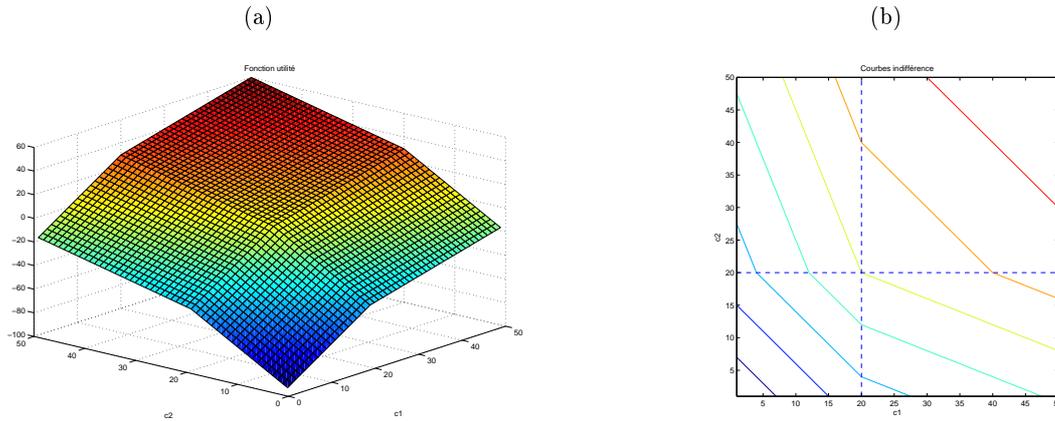
good, and thus it is optimal for the consumer to set the consumption of one of the two goods at the corresponding reference point, and to adjust the consumption of the other good accordingly. Figure 4 shows an example of a classical corner solution, and an example of the new solution type.

Figure 4: Linear Utility with Loss Aversion – Optimal Solutions

(a) (b)
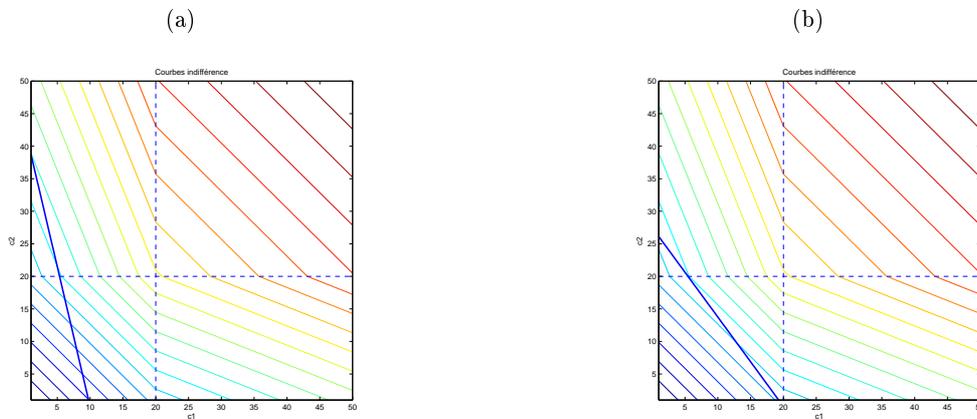
2.2   Prospect Theory

I will now consider the case when the consumer has a Prospect Theory utility function :

$$U(c_1, c_2) = u_1(c_1) + \beta * u_2(c_2)$$

$$s.t. \ p_1 c_1 + p_2 c_2 = W$$

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with β the relative weight of the good 2 in terms of the good 1, λ the loss aversion coefficient, W the consumer's budget, and

$$u_i(c_i) = (c_i - r_i)^\alpha \quad \text{si } c_i \geq r_i$$

$$u_i(c_i) = -\lambda(r_i - c_i)^\alpha \quad \text{si } c_i < r_i$$

Figure 5 shows the utility function and the corresponding indifference curves, for a vector of reference points $(20, 20)$. I took $\alpha = 0.5$ to make more salient the convex and concave parts of the utility function, even though the standard value in the literature is $\alpha = 0.88$.

Figure 5: Utility Function of the Prospect Theory

(a)

(b)
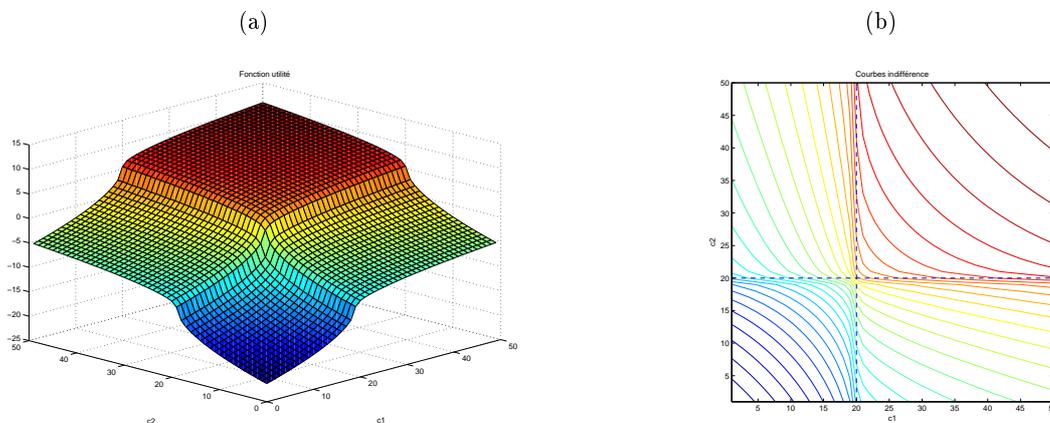
The solution of the consumer's program is not obvious : the shape of the utility function changes according to the position of the consumption vector (c_1, c_2) with respect to the reference points vector (r_1, r_2). In the general case, there are four possible situations, corresponding to the four "quadrants" defined by the reference points vector :

- $c_1 \geq r_1$ and $c_2 \geq r_2$;
- $c_1 \geq r_1$ and $c_2 \leq r_2$;
- $c_1 \leq r_1$ and $c_2 \geq r_2$;
- $c_1 \leq r_1$ and $c_2 \leq r_2$;

As the shape of the utility function changes from one quadrant to the next, instead of being everywhere concave, we cannot determine the optimal solution for the consumer program directly through variational methods (first and second order conditions). For each quadrant, we need to calculate the locally optimal solutions, whether they are interior, corner, or "interior corner" solutions ; then we will be able to determine the global optimum by comparing all the local optima that are actually inside the budget set.

