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Information in Hierarchies

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Abstract

We determine the optimal policy to cope with information concealment in a hierarchy where a principal relies on a supervisor to obtain verifiable information about an agent's output. Depending on the information he has obtained, the informed supervisor may either collude with the agent or with the principal and conceal information. The principal has the choice of four policies to cope with information concealment: it can prevent both types of information concealment, allow both of them, or prevent one of them and allow the other one. We characterize the incentive contracts in this environment and show that it is not optimal to allow information concealment, that is, the optimal policy of a hierarchy exposed to multiple types of information concealment is to prevent them all.

JEL Classification: D20; D73; L20; M50.

Keywords: Hierarchy; Information concealment.

Résumé

Cet article détermine la politique optimale pour gérer la possibilité de la rétention d'informations dans une firme hiérarchique à l'intérieur de laquelle un principal emploie un superviseur pour être informé sur la performance de son agent. La firme hiérarchique est vulnérable à la rétention d'informations par une coalition principal/superviseur ainsi qu'une coalition superviseur/agent. Pour gérer la possibilité de la rétention d'informations, la firme a le choix entre une politique totalement préventive, une politique partiellement préventive/permissive et une politique totalement permissive. Je montre que la politique optimale d'une firme hiérarchique vulnérable à de multiples formes de rétention d'informations est la prévention totale ou, de manière équivalente, qu'il n'est pas optimal de tolérer la rétention d'informations à l'intérieur de la firme.

Mots-clés : Hiérarchie ; Rétention d'informations.

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1 Introduction

While fair and transparent information dissemination is crucial for the functioning of firms, information concealment in firms is a well-documented phenomenon.[1] An important question for social scientists is thus: what is the optimal policy to cope with information concealment in hierarchical firms? This paper aims at providing an answer to this question.

Opportunities for information concealment arise when a principal relies on a supervisor to obtain information about an agent's performance. Depending on the information he has obtained, the informed supervisor may either collude with the agent or with the principal, and hence conceal information. Following the seminal work of Tirole (1986), an extensive literature has been devoted to the analysis of collusion between the supervisor and the agent.[2] A central result of this literature is that it is optimal to deter supervisor/agent collusion, and therefore to prevent information concealment associated with this type of collusion. The principal may optimally restrict attention to contracts that do not leave scope for collusion between the supervisor and the agent. This result is known as the *collusion-proofness principle*.

While a supervisor's benevolence for her/his subordinate may result in supervisor/agent collusion, empirical studies show that a supervisor's loyalty to her/his principal may also result in principal/supervisor collusion, and hence in an unfair treatment of the supervisor's subordinate (e.g., Crozier, 1967; Crozier and Friedberg, 1980). To date, almost nothing is known about this latter type of collusion. This is because, in order to investigate supervisor/agent collusion, the focus of the prevailing literature has been specific models of hierarchies where principal/supervisor collusion is either artificially ruled out or harmless.

We depart from the existing literature by considering a more general environment where both

[1] For example, Crozier (1967), Crozier and Friedberg (1980), Johnson and Libecap (1989), Lev (2003), Lee, Lev and Yeo (2007), Zhou (2010), Wagner (2011), among others.

[2] For non-exhaustive surveys and discussions of this literature, see Tirole (1992, 1994). While economists have investigated other forms of unofficial activity in vertical agency/business relationships, such as influence activities (Milgrom, 1988), managerial entrenchment (Shleifer and Vishny, 1989), and abuse of authority (Vafaï, 2002), this paper focuses on collusion in hierarchies in the tradition of Tirole (1986, 1992).

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types of collusion may occur. The principal's concern is then to design optimal contracts in a hierarchical organization exposed to multiple types of collusion, or, equivalently, to multiple types of information concealment. We characterize the different possible contractual responses to collusion and show that the optimal policy is to deter both types of collusion. We therefore provide a general result for the management of information in hierarchies.

For our analysis, we consider a hierarchical principal-supervisor-agent organization with moral hazard where the role of the supervisor is to make a verifiable report on the agent's output. The supervision technology is imperfect, and thus the supervisor obtains information only with a certain probability. The informed supervisor then has the possibility to conceal information and claim that supervision has not been conclusive.[3] If supervision reveals that the agent has produced a high output, that is, if the information obtained by the supervisor is unfavorable to the principal - in the sense that the principal has then to pay a higher wage to the agent than in the case where supervision is inconclusive - the supervisor may collude with the principal and, in exchange for a bribe, make an uninformative report. If instead supervision reveals evidence that a low output has been produced, that is, if the information obtained by the supervisor is unfavorable to the agent, the supervisor may collude with the agent and, in exchange for a bribe, report that supervision has not been conclusive.

The principal has the choice of four policies to cope with collusion: it can deter both types of collusion, allow both of them, or deter one of them and allow the other one. Comparing the expected costs of these policies, we find that the optimal policy to cope with multiple types of collusion is to prevent them all. We thus establish a *multi-collusion-proofness principle*.

The plan of the paper is as follows. The model is presented in Section 2. Section 3 investigates the four possible policies to cope with collusion/information concealment. The optimal policy of a hierarchy exposed to multiple types of collusion is determined in Section 4. Section 5 concludes the paper. Proofs are relegated to an Appendix.

[3] Because it is hard evidence, the supervisor's information can only be concealed but not forged.

2 The model

A hierarchical agency under moral hazard is composed of three players: a principal (P, it), a supervisor (S, he) and an agent (A, she).

The agent, who is the productive unit, can choose between two effort levels, $e \in \{0, 1\}$; i.e., she may either shirk ($e = 0$) or work ($e = 1$). Neither the principal nor the supervisor can observe the agent's effort level. The production technology is such that if the agent decides to work, she produces a high output $x_H > 0$ with probability $\pi \in (0, 1]$ and a low output $x_L \equiv 0$ with probability $1 - \pi$. If instead the agent decides to shirk, she produces x_L.

Following the literature on collusion, we assume that hard information, or equivalently, hard evidence (i.e., verifiable), about the output produced by the agent can only be obtained through supervision (see Tirole, 1992, and references therein). The principal and the agent are unable - due, for example, to a lack of time or expertise - to perform the supervisory task. The principal thus relies on the supervisor to obtain a verifiable report on the agent's output. Evidence on output obtained by the supervisor is his private information, but, once publicly revealed it is verifiable.[4] Given that the supervision technology is imperfect, the supervisor obtains information about output only with probability $p \in (0, 1)$. The supervisor's report, r, therefore belongs to $I = \{x_L, \emptyset, x_H\}$, where $r = \emptyset$ indicates that no information has been obtained. Since the information obtained by the supervisor is hard/verifiable evidence, it can be *concealed* but *not forged*.[5]

The agent and the supervisor are both risk neutral and their utility functions are respectively, $U^A(w, e) = w - \gamma e$ and $U^S(s) = s$, where w and s are the transfers received from the principal and $\gamma > 0$ is the agent's disutility of effort. For simplicity and without loss of generality, the agent's and supervisor's reservation utilities are normalized to zero.

