PAC vs. DEMAL. A Dialogical Reconstruction of Public Announcement Logic with Common Knowledge

Sébastien Magnier

PAC vs. DEMAL

A Dialogical Reconstruction of Public Announcement Logic with Common Knowledge

SÉBASTIEN MAGNIER*

ABSTRACT. Since the work of Plaza (1989) about acts of public communication, a lot of dynamic epistemic logic systems have emerged. A general state of the art can be found in van Ditmarsch et al. (2007). Such logics model situations in which some announcements can be made and after an announcement, the situations which are incompatible with it are deleted from the model. In this paper we propose a reconstruction of the logic **PAC** (*Public Announcement logic with Common knowledge*, see van Ditmarsch et al. (2007)) through the dialogical framework. The idea of this work is to rediscover announcements as acts: acts of an arguer during an argumentative dialogue about knowledge change of agents instead of "model-modifiers". We name this reconstruction **DEMAL** for *Dialogical Epistemic Multi-Agent Logic*.

Introduction

Epistemic logics are often used to explore different philosophical problems in epistemic area. Thanks to Plaza (1989), epistemic logics became dynamic, but in the same time they got closer to artificial intelligence and computational sciences than 'traditional' philosophy. So, when we try to understand what the philosophical ground or the signification of epistemic operators (knowledge and announcement operators) is, answers are a bit evasive. Hendricks (2005) and Hendricks and Symons (2010) have shown the connection between these two traditions but not directly through the meaning of what knowledge operators are. To sum up, in model theory, epistemic operators allow some particular moves through accessibility relations between different states of a model and

*Univ Lille Nord de France, F-59000 Lille France;
UdL3, STL, F-59653 Villeneuve d'Ascq France;
CNRS UMR8163.

announcements are considered as *truthful* operations on the model. In that case, it is not the announcement itself which brings new information: knowledge of agents grows up with announcements because the model is restricted as a consequence of announcements. Agents' knowledge emerges because the announcement *cuts* some accessibilities. Moreover, each knowledge operator receives a semantic definition which represents particular conditions in terms of truth. Indeed, with the model theoretic approach of epistemic problems, every operator is defined in terms of truth value. The problem is that defining operators in such way does not allow us to understand their meaning behind their truth value. Moreover they can be subject to attacks of scepticism about the question of truth.

Through our dialogical approach, we want to propose another way to define the epistemic operators. Rebuschi and Lihoreau (2008) and Rebuschi (2009) already present some interesting works in dialogical logic in epistemic fields. In dialogical logic, operators are *playable-defined* instead of truth-defined.[1] It means that we leave aside the question of their truth in benefits of their meaning through their conditions of use during an argumentative process. Thanks to the dialogical framework, i.e. a game built around the interaction between two players, we can explore the signification of utterances of players about agents' change of knowledge in an argumentative debate. In that specific case, the signification of those operators follows from the debate itself, from the way to *challenge* and *defend* them. In the course of the debate, epistemic operators represent either a commitment or a choice (sometimes both). They are commitments insofar as when a player utters an argument which implies the knowledge of one agent (or more), the other player will challenge the utterance, and the first shall defend his argument. Consequently, uttering an argument based on the knowledge of one agent leads to defend agent's knowledge. Epistemic operators also represent choices because when a player challenges any form of knowledge uttered during the debate, he has to choose different parameters for his challenge. According to the operator at stake he must choose: labels, agents or sequences of agents. In fact thanks to the dialogical framework, we can take literally Mackenzie's remark and start dynamical studies of the meaning of all knowledge operators defined

[1]Playable-defined is an original way to understand the notion of validity independently of the truth. In that case, validity is defined in terms of winning strategies instead of truth.

by the interaction between the two players of the dialogue, that is in a communicative context.[2]

First, we present the epistemic operators and the announcement operators with their intended interpretations and their model theoretical semantics. After that, we expose our dialogical reconstruction of these operators and some examples.

1 Public Announcement Logic

But first of all, some lexical definitions are required to avoid confusions. As we have already begun, we will deal with *agents* and *players*. The term *agent* refers to the possessor of knowledge and the term *player* is used when we speak about the arguer of the dialogue. Arguer and player can be synonymously used, the latter standing for an abbreviation of the 'arguer of the dialogue'. But we must be careful not to confuse agents and players: agents belong to the level of the logical language whereas players belong to the argumentative level of the dialogue.

Let us start by briefly explaining what we have in mind when we speak about knowledge operators. In fact, there are three knowledge operators and we can consider that each of them expresses a higher level of complexity.

The first knowledge operator represents a weak degree of complexity because it involves only one agent: it is named *individual knowledge operator*. It is a kind of link between two different things: *an agent* and *a proposition*. Somehow, we can consider that the agent is the bearer of the knowledge and the proposition is the content of it. This operator represents a particular way in which the agent turns himself towards the proposition. For an agent, a proposition can be judged known if and only if certain properties are respected. It is that judgement (involved by the properties) which is expressed by the epistemic operator.

If we want to treat the knowledge of several agents, other operators are required. We need one operator which is able to express the knowledge of all agents and another one which allows us to express the knowledge that agents can have about their own knowledge and

[2]In Mackenzie (1985), the author claims that although "Robinson Crusoe may have constructed derivations or proofs for his own edification, he could not engage in argument until Man Friday joined him." Mackenzie points out the fact that logic becomes interesting and really dynamic when it is studied in an argumentative context, otherwise it is a computational game without consideration on meaning.

about other agent's knowledge. Those operators are already expressed in **PAC**.[3]

The second knowledge operator expresses a quantification on agents; that is, relatively to a determined proposition and a determined group of agents, all agents of the group know the proposition: it is named *sharing knowledge operator*.[4] Indeed the knowledge of the proposition is shared by all agents, that is: everybody in the group knows the proposition. Beware: the knowledge about the proposition is the only thing which is shared. They do not share the knowledge about this knowledge.