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2.2.1 North East Quadrant : $c_1 \geq r_1$ and $c_2 \geq r_2$

Interior Solution In the NE quadrant of the consumption space, we have

$$U(c_1, c_2) = (c_1 - r_1)^\alpha + \beta(c_2 - r_2)^\alpha \; s.t. \; p_1 c_1 + p_2 c_2 = W$$

Or, if we remplace c_2 by its expression in the budget constraint

$$U(c_1) = (c_1 - r_1)^\alpha + \beta \left(\frac{W - p_1 c_1}{p_2} - r_2 \right)^\alpha$$

We take the first derivative :

$$\frac{\partial U}{\partial c_1}(c_1) = \alpha(c_1 - r_1)^{\alpha-1} + \alpha\beta \left(\frac{-p_1}{p_2} \right) \left(\frac{W - p_1 c_1}{p_2} - r_2 \right)^{\alpha-1}$$

This derivative is equal to zero if

$$\alpha(c_1^* - r_1)^{\alpha-1} = \alpha\beta \left(\frac{p_1}{p_2} \right) \left(\frac{W - p_1 c_1^*}{p_2} - r_2 \right)^{\alpha-1}$$

$$c_1^* - r_1 = \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{\alpha-1}} \left(\frac{W - p_1 c_1^*}{p_2} - r_2 \right)$$

$$c_1^* - r_1 = \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{\alpha-1}} \left(\frac{W}{p_2} - r_2 \right) - \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right) c_1^*$$

$$c_1^* + \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right) c_1^* = r_1 + \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{\alpha-1}} \left(\frac{W}{p_2} - r_2 \right)$$

$$\left[1 + \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{\alpha}{\alpha-1}} \right] c_1^* = r_1 + \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{\alpha-1}} \left(\frac{W}{p_2} - r_2 \right)$$

$$c_1^* = \left[1 + \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{\alpha}{\alpha-1}} \right]^{-1} r_1 + \frac{\beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{\alpha-1}}}{1 + \beta^{\frac{1}{\alpha-1}} \left(\frac{p_1}{p_2} \right)^{\frac{\alpha}{\alpha-1}}} \left(\frac{W}{p_2} - r_2 \right)$$

We can notice that for $\beta = p_1 = p_2 = 1$ and $r_1 = r_2$ (the goods are perfectly symmetrical), we have $c_1^* = c_2^* = \frac{W}{2}$, which is consistent with the intuition. If $\beta = p_1 = p_2 = 1$ but $r_1 \neq r_2$, we have

$$c_1^* = \frac{1}{2}(r_1 - r_2) + \frac{W}{2}, \text{ and thus } c_1^* - r_1 = c_2 - r_2 = \frac{W}{2} - \frac{1}{2}(r_1 + r_2)$$

That is to say, the consumer has an equal "net consumption" (consumption minus reference point) for each good.

Let's take the second order derivative :

$$\frac{\partial^2 U}{\partial (c_1)^2}(c_1) = \alpha(\alpha - 1)(c_1 - r_1)^{\alpha-2} + \alpha(\alpha - 1)\beta \left(\frac{-p_1}{p_2} \right)^2 \left(\frac{W - p_1 c_1}{p_2} - r_2 \right)^{\alpha-2}$$

The corresponding second order condition (after simplifying by $\alpha(\alpha - 1) < 0$) is

$$(c_1^* - r_1)^{\alpha-2} + \beta \left(\frac{p_1}{p_2}\right)^2 \left(\frac{W - p_1 c_1^*}{p_2} - r_2\right)^{\alpha-2} \geq 0$$

This condition is fulfilled for $c_1 \geq r_1$, $c_2 \geq r_2$ and $\alpha \leq 1$.

This means we can exclude "frontier" solutions $c_1 = r_1$, $c_2 > r_2$ et $c_1 > r_1$, $c_2 = r_2$. Indeed, if we consider the first possibility ($c_1 = r_1$, $c_2 > r_2$), we have

$$U(r_1) \quad = U\left(c_1^*\right) + U'\left(c_1^*\right)\left(r_1 - c_1^*\right) + U''\left(c_1^*\right)\left(r_1 - c_1^*\right)^2$$

$$= U\left(c_1^*\right) + U''\left(c_1^*\right)\left(r_1 - c_1^*\right)^2 < U\left(c_1^*\right)$$

by concavity of the utility function in this quadrant. Therefore, we cannot have a frontier solution between this quadrant and another one, as the interior solution gives a higher utility. We can exclude the second type of frontier solutions with a similar line of reasoning.

2.2.2 South-East Quadrant: $c_1 \geq r_1$ and $c_2 < r_2$

Interior Solution In the SE quadrant, we have

$$U(c_1, c_2) = (c_1 - r_1)^{\alpha} - \beta\lambda(r_2 - c_2)^{\alpha} \ s.t. \ p_1 c_1 + p_2 c_2 = W$$

or, if we replace c_2 by its expression in the budget constraint :

$$U(c_1) = (c_1 - r_1)^{\alpha} - \beta\lambda\left(r_2 - \frac{W - p_1 c_1}{p_2}\right)^{\alpha}$$

We take the first derivative :

$$\frac{\partial U}{\partial c_1}(c_1) = \alpha(c_1 - r_1)^{\alpha-1} - \alpha\beta\lambda\left(\frac{p_1}{p_2}\right)\left(r_2 - \frac{W - p_1 c_1}{p_2}\right)^{\alpha-1}$$

The corresponding first order condition (after simplifying by $\alpha > 0$) is

$$(c_1 - r_1)^{\alpha-1} = \beta\lambda\left(\frac{p_1}{p_2}\right)\left(r_2 - \frac{W - p_1 c_1}{p_2}\right)^{\alpha-1}$$

that is

$$\left(r_2 - \frac{W - p_1 c_1}{p_2}\right)^{1-\alpha} \quad = \beta\lambda\left(\frac{p_1}{p_2}\right)(c_1 - r_1)^{1-\alpha}$$

$$r_2 - \frac{W - p_1 c_1}{p_2} \quad = \left[\beta\lambda\left(\frac{p_1}{p_2}\right)\right]^{\frac{1}{1-\alpha}}(c_1 - r_1)$$

$$r_2 + \left[\beta\lambda\left(\frac{p_1}{p_2}\right)\right]^{\frac{1}{1-\alpha}} r_1 - \frac{W}{p_2} \quad = \left[\beta\lambda\left(\frac{p_1}{p_2}\right)\right]^{\frac{1}{1-\alpha}} c_1 + \left(\frac{p_1}{p_2}\right) c_1$$