[4] That is, evidence is verifiable only by the person(s) to whom the supervisor reveals it. Evidence is publicly verifiable only when the supervisor produces his report.

[5] This standard assumption means that the supervisor cannot misreport the high output as low output or the low output as high output. Upon observing the agent's output, the supervisor can conceal his information and claim that the monitoring was inconclusive (i.e., $r = \emptyset$).

Since hard information about the output is only obtained through supervision, contracts are contingent on the supervisor's report. The principal offers a contract $(w_L, w_\emptyset, w_H)$ to the agent, where w_L and w_H are the wages she receives when $r = x_L$ and $r = x_H$, respectively, and $w_\emptyset$ is the wage she receives when $r = \emptyset$. Similarly, the principal offers a contract $(s_L, s_\emptyset, s_H)$ to the supervisor. The agent and the supervisor are protected by limited liability. We simply make the assumption that negative wages cannot be imposed on them.

The high output, x_H, is assumed to be large enough for it to be in the principal's interest to engage in production.

The principal's concern is thus to design contracts that both elicit the effort level $e = 1$ and minimize the expected cost of production and supervision $p\left[\pi(w_H + s_H) + (1-\pi)(w_L + s_L)\right] + (1-p)(w_\emptyset + s_\emptyset)$.

Given that the supervision technology is imperfect and information is hard, the supervisor has discretion to conceal information from the principal. The supervisor's discretion allows him to engage in two types of collusion. We assume that, when engaging in collusion, the supervisor unofficially shows the private evidence obtained to the other involved party. That is, in line with Tirole's (1986, 1992) standard models and most of the existing literature, collusion occurs under symmetric information on evidence among involved parties.

When supervision reveals evidence that the agent has produced a low output, the supervisor may collude with the agent and, in exchange for a bribe, report $r = \emptyset$ instead of $r = x_L$ to the principal. The agent then receives $w_\emptyset$ from the principal and pays the promised bribe to the supervisor. When supervision reveals evidence that a high output has been produced, the principal may collude with the supervisor. In this case, the supervisor reports $r = \emptyset$ instead of $r = x_H$ and the principal pays him the promised bribe.

In accordance with the existing literature, we make the following standard assumptions regarding collusion (see, for example, Tirole, 1992). First, collusion is only observable to the involved

parties. Second, the technology used to transfer bribes, which we refer to as the side transfer technology, may be less or equally efficient to the official transfer technology (i.e., the transfer technology used by the principal to pay its employees). This means that unofficial income can be transferred to the supervisor at a rate $k \in (0, 1]$. If $k \in (0, 1)$, there are transaction costs connected to side contracting. That is, a side transfer of size t is only worth kt to the supervisor. This may be, for example, because collusion is costly to organize. If instead $k = 1$, the side transfer technology is totally efficient. Third, the supervisor has all the bargaining power when engaging in collusion. Fourth, side transfers are enforceable. Fifth, the supervisor does not engage in collusion when indifferent, that is, when payoffs associated with colluding and not colluding are identical.

The timing is as follows: (1) The principal offers a contract $(w_L, w_\emptyset, w_H)$ to the agent and a contract $(s_L, s_\emptyset, s_H)$ to the supervisor. (2) The agent and the supervisor decide whether to accept or refuse the contract. If either refuses, the game ends and they both get their reservation utility. If instead contracts are accepted, the game continues as follows. (3) Supervision takes place and the agent decides whether to work or to shirk. (4) Hard information about the output produced by the agent is obtained with probability p and decisions of whether or not to engage in collusion are made. (5) The supervisor produces a report. (6) Transfers and side transfers take place.

We look for a *Subgame Perfect Equilibrium* of this game.

3 Policies to cope with information concealment

To cope with information concealment, the principal has the choice of the following four policies: (1) Deterring both types of collusion (policy 1). (2) Allowing supervisor/agent collusion and deterring principal/supervisor collusion (policy 2). (3) Deterring supervisor/agent collusion and allowing principal/supervisor collusion (policy 3). (4) Allowing both types of collusion (policy 4).

In order to determine the optimal policy of a hierarchy regarding collusion, we first need to characterize the optimal contracts and expected cost of each of the above four policies. The optimal

policy is then that which has the lowest expected cost.

3.1 Both types of collusion deterred

When both types of collusion are prevented, the agent's incentive compatibility constraint is $p[\pi w_H + (1-\pi)w_L] + (1-p)w_\emptyset - \gamma \geq pw_L + (1-p)w_\emptyset$, or equivalently,

$$w_H - w_L \geq \frac{\gamma}{p\pi}. \tag{1}$$

This equation makes the agent prefer to exert effort in equilibrium.[6]

Given that the agent and the supervisor are protected by limited liability,

$$w_L \geq 0, w_\emptyset \geq 0, w_H \geq 0, s_L \geq 0, s_\emptyset \geq 0, s_H \geq 0. \tag{2}$$

The agent's contract must also satisfy her participation constraint, $p[\pi w_H + (1-\pi)w_L] + (1-p)w_\emptyset - \gamma \geq 0$. However, transfers must be nonnegative, and hence the agent's participation constraint is redundant and will be disregarded in the rest of the paper.