The last knowledge operator considered allow us to express such form of knowledge, i.e., sharing knowledge about sharing knowledge. Indeed, this operator expresses an infinite iteration of the sharing knowledge operator. In other words, it means that everybody knows that everybody knows that everybody knows and so on ad infinitum.

In addition to those epistemic operators, we also have public announcement operators. Public announcement operators differ from the preceding operators because they do not directly concern the agents or their knowledge but firstly affect the model. Indeed, this kind of operator restricts the model $\mathcal{M}$ in which the knowledge of agents is evaluated to a submodel $\mathcal{M}^{announcement}$ in which the announcement holds.[5] That is, all states in which the announcement does not hold must be removed from the model $\mathcal{M}$. It is in that sense that an announcement can modify the knowledge of agents because it can change the accessibility relations by deleting states which do not respect the announcement. Those states are said to be incompatible with the epistemic alternatives of agents.

1.1 The Syntax

Given a finite set of agents Ag and a countable set of atoms $\mathcal{P}$, the language $\mathcal{L}_{KC[\,]}(Ag, \mathcal{P})$ is inductively defined by the BNF as follows:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi \mid E_G\varphi \mid C_G\varphi \mid [\varphi]\varphi$$

where $a \in Ag, G \subseteq Ag$, and $p \in \mathcal{P}$. We denote the dual of $[\varphi]\,\varphi$ by $\langle\varphi\rangle\varphi$; so we have $\langle\varphi\rangle\varphi$ as an abbreviation of $\neg[\varphi]\neg\varphi$.

3See van Ditmarsch et al. (2007) pp. 90-91 for a detailed presentation.

4Also well known as *Everybody knows operator*, see Fagin et al. (1995) ch. 2.

5For convenient reason, we will write $\mathcal{M}^{\mathcal{A}}$ instead of $\mathcal{M}^{announcement}$.

1.2 Intended interpretations

- K_a: Individual Knowledge
  The intended interpretation of a formula $K_a\varphi$ is: "agent a knows φ".

- E_G: Sharing Knowledge
  The intended interpretation of a formula $E_G\varphi$ is: "all members of the group G know φ".

- C_G : Common Knowledge
  The intended interpretation of a formula $C_G\varphi$ is: "all members of the group G know that all members of the group... know φ".[6]

- $[\varphi]\psi$: Public Announcement
  The intended interpretation of a formula $[\varphi]\,\psi$ is: "after every announcement of φ, ψ holds".

- $\langle\varphi\rangle\psi$: Dual of an Announcement
  The intended interpretation of a formula $\langle\varphi\rangle\psi$ is "after a possible announcement of φ, ψ holds".

1.3 Model Theoretic Semantics

An epistemic model is a triple $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}_{a\{a\in Ag\}}, \mathcal{V}\rangle$ such that $\mathcal{W}$ is a set of states w, each $\mathcal{R}_a$ is a binary equivalence relation over $\mathcal{W}$, and $\mathcal{V}$ is a valuation function such that for every $p \in \mathcal{P}$ yields the set $\mathcal{V}(p) \subseteq \mathcal{W}$ of states in which p is true. Standard connectives have the usual semantics definition; operators mentioned above receive the following ones:

- *K_a operator*:
  $\mathcal{M}, w \models K_a\varphi$ iff $\mathcal{M}, w' \models \varphi$ for all $w' \in \mathcal{W}$ such as $w\mathcal{R}_a w'$.

- *E_G operator*:
  $\mathcal{M}, w \models E_G\,\varphi$ iff $\mathcal{M}, w \models K_a\varphi$ for all $a \in G$.

- *C_G operator*:
  $\mathcal{M}, w \models C_G\varphi$ iff $\mathcal{M}, w \models K_{a_1} \ldots K_{a_n}\varphi$ for each sequence of agents $\langle a_1 \ldots a_n\rangle \in G^*$.[7]

[6] There are different semantic definitions of common knowledge (see Gerbrandy (1999) ch. 3 and Fagin et al. (1995) ch. 11 for more details on that issue) but we choose to use the 'iterated approach'.

[7] G^* stands for the set $G0 \cup G1 \cup G2 \cup \ldots$

- $[\varphi]\,\psi$ *operator*:
  $\mathcal{M}, w \models [\varphi]\,\psi$ iff $\mathcal{M},w \models \varphi$ implies $\mathcal{M}^{\varphi}, w \models \psi$.
- $\langle\varphi\rangle\psi$ *operator*:
  $\mathcal{M},w \models \langle\varphi\rangle\psi$ iff $\mathcal{M},w \models \varphi$ and $\mathcal{M}^{\varphi}, w \models \psi$.

where $\mathcal{M}^{\varphi} = \langle \mathcal{W}', \mathcal{R}'_{a\{a\in Ag\}}, \mathcal{V}' \rangle$, is defined as follows:
$\mathcal{W}' = \{w' \in \mathcal{W} \mid \mathcal{M}, w' \models \varphi\}$
$\mathcal{R}'_a = \mathcal{R}_a \cap (\mathcal{W}' \times \mathcal{W}')$
$\mathcal{V}'(p) = \mathcal{V}p \cap \mathcal{W}'$

The special feature of the two last operators is to modify the model by eliminating states in which the announcement does not hold. It is precisely in that sense that those operators are considered as dynamic because they are able to restrict the model to a submodel. The problem is that in dialogical logic we do not have any model, so obviously we cannot restrict the model. But, in dialogical logic, players have to do some choices; hence we can impose some constraints on them. That point is further develop in the next section.

2 DEMAL: A Dialogical Reconstruction of PAC

In Rahman and Keiff (2005), Fontaine and Redmond (2008) and in Keiff (2009), the reader can find a detailed presentation of what the dialogical logic is. Here we just succinctly expose the purpose of such framework. In dialogical logic two players confront each other around an argument. We note d_{Δ} the dialogue which starts with a thesis Δ (also named the initial argument). On the one hand we have the **P**roponent who defends the thesis Δ; and on the other hand the **O**pponent who tries to challenge Δ in accordance with of the rules.[8] The **O**pponent's aim is to invalidate the thesis uttered by the **P**roponent by building a counter-argument to Δ. If the **O**pponent does not succeed in that task, we can consider that the **P**roponent has a winning strategy for Δ because he manages to defend against all possible challenges of the **O**pponent (still in accordance with the rules). Then it means that the **P**roponent proves the validity of his initial argument.

