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Taxation of early retirement windows and delaying retirement: the French experience *

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Abstract

This paper investigates the effect of the 2003 French pension reform on hiring, firing and employment rates among older workers. This reform increased the mandatory retirement age and simultaneously it set a tax levied on early retirement windows payed by firms to their older workers, to encourage them to leave their job early. We use a matching model with endogenous job destruction extended to account for a mandatory retirement age and we calibrate the model with data drawn from the French Labor Force Surveys for the years 2001 and 2002. We show that in the case of a high tax rate, delaying retirement raises job separation rates, which partially offsets its positive effect on job finding rates. Consequently, the combination of an increase in the retirement age and a taxation on early retirement windows may have perverse effects on the employment rate among older workers.

Keywords : Delaying retirement, early retirement windows, job matching models, employment protection

JEL Classification J23, J63, J65

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1 Introduction

Over the last three decades, most European countries experienced an ageing of their population due to a fall in fertility rates since 1970 in a setting of steadily increasing life expectancy. Furthermore during the same period, the employment rate among workers aged 55-64 years has been decreasing, leading to an important rise in dependency rates and putting therefore a strong pressure on budgetary balance of PAYG retirement schemes.

The creation of early retirement schemes in the 1970's may be an explanation of the fall in employment of elderly observed in Europe during the same period. Zaidmann (2000) shows that in France these schemes led to a consensus between older workers, firms and government. This phenomenon may be due to two main reasons. First, early retirement schemes were partly financed by the government so they could be viewed by firms as a layoff subsidy (Hutchens, 1999, Tuulia and Uusitalo 2005). Second, the government encouraged early retirement to make more room for youngsters in the labor market in a setting of high youth unemployment (Zaidman, 2000).

Nevertheless thirty years later, since the employment rate among workers aged more than 55 in France was one of the lower in the European Union (29.9% in France with respect to an European average of 37.8%[1]), the French government implemented important changes to constrain early retirement. These changes aimed simultaneously at tightening the access conditions to early retirement schemes for workers and firms and at increasing the share of early retirement expenses charged to employers. However, in spite of the increase of early retirement costs for firms, employers continue to encourage their older workers to leave early their job, offering them generous financial incentives called "early retirement windows". In the face of this widespread phenomenon, notably in the case of big firms, the French government set in 2003 a tax levied on the amount of early retirement windows payed by firms. The tax rate amounted to 23.85% in 2003 and in 2008 it raised to 50%. In addition, to prevent firms from pushing their older workers into retirement too early, the 2003 reform also led to an increase in the mandatory retirement age. Initially, this age was 60 which means that when a worker reached 60 and if his insurance period was sufficient to allow him to draw a full pension, an employer could push him into retirement paying him a low retirement allowance. In 2003 this age has been rising to 65 and since 2008 it has been setting to 70.

The goal of this paper is to investigate the effect of the combination of these two reforms on the hiring rate, the firing rate and the employment rate

[1]Source: Eurostat

among older workers. The effect of an increase in legal retirement age on employment among older workers has been already considered in the literature. Previous studies point out that delaying retirement may have a positive "horizon effect" on job creation, when labor is viewed as a quasi-fixed factor that implies fixed costs (Oi, 1962). These costs result from either a bilateral monopoly problem (Hutchens, 1986), or from an accumulation of specific human capital through training (cf Hashimoto 1981) and imply that firms are more reluctant to hire a worker close to the retirement age. Furthermore focussing on job destruction, Aubert et al. (2006) show that there is an age-bias technological change, so employers are less likely to keep an older worker in the case of a shock on his job, if his employment duration is too short. Extending the model of Mortensen and Pissarides (1994) in order to account for the life cycle of the worker with a bounded retirement age, Chéron et al. (2007) draw similar conclusions and argue that an increase in retirement age may have a positive effect on hiring rates and may reduce firing rates by lengthening the employment duration of older workers.

In addition, the effect of an increase in age-specific employment protection on employment has also been considered in previous literature. Investigating the effect of the Contribution Delalande, a tax payed by firms who fire workers aged more than 50, Behaghel (2007) shed a new light on a potential perverse effect of such a partial employment protection on the hiring rate of the targeted age group of workers and also on the firing rate of the previous cohort of workers. Indeed, although this tax aims at discouraging firms from laying older workers off, firms may have interest to fire their workers before they reach the threshold age and they may be reluctant to hire workers just below this age. Studying empirically the magnitude of these effects using Labor Force Surveys from 1982 to 2002, Behaghel et al. (2008) show that the tax has a strong negative effect on the hiring probability of unemployed workers in the protected age group.

While the effects of an increase in legal retirement age and of an age-specific tax on firing on employment have already been studied separately, there does not exist as far as we know a paper that examines the impact of the combination of both these reforms on elderly employment. Borrowing the Behaghel's theoretical framework (2007), we distinguish two age groups of workers, the middle-age and the older workers, considering that middle-age workers are protected according to the French Employment Protection Legislation, while older workers are protected by a tax levied on early retirement windows offered by their employer to encourage them to leave their job early.

The key result of this study can be summarized as follows : in the case of a high tax rate, delaying retirement may raise the separation rate for the protected group of workers through an "impatience effect". Indeed, in the case of a negative productivity shock on a job filled by an older worker, postponing retirement may encourage an employer to offer his worker a generous early retirement window to force him to leave, rather than waiting for him to reach the new mandatory retirement age. So, in this paper we determine a critical value of the tax rate, above which delaying retirement may raise job separations for older workers. The impatience effect may partially offset the positive horizon effect exerted by a rise in the retirement age on the hiring probability of older workers. In other words, when the government set a tax levied on early retirement windows it attenuates the positive effect of delaying retirement on job finding rates of the protected age group.

In addition, as an increase in the retirement age reduces transitions from unemployment to retirement, it exerts a negative effect on employment among older workers if the horizon effect is not sufficiently strong. Consequently, the higher the tax rate on early retirement windows, the lower the horizon effect and the stronger the negative effect of postponing retirement on employment among the protected age group.

Calibrating our model using data drawn from the French Labor Force Survey for the years 2001-2002, we provide a numerical illustration of these findings. We show that the change in the job separation rate for older workers after an increase in the retirement age strongly depends on the tax rate. Consequently, the effect of delaying retirement on employment among the protected age group is sensitive to the level of taxation of early retirement windows.

The remainder of the paper is structured as follows. In the next section, using the data drawn from the two waves of the European survey SHARE (Survey on Health Ageing and Retirement in Europe), we study to what extent older workers who retired early have been given early retirement windows from their employer. Then in section 3, we describe in detail the theoretical model, following the specification of Behaghel (2007). In section 4 we present our main theoretical findings on the effect of an increase in the mandatory retirement age on hiring rates and firing rates of middle-age and older workers in a setting of partial employment protection. In the section 5, we describe our quantitative analysis based on the French Labor Force Surveys for the years 2001 and 2002 and we present our results. Section 6 concludes.

2 Early retirement windows: a new pattern in Europe

While the incidence of early retirement windows has been already considered in the American case[2], too few studies examine this pattern in the European case. Using data from the 1997 International Social Survey Program, Dorn and Sousa-Poza (2007) investigate the incidence of involuntary early retirement for 19 countries[3], asking retired respondents if they retired early "by choice" or "not by choice". Their analysis covers the early retirement of individuals aged between 45 and 64 who retired between 1983 and 1997. Providing some descriptive statistics, they show that in some European countries like Germany or Portugal, more than half of all retired respondents state that they retire early "not by choice". In France this proportion amounts to 41%, which is also very high.

However, a dummy equal to one if the individual reports that he retires early "not by choice" does not provide some accurate information about the motivations of his forced early retirement. In addition, this variable raises an empirical issue. Indeed, a respondent may mention that he retired "not by choice" to legitimate his own choice, leading to a potential justification bias (Parsons, 1980. To address both these issues we use the two first waves 2004 and 2006 of the Survey on Health Ageing and Retirement in Europe (SHARE). This data provides some information for eleven countries[4] on the factors that led respondents to retire. We proceed in two steps. In the first step, we include in our sample only retired respondents aged between 57 and 69 in 2006, who were not entitled to a public or private pension when they left their activity. We obtain a sample made up of 1735 individuals. We remark in the table 1 that in Europe early exit stems from 3 main reasons. First, almost one third of the sample report that they left their job after they had been given early retirement windows from their employer. Second, around 15% of the respondents mention that they had been laid off. Third, around 30% of the individuals report that they left early their job due to their bad health status.

When we examine the self-reported reasons for early retirement by countries in the figure 1 we observe that the relative weight of each factor to explain early exit differs among countries. Nevertheless in most countries,

[2]See for instance Brown (1999)

[3]Canada, Cyprus, Denmark, France, Germany, Great Britain, Hungary, Italy, Japan, the Netherlands, New Zealand, Norway, Poland, Portugal, Slovenia, Spain, Sweden, Switzerland and USA

[4]Austria, Germany, Sweden, Netherlands, Spain, Italy, France, Denmark, Greece, Switzerland and Belgium

Table 1: Reasons for early retirement for 11 European countries

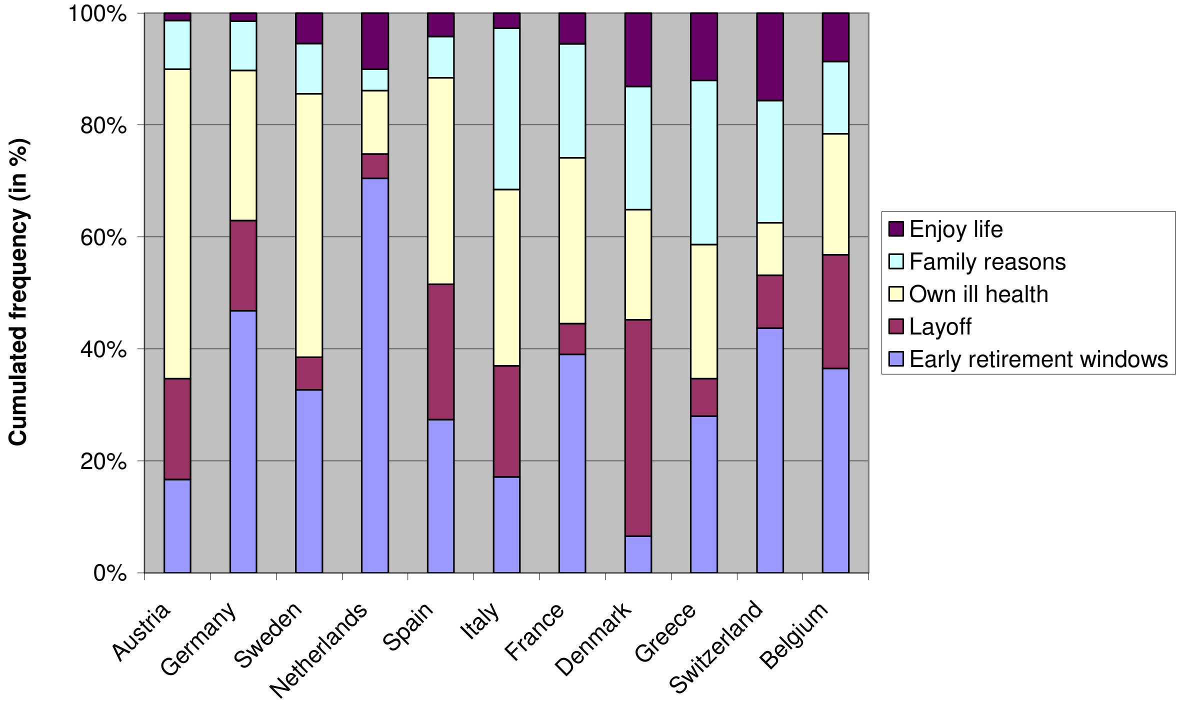
Reasons for early exit	**Number of observations**	**Frequency (in %)**
Early retirement windows	589	33.95
Layoff	271	15.62
Own ill health	520	29.97
Ill health of a relative	43	2.48
Retire at same time as spouse	65	3.75
To spend more time with family	133	7.67
To enjoy life	114	6.57
Total	1735	100

Note: Our sample contains all retired individuals aged between 55 and 69 in 2006 who were not entitled to a public or private pension when they left their activity
Source: SHARE (waves 2004 and 2006)

more than one third individuals state that they left early their job after they received early retirement windows from their employer. This fraction is particularly high in Netherlands, in Germany, in Switzerland in Belgium or in France. In contrast, in Austria, Sweden or Spain, it is rather a bad health condition that represents the major part of the reasons for early exit.

It may be argued that the high fraction of individuals reporting an early exit due to financial incentives payed by their employer may result from a simple justification bias. The respondents could invoke that reason to justify their decision of leaving early their job. To address this empirical issue we select in a second step only retired individuals in 2006 who were working in 2004. These respondents were asked in 2004 the following question: "Thinking about your present job, would you like to retire as early as possible?". This measure provides us a good measure of the individual propensity to early retirement and it has been already used in other empirical studies to examine the factors of early retirement (Debrand and Blanchet, 2007, Siegrist et al., 2007). So we group the respondents into two categories: the group 1 is made up of respondents who answered "Yes" to this question that we view as workers with high propensity to early retirement and the group 2 includes individuals who answered "No". For each group, we study the reasons for early exit. In the table 2, we remark that the fraction of respondents who mention early retirement windows as a motivation for early exit does not differ across groups, so the justification bias appears to be negligible when we study the incidence of early retirement windows in the individual retirement decision. Consequently, descriptive statistics of the figure are robust to a jus-

Figure 1: Reasons for early exit by countries



Source: SHARE (waves 2004 and 2006)

tification bias. We observe that in France, the incidence of early retirement windows amounts to around 40% of early exits, which explains the willing of the French government to discourage these practices by setting a tax on the whole amount of financial incentives offered by employers to their older workers.

