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Abstract

With the emergence of the chaos theory and the method of surrogates data, nonlinear approaches employed in analysing time series typically suffer from high computational complexity and lack of straightforward explanation. Therefore, the need for methods capable of characterizing time series in terms of their linear, nonlinear, deterministic and stochastic nature are preferable. In this paper, we provide a signal modality analysis on a variety of exchange rates. The analysis is achieved by using the recently proposed 'delay vector variance' (DVV) method, which examines local predictability of a signal in the phase space to detect the presence of determinism and nonlinearity in a time series. Optimal embedding parameters used in the DVV analysis are obtain via a differential entropy based method using wavelet-based surrogates. A comprehensive analysis of the feasibility of this approach is provided. The empirical results show that the DVV method can be opted as an alternative way to understanding exchange rates dynamics.

Keywords: Nonlinearity analysis, Exchange Rates, Surrogates, Delay vector variance (DVV) method, Wavelets
JEL: C14, C22, C40, F31

1. Introduction

In real-world applications of economic time series analysis, the process underlying the generated signal, which is the time series, are a priori unknown. These signals usually contain

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both linear and nonlinear, as well as deterministic and stochastic components, yet it is a common practice to model such processes using suboptimal, but mathematically tractable models. In general, performing a nonlinearity analysis in a modelling or signal processing context can lead to a significant improvement of the quality of the results, since it facilitates the selection of appropriate processing methods, suggested by the data itself. Interestingly, there has been an increasing concerns on the forecasting performance of some nonlinear models in modelling economic variables. Nonlinear models often provide superior in-sample fit, but rather poor out-of-sample forecast performance (Stock and Watson, 1999). In cases were the nonlinearity is suprious or relevant for only a small part of the observations, the use of nonlinear models will lead to forecast failure (Terasvirta, 2011, Buncic, 2009). It is, therefore, essential to investigate the intrinsic dynamical properties of economic time series in terms of its deterministic/stochastic and nonlinear/linear components which reveal important information that otherwise remains not clear in using conventional linear methods of time series analysis.

Several methods for detecting nonlinear nature of a signal have been proposed over the past few years. The classic ones include the 'deterministic versus stochastic' (DVS) plots (Casdagli, 1994), the Correlation Exponent and 'δ-ϵ' method (Kaplan, 1994). For our purpose, it is desirable to have a method which is straightforward to visualize, and which facilitates the analysis of predictability, which is a core notion in online learning. In this paper, we adopt to the recently proposed phase space based 'delay vector variance' (DVV) method (Gautama, 2004a), for signal characterisation, which is more suitable for signal processing application because it examines the nonlinear and deterministic signal behaviour at the same time. This method has been used for the qualitative assessment of machine learning algorithms, analysis of functional magnetic resonance imaging (fMRI) data, as well as analysing nonlinear structures in brain electrical activity and heart rate variability (HRV) (Gautama, 2004b). Optimal embedding parameters will be obtained using a differential entropy based method proposed in Gautama (2003), which allows for simultaneous determination of both the embedding dimension and time lag. Surrogate generation used in this study will be based on a recently refined Iterative Amplitude Adjusted Fourier Transform (iAAFT) using a wavelet-based approach, denoted WiAAFT (Keylock, 2006).

In this paper, we provide a novel procedure in understanding exchange rate dynamics based on a phase space 'delay vector variance' (DVV) method. The empirical results on a variety of exchange rates shows that this new approach can be opted as an alternative way to understanding exchange rates dynamics. The paper is organised as follows: In section2, we discuss wavelets and wavelet transforms and then give an overview on recent types of surrogate generation. An entropy-based method for determining the embedding parameters of the phase-space of a time series is presented. We then provide the 'Delay Vector Variance' methodology with an illustration. In section3, we present a comprehensive analysis of the feasibility of this approach to characterizing a variety of exchange rates: real effective exchange rate of euro, five bilateral real exchange rate series relative to US dollar and then eight bilateral real exchange rate series relative to the euro.

2. Background and 'Delay Vector Variance' Method

The Characterization of signal nonlinearities, which emerged in physics in the mid–1990s, have been successfully applied in predicting survival in heart failure cases and also adopted in practical engineering applications (Ho et al. (1997); Chambers and Mandic (2001)). We adopt to some terminologies given by Gautama (2004b), that we will refer to in the rest of the manuscript:

The "nature" of a signal refers to two sets of signal properties: linear/nonlinear and deterministic/stochastic. We will often refer to a time series as a signal. A linear signal is generated by a linear time-invariant system, driven by white Gaussian noise, measured by a static, monotonic, and possibly nonlinear observation function. For instance, a linear signal can be generated by an autoregressive (AR) model driven by normally distributed, white noise. Signals that cannot be generated in such a way are considered nonlinear. A signal is considered deterministic if it can be precisely described by a set of equations; otherwise it is considered stochastic. Surrogate time series, or 'surrogate' for short, is non-parametric randomised linear version of the original data which preserves the linear properties of the original data. There exist many methods for generating surrogates (for an overview, Schreiber and Schmitz (2000)).