The debate which follows after the thesis offers to dialogical logic the possibility to define the meaning of the logical constants via the notions

[8]The set of rules determines the dialogical system in which the thesis can be challenged and defended.

of commitment and choice during the argumentative process between the two players. It is that very point which allows us to explore and to provide an original theory of the meaning of the different epistemic operators based on the use of these logical constants during an exchange on the argument.
Our dialogical language is obtained from $\mathcal{L}_{KC[\,]}(Ag, \mathcal{P})$ by the addition of:

- two meta-logical symbols **?** and **!**
- two labels **O** and **P** standing for the players.
- a label i which indicates the context of the utterance. Labels are finite sequences of positive integers such as 1_a1 and 1_a2 indicate (from the model-theoretic point of view) that contexts 1 and 2 are reachable from 1 for the agent a.

2.1 A new form of Labels

In standard dialogical logic, labels are finite sequences of positive integers. If the label i is a sequence of length > 1, the positive integers of the sequence will be separated by periods. But in our work, sequences are separated by the 'name' of agents $(a_1, ..., a_n)$. Thus, if i is a label and n is a positive integer, then i_an is also a label of the agent a, called an extension of i.

The problem is that we want to treat about a logic with public announcement, hence this formulation of labels is not sufficient and we must enrich it. The idea is to keep track of the announcement on labels. For this reason, we use another way to denote a label. A labelled formula is now defined as a triple:

$\mathcal{A}|n : \Sigma$, such that:

- $\mathcal{A} = \sigma_1\sigma_2...\sigma_n$ is a sequence of announced formulas. If there is no announced formula, the sequence remains empty and we denote the list by ϵ.[9]
- n is a sequence of labels such as $i_{a_1}i'_{a_2}...i''_{a_n}$,
- Σ is either a $\mathcal{L}_{KC[\,]}$ formula or a dialogical symbolic challenge.[10]

[9]The idea of the sequence of announcement is taken from Balbiani et al. (2010).

[10]Dialogical symbolic challenges are challenges of the form "?...", see particle rules column "**Y**'s-Challenge" section 2.2-2.4.

For short, we add the new argument carried by the announcement to the list $\mathcal{A}$, but strictly speaking no new label appears. The label remains the same; we just add more information on it, adding announcements as a prefixed list to the label.

2.2 The standard Particle Rules

As every game, a dialogical game obeys some rules. Two kinds of rules are required: particle rules and structural rules. Particle rules define how to challenge and defend each logical constant during the dialogue whereas structural rules regulate the course of the dialogue itself. Consequently, the first ones provides the local meaning of logical constants and the second ones the global meaning of the game. The latter kind of rules will be exposed in section 2.6.

Particle rules are symmetric, that is, they are strictly the same for the **O**pponent and for the **P**roponent. For this reason, we use the notation **X** and **Y** in their formulation. Table for particle rules must be read as "*In the course of the dialogue if the player* **X** *does the utterance* φ (**X**-Utterance); *then the player* **Y** *challenges it with the corresponding attack* (**Y**-Challenge) *and* **X** *must produce the corresponding defence* (**X**-Defence)". The Particle rules for standard connectives are defined below:

Logical constants	**X** - Utterance	**Y** - Challenge ?	**X** - Defence !
$\neg$, no possible defence	$\mathcal{A}\|i$: $\neg\varphi$	$\mathcal{A}\|i : \varphi$	$\otimes$
$\wedge$, the challenger has the choice on the conjunct to challenge	$\mathcal{A}\|i : \varphi \wedge \psi$	$\mathcal{A}\|i : ?_{\wedge 1}$ *or* $\mathcal{A}\|i : ?_{\wedge 2}$	$\mathcal{A}\|i : \varphi$ *respectively* $\mathcal{A}\|i : \psi$
$\vee$, the defender has the choice on the disjunct to defend	$\mathcal{A}\|i : \varphi \vee \psi$	$\mathcal{A}\|i : ?$	$\mathcal{A}\|i : \varphi$ *or* $\mathcal{A}\|i : \psi$

Particle Rules for Standard Connectives (PR−SC)

There is no possible defence for the negation, which is symbolized by "$\otimes$". The play on the conjunction and the disjunction shows the difference between having the burden of the choice or not. In case of a conjunction, it is the challenger who chooses the conjunct and the defender is subjected to his choice, whereas the roles are reversed with the disjunction, it is the defender who has the burden of the choice.

2.3 Epistemic Operators

As we have already said, our aim is to reconstruct the epistemic operators in dialogical logic, using the notion of choice. This is easily done by the interaction between the two arguers of the dialogue. Let us take the semantics of the individual knowledge operator as an example.

1. Individual knowledge's semantics stipulates that in a model $\mathcal{M}$ and in a state w, a formula $K_a\varphi$ is satisfied provided φ is satisfied in every state w' of $\mathcal{M}$ reachable for the agent a from w.

2. Consider $\langle \mathcal{A}|i : K_a\varphi \rangle \in d_\Delta$.The player **X** who makes this utterance, utters in fact that φ must hold in any label reachable for a. So, if **Y** wants to challenge it, he can arbitrary choose a label i' reachable for a in which **X** would be able to utter φ. And then the label of **X**'s defence turns into $\mathcal{A}|i_a i'$.[11]

From a general point of view, in every case in which a player **X** utters something like *"for all things, something is the case"*, the player **X** exposes himself to the choice of the player **Y**. And after that **X** has to play taking into account **Y**'s choice. If he is not able to do this, his utterance fails and he will not be able to defend his utterance.

