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Abstract

This paper suggests a new explanation for changes in economic and population growth with a long run perspective, emphasizing the role of land in the development process. Starting from a pre-industrialization state called the "Malthusian regime", land and labor are the main production factors. The size of population is limited by the quantity of land available for households and by incomes. Technical progress driven by a "Boserupian effect" may push the economy towards a take-off regime. In this regime, capital accumulation begins and a "learning-by-doing" effect in production takes over from the "Boserupian effect". If this effect is strong enough, the economy can reach an "ultimate growth regime". In the different phases, land plays a crucial role.

Keywords: endogenous fertility, land, endogenous growth.
JEL Classification: D9, J13, O11, R21.

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1 Introduction

This paper is a contribution to the unified growth theory emphasizing the role of land and technological progress in economic and population growth. In a long run perspective, land seems a very important variable in the growth process, that deserves a particular study. Our approach is particularly related to two recent articles that have made a breakthrough on the theory of population evolution and growth: Galor and Weil (2000) and Hansen and Prescott (2002).

Galor and Weil (2000) develop a growth model that may explain the joint historical dynamics of education, population, and technology. They are able to reproduce changes in economic growth and population through three regimes: Malthusian, Post-Malthusian, and Modern Growth. Our approach differs from theirs in two key ways. First, land plays a limited role in their model: they assume that the return on land is zero and that the ownership of land is public. Secondly, their model allows population to increase with no bound and population density is not a brake on population growth. We depart from Galor and Weil (2000) by including a true land market with endogenous rent and prices. This true market, combined with a congestion effect on the use of land, induces an upper bound on population size.

Hansen and Prescott (2002) give an explanation of fertility behaviors during the industrialization process that emphasizes the role of land. This process is due to the substitution of capital to land in production, driven by biased technical progress in favor of the less land-intensive technology. In their story, population growth is based on an inverse U-shaped functional form of consumption inspired by Malthus (1798). Their model includes a true land market, but both sectors have technologies based on specific exogenous technical progress. Our approach complements Hansen and Prescott (2002) in two ways. First, we introduce endogenous fertility behaviors. The fertility decisions depend on different parameters of cost including a cost in time for parents and a housing cost related to the price of land. Secondly, we also depart from Hansen and Prescott (2002) by incorporating endogenous technical progress. This technical progress results from the increase in population density, that stimulates innovations, and from learning by doing. It induces a substitution of capital to land as in Hansen and Prescott (2002).

Our approach can also be related to Kremer (1993) who developed a model emphasizing interactions between technology and population. This model leads to a testable law of population dynamics that is not rejected by the data on a very long historical period. Our model develops the micro-foundations of behaviors that underlie the interactions between technology and population. This allow us to generate a demographic transition without

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assuming that the population growth rate increases with the level of population at low levels of income and decreases with the level of population at high levels of income.

We develop an overlapping generations model in which fertility is endogenous. The utility of the parents is a function of good consumptions, of the number of their children, and of the consumption of a fixed asset: land. Each child implies a financial cost and induces a congestion effect on the utility of land. In our analysis, land can be used both as a production factor and as housing services for households. As a production factor, land is an income source for households. Under the form of housing services, land gives utility to households. Moreover, as the demand for housing services depends on the number of children, land is also related to fertility behaviors.

To complement our model we introduce two types of survival probabilities: a child survival rate and an adult survival rate. An increase in the child survival rate decreases the cost of each surviving child. A rise in adult survival rates increases the propensity to save of households. As shown in Aghion *et al.* (2010), improvement in life expectancy has a significantly positive impact on per capita GDP growth.

The production technology uses three factors: capital, land, and labor. The productivity of capital benefits from technical progress. Technical progress is driven by two effects: a "Boserupian effect" and a "learning-by-doing effect". The first effect follows Boserup (1965), for whom the density of population may stimulate the incentives to innovate, in order to increase productivity. The second effect is inspired by the Romer (1986) model: the knowledge that was acquired in production in the past increases current productivity. As in Romer (1986), this knowledge can be measured by a proxy variable which is the capital stock.

The economy in our model experiences different stages of development as suggested by Rostow (1959). The analysis firstly focuses on a country starting from a pre-industrialization state. Land and labor are the main production factors. The size of population is limited by the quantity of land available for households and by incomes. This pre-industrialization state is called the "Malthusian regime".

During this phase, some innovations can appear, driven by the "Boserupian effect". If this technical progress is marked enough, the economy can jump out of this Malthusian regime, to undergo a take-off phase. But in the converse case, the economy remains trapped in the stationary "Malthusian regime".

In the take-off regime, the economy begins to accumulate physical capital. The role of land in production becomes less important. A "learning-by-

doing" effect in production takes over from the "Boserupian effect". Productivity increases as incomes and population rise. If this "learning-by-doing" effect is strong enough, the economy can reach an "ultimate growth regime".

In the "ultimate growth regime", the economy grows at a constant positive rate. Population converges towards a constant size, as its expansion is limited by land.

In the different phases, land plays a crucial role.[1] In the "Malthusian regime", a high population density gives incentives to innovate by the "Boserup effect". Therefore, starting with a lower endowment of land or with a higher population size allows a country to reach the take-off phase earlier. In the second phase, when the "learning-by-doing" effect becomes the engine of growth, the size of land has a positive effect on development and thus on the possibility of reaching the third phase. For given technological parameters, a minimum endowment of land is required to reach the ultimate growth regime associated with a positive growth rate. In the third phase, the value of the long run growth increases with the land endowment. The interpretation of this result is that our production technology exhibits the usual property of a scale effect, as in many endogenous growth models. As returns to scale are increasing, the size of population has a positive effect on the long run growth rate. And population size is determined by land endowment in our model.

Mortality rates also play a key role in take-off. Mortality is introduced in the model through two survival rates: the survival rate of children and the survival rate of adults. An increase in the survival rate of children reduces the cost of a surviving child. An increase in the survival rate of adults increases their propensity to save, and thus favours capital accumulation. Historically, the Malthusian demographic regime had been characterized by high levels of both fertility and mortality. A decrease in mortality rates can induce both transitions: from the Malthusian to the take-off regime and from the take-off to the ultimate growth regime.