2.1. Wavelet-based Surrogates

For identification of nonlinear/linear behavior in a given time series, the null hypothesis that the original data conform to a linear Gaussian stochastic process is formulated. An established method for generating constrained surrogates conforming to the properties of a linear Gaussian process is the Iterative Amplitude Adjusted Fourier Transform (iAAFT), which has become quite popular (Teolis (1998); Schreiber and Schmitz (1996, 2000); Kugiumtzis (1999)). This type of surrogate time series retains the signal distribution and amplitude spectrum of the original time series, and takes into account a possibly nonlinear and static observation function due to the measurement process. The method uses a fixed point iteration algorithm for achieving this, for the details of which we refer to Schreiber and Schmitz (1996, 2000).

Wavelet-based surrogate generation is a fairly new method of constructing surrogate for hypothesis testing of nonlinearity which applies a wavelet decomposition of the time series. The main difference between Fourier transform and wavelet transform[1] is that the former is only localized in frequency, whereas the latter is localized both in time and frequency. The idea of a wavelet representation is an orthogonal decomposition across a hierarchy of temporal and spatial scales by a set of wavelet and scaling functions.

The iAAFT-method has recently been refined using a wavelet-based approach, denoted by WiAAFT (Keylock (2006)), that provides for *constrained realizations*[2] of surrogate data that resembles the original data while preserving the local mean and variance as well as the power spectrum and distribution of the original except for randomizing the nonlinear properties of the signal. The WiAAFT-procedure follows the iAAFT-algorithm but uses the Maximal Overlap Discrete Wavelet Transform (MODWT) where the iAAFT-procedure is applied to each set of wavelet detail coefficients $D_j(n)$ over the dyadic scales 2^{j-1} for $j = 1, \cdots, J$, i.e., each set of $D_j(n)$ is considered as a time series of its own.

2.2. Optimal Embedding Parameters

In the context of signal processing, an established method for visualising an attractor of an underlying nonlinear dynamical signal is by means of time delay embedding (Hegger and Schreiber (1999)). By time-delay embedding, the original time series $\{x_k\}$ is represented in the

[1] There are essentially two distinct classes of Wavelet transforms: the continuous wavelet transform and the discrete wavelet transform. We refer the reader to Addison (2005); Walden and Percival (2000) for a review on Wavelet transforms.

[2] These are surrogate realizations that are generated from the original data to conform to certain properties of the original data, e.g., their linear properties, i.e., mean, standard deviation, distribution, power spectrum and autocorrelation function (Schreiber and Schmitz (1996, 2000)).

so-called 'phase space' by a set of delay vectors (DVs) of a given embedding dimension, m, and time lag, τ : $x(k) = [x_{k-\tau}, \cdots, x_{k-m\tau}]$. Gautama (2003) proposed a differential entropy based method for determining the optimal embedding parameters of a signal. The main advantage of this method is that a single measure is simultaneously used for optimising both the embedding dimension and time lag. We provide below an overview of the procedure:

The "Entropy Ratio" is defined as

$$R_{ent}(m,\tau) = I(m,\tau) + \frac{m \ln N}{N}, \tag{1}$$

where N is the number of delay vectors, which is kept constant for all values of m and τ under consideration,

$$I(m,\tau) = \frac{H(x,m,\tau)}{\langle H(x_{s,i},m,\tau)\rangle_i} \tag{2}$$

where x is the signal, $x_{s,i}$ $i = 1, \cdots, T_s$ surrogates of the signal x, $\langle\cdot\rangle_i$ denotes the average over i, $H(x,m,\tau)$ denotes the differential entropies estimated for time delay embedded versions of a time series, x, which an inverse measure of the *structure* in the phase space. Gautama (2003) proposed to use the Kozachenko-Leonenko (K-L) estimate (Leonenko and Kozachenko (1987)) of the differential entropy given by

$$H(x) = \sum_{j=1}^{T} \ln(T\rho_j) + \ln 2 + C_E \tag{3}$$

where T is the number of samples in the data set, ρ_j is the Euclidean distance of the j-th delay vector to its nearest neighbour, and $C_E(\approx 0.5772)$ is the Euler constant. This ratio criterion requires a time series to display a clear structure in the phase space. Thus, for time series with no clear structure, the method will not yield a clear minimum, and a different approach needs to be adopted, possibly one that does not rely on a phase space representation. When this method is applied directly to a time series exhibiting strong serial correlations, it yields embedding parameters which have a preference for $\tau_{opt} = 1$. In order to ensure robustness of this method to the dimensionality and serial correlations of a time series, Gautama (2003) suggested to use the iAAFT method for surrogate generation since it retains within the surrogate both signal distribution and approximately the autocorrelation structure of the original signal. In this Paper, we opt to use wavelet-based surrogate generation method, WiAAFT by in Keylock (2006), for reasons already discussed in the previous section.