Operators of sharing knowledge and common knowledge behave in a similar way. A player **X** who utters that φ is a shared knowledge into the group G ($E_G\varphi$), utters in fact that any agent in G individually knows the proposition φ. In that case, **Y** has the choice and can arbitrarily choose any agent in G in order to challenge **X**'s utterance. Consequently, after **Y**'s challenge, **X** must utter φ in the scope of the individual knowledge operator of the corresponding agent chosen by **Y**.

A player **X**, who utters that a proposition φ is commonly known by the group G ($C_G\varphi$), utters in fact that for any possible sequence of agents the proposition φ holds. Hence, **Y** can choose any arbitrary sequence of agents in G; and **X** must defend φ in the scope of the corresponding sequence of agents chosen by **Y**. The particle rules giving the meaning of all these operators are given in the following table.

[11]Due to the list of announcement, this must be completed by a specific rule which ensures that i' respects the list, see section Structural Rules (SR-A.1) for more details on that point.

Logical constants	**X**-Utterance	**Y**-Challenge ?	**X**-Defence !
K_a, the challenger can choose any label i' for a	$\mathcal{A}\|i : K_a\varphi$	$\mathcal{A}\|i : ?^a_{i'}$	$\mathcal{A}\|i_a i' : \varphi$
E_G, the challenger can choose any agent $a \in G$	$\mathcal{A}\|i : E_G\varphi$	$\mathcal{A}\|i : ?a \in G$	$\mathcal{A}\|i : K_a\varphi$
C_G, the challenger can choose any sequence of agents, this sequence can be empty	$\mathcal{A}\|i : C_G\varphi$	$\mathcal{A}\|i : ?\langle a_1...a_n\rangle \in G^*$	$\mathcal{A}\|i : K_{a_1}...K_{a_n}\varphi$

Particle Rules for Epistemic Operators (PR−KO)

Through those rules, a noticeable thing appears: *eventually*, exchanges starting with $E_G\varphi$ or $C_G\varphi$ will proceed according to the rule for K_a. Moreover, thinking about the rules for epistemic operators in that way gives us a better understanding on how each of them are constructed from an individual knowledge operator. In fact, it is always the notion of choice which leads to φ from any epistemic operators. The type of the choice simply depends on the operator at stake.

Note: Since we are in a multi-agent setting some notational requirements must be clarified. Let us consider a formula $\varphi \in d_\Delta$ where φ stands for i: $K_{a_1}K_{a_2}\psi$. As the challenger will have to choose a label for each different individual knowledge operator, the latest label is noted "$i_{a_1}i'_{a_2}i''$". In this way, we are able to retrace the "story" of the formula at stake.[12] And indeed, it is easy to see that the labelled formula "$i_{a_1}i'_{a_2}i''$: ψ" follows from i: $K_{a_1}K_{a_2}\psi$ because each a_n between labels represents a choice of a challenger for a K_a operator.

2.4 Announcement & Dialogue

We are now ready to present dynamic operators of public announcements in the dialogical framework. Public announcement operators can also be understood through the notion of choice plus the notion of commitment. Indeed, as in dialogical logic we assume that we do not have any model, the eliminating path of an announcement must be on choices of players. It means that after a play on an announcement

[12]This point is important for the application of the rule **(SR-5*)**, see section 2.6 Structural Rules for DEMAL for more details.

operator, players must take into account the announcement. One interesting point here is to understand the idea of migration to a sub-model through a commitment of a player during the debate instead of an operation on a model. It works for public announcement operator as well as for its dual. The difference between the public announcement operator and its dual is captured by a difference in the possible defence (after the challenge on the announcement operator).

In the case of the public announcement operator the defender can choose to commit himself or not in the announced formula, but in case of its dual he has not the choice, he must commit himself in the announced formula. That public announcement operator represents, in the dialogical framework, a kind of commitment in terms of concession. And in fact it is: the player who challenges the announcement can be constraint to concede the announcement if the defender does not commit himself in its defence (by challenging the negation, see the rule below). Whereas the dual of public announcement operator represents a commitment in terms of assertion: the player who utters a dual of a public announcement must be able to take in charge the utterance of the announced formula. It means that he will be able to defend that proposition against any challenge (allowed by the set of rules) of his adversary.

Following Walton and Krabbe (1995), Yamada (2012) proposes a logic that also brings out a such distinction between concession commitment and assertion commitment. It would seem that there exists some interesting connexions between Yamada's approach and **DEMAL**, but it is not our purpose to further expose them here.

The meaning of announcement operator is given by the rules in the following table (the symbol "+" means that the right part is added to the left part of the list):

Logical constants	**X** Utterance	**Y** Challenge ?	**X** Defence !
$[\varphi]\,\psi$, the defender has the choice for his defence	$\mathcal{A}\|i : [\varphi]\,\psi$	$\mathcal{A}\|i : ?$	$\mathcal{A}\|i : \neg\varphi$ *or* $\mathcal{A} + \varphi\|i : \psi$
$\langle\varphi\rangle\psi$, the challenger has the choice for his challenge	$\mathcal{A}\|i : \langle\varphi\rangle\psi$	$\mathcal{A}\|i : ?_{\langle 1\rangle}$ *or* $\mathcal{A}\|i : ?_{\langle 2\rangle}$	$\mathcal{A}\|i : \varphi$ *respectively* $\mathcal{A} + \varphi\|i : \psi$

Particle Rules for Announcement Operators (PR−AO)

The particle rule for public announcement operator can be intuitively understood as the following exchange:

> in accordance to announcements in $\mathcal{A}$, the player **X** says: "in the situation *i*, if I commit myself in the defence of the argument φ, then I will utter ψ in accordance to φ". So, player **Y** asks him what he will do, hence **X** has the choice:
>
> - he can reply "No I don't want to commit myself in the utterance of the argument φ", *or*
> - he can utter ψ assuming φ.