Table 2: The reasons for early exit according to the propensity to early retirement of respondents

	Frequency (in %)	
Reasons for early exit	**Group 1**	**Group 2**
Early retirement windows	45.54	45.31
Layoff	13.39	9.38
Own ill health	11.61	7.81
Family reasons	16.07	28.13
To enjoy life	13.39	9.38
Total	100	100

Note: Our sample contains all individuals retired in 2006 aged between 55 and 69 in 2006 who were not entitled to a public or private pension when they left their activity and who were employed in 2004

Lecture: The group 1 includes agents with an high propensity to early retirement. The group 2 includes individuals with a low propensity to early retirement

Source: SHARE (waves 2004 and 2006)

3 The model

3.1 The economy

Following the specification of Mortensen and Pissarides (1994), we consider an economy in which firms produce one type of good using only one factor of production: the labor. For a sake of simplicity, we assume that a firm can not employ more than one worker and the number of jobs is endogenous. Following Behaghel (2007), we consider two age groups of workers: the middle-age workers (group C_1) and the older workers (group C_2). A middle-age worker may switch to the next age group at a Poisson arrival rate η_1. Similarly, an older worker may reach the mandatory retirement age at a Poisson arrival rate η_2.

In our model the Poisson arrival rate η_2 is a key parameter to determine the horizon of older workers denoted by H. We define H in the following way:

$$H = \int_0^{+\infty} t\eta_2 e^{-\eta_2 t} dt = \frac{1}{\eta_2}$$

In the remainder of the paper we view an increase in the mandatory retirement age as a decrease in η_2.

We consider an economy "à la" Mortensen Pissarides (1994) with endogenous job creation and destruction. In this model, workers and firms with vacant jobs meet each other according to a matching function $m(u_i, v_i)$, that denotes the number of matches as a function of the unemployment rate u_i among the group C_i and the vacancy rate v_i targeted to job-seekers belonging to the group C_i. Here the matching technology is age-discriminating, which means that firms are free to target their hirings by age[5]. We also assume that the matching function is increasing, concave in each argument and linear homogeneous. Let θ_i be the labor market tightness namely the number of vacancies per worker, so we can define the Poisson arrival rate $q(\theta_i)$ of a match for an employer posting a vacancy targeted to job seekers belonging to the group C_i.

$$q(\theta_i) = \frac{m(u_i, v_i)}{v_i} = m(\frac{1}{\theta_i}, 1) \quad i = 1, 2 \tag{1}$$

Therefore $q(\theta_i)$ is a decreasing function of the tightness θ_i. Furthermore we can define the Poisson arrival rate $p(\theta_i)$ of a match for a job seeker belonging to the group C_i:

$$p(\theta_i) = \frac{m(u_i, v_i)}{u_i} = \theta_i q(\theta_i) \quad i = 1, 2 \tag{2}$$

So $p(\theta_i)$ is an increasing function of the tightness θ_i. Consequently, θ_i is an endogenous key variable to determine the job-finding rates of each age group of workers in our economy.

3.2 The firms' behaviour

In our model each firm has one job that can be either filled and producing or vacant and searching. As long as the job is vacant, firms pay a cost c of maintaining a vacancy. When the vacancy is matched with a worker, his idiosyncratic productivity $y\epsilon$ is drawn randomly from the fixed distribution $G(\epsilon)$ with $\epsilon \in [0, \overline{\epsilon}]$. The firm hires the worker if ϵ is higher than the productivity threshold ϵ_i^c. Consequently, ϵ_i^c is an other endogenous key variable to determine the hiring rate of each age group of workers.

[5]This assumption holds in a setting of weak legislation prohibiting age-discrimination

Let J_v^i be the value to an employer of posting a vacancy targeted on workers belonging to the group C_i. At steady-state, we obtain the following Bellman equations:

$$rJ_v^i = -c + q(\theta_i)[\int_0^{\overline{\epsilon}} \max\{J_i(x), 0\} dG(x) - J_v^i] \quad i = 1, 2 \tag{3}$$

where $J_i(\epsilon)$ is the asset value of a job filled by a worker belonging to the age group C_i with a productivity level ϵ. Under the free-entry condition, the flow value to the employer from opening a new vacancy is equal to zero at steady-state equilibrium. Therefore we get :

$$\int_{\epsilon_i^c}^{\overline{\epsilon}} J_i(x) dG(x) = \frac{c}{q(\theta_i)} \quad i = 1, 2 \tag{4}$$

This first condition implies that the mean search cost must be equal to the value to an employer of a filled job. An increase in this value encourages therefore employers to open more vacancies.

When a job is filled, a worker belonging to the age group C_i starts producing an output $y\epsilon$, where ϵ is the random component of the productivity, and he receives a productivity-contingent wage $w_i(\epsilon)$. Then the job can be hit by an idiosyncratic shock at a Poisson arrival rate λ. In that case, a new random productivity level ϵ is drawn according to a cumulative distribution function $G(x)$ and the employer has no other choice either to close down the job or to keep the worker. Existing filled jobs are destroyed if the productivity level falls below a productivity threshold ϵ_i^d. Consequently, a job occupied by a worker belonging to the age group C_i may be destroyed at a Poisson arrival rate $\lambda G(\epsilon_i^d)$. We assume that λ does not differ across age groups so ϵ_i^d is a key endogenous variable to determine job destruction among each age group of workers.

In our model, we allow employers to offer their older workers early retirement windows to encourage them to leave their job and to avoid a layoff. As we mentioned in the introduction, this firm's behavior is the response of employers to the tightening of access conditions for publicly financed early retirement schemes and an increasingly part of employers offer early retirement windows since the early 2000's. Consequently, in our model, we consider that the firing cost for an employer depends on the age of the worker. If an employer fires a middle-age worker, he has to pay him severance pay denoted by f_1 according to the French Employment Protection Legislation. However, if an employer fires an older worker, he offers him early retirement windows denoted by f_2. Reproducing the 2003 French pension reform, we set a tax levied on the amount of the early retirement window payed by the employer at a rate τ.

We can discuss the motivations that lead employers to offer early retirement windows to their older workers rather than firing them. To justify our specification, we argue that the legislation regarding firing older workers implies a cumbersome and often costly procedure for employers. Indeed, studying more than 300 court rulings of the Court of Cassation during the period 1994-2004 regarding job separations for older workers, Amauger-Lattes and Desbarrats (2006) highlight the fact that the 2003 French pension reform has not increased layoffs but it has encouraged dismissals. They explain this phenomenon stating that employers strike a mutual agreement with their older workers, offering them generous early retirement windows and reporting a "dismissal for serious misconduct", although it is not the case. Consequently, we consider two ways for an employer to get rid of his older worker: either he offers him early retirement windows or he waits for his worker to reach mandatory retirement age. In that second case, the employer has to pay the worker a retirement allowance f_r.

In addition, recall that in our model a middle-age worker may switch to the next age group at a Poisson arrival rate η_1. In that case, if his random component of productivity ϵ is lower than the productivity threshold ϵ_2^d, his job breaks up. So for each age group of workers C_i $(i \in 1, 2)$, the value to an employer of hiring a worker with a random component of productivity level equal to ϵ is defined by the following Bellman equations:

$$rJ_1(\epsilon) = y\epsilon - w_1(\epsilon) + \lambda[\int_0^{\bar{\epsilon}} \max\{J_1(x), -f_1\}dG(x) - J_1(\epsilon)]$$
$$+\eta_1[\max\{J_2(\epsilon), -f_2(1+\tau)\} - J_1(\epsilon)] \tag{5}$$

And:

$$rJ_2(\epsilon) = y\epsilon - w_2(\epsilon) + \lambda[\int_0^{\bar{\epsilon}} \max\{J_2(x), -f_2(1+\tau)\}dG(x) - J_2(\epsilon)]$$
$$+\eta_2[(J_v^2 - f_r) - J_2(\epsilon)] \tag{6}$$

3.3 Rent-sharing rules

We assume that the wage is set to split the match surplus between the firm and the worker at all times and in fixed proportions, as in the case of a standard Nash wage bargaining. The worker's share is β. In a setting of employment protection, we have to consider two rent-sharing rules. Indeed, when a worker is matched with a vacancy, no severance payment has to be paid if negotiation fails. However, following the standard model of Mortensen and Pissarides (1994), we assume that wages are renegotiated continuously so that the wage received by a worker accounts for the employment protection he will benefit from in the case of a layoff. So, we may define a first rent-sharing rule when the worker is hired, that determines a potential wage w_i^0 in the following way:

$$w_i^0(\epsilon) = argmax\{[W_i(\epsilon) - U_i]^\beta [J_i(\epsilon)]^{(1-\beta)}\} \quad i = 1, 2 \tag{7}$$

where $W_i(\epsilon)$ is the flow value to a worker belonging to the age group C_i from employment and U_i is the flow value to an unemployed worker belonging to the age group C_i. Solving the program (7), we get the following first rent-sharing rule:

$$W_i(\epsilon) - U_i = \beta S_i^0(\epsilon) = \beta[J_i(\epsilon) + W_i(\epsilon) - U_i] \tag{8}$$

where $S_i^0(\epsilon)$ is the match surplus from a job creation targeted to workers belonging to the age group C_i. As the wage is assumed to be renegotiated immediately, we get the new maximization program:

$$w_i(\epsilon) = argmax\{[W_i(\epsilon) - U_i - f_i]^\beta [J_i(\epsilon) + f_i(1+\tau_i)]^{(1-\beta)}\} \quad \text{with } \tau_1 = 0 \text{ and } \tau_2 = \tau \tag{9}$$

Solving (9) we get the following rent-sharing rule:

$$W_i(\epsilon) - U_i - f_i = \beta S_i(\epsilon) = \beta[J_i(\epsilon) + \tau_i f_i W_i(\epsilon) - U_i] \tag{10}$$

We remark that a job filled by a middle-age worker breaks up if $J_1(\epsilon) \leq -f_1$ which implies $W_1 \leq U_1 + f_1$. In a similar way, a job filled by an older worker breaks up if $J_2(\epsilon) \leq -f_2(1 + \tau)$, which implies $W_2 \leq U_2 + f_2$.

Let us first define the flow value from employment to a worker of the group C_i. When he is hired and as long as his job is not hit by an idiosyncratic shock, he receives the productivity-contingent wage $w_i(\epsilon)$. When a shock occurs at a Poisson arrival rate λ, the productivity level changes and the worker may be fired. In that case he receives a payment from his employer f_i (f_1 corresponding to severance payment and f_2 corresponding to the amount of the early retirement window received). If he remains employed despite the shock, he receives a new wage $w_i(\epsilon)$, which changes the value of his job $W_i(\epsilon)$. Furthermore, a middle age-worker occupying a job with a random component of productivity level equal to ϵ may switch to the next age group at a Poisson arrival rate η_1 and if his job does not break up he benefits from the discounted income flows an older worker derives from the same job. Similarly, an older worker may reach the mandatory retirement age at a Poisson arrival rate η_2 and then his job breaks up and he receives a retirement allowance f_r. In that case, the worker is retired and he benefits from a pension P discounted over an infinity of time. So for each age group of workers, the flow value from employment to a worker satisfies the following Bellman equations:

$$rW_1(\epsilon) = w_1(\epsilon) + \lambda[\int_0^{\overline{\epsilon}} \max\{W_1(x), U_1 + f_1\} dG(x) - W_1(\epsilon)]$$
$$+\eta_1[\max\{W_2(\epsilon), U_2 + f_2\} - W_1(\epsilon)] \quad (11)$$

and:

$$rW_2(\epsilon) = w_2(\epsilon) + \lambda[\int_0^{\overline{\epsilon}} \max\{W_2(x), U_2 + f_2\} dG(x) - W_2(\epsilon)]$$
$$+\eta_2[(f_r + U_3) - W_2(\epsilon)] \quad \text{where } rU_3 = P \quad (12)$$

Furthermore, we determine the present flow value from unemployment to a worker belonging to the age group C_i, denoted by U_i by the following equation:

$$rU_i = z_i + p(\theta_i)[\int_0^{\overline{\epsilon}} \max\{W_i^0(x), U_i\} dG(x) - U_i] + \eta_i(U_{i+1} - U_i) \quad i = 1, 2 \quad (13)$$

where z_i is the non-labor income received by an unemployed worker belonging to the age group C_i. It is noteworthy that an older unemployed worker who retires does not receive any retirement allowance f_r.

In the appendix (7.1), we determine the wage equations for each age group of workers:

$$w_1(\epsilon) = (1 - \beta)z_1 + \beta(y\epsilon + c\theta_1 - \eta_1 f_2 \tau) + f_1(r + \eta_1) - \eta_1 f_2 \quad (14)$$

$$w_2(\epsilon) = \beta y\epsilon - (1 - \beta)z_2 - \beta c\theta_2 + (r + \eta_2)[f_2(1 + \beta\tau)] - \eta_2 f_r \quad (15)$$

We observe first that the wage received by a worker belonging to the group C_i is an increasing function of the non-labor income z_i and of the probability to be matched with a job $p(\theta_i)$, given that these variables raise the worker's threat point, allowing him to extract a higher share of the match surplus from wage bargaining. Furthermore, we remark that the wage of a middle-age worker decreases with the amount of firing costs f_2 and that the wage received by an older worker decreases with the amount of retirement allowance f_r. These results are consistent with Lazear's findings (1990) that show that the higher the employment protection of a worker the lower his wage has to be in the beginning of his career.

3.4 Job destruction and job creation at steady-state

As we already mentioned in the subsection (3.2), in the case of a shock on a job, the employer has no other choice either to retain his worker with the new value of ϵ, the random component of productivity drawn from a distribution $G(x)$, or to fire the worker. In the appendix 7.2, we determine two productivity thresholds, below which firms close down existing filled jobs.

$$y\epsilon_1^d = z_1 + \frac{\beta c}{1-\beta}\theta_1 - \lambda \int_{\epsilon_1^d}^{\overline{\epsilon}} S_1(x)dG(x) - \eta_1 \max\{S_2(\epsilon_1^d), 0\} + \eta_1 f_2 \tau \quad (16)$$

and:

$$y\epsilon_2^d = z_2 + \frac{\beta c}{1-\beta}\theta_2 - \lambda \int_{\epsilon_2^d}^{\overline{\epsilon}} S_2(x)dG(x) - (r+\eta_2) f_2 \tau \quad (17)$$

We observe that the productivity threshold is less than the opportunity cost of employment, composed of non-labor income z_i and of the expected gain to search for a job. Indeed, the third term on the right-hand side of (16) and (17) represents the option value of retaining an existing match despite a shock. This labor-hoarding phenomenon is due to the fact that firms are faced with a positive cost of maintaining a vacancy and therefore they accept to incur a loss in anticipation of a future improvement in the value of the match's product[6].