2.3. 'Delay Vector Variance' method

The 'delay vector variance' (DVV) method (Gautama (2004a)) is a recently proposed phase space based method for signal characterisation. It is more suitable for signal processing application because it examines the nonlinear and deterministic signal behaviour at the same time. The algorithm is summarized below:

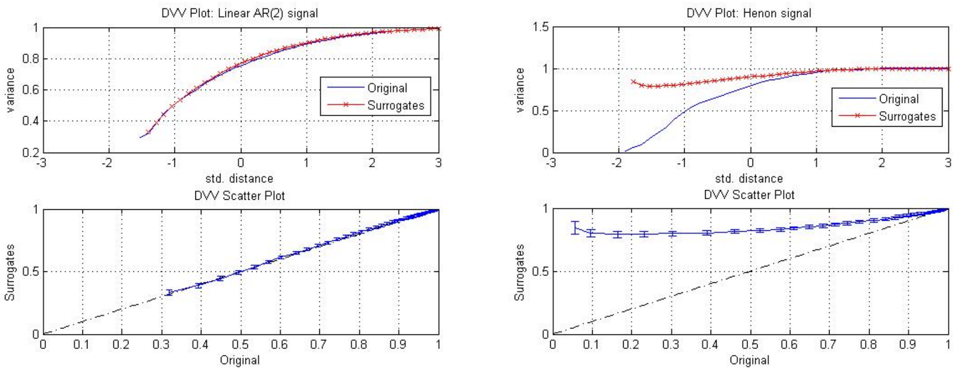
- For an optimal embedding dimension m and time lag τ, generate delay vector (DV): $x(k) = [x_{k-\tau}, \cdots, x_{k-m\tau}]$ and corresponding target x_k
- The mean μ_d and standard deviation, σ_d, are computed over all pairwise distances between DVs, $\|x(i) - x(j)\|$ for $i \neq j$.

- The sets Ω_k are generated such that $\Omega_k = \{x(i) || \|x(k) - x(i)\| \leq \varrho_d\}$, i.e., sets which consist of all DVs that lie closer to $x(k)$ than a certain distance ϱ_d, taken from the interval $[\min\{0, \mu_d - n_d\sigma_d\}; \mu_d + n_d\sigma_d]$, e.g., uniformly spaced, where n_d is a parameter controlling the span over which to perform the DVV analysis.

- For every set Ω_k, the variance of the corresponding targets, σ_k^2, is computed. The average over all sets Ω_k, normalised by the variance of the time series, σ_x^2, yields the *target variance*, σ^{*2} :

$$\sigma^{*2}(\varrho_d) = \frac{\frac{1}{N}\sum_{k=1}^{N} \sigma_k^2(\varrho_d)}{\sigma_x^2} \tag{4}$$

where N denotes the total number of sets $\Omega_k(\varrho_d)$



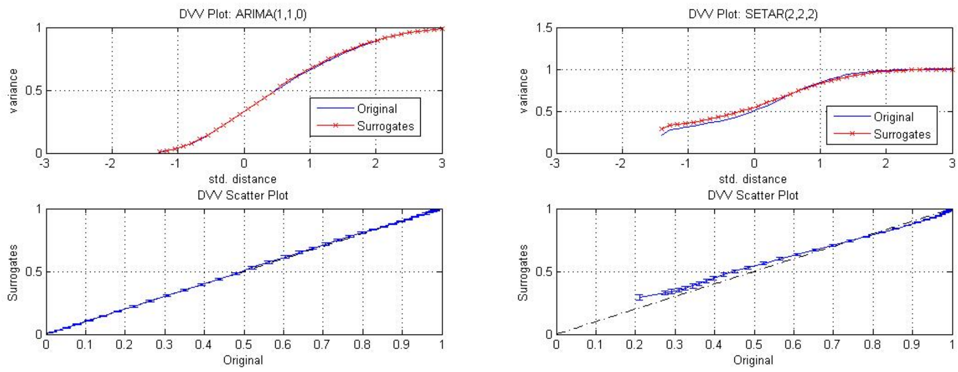
(a) AR(2) signal

(b) Henon signal

Figure 1: Nonlinear and deterministic nature of signals. The first row of Diagrams 1a and 1b are DVV plots for a linear benchmark signal: AR(2) signal and a nonlinear benchmark signal: Henon signal, where the red line with crosses denotes the DVV plot for the average of 25 WiAAFT-based surrogates while the blue line denotes that for the original signal. The second row of Diagrams 1a and 1b denote the DVV scatter diagrams for those two signals, where error bars denote the standard deviation of the target variances of surrogates.