Collusion between the supervisor and the agent may take place when supervision reveals evidence that a low output has been produced. This is the case either when the agent works hard but is unlucky or when she shirks. In either case, the supervisor may collude with the agent and, in exchange for a bribe, report $r = \emptyset$ instead of $r = x_L$. If collusion occurs, the agent's utility is $w_\emptyset - b_{S/A}$ if she has shirked, and $w_\emptyset - b_{S/A} - \gamma$ if she has worked but has been unlucky, where $b_{S/A}$ is the bribe offered to the supervisor. If instead collusion does not take place, the agent's utility is w_L if she has shirked, and $w_L - \gamma$ if she has worked but has been unlucky. The agent is then ready to collude with the supervisor if $w_\emptyset - b_{S/A} \geq w_L$, that is, if $b_{S/A} \leq w_\emptyset - w_L$. The maximum bribe, $b^M_{S/A}$, the agent is willing to offer for the report, $r = \emptyset$, is therefore $b^M_{S/A} \equiv w_\emptyset - w_L$. We have made the standard assumption that the supervisor does not engage in collusion when indifferent. Collusion between the supervisor and the agent will thus not take place if the supervisor's utility

[6] We make the standard assumption that the agent chooses to work when she is indifferent.

from making a truthful report, s_L, exceeds his utility from concealing information, $s_\emptyset + kb^M_{S/A}$, that is, if

$$s_L \geq s_\emptyset + k(w_\emptyset - w_L). \tag{3}$$

When supervision reveals evidence that a high output has been produced, the supervisor may collude with the principal and, in exchange for a bribe, report $r = \emptyset$ instead of $r = x_H$. If the principal does not collude with the supervisor it has to pay $w_H + s_H$ to its employees whereas if it colludes with him it only has to pay $w_\emptyset + s_\emptyset + b_{P/S}$, where $b_{P/S}$ denotes the bribe paid for information concealment. The principal will thus collude with the supervisor if $w_\emptyset + s_\emptyset + b_{P/S} \leq w_H + s_H$. The maximum bribe, $b^M_{P/S}$, the principal is then ready to pay for the report $r = \emptyset$ is $b^M_{P/S} \equiv w_H + s_H - w_\emptyset - s_\emptyset$. Principal/supervisor collusion will therefore not occur if the supervisor's utility from making a truthful report, s_H, exceeds his utility from concealing information, $s_\emptyset + kb^M_{P/S}$. That is, if

$$s_H \geq s_\emptyset + \frac{k}{1-k}(w_H - w_\emptyset) \text{ for } k \in (0,1) \text{ or } w_\emptyset \geq w_H \text{ for } k = 1. \tag{4}$$

We refer to constraints (3) and (4) respectively as the supervisor/agent and the principal/supervisor no-collusion constraints.

Thus, when both types of collusion are deterred, the program of the hierarchy can be written as

$$[P_1] \qquad \min_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} \quad p\left[\pi(w_H + s_H) + (1-\pi)(w_L + s_L)\right] + (1-p)(w_\emptyset + s_\emptyset)$$

$$\text{s.t. (1), (2), (3), and (4).}$$

Define $\overline{\pi} \equiv \frac{(1-k)[1-p(1-k)]}{pk(2-k)}$. The following proposition then summarizes the solution to this program.

Proposition 1. A hierarchy where both types of collusion are deterred has the following properties:

(1) The optimal contracts are $(w_L, w_\emptyset, w_H) = (0, 0, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, \frac{k\gamma}{(1-k)p\pi})$ for $k < 1$, (a) $p \leq 1-k$; (b) $p > 1-k$ and $\pi \leq \overline{\pi}$. The expected cost of production and supervision is $C^1 = \frac{\gamma}{1-k}$.

(2) The optimal contracts are $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (\frac{k\gamma}{p\pi}, 0, 0)$ for $k < 1$, $p > 1-k$ and $\pi > \overline{\pi}$. The expected cost of production and supervision is $C^2 = \frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi}$.

(3) The optimal contracts are $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (\frac{\gamma}{p\pi}, 0, 0)$ for $k = 1$. The expected cost of production and supervision is $C^3 = \frac{\gamma}{p\pi}$.

The principal optimally sets $w_L = 0$, and hence $w_H = \frac{\gamma}{p\pi}$ from the agent's incentive compatibility constraint. To set $w_\emptyset$, the principal faces a trade-off between reducing the cost of preventing one type of collusion and increasing the cost of preventing the other type of collusion. The principal has the choice of two strategies to deter principal/supervisor collusion.

The first strategy consists in setting $w_\emptyset - w_H < 0$ - that is, here, in setting $w_\emptyset < \frac{\gamma}{p\pi}$ and more precisely $w_\emptyset = 0$ - and then creating incentive payments for the supervisor, s_H, to reveal his information. We refer to this strategy as *incentive strategy*. An incentive strategy reduces the cost, s_L, of preventing supervisor/agent collusion but increases the cost, s_H, of preventing principal/supervisor collusion. In fact, the incentive strategy - which, as just stressed, consists here in setting $w_\emptyset = 0$ - deters collusion between the supervisor and the agent at no additional cost, s_L, to the principal. Note that throughout the paper we will term incentive strategy a strategy that deters principal/supervisor collusion by offering $w_\emptyset < w_H$. Hence, two strategies against principal/supervisor collusion which both set $w_\emptyset < w_H$ but involve the use of distinct contracts may both be termed incentive strategy.

The second strategy, termed *stake-eliminating strategy*, consists in setting $w_\emptyset = w_H = \frac{\gamma}{p\pi}$.[7]

[7] The terms incentive strategy and stake-eliminating strategy are clearly used with respect to collusion between the principal and the supervisor.

This strategy destroys the stake, $w_H - w_\emptyset$, of principal/supervisor collusion. Unlike the incentive strategy, the stake-eliminating strategy reduces the cost, s_H, of deterring principal/supervisor collusion but increases the cost, s_L, of deterring supervisor/agent collusion. More precisely, as obvious, the stake-eliminating strategy deters principal/supervisor collusion at no extra cost, s_H.

The optimal strategy against collusion will hence depend on the probability of occurrence of each type of collusion. When it is likely that supervisor/agent collusion takes place, the optimal strategy is the incentive strategy. When instead it is likely that principal/supervisor collusion occurs, the optimal strategy is the stake-eliminating strategy. Notice that the optimal strategy takes into account the extra cost, $w_\emptyset > 0$, of the stake-eliminating strategy. Since for this strategy $\frac{\partial w_\emptyset}{\partial \pi} < 0$, setting $w_\emptyset > 0$ to reduce s_H is profitable when principal/supervisor collusion is likely to take place (i.e., π relatively large).

When the side transfer technology is not totally efficient (i.e., $k < 1$), three cases must then be considered.

If $p \leq 1-k$, the probability that supervision will not reveal anything, and thus supervisor/agent and principal/supervisor collusion will not take place is relatively high. That is, the probability that $w_\emptyset$ will be paid to the agent is relatively high. The optimal strategy is then the incentive strategy.

If instead the supervision technology is relatively efficient (i.e., $p > 1 - k$), and it is therefore likely that supervisor/agent and principal/supervisor collusion will take place, the optimal strategy will depend on the quality of the production technology. When $\pi \leq \overline{\pi}$, that is, when the production technology is relatively inefficient, the incentive strategy is the optimal strategy. This is because it is then likely that the agent produces a low output, and thus the principal's main concern becomes the possibility of collusion between the supervisor and the agent. When instead the production technology is relatively efficient, it is likely that the agent produces a high output, and therefore the principal is mainly concerned with principal/supervisor collusion. The principal then opts for

the stake-eliminating strategy.