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And eventually

$$c_1^* = \left[\left(\frac{p_1}{p_2}\right) + \left(\beta\lambda\left(\frac{p_1}{p_2}\right)\right)^{\frac{1}{1-\alpha}}\right]^{-1}\left[r_2 + \left(\beta\lambda\left(\frac{p_1}{p_2}\right)\right)^{\frac{1}{1-\alpha}} r_1 - \frac{W}{p_2}\right]$$

Let's take the second order derivative and calculate the second order condition for this equation to define a local maximum :

$$\frac{\partial^2 U}{\partial(c_1)^2}(c_1) = \alpha(\alpha-1)(c_1-r_1)^{\alpha-2} - \alpha(\alpha-1)\beta\lambda\left(\frac{p_1}{p_2}\right)^2\left(r_2 - \frac{W-p_1c_1}{p_2}\right)^{\alpha-2}$$

The SOC in c_1^* is $\frac{\partial^2 U}{\partial(c_1)^2}(c_1^*) \leq 0$, or if we simplify by $\alpha(\alpha-1) < 0$,

$$(c_1-r_1)^{\alpha-2} - \beta\lambda\left(\frac{p_1}{p_2}\right)^2\left(r_2 - \frac{W-p_1c_1}{p_2}\right)^{\alpha-2} \geq 0$$

We can replace $(c_1^*-r_1)^{\alpha-1}$ by its expression in the FOC :

$$(c_1^*-r_1)^{-1}\left[\beta\lambda\left(\frac{p_1}{p_2}\right)\left(r_2 - \frac{W-p_1c_1^*}{p_2}\right)^{\alpha-1}\right] - \beta\lambda\left(\frac{p_1}{p_2}\right)^2\left(r_2 - \frac{W-p_1c_1^*}{p_2}\right)^{\alpha-2} \geq 0$$

which means we can greatly simplify the inequality :

$$(c_1^*-r_1)^{-1} - \left(\frac{p_1}{p_2}\right)\left(r_2 - \frac{W-p_1c_1^*}{p_2}\right)^{-1} \geq 0$$

or

$$r_2 - \frac{W-p_1c_1^*}{p_2} \geq \left(\frac{p_1}{p_2}\right)(c_1^*-r_1)$$

And eventually (after multiplying by p_2) :

$$p_2r_2 + p_1r_1 \geq W$$

This inequality is the SOC for the result of the FOC to be a local maximum. If it is not fulfilled, the locally optimal solution is a corner solution. This inequality has a straightforward interpretation : for the optimal solution to imply $c_2 < r_2$, a necessary condition is that the basket (r_1, r_2) is not part of the budget set for prices p_1 and p_2, and wealth W.

Corner Solutions The only possible corner solution in this quadrant is $c_1 \geq r_1$ and $c_2 = 0$. Let us calculate the first order derivative of the utility function in c_1 for $c_1 = W/p_1$, $c_2 = 0$:

$$\frac{\partial U}{\partial c_1}\left(\frac{W}{p_1}\right) = \alpha\left(\frac{W}{p_1} - r_1\right)^{\alpha-1} - \alpha\beta\lambda\left(\frac{p_1}{p_2}\right)(r_2)^{\alpha-1}$$

A necessary condition for a corner solution is that this derivative be positive :

$$\frac{\partial U}{\partial c_1}\left(\frac{W}{p_1}\right) \geq 0$$

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or

$$\alpha \left(\tfrac{W}{p_1} - r_1\right)^{\alpha-1} - \alpha\beta\lambda \left(\tfrac{p_1}{p_2}\right) r_2^{\alpha-1} \;\geq 0$$

$$\left(\tfrac{W}{p_1} - r_1\right)^{\alpha-1} \;\geq \beta\lambda \left(\tfrac{p_1}{p_2}\right) r_2^{\alpha-1}$$

$$r_2^{1-\alpha} \;\geq \beta\lambda \left(\tfrac{p_1}{p_2}\right) \left(\tfrac{W}{p_1} - r_1\right)^{1-\alpha}$$

$$r_2 \;\geq \left[\beta\lambda \left(\tfrac{p_1}{p_2}\right)\right]^{\frac{1}{1-\alpha}} \left(\tfrac{W}{p_1} - r_1\right)$$

we can see that for $\beta = p_1 = p_2 = 1$, this result yields

$$(c_1 - r_1)^{\alpha-1} \geq \lambda r_2^{\alpha-1}$$

This conclusion has an interpretation in terms of the utility function curvature : in the standard case, utility is concave with respect to each good, therefore the more we diminish the consumption of one good, the more the marginal utility corresponding to this good increases, and the more painful any additional decrease in the consumption of this good. But here, utility is convex with respect to the good 2. This means that the more we diminish the consumption of this good, the more the marginal utility of this good decreases. If we cross the threshold when marginal utility for both goods is equal (e.g. when the reference point for good 2 is very high), the consumer would be better off suppressing all consumption of good 2 and allocating all her budget to the consumption of good 1. Figure 6 illustrates this phenomenon.

Figure 6: Corner solution in the SE quadrant

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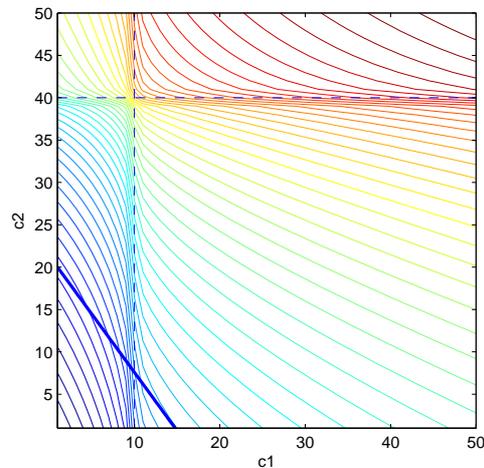
Frontier Solutions We don't need to consider the frontier solution between the SE and NE quadrants. Indeed, even if this frontier solution grants the consumer a higher utility than the interior solution of the SE quadrant, the interior solution of the NE quadrant dominates this frontier solution, which therefore cannot be a global maximum.