As a result of the standardisation of the distance axis, the resulting *DVV plots* are easy to interpret, as illustrated in the first row of Figure 1a and Figure 1b. The minimum target variance, which corresponds to the lowest point of the curve, is a measure for the amount of noise which is present in the time series. The presence of a strong deterministic component will lead to small target variances for small spans, n_d. At the extreme right, the DVV plots smoothly converge to unity, as illustrated in Figure 1a and Figure 1b. The reason behind this is that for maximum spans, all DVs belong to the same set, and the variance of the targets is equal to the variance of the time series. In the following step, the linear or nonlinear nature of the time series is examined by performing DVV analysis on both the original and 25 WiAAFT surrogate time series. Due to the standardisation of the distance axis, these plots can be conveniently combined within a scatter diagram, where the horizontal axis corresponds to the DVV plot of the original time series, and the vertical to that of the surrogate time series. If the surrogate time series yield DVV plots similar to that of the original time series, as illustrated by the first row of Figure 1a, the DVV scatter diagram coincides with the bisector line, and the original time series is judged to be linear, as shown in second row of Figure 1a. If not, as illustrated by first row of Figure 1b, the DVV scatter diagram will deviate from the bisector line and the original time series is judged to be nonlinear, as depicted in the second row of Figure 1b.

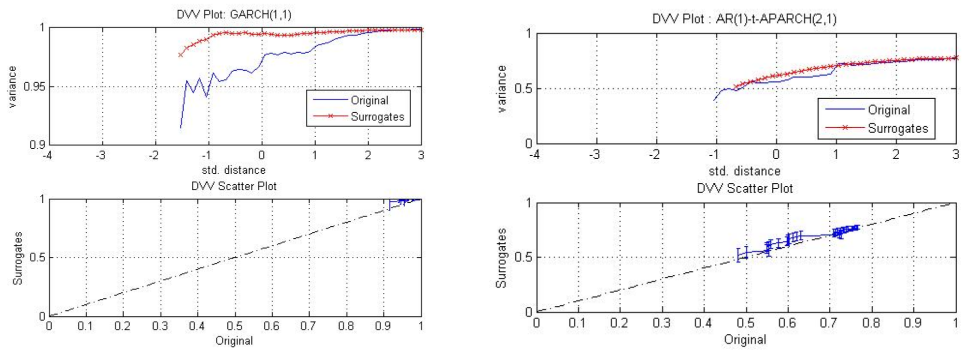
In Figure 2 and Figure 3, we provide the structure of the DVV analysis on some simulated processes such as: a self-exciting threshold autoregressive process (SETAR), linear autoregressive integrated moving average (ARIMA) signal, a Generalised autoregressive conditional heteroskadastic process (GARCH), and a signal with a mean equation as Autoregressive (AR) process and the innovations generated from a skewed Student-t APARCH (asymmetric power autoregressive conditional heteroskadastic) process. The interpretation of the DVV analysis is not different from the previous illustration.



(a) DVV analysis on ARIMA(1,1,0) signal

(b) DVV analysis on SETAR(2,2,2) signal

Figure 2: DVV analysis on ARIMA and SETAR signals



(a) DVV analysis on GARCH(1,1) signal

(b) DVV analysis on AR(1)-t-APARCH(2,1) signal

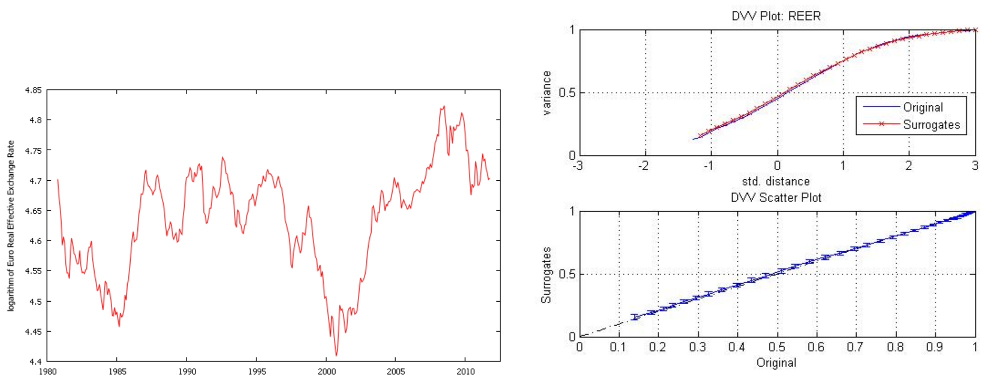
Figure 3: DVV analysis on GARCH and AR(1)-t-APARCH(2,1) signals