Our analysis is related to several strands of literature. First, different authors have stressed the importance of an unbalanced growth process. Galor and Moav (2004) develop a growth theory that describes the replacement of physical capital accumulation by human capital accumulation as a prime engine of growth along the process of development. Kongsamut *et al.* (2001)

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[1]Allowing for capital accumulation and property rights over land considerably complicates the model, compared to Galor and Weil (2000). The quantity of capital has to be set to equalize its marginal product to the equilibrium interest rate, whereas the price of land has to follow a path such that the total return on land (rent plus net price appreciation) is also equal to the interest rate.

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propose a model of unbalanced growth, in which the growth process leads to a massive reallocation of labor from agriculture to manufacturing and services.

Secondly, other papers give a crucial role to land in the growth process. Galor *et al.* (2009) suggest that inequality in the distribution of land ownership may postpone or prevent take-off. Landowners affect the political process and postpone the implementation of education. Brunt and Garcia-Penalosa (2010) study the interactions between industrialization and urbanization. They point out a new mechanism that could drive technological change: the population density in cities may trigger the creation and diffusion of knowledge. This paper provides a new interpretation of the "Boserup effect" introduced in our framework: a high density of population leads to more innovation and technological progress.

Section Two presents the model. Section Three analyzes the dynamics of the intertemporal equilibrium. Section Four shows how the dynamics allow us to isolate different phases in the development process. Section Five presents the results of simulations obtained through a calibrated model. Section Six concludes and Section Seven gives references. A last section is devoted to technical appendixes.

2   The Model

We develop a two-period overlapping-generations model *à la* Diamond (1965) where fertility is endogenous. In every period the economy produces a single homogenous good, using land, labor, and capital as inputs. The single good is used both for consumption and capital accumulation. Land is a fixed factor, that includes agricultural land, business building, and housing. Services of land may be used both by firms as an input in the production process and by households. For the sake of simplicity, its supply is assumed to be constant and exogenous. The number of units of labor is equal to the number of young people and thus determined by households' decisions in the preceding period.

2.1   The firm

Production occurs according to a constant-returns-to-scale technology that is subject to endogenous technological progress. The output produced at time t, Y_t, is

$$Y_t = F(A_t K_t, L_t, X_t) \tag{1}$$

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where K_t, L_t, and X_t are the quantities of capital, labor, and land employed in production at time t. A simple form is assumed for the theoretical model:

$$F(A_t K_t, L_t, X_t) = [\lambda A_t K_t + (1 - \lambda) X_t]^\alpha \, L_t^{1-\alpha} \qquad (2)$$

with $0 < \alpha < 1$, $0 < \lambda < 1$, and $A_t > 0$. A_t represents the endogenously determined technological level at time t. Technological progress does not apply to land but to capital. This is a shortcut to point out that technological progress is particularly capital-oriented. This is in line with Hansen and Prescott (2002) who assume that the technology using land as a production factor has a lower growth rate in technical progress. As technical progress increases, capital is substituted to land in the production process. Perfect substitution between capital and land is assumed in order to get some tractability of the model. This theoretical case can be viewed as a limit case of a more general Constant Elasticity Substitution production function

$$F(A_t K_t, L_t, X_t) = \left[\lambda (A_t K_t)^{1-\frac{1}{\varepsilon}} + (1 - \lambda) X_t^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon \alpha}{\varepsilon - 1}} L_t^{1-\alpha} \qquad (3)$$

that will be examined in the section on simulations.

The capital is fully depreciated in one period. The number of units of labor is determined by households' decisions in the preceding period regarding the number of their children. Households have property rights over land. The land used as an input by the firm is rented to households. The return on land has therefore to be computed and compared to its equilibrium price taken as given by the firm.

With the production technology (2), capital will be used in production, only if the technical progress A_t is high enough. As A_t increases, the demand for capital becomes positive, and the demand for land X_t decreases and may even cancel out. This lower bound on X_t could be set to any positive value with a small modification of the production function and has been kept to zero only for tractability. This last point is due to the assumption of perfect substitutability between capital and land which is convenient to allow some tractability of the model. With production (3) the demand for land remains positive, even if it may go to zero.

The firm maximizes its profit, taken the wage rate, the interest rate, the rent rate, and the level of technology as given.

First order conditions for the optimization problem are derived below. All markets are perfectly competitive. On the labor market the quantity of

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labor used in production L_t is equal to N_t the number of young households at period t.[2] Defining, $k_t \equiv \frac{K_t}{N_t}$ and $x_t \equiv \frac{X_t}{N_t}$, the competitive wage is:

$$w_t = (1 - \alpha) \left[\lambda(A_t k_t) + (1 - \lambda) x_t \right]^\alpha \qquad (4)$$

Marginal productivities of capital and land are respectively defined as:

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\begin{aligned}
R_t &\equiv \alpha \lambda A_t \left[\lambda(A_t k_t) + (1 - \lambda) x_t \right]^{\alpha - 1} &(5) \\
\chi_t &\equiv \alpha(1 - \lambda) \left[\lambda(A_t k_t) + (1 - \lambda) x_t \right]^{\alpha - 1} &(6)
\end{aligned}
$$

Define π_t as the competitive rent rate on land and ρ_t as the competitive gross return on savings. The solution of the optimization problem is the following. If for $k_t = 0$, $R_t = \alpha \lambda A_t \left[(1 - \lambda) x_t \right]^{\alpha - 1} < \rho_t$, the firm does not use any capital. If $\alpha \lambda A_t \left[(1 - \lambda) x_t \right]^{\alpha - 1} > \rho_t$, $k_t > 0$ at the equilibrium and $R_t = \rho_t$.

If for $x_t = 0$, $\chi_t = \alpha(1 - \lambda) \left[\lambda(A_t k_t) \right]^{\alpha - 1} < \pi_t$, the firm does not use land. If $\alpha(1 - \lambda) \left[\lambda(A_t k_t) \right]^{\alpha - 1} > \pi_t$, $x_t > 0$ at the equilibrium and $\chi_t = \pi_t$.