Regarding the effects on employment protection in the case of the workers belonging to the group C_1, we draw the same conclusions as Lazear (1990): any severance payment arrangement is neutral on the firing decision of firms through an optimal labor contract, in which a worker is willing to pay a fee when he signs the contract to buy the protection of his job. In a similar way, the amount of retirement allowance f_r is also neutral on the firing decision of firms regarding the older workers.

Furthermore in the case of the older workers, we observe that the amount of early retirement window payed by the firm f_2 has a negative impact on the threshold productivity ϵ_2^d. This may be due to the fact that a third agent, the government, receives a part of this payment through a tax at a rate τ, which implies that the worker is not given the whole payment when he is fired: in that case, following the Lazear's theory (1990), firing incentives are distorted. Indeed, as firms expect that firing older workers is more costly, they are more reluctant to close down their jobs. However, we also observe a threshold effect regarding the younger cohort of workers, in the sense that

[6] this type of labor-hoarding behavior has been well investigated by Mortensen Pissarides (1994)

firms have interest to lay middle-age workers off before they switch to the next age group [7].

Regarding hiring rate of each age group of workers, we determine the productivity threshold ϵ_i^c below which the employer does not recruit an unemployed worker belonging to the group C_i, so $S_i^0(\epsilon_i^c) = 0$. Using (8) and (10), we deduce:

$$S_i(\epsilon) = S_i^0(\epsilon) + f_i\tau_i \quad \text{with } \tau_1 = 0 \text{ and } \tau_2 = \tau \tag{18}$$

Using the expression (18) we determine the productivity threshold ϵ_i^c as a function of ϵ_i^d:

$$\epsilon_i^c = \epsilon_i^d + (r + \eta_i + \lambda) f_i\tau_i \quad i = \{1, 2\} \tag{19}$$

Consequently, an increase in the firing costs f_i reduces the hiring rate of the workers belonging to the group C_i, only if the tax rate τ_i is higher than 0. Furthermore, the free-entry condition (4) and the two rent-sharing rules (8) and (10) imply:

$$(1-\beta)\int_{\epsilon_i^c}^{\bar{\epsilon}} S_i^0(x)dG(x) = \frac{c}{q(\theta_i)} \tag{20}$$

At steady-state equilibrium, θ_2 and ϵ_2^d solve the following equation system:

$$\begin{cases} \frac{c}{q(\theta_2)} = & (1-\beta)\int_{\epsilon_2^c}^{\bar{\epsilon}} [\frac{y(x-\epsilon_2^d)}{(r+\eta_2+\lambda)} - f_2\tau]dG(x) \\ y\epsilon_2^d = z_2 + \frac{\beta c}{1-\beta}\theta_2 - & \frac{\lambda}{(r+\eta_2+\lambda)}\int_{\epsilon_2^d}^{\bar{\epsilon}} [y(x-\epsilon_2^d)]dG(x) - (r+\eta_2)f_2\tau \\ y\epsilon_2^c = & y\epsilon_2^d + (r+\eta_2+\lambda)f_2\tau \end{cases}$$

As the first and the second equation describe respectively a downward-sloping and an upward-sloping curve, there exists one unique solution (θ_2, ϵ_2^d) to this problem.

Regarding the workers belonging to the age group C_1, there may be two cases, depending on whether their job may break up when they are ageing or not. In the case 1, the worker keeps his job even though he switches to the next age group. It implies that the reservation productivity ϵ_1^d is higher than ϵ_2^d. Determining the match surplus in appendix 7.3, we show that $(\tilde{\epsilon_1^d}, \tilde{\theta_1})$ solves the following equation system:

$$\begin{cases} \frac{c}{q(\tilde{\theta_1})} = & (1-\beta)\frac{(r+\eta_1+\eta_2+\lambda)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}\int_{\tilde{\epsilon_1^d}}^{\bar{\epsilon}} y(x-\tilde{\epsilon_1^d})dG(x) \\ y\tilde{\epsilon_1^d} = z_1 + \frac{\beta c}{1-\beta}\tilde{\theta_1} - & \frac{\lambda(r+\eta_1+\eta_2+\lambda)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}\int_{\tilde{\epsilon_1^d}}^{\bar{\epsilon}} y[x-\tilde{\epsilon_1^d}]dG(x) - \eta_1 y\frac{\tilde{\epsilon_1^d}-\epsilon_2^d}{(r+\eta_2+\lambda)} + \eta_1 f_2\tau \end{cases}$$

[7] for similar results, see Behaghel (2007)

In the case 2, the worker does not keep necessarily his job when he switches to the next age group. It implies that $\epsilon_1^d \leq \epsilon_2^d$. Determining the match surplus in appendix 7.3 we show that $(\hat{\epsilon_1^d}, \hat{\theta_1})$ solves the following equation system:

$$\begin{cases} \frac{c}{q(\hat{\theta_1})} = \frac{(1-\beta)}{(r+\eta_1+\lambda)} \int_{\hat{\epsilon_1^d}}^{\bar{\epsilon}} y(x - \hat{\epsilon_1^d}) dG(x) + (1-\beta) \frac{\eta_1}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)} \int_{\epsilon_2^d}^{\bar{\epsilon}} [y(x - \epsilon_2^d)] dG(x) \\ \\ y\hat{\epsilon_1^d} = z_1 + \frac{\beta c}{1-\beta} \hat{\theta}_1 - \frac{\lambda}{(r+\eta_1+\lambda)} \int_{\hat{\epsilon_1^d}}^{\bar{\epsilon}} [y(x - \hat{\epsilon_1^d})] dG(x) - \frac{\lambda \eta_1}{(r+\eta_1+\lambda)} \int_{\epsilon_2^d}^{\bar{\epsilon}} [\frac{y(x-\epsilon_2^d)}{(r+\eta_2+\lambda)}] dG(x) + \eta_1 f_2 \tau \end{cases}$$

In the appendix 7.4 we show that the equilibrium solves either the first system or the second. Furthermore as in each system the two first equations describe respectively a downward-sloping and an upward-sloping curve, we deduce that there exists one unique solution, that may be either the couple $(\tilde{\epsilon_1^d}, \tilde{\theta_1})$ or the couple $(\hat{\epsilon_1^d}, \hat{\theta_1})$.

3.5 Unemployment at steady-state equilibrium

We determine the equilibrium values of the unemployment rate u_i and the vacancy rate v_i among each group of workers, using two sets of steady-state conditions. The first implies that, for each age group C_i of workers, the labor force N_i is constant so we get:

$$N_0 \eta_0 = N_1 \eta_1 = N_2 \eta_2 \tag{21}$$

These two equations characterize N_1 and N_2 for exogenous values of N_0, η_0, η_1 and η_2. The second condition implies that for each age group, the flow of workers out of unemployment equals the flow of workers back into unemployment:

$$\lambda G(\epsilon_i^d)(1 - u_i) N_i + N_{i-1} \eta_{i-1} u_{i-1} = p(\theta_i) u_i N_i [1 - G(\epsilon_i^c)] + \eta_i u_i N_i \tag{22}$$

So combining (21) and (22), we determine the unemployment rate u_i in the following way:

$$u_i = \frac{\lambda G(\epsilon_i^d) + u_{i-1} \eta_i}{\lambda G(\epsilon_i^d) + p(\theta_i)[1 - G(\epsilon_i^c)] + \eta_i} \tag{23}$$

As $p(\theta_i)$ is an increasing function of θ_i and therefore of the vacancy rate v_i, we find the expression of the Beveridge curve [8] for each generation of workers, that is an inverse relation between vacancy and unemployment rate. Furthermore, defining the labor market tightness at steady-state equilibrium, we find an other relation between u_i and v_i such that $v_i = \theta_i u_i$. Therefore the equilibrium unemployment exists and is unique, at the intersection between the Beveridge Curve and the increasing curve whose equation is $v_i = \theta_i u_i$.

[8]see notably Blanchard et al. (1989)

4 The effect of the 2003 French pension reform on hiring and job separation rates by age group of workers

To study the effect of a tax on early retirement windows payed by firms combined with an increase in the mandatory retirement age on hiring and job separation rates by age, we make some assumptions regarding the functional forms of the matching function and of the distribution of the component ϵ of the productivity levels. First, we assume that matching function is Cobb-Douglas such that:

$$m(u_i, v_i) = u_i^\alpha v_i^\alpha$$

where α is the elasticity of the matching function. Furthermore, we assume that ϵ_i follows an uniform distribution in the interval $[0, 1]$.

In this section, we study in a first subsection the effect of an increase in the tax rate τ on hiring and job separation rates of older workers. Then, we examine the effect of an increase in the mandatory retirement age in a setting of a taxation of early retirement windows, to investigate the effect of the combination of both these reforms on transition rates among older workers. In a second subsection, we investigate the effect of an increase in the tax rate τ on job finding and separation rates among middle-age workers, then we examine the effect of a combination of this tax with an increase in the mandatory retirement age on these rates.

4.1 A qualitative analysis for older workers

Under the assumptions defined in the beginning of this section, we determine the job creation condition C^2 and the job destruction condition D^2 such that:

$$\begin{cases} C^2(\theta_2, \epsilon_2^d, \tau, \eta_2) = \frac{(1-\beta)y}{2(r+\eta_2+\lambda)}(1 - \epsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y})^2 - c\theta_2^\alpha = 0 \\ D^2(\theta_2, \epsilon_2^d, \tau, \eta_2) = z_2 + \frac{\beta c}{(1-\beta)}\theta_2 - \frac{\lambda y}{2(r+\eta_2+\lambda)}(1 - \epsilon_2^d)^2 - (r + \eta_2)f_2\tau - y\epsilon_2^d = 0 \end{cases}$$

Let C_i^2 and D_i^2 be respectively the partial derivatives of C^2 and D^2 with respect to their i-th argument. Differentiating this equations system with respect to ϵ_2^d, we obtain:

$$\begin{cases} \frac{\partial \epsilon_2^d}{\partial \tau} = \frac{C_3^2 D_1^2 - D_3^2 C_1^2}{D_2^2 C_1^2 - C_2^2 D_1^2} \\ \frac{\partial \epsilon_2^d}{\partial \eta_2} = \frac{C_4^2 D_1^2 - D_4^2 C_1^2}{D_2^2 C_1^2 - C_2^2 D_1^2} \end{cases}$$

We show in the appendix (7.5) that an increase in τ reduces job separation rates among older workers, which is consistent with previous Behaghel's findings.

Then, we examine the effect of a decrease in η_2 that we view as an increase in the mandatory retirement age on separation rates, in a setting of a taxation of early retirement windows at a rate $\tau \neq 0$. We show that in that case, delaying mandatory retirement age has two offsetting effects on the productivity threshold ϵ_2^d. On the one hand, an increase in the mandatory retirement age raises the option value of the employer to retain an existing match despite a shock, given that the employer expects a higher duration of his job[9]. This first effect reinforces the labor-hoarding effect of the tax and reduces job destruction holding all other things fixed.

On the other hand in the case of a high tax rate τ, an increase in the mandatory retirement age may encourage employers to dismiss their older workers, offsetting therefore the dissuasive effect of the tax. The idea is the following: in the case of a high taxation of early retirement windows, an employer has interest to retain his older worker, even though the present value of his job to the employer is negative after a productivity shock. Indeed, as long as the loss in profits does not exceed the separation costs due to the tax, the employer prefers waiting for his worker to reach the mandatory retirement age. In this setting, when the government raises the mandatory retirement age, the horizon along which the firm will incur loss in profits is longer and the employer may have interest to dismiss his older worker earlier, offering him early retirement windows. This impatience effect of firms will therefore raise job separations among older workers, offsetting the initial labor-hoarding effect of the tax.

Consequently, the effect of postponing retirement on job separations among older workers is ambiguous and depends widely on the tax rate τ. Indeed the higher the tax rate, the more likely employers to accept to incur important loss in profits, waiting for their workers to reach the mandatory retirement age and the stronger the impatience effect after an increase in the mandatory retirement age.

Proposition 1. *For values of the tax rate τ sufficiently high such that $\tau > \tau^c = \frac{2\lambda y(1-\epsilon_2^d)^2}{f_2[2(r+\eta_2+\lambda)]^2}$, then $\partial\epsilon_2^d/\partial\eta_2 < 0$.*

Proof: Computing the partial derivative $\partial\epsilon_2^d/\partial\eta_2$, we deduce the following condition:

$$\frac{\partial \epsilon_2^d}{\partial \eta_2} < 0 \Leftrightarrow D_4^2 < 0$$

[9] See also Chéron et al. (2007) for similar results

In the appendix (7.5), we determine D^2_4, and we find a critical value of the tax rate τ^c, above which delaying retirement may lead to raise the productivity threshold ϵ^d_2:

$$\tau^c = \frac{2\lambda y(1-\varepsilon^d_2)^2}{[2(r+\eta_2+\lambda)]^2 f_2} \tag{24}$$

∎

First, we observe that the critical value τ^c falls when the amount of the early retirement window f_2 is high. Indeed, this tax is levied on the whole amount of financial incentives payed by the employer to his older worker. Consequently, the higher the amount of early retirement window the more likely an increase in mandatory retirement age to offset the labor-hoarding of the tax.

Second, we remark that τ^c is a decreasing function of the productivity threshold ϵ^d_2. So, in the setting of a high job separation rate of older workers, delaying retirement may lead to more dismissals among this age group of workers.

Third, τ^c also decreases with η_2. We deduce that in the case where the mandatory retirement age is initially low, increasing this age may encourage employers to get rid of their older workers.

Furthermore, a rise in the productivity threshold ϵ^d_2 may lead to reduce the hiring rate of older workers, all other things being equal and may therefore attenuate and even offset the positive horizon effect due to an increase in the mandatory retirement age. So we have to determine to what extent a combination of a tax on early retirement windows and of an increase in the mandatory retirement age affects the tightness θ_2. Differentiating the job destruction condition (D^2) and the job creation condition (C^2) with respect to θ_2 we get:

$$\begin{cases} \frac{\partial\theta_2}{\partial\tau} = \frac{D^2_3 C^2_2 - C^2_3 D^2_2}{D^2_2 C^2_1 - C^2_2 D^2_1} \\ \\ \frac{\partial\theta_2}{\partial\eta_2} = \frac{D^2_4 C^2_2 - C^2_4 D^2_2}{D^2_2 C^2_1 - C^2_2 D^2_1} \end{cases}$$

We show in the appendix (7.5) that an increase in the tax rate τ reduces θ_2. This is a first perverse effect of a partial employment protection targeted to older workers already highlighted by Behaghel (2007). In addition, recall that job creation is increasing with θ_2 but it is also decreasing with the productivity threshold $\epsilon^c_2 = \epsilon^d_2 + (r+\eta_2+\lambda)f_2\tau$. Consequently, an increase in the tax rate τ also raises ϵ^c_2, which reinforces the negative effect of an increase in τ on the job finding rate of older workers.