The only remaining possible frontier solution is $c_1 = r_1$, $c_2 < r_2$. We can apply the same line of reasoning as for the NE quadrant :

$$
\begin{aligned}
U(r_1) \quad &= U\left(c_1^*\right) + U'\left(c_1^*\right)\left(r_1 - c_1^*\right) + U''\left(c_1^*\right)\left(r_1 - c_1^*\right)^2 \\
\\
&= U\left(c_1^*\right) + U''\left(c_1^*\right)\left(r_1 - c_1^*\right)^2 < U\left(c_1^*\right)
\end{aligned}
$$

if the previous SOC is fulfilled. In this case, there cannot be any frontier solution because the interior solution grants the consumer a higher utility.

2.2.3 North-West quadrant: $c_1 < r_1$ et $c_2 \geq r_2$

The treatment of the NW quadrant is similar to the treatment of the SE quadrant. The utility function is

$$
U(c_1, c_2) = -\lambda(r_1 - c_1)^\alpha + \beta(c_2 - r_2)\alpha \ s.t. \ p_1 c_1 + p_2 c_2 = W
$$

Which means that

$$
U(c_1) = -\lambda(r_1 - c_1)^\alpha + \beta \left(\frac{W - p_1 c_1}{p_2} - r_2 \right)^\alpha
$$

The first derivative of the utility function is

$$
\frac{\partial U}{\partial c_1}(c_1) = \lambda\alpha(r_1 - c_1)^{\alpha-1} - \alpha\left(\frac{p_1}{p_2} \right)\beta\left(\frac{W - p_1 c_1}{p_2} - r_2 \right)^{\alpha-1}
$$

The corresponding FOC is

$$
\begin{aligned}
\lambda(r_1 - c_1)^{\alpha-1} \quad &= \left(\tfrac{p_1}{p_2} \right)\beta\left(\tfrac{W - p_1 c_1}{p_2} - r_2 \right)^{\alpha-1} \\
\\
\left(\tfrac{W - p_1 c_1}{p_2} - r_2 \right)^{1-\alpha} \quad &= \left(\tfrac{p_1}{p_2} \right)\left(\tfrac{\beta}{\lambda} \right)(r_1 - c_1)^{1-\alpha} \\
\\
W - \tfrac{p_1}{p_2} c_1 - r_2 \quad &= \left(\tfrac{p_1}{p_2} * \tfrac{\beta}{\lambda} \right)^{\frac{1}{1-\alpha}}(r_1 - c_1) \\
\\
W - r_2 - \left(\tfrac{p_1}{p_2} * \tfrac{\beta}{\lambda} \right)^{\frac{1}{1-\alpha}} r_1 \quad &= \tfrac{p_1}{p_2} c_1 - \left(\tfrac{p_1}{p_2} * \tfrac{\beta}{\lambda} \right)^{\frac{1}{1-\alpha}} c_1 \\
\\
c_1^* \quad &= \left[\tfrac{p_1}{p_2} - \left(\tfrac{p_1}{p_2} * \tfrac{\beta}{\lambda} \right)^{\frac{1}{1-\alpha}} \right]^{-1}\left[W - r_2 - \left(\tfrac{p_1}{p_2} * \tfrac{\beta}{\lambda} \right)^{\frac{1}{1-\alpha}} r_1 \right]
\end{aligned}
$$

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The second derivative of the utility function is

$$\frac{\partial^2 U}{\partial (c_1)^2}(c_1) = -\lambda\alpha(\alpha-1)(r_1-c_1)^{\alpha-2} + \alpha(\alpha-1)\left(\frac{p_1}{p_2}\right)^2 \beta\left(\frac{W-p_1 c_1}{p_2}-r_2\right)^{\alpha-2}$$

The corresponding SOC is

$$\lambda(r_1-c_1)^{\alpha-2} - \left(\frac{p_1}{p_2}\right)^2 \beta\left(\frac{W-p_1 c_1}{p_2}-r_2\right)^{\alpha-2} \leq 0$$

Here again, we can replace $\lambda(r_1-c_1)^{\alpha-1}$ by its expresssion in the FOC when $c_1 = c_1^*$:

$$\left(\frac{p_1}{p_2}\right)\beta\left(\frac{W-p_1 c_1^*}{p_2}-r_2\right)^{\alpha-1}(r_1-c_1^*)^{-1} - \left(\frac{p_1}{p_2}\right)^2 \beta\left(\frac{W-p_1 c_1^*}{p_2}-r_2\right)^{\alpha-2} \leq 0$$

Then we simplify :

$$(r_1-c_1^*)^{-1} - \left(\frac{p_1}{p_2}\right)\left(\frac{W-p_1 c_1^*}{p_2}-r_2\right)^{-1} \leq 0$$

$$\frac{W-p_1 c_1^*}{p_2} - r_2 \leq \left(\frac{p_1}{p_2}\right)(r_1-c_1^*)$$

$$W - p_1 c_1^* - p_2 r_2 \leq p_1(r_1-c_1^*)$$

$$W \leq p_1 r_1 + p_2 r_2$$

We find the same SOC as for the SE quadrant, which is logical as the 2 goods are symmetrical.

Corner Solutions   The only possible corner solution is $c_1 = 0$. Let us calculate the derivative of the utility function w.r.t. c_1 when $c_1 = 0$, $c_2 = W/p_2$:

$$\frac{\partial U}{\partial c_1}(0) = \lambda\alpha r_1^{\alpha-1} - \alpha\beta\left(\frac{p_1}{p_2}\right)\left(\frac{W}{p_2}-r_2\right)^{\alpha-1}$$

The necessary condition for a corner solution is that this derivative be negative :

$$\frac{\partial U}{\partial c_1}(0) \leq 0$$

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or

$$\lambda \alpha r_1^{\alpha-1} \leq \alpha \beta \left(\frac{p_1}{p_2}\right) \left(\frac{W}{p_2} - r_2\right)^{\alpha-1}$$

$$r_1^{\alpha-1} \leq \left(\frac{\beta}{\lambda} \times \frac{p_1}{p_2}\right) \left(\frac{W}{p_2} - r_2\right)^{\alpha-1}$$

$$\left(\frac{W}{p_2} - r_2\right)^{1-\alpha} \leq \left(\frac{\beta}{\lambda} \times \frac{p_1}{p_2}\right) r_1^{1-\alpha}$$

$$\frac{W}{p_2} - r_2 \leq \left(\frac{\beta}{\lambda} \times \frac{p_1}{p_2}\right)^{\frac{1}{1-\alpha}} r_1$$

This condition has the same interpretation in terms of the utility function curvature as the condition in the SE quadrant.