2.2 Technology

We assume the existence of two sources of technical progress. The first one, called the "Boserup effect", assumes that the density of population favors technical progress, as demographic pressure stimulates innovation. The second one, called the "Learning by doing effect", posits that the knowledge acquired in past production increases current productivity. This knowledge, as suggested by Romer (1986), is represented by the aggregate capital stock of the economy (in the spirit of AK models). Thus, technical progress is an increasing function of both population density and K_t.

$$A_t = g(\frac{N_t}{\overline{X}}, K_t) \qquad (7)$$

We use the particular function below:

$$A_t = \left[\mu \left(A_N \left(\frac{N_t}{\overline{X}} \right)^\beta \right)^{1 - \frac{1}{\nu}} + (1 - \mu) \left(A_K^{\frac{1}{\alpha}} K_t^{(\frac{1}{\alpha} - 1)} \right)^{1 - \frac{1}{\nu}} \right]^{\frac{\nu}{\nu - 1}} \qquad (8)$$

with $\nu > 1$ and $0 < \mu < 1$. $\overline{X}$ is the total amount of land available in the economy.

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[2]Each young agent provides one unit of labor. The old are retired.

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A_N and A_K are positive parameters. At the beginning of the growth process, the level of capital is low, and technological progress increases with the size of the population. Then, as the technological progress increases, the level of capital increases itself and magnifies the increase in the level of technology. As the size of the population stabilizes, technological progress is only driven by capital accumulation.

2.3   Households

In each period t a generation consists of N_t identical individuals. Members of generation t live with probability p_t for two periods and die with probability $(1 - p_t)$ at the end of the first period. They work in the first period and are retired during the second one if they survive. Members of generation t choose at date t consumption while young (c_t) and old (d_{t+1}), as well as the number of their children per adult (m_t), and their consumption of land (v_t). Only a fraction η_t of the children m_t survives.

2.3.1   The optimization program

The preferences of members of generation t are represented by the utility function

$$U(c_t, d_{t+1}, m_t, v_t) = \Gamma_1 \ln c_t + p_t \Gamma_2 \ln d_{t+1} + \Gamma_3 \ln \eta_t m_t + \Gamma_4 \ln(v_t - \xi \eta_t m_t) \quad (9)$$

where ξ is a positive parameter and $\Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 = 1$.

Households maximize their expected utility taking into account the probability of reaching the second period. One can define $\zeta_t \equiv v_t - \xi \eta_t m_t$ that measures the services of land per adult. It is increasing with the total amount of land per adult and decreasing with the number of surviving children per adult.

Each newborn child entails a rearing cost of $\phi^1 w_t$. Moreover, for each surviving child, an additional cost of $\phi^2 w_t$ is borne. The dependence with respect to w_t is consistent with the fact that the costs of rearing children are proportional to the standard of living of their parents. This may also be viewed as the cost of children in time. The total cost of children per adult is thus

$$(\phi^1 + \phi^2 \eta_t) w_t m_t$$

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The number of surviving children per adult is $m'_t \equiv \eta_t m_t$. The corresponding cost is $\phi_t w_t m'_t$ with

$$\phi_t = \frac{\phi^1}{\eta_t} + \phi^2$$

Members of generation t maximize their intertemporal utility function under the following budget constraints:

$$c_t + s_t + \phi w_t m'_t + \pi_t v_t = w_t \tag{10}$$

$$d_{t+1} = \frac{\rho_{t+1}}{p_t} s_t \tag{11}$$

where s_t is the amount of savings per young household and ρ_{t+1} is the expected gross rate of return on savings. The actual return is thus $\rho'_{t+1} \equiv \frac{\rho_{t+1}}{p_t}$ as the savings of the dead agents are redistributed to the surviving ones.

Note that using ζ_t (the services of land per adult), one can easily make clear the real cost of one surviving child $(\phi_t w_t + \xi \pi_t)$ which can be broken down as the sum of the cost in time and the cost in land. The intertemporal budget constraint may be rewritten as:

$$c_t + \frac{d_{t+1}}{\rho'_{t+1}} + (\phi_t w_t + \xi \pi_t) m'_t + \pi_t \zeta_t = w_t \tag{12}$$

First order conditions for the optimization problem lead to the following solutions:

$$c_t = \gamma_{1,t} w_t \tag{13}$$

$$s_t = \gamma_{2,t} w_t \tag{14}$$

$$d_{t+1} = \gamma_{2,t} w_t \rho'_{t+1} \tag{15}$$

$$m'_t = \frac{\gamma_{3,t} w_t}{(\phi_t w_t + \xi \pi_t)} \tag{16}$$

$$v_t = \frac{\xi \gamma_{3,t} w_t}{(\phi_t w_t + \xi \pi_t)} + \gamma_{4,t} \frac{w_t}{\pi_t} \tag{17}$$

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with

$$\gamma_{1,t} = \frac{\Gamma_1}{\Gamma_1 + p_t\Gamma_2 + \Gamma_3 + \Gamma_4} \tag{18}$$

$$\gamma_{2,t} = \frac{p_t\Gamma_2}{\Gamma_1 + p_t\Gamma_2 + \Gamma_3 + \Gamma_4} \tag{19}$$

$$\gamma_{3,t} = \frac{\Gamma_3}{\Gamma_1 + p_t\Gamma_2 + \Gamma_3 + \Gamma_4} \tag{20}$$

$$\gamma_{4,t} = \frac{\Gamma_4}{\Gamma_1 + p_t\Gamma_2 + \Gamma_3 + \Gamma_4} \tag{21}$$

At shown in equations (18), (19), (20), and (21), a rise in life expectancy (p_t) increases savings and the propensity to consume in the second period. It decreases first period consumption, fertility, and demand for land.