When the side transfer technology is totally efficient (i.e., $k = 1$), deterring principal/supervisor collusion requires $w_\emptyset \geq w_H$. Since the objective function is increasing in $w_\emptyset$, it is optimal to set $w_\emptyset = w_H = \frac{\gamma}{p\pi}$, or equivalently, to adopt the stake-eliminating strategy.

3.2 S/A collusion allowed and P/S collusion deterred

If the principal decides to allow collusion between the supervisor and the agent in the case where the supervision reveals evidence that a low output has been produced - that is, if it decides to offer contracts which do not satisfy the supervisor/agent no-collusion constraint $s_L \geq s_\emptyset + k(w_\emptyset - w_L)$ - supervisor/agent collusion takes place. Indeed, as explained above, given that for the bribe $b^M_{S/A} \equiv w_\emptyset - w_L$ both the supervisor and the agent are then willing to collude, supervisor/agent collusion occurs. Consequently, when supervisor/agent collusion is allowed, the supervisor reports $r = \emptyset$ and the principal must then offer $w_\emptyset$ instead of w_L to the agent. The agent then pays a bribe $b^M_{S/A} \equiv w_\emptyset - w_L$ to the supervisor.

The agent's incentive compatibility constraint is however identical to that in the case where supervisor/agent collusion is deterred. Indeed, the agent's incentive compatibility constraint is now $p\left[\pi w_H + (1-\pi)(w_\emptyset - b^M_{S/A})\right] + (1-p)w_\emptyset - \gamma \geq p(w_\emptyset - b^M_{S/A}) + (1-p)w_\emptyset$, that is, after substituting for $b^M_{S/A}$, constraint (1).

As for the principal's objective function, it writes $p\left[\pi(w_H + s_H) + (1-\pi)(w_\emptyset + s_\emptyset)\right] + (1-p)(w_\emptyset + s_\emptyset)$, or equivalently, $p\pi(w_H + s_H) + (1 - p\pi)(w_\emptyset + s_\emptyset)$.

We have made the standard assumption that the supervisor does not engage in collusion when indifferent. Thus, if the principal decides to allow supervisor/agent collusion, it must set $b^M_{S/A} \equiv w_\emptyset - w_L$ such that the supervisor receives a strictly positive - and, clearly, as small as possible - bribe. That is, $w_\emptyset - w_L \geq \varepsilon$, where $\varepsilon > 0$ and $\varepsilon \to 0$. The optimal contracts must also satisfy $s_L < s_\emptyset + k(w_\emptyset - w_L)$ with $k \leq 1$. To obtain a not strict inequality constraint, we rewrite this

constraint as $s_L \leq s_\emptyset + k(w_\emptyset - w_L - \varepsilon)$. We then have

$$w_\emptyset - w_L \geq \varepsilon. \tag{5}$$

$$s_L \leq s_\emptyset + k(w_\emptyset - w_L - \varepsilon). \tag{6}$$

Therefore, if the principal allows supervisor/agent collusion, the program of the hierarchical organization is

$$[P_2] \qquad \min_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} \quad p\pi(w_H + s_H) + (1 - p\pi)(w_\emptyset + s_\emptyset)$$

$$\text{s.t. (1), (2), (4), (5), and (6).}$$

Define $\widehat{\pi} \equiv \frac{1-k}{p}$. Proposition 2 summarizes the solution to this program.

Proposition 2. A hierarchy where supervisor/agent collusion is allowed and principal/supervisor collusion is deterred has the following properties:

(1) The optimal contracts are $(w_L, w_\emptyset, w_H) = (0, \varepsilon, \frac{\gamma}{p\pi})$, $s_L = s_\emptyset = 0$ and $s_H = \frac{k}{1-k}(\frac{\gamma}{p\pi} - \varepsilon)$ for $k < 1$, (a) $p \leq 1 - k$; (b) $p > 1 - k$ and $\pi \leq \widehat{\pi}$. The expected cost of production and supervision is $C^1(\varepsilon) = \frac{\gamma}{1-k} + \frac{(1-k-p\pi)\varepsilon}{1-k} = C^1 + \frac{(1-k-p\pi)\varepsilon}{1-k}$.

(2) The optimal contracts are $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$, $s_L \in \left[0, k(\frac{\gamma}{p\pi} - \varepsilon)\right]$ and $s_\emptyset = s_H = 0$ for $k = 1$ and $k < 1$, $p > 1 - k$ and $\pi > \widehat{\pi}$. The expected cost of production and supervision is C^3.

Before providing intuition for this proposition, note that, since collusion between the supervisor and the agent is allowed, the incentive strategy adopted here is slightly different from that adopted in the case where both types of collusion are deterred. Also observe that for the parameter values of case (1) of Proposition 2, one has $\frac{1-k-p\pi}{1-k} \geq 0$.

Given that $\widehat{\pi} > \overline{\pi}$ when $p > 1 - k$, the stake-eliminating strategy is less often adopted here than in the case where both types of collusion are deterred. This is because, unlike in the case where

supervisor/agent collusion is prevented, the principal must now also pay a wage $w_\emptyset$ to the agent when supervision reveals evidence that the agent has produced a low output. In other words, the stake-eliminating strategy has now an extra cost. Thus, compared with the case where both types of collusion are prevented, when supervisor/agent collusion is allowed, an incentive strategy has an additional advantage over the stake-eliminating strategy.

Finally, note that since when adopting the present policy the principal must pay $w_\emptyset$ and $s_\emptyset$ respectively to the agent and the supervisor both when supervision has been inconclusive and when a low output has been produced, the expected cost of the stake-eliminating strategy is different here from that in the first policy where both types of collusion are deterred.

3.3 S/A collusion deterred and P/S collusion allowed

By the same argument as in the previous subsection, if the principal decides to allow collusion between itself and the supervisor, principal/supervisor collusion occurs. Hence, when the supervision reveals that the agent has produced a high output and principal/supervisor collusion is allowed, the supervisor receives a bribe $b^M_{P/S} \equiv w_H + s_H - w_\emptyset - s_\emptyset$ from the principal to report $r = \emptyset$ instead of $r = x_H$.