Frontiers Solutions   The only frontier solution we need to consider is $c_1 < r_1$, $c_2 = r_2$, because we have seen that the frontier solutions with the NE quadrant cannot be global maxima.

We can apply the same line of reasoning as in the SE quadrant (with c_1^* being now the local maximum in the NO quadrant) :

$$U\left(\frac{W-p_2r_2}{p_1}\right) = U(c_1^*) + U'(c_1^*)\left(\frac{W-p_2r_2}{p_1} - c_1^*\right) + U''(c_1^*)\left(\frac{W-p_2r_2}{p_1} - c_1^*\right)^2$$

$$= U(c_1^*) + U''(c_1^*)\left(\frac{W-p_2r_2}{p_1} - c_1^*\right)^2 \leq U(c_1^*)$$

if the former SOC is fulfilled. In this case, there cannot be any frontier solution.

2.2.4   SO quadrant : $c_1 < r_1$ et $c_2 < r_2$

Interior Solution   In the SO quadrant, we have

$$U(c_1, c_2) = -\lambda(r_1 - c_1)^\alpha - \beta\lambda(r_2 - c_2)^\alpha \ s.t. \ p_1c_1 + p_2c_2 = W$$

Then I replace c_2 by its expression in the budgetary constraint :

$$U(c_1) = -\lambda(r_1 - c_1)^\alpha - \beta\lambda\left(r_2 - \frac{W - p_1c_1}{p_2}\right)^\alpha$$

I take the derivative :

$$\frac{\partial U}{\partial c_1}(c_1) = \alpha\lambda(r_1 - c_1)^{\alpha-1} - \alpha\beta\lambda\left(\frac{p_1}{p_2}\right)\left(r_2 - \frac{W - p_1c_1}{p_2}\right)^{\alpha-1}$$

And then the second derivative

$$\frac{\partial^2 U}{\partial (c_1)^2}(c_1) = -\alpha(\alpha-1)\lambda(r_1 - c_1)^{\alpha-2} - \alpha(\alpha-1)\beta\lambda \left(\frac{p_1}{p_2}\right)^2 \left(r_2 - \frac{W - p_1 c_1}{p_2}\right)^{\alpha-2}$$

The associated SOC (after simplifying by $-\alpha(\alpha-1)\lambda > 0$) is

$$(r_1 - c_1)^{\alpha-2} + \beta \left(\frac{p_1}{p_2}\right)^2 \left(r_2 - \frac{W - p_1 c_1}{p_2}\right)^{\alpha-2} \leq 0$$

This condition is not verified for $c_1 \leq r_1$ et $c_2 \leq r_2$, which implies that the optimal solution cannot be interior to the SE quadrant.

Corner Solution   Both types of corner solution are possible : $c_1 = 0$, $c_2 = W/p_2$ and $c_1 = W/p_1$, $c_2 = 0$. By convexity of the utility function, these solutions give the consumer a higher utility than the interior solution. To determine the optimal solution, we only have to compare the two corner solutions : the optimal solution depends on the relative price and β. Indeed, we have :

$$U(0) = -\beta\lambda \left(r_2 - \frac{W}{p_2}\right)^{\alpha} \text{ et } U\left(\frac{W}{p_1}\right) = -\lambda \left(r_1 - \frac{W}{p_1}\right)^{\alpha}$$

2.2.5   Global Optima

Graphical Analysis   We can analyze graphically the consumer's optimal behavior, by considering that she possesses two "utility pumps", one for each good, and that she allocate her budget by introducing 1-euro coins one by one in one of the two pumps. At the beginning, both pumps are empty, and the consumer must choose to which pump allocate her first euro. If she allocates it to pump 1, the generated utility will be approximately equal to $u_1'(0) = \lambda\alpha (r_1)^{\alpha-1}$, and similarly for pump 2.

Let us assume that $u_1'(0) \geq u_2'(0)$ and that the consumer allocate her first euro to pump 1. As the function u_1 is convex in losses, marginal utility is increasing, so it is in the consumer's best interest to also allocate her second euro to pump 1, and so on, as the consumer "climbs" the curve of the utility function. Once the consumer reaches the gain zone, marginal utility becomes decreasing, but it is still high enough so that the consumer continues to allocate her budget to pump 1. When the consumer reaches the point $\bar{c}_1(r_2)$ such that

$$u_1' (\bar{c}_1(r_2)) = u_2'(0)$$

then the marginal utility of pump 2 becomes higher than the marginal utility of pump 1, and by convexity of the utility function with respect to the good 2, the consumer will allocate a

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Figure 7: Representation of the utility pump 1
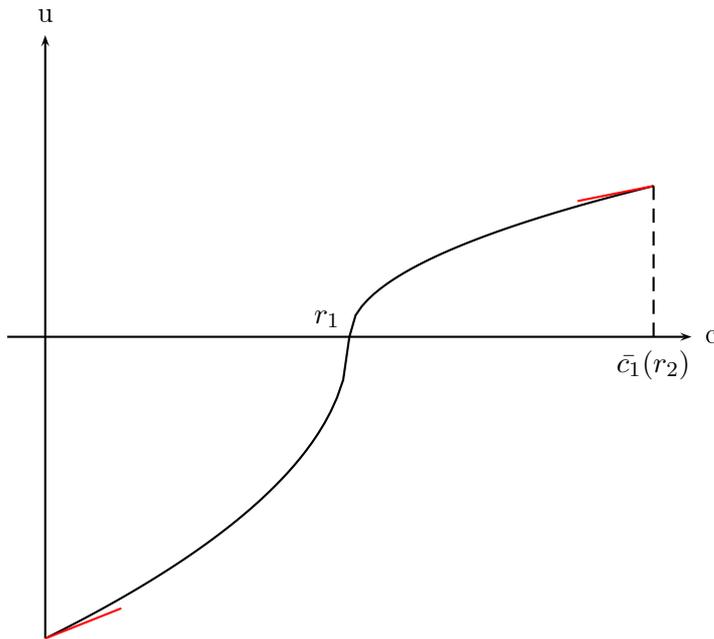
succession of euros to pump 2. She will start allocate euros to pump 1 again only when the marginal utility for both pumps will be equal. From this point on, she will allocate her euros alternatively to each pump until she has exhausted her budget.