2.3.2 Population density

The number of young households at date $t + 1$ is by definition equal to:

$$N_{t+1} \equiv m'_t N_t \tag{22}$$

From now on, the lower case designates the upper case variable divided by the number of young households. For instance, $\overline{x}_t$ is defined as $\frac{X}{N_t}$ the quantity of land available per young living agent. The evolution of land per young alive can thus be described by the following equation:

$$\overline{x}_{t+1} = \frac{\overline{x}_t}{m'_t} \tag{23}$$

2.4 Market equilibrium

2.4.1 Land markets

Land has two prices: the rent rate π_t and the price for sale q_t. It is the rent rate that determines the allocation of land between firms and consumers. The price for sale determines the allocation of savings between capital and land (see below). The equilibrium on the rent market expressed per head of young household is:

$$v_t + x_t = \overline{x}_t \tag{24}$$

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2.4.2 Capital and financial markets

Total savings have to be shared between physical capital and land.

$$\gamma_{2,t} w_t = m'_t k_{t+1} + q_t \overline{x}_t \tag{25}$$

where k_{t+1} stands for the capital per young household at date $t + 1$. The amount of physical capital per young household available in the economy in $t + 1$ is thus depending on the value of land ($b_t \equiv q_t \overline{x}_t$).

At the equilibrium, savings are at least composed of land[3] and the return on savings is equal to the return on land:

$$\rho_{t+1} = \frac{q_{t+1} + \pi_{t+1}}{q_t} \tag{26}$$

If capital is used in $t + 1$, household savings have to be split into physical capital and land. Thus, the gross return on physical capital has to be equal to the return on land. The arbitrage condition is written as follows:

$$R_{t+1} = \frac{q_{t+1} + \pi_{t+1}}{q_t} \tag{27}$$

If there is no capital in $t + 1$, the arbitrage condition does not hold and $R_{t+1} < \frac{q_{t+1}+\pi_{t+1}}{q_t}$.

3 Intertemporal equilibrium

In this section we analyze the dynamics without capital before turning to dynamics with capital.

3.1 The dynamics without capital

We assume that at the beginning of the economy, there is no capital ($k_0 = 0$). As we have mentioned before, this phase occurs as long as the marginal productivity of capital is smaller than the return on land $\alpha \lambda A_t \left[(1 - \lambda)x_t\right]^{\alpha-1} < \rho_t$. For $K_t = 0$, equation (8) becomes:

$$A_t = \mu^{\frac{\nu}{\nu-1}} \left(\frac{A_N}{\overline{x}_t^{\beta}}\right) \tag{28}$$

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[3]This is true even if firms do not use any land, as consumers rent it for housing.

Thus, as long as $K_t = 0$, A_t increases only by the Boserup effect which depends only on the density of population. The competitive wage is given by

$$w_t = (1 - \alpha) \left[(1 - \lambda) x_t \right]^{\alpha} \tag{29}$$

and marginal productivity of land is

$$\chi_t \equiv \alpha(1 - \lambda) \left[(1 - \lambda) x_t \right]^{\alpha - 1} \tag{30}$$

The firm takes the price of land as given, thus as long as the firm uses land,

$$\chi_t = \pi_t \tag{31}$$

Equations (29), (30), and (31) are used to obtain new expressions for m_t' and v_t from equations (16) and (17).

$$m_t' = \frac{\gamma_{3,t}(1 - \alpha) x_t}{(\phi_t(1 - \alpha) x_t + \xi \alpha)} \tag{32}$$

$$v_t = \frac{\xi \gamma_{3,t}(1 - \alpha) x_t}{(\phi_t(1 - \alpha) x_t + \xi \alpha)} + \gamma_{4,t} \frac{(1 - \alpha)}{\alpha} x_t \tag{33}$$

Using v_t in (24), we obtain:

$$\left(1 + \gamma_{4,t} \frac{(1 - \alpha)}{\alpha} \right) x_t + \frac{\xi \gamma_{3,t}(1 - \alpha) x_t}{\phi_t(1 - \alpha) x_t + \xi \alpha} = \bar{x}_t \tag{34}$$

Using m_t in (23), we obtain:

$$\bar{x}_{t+1} = \frac{\phi_t(1 - \alpha) x_t + \xi \alpha}{\gamma_{3,t}(1 - \alpha) x_t} \bar{x}_t \tag{35}$$

Rearranging (34) and (35), the dynamics of x_t is:

$$
\begin{aligned}
& \left(1 + \gamma_{4,t} \frac{(1 - \alpha)}{\alpha} \right) \frac{\phi_t}{\gamma_{3,t}} x_t \\
= \; & \left(1 + \gamma_{4,t+1} \frac{(1 - \alpha)}{\alpha} \right) x_{t+1} + \frac{\xi \gamma_{3,t+1}(1 - \alpha) x_{t+1}}{\phi_{t+1}(1 - \alpha) x_{t+1} + \xi \alpha} \\
& - \frac{\xi}{\gamma_{3,t}} \left(\gamma_{3,t} + \gamma_{4,t} + \frac{\alpha}{(1 - \alpha)} \right)
\end{aligned}
\tag{36}
$$

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To characterize the dynamics of the economy, we first assume that the survival probabilities are constant ($\eta_t = \eta$ and $p_t = p$). Thus, ϕ_t, $\gamma_{1,t}$, $\gamma_{2,t}$, $\gamma_{3,t}$, and $\gamma_{4,t}$ are also constant and respectively equal to ϕ, γ_1, γ_2, γ_3, and γ_4. Equation (36) defines the dynamics of x_t of the form $x_t = f(x_{t+1})$. f is a monotonic increasing function, with $f(0) < 0$. Under the assumption of $\gamma_3 > \phi$, $f'(x) > 1$, and there exists a unique positive steady state computed from (35).

$$x^* = \frac{\xi\alpha}{1-\alpha}\frac{1}{\gamma_3 - \phi} \tag{37}$$

Following theses properties, the dynamics of x_t are determinate and monotonically converge towards x^*.

If one assumes a low size of population at the beginning of the economy, $\bar{x}_0$ is high. From equation (34), it implies that x_0 is high and greater than x^*, therefore x_t, the amount of land per young household used as an input, monotonically decreases from x_0 to x^*.