The principal's objective function becomes $p\left[\pi(w_\emptyset + s_\emptyset + b^M_{P/S}) + (1-\pi)(w_L + s_L)\right] + (1-p)(w_\emptyset + s_\emptyset)$, that is, after substituting for $b^M_{P/S}$, the same objective function as that of program $[P_1]$.

The agent's incentive compatibility constraint is now $p\left[\pi w_\emptyset + (1-\pi)w_L\right] + (1-p)w_\emptyset - \gamma \geq pw_L + (1-p)w_\emptyset$, or equivalently,

$$w_\emptyset - w_L \geq \frac{\gamma}{p\pi}. \tag{7}$$

We have assumed that the supervisor does not engage in collusion when indifferent. Hence, if the principal decides to allow principal/supervisor collusion, it must set $b^M_{P/S} \equiv w_H + s_H - w_\emptyset - s_\emptyset$ such that the supervisor receives a strictly positive - and, obviously, as small as possible - bribe. That

is, $w_H + s_H - w_\emptyset - s_\emptyset \geq \varepsilon$. The optimal contracts must also satisfy $s_H < s_\emptyset + k(w_H + s_H - w_\emptyset - s_\emptyset)$, that is, $s_H < s_\emptyset + \frac{k}{1-k}(w_H - w_\emptyset)$. To obtain a not strict inequality constraint, we rewrite this constraint as $s_H \leq s_\emptyset + \frac{k}{1-k}(w_H - w_\emptyset - \varepsilon)$. We therefore have

$$w_H + s_H - w_\emptyset - s_\emptyset \geq \varepsilon. \tag{8}$$

$$s_H \leq s_\emptyset + \frac{k}{1-k}(w_H - w_\emptyset - \varepsilon) \text{ if } k \in (0,1) \text{ or } w_H \geq w_\emptyset + \varepsilon \text{ if } k = 1. \tag{9}$$

When the principal allows collusion between itself and the supervisor, the program of the hierarchy is

$$[P_3] \qquad \min_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} \quad p\left[\pi(w_H + s_H) + (1-\pi)(w_L + s_L)\right] + (1-p)(w_\emptyset + s_\emptyset)$$

$$\text{s.t. (2), (3), (7), (8), and (9).}$$

The following proposition then summarizes the solution to this program.

Proposition 3. A hierarchy where supervisor/agent collusion is deterred and principal/supervisor collusion is allowed has the following properties: the optimal contracts are such that $w_L = s_\emptyset = 0$, $w_\emptyset = \frac{\gamma}{p\pi}$, $w_H = \frac{\gamma}{p\pi} + \varepsilon$, $s_L = \frac{k\gamma}{p\pi}$ and $s_H = 0$ for $k \leq 1$. The expected cost of production and supervision is $C^2(\varepsilon) = \frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi} + p\pi\varepsilon = C^2 + p\pi\varepsilon$.

Compared with the full preventive policy to cope with collusion, that is, policy 1, deterring supervisor/agent collusion is now always costly. This is because, when allowing collusion between the supervisor and itself, the principal systematically sets $w_\emptyset = \frac{\gamma}{p\pi}$, and hence $s_L = \frac{k\gamma}{p\pi}$.

3.4 Both types of collusion allowed

If the principal decides to allow both types of collusion, its objective function becomes

$p\left[\pi(w_\emptyset + s_\emptyset + b^M_{P/S}) + (1-\pi)(w_\emptyset + s_\emptyset)\right] + (1-p)(w_\emptyset + s_\emptyset)$, that is, the objective function of program $[P_2]$. The principal must minimize this function under constraints (2), (6), (7), (8), and (9). Given that $\varepsilon < \frac{\gamma}{p\pi}$, constraint (5) is strictly less restrictive than constraint (7), and hence is redundant.

When both types of collusion are allowed the program of the hierarchy is thus

$$[P_4] \qquad \min_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} \quad p\pi(w_H + s_H) + (1-p\pi)(w_\emptyset + s_\emptyset)$$

$$\text{s.t. (2), (6), (7), (8), and (9).}$$

The solution to this program is summarized in the following proposition.

Proposition 4. A hierarchy where both types of collusion are allowed has the following properties: the optimal contracts are such that $w_L = s_\emptyset = 0$, $w_\emptyset = \frac{\gamma}{p\pi}$, $w_H = \frac{\gamma}{p\pi} + \varepsilon$, $s_L \in \left[0, k(\frac{\gamma}{p\pi} - \varepsilon)\right]$ and $s_H = 0$ for $k \leq 1$. The expected cost of production and supervision is $C^3(\varepsilon) = \frac{\gamma}{p\pi} + p\pi\varepsilon = C^3 + p\pi\varepsilon$.

We now turn to the determination of the optimal policy to deal with collusion.

4 Optimal information management in firms

To determine the optimal policy to cope with collusion/information concealment in firms, we compare the expected costs of the above four policies. These cost comparisons yield:

Theorem. It is not optimal to allow information concealment in hierarchical firms, or, equivalently, the optimal policy to cope with multiple types of collusion in hierarchical firms is to prevent them all.

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This theorem establishes a multi-collusion-proofness principle and can be explained in the following way. We first consider the case where $k \in (0,1)$. In this case, policy 3 (preventing supervisor/agent collusion and allowing principal/supervisor collusion) is strictly less expensive than policy 4 (allowing both types of collusion). Indeed, the only difference between these two policies is that when the supervision reveals that a low output has been produced, the principal pays $s_L = \frac{k\gamma}{p\pi}$ to the supervisor if it decides to prevent supervisor/agent collusion, whereas if allowing this type of collusion, the principal pays $w_\emptyset = \frac{\gamma}{p\pi}$ to the agent. The extra costs of supervisor/agent collusion deterrence and allowance are then respectively $p(1-\pi)\frac{k\gamma}{p\pi}$ and $p(1-\pi)\frac{\gamma}{p\pi}$. Since there are transaction costs connected to side contracting, that is, $k \in (0,1)$, preventing supervisor/agent collusion is optimal.

Three cases must be distinguished when comparing policy 3 with policy 1 (preventing both types of collusion). If $p > 1-k$ and $\pi > \overline{\pi}$, policy 1 is strictly less expensive since w_H is larger when adopting policy 3. If $p \leq 1-k$, the probability that supervision will not be conclusive, and therefore $w_\emptyset$ will be paid to the agent is relatively high. Allowing principal/supervisor collusion, and thus setting $w_\emptyset = \frac{\gamma}{p\pi}$ is then more expensive than deterring both types of collusion through an incentive strategy with $w_\emptyset = 0$. If $p > 1-k$ and $\pi \leq \overline{\pi}$, it is likely that a low output will be produced, and hence s_L will be paid to the supervisor. It is then strictly less expensive to prevent principal/supervisor collusion, and thus set $s_L = 0$, than to allow this type of collusion and set $s_L = \frac{k\gamma}{p\pi}$. To sum up policy 1 is strictly preferable to policy 3.