Furthermore, even though delaying retirement encourages the creation of jobs targeted to older workers through an horizon effect, we have showed that for a high tax rate τ, such that $\tau \geq \tau^c$, this reform may increase ϵ_2^d. This impatience effect may reduce the present value to an employer of a job filled by an older worker making firms more reluctant to hire older workers. Consequently, we highlight the fact that at some values of the tax rate τ an increase in the mandatory retirement age reduces job creation among older workers.

Proposition 2. *For a high value of the tax rate τ, such that $\tau > \tau^{cc}$ where $\tau^{cc} = \frac{y\{(1-\varepsilon_2^d)[(r+\eta_2)+\lambda\varepsilon_2^d]+(1-\varepsilon_2^d)^2\}}{f_2(r+\eta_2+\lambda)[\lambda(1-\varepsilon_2^d)+(r+\eta_2+\lambda)]}$, then $\partial\theta_2/\partial\eta_2 > 0$.*

Proof: see appendix (7.5) ∎

In addition, recall that the job finding rate of older workers also depends on the productivity threshold $\epsilon_2^c = \epsilon_2^d + (r + \eta_2 + \lambda)(f_2\tau/y)$. It is noteworthy that even though $\tau > \tau^c$, which implies $\partial\epsilon_2^d/\partial\eta_2 < 0$, ϵ_2^c does not necessarily rise after an increase in retirement age. Indeed, calculating the derivative of ϵ_2^c with respect to η_2 we get the following expression :

$$\frac{\partial\epsilon_2^c}{\partial\eta_2} = \frac{\partial\epsilon_2^d}{\partial\eta_2} + (f_2\tau/y) \tag{25}$$

This expression may be negative only if $-\frac{\partial\epsilon_2^d}{\partial\eta_2} > (f_2\tau/y)$. The right-hand side of this inequality is due to the fact that after an increase in retirement age, the expected duration of a job is higher, so an employer may be less reluctant to hire an unemployed worker aged 55-59 years.

Let Π_2 be the job finding rate among older workers, we define Π_2 in the following way :

$$\Pi_2 = \theta_2^{1-\alpha}[1 - \epsilon_2^d - (r + \eta_2 + \lambda)\frac{f_2\tau}{y}]$$

Determining the partial derivative of Π_2 with respect to η_2, we obtain the following expression :

$$\frac{\partial\Pi_2}{\partial\eta_2} = \theta_2^{-\alpha}[(1-\alpha)\frac{\partial\theta_2}{\partial\eta_2}(1-\epsilon_2^c) - \theta_2(\frac{\partial\epsilon_2^d}{\partial\eta_2} + \frac{f_2\tau}{y})]$$

Although this expression seems to be complicated, its sign depends strongly on the tax rate τ. We highlight three cases. In the first case, the tax rate is so low that $\tau < \tau^c$ and an increase in the mandatory retirement age raises unambiguously the job finding rate of an unemployed older worker. In the second case, the tax rate belongs to the interval $[\tau^c, \tau^{cc}]$. In this case, an increase in mandatory retirement age leads to a rise in ϵ_2^d but also in the tightness θ_2. Consequently if the impatience effect is sufficiently high, then $\partial\Pi_2/\partial\eta_2 > 0$ which means that delaying mandatory retirement age reduces the job finding rate of an unemployed older worker. But it is the reverse story if the horizon effect dominates the impatience effect.

Last but not least if the tax rate τ is higher that the critical value τ^{cc}, then delaying retirement increases the productivity threshold ϵ_2^d and reduces simultaneously the tightness θ_2, leading unambiguously to a fall in the job finding rate of an older job-seeker.

4.2 A qualitative analysis for middle-age workers

As mentioned in the previous section, we have to distinguish two cases: the case 1, where $\max\{S_2(\epsilon_1^d), 0\} = S_2$ and the case 2 where $\max\{S_2(\epsilon_1^d), 0\} = 0$. In the case 1, the tightness θ_1 and the productivity threshold ϵ_1^d are defined at equilibrium by the following equations system:

$$\begin{cases} C^1(\theta_1, \epsilon_1^d, \tau, \eta_2, \epsilon_2^d) = -c\theta_1^\alpha + \frac{(1-\beta)(r+\eta_1+\eta_2+\lambda)y}{2(r+\eta_1+\lambda)(r+\eta_2+\lambda)}(1-\epsilon_1^d)^2 \\ D^1(\theta_1, \epsilon_1^d, \tau, \eta_2, \epsilon_2^d) = -y\epsilon_1^d + z_1 + \frac{\beta c\theta_1}{(1-\beta)} - \frac{\lambda(r+\eta_1+\eta_2+\lambda)y}{2(r+\eta_1+\lambda)(r+\eta_2+\lambda)}(1-\epsilon_1^d)^2 - \eta_1 y\frac{\epsilon_1^d-\epsilon_2^d}{(r+\eta_2+\lambda)} + \eta_1 f_2\tau \end{cases}$$

And in the case 2, the tightness θ_1 and the productivity threshold ϵ_1^d are defined at equilibrium by the following equations system:

$$\begin{cases} C^1(\theta_1, \epsilon_1^d, \tau, \eta_2, \epsilon_2^d) = -c\theta_1^\alpha + \frac{(1-\beta)y}{2(r+\eta_1+\lambda)}[1-\epsilon_1^d]^2 + \frac{(1-\beta)\eta_1 y}{2(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d]^2 \\ D^1(\theta_1, \epsilon_1^d, \tau, \eta_2, \epsilon_2^d) = -y\epsilon_1^d + z_1 + \frac{\beta c\theta_1}{(1-\beta)} - \frac{\lambda y[1-\epsilon_1^d]^2}{2(r+\eta_1+\lambda)} - \frac{\eta_1 y}{2(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d]^2 + \eta_1 f_2\tau \end{cases}$$

We observe that in the case 1, the job creation condition does not depend on ϵ_2^d, contrarily to the case 2. This difference is noteworthy notably when we study the effect of an increase in the tax rate τ on the productivity threshold ϵ_1^d. We show in the appendix (7.5) that in the case 1, an increase in τ raises ϵ_1^d if the following condition holds:

$$\eta_1 f_2 > -\frac{\eta_1 y}{(r+\eta_2+\lambda)}\frac{d\epsilon_2^d}{d\tau} \tag{26}$$

The term on the left-hand side of the inequality represents the direct effect of an increase in τ on the firing rate of middle-age workers, already highlighted by Behaghel (2007). Indeed, if it is more costly for an employer to get rid of an older worker, he has interest to lay his worker off before he switches to next age group. The right-hand side of the inequality represents the indirect effect of an increase in τ. Indeed, given that an increase in the tax rate reduces job destruction among older workers, so it raises the present value to the employer of a job filled by a middle-age worker who will switch to the next age group. Consequently, in the case of a shock on a job, firms have interest to retain the existing match.

Furthermore, in the case 2, we obtain in the appendix (7.5) a sufficient condition such that an increase in τ raises ϵ_1^d:

$$\eta_1 f_2 \alpha c \theta_1^{\alpha-1} > -\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d]\frac{d\epsilon_2^d}{d\tau}[\alpha c \theta_1^{\alpha-1} - \beta c] \tag{27}$$

The term on the left-hand side of the inequality represents the direct effect and the term on the right-hand side corresponds to the indirect effect. Contrarily to the condition (26), the direct effect and the indirect effect have not necessarily opposite signs. Indeed, if the bargaining power of workers is too high, such that $\beta > \alpha\theta_1^{\alpha-1}$, then the inequality (27) is necessarily true. The idea is the following. If β is too high, then the decrease in ϵ_2^d raises θ_1, leading to a strong increase in wage and it may encourage employers to lay more middle-age workers off. However, if we assume that β is relatively low, such that $\beta < \alpha\theta_1^{\alpha-1}$, then in the case 1 or in the case 2, an increase in τ leads to a rise in ϵ_1^d provided that the direct effect offsets the indirect effect (through the fall in ϵ_2^d).

In addition, we show in the appendix (7.5) that in the case 1, the effect of an increase in the mandatory retirement age on ϵ_1^d depends on the sign of the following expression:

$$\frac{d\epsilon_1^d}{d\eta_2} > 0 \Leftrightarrow \frac{2\eta_1(r+\eta_1+\lambda)y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}(\lambda\alpha c\theta_1^{\alpha-1} - \beta c)$$

$$+\alpha c\theta_1^{\alpha-1}[\eta_1 y\frac{(\epsilon_1^d - \epsilon_2^d)}{(r+\eta_2+\lambda)^2} + \frac{\eta_1 y}{r+\eta_2+\lambda}\frac{d\epsilon_2^d}{d\eta_2}] > 0 \tag{28}$$

The first term of this sum corresponds to the direct effect, that reduces job destruction among middle-age workers if $\beta < \lambda\alpha\theta_1^{\alpha-1}$. Indeed if the bargaining power is too high, then delaying mandatory retirement age may raise the job finding rate of middle-age workers, increasing therefore wages and encouraging employers to close down more jobs hit by a shock. However, if β is relatively low, a decrease in η_2 raises the option value of retaining an existing match in the case of a shock.

The second term corresponds to the indirect effect that depends directly on the tax rate τ. Indeed, we have previously shown that a decrease in η_2

raises job destruction among older workers if $\tau > \tau^c$. In that case, a firm employing a middle-age worker expects his job is more likely to break up when he will switch to the next age group, which makes his employer more reluctant to retain his job in the case of a negative productivity shock.

It is the same story in the case 2. Indeed, we show in the appendix (7.5) that the effect of a decrease in η_2 on ϵ_1^d depends on the sign of the following expression:

$$\frac{2(r+\eta_1+\lambda)\eta_1 y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}[1-\epsilon_2^d]^2[\alpha c\theta_1^{\alpha-1}-\beta c]$$
$$+\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d][\alpha c\theta_1^{\alpha-1}-\beta c]\frac{d\epsilon_2^d}{d\eta_2} \quad (29)$$

We find again both direct and indirect effects. Consequently, under the condition $\beta < \alpha\theta_1^{\alpha-1}$, delaying retirement raises the option value to the employer to retain an existing match but this direct effect may be attenuated if $\tau > \tau^c$.

Now, we investigate the effect of an increase in τ on the tightness θ_1 in the case 1. We show in the appendix (7.5) that an increase in τ reduces θ_1 under the following condition:

$$\eta_1 f_2 > -\frac{\eta_1 y}{r+\eta_2+\lambda}\frac{d\epsilon_2^d}{d\tau} \quad (30)$$

This condition ensures that an increase in τ raises job destruction among middle-age workers. Consequently, as $\epsilon_1^d = \epsilon_1^c$, a rise in ϵ_1^d reduces the job finding rate of middle-age workers.

We draw similar conclusions in the case 2. Indeed in the appendix (7.5), we show that an increase in τ reduces θ_1 under the following condition:

$$\frac{-(1-\beta)y}{(r+\eta_1+\lambda)}[1-\epsilon_1^d]\eta_1 f_2+\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d][\alpha c\theta_1^{\alpha-1}-\beta c]\frac{d\epsilon_2^d}{d\tau}<0 \quad (31)$$

So provided that β is sufficiently low, such that $\beta < \alpha\theta_1^{\alpha-1}$, then the hiring rate of middle-age workers decreases with τ only if the direct effect through the increase in ϵ_1^c offsets the indirect effect through the fall in ϵ_2^d.

In addition, we have to determine the effect of an increase in the mandatory retirement age on the tightness θ_1. In the appendix (7.5), we show that it has a positive direct effect on θ_1. Nevertheless, a decrease in η_2 may lead to more separations among older workers and therefore to an increase in ϵ_1^d, if the tax rate τ is higher than τ^c. In that case, this indirect effect raises

ϵ_1^c and attenuates therefore the direct positive effect of an increase in the mandatory retirement age on θ_1.

To sum up our findings, we have shown that in the case of an high taxation of early retirement windows, an increase in the mandatory retirement age may encourage employers to close down more jobs hit by idiosyncratic shocks which attenuates the positive horizon effect on job creation. To investigate the magnitude of these effects and to study their impact on employment rate among both age groups of workers, we implement a numerical illustration.

5 A numerical illustration

5.1 The effects of an increase in the mandatory retirement age on hiring and separation rates for both age groups of workers

For this numerical illustration, we consider two age groups of workers : the workers aged 55-59 years and the workers aged 50-54 years. Considering that one period is a year, we set initially $\eta_1 = \eta_2 = 0.2$, so the average duration of each age group equals 5 years and the mandatory retirement age is set to 60 before the 2003 French pension reform.

This numerical illustration aims at determining the magnitude of the effects of an increase in the mandatory retirement age raising from 60 to 65, namely a decrease in η_2 from 0.2 to 0.1, on the job finding and separation rates among each group of workers, for different values of the tax rate τ. The values chosen for this numerical illustration are reported in the table 3.

A first set of parameters $\Phi_1 = \{\alpha, \beta, \lambda, r, z_1, z_2, f_1, f_2\}$ is based on external information. For the values of the elasticity of the matching function α we choose $\alpha = 0.5$ as in the standard literature. The bargaining power of workers β is set to 0.5 so that the Hosios' condition (1990) holds. The annual interest rate r is set to 3%. The amounts of unemployment benefits z_1 and z_2 are computed from the values of the average wages of each group of workers. Using empirical results of Aubert (2005) drawn from a firm-level survey called DADS (*Déclaration Annuelle des Données Sociales*) for the year 2001, we set the gross hourly wage of a worker aged 50-54 to 16 euros and the gross hourly wage of a worker aged 55-59 to 17 euros. We determine therefore the gross yearly average wage for each group of workers, considering a basis of 35 hours per week. We obtain $w_1 = 29121$ euros and $w_2 = 30941$ euros. Setting a replacement rate to 50% we deduce that $z_1 = 14561$ euros and $z_2 = 15471$ euros.