3. Data Analysis

In this section, we provide a comprehensive analysis of the feasibility of the DVV method in investigating the intrinsic dynamical properties of exchange rates. Optimal embedding parameters used in the DVV analysis are obtained via the differential-entropy method. Wavelet-based surrogate generation will be employed using the WiAAFT algorithm (Keylock (2006)). In this study, we present results on a variety of exchange rates : real effective exchange rate of euro, five bilateral real exchange rate series relative to US dollar and then eight bilateral real exchange rate series relative to euro. We remark that all exchange rate time series considered in this study are of monthly frequency. However, our approach is still applicable to understanding the dynamics of exchange rates at different frequencies.

3.1. Application to Real Effective Exchange Rate of Euro

The monthly real effective exchange rate CPI deflated[3] time series for the Euro. The data[4] spans from 1980:10 to 2011:10 implying 373 observations. The logged time series is depicted in Figure 4a. To choose the optimal embedding parameters for the DVV method, we opted for the differential-entropy based method, previously discussed in section 2.2, which yields $m = 2$, $\tau = 4$ and $R_{ent}(m, \tau) = 1.0350$. From Figure 4b, the DVV scatter diagram coincides with bisector line, indicating linear dynamics of this exchange rate series. The DVV plot also reveals that this exchange rate series is neither strictly deterministic nor strictly stochastic, but exhibits both characteristics. This findings suggest the use of linear models in explaining the dynamics of the euro real effective exchange rate series.



(a) Logarithm of real effective exchange rate of Euro

(b) DVV analysis

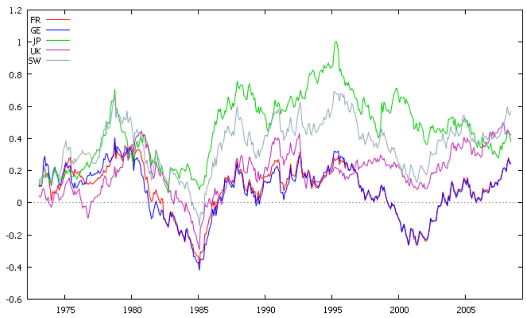
Figure 4: The figure displays the time series plot of the Logarithm of euro real effective exchange rate (October, 1980 to October, 2011) and the associated plots from the DVV analysis which indicates both deterministic and stochastic characteristics with linear dynamics.

[3]The effective exchange rates (EERs) of the euro are geometrically weighted averages of the bilateral exchange rates of the euro against the currencies of the euro area's main trading partners. For additional information, see the "Daily nominal effective exchange rate of the euro" section of the ECB's website

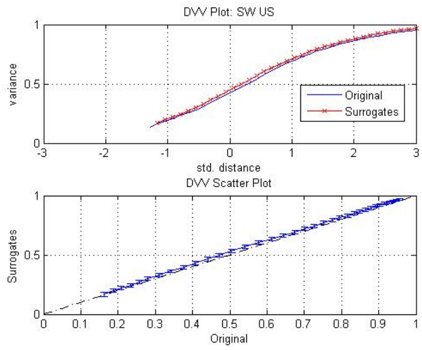
[4]Data source: European Central Bank and code: EXR.M.Z08.EUR.ERC0.A

3.2. Application to Five Bilateral real exchange rate series relative to US Dollar

For this application, we apply our proposed procedure on the same dataset used in the paper of Buncic (2009), in which the author discusses the forecasting failures of exponential smooth transition autoregressive (ESTAR) models to exchange rates. Our objective is to verify if indeed the ESTAR model, which is a nonlinear model, was appropriate for modelling the real exchange rates considered by Buncic, 2009. The data [5] consists of five real exchange rates relative to the US dollar corresponding to the UK, Japan, German, France and Switzerland, from January 1973 to June 2008. These real exchange rates are constructed in the standard way as $q_t \equiv log(\frac{CPI_t^{home}}{CPI_t^{US} S_t})$, where S_t is the home currency price of one US dollar. The time series plot of the normalised real exchange rates over the period is shown in Figure 5a.



(a) Time series plot of the normalised real exchange rates over the period from January 1973 to June 2008.