Finally, as above, three cases should be considered when comparing policy 1 with policy 2 (allowing supervisor/agent collusion and preventing principal/supervisor collusion). If $p \leq 1-k$, or $p > 1-k$ and $\pi \in (0, \overline{\pi}]$, both policies use an incentive strategy to deter principal/supervisor collusion. However, policy 1 is strictly less expensive than policy 2 given that, as explained, policy 2 requires $w_\emptyset = \varepsilon$ and $s_H = \frac{k}{1-k}(\frac{\gamma}{p\pi} - \varepsilon)$ instead of $w_\emptyset = 0$ and $s_H = \frac{k\gamma}{(1-k)p\pi}$. If $p > 1-k$ and $\pi \in (\overline{\pi}, \widehat{\pi}]$, policy 1 uses a stake-eliminating strategy (i.e., $s_H = 0$) to prevent principal/supervisor

collusion while policy 2 uses an incentive strategy (i.e., $s_H = \frac{k}{1-k}(\frac{\gamma}{p\pi} - \varepsilon)$). In this case, a stake-eliminating strategy is strictly preferable since it is likely that supervision reveals that a high output has been produced, and hence s_H will be paid to the supervisor. If $p > 1 - k$ and $\pi > \widehat{\pi}$, both policies use a stake-eliminating strategy to deter principal/supervisor collusion. Policy 1 is however strictly less expensive. The reason is that provided above for the comparison of policies 3 and 4.

In the case where $k = 1$, policies 1 and 2 have the same cost and are less expensive than policies 3 and 4. Policy 1 is therefore weakly optimal.

To summarize the findings, we have that preventing collusion is strictly preferable in the most likely case where $k < 1$ and weakly preferable in the case where $k = 1$.

5 Conclusion

This paper has investigated the optimal response of a hierarchical agency relationship to the threat of multiple types of information concealment. Departing from the existing literature, we have presented a principal-supervisor-agent hierarchy vulnerable to supervisor/agent and principal/supervisor collusion, and proved that both types of collusion/information concealment should be prevented.

Appendix

Proof of Proposition 1. Before turning to the derivation of the optimal incentive contracts of the four policies, it is straightforward to see that, for each policy, the principal optimally sets $s_\emptyset$ as low as allowed by the limited liability constraint, that is, $s_\emptyset = 0$. This is because an uninformative report is undesirable from the principal's point of view.

By reducing w_H the principal may both soften constraints and lower the expected cost of the hierarchy. We therefore have $w_H = \frac{\gamma}{p\pi} + w_L$ from the agent's binding incentive compatibility constraint. Two cases must then be distinguished with respect to k.

1. $k < 1$. The equation $w_H = \frac{\gamma}{p\pi} + w_L$ should be substituted into the principal/supervisor no-collusion constraint. This constraint then writes $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$. The principal may relax this constraint by increasing $w_\emptyset$. However, since setting $w_\emptyset > \frac{\gamma}{p\pi} + w_L$ instead of $w_\emptyset \leq \frac{\gamma}{p\pi} + w_L$ both increases the expected cost of the hierarchy (because this cost is increasing in $w_\emptyset$) and makes the supervisor/agent no-collusion constraint $s_L \geq k(w_\emptyset - w_L)$ more severe without allowing to reduce s_H below 0 (because of the limited liability constraint $s_H \geq 0$), we have $w_\emptyset \leq \frac{\gamma}{p\pi} + w_L$. The relevant constraint on s_H is then $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$, that is, the limited liability constraint $s_H \geq 0$ is redundant. A similar argument proves that the relevant constraint on s_L is not the limited liability constraint $s_L \geq 0$ but the supervisor/agent no-collusion constraint $s_L \geq k(w_\emptyset - w_L)$. Indeed, the principal may relax the supervisor/agent no-collusion constraint by increasing w_L. However, since setting $w_L > w_\emptyset$ instead of $w_L \leq w_\emptyset$ both increases the expected cost of the hierarchy (because this cost is increasing in w_L) and makes the principal/supervisor no-collusion constraint $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$ more severe without allowing to reduce s_L below 0 (because of the limited liability constraint $s_L \geq 0$), we have $w_L \leq w_\emptyset$. To summarize, we have $w_\emptyset \leq \frac{\gamma}{p\pi} + w_L$ and $w_L \leq w_\emptyset$, that is $w_\emptyset \in \left[w_L, \frac{\gamma}{p\pi} + w_L\right]$ with $w_L \geq 0$, and thus the relevant constraints on s_H and s_L are respectively $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$ and $s_L \geq k(w_\emptyset - w_L)$.

Given that the objective function is increasing in s_L and s_H and lowering these wages does not

make the other constraints more severe, we have $s_L = k(w_\emptyset - w_L)$ and $s_H = \frac{k}{1-k}(\frac{\gamma}{p\pi} + w_L - w_\emptyset)$. Substituting these equations into the objective function of program $[P_1]$, this program becomes:

$$\min_{w_L, w_\emptyset} \quad \frac{\gamma}{1-k} + \frac{p[(1-k)[1-(1-\pi)k]+\pi k]}{1-k} w_L + \frac{(1-k)[1-p[1-(1-\pi)k]]-p\pi k}{1-k} w_\emptyset$$

$$\text{s.t. } w_L \geq 0 \text{ and } w_\emptyset \in \left[w_L, \frac{\gamma}{p\pi} + w_L\right].$$

Two cases have now to be considered.

Let $\Gamma \equiv (1-k)\left[1-p\left[1-(1-\pi)k\right]\right] - p\pi k$. Then if $\Gamma \geq 0$, that is, if $\pi \leq \overline{\pi} \equiv \frac{(1-k)[1-p(1-k)]}{pk(2-k)}$, the objective function is increasing in $w_\emptyset$. It is then optimal to set $w_\emptyset$ as low as possible, that is, $w_\emptyset = w_L$. Substituting $w_\emptyset = w_L$ into the objective function of the above program, this function writes $\frac{\gamma}{1-k} + w_L$. Since this function is increasing in w_L, the principal sets w_L as low as possible, that is, $w_L = 0$, and hence $w_\emptyset = 0$. We then have $w_H = \frac{\gamma}{p\pi}$, $s_L = 0$ and $s_H = \frac{k\gamma}{(1-k)p\pi}$.