Figure 7 illustrates this reasoning.

Global Optima   Based on the former graphical analysis, I will determine global maxima, by examining the different possible situations, depending on the quadrants across which the budget constraint passes. Indeed, the budget constraint can pass :

1. only across the SO quadrant,

2. across the SO and NO quadrants,

3. across the SO and SE quadrants,

4. across the SO, NO and SE quadrants,

5. across the NE, NO and SE quadrants,

Figure 8 illustrates these different cases. Let us consider them successively.

SO quadrant   If the budget constraint passes only across the SO quadrant, the optimal solution is one of the two corner solutions, the one with the highest utility. If $\beta = 1$, $p_1 = p_2$ and $r_1 = r_2$,

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Figure 8: Possible budget constraints
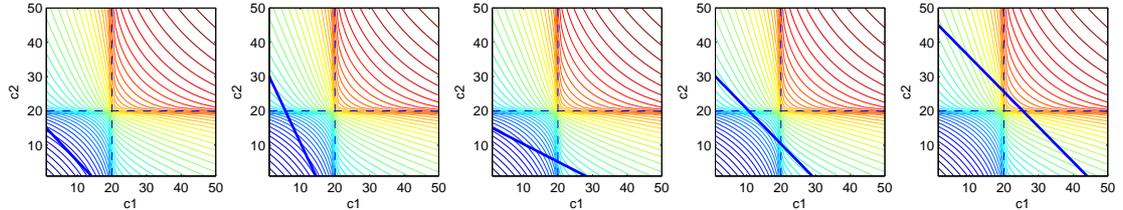
the consumer is indifferent between both corner solutions. If $\beta = 1$, $p_1 = p_2$ but $r_1 \neq r_2$, the consumer devotes all her budget to the good with the lowest reference point : by convexity of the utility function, it is the good with the highest marginal utility.

SO and NO quadrants   If the budget constraint passes across the SO and NO quadrants, we have three candidate solutions :

  – the corner solution of the SO quadrant,
  – the corner solution of the NO quadrant,
  – the interior solution of the NO quadrant .

We can notice immediately that if the budget constraint passes across the SO and NO quadrants, the point (r_1, r_2) is not part of the budget set, and therefore the SOC for the interior solution is fulfilled.

The consumer chooses the one of the two goods with the highest marginal utility per euro. If it is the good 1, as the consumer's budget is insufficient to reach the concave zone of her utility function, the consumer will devote all her budget to good 1, which corresponds to the corner solution in the SO quadrant.

If it is the good 2, it is cheap enough for the consumer to reach the concave zone of her utility function. If the consumer's budget is insufficient to reach the point $\bar{c}_1(r_2)$, she will allocate all her budget to good 1, which corresponds to the corner solution in the NO quadrant. If the

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consumer's budget is sufficient, she will allocate some of her budget to good 2, which corresponds to the interior solution in the NO quadrant.

SO and SE quadrants   When the budget constraint passes across the SO and SE quadrants, the reasoning is exactly the same as in the previous case, with good 1 and 2 inversed.

SO, NO and SE quadrants   In this case, we can immediately eliminate the interior extremum of the SO quadrant, because we have seen that it is a minimum and not a maximum. The choice between the NO and SE quadrants is based on the relative price between the two goods and the consumer's preference. Once the consumer has selected her "favorite" quadrant (i.e. the good to which the consumer will first allocate her money), the choice between the interior solution and the corner solution is based on the same criterion as in the previous cases.

NE, NO and SE quadrants   In this case, the consumer's behavior will depends on whether her budget is sufficient to reach the point $\bar{c}_1(r_2)$ (if she allocates her budget first to the good 1 ; the reasoning is reversed if she chooses the good 2 first). If it is insufficient, the consumer allocates all her budget to good 1, and the optimal solution is the corner solution of the NO quadrant. If it is sufficient, the consumer will start allocating some of her budget to good 2, which corresponds to the interior solution of the NO quadrant. Beyond a certain threshold, the consumer will reach her concave zone for the second good, which corresponds to the interior solution of the NE quadrant.

Conclusion   We can sum up the general behavior of the consumer by plotting the income expansion path, i.e. the set of consumption vectors chosen for all the possible levels of wealth, for given relative prices. The figure 9 represents the income expansion path when the consumer consumes in priority the good 1.

In other words, the classical result of the consumer theory stating that consumers have a taste for variety and prefer splitting their consumption rather homogeneously between goods appears as a particular case, which applies only when consumers are rich enough to keep their consumption above their reference point for all goods.

2.3   Concave Utility

Finally, I consider the case of a utility function which is everywhere concave, but with loss aversion. This functional form is the one observed empirically by Abdellaoui, Bleichrodt, and

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Figure 9: Income Expansion Path
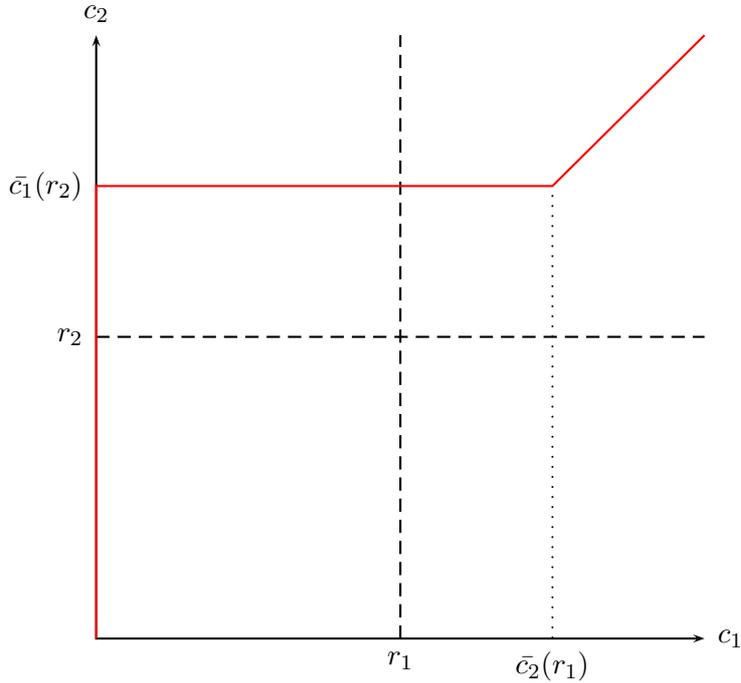
L'Haridon (2008) in risky choices, even though most empirical results for choices under risk are rather heading in the direction of a utility function which is slightly convex in losses (e.g. Abdellaoui, Bleichrodt, and Paraschiv (2007)). As far as I know, this possibility has not been considered for choices under certainty in the behavioral economics literature, and loss aversion has always been associated with a linear or S-shaped (first convex then concave) utility function. The only article which discuss this possibility is Lapidus and Sigot (2000) in the History of Economic Thought, in an analysis of the benthamian theory of pleasure and pain.