The particle rule for the dual of a public announcement operator can be understood following the same exchange except on who has the burden of the choice:

> in accordance to announcements in $\mathcal{A}$, the player **X** says: "in the situation *i*, I commit myself in the defence of the argument φ and I will utter ψ in respect of φ". Hence, now **Y** has the choice, he can compel **X** to utter:
>
> - φ, *or*
> - ψ assuming φ.
>
> And of course, **X** must answer following **Y**'s choice.

2.5 The set *PartRules*

All of the particle rules presented above (particle rules for standard connectives, for knowledge operators and for public announcement operators) are brought together in a set named *PartRules*:

$$\textit{PartRules} := \text{PR-SC} \cup \text{PR-EO} \cup \text{PR-AO}$$

2.6 Structural Rules

As mentioned in section 2.2, there are two kinds of rules in dialogical logic. We have seen the particle rules which are the rules for the logical constants, now we must expose the rules which govern the dialogue itself: the structural rules. These rules concern the process of the dialogue, they deal with how to play the dialogue. Firstly, we begin by presenting what we can name the standard structural rules. They can be said *Standard* because those rules are the base of any dialogical system.

Standard Structural Rules

- **Starting Rule (SR-0)**: Any dialogue d_Δ starts with **P** uttering Δ (the thesis). After the thesis has been uttered, players alternately choose a *repetition rank* n and m. A repetition rank is a positive integer number which corresponds to the number of defences or challenges that a player can perform.

- **Game-playing (SR-1)**: Players act alternately. Whenever he has a turn to play, player **X** can challenge any previous **Y** move or defend against any previous **Y** challenge up to his repetition rank.

- **Formal Rule (SR-2)**: **P** is allowed to utter an atomic formula only if **O** has uttered it first.

- **Winning Rule (SR-3)**: Player **X** wins a dialogue if and only if it is **Y**'s to play but he cannot.

Those rules are for a basic dialogue on propositional logic. But epistemic logic requires a dialogical modal logic, so we need to complete these standard rules. Rahman and Rückert (1999) and Rahman and Keiff (2005) have developed dialogical modal systems. For **DEMAL** we propose the following two rules: one for the condition of the choice of labels and another for the announcement operators.

Structural Rules for DEMAL

- **(SR-5*) A chosen label for agent *a***:[13] to challenge a K_a *operator* from a label $\mathcal{A}|i...i'$, **P** can choose for a any label i'' in the sequence $\mathcal{A}|i...i'$ such that:
  - $\mathcal{A}|i'' = \mathcal{A}|i'$ *or*
  - we have both:
    - ∗ **O** has already chosen the label $\mathcal{A}|i''$ for a, and
    - ∗ there is no b in the sequence $\mathcal{A}|i'...j_b...i''$ where $a \neq b$.

The first requirement stipulates that **P** can use the reflexivity for his choice whereas the two other requirements express the symmetric and transitive closure for the choice made on a's label.

[13]The rule **(SR-5*)** is a slight adaptation of the rule **(SR-ST9.2S5)** found in Rahman and Keiff (2005). The rule **(SR-ST9.2S5)** characterizes a S5 accessibility between dialogical labels, **(SR-5*)** privatises that access to each agent $a \in Ag$.

- **Announcement & Label (SR-A.1)**: If there is a move $\langle \mathbf{X} - \mathcal{A}|i : e\rangle$ such that the sequence $\mathcal{A}$ is non-empty, then **Y** can compel **X** to utter the last element of $\mathcal{A}$ in the label i or in a label j if $e = ?_j$.
- **Step back & Step forward (SR-A.2)**: After an utterance φ that follows from a play on an announcement $[\varphi]$... (or $\langle\varphi\rangle$...) in label i, **P** can defend an atomic formula p in a label:
  - i if **O** has uttered p in $\varphi|i$ (SB), *or*
  - $\varphi|i$ if **O** has uttered φ and p in i (SF).

All of the structural rules that we have presented above are brought together in a set named *StrucRules*:

$$StrucRules = \text{SR-0} \cup \text{SR-1} \cup \text{SR-2} \cup \text{SR-3} \cup \text{SR-5}^* \cup \text{SR-A}$$

2.7 DEMAL

DEMAL is defined as being the set *PartRules* and the set *StrucRules*.

$$\textbf{DEMAL} = PartRules \cup StrucRules$$

With respect to the rules of **DEMAL**, there is a winning strategy for the **P**roponent for all of the formulas in Table 1 which characterize the system **PAC**. A soundness and completeness proof of **DEMAL** has been presented at the XIVth CLMPS and it is to appear.[14]

3 Examples

Let us take some examples to illustrate how to play with **DEMAL**. The first one only deals with individual knowledge operator, the next one a group of agents and the last one a public announcement. In the following examples, with respect to (SR-0), **P** always utters the thesis at move 0; and as the arguers must play alternatively, **P**'s moves are even-numbered whereas **O**'s moves are odd-numbered (see external columns). Challenged moves are noted in the middle columns (middle left column for **O** and middle right column for **P**).

[14] See Magnier and de Lima (2012).

$K_a(\varphi \to \psi) \to (K_a\varphi \to K_a\psi)$	distribution K_a over $\to$
$K_a\varphi \to \varphi$	truth
$K_a\varphi \to K_a\, K_a\, \varphi$	introspection positive
$\neg K_a\varphi \to K_a\neg K_a\varphi$	introspection negative
$[\varphi]\, p \to (\varphi \to p)$	atomic permanence
$[\varphi]\, \neg\psi \leftrightarrow (\varphi \to \neg\, [\varphi]\, \psi)$	announcement and negation
$[\varphi]\, (\varphi \wedge \chi) \leftrightarrow ([\varphi]\, \psi \wedge [\varphi]\, \chi)$	announcement and conjunction
$[\varphi]\, K_a\psi \leftrightarrow (\varphi \to K_a\, [\varphi]\, \psi)$	announcement and knowledge
$[\varphi]\, [\psi]\, \chi \leftrightarrow [\varphi \wedge [\varphi]\, \psi]\, \chi$	announcement and composition
$C_G(\varphi \to \psi) \to (C_G\varphi \to C_G\psi)$	distribution C_G over $\to$
$C_G\varphi \to (\varphi \wedge E_G C_G\varphi)$	mix of common knowledge
$C_G(\varphi \to E_G\varphi) \to (\varphi \to C_G\varphi)$	induction of common knowledge

Table 1: Axioms of PAC

3.1 Example 1

First, we take the formula $\neg K_a p \vee K_a K_b K_a p$ in order to show how the rule (SR-5*) works.