We consider age-dependent firing costs. In the case of a layoff of a worker aged 50-54 years, the employer has to give him severance payment according to the French Employment Legislation. We have shown previously that this payment had no effect on the hiring or firing behavior of firms. Regarding the 55-59 years old, we assume that employers prefer dismissing a worker invoking the serious misconduct and paying him early retirement window f_2 as highlighted by Amauger-Lattes and Desbarats (2006). A little is known about this payment so we set it in a first step to 50% of the yearly gross wage ($f_2 = 15470.5$ euros) and in a second step we implement a sensitivity analysis to check whether our results are robust to different values of f_2. Similarly, as we expect that our results are sensitive to the value of the Poisson arrival rate of idiosyncratic shock λ, we compute the effects of an increase in mandatory retirement age on transition rates for different values of this parameter. So we will distinguish three cases. In the case 1, the benchmark case, f_2 is set to 15470.5 euros and λ is set to 0.2. In the case 2, f_2 is set to only 25% of the average wage so $f_2 = 7735.25$ and λ remains unchanged. In the case 3, f_2 is equal to its value in the benchmark case but $\lambda = 0.4$, a higher value than in our baseline case.

Lastly, we choose to calibrate a second set of parameters $\Phi_2 = \{y, c\}$ to reproduce some stylized facts about hiring and separation rates among workers aged 55-59 years. According to the French Labor Force Survey for the years 2001 and 2002, the job finding rate of an unemployed worker aged 55-59 equals 6.14%. In addition, the job separation rate for this cohort of workers equals 9.29%. Solving our model using these values, we obtain an average productivity of job y equal to 30213 and a cost of maintaining a vacancy c equal to 43931 euros. Using the French Labor Force Survey for the years 2001 and 2002, we remark that these values allow us to match in a satisfying way the observed job finding and job destruction rates among the workers aged 50-54. Indeed, solving the equations system composed of the job creation and the job destruction condition for the workers aged 50-54 setting $c = 43931$ and $y = 30213$, we obtain a job finding rate equal to 17.25% and a job destruction rate equal to 6,25% for this group while the observed rates equal respectively 17.08% and 3.55% according to the French Labor Force Survey 2001-2002 (cf figure 2). In addition, the expression of the steady-state unemployment rate (23) shows that the employment rate among the group C_1 of workers depends on the employment rate among the previous cohort of workers. According to the French Labor Force Survey, we set the employment rate among workers aged 45-49 years to 81.21%. From the steady-state expression (23), we compute employment rates and we see that the values obtained are close to those observed from the data. Indeed, we find an employment rate equal to 77.33% among the 50-54 years old and equal to 60.98% among the workers aged 55-59, while the observed rates equal respectively 75.71% and 57.77%.

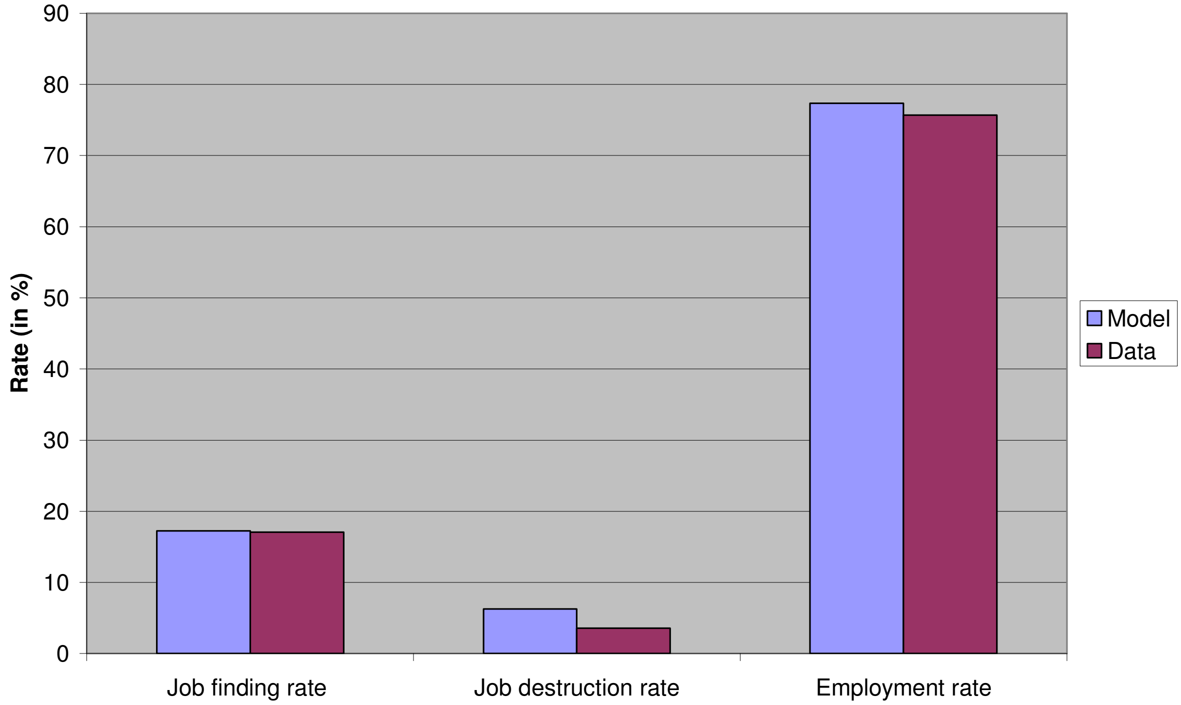
Table 3: Values of parameters

Elasticity of the matching function	α	0.5
Bargaining power	β	0.5
Poisson arrival rate of shocks	λ	0.2
Annual interest rate	r	0.03
Workers aged between 50 and 54		
Unemployment benefits		
(as a fraction of the average wage)	z_1	50%
Probability to switch to the next age group	η_1	0.1
Workers aged between 55 and 59		
Unemployment benefits		
(as a fraction of the average wage)	z_2	50%
Early retirement windows (as a fraction of the average wage)	f_2	50%
Probability to reach the mandatory retirement age	η_2	0.2
Calibrated values		
Cost of maintaining a vacancy	c	43931
Average productivity	y	30213

From this numerical illustration, we seek to highlight the effect due to a tax on early retirement windows offered by firms, when the mandatory retirement age rises. We focus first on the workers aged 55-59 years. We observe in the figure 3 that in the case where early retirement windows are not taxed, the labor-hoarding effect of delaying mandatory retirement age is very low. Indeed, focussing first on the red curve, corresponding to the case 1 where $f_2 = 15470.5$ and $\lambda = 0.2$, for $\tau = 0$ a decrease in η_2 from 0.2 à 0.1 reduces by less than 1% the firing rate of the workers aged 55-59 years. We also remark that this labor-hoarding effect is offset by the impatience effect highlighted in this paper, when the tax rate τ rises. For instance, for a tax rate equal to 50%, as it is the case in France after the 2008 reform (an extension of the 2003 reform), an increase in mandatory retirement age from 60 to 65 may increase job separation rate for the workers aged 55-59 years by 5%.

Our results are qualitatively robust to other values of f_2 and λ. Nevertheless, in the case 2 where f_2 is twice lower than in the benchmark case, the increasing dashed blue curve is flatter than the red curve. So in this setting, an increase in the retirement age leads to a rise in job destruction among the workers aged 55-59 years for a higher value of the tax rate than in the case 1. In addition in the case 3, we see that the green circled curve is as steep as the

Figure 2: Job creation, job destruction and employment among the workers aged between 50 and 54 over the period 2001-2002

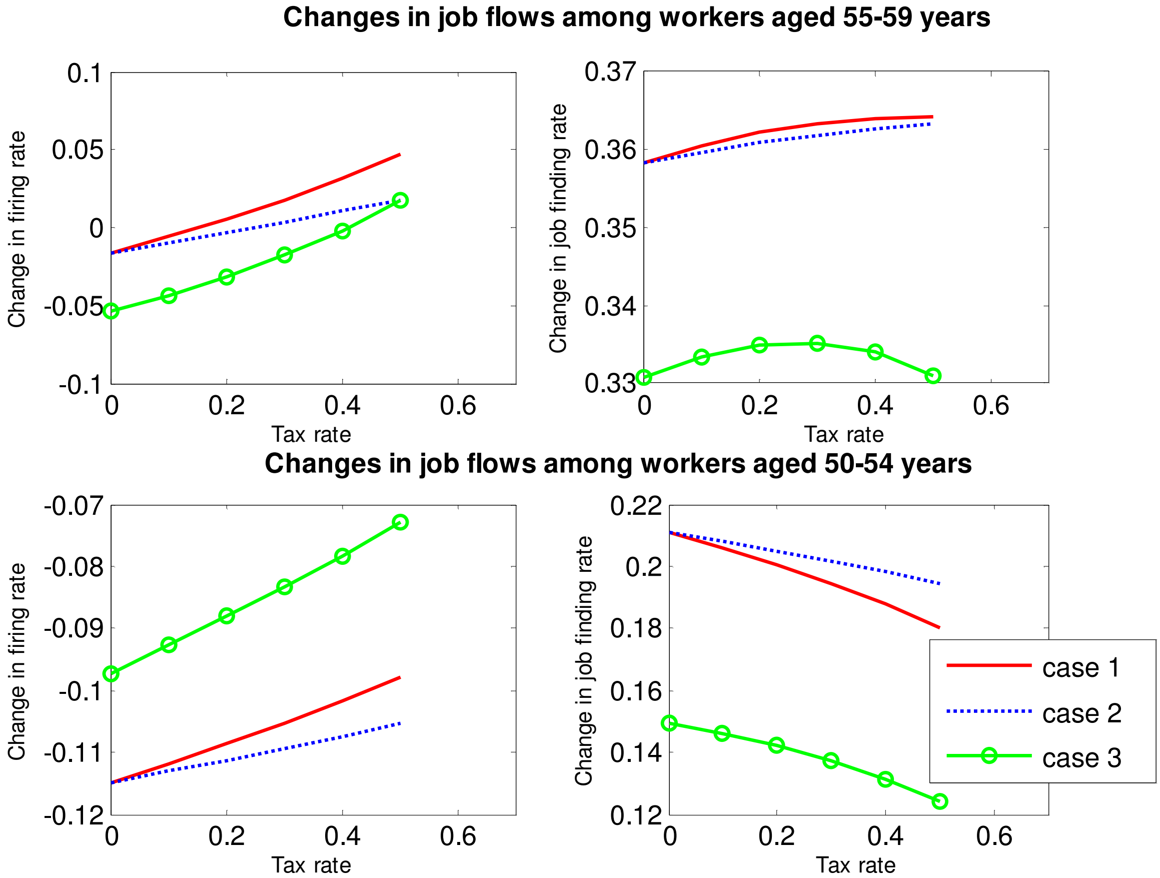


Lecture: The job finding rate is the fraction of workers aged 50-54 unemployed in 2001 who find a job in 2002

The firing rate is the fraction of workers aged 50-54 employed in 2001 who get into unemployment in 2002

Source: French Labor Force Survey (2001-2002)

Figure 3: The effect of a decrease in η_2 from 0.2 to 0.1 on job flows among each age group of workers for different values of the tax rate τ



Lecture: Each graph represents the relative variation in job flows after a decrease in η_2 from 0.2 to 0.1 as a function of the tax rate τ.
The red curve corresponds to the case 1 (benchmark case) where $\lambda = 0.2$ and $f_2 = 15470.5$. The dashed blue curve corresponds to the case 2 where $f_2 = 7735.25$ and $\lambda = 0.2$. The circled green curve corresponds to the case 3 where $f_2 = 15470.5$ and $\lambda = 0.4$.

red curve, but the value of the tax rate above which delaying retirement age raises firings among the workers aged 55-59 years is higher than in the case 1. This result is due to the fact that the labor-hoarding effect of an increase in mandatory retirement age in absence of a tax on early retirement windows is sharply higher in the case 3 than in the case 1. However, the higher the tax rate τ and the lower is this effect which is consistent with our theoretical findings.

Furthermore, we remark in the figure 3 that delaying retirement strongly encourages job creation among the workers aged 55-59 years. Indeed, without a tax on early retirement windows, a decrease in η_2 from 0.2 to 0.1 raises the job finding rate of these workers by more than 35%. We remark that this effect is weakly sensitive to the tax rate τ. Nevertheless, it can appear to be surprising to obtain increasing curves regarding the cases 1 and 2 (red and dashed blue curves). We would expect that the higher the impatience effect the lower is the horizon effect. However, recall that the effect of a decrease in η_2 on the job creation threshold ϵ_2^c is ambiguous and is defined by the expression (25). We observe that the higher the tax rate τ, the higher the second term of the derivative and therefore delaying retirement may reduce the threshold productivity ϵ_2^c while it raises the threshold ϵ_2^d. In addition we observe that in the case 3, when the Poisson arrival rate λ is higher than in the benchmark case, if the tax rate is sufficiently high the horizon effect decreases with τ which yields an inverted U-shaped circled green curve. Nevertheless, the magnitude of all these effects is close to zero and appear to be negligible. We conclude that the positive horizon effect exerted by an increase in the retirement age on the job finding rate of workers aged 55-59 years is very strong whatever the value of the tax rate τ.

Then we investigate the effect of delaying retirement age on job flows among the workers aged 50-54 years. We see first that increasing the mandatory retirement age may reduce job destruction rate among this group of workers by more than 11% in the benchmark case. This labor-hoarding effect is weakly sensitive to the value of the tax rate τ. Indeed, even though the tax rate is set to $\tau = 0.5$, the relative variation in the firing rate after an increase in retirement age equals -10% very close to -11%. And if f_2 is lower than in the benchmark case (blue dashed curve) the effect of the tax on this labor-hoarding effect appear to be negligible. Nevertheless, when looking at the circled green curve, it is noteworthy that the labor-hoarding effect may be attenuated by a taxation of early retirement windows in a significative way when the Poisson arrival rate λ is sufficiently high. Indeed, in absence of taxation, a decrease in η_2 from 0.2 to 0.1 reduces firing rate among workers aged 50-54 years by 10%. When τ is set to 0.5, the same reform reduces the job destruction rate among these workers by only 7%.