Therefore, I suggest the following functional form, which yields the desired shape while staying close to the spirit of the initial Prospect Theory.

$$U(c_1, c_2) = u_1(c_1) + \beta * u_2(c_2)$$

$$s.t. \ p_1 c_1 + p_2 c_2 = W$$

with β the relative weight of good 2 with respect to good 1, λ the coefficient for loss aversion,

W the consumer's budget, and

$$u_i(c_i) = \lambda * c_i^\alpha \quad \text{si } c_i < r_i$$

$$u_i(c_i) = (\lambda - 1) * r^\alpha + c_i^\alpha \quad \text{si } c_i \geq r_i$$

The figure 10 shows the utility function and the corresponding indifference curves, with a vector of reference points $(20, 20)$.

Figure 10: Concave Utility and Loss Aversion

(a)   (b)
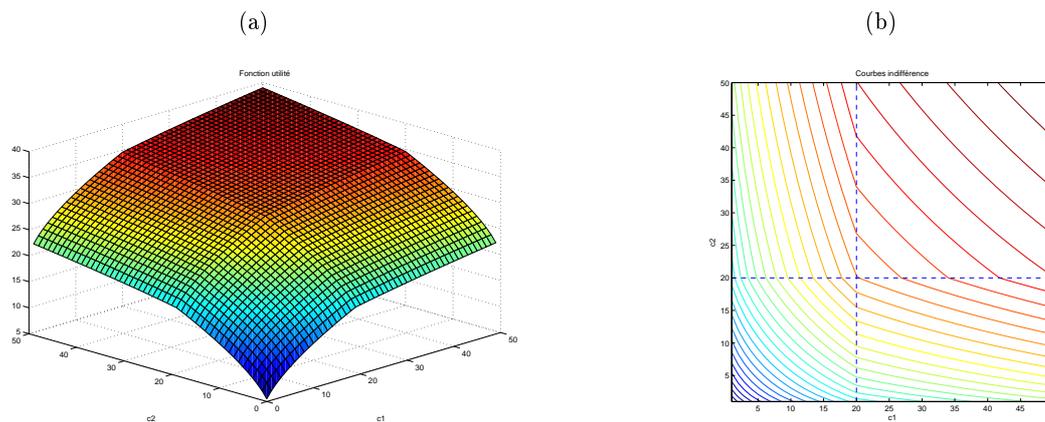
Let us solve the consumer's program. The utility function is not everywhere differentiable: for $c_i = r_i$, the utility funciton is left- and right-differentiable, but the derivatives are not equal. As a result, the optimal solution is characterized by

$$\frac{u_1'(c_1^*)}{\beta u_2'(c_2^*)} = \frac{p_1}{p_2}$$

if the utility function is differentiable in c_1^* and c_2^*, but

$$\frac{u_{1_d}'(c_1^*)}{\beta u_{2_d}'(c_2^*)} \leq \frac{p_1}{p_2} \leq \frac{u_{1_g}'(c_1^*)}{\beta u_{2_g}'(c_2^*)}$$

otherwise.

As the utility function is everywhere concave, the consumer's behavior becomes again roughly "classical": except if the preferences or relative prices are extreme, the optimal solutions are interior solutions. However, there is still some "rigidity": as the slope of the indifference curves is different on the left and on the right of a reference point, if the optimal solution implies that the consumption is equal to the reference point for one of the two goods, there exists an interval for the relative price such that the consumption of this good remains constant at the level of the reference point, and only the consumption of the other good adjusts. This situation is illustrated by the figure 11 (the right side of the figure is an enlargment of the left side).

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Figure 11: Loss Aversion-induced Rigidity

(a)   (b)
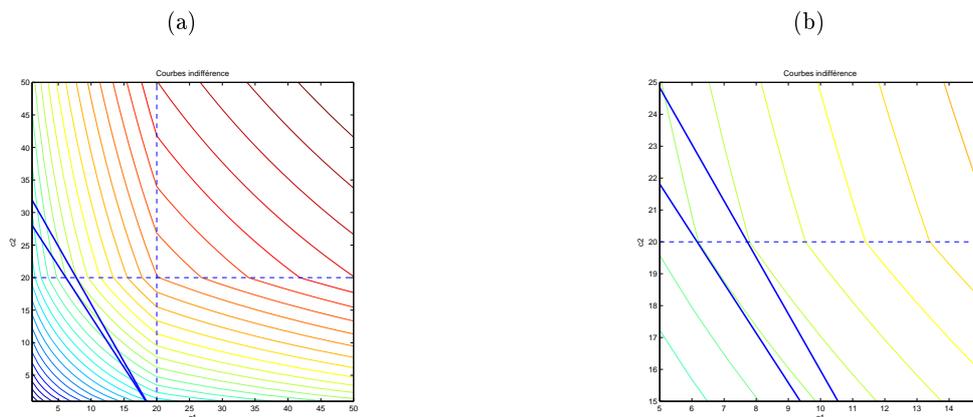
2.4   Discussion

I considered the effects of loss aversion for different functional forms. This effects are qualitatively homogeneous : loss aversion induces solutions where consumption is equal to the reference point for (at least) one of the goods. But these effects change quantitatively depending on the functional form : for a concave utility function, there exists only a rigidity of the consumption in the neighbourhood of a reference point, but the optimal solutions are always interior solutions and they are generally responsive to a relative price variation. For a linear utility function, and *a fortiori* for a convex utility function (at least in losses), the only possible solutions are corner solutions or solutions where consumption equals the reference point.