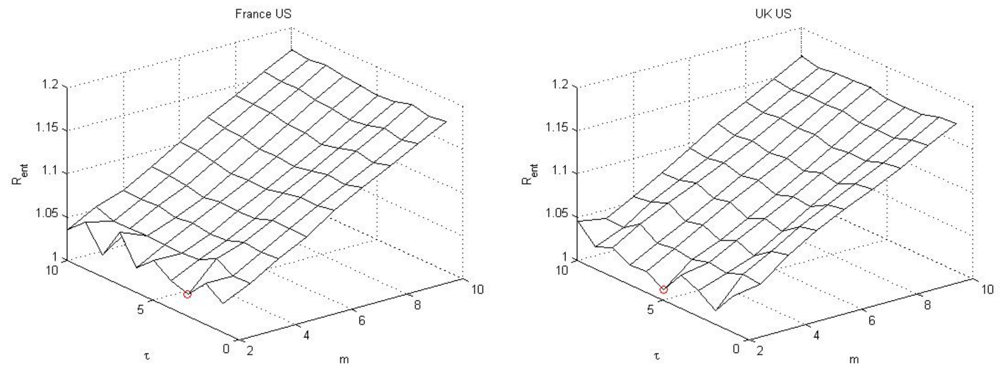
(b) Bilateral exchange rate of Switzerland- US

Figure 5: The DVV scatter diagram coincides with the bisector, indicating linear nature

To begin with the DVV method, we first opt to use the Differential-Entropy based method with wavelet-based surrogates to obtain the optimal embedding parameters. In general, we obtain embedding dimension parameter, $m = 2$ with different time delays, τ for all the exchange rates considered in this study. Figure 6 shows the structure of the Differential-Entropy method for two bilateral exchange rates: France-US and UK - US.

Looking at Figures 5b, 7, 8, the DVV analysis reveals that the five real exchange rates relative to the US dollar are driven by linear dynamics. The DVV analysis, looking at the DVV plots, reveals that the five exchange rate series are neither strictly deterministic nor strictly stochastic, but exhibits both characteristics. These findings provide no support for the use of nonlinear models such as ESTAR model to forecast such real exchange rates since they exhibit a linear nature. In our opinion, a possible nonlinear alternative such as the SETAR model could be considered in modelling the UK-US exchange rate series since the DVV analysis shown in Figure 8b yields a slight deviation from the bisector around the central region of the plots. Hence, our findings are consistent with the results of Buncic (2009) on no forecast gained by ESTAR model over linear autoregressive model.

[5]The data can be downloaded from http://www.mathstat.unisg.ch/buncic/data/rer_data.xls



(a) Bilateral exchange rate of France - US.
(b) Bilateral exchange rate of UK - US.

Figure 6: The Optimal embedding parameters of the time series obtain via the Differential-Entropy based method using wavelet-based surrogates. We observe a clear structure with the parameter values indicated as an open circle.

3.3. Application to Bilateral real exchange rate series relative to Euro

We now consider eight bilateral monthly exchange rates of Australian dollar, Canadian dollar, Swiss franc, UK pound sterling, Japanese yen, US dollar, Hong Kong dollar and SouthAfrican rand relative to the Euro. Our data is taken from European Central Bank [6] and spans from 1999:01 to 2011:11 implying 155 observations. We obtain the same embedding dimension of $m = 2$ and different time delay, τ for all the exchange rates under this section.

The results based on the DVV analysis indicates nonlinear dynamics for the bilateral exchange rates relative to euro: Australian dollar, Canadian dollar, Swiss franc, and SouthAfrican rand, as shown in Figures 9a, 9b, 10a and 12b respectively. Similar interpretation of Figures 10b, 11a, 11b and 12a, provides evidence of linear dynamics[7] for the other exchange rates considered in this section.

4. Conclusion

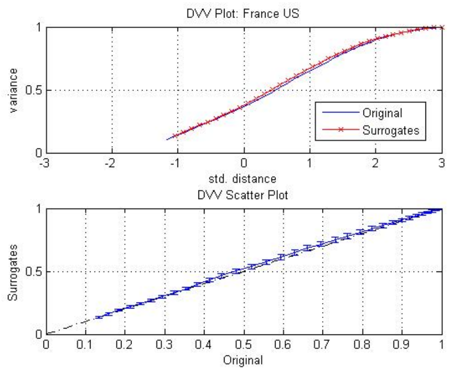
In this study, we have provided a new procedure to characterizing the dynamics of exchange rates. A comprehensive analysis of the feasibility of this approach is provided. The empirical results on a variety of exchange rates shows that the 'delay vector variance' (DVV) method can be opted as an alternative way to understanding exchange rates dynamics.