The stationary state is characterized by the following set of equations. Using (37) in (32), (33), (24), (29), (30), and (31), one gets m'^*, $\bar{x}^*$, w^*, and π^*.

$$m'^* = 1 \tag{38}$$

$$\bar{x}^* = \xi + \frac{\xi\alpha}{1-\alpha}\frac{1}{\gamma_3 - \phi}\left(1 + \gamma_4\frac{(1-\alpha)}{\alpha}\right) \tag{39}$$

$$w^* = (1-\alpha)\left[(1-\lambda)\frac{\xi\alpha}{1-\alpha}\frac{1}{\gamma_3 - \phi}\right]^\alpha \tag{40}$$

$$\pi^* = \alpha(1-\lambda)\left[(1-\lambda)\frac{\xi\alpha}{1-\alpha}\frac{1}{\gamma_3 - \phi}\right]^{\alpha-1} \tag{41}$$

As $\bar{x}_t \equiv \frac{\overline{X}}{N_t}$, we also have

$$N^* = \frac{\overline{X}}{\bar{x}^*} \tag{42}$$

Note that $m'^* = 1$ means that the growth rate of population is null as each adult has only one surviving child. In this regime, there is no technical progress. Fertility levels can increase only if wages increase (cf. equation (16)). As global income is bounded by the decreasing returns on land, and utility increases with the services of land per adult, in the long run output

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and population are stationary. Note that at the stationary state, population size is proportional to the global endowment in land.

This stationary state exists only if the marginal productivity of capital is smaller than the return on land (i.e. for each t, $\alpha \lambda A_t \left[(1-\lambda)x_t\right]^{\alpha-1} < \rho_t$), otherwise the firm starts to accumulate physical capital and we switch to a new regime with capital.

We characterize this condition at the stationary state in order to determine the range of parameters for which the economy stays at the stationary state without capital. From (26) and (23), ρ_{t+1} can be expressed as

$$\rho_{t+1} = \frac{q_{t+1}\bar{x}_{t+1} + \pi_{t+1}\bar{x}_{t+1}}{q_t \bar{x}_t} m'_t \tag{43}$$

From (25), $q_t \bar{x}_t = \gamma_2 w_t$. At the stationary state, $m'_t = 1$ and from (29), (30), and (31),

$$\rho = 1 + \frac{\alpha}{\gamma_2(1-\alpha)}\frac{\bar{x}^*}{x^*}$$

Finally,

$$\rho^* = \frac{1}{\gamma_2}\left(\frac{1}{(1-\alpha)} - \gamma_1 - \phi\right)$$

The condition $\alpha \lambda A \left[(1-\lambda)x\right]^{\alpha-1} < \rho$, can thus be written as:

$$\alpha\lambda\mu^{\frac{\nu}{\nu-1}} A_N \times \left(\xi + \frac{\xi\alpha}{1-\alpha}\frac{1}{\gamma_3 - \phi}\left(1 + \gamma_4\frac{(1-\alpha)}{\alpha}\right)\right)^{-\beta}$$
$$\times \left[(1-\lambda)\left(\frac{\xi\alpha}{1-\alpha}\frac{1}{\gamma_3 - \phi}\right)\right]^{\alpha-1}$$
$$-\frac{1}{\gamma_2}\left(\frac{1}{(1-\alpha)} - \gamma_1 - \phi\right) < 0 \tag{44}$$

The condition under which the economy stays at the stationary state without capital depends on the relative value of fourteen parameters determining: the utility (γ_1, γ_2, γ_3, γ_4, ξ), the cost of children (ϕ), and the production technology (α, λ, μ, ν, A_N, β) with the cost of children ϕ depending on the survival probability of young agents η and on the specific costs of children ϕ^1 and ϕ^2 and the γ_i's depending on the adult survival probability p.

If we consider that all parameters are fixed except A_N, η, and p, this condition (44) can be rewritten as

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$$H(A_N, \eta, p) < 0 \tag{45}$$

This condition defines a threshold level for A_N that is denoted by $\bar{A}_N$ such that $H(A_N, 1, 1) = 0$. Our analysis has produced the following result:

Proposition 1 • *If $A_N < \bar{A}_N$, the economy converges toward a Malthusian underdeveloped equilibrium without capital.*

• *If $A_N > \bar{A}_N$, it is still the case if η or p are low enough. If η and p are high enough, the economy switches to a regime with capital in a finite time. As technical progress increases, capital productivity reaches a level such that investing in capital becomes profitable.*

Proof. First we prove that if η is low enough, condition (44) is true. $\phi = \frac{\phi^1}{\eta} + \phi^2$ depends on the infant survival rate parameter η. As $\phi < \gamma_3$, there exists a lower bound on η: $\eta_{\inf} = \frac{\phi^1}{\gamma_3 - \phi^2}$ and as η goes to $\eta_{\inf}$, $H(A_N, \eta, p) \to -\frac{1}{\gamma_2}\left(\frac{1}{(1-\alpha)} - \gamma_1 - \gamma_3\right) < 0$. For small values of η (η close to $\eta_{\inf}$), the economy converges toward a Malthusian underdeveloped equilibrium without capital.

Secondly, we prove that if p is low enough, condition (44) is true. γ_1, γ_3, and γ_4 are decreasing with p and γ_2 increases from 0 to Γ_2 when p increases from 0 to 1. If p goes to 0, $H(A_N, \eta, p) \to -\infty$. For small values of p, the economy converges toward a Malthusian underdeveloped equilibrium without capital.

Third, if p and η are high enough, the economy switches to a regime with capital. Indeed, as $H(A_N, 1, 1) > 0$, the Malthusian stationary state (without capital) no longer exists when p and η are sufficiently close to one. ∎

Three parameters may explain a switch from a Malthusian regime without capital to a regime with capital: a technology shock on A_N, or an increase in the survival parameters η and p.

An increase of A_N has a direct effect on the marginal productivity of capital.