	O			**P**	
				$\epsilon\mid 0$: $\neg K_a p \vee K_a K_b K_a p$	0
	$n := 1$			$m := 2$	
1	$\epsilon\mid 0$: $?_\vee$	0		$\epsilon\mid 0$: $\neg K_a p$	2
3	$\epsilon\mid 0$: $K_a p$	2		$\otimes$	
				$\epsilon\mid 0$: $K_a K_b K_a p$	4
5	$\epsilon\mid 0$: $?^a_1$	4		$\epsilon\mid 0_a 1$: $K_b K_a p$	6
7	$\epsilon\mid 0_a 1$: $?^b_2$	6		$\epsilon\mid 0_a 1_b 2$: $K_a p$	8
9	$\epsilon\mid 0_a 1_b 2$: $?^a_3$	8		$--$	
11	$\epsilon\mid 0_a 1$: p		3	$\epsilon\mid 0$: $?^a_1$	10

***Explanations*:** at move 1, **O** challenges the disjunction that **P** defends move 2. Move 3: **O** challenges the negation from move 2. **P** has no defence but he can either challenge move 3 or thanks to his rank of repetition ($m := 2$), he can repeat his defence, which he does at move 4. Move 5: in order to challenge the K operator from move 4, **O** chooses the label 1 for the agent a, from this label he chooses the label 2 for the agent b (move 7); and at move 9, **O** chooses the label 3 for a. Consequently, **P** should utter his defence in the label $\epsilon|0_a 1_b 2_a 3$.

Owing to (SR-2), **P** cannot answer for the moment, he only can counter-attack move 3 to have p in the appropriate label. The problem is that the rule (SR-5*) forbids this path. From the label $\epsilon|0$, **P** can choose for the agent a any label in the sequence $0_a1_b2_a3$ if and only if the label is not new for a (that is the case here for the label 3 introduced move 9) and if there is no b between the label where the choice is produced and the chosen label, that is between 0 and 3 such that $a \neq b$. But there is a b between 1 and 2 which breaks off the sequence 0...3 for a. So, **P** cannot reuse it and due to (SR-3) **O** wins the dialogue at move 11.

Thanks to this example, it clearly appears that in order to win **P** must violate the S5 closure of the agent a to mix it up with S5 closure of agent b.

3.2 Example 2

The next example presents the axiom called *mix of common knowledge* $C_Gp \rightarrow (p \wedge E_GC_Gp)$. We choose that axiom because it requires to use all epistemic particle rules of **DEMAL**. This axiom is translated in its disjunctive form: $\neg C_Gp \vee (p \wedge E_GC_Gp)$.

	O			P	
				$\epsilon\|0$:$\neg C_Gp \vee (p \wedge E_GC_Gp)$	0
	$n := 1$			$m := 2$	
1	$\epsilon\|0$: $?_\vee$	0		$\epsilon\|0$:$\neg C_Gp$	2
3	$\epsilon\|0$: C_Gp	2		$\otimes$	
				$\epsilon\|0$: $p \wedge E_GC_Gp$	4
5	$\epsilon\|0$: $?_{\wedge 2}$	4		$\epsilon\|0$: E_GC_Gp	6
7	$\epsilon\|0$: ? $a \in G$	6		$\epsilon\|0$: K_aC_Gp	8
9	$\epsilon\|0$: $?^a_1$	8		$\epsilon\|0_a1$: C_Gp	10
11	$\epsilon\|0_a1$: ? $\langle b, c\rangle \in G^*$	10		$\epsilon\|0_a1$: K_bK_cp	12
13	$\epsilon\|0_a1$: $?^b_2$	12		$\epsilon\|0_a1_b2$: K_cp	14
15	$\epsilon\|0_a1_b2$: $?^c_3$	14		$\epsilon\|0_a1_b2_c3$: p	24
17	$\epsilon\|0$: $K_aK_bK_cp$		3	$\epsilon\|0$: ? $\langle a, b, c\rangle \in G^*$	16
19	$\epsilon\|0_a1$: K_bK_cp		17	$\epsilon\|0$: $?^a_1$	18
21	$\epsilon\|0_a1_b2$: K_cp		19	$\epsilon\|0_a1$: $?^b_2$	20
23	$\epsilon\|0_a1_b2_c3$: p		21	$\epsilon\|0_a1_b2$: $?^c_3$	22

***Explanations*:** moves 1 to 3 are similar to the example 1. Move 5, in accordance to the particle rule of the conjunction and his rank of repetition 1

($n := 1$), **O** has the choice in his challenge but he can play only once. Let us consider that he chooses the second conjunct (we shall consider the other case below). Then **P** utters $E_G C_G p$ in the label $\epsilon|0$ move 6. Thereafter (moves 7-9), **O** chooses the agent a and then the label 1 for a. Obviously, **P** does the corresponding defence uttering $C_G p$ in that label. At move 11, **O** arbitrary chooses the sequence $\langle b, c \rangle$ of agents. According to the particle rule for the common knowledge operator, **P** utters the corresponding embedded sequence of individual knowledge operators. Moves 11 and 13: **O** respectively chooses the label 2 and 3 for agents b and c. Doing so, he compels **P** to utter p in the label $\epsilon|0_a 1_b 2_c 3$, but due to (SR-2), **P** cannot produce that defence for the moment. Hence, **P** counter-attacks move 3 with the sequence $\langle a, b, c \rangle$ of agents at move 16. From moves 18 to 22, he reuses labels chosen by **O** (moves 9, 13 and 15). In his turn, at move 22, **P** forces **O** to utter p in the label $\epsilon|0_a 1_b 2_c 3$. Now, **P** is allowed to produce his defence against the challenge that has not been defended yet. Then he utters $\epsilon|0_a 1_b 2_c 3$: p at move 24 and wins.