Appendix A. Results from Differential-Entropy method

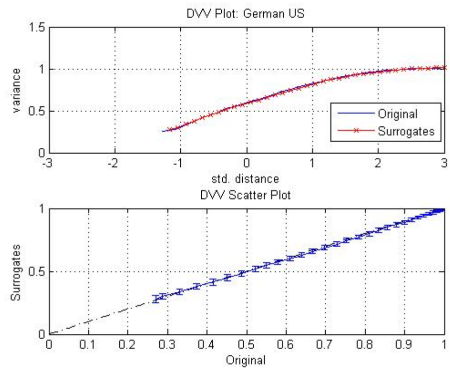
We provide a table A.1 of reported results on the optimal embedding parameters, (m, τ) and associated entropy-ratio, $R_{ent}(m, \tau)$ for the exchange rates considered in this study.

[6]Data source: ECB's website http://www.ecb.int/stats/services/downloads/html/index.en.html

[7]We obtained the same dynamics for the U.S.- Euro Foreign Exchange Rate when our approach is applied to daily rates. This unreported results is available from authors upon request.

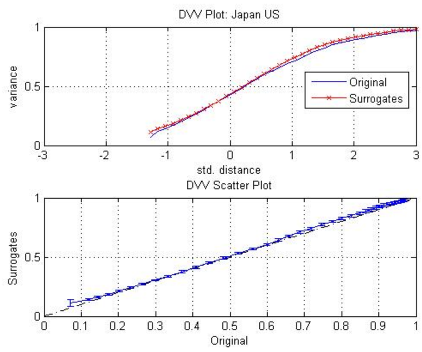


(a) Bilateral exchange rate of France - US

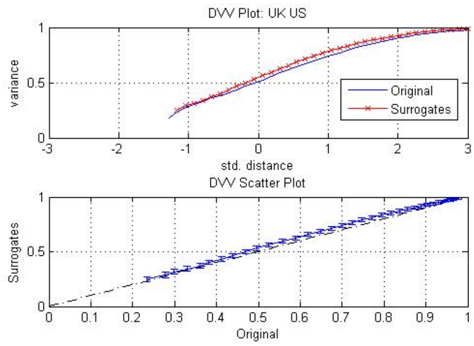


(b) Bilateral exchange rate of German - US

Figure 7: The DVV Analysis plot reveals that both bilateral exchange rates exhibit linear dynamics. It is clear that the DVV scatter diagrams coincides with the bisector.

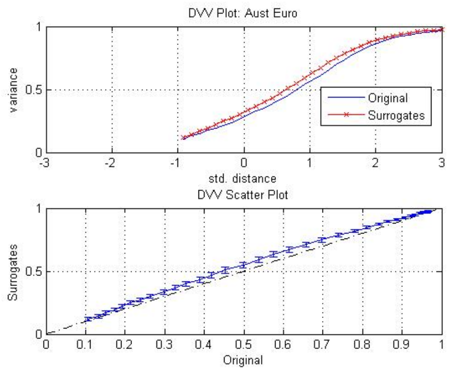


(a) Bilateral exchange rate of Japan- US

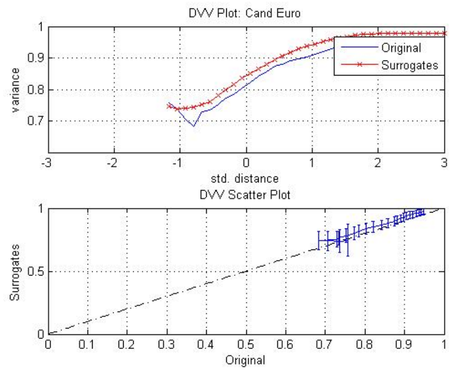


(b) Bilateral exchange rate of UK - US

Figure 8: The DVV scatter diagrams for both exchange rates series almost coincides with bisector, indicating linear dynamics.

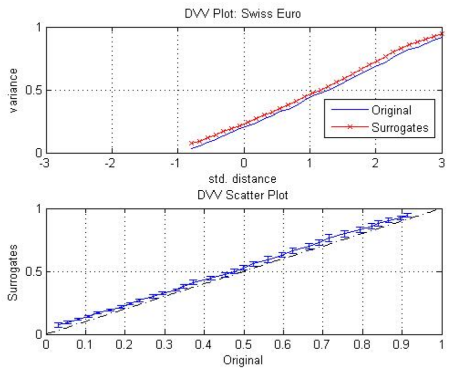


(a) Bilateral exchange rate of Australia- Euro

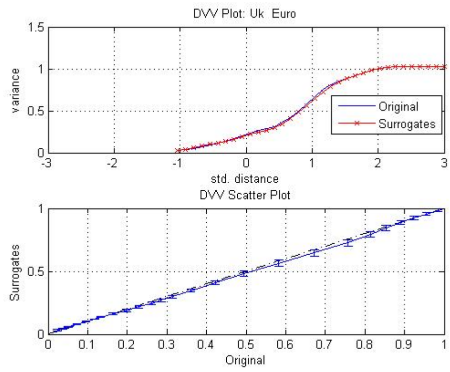


(b) Bilateral exchange rate of Canada- Euro

Figure 9: The DVV analysis reveals nonlinear dynamics of both exchange rate series