An increase of η induces an increase in population size that increases capital productivity by the Boserup effect. As population size increases, the marginal productivity of land also increases as the quantity of land available for production is lower. The first effect always dominates, for η high enough. Thus, a reduction of infant mortality may push the economy toward development.

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An increase of p (thus a decrease in the adult mortality rate) also induces a switch to a regime with capital. The main effect of an increase in p is an increase in savings. When p is small, the amount of savings is very low. As land is the only asset in which savings can be invested its price is very low. Hence, the return on land is quite high. When p increases the return on land decreases. The return on land cannot be higher than the return on capital any longer when p is high enough. The economy starts to accumulate capital. Thus, a reduction of adult mortality may push the economy towards development.

3.2 The dynamics with capital

We first study the existence of a long run path of endogenous growth, and secondly how this growth rate depends on various parameters.

3.2.1 Existence of the long run growth rate

What are the dynamics of the economy if (44) is not true any longer? As the capital stock increases, the economy benefits from technical progress driven by two effects: the previous Boserup effect and the learning by doing effect. The analysis of the dynamics will show the following properties: depending on the value of A_K that governs the learning by doing effect, the economy converges either towards a steady state with a constant value for capital, or towards a long run state with a constant endogenous growth rate of capital. In both cases, land is only used by households, and not by the productive sector any more. In the extended model with the CES production function given by (3) simulated in Section 5, the use of land decreases along the growth process.

We start by proving a first result:

Lemma 1 *If (44), there does not exist a steady state with both capital and land used in production.*

Proof. See Appendix 1 ∎

The consequence of this lemma is that two types of long run equilibria can exist in this model. In both cases, no land is used in the production sector. Either the economy converges toward a steady state value of k_t, or k_t undergoes endogenous growth at a rate that tends to some constant value.

Lemma 2 *In the long run the dynamics of capital are characterized by*

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$$k_{t+1} = (\gamma_2 - B)(1-\alpha)\lambda^\alpha \left[\mu(\frac{A_N}{\overline{x}_\infty^\beta})^{1-\frac{1}{\nu}} + (1-\mu)\left(A_K^{\frac{1}{\alpha}} N_\infty^{(\frac{1}{\alpha}-1)} k_t^{(\frac{1}{\alpha}-1)}\right)^{1-\frac{1}{\nu}} \right]^{\frac{\alpha\nu}{\nu-1}} k_t^\alpha$$

with B, $\overline{x}_\infty$ and N_∞ depending on α, γ_2, γ_3, γ_4, $\bar{X}$, ξ and ϕ (equations (87), (8.2), and (80)).
Proof. See Appendix 2. ∎

Proposition 2 *Let us define $\bar{A}_K$ as*

$$\bar{A}_K = \frac{1}{(\gamma_2 - B)(1-\alpha)\lambda^\alpha(1-\mu)^{\frac{\alpha\nu}{\nu-1}} N_\infty^{1-\alpha}} \tag{46}$$

If $A_K < \bar{A}_K$, k_t converges in the long run toward a constant value; if $A_K > \bar{A}_K$, k_t increases at a positive rate in the long run, and the factor growth rate converges toward a constant value G^* with

$$G^* = (\gamma_2 - B)(1-\alpha)\lambda^\alpha(1-\mu)^{\frac{\alpha\nu}{\nu-1}} A_K N_\infty^{1-\alpha} \tag{47}$$

Proof. The proof is straightforward from Lemma 2. In the case $A_K > \bar{A}_K$, k_t increases endogenously with a positive growth rate. In the long run, as the Boserup effect becomes negligible with respect to the learning by doing term, in technical progress (A_t)

$$A_t = (1-\mu)^{\frac{\nu}{\nu-1}} A_K^{\frac{1}{\alpha}} K_t^{(\frac{1}{\alpha}-1)} \tag{48}$$

the growth factor of k_t tends to $G^* = (\gamma_2 - B)(1-\alpha)\lambda^\alpha(1-\mu)^{\frac{\alpha\nu}{\nu-1}} A_K N_\infty^{1-\alpha}$. ∎

3.2.2 Capital accumulation threshold characteristics

Note that $\bar{A}_K$ depends on

$$N_\infty = \frac{\bar{X}}{\xi} \frac{\gamma_3 - \phi}{\gamma_3 + \gamma_4 - \phi}$$

Thus, condition $A_K < \bar{A}_K$ depends on the size of land available in the economy $(\bar{X})$. If this size is too low, no endogenous growth can arise in the long run, as land determines an upper bound to the size of the population.

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3.2.3 The growth rate characteristics

As usual in endogenous growth models with learning by doing driving technical progress, the size of population has a positive effect on the growth rate. This effect is bounded here as population tends to be constant in the long run. This is due to the fixed factor, as in the long run the size of the population is limited by the quantity of land available.

As $N_\infty = \frac{\bar{X}}{\xi} \frac{\gamma_3 - \phi}{\gamma_3 + \gamma_4 - \phi}$, the ultimate growth rate increases with the quantity of land available $\bar{X}$. This is in line with Romer (1986), and more generally true of all endogenous growth models with a scale effect, where growth depends on the size of the population.

Growth (G^*) also depends on mortality rates as γ_2, B, and N_∞ depend on either or both η and p. The intuitions are the following.

An increase in η induces a decrease in ϕ. This has two consequences. Firstly, N_∞ decreases with ϕ: population size in the long run is higher when the cost of children is smaller.

Secondly, from (86), one gets

$$\gamma_2 - B = \frac{B}{B\frac{1-\alpha}{\alpha} + (\gamma_3 + \gamma_4 - \phi)\frac{1-\alpha}{\alpha}} \tag{49}$$

This expression shows that B decreases with ϕ. Indeed, if B increases with ϕ, the left hand side of (49) decreases with ϕ, when the right hand side increases, which is a contradiction. Thus $\gamma_2 - B$ increases with ϕ. Recall that γ_2 is the saving rate and that only $\gamma_2 - B$ is invested in capital, while B is invested in land. When the cost of children ϕ is smaller, the population size N_∞ in the long run is higher and the price of land is higher. Thus the share $\gamma_2 - B$ invested in capital is lower.