Regarding the job creation among the 50-54 years, we see that delaying retirement strongly increases job finding rates. Indeed, for $\tau = 0$ a decrease in η_2 from 0.2 to 0.1 increases the job finding rate by 21% in the benchmark case. However, when the tax rate τ is set to 50% we see that this horizon effect falls from +21% to +18%. So we conclude that in the case of a taxation of early retirement windows, the initial horizon effect exerted by an increase in retirement age on job creation may be attenuated in a significative way. However, when we look at the blue dashed curve corresponding to the case 2 where the value of f_2 is lower, we see that this result is true only for sufficiently high values of the amount of early retirement windows.

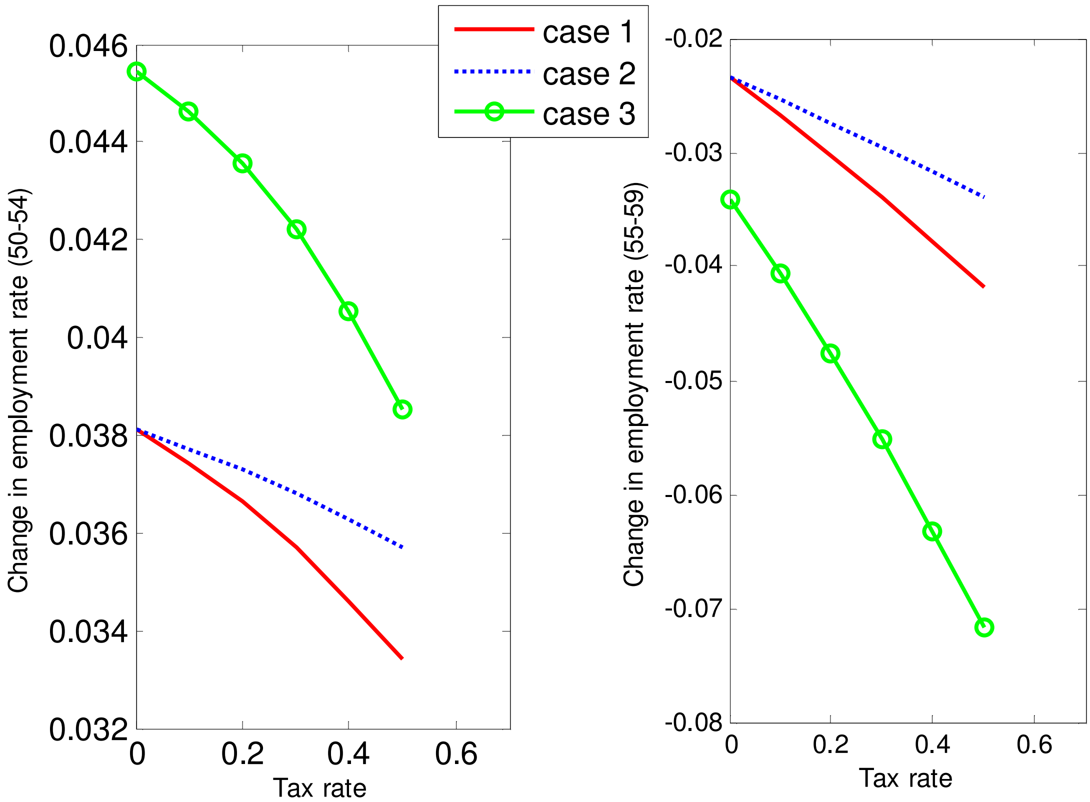
5.2 The effects of an increase in retirement age on employment rates among both age groups of workers

Using the expression (23) of the steady-state unemployment rate among the age group of workers C_i, we deduce the effects of an increase in retirement age on employment rates and we investigate to what extent a taxation on early retirement windows alters this effect.

Focussing first on the workers aged 50-54 years, we observe in the figure 4 that in the absence of taxation of early retirement windows, a decrease in η_2 from 0.2 to 0.1 leads to a rise in employment rate by 3.8% in the benchmark case. In the case 1, when the tax rate is as high as 50%, delaying retirement still raises employment rate by 3.4%. So a high taxation of early retirement windows may attenuate by 10% the positive effect exerted by an increase in retirement age on employment rate among workers aged 50-54 years. This attenuation may be less important for lower values of f_2 (the dashed blue curve is quite flatter than the red curve) but it may be more important for higher values of λ holding f_2 at this benchmark value. Indeed, in the case 3, we see that for $\tau = 0$ the relative variation in employment rate after a decrease in η_2 from 0.2 to 0.1 is +4.6% and it drops to +3.8% for $\tau = 0.5$, so high values of the tax rate τ may attenuate the positive effect of a increase in retirement age on employment rate among workers aged 50-54 years by 17%.

Then when we investigate the effect of an increase in retirement age on employment rate among the workers aged 55-59 years, we see that the tax rate τ plays a more important role. Using the expression (23), we observe that a decrease in η_2 exerts a direct negative effect on employment rate among workers aged 55-59 years. Indeed, when the mandatory retirement age raises, it leads to reduce flows out of unemployment regarding job seekers aged 55-59 years. In addition, a decrease in η_2 may have indirect effects on

Figure 4: The effect of a decrease in η_2 from 0.2 to 0.1 on employment rates among each age group of workers



Lecture: Each graph represents the relative variation in steady-state employment rates after a decrease in η_2 from 0.2 to 0.1. as a function of the tax rate τ.
The red curve corresponds to the case 1 (benchmark case) where $\lambda = 0.2$ and $f_2 = 15470.5$. The dashed blue curve corresponds to the case 2 where $f_2 = 7735.25$ and $\lambda = 0.2$. The circled green curve corresponds to the case 3 where $f_2 = 15470.5$ and $\lambda = 0.4$.

the employment rate. First, it may have a positive effect on employment through an increase in the job finding rate of workers aged 55-59 years. But this horizon effect may be partially offset by the impatience effect for high values of the tax rate τ. Indeed, in this setting, delaying retirement age leads to a rise in job destruction among the workers aged 55-59 years, which exerts a negative effect on the employment rate among this group of workers.

We see in the figure 4 that in the absence of a taxation of early retirement windows, a decrease in η_2 from 0.2 to 0.1 reduces by 2.5% the employment rate among workers aged 55-59 years. In that case, even though the direct effect of a decrease in η_2 on employment dominates, it is strongly attenuated by the horizon effect and therefore, the decrease in employment rate after an increase in retirement age is quite low. However, for a tax rate τ equal to 50%, the impatience effect exerts a supplementary negative impact on the employment rate. Consequently, after an increase in retirement age, the employment rate falls by 4%. So in the case of a high taxation of early retirement windows, the decrease in the employment rate that results from an increase in retirement age is twice higher than in the case where early retirement windows are not taxed. We deduce therefore that the combination of a rise in the mandatory retirement age and of a taxation of the financial incentives to retire payed by firms to their workers aged 55-59 years may have perverse effects on employment among this age group of workers.

6 Conclusion

The goal of this paper is to study the effects of postponing retirement in a setting of an age-specific employment protection on the hiring and separation rates of older workers and also on employment of the elderly. Reproducing the 2003 French pension reform, we set a tax levied on early retirement windows payed by firms to their older workers to dismiss them. We provide some theoretical findings considering a matching model with endogenous destruction extended to account for a mandatory retirement age.

We highlight that in the case of a high tax rate, delaying retirement may raise separations among the targeted age group of workers through an impatience effect. Indeed, a high tax rate discourages firms from dismissing older workers paying them financial incentives, so employers prefer waiting for their workers to reach the mandatory retirement age. In this setting, delaying retirement forces employers to retain their workers for a longer time, and they could be interest to dismiss them before they reach the retirement age in spite of the cost induced by the tax. We point out that there exists a critical value of the tax rate above which the impatience effect offsets the labor-hoarding effect of postponing retirement. Calibrating our data using the French Labor Surveys for the years 2001 and 2002, we show that this critical value is negatively correlated to the amount of early retirement windows but it is positively correlated to the Poisson arrival rate of idiosyncratic shocks. Nevertheless, for reasonable values of these parameters, the relative variation in the job separation rate among the targeted age group of workers after an increase in the mandatory retirement age from 60 to 65 equals to -1% in the absence of taxation on early retirement windows and to 5% in the case where the tax rate equals 50%.

In addition, the impatience effect that we highlight in this paper may partially offset the positive impact of an increase in the retirement age on the hiring rate of these workers. Theoretically, we determine a second critical value of the tax rate above which delaying retirement reduces the hiring rate among the group of workers targeted by the tax. However, we see in our numerical illustration that the extent to which the tax rate influences the effect of postponing retirement on the job finding rate is negligible.

Consequently, the relative variation in the employment rate among the workers targeted by the tax strongly depends on the taxation level. Indeed, in the case of a high tax rate delaying retirement leads to more separations, which exerts a perverse effect on the employment.

Similarly, regarding the previous cohort of workers, we show theoretically that the impatience effect may affect the impact of delaying retirement on the firing and the hiring rates but we show in our numerical illustration that the role of the impatience effect may be quite significative only for high values of the Poisson arrival rate of idiosyncratic shocks. Consequently, the relative variation in the employment rate among this cohort of workers after an increase in the mandatory retirement age is sensitive to the tax rate on early retirement windows only if the Poisson arrival rate of shocks is sufficiently high.

However, these results have to be considered with caution. Indeed, in our model we consider an exogenous and constant amount of early retirement windows, while we can expect that the amount offered by the employer to his worker depends on the characteristics of this worker. Too few informations are still available about these early retirement windows, however since the 2008 reform, an employer who offers such financial incentives to his older workers have to indicate their names, their ages and the amount that they received prior to exit. Using this data, we could carry out an empirical study aiming at better understanding the factors that lead employers to offer such financial incentives and the determinants of the amount offered. We leave this issue for further investigation.

7 Appendix

7.1 The wage equations

Using the sharing rule (10) and rearranging terms, we obtain:

$$-(r+\eta_i+\lambda)(1-\beta)U_i = (r+\eta_i+\lambda)[\beta[J_i(\epsilon)+W_i(\epsilon)]-W_i(\epsilon)+f_i(1+\beta\tau_i)] \quad (32)$$

Let us first define the wage equation for the middle-age workers belonging to the age group C_1. Bellman equations (5) and (11) imply:

$$-(r+\eta_1+\lambda)(1-\beta)U_1 = \beta[y\epsilon+\lambda\int_{\epsilon_1^d}^{\bar{\epsilon}} S_1(x)dG(x)+\lambda U_1+\eta_1\max\{S_2(\epsilon),0\}+\eta_1(U_2-f_2\tau)]$$

$$-w_1(\epsilon)-\lambda\int_{\epsilon_1^d}^{\bar{\epsilon}}[W_1(x)-U_1-f_1]dG(x)-\lambda(U_1+f_1)-\eta_1\max\{W_2(\epsilon)-U_2-f_2,0\}-\eta_1(U_2+f_2)$$

$$+(r+\lambda+\eta_1)f_1 \quad (33)$$

Using the sharing-rule (10) we obtain:

$$\lambda\beta\int_{\epsilon_1^d}^{\bar{\epsilon}} S_1(x)dG(x) = \lambda\int_{\epsilon_1^d}^{\bar{\epsilon}}[W_1(x)-U_1-f_1]dG(x)$$

And:

$$\beta\max\{S_2(\epsilon),0\} = \max\{W_2(\epsilon)-U_2-f_2,0\}$$

Therefore we get:

$$-(r+\eta_1+\lambda)(1-\beta)U_1 = \beta y\epsilon - w_1(\epsilon) - U_1\lambda[1-\beta]$$

$$-(1-\beta)\eta_1(U_2+f_2) - \eta_1\beta[f_2(1+\tau)] + f_1(r+\eta_1) \quad (34)$$

Substituting the Bellman equation (13) into this expression, we obtain:

$$-(1-\beta)[p(\theta_1)[\int_0^{\bar{\epsilon}}\max\{W_1(x),U_1\}dG(x)-U_1]+z_1] = \beta\epsilon$$

$$-w_1(\epsilon)-(1-\beta)\eta_1 f_2-\eta_1\beta[f_2(1+\tau)]-\lambda f_1 \quad (35)$$

Combining the rent-sharing rule (8) and the free-entry condition (4) we get:

$$\int_{\epsilon_i^c}^{\bar{\epsilon}}[W_1(x)-U_1]dG(x) = \frac{\beta}{(1-\beta)}\frac{c}{q(\theta_1)} \quad (36)$$

So, substituting this expression into (35), we deduce the following wage equation:

$$w_1(\epsilon) = (1-\beta)z_1 + \beta(y\epsilon + c\theta_1 - \eta_1 f_2\tau) + f_1(r+\eta_1) - \eta_1 f_2 \qquad (37)$$

Then we determine the wage equations for the older workers belonging to the group C_2. Substituting Bellman equations (6) and (12) into the expression (10), we get:

$$-(r+\eta_2+\lambda)(1-\beta)U_2 = \beta\{y\epsilon + \lambda\int_{\epsilon_2^d}^{\overline{\epsilon}} S_2(x)dG(x) + \lambda(U_2 - f_2\tau) + \eta_2\frac{P}{r}\}$$

$$-w_2(\epsilon) - \lambda\int_{\epsilon_2^d}^{\overline{\epsilon}}[W_2(x) - U_2 - f_2]dG(x) - \lambda(U_2+f_2) - \eta_2(\frac{P}{r} + f_r)$$

$$+(r+\eta_2+\lambda)f_2(1+\beta\tau) \qquad (38)$$

The rent-sharing rule (10) implies:

$$\lambda\beta\int_{\epsilon_2^d}^{\overline{\epsilon}} S_2(x)dG(x) = \lambda\int_{\epsilon_2^d}^{\overline{\epsilon}}[W_2(x) - U_2 - f_2]dG(x)$$