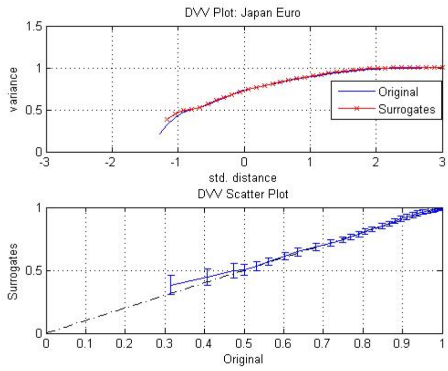


(a) Bilateral exchange rate of Swiss -Euro

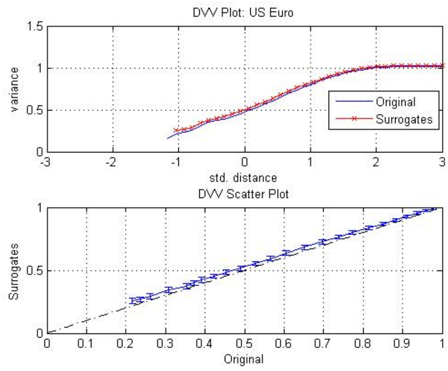


(b) Bilateral exchange rate of UK-Euro

Figure 10: In diagram 10a, the DVV analysis indicates a nonlinear dynamics for the Swiss - Euro exchange rate series. The linear nature of the UK-Euro exchange rate series is shown in diagram 10b. The DVV analysis also indicates a presence of strong deterministic components in the dynamics of both exchange rates.

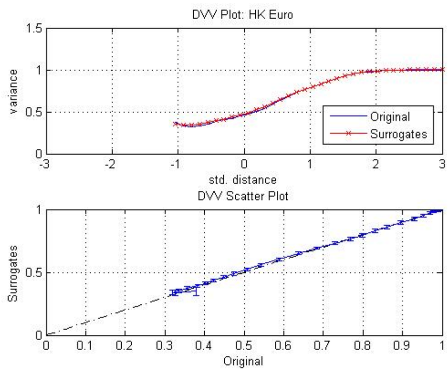


(a) Bilateral exchange rate of Japan-Euro

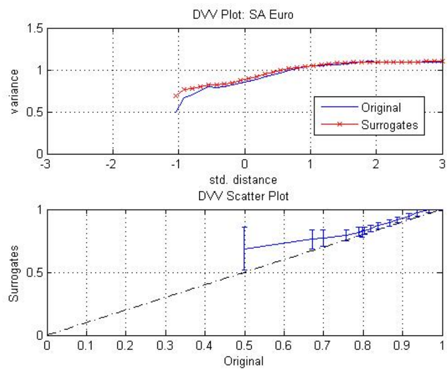


(b) Bilateral exchange rate of US - Euro

Figure 11: The DVV analysis indicates linear dynamics for the Japan-Euro exchange rates. The DVV scatter diagram of the US-Euro exchange rate series almost coincides with bisector, indicating linear dynamics as shown in diagram 11b. The DVV plots reveals that both exchange rate series are neither strictly deterministic nor strictly stochastic, but exhibits both characteristics.



(a) Bilateral exchange rate of Hong-Kong - Euro



(b) Bilateral exchange rate of South Africa - Euro

Figure 12: The DVV scatter plots indicates linear nature for the Hong-Kong - Euro exchange rate series and nonlinear dynamics for the South Africa - Euro exchange rate series.

Exchange Rate	m	τ	$R_{ent}(m,\tau)$
REER	2	4	1.0350
France-US	2	3	1.0247
German-US	2	8	1.0168
Japan-US	2	4	1.0162
UK-US	2	5	1.0119
SW-US	2	3	1.0115
Aust-Euro	2	1	1.1273
Cand-Euro	2	5	1.1385
Swiss-Euro	2	1	0.9862
Uk-Euro	2	2	1.1071
Jap-Euro	2	9	1.0804
US-Euro	2	8	1.0788
HK-Euro	2	7	1.1294
SA-Euro	2	10	1.0466

Table A.1: The Differential-Entropy based method for the Exchange Rates. The real effective exchange rate of euro is denoted REER on the first row. The second row denotes five bilateral real exchange rate series relative to US dollar corresponding to the UK, Japan, German, France and Switzerland respectively. The last row represents eight bilateral real exchange rate series relative to the euro : Australian dollar, Canadian dollar, Swiss franc, UK pound sterling, Japanese yen, US dollar, Hong Kong dollar and SouthAfrican rand.

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