Now let us consider that instead of the second conjunct, **O** has chosen the first. It provides the dialogue below:

	O			**P**	
				$\epsilon\|0{:}\neg C_G p \vee (p \wedge E_G C_G p)$	0
	$n := 1$			$m := 2$	
1	$\epsilon\|0$: $?_\vee$	0		$\epsilon\|0{:}\neg C_G p$	2
3	$\epsilon\|0$: $C_G p$	2		$\otimes$	
				$\epsilon\|0$: $p \wedge E_G C_G p$	4
5	$\epsilon\|0$: $?_{\wedge 1}$	4		$\epsilon\|0$: p	8
7	$\epsilon\|0$: p		3	$\epsilon\|0$: $? \langle\,\rangle \in G^*$	6

Explanations: In the course of that dialogue, **O** forces **P** to utter p in the label $\epsilon|0$. For the same reason as before, **P** cannot defend yet. But at move 6, **P** challenges the move 3 choosing an empty sequence of agent. Doing this, he reverses the charge and then it is **O**'s turn to utter p in the label $\epsilon|0$. Thereafter, **P** can produce his defence in the move 8. It is **O**'s turn to play but there is no other move that he can do. So according to (SR-3) the game is over and **P** wins this dialogue.

Consequently whatever **O**'s choice was on the conjunction, **P** is able to win.

3.3 Example 3

van Ditmarsch (2010) presents an interesting understanding of *Moore Sentence*. The special feature of such sentences is to become false just by their own announcement. Such propositions are true before their announcement but false after that. We now consider such an example in order to test the ability of our rules to deal with case of what is commonly named *unsuccessful update*.[15] Moreover sentences like *Moore Sentence* can help the reader to correctly understand how to play with **DEMAL**'s rules, in particular (SR-A.1).

	O			**P**	
				$\epsilon\mid 0$: $[p \wedge \neg K_a p]\,(p \wedge \neg K_a p)$	0
	$n := 1$			$m := 2$	
1	$\epsilon\mid 0$: $?_{[\ ]}$	0		$\epsilon\mid 0 : \neg(p \wedge \neg K_a p)$	2
3	$\epsilon\mid 0 : p \wedge \neg K_a p$	2		$\otimes$	
5	$\epsilon\mid 0$: p		3	$\epsilon\mid 0$: $?_{\wedge_1}$	4
7	$\epsilon\mid 0$: $\neg K_a p$		3	$\epsilon\mid 0$: $?_{\wedge_2}$	6
	$\otimes$		7	$\epsilon\mid 0$: $K_a p$	8
9	$\epsilon\mid 0$: $?^a_1$	8			
				$p \wedge \neg K_a p\mid 0$: $p \wedge \neg K_a p$	10
11	$p \wedge \neg K_a p\mid 0$: $?_{\wedge_2}$	10		$p \wedge \neg K_a p\mid 0$: $\neg K_a p$	12
13	$p \wedge \neg K_a p\mid 0$: $K_a p$	12		$\otimes$	
			13	$p \wedge \neg K_a p\mid 0$: $?^a_1$	14
15	$\epsilon\mid 0_a 1$: $!_{(p \wedge \neg K_a p)}$	14		$\epsilon\mid 0_a 1$: $p \wedge \neg K_a p$	16
17	$\epsilon\mid 0_a 1$: $?_{\wedge_1}$	16		$--$	

***Explanations*:** move 1, **O** challenges the announcement and **P** chooses to reject to commit himself in the announcement. Move 3: **O** challenges the negation. At moves 4 and 6, thanks to his rank of repetition ($m := 2$), **P** plays twice over his challenge on the conjunction of move 3 and he obtains both conjuncts. Move 8: **P** challenges the negation of move 7. As **O** has no possible defence, he challenges move 8 by choosing the label 1, but owing to (SR-2) **P** cannot produce the corresponding defence for the moment. Hence at move 10, he uses his rank of repetition ($m := 2$) in order to change his defence from the challenge on announcement (challenged by **O** at move 1 and already defended by **P** at move 2). Moves 11 and 13 are respectively about conjunction and negation. Move 14: in the label in which both player are committed in

[15]See van Ditmarsch and Kooi (2006) for various definitions on *successful* formula and update.

the announcement, **P** chooses to challenge move 13 with the label 1 in order to produce the corresponding defence of the challenge move 9. But **O** has never uttered the announcement in the label 0_a1. Then at move 15, **O** uses the rule (SR-A.1) and compels **P** to utter the announcement in the label 0_a1, what he does at move 16. Move 17, **O** challenges the conjunction of move 16, but once again due to (SR-2), **P** is not able to produce this defence. And as he has no other possible move, he loses and **O** is the winner of the dialogue.

Hence, in **DEMAL**, **P** cannot win a dialogue which states that a Moore Sentence is successful.

4 Conclusion and further developments

4.1 Choice, Announcement and Commitment

As we have tried to underline, the dialogical framework offers an alternative understanding of epistemic operators. On the one hand, all knowledge operators are interpreted in terms of choice: a label for K-operator, an agent for E-operator and a sequence of agents for C-operator. On the other hand, announcement operators represent acts of commitment, either a concession or an assertion taken in charge by the arguers.

Thanks to the dialogical framework the meaning of these operators is dynamically given through the game of challenges and defences between the two arguers of the dialogue. They are defined in their use in an argumentative context, not from an abstract point of view in terms of truth-conditions. The construction of the epistemic operators and the difference between the public announcement and its dual appear in new lights. Through the burden of choice, all epistemic operators lead to a boolean proposition: common knowledge operator changes in a sequence of embedded individual knowledge operator; sharing knowledge operator becomes an individual knowledge operator and the individual knowledge operator simply gives the proposition in a chosen label. Moreover, as it is shown, announcement operators are not operations on models but are acts of commitment of the players of the dialogue. That gives us a new light on what public announcement can be, and we hope to get closer to Plaza (1989)'s intuition about exchange in communicative contexts.