Therefore we get:

$$-(r+\eta_2+\lambda)(1-\beta)U_2 = \beta y\epsilon - w_2(\epsilon) - U_2\lambda[1-\beta] - (1-\beta)\eta_2\frac{P}{r}$$

$$-\eta_2 f_r + (r+\eta_2)[f_2(1+\beta\tau)] \qquad (39)$$

Substituting the Bellman equation (13) into this expression and rearranging terms, we obtain:

$$-(1-\beta)[z_2 + p(\theta_2)[\int_0^{\overline{\epsilon}} max\{W_2(x), U_2\} - U_2]dG(x)] = \beta y\epsilon - w_2(\epsilon)$$

$$+(r+\eta_2)[f_2(1+\beta\tau)] - \eta_2 f_r \qquad (40)$$

Using the sharing rule (8) and the free-entry condition (4), we deduce the following wage equation:

$$w_2(\epsilon) = \beta y\epsilon - (1-\beta)z_2 - \beta c\theta_2 + (r+\eta_2)[f_2(1+\beta\tau)] - \eta_2 f_r \qquad (41)$$

7.2 The productivity thresholds

We have to determine the productivity threshold ϵ_i^d, below which the firm closes down the job filled by a worker belonging to the group C_i. First we define ϵ_1^d. Substituting the wage equation (14) into the expression (5) we get:

$$(r+\eta_1+\lambda)J_1(\epsilon) = (1-\beta)[y\epsilon - z_1] - \beta c\theta_1 - f_1(r+\eta_1+\lambda) + \eta_1 f_2(1+\beta\tau)$$
$$+\lambda\int_{\epsilon_1^d}^{\overline{\epsilon}}[J_1(x)+f_1]dG(x) + \eta_1\max\{J_2(\epsilon)+f_2(1+\tau),0\} - \eta_1 f_2(1+\tau)$$

Simplifying this expression we obtain:

$$(r+\eta_1+\lambda)S_1(\epsilon)(1-\beta) = (1-\beta)[y\epsilon - z_1] - \beta c\theta_1 - (1-\beta)\eta_1 f_2\tau$$
$$+\lambda(1-\beta)\int_{\epsilon_1^d}^{\overline{\epsilon}} S_1(x)dG(x) + \eta_1(1-\beta)\max\{S_2(\epsilon),0\} \quad (42)$$

Evaluating (42) at $\epsilon = \epsilon_1^d$ gives the following productivity threshold:

$$y\epsilon_1^d = z_1 + \frac{\beta c}{1-\beta}\theta_1 - \lambda\int_{\epsilon_1^d}^{\overline{\epsilon}} S_1(x)dG(x) - \eta_1\max\{S_2(\epsilon_1^d),0\} + \eta_1 f_2\tau \quad (43)$$

We proceed in a similar way to determine ϵ_2^d, substituting the wage equation (15) into the expression (6) we get:

$$(r+\eta_2+\lambda)J_2(\epsilon) = (1-\beta)[y\epsilon - z_2] - \beta c\theta_2 - f_2(1+\tau)(r+\eta_2) + (1-\beta)(r+\eta_2)f_2\tau$$
$$+\lambda\int_{\epsilon_2^d}^{\overline{\epsilon}}[J_2(x)+f_2(1+\tau)]dG(x) - \lambda f_2(1+\tau)$$

Simplifying this expression we obtain:

$$(r+\eta_2+\lambda)(1-\beta)S_2(\epsilon) = (1-\beta)[y\epsilon - z_2] - \beta c\theta_2 + (1-\beta)f_2\tau(r+\eta_2)$$
$$+\lambda(1-\beta)\int_{\epsilon_2^d}^{\overline{\epsilon}} S_2(x)dG(x) \quad (44)$$

So evaluating (44) at $\epsilon = \epsilon_2^d$, we get the following job destruction condition:

$$y\epsilon_2^d = z_2 + \frac{\beta c}{1-\beta}\theta_2 - \lambda\int_{\epsilon_2^d}^{\overline{\epsilon}} S_2(x)dG(x) - (r+\eta_2)f_2\tau \quad (45)$$

Furthermore, rent-sharing rules (8) and (10) imply:

$$S_i(\epsilon) = S_i^0(\epsilon) + f_i\tau_i \quad \tau_1 = 0, \tau_2 = \tau \quad (46)$$

So using (46) we can deduce the productivity threshold ϵ_i^c, such that:

$$\epsilon_i^c = \epsilon_i^d + (r+\eta_i+\lambda)f_i\tau_i \quad (47)$$

7.3 Match surpluses

Given that $S_2(\epsilon_2^d)0$, we get:

$$S_2(\epsilon) - S_2(\epsilon_2^d) = \frac{y(\epsilon - \epsilon_2^d)}{r + \eta_2 + \lambda} \tag{48}$$

Furthermore, in the case where $\epsilon_1^d > \epsilon_2^d$, given that $S_1(\epsilon_1^d) = 0$, we obtain:

$$(r + \eta_1 + \lambda)[\tilde{S}_1(\epsilon) - \tilde{S}_1(\epsilon_1^d) = y(\epsilon - \epsilon_1^d) + \eta_1[S_2(\epsilon) - S_2(\epsilon_1^d)] \tag{49}$$

The expression (48) allows us to simplify this expression so we get:

$$\tilde{S}_1(\epsilon) = \frac{y(\epsilon - \epsilon_1^d)}{(r + \eta_1 + \lambda)}(1 + \frac{\eta_1}{r + \eta_2 + \lambda}) \tag{50}$$

In the case where $\epsilon_1^d \leq \epsilon_2^d$, in a similar way we obtain:

$$\hat{S}_1(\epsilon) = \frac{y(\epsilon - \epsilon_1^d)}{(r + \eta_1 + \lambda)} + \frac{\eta_1}{r + \eta_1 + \lambda} \max\{\frac{y(\epsilon - \epsilon_2^d)}{(r + \eta_2 + \lambda)}, 0\} \tag{51}$$

7.4 The equilibrium for the middle-age workers

In the case 1, where $\max\{S_2(\epsilon_1^d), 0\} = S_2(\epsilon_1^d)$, an unique equilibrium $(\tilde{\epsilon_1^d}, \tilde{\theta}_1)$ is defined by the following equation system:

$$\begin{cases} \frac{c}{q(\tilde{\theta}_1)} = & (1-\beta)\frac{(r+\eta_1+\eta_2+\lambda)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)} \int_{\tilde{\epsilon_1^d}}^{\bar{\epsilon}} y(x - \tilde{\epsilon_1^d})dG(x) \\ y\tilde{\epsilon_1^d} = z_1 + \frac{\beta}{(1-\beta)}c\tilde{\theta}_1 - \frac{\lambda}{(r+\eta_1+\lambda)}(1 + \frac{\eta_1}{(r+\eta_2+\lambda)}) \int_{\tilde{\epsilon_1^d}}^{\bar{\epsilon}} y(x - \tilde{\epsilon_1^d})dG(x) + \eta_1 f_2 \tau - \eta_1 y \frac{(\tilde{\epsilon_1^d} - \epsilon_2^d)}{(r+\lambda+\eta_2)} \end{cases}$$

And in the case 2, where $\max\{S_2(\epsilon_1^d), 0\} = 0$, an unique equilibrium $(\hat{\epsilon_1^d}, \hat{\theta}_1)$ is defined by the following equation system:

$$\begin{cases} \frac{c}{q(\hat{\theta}_1)} = & \frac{(1-\beta)}{(r+\eta_1+\lambda)} \int_{\hat{\epsilon_1^d}}^{\bar{\epsilon}} y(x - \hat{\epsilon_1^d})dG(x) + (1-\beta)\frac{\eta_1}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)} \int_{\epsilon_2^d}^{\bar{\epsilon}} y[x - \epsilon_2^d]dG(x) \\ y\hat{\epsilon_1^d} = z_1 + \frac{\beta}{(1-\beta)}c\hat{\theta}_1 - \frac{\lambda \int_{\hat{\epsilon_1^d}}^{\bar{\epsilon}} y(x - \hat{\epsilon_1^d})dG(x)}{(r+\eta_1+\lambda)} - \frac{\eta_1}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)} \int_{\epsilon_2^d}^{\bar{\epsilon}} y(x - \epsilon_2^d)dG(x) + \eta_1 f_2 \tau \end{cases}$$

So, we have to show that the equilibrium for the younger generation is either the couple $(\tilde{\epsilon_1^d}, \tilde{\theta}_1)$ or the couple $(\hat{\epsilon_1^d}, \hat{\theta}_1)$. In other words, we have to show that there can be neither 0 solutions nor 2 solutions to this problem. We borrow the proof of Behaghel (2007). Indeed, if the problem had 0 solutions, it would imply : $\tilde{\epsilon_1^d} < \epsilon_2^d < \hat{\epsilon_1^d}$. So subtracting the two job creation conditions each other we would get:

$$\frac{c}{q(\tilde{\theta_1})} - \frac{c}{q(\hat{\theta_1})} = \frac{(1-\beta)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[(r+\eta_1+\eta_2+\lambda)\int_{\tilde{\epsilon_1^d}}^{\epsilon_2^d} y(x-\tilde{\epsilon_1^d})dG(x)$$

$$+\eta_1[y\epsilon_2^d - y\tilde{\epsilon_1^d}](1-G(\epsilon_2^d))] + \frac{(1-\beta)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[(r+\eta_2+\lambda)\int_{\epsilon_2^d}^{\hat{\epsilon_1^d}} y(x-\tilde{\epsilon_1})dG(x)$$

$$+(r+\eta_2+\lambda)[y\hat{\epsilon_1^d} - y\tilde{\epsilon_1^d}](1-G(\hat{\epsilon_1^d}))] > 0 \qquad (52)$$

Therefore, we deduce that $\tilde{\theta_1} > \hat{\theta_1}$.

Furthermore, subtracting the two job destruction conditions each other we would get:

$$\frac{\beta}{(1-\beta)}c(\tilde{\theta_1}-\hat{\theta_1}) = (y\tilde{\epsilon_1^d} - y\hat{\epsilon_1^d}) + \lambda[\frac{\int_{\tilde{\epsilon_1^d}}^{\bar{\epsilon}}(y\hat{\epsilon_1^d} - y\tilde{\epsilon_1^d})dG(x)}{(r+\eta_1+\lambda)} + \frac{\eta_1 y\int_{\tilde{\epsilon_1^d}}^{\bar{\epsilon}}(\epsilon_2^d - \tilde{\epsilon_1^d})dG(x)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}]$$

$$+\eta_1 y\frac{(\tilde{\epsilon_1^d} - \epsilon_2^d)}{(r+\eta_2+\lambda)}$$

$$\Leftrightarrow \frac{\beta}{(1-\beta)}c(\tilde{\theta_1}-\hat{\theta_1}) = (y\tilde{\epsilon_1^d} - y\hat{\epsilon_1^d})[1 - \frac{\lambda(1-G(\tilde{\epsilon_1^d}))}{(r+\eta_1+\lambda)}]$$

$$+\frac{\eta_1 y(\tilde{\epsilon_1^d} - \epsilon_2^d)}{(r+\eta_2+\lambda)}[1 - \frac{\lambda(1-G(\tilde{\epsilon_1^d}))}{(r+\eta_1+\lambda)}] < 0 \qquad (53)$$

So we deduce that $\tilde{\theta_1} < \hat{\theta_1}$. This result is not possible given the previous result, so it is the proof that there exists at least one solution to the problem.

Furthermore, we proceed in a similar way to show that the problem does not admit two solutions. Indeed, if he admitted two solutions, it would imply: $\tilde{\epsilon_1^d} > \epsilon_2^d > \hat{\epsilon_1^d}$. In that case, the equation (52) would imply $\tilde{\theta_1} < \hat{\theta_1}$ and the equation (53) would imply $\tilde{\theta_1} > \hat{\theta_1}$, so this case is absurd. Consequently, the problem admits one unique solution: either the couple $(\tilde{\epsilon_1^d}, \tilde{\theta_1})$ if $\epsilon_1^d > \epsilon_2^d$, or the couple $(\hat{\epsilon_1^d}, \hat{\theta_1})$ if $\epsilon_1^d < \epsilon_2^d$.

7.5 Effects of the tax rate and the retirement age on hiring and separation rates among older workers

We determine the partial derivatives of the job creation condition:

$$\begin{cases} C_1^2 = -c\alpha\theta_2^{(\alpha-1)} < 0 \\ C_2^2 = \frac{-(1-\beta)}{(r+\eta_2+\lambda)}[y(1-\varepsilon_2^d) - (r+\eta_2+\lambda)f_2\tau] < 0 \\ C_3^2 = -(1-\beta)f_2[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}] < 0 \\ C_4^2 = \frac{-2(1-\beta)y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y})^2 - f_2\tau\frac{(1-\beta)}{(r+\eta_2+\lambda)}[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}] < 0 \end{cases}$$

Then we determine the partial derivatives for the job destruction condition:

$$\begin{cases} D_1^2 = \frac{\beta c}{(1-\beta)} > 0 \\ D_2^2 = \frac{\lambda y}{(r+\eta_2+\lambda)}(1-\varepsilon_2^d) - y = \frac{-[(r+\eta_2)y+\lambda y\varepsilon_2^d]}{(r+\eta_2+\lambda)} < 0 \\ D_3^2 = -(r+\eta_2)f_2 < 0 \\ D_4^2 = \frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau \end{cases}$$

We examine the effect of an increase in τ on ϵ_2^d:

$$\frac{d\varepsilon_2^d}{d\tau} = \frac{(C_3^2D_1^2 - D_3^2C_1^2)}{(D_2^2C_1^2 - C_2^2D_1^2)} < 0$$

Then we investigate the effect of a decrease in η_2 (namely an increase in the mandatory retirement age) on ϵ_2^d:

$$\frac{d\varepsilon_2^d}{d\eta_2} = \frac{(C_4^2D_1^2 - D_4^2C_1^2)}{(D_2^2C_1^2 - C_2^2D_1^2)}$$

We know that: $C_4^2D_1^2 < 0$ and $C_1^2 < 0$. So if $D_4^2 < 0$ then $\frac{d\varepsilon_2^d}{d\eta_2} < 0$. We have to study the sign of D_4^2:

$$D_4^2 = \frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau$$

Consequently, a sufficient condition such that $D_4^2 < 0$ may be expressed as follows:

$$\frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau \quad < \quad 0$$

$$\Leftrightarrow \quad \tau > \frac{2\lambda y(1-\varepsilon_2^d)^2}{[2(r+\eta_2+\lambda)]^2 f_2}$$