If instead $\Gamma < 0$, that is, if $\pi > \overline{\pi}$, the objective function is decreasing in $w_\emptyset$, and it is hence optimal to set $w_\emptyset$ as high as possible. The principal then sets $w_\emptyset = \frac{\gamma}{p\pi} + w_L$. Substituting this equation into the objective function of the above program, this function becomes $\frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi} + w_L$. Given that this function is increasing in w_L, the principal sets w_L as low as possible, that is, $w_L = 0$, and hence $w_H = w_\emptyset = \frac{\gamma}{p\pi}$. We then have $s_L = \frac{k\gamma}{p\pi}$ and $s_H = 0$.

Since we have $\overline{\pi} \geq 1$ if $p \leq 1-k$, and thus we then systematically have $\Gamma \geq 0$, the optimal contracts are: a. $(w_L, w_\emptyset, w_H) = (0, 0, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, \frac{k\gamma}{(1-k)p\pi})$ for (α) $p \leq 1-k$; (β) $p > 1-k$ and $\pi \leq \overline{\pi}$. The expected cost of the hierarchy is then $C^1 = \frac{\gamma}{1-k}$. b. $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (\frac{k\gamma}{p\pi}, 0, 0)$ for $p > 1-k$ and $\pi > \overline{\pi}$. The expected cost of the hierarchy is then $C^2 = \frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi}$.

2. $k = 1$. Given that the principal's objective function is increasing in $w_\emptyset$, and lowering this wage does not make constraints more severe in this case, we optimally have $w_\emptyset = w_H = \frac{\gamma}{p\pi} + w_L$ from the principal/supervisor no-collusion constraint and the agent's binding incentive compatibility constraint. Substituting $w_\emptyset = w_H = \frac{\gamma}{p\pi} + w_L$ both into the objective function of

program $[P_1]$ and into the supervisor/agent no-collusion constraint, we have:

$$\begin{aligned} \min_{w_L, s_L, s_H} \quad & \tfrac{[1-p(1-\pi)]\gamma}{p\pi} + w_L + p\left[(1-\pi)s_L + \pi s_H\right] \\ \text{s.t. } & w_L \geq 0,\ s_L \geq \tfrac{\gamma}{p\pi} \text{ and } s_H \geq 0. \end{aligned}$$

We clearly have $w_L = 0$, $s_L = \frac{\gamma}{p\pi}$ and $s_H = 0$. The expected cost of the hierarchy is then $C^3 = \frac{\gamma}{p\pi}$.

Proof of Proposition 2. Since w_L does not enter the objective function and reducing this wage relaxes the constraint $w_H - w_L \geq \frac{\gamma}{p\pi}$, we have $w_L = 0$. This constraint thus writes $w_H \geq \frac{\gamma}{p\pi}$. Given that reducing w_H lowers the expected cost of the hierarchy without making constraints more severe, we have $w_H = \frac{\gamma}{p\pi}$. Two cases should then be considered with respect to k.

1. $k < 1$. Substituting $w_H = \frac{\gamma}{p\pi}$ into the principal/supervisor no-collusion constraint, this constraint writes $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} - w_\emptyset)$. The principal may then relax this constraint by increasing $w_\emptyset$. However, since setting $w_\emptyset > \frac{\gamma}{p\pi}$ instead of $w_\emptyset \leq \frac{\gamma}{p\pi}$ increases the expected cost of the hierarchy (because this cost is increasing in $w_\emptyset$) without allowing to reduce s_H below 0 (because of the limited liability constraint $s_H \geq 0$), we have $w_\emptyset \leq \frac{\gamma}{p\pi}$. The relevant constraint on s_H is then $s_H \geq \frac{k}{1-k}(\frac{\gamma}{p\pi} - w_\emptyset)$, that is, the limited liability constraint $s_H \geq 0$ is redundant. Note that since $w_\emptyset - w_L \geq \varepsilon$ and $w_L = 0$, we also have $w_\emptyset \geq \varepsilon$. Therefore $w_\emptyset \in \left[\varepsilon, \frac{\gamma}{p\pi}\right]$. Program $[P_2]$ then writes:

$$\begin{aligned} \min_{w_\emptyset, s_L, s_H} \quad & \gamma + (1-p\pi)w_\emptyset + p\pi s_H \\ \text{s.t. } & w_\emptyset \in \left[\varepsilon, \tfrac{\gamma}{p\pi}\right],\ s_L \in [0, k(w_\emptyset - \varepsilon)] \text{ and } s_H \geq \tfrac{k}{1-k}(\tfrac{\gamma}{p\pi} - w_\emptyset). \end{aligned}$$

Given that the objective function is increasing in s_H, we have $s_H = \frac{k}{1-k}(\frac{\gamma}{p\pi} - w_\emptyset)$. Substituting $s_H = \frac{k}{1-k}(\frac{\gamma}{p\pi} - w_\emptyset)$ into the objective function and expressing the objective function with respect to $w_\emptyset$, the optimal wage $w_\emptyset$ is thus the solution to

$$\min \quad \tfrac{\gamma}{1-k} + \tfrac{1-k-p\pi}{1-k} w_\emptyset$$

$$w_\emptyset, s_L$$

$$\text{s.t. } w_\emptyset \in \left[\varepsilon, \tfrac{\gamma}{p\pi}\right] \text{ and } s_L \in [0, k(w_\emptyset - \varepsilon)].$$

Let $\Phi \equiv 1-k-p\pi$. Two cases should then be distinguished. If $\Phi \geq 0$, that is, if $\pi \leq \widehat{\pi} \equiv \frac{1-k}{p}$, the objective function is increasing in $w_\emptyset$ and it is thus optimal to set $w_\emptyset = \varepsilon$. In this case, $s_L = 0$ and the expected cost of the hierarchy is $C^1(\varepsilon) = C^1 + \frac{(1-k-p\pi)\varepsilon}{1-k}$. If instead $\Phi < 0$, that is, if $\pi > \widehat{\pi}$, the objective function is decreasing with $w_\emptyset$ and it is therefore optimal to set $w_\emptyset = \frac{\gamma}{p\pi}$. In this case, $s_L \in \left[0, k(\frac{\gamma}{p\pi} - \varepsilon)\right]$and the expected cost of production and supervision is $C^3 = \frac{\gamma}{p\pi}$.

Since $\widehat{\pi} \geq 1$ if $p \leq 1-k$, we have the different cases of Proposition 2 for $k < 1$.