4.2 Further Developments

Therefore, this dialogical reconstruction represents a starting point of further works and researches. As previously said, a soundness and completeness proof of **DEMAL** is given in Magnier and de Lima (2012). One interesting field of further research is found in direction of Yamada's works and principally consists in the exploration in the possible extension of **DEMAL** with an operator that deletes commitments that occur in course of a dialogue.

Another interesting issue using the logic **DEMAL** is on Leibniz's studies about conditional law. Thiercelin (2010) and Thiercelin (2011) underline the fact that Leibniz provides very precise criteria for the juridical modality of the conditional law. The restrictions that Leibniz provides on conditional for his studies in law seem to be (and they are!) very close to announcement operator. However surprising it may seem, most of his criteria exactly correspond to how a public announcement is understood in **DEMAL**. Indeed, it is possible to interpret a sentence like *B is suspended to A* (which corresponds to the juridical modality of the conditional law) as $[A]B$ in the course of a dialogue between what Thiercelin named the 'conditioner' and the 'conditionee'[16] that can respectively be understood as the arguer who utters an announcement and the other one who challenges it. The first step of this exploration is already done in a jointed work in Magnier and Rahman (2012).

Acknowledgements. Special thanks are addressed to all those who have supported and encouraged this field of research, in particular T. de Lima for his useful help and feedback, to V. Fiutek for her many advices and to anonymous referees for their encouraging comments.

Bibliography

Balbiani, P., van Ditmarsch, H., Herzig, A., and de Lima, T., 2010. Tableaux for Public Announcement Logic. *Journal of Logic and Computation*, 20:55–76.

Fagin, R., Halpern, J. Y., Moses, Y., and Vardi, M. Y., 1995. *Reasoning about knowledge*. MIT Press, Cambridge, Massachusetts.

Fontaine, M. and Redmond, J., 2008. *Logique Dialogique : une introduction - Première partie 1 : Méthode de la dialogique : régles et exercices*, volume 5 of *col. Cahiers de logique et Epistémologie*. College Publications, London.

Gerbrandy, J., 1999. *Bisimulation on Planet Kripke*. PhD thesis, University of Amsterdam, ILLC. URL http://www.di.unito.it/~gerbrand/papers/DS-1999-01.text.pdf.

[16]see Thiercelin (2011) pp. 251-253.

Hendricks, V. F., 2005. *Mainstream and Formal Epistemology*. Cambridge University Press.

Hendricks, V. F. and Symons, J., 2010. Where's the bridge? epistemology and epistemic logic. *Philosophical Studies*, 128:137–167.

Keiff, L., 2009. Dialogical logic. URL {http://plato.stanford.edu/entries/logic-dialogical/}.

Mackenzie, J., 1985. No logic before friday. *Synthese*, 63(3):329–341.

Magnier, S. and de Lima, T. A Soundness & Completeness Proof on Dialogues and Dynamic Epistemic Logic. 2012 - forthcoming.

Magnier, S. and Rahman, S. Leibniz's Notion of Conditional Right and the Dynamics of Public Announcement. *Logos & Episteme*. 2012 - forthcoming.

Plaza, J., 1989. Logics of public announcements. In Pfeifer, M. S., Hadzikadic, M., and Ras, Z. W. (ed.), *Proceedings 4th International Symposium on Methodologies for Intelligent Systems*, pages 201–216.

Rahman, S. and Keiff, L., 2005. On how to be a dialogician. In Vanderveken, D. (ed.), *Logic, Thought and Action*, pages 359–408. Springer, Dordrecht.

Rahman, S. and Rückert, H., 1999. Dialogische Modallogik (für T, B, S4, und S5). *Logique et Analyse*, 167(168):243–282.

Rebuschi, M., 2009. Implicit vs. explicit knowledge in dialogical logic. In Majer, O., Pietarinen, A. V., and Tulenheimo, T. (ed.), *Games: Unifying Logic, Language, and Philosophy*, pages 229–246. Springer.

Rebuschi, M. and Lihoreau, F., 2008. Contextual epistemic logic. In Dégremont, C., Keiff, L., and Rückert, H. (ed.), *Dialogues, Logics and Other Strange Things. Essays in Honour of Shahid Rahman*, pages 305–335. College Publications, London.

Thiercelin, A. Conditions, conditionnels, droits conditionnels : l'articulation du jeune Leibniz. unpublished manuscript, 2010.

Thiercelin, A., 2011. Epistemic and practical aspects of conditionals in leibniz's legal theory of conditions. In Gabbay, D., Canivez, P., Rahman, S., and Thiercelin, A. (ed.), *Approaches to Legal Rationality*, volume 20 of *Logic, Epistemology, and the Unity of Science (LEUS)*, pages 203–215. Springer, Dordrecht.

van Ditmarsch, H. Dynamic epistemic logic, the Moore sentence, and the Fitch paradox, 2010. URL http://arche-wiki.st-and.ac.uk/~ahwiki/pub/Arche/ArcheLogicGroup10Jun2009/slidesMooreFitch.pdf.

van Ditmarsch, H., van der Hoek, W., and Kooi, B., 2007. *Dynamic Epistemic Logic*, volume 337 of *Synthese Library : Studies in Epistemology, Logic, Methodology, and Philosophy of Science*. Springer, Dordrecht.

van Ditmarsch, H. and Kooi, B., 2006. The secret of my success. *Synthese*, 153(2): 339–339.

Walton, D. N. and Krabbe, E. C. W., 1995. *Commitment in Dialogue: Basic Concepts of Interpersonal Reasoning*. Albany State University of New York Press, New York.