In the end, η has two antagonistic effects on G^*: a negative impact by $\gamma_2 - B$ and a positive by N_∞. But for plausible numerical values of the parameters the increase in η always induces an increase in G^*.

An increase in p induces an increase in γ_2, and a decrease in γ_1, γ_3, and γ_4. This has two consequences. Firstly, N_∞ decreases with p: the number of young households in the long run is smaller when the adult life expectancy increases. This is a consequence of the change in households' preferences in favor of second period consumption and to the detriment of other "goods" such as first period consumption, children, and land.

Secondly, $\gamma_2 - B$ increases with p. Indeed, γ_2 increases with p. If B decreases with p, the result is obtained. If B increases with p, from the right hand side of (49), as γ_3 and γ_4 decrease, $\gamma_2 - B$ increases. This is also a

consequence of the change in households' preferences: the increase in life expectancy induces a higher propensity to save in capital.

In the end, p has two antagonistic effects on G^*: a positive impact by $\gamma_2 - B$ and a negative by N_∞. But for plausible numerical values of the parameters the increase in p always induces an increase in G^*.

4 The three stages of economic development

Our framework allows us to describe three phases of economic development: the "Malthusian Regime", the "Take-off Regime", and the "Ultimate Growth Regime".

4.1 The first phase: the "Malthusian Regime"

In the "Malthusian Regime", land and labor are the only production factors. There is no capital. Technical progress may increase with the density of population. This "Boserup effect" emphasizes that when land becomes scarcer, people try to improve technology. Unfortunately, as long as technical progress is not great enough, it is not incorporated in the production process. Thus, if A_N is not high enough (equation (44)), the economy converges towards a steady state in the "Malthusian Regime" with low population, and no capital. The size of land does not affect the consumption levels per agent, as the population is proportional to the land available and as the technology exhibits constant return to scale.

If A_N is high enough, the economy starts to accumulate capital, there is no stationary state without capital, and the economy evolves through the second phase.

4.2 The second phase: The take-off

In the second phase, as A_N is high enough, the economy starts to accumulate capital. Capital is not very high at the beginning and the main driving force of technical progress is the "Boserupian effect". The density of population plays a key role in the accumulation of technical progress and in the take-off of capital accumulation. Nevertheless, this phenomenon is bounded as it relies on population density. Indeed, when population density is too high, utility is low and fertility falls. As this effect is bounded, it cannot explain long run growth. But if capital is high enough a "learning by doing" effect arises and technical progress keeps growing, at least for a while.

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If A_K is not high enough ($A_K < \bar{A}_K$), the economy converges towards a steady state with constant levels of population, capital and technology. The level of capital may be quite low or quite high depending on the empirical value of A_K. If A_K is low, the level of technology achievable is mainly determined by the "Boserupian effect". If A_K is higher but still smaller than $\bar{A}_K$, the level of capital and technology achieve a higher level, due to a "learning by doing effect". But this last effect is bounded.

If A_K is high enough ($A_K > \bar{A}_K$), the economy goes through the "Boserupian phase" and reaches the "learning by doing" phase. Contrarily to the previous case, the technology does not reach a stationary state. Technical progress grows at a positive rate thanks to the "learning by doing" effect. The capital also keeps growing. This leads to the third phase where capital grows at a constant rate and population is stationary.

4.3   The third phase: the "Ultimate Growth Regime"

If A_K is high enough ($A_K > \bar{A}_K$), the economy converges to a state with stationary population and a constant growth rate of capital accumulation defined by equation (47). Land plays a twofold role in this last phase. As the density of population in the long run is $\frac{N_\infty}{X} = \frac{1}{\xi}\frac{\gamma_3 - \phi}{\gamma_3 + \gamma_4 - \phi}$, the long run size of population is proportional to the land endowment. $\bar{A}_K$ decreases with the size of population (equation (46)) and the growth rate of capital increases with it (equation (47)). Thus, a high land endowment reduces the threshold $\bar{A}_K$ to reach the growth regime and increases the long run growth rate when this regime is achieved. This result comes from the scale effect existing in many endogenous growth models as in Romer (1986).

5   Simulations

The production function studied above can be viewed as a limit case of a more general Constant Elasticity Substitution production function as mentioned in (3)

$$F(K_t, L_t, X_t) = \left[\lambda (A_t K_t)^{1 - \frac{1}{\varepsilon}} + (1 - \lambda) X_t^{1 - \frac{1}{\varepsilon}}\right]^{\frac{\varepsilon \alpha}{\varepsilon - 1}} L_t^{1 - \alpha}$$

We choose ε high enough (at least $\varepsilon > 1$) in order to be close to the theoretical case studied in the previous sections: substitutability between capital (K_t) and land (X_t) is supposed to be high.

As Inada conditions are satisfied for this production function, the firm's problem always has an interior solution.

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$$R_t = F'_K(K_t, L_t, X_t) \qquad (50)$$
$$w_t = F'_L(K_t, L_t, X_t) \qquad (51)$$
$$\pi_t = F'_X(K_t, L_t, X_t) \qquad (52)$$

Nevertheless, if one considers numerical values, the economy starts from a situation where K_t is nearly null and goes to a situation where X_t is nearly null.