We can recall that the shape of the utility function has two possible interpretations. From the standpoint of the consumer theory, concavity means a decreasing marginal utility ; from the standpoint of the theory of choice under uncertainty, concavity means risk aversion. Therefore, our current results suggest that loss aversion should not be considered in and by itself, but in conjonction with an analysis of the context and of the consumer's psychological characteristics, in order to determine the appropriate functional form of the utility function (at least temporarily and locally).

The standard theory of the consumer's behavior relies on concave utility functions, which induces a preference for mixing rather than for extreme allocations, but some empirical results suggest that consumers might actually demonstrate mixing aversion and a taste for extreme allocations (e.g. Dhar and Simonson (1999)). In the next section, I present several examples in more detail.

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3 Applications

3.1 Cabdrivers labor supply

In an often quoted article, Camerer, Babcock, Loewenstein, and Thaler (1997) observe that New-York cabdrivers show a "wage"-elasticity of their labor supply which is negative: they drive less on good days (i.e. days when the demand for taxis is high, hence a driver spends more of his time actually driving clients instead of looking for one, which implies a higher income per hour) than on bad days[1]. This behavior cannot be explained if drivers maximize a standard utility function: it would be more efficient to work more hours on good days and less on bad days. Even if the income effect is stronger than the substitution effect, as soon as the time horizon of the drivers exceeds one day, it would be better to work more on good days and take much more leisure on bad days. This result has been confirmed by Fehr and Goette (2007) during a randomized field experiment, which offers a more controlled environment than a natural experiment.

The interpretation that the authors offer is that, first, the drivers's time horizon is a day, and second, they have a daily income target that they try to reach, and they feel loss aversion when they can't reach it. This interpretation is consistent with my results, but is is incomplete, as it doesn't take into account the curvature of the utility function. And yet, I have showed that the curvature of the utility function strongly affect the consumers's behavior. Moreover, both Camerer, Babcock, Loewenstein, and Thaler (1997) and Fehr and Goette (2007) consider only loss aversion with respect to the income, and not effort (or reversely, leisure), which restrain the generality of their analysis.

Let us get back to the models I introduced *supra*, and consider that one good represents leisure and the other one represents aggregate consumption. We also assume, in line with Camerer, Babcock, Loewenstein, and Thaler (1997), that the reference points correspond to the agent's average leisure and average consumption in the long run. We observe that indeed, whatever the shape of the utility function, a counterclockwise rotation of the budget line around the point L_{max} (i.e. the agent's initial endowment of time) leads the agent to keep constant her consumption and to decrease only her leisure. But if the hourly labor income decrases strongly, the agent's optimal behavior will depend on the curvature of the utility function in losses: if the utility function is linear or convex, the agent will try to keep her consumption equal to the reference point as long as possible, but if the hourly labor income gets under a certain threshold,

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1. Farber (2005) reaches different results with the same data by using a different econometric strategy

her behavior will change completely, and she will "jump" back to the reference point for leisure, by strongly decreasing her consumption. On the contrary, for a concave utility function, when the hourly labor income gets lower, the agent will start to decrease progressively her consumption while also decreasing her leisure.

Similarly, with a clockwise rotation of the budget line (i.e. the hourly labor income gets above average), the agents optimal behavior will depend on the curvature of the utility function. If utility is linear, the agent will increase her consumption while keeping her leisure equal to the reference point. If utility is concave (whether it is globally concave, or only concave in the gains *à la* Prospect Theory), the agent will split her additional income between consumption and leisure, and both will increase.

As a conclusion, this particular case illustrates how much the impact of loss aversion on the consumer's behavior depends on the curvature of the utility function.

3.2 The "What the Hell" Effect

People on a diet often set themselves numerical targets (e.g. a certain number of calories per day). Their situation can be understood as a tradeoff between two goals: on the one hand caloric restriction (in order to lean out), and on the other hand the pleasure coming from eating tasteful food and taming their hunger. If these people had utility functions for these goals that were concave, a modification of the "relative price" between the two goals or of the "budget constraint", such as being forced unwillingly into the loss zone for one of the goals, would imply only small changes in behavior. Yet, as suggest Cochran and Tesser (1996), a minor change in the situation can lead to a major change in behavior:

> Gunilla found the beautiful formal she wanted for the prom, but she needed to lose some weight. Her friend, fresh from a social psychology course, recommended that she set a specific, daily caloric goal. On the third day, after eating a serving of "lite" spaghetti, Gunilla read the package and realized that she was slightly over her daily goal. Her response was interesting: She said to herself "What the hell. Since I'm already over my goal it doesn't matter what I eat". And she proceeded to consume half of her mom's apple pie.

Cochran and Tesser (1996) call this phenomenon the "What the Hell Effect".

This effect is not restrained to food diets, but also applies to all situations when a precise numerical goal cannot be reached. Another example is money budgetting and credit card use:

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Soman and Cheema (2004) find that consumers who have exceeded their monthly budget are more prone to superfluous expenses compared to consumers who are still in the limits of their budget. And Wilcox, Block, and Eisenstein (2011) find that for consumers with a higher degree of self-control (and hence consumers who can refrain most of the time from using their credit card), having an outstanding credit on their card leads them to use their card more.

These different empirical results are consistent with an utility function which is linear, or convex in losses. Moreover, when a consumer ends up just below a numerical goal with no possibility to get back into the gain zone for this dimension, she has a tendency to go further away from the goal by deepening her losses, which suggests that in riskless choice, the convexity of the utility function has a stronger effect than loss aversion. This behavior is the counterpart for riskless choice of the lower risk-aversion in losses for risky choice.

3.3 Convexity and Self-Control

The examples I presented in the previous section could also be interpreted in terms of self-control. An argument against this interpretation and in favour of my interpretation is that in other situations where self-control does not intervene, we observe similar behaviors. For example, Dörner (1989) finds that when facing a complex situation with multiple and contradictory goals, where it is not possible to reach all of them, decision-makers often choose to concentrate on only one goal, even if it means higher losses for the other goals.

4 Conclusion

In this article, I showed that loss aversion has observable consequences which depends on the curvature of the utility function. These observable consequences are consistent with several empirical phenomena which cannot be explained by the standard theory of consumer's behavior.

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