Now, we determine the effect of an increase in τ on the tightness θ_2:

$$\frac{d\theta_2}{d\tau} = \frac{D_3^2 C_2^2 - C_3^2 D_2^2}{(C_1^2 D_2^2 - D_1^2 C_2^2)}$$

$$\begin{aligned} D_3^2 C_2^2 - C_3^2 D_2^2 \quad = \quad & \frac{(1-\beta)}{(r+\eta_2+\lambda)}[y(1-\varepsilon_2^d) - (r+\eta_2+\lambda)f_2\tau](r+\eta_2)f_2 - \\ & (1-\beta)f_2[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}]\frac{(r+\eta_2)y+\lambda y\varepsilon_2^d}{(r+\eta_2+\lambda)} \end{aligned}$$

Factorizing this expression by $\frac{(1-\beta)}{(r+\eta_2+\lambda)}$ we deduce that its sign is the same as the sign of the following expression:

$$[y(1-\varepsilon_2^d)-(r+\eta_2+\lambda)f_2\tau](r+\eta_2)f_2 - f_2[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}][(r+\eta_2)y+\lambda y\varepsilon_2^d]$$

Simplifying and rearranging terms, we find that this expression equals:

$$-f_2\lambda\varepsilon_2^d y[(1-\varepsilon_2^d) - \frac{(r+\eta_2+\lambda)f_2\tau}{y}] < 0$$

Then we determine the effect of an increase in the mandatory retirement age on θ_2:

$$\frac{d\theta_2}{d\eta_2} = \frac{D_4^2 C_2^2 - C_4^2 D_2^2}{(C_1^2 D_2^2 - D_1^2 C_2^2)}$$

If $D_4^2 > 0$, then $\frac{d\theta_2}{d\eta_2} < 0$. However, if $D_4^2 < 0$, then :

$$D_4^2 C_2^2 - C_4^2 D_2^2 = -[\frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau]\frac{(1-\beta)}{(r+\eta_2+\lambda)}[y(1-\varepsilon_2^d)-(r+\eta_2+\lambda)f_2\tau]$$

$$-\{\frac{2(1-\beta)y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y})^2$$

$$+f_2\tau\frac{(1-\beta)}{(r+\eta_2+\lambda)}[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}]\}\frac{[(r+\eta_2)y+\lambda y\varepsilon_2^d]}{(r+\eta_2+\lambda)}$$

Factorizing by $\frac{(1-\beta)}{(r+\eta_2+\lambda)}[(1-\varepsilon_2^d)-\frac{(r+\eta_2+\lambda)f_2\tau}{y}]$ we deduce that this expression has the same sign as the following expression:

$$-y[\frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2-f_2\tau]-f_2\tau\frac{[(r+\eta_2)y+\lambda y\varepsilon_2^d]}{(r+\eta_2+\lambda)}$$
$$-\frac{2y}{[4(r+\eta_2+\lambda)]}(1-\varepsilon_2^d-\frac{(r+\eta_2+\lambda)f_2\tau}{y})\frac{[(r+\eta_2)y+\lambda y\varepsilon_2^d]}{(r+\eta_2+\lambda)}$$

The sign of this expression may be ambiguous given that at $\tau>\tau^c$, $\frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2-f_2\tau<0$. We can define a sufficient condition such that $\frac{\partial\theta_2}{\partial\eta_2}>0$:

$$f_2\tau\frac{[\lambda y(1-\varepsilon_2^d)]}{(r+\eta_2+\lambda)} > \frac{y^2}{2(r+\eta_2+\lambda)^2}\{(1-\varepsilon_2^c)[(r+\eta_2)+\lambda\varepsilon_2^d]+(1-\varepsilon_2^d)^2\}$$
$$\Leftrightarrow \tau>\frac{y\{(1-\varepsilon_2^d)[(r+\eta_2)+\lambda\varepsilon_2^d]+(1-\varepsilon_2^d)^2\}}{f_2(r+\eta_2+\lambda)[\lambda(1-\varepsilon_2^d)+(r+\eta_2+\lambda)]}$$

Effects of the tax rate and the retirement age on hiring and separation rates among middle-age workers

We consider first the case 1 where $\max\{S_2(\epsilon_1^d),0\}=S_2(\epsilon_1^d)$. We determine the effect of an increase in τ and of a decrease in η_2 on the productivity threshold ϵ_1^d. Differentiating this equations system we find the two following expressions:

$$\begin{cases}\frac{\partial\epsilon_1^d}{\partial\tau}=\frac{(C_3^1D_1^1-D_3^1C_1^1)+(C_5^1D_1^1-D_5^1C_1^1)\frac{d\epsilon_2^d}{d\tau}}{D_2^1C_1^1-C_2^1D_1^1}\\ \\ \frac{\partial\epsilon_1^d}{\partial\eta_2}=\frac{(C_4^1D_1^1-D_4^1C_1^1)+(C_5^1D_1^1-D_5^1C_1^1)\frac{d\epsilon_2^d}{d\eta_2}}{D_2^1C_1^1-C_2^1D_1^1}\end{cases}$$

We determine the partial derivatives for the job creation condition:

$$\begin{cases}C_1^1=-\alpha c\theta_1^{\alpha-1}<0\\ \\ C_2^1=\frac{-(1-\beta)(r+\eta_1+\eta_2+\lambda)y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}(1-\epsilon_1^d)<0\\ \\ C_3^1=0\\ \\ C_4^1=\frac{-2\eta_1(r+\eta_1+\lambda)(1-\beta)y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}(1-\epsilon_1^d)^2<0\\ \\ C_5^1=0\end{cases}$$

Then we calculate the partial derivatives for the job destruction condition:

$$\begin{cases} D_1^1 = \frac{\beta c}{(1-\beta)} > 0 \\ D_2^1 = \frac{-(r+\eta_1+\eta_2+\lambda)y[(r+\eta_1)+\lambda\epsilon_1^d]}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)} < 0 \\ D_3^1 = \eta_1 f_2 > 0 \\ D_4^1 = (1-\epsilon_1^d)^2 \frac{2\eta_1(r+\eta_1+\lambda)\lambda y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2} + \eta_1 y \frac{(\epsilon_1^d-\epsilon_2^d)}{(r+\eta_2+\lambda)^2} > 0 \\ D_5^1 = \frac{\eta_1 y}{(r+\eta_2+\lambda)} > 0 \end{cases}$$

Consequently:

$$\frac{d\epsilon_1^d}{d\tau} > 0 \Leftrightarrow \eta_1 f_2 > -\frac{\eta_1 y}{(r+\eta_2+\lambda)}\frac{d\epsilon_2^d}{d\tau}$$

Furthermore:

$$\frac{d\epsilon_1^d}{d\eta_2} > 0 \Leftrightarrow \frac{2\eta_1(r+\eta_1+\lambda)y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}(\lambda\alpha c\theta_1^{\alpha-1} - \beta c)$$

$$+\alpha c\theta_1^{\alpha-1}[\eta_1 y\frac{(\epsilon_1^d-\epsilon_2^d)}{(r+\eta_2+\lambda)^2} + \frac{\eta_1 y}{r+\eta_2+\lambda}\frac{d\epsilon_2^d}{d\eta_2}] > 0$$

We determine then the effect of an increase in τ and of a decrease in η_2 on the tightness θ_1. We obtain the following equations system:

$$\begin{cases} \frac{d\theta_1}{d\tau} = \frac{(D_3^1C_2^1 - C_3^1D_2^1) + (D_5^1C_2^1 - C_5^1D_2^1)\frac{d\epsilon_2^d}{d\tau}}{C_1^1D_2^1 - D_1^1C_2^1} \\ \frac{d\theta_1}{d\tau} = \frac{(D_4^1C_2^1 - C_4^1D_2^1) + (D_5^1C_2^1 - C_5^1D_2^1)\frac{d\epsilon_2^d}{d\eta_2}}{C_1^1D_2^1 - D_1^1C_2^1} \end{cases}$$

Consequently:

$$\frac{d\theta_1}{d\tau} < 0 \Leftrightarrow \eta_1 f_2 + \frac{\eta_1 y}{r+\eta_2+\lambda}\frac{d\epsilon_2^d}{d\tau} > 0$$

Furthermore:

$$\frac{d\theta_1}{d\eta_2} < 0 \Leftrightarrow (D_4^1C_2^1 - C_4^1D_2^1) + D_5^1C_2^1\frac{d\epsilon_2^d}{d\eta_2} < 0$$

As $D_4^1C_2^1 < 0$, $C_4^1D_2^1 > 0$ and $D_5^1C_2^1 < 0$, so the direct effect implies that a decrease in η_2 raises θ_1. The indirect effect through $\partial\epsilon_2^d/\partial\eta_2$ may reinforce the direct effect if $\tau < \tau^c$, but it may attenuate the direct effect if $\tau > \tau^c$.

In a second step, we consider the case 2 where $\max\{S_2(\epsilon_1^d), 0\} = 0$. We calculate the partial derivatives for the job creation condition:

$$\begin{cases} C_1^1 = -\alpha c\theta_1^{\alpha-1} < 0 \\ C_2^1 = \frac{-(1-\beta)y}{(r+\eta_1+\lambda)}[1-\epsilon_1^d] < 0 \\ C_3^1 = 0 \\ C_4^1 = -[1-\epsilon_2^d]^2 \frac{2(1-\beta)\eta_1 y(r+\eta_1+\lambda)}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2} < 0 \\ C_5^1 = \frac{-(1-\beta)\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d] < 0 \end{cases}$$

We also calculate the partial derivatives for the job destruction condition:

$$\begin{cases} D_1^1 = \frac{\beta c}{(1-\beta)} > 0 \\ D_2^1 = \frac{y[-(r+\eta_1)-\lambda\epsilon_1^d]}{(r+\eta_1+\lambda)} < 0 \\ D_3^1 = \eta_1 f_2 > 0 \\ D_4^1 = \frac{2(r+\eta_1+\lambda)\eta_1 y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}[1-\epsilon_2^d]^2 > 0 \\ D_5^1 = \frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d] > 0 \end{cases}$$

We determine first the effect of an increase in τ and of a decrease in η_2 on the productivity threshold ϵ_1^d. Differentiating our equations system, we obtain the two following expressions:

$$\begin{cases} \frac{d\epsilon_1^d}{d\tau} = \frac{(C_3^1 D_1^1 - D_3^1 C_1^1) + (C_5^1 D_1^1 - D_5^1 C_1^1)\frac{d\epsilon_2^d}{d\tau}}{D_2^1 C_1^1 - C_2^1 D_1^1} \\ \frac{d\epsilon_1^d}{d\eta_2} = \frac{(C_4^1 D_1^1 - D_4^1 C_1^1) + (C_5^1 D_1^1 - D_5^1 C_1^1)\frac{d\epsilon_2^d}{d\eta_2}}{D_2^1 C_1^1 - C_2^1 D_1^1} \end{cases}$$

We determine a sufficient condition under which an increase in the tax rate τ raises ϵ_1^d:

$$\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d]\frac{d\epsilon_2^d}{d\tau}[\alpha c\theta_1^{\alpha-1} - \beta c] + \eta_1 f_2 \alpha c\theta_1^{\alpha-1} > 0 \quad (54)$$

Furthermore, $\frac{d\epsilon_1^d}{d\eta_2}$ has the same sign as the following expression:

$$\frac{2(r+\eta_1+\lambda)\eta_1 y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}[1-\epsilon_2^d]^2[\alpha c\theta_1^{\alpha-1}-\beta c]$$

$$+\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d][\alpha c\theta_1^{\alpha-1}-\beta c]\frac{d\epsilon_2^d}{d\eta_2}$$

Then, we determine the effect of an increase in τ and of a decrease in η_2 on the tightness θ_1. We obtain the two following expressions:

$$\begin{cases} \frac{d\theta_1}{d\tau} = \frac{(D_3^1C_2^1-C_3^1D_2^1)+(D_5^1C_2^1-C_5^1D_2^1)\frac{d\epsilon_2^d}{d\tau}}{C_1^1D_2^1-D_1^1C_2^1} \\ \\ \frac{d\theta_1}{d\eta_2} = \frac{(D_4^1C_2^1-C_4^1D_2^1)+(D_5^1C_2^1-C_5^1D_2^1)\frac{d\epsilon_2^d}{d\eta_2}}{C_1^1D_2^1-D_1^1C_2^1} \end{cases}$$

Consequently, an increase in τ leads to a fall in the tightness θ_1 if the following condition holds:

$$\frac{-(1-\beta)}{(r+\eta_1+\lambda)}[1-\epsilon_1^d]\eta_1 f_2+\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d][\alpha c\theta_1^{\alpha-1}-\beta c]\frac{d\epsilon_2^d}{d\tau}<0 \quad (55)$$

Furthermore, regarding the effect of a decrease in η_2 on θ_1, we draw similar conclusions as in the case 1.

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The CNRS **Institute for Labor Studies and Public Policies** (the TEPP Institute, FR n°3126 CNRS) gathers together research centres specializing in economics and sociology:

- the **Centre d'Etudes de l'Emploi** (Centre for Employment Studies), **CEE**, Public Administrative Organization, under the leadership of the Ministers of Work and Research
- l'**Equipe de Recherche sur les Marchés, l'Emploi et la Simulation** (Research Team on Markets, Employment and Simulation), **ERMES**, University of Paris II Panthéon-Assas
- the **Centre d'Etudes des Politiques Economiques de l'université d'Evry** (Research Centre focused on the analysis of economic policy and its foundations and implications), **EPEE**, University of Evry Val d'Essonne
- the **Centre Pierre Naville** (Research on Work and Urban Policies), **CPN**, University of Evry Val d'Essonne
- l'**Equipe de Recherche sur l'Utilisation des Données Temporelles en Economie** (Research Team on Use of Time Data in Economics), **ERUDITE**, University of Paris-Est Créteil and University of Paris-Est Marne-la-Vallée
- the **Groupe d'Analyse des Itinéraires et des Niveaux Salariaux** (The Group on Analysis of Wage Levels and Trajectories), **GAINS**, University of the Maine

The TEPP Institute brings together 115 researchers and research professors, 140 PhD students and 40 associate researchers who study changes in work and employment in relation to the choices made by firms and analyse public policies using new evaluation methods.