5.1 Dynamics

Assuming the same behaviors for consumers as in the previous sections, the dynamics of the economy are fully characterized by a system given in Appendix 2 with twelve variables R_t, A_t, k_t, x_t, w_t, π_t, K_t, m'_t, v_t, $\overline{x}_t$, q_t, N_t. Rearranging the equations without K_t and N_t, and deflating w_t, q_t, k_t, π_t by G^{*t} and A_t by $(G^{*t})^{\frac{1}{\alpha}-1}$, we get a dynamic system converging towards a stationary state. Substituting x_t by $x_t = y_t \left(\frac{1}{G^{*t}}\right)^{\frac{\varepsilon-1}{\alpha}}$ for numerical reasons, one gets the simulated system of eleven equations:

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$$m'_t = \frac{\gamma_{3,t}\tilde{w}_t}{(\phi_t\tilde{w}_t + \xi\tilde{\pi}_t)} \tag{53}$$

$$v_t = \frac{\xi\gamma_{3,t}\tilde{w}_t}{(\phi_t\tilde{w}_t + \xi\tilde{\pi}_t)} + \gamma_{4,t}\frac{\tilde{w}_t}{\tilde{\pi}_t} \tag{54}$$

$$\overline{x}_t = y_t\left(\frac{1}{G^{*t}}\right)^{\frac{\varepsilon-1}{\alpha}} + v_t \tag{55}$$

$$\gamma_{2,t}\tilde{w}_t = \tilde{q}_t\overline{x}_t + m'_t G^*\tilde{k}_{t+1} \tag{56}$$

$$\tilde{q}_t\overline{x}_t = \frac{G^*}{R_{t+1}}(m'_t\tilde{q}_{t+1}\overline{x}_{t+1} + \tilde{\pi}_{t+1}\overline{x}_t) \tag{57}$$

$$G_{t+1} = G \times G_t \tag{58}$$

$$\tilde{\pi}_t = \alpha(1-\lambda)y_t^{-\frac{1}{\varepsilon}}\left[\lambda(\tilde{A}_t\tilde{k}_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda)(y_t)^{(1-\frac{1}{\varepsilon})}\left(\frac{1}{G_t}\right)^{\frac{\varepsilon-1}{\alpha}}\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}-1} \tag{59}$$

$$\tilde{w}_t = (1-\alpha)\left[\lambda(\tilde{A}_t\tilde{k}_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda)\left(y_t\left(\frac{1}{G_t}\right)^{\frac{\varepsilon}{\alpha}}\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} \tag{60}$$

$$R_t = \alpha\lambda\tilde{A}_t^{(1-\frac{1}{\varepsilon})}\tilde{k}_t^{-\frac{1}{\varepsilon}}\left[\lambda(\tilde{A}_t\tilde{k}_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda)\left(y_t\left(\frac{1}{G_t}\right)^{\frac{\varepsilon}{\alpha}}\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}-1} \tag{61}$$

$$\tilde{A}_t = \left[\mu\left(\left(\frac{1}{G_t}\right)^{\frac{1-\alpha}{\alpha}}(\frac{A_N}{\overline{x}_t^\beta})\right)^{1-\frac{1}{\nu}} + (1-\mu)\left(A_K^{\frac{1}{\alpha}}\left(\tilde{k}_t\frac{\overline{X}}{\overline{x}_t}\right)^{(\frac{1}{\alpha}-1)}\right)^{1-\frac{1}{\nu}}\right]^{\frac{\nu}{\nu-1}} \tag{62}$$

$$\overline{x}_{t+1} = \frac{\overline{x}_t}{m'_t} \tag{63}$$

with eleven variables: R_t, $\tilde{A}_t$, $\tilde{k}_t$, y_t, $\tilde{w}_t$, $\tilde{\pi}_t$, m'_t, v_t, $\overline{x}_t$, $\tilde{q}_t$, and G_t. The last one G_t is exogenous and used only to compute deflated variables.

5.2   Calibration and computation

The parameters used to simulate the dynamics to the ultimate growth regime are the following:

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Production	$\lambda = 0.5$	$\varepsilon = 4$	$\alpha = 0.33$		
Technology	$\mu = 0.5$	$A_N = 599$	$\beta = 0.5$	$\nu = 2$	$A_K = 50$
Utility	$\Gamma_1 = 0.25$	$\Gamma_2 = 0.25$	$\Gamma_3 = 0.25$	$\Gamma_4 = 0.25$	$\xi = 1$
Cost of a child	$\phi^1 = 0.02$	$\phi^2 = 0.08$			
Fixed asset	$\overline{X} = 2.66$				
Life Expectancy	$\eta = 1$	$p = 1$			

Recall that the cost in time of one child is $\phi^1 + \phi^2 \eta$. Thus, the total cost of one child is ten per cent of wages when all children survive. This is in line with the calculations of Glaude and Moutardier (1991) for the cost of a child.

$\eta = 1$ and $p = 1$ are the long run values for survival probabilities. Some simulations with increasing survival probability scenarios are computed below.

The parameters have been chosen in order to get in the long run $G^* = 2$ and $N_\infty = 1$. If one assumes that a period is thirty years, this leads to an annual growth rate of 2.33%. The population of one generation has been arbitrarily normalized to one in the long run by an appropriate choice of the unit of measure of $\overline{X}$. Note that total population $(N_{t+1} + N_t + p_{t-1}N_{t-1})$ converges toward three in the long run, as our model has three generations living at the same time (children, young agents, and retired people).

Simulations are computed with Dynare, details are given in Adjemian *et al.* (2010).

We start with the parameter values given above that allow sustained growth at a positive rate $(A_K > \bar{A}_K)$. Three difficulties arise in the computation of the dynamic path.

First, one has to compute the long run stationary state. As we simulate an endogenous growth model, there is an hysteresis problem on the long run value $\tilde{k}^*$ of $\tilde{k}_t$ that depends on the whole path. To compute $\tilde{k}^*$, it is necessary to make a loop. We choose an arbitrary value for $\tilde{k}^*$ as a terminal condition on $\tilde{k}_T$ at period T (the last period of simulations). Starting from the initial value $\tilde{k}_0 = \tilde{k}^*$, the simulation until period T gives a new value of $\tilde{k}^*$. This routine is iteratively run until the new calculated $\tilde{k}^*$ matches the previous $\tilde{k}^*$.

Secondly, we compute trajectories that start from initial conditions away from the stationary state path. This is implemented step by step in order to obtain convergence of the algorithm. At each step, we need new initial values for the computed endogenous variables. We take the ones from the last computed path. Obviously, at each step $\tilde{k}^*$ is modified as it depends on the whole path and is computed again with a loop.

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In the end, initial conditions are set to values corresponding to a steady state with no growth for parameters such that $A