2. $k = 1$. Since $\frac{\gamma}{p\pi} > \varepsilon$, constraint $w_\emptyset \geq w_H$, that is, $w_\emptyset \geq \frac{\gamma}{p\pi}$, is more restrictive than constraint $w_\emptyset \geq \varepsilon$. Program $[P_2]$ thus writes:

$$\min \quad \gamma + (1-p\pi) w_\emptyset + p\pi s_H$$

$$w_\emptyset, s_L, s_H$$

$$\text{s.t. } w_\emptyset \geq \tfrac{\gamma}{p\pi},\ s_L \in [0, w_\emptyset - \varepsilon] \text{ and } s_H \geq 0.$$

Given that the objective function of this program is increasing in $w_\emptyset$, $w_\emptyset = \frac{\gamma}{p\pi}$ $(= w_H)$. We obviously have $s_L \in \left[0, \frac{\gamma}{p\pi} - \varepsilon\right]$ and $s_H = 0$. Hence, when $k = 1$, the expected cost of production and supervision is C^3.

Proof of Proposition 3. Since reducing $w_\emptyset$ and s_L lowers the expected cost of the hierarchy without making constraints more severe, we have $w_\emptyset = \frac{\gamma}{p\pi} + w_L$, and hence $s_L = \frac{k\gamma}{p\pi}$ with $k \leq 1$. We then obviously have $w_L = 0$, and thus $w_\emptyset = \frac{\gamma}{p\pi}$ and $w_H + s_H \geq \frac{\gamma}{p\pi} + \varepsilon$. Program $[P_3]$ becomes:

$$\min \quad \tfrac{[1-p[1-(1-\pi)]k]\gamma}{p\pi} + p\pi(w_H + s_H)$$

$$w_H, s_H$$

$$\text{s.t. } w_H + s_H \geq \tfrac{\gamma}{p\pi} + \varepsilon \text{ and } s_H \leq \tfrac{k}{1-k}(w_H - \tfrac{\gamma}{p\pi} - \varepsilon) \text{ if } k < 1 \text{ or } w_H \geq \tfrac{\gamma}{p\pi} + \varepsilon \text{ if } k = 1.$$

Clearly, we then have $w_H + s_H = \frac{\gamma}{p\pi} + \varepsilon$ and $s_H = 0$. Therefore, $w_H = \frac{\gamma}{p\pi} + \varepsilon$ and the expected cost of production and supervision is $C^2(\varepsilon) = C^2 + p\pi\varepsilon$.

Proof of Proposition 4. As above, obviously $w_L = 0$, and hence $w_\emptyset = \frac{\gamma}{p\pi}$ and $s_L \in \left[0, k(\frac{\gamma}{p\pi} - \varepsilon)\right]$ for $k \leq 1$. Program $[P_4]$ then writes:

$$\min_{w_H, s_H} \quad \frac{(1-p\pi)\gamma}{p\pi} + p\pi(w_H + s_H)$$

$$\text{s.t. } w_H + s_H \geq \frac{\gamma}{p\pi} + \varepsilon,\ s_H \leq \frac{k}{1-k}(w_H - \frac{\gamma}{p\pi} - \varepsilon) \text{ if } k < 1 \text{ or } w_H \geq \frac{\gamma}{p\pi} + \varepsilon \text{ if } k = 1.$$

As in the preceding proof, $w_H + s_H = \frac{\gamma}{p\pi} + \varepsilon$ and $s_H = 0$. Therefore, $w_H = \frac{\gamma}{p\pi} + \varepsilon$ and the expected cost of the hierarchy is $C^3(\varepsilon) = C^3 + p\pi\varepsilon$.

Proof of the Theorem. We start by comparing the various policies in the case where $k < 1$. First, let us compare the cost of policy 3, that is, the cost of preventing supervisor/agent collusion and allowing principal/supervisor collusion, with the cost of policy 4, that is, the cost of allowing both types of collusion. Since $C^2 < C^3$ for $k \in (0, 1)$, we clearly have $C^2(\varepsilon) < C^3(\varepsilon)$, and hence, as explained in the text, policy 4 is strictly more expensive than policy 3.

Next, let us compare the cost of policy 3 with the cost of policy 1, that is, the cost of preventing both types of collusion. As clear from Proposition 1, when $p > 1 - k$ and $\pi > \overline{\pi}$, policy 3 is strictly more expensive than policy 1. Given that if $\pi \leq \overline{\pi}$, $C^1 \leq C^2$, and hence $C^1 < C^2(\varepsilon)$, and recalling that $\overline{\pi} \geq 1$ if $p \leq 1 - k$, we have that when $p \leq 1 - k$, or $p > 1 - k$ and $\pi \leq \overline{\pi}$, policy 1 is strictly less expensive than policy 3. To sum up, policy 1 is strictly preferable to policy 3.

Finally, we must compare the cost of policy 1 with the cost of policy 2, that is, the cost of allowing supervisor/agent collusion and preventing principal/supervisor collusion. Before proceeding to this comparison, note that $\overline{\pi} < \widehat{\pi}$ if $p > 1 - k$, and recall that $\widehat{\pi} \geq 1$ if $p \leq 1 - k$. When

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$p \leq 1-k$, or $p > 1-k$ and $\pi \in (0, \overline{\pi}]$, we must compare C^1 (policy 1) with $C^1(\varepsilon)$ (policy 2). Given that $\frac{(1-k-p\pi)\varepsilon}{1-k} \geq 0$ if $\pi \leq \widehat{\pi}$, and hence $C^1 \leq C^1(\varepsilon)$ if $\pi \leq \widehat{\pi}$, the optimal policy is policy 1. When $p > 1-k$ and $\pi \in (\overline{\pi}, \widehat{\pi}]$, we must compare C^2 (policy 1) with $C^1(\varepsilon)$ (policy 2). Since $C^1 \leq C^1(\varepsilon)$ if $\pi \leq \widehat{\pi}$ and $C^2 < C^1$ if $\pi > \overline{\pi}$, policy 1 is then optimal. When $p > 1-k$ and $\pi > \widehat{\pi}$, we must compare C^2 (policy 1) with C^3 (policy 2). Given that $C^2 < C^3$, the principal then opts for policy 1.

As it is straightforward to see, for $k = 1$ policies 1 and 2 have the same cost and are less expensive than policies 3 and 4. Deterring all types of collusion is then weakly optimal.

Summarizing these comparisons, we have that deterring both types of collusion is strictly less expensive than the other three possible policies if $k < 1$ and weakly less expensive than the other three policies if $k = 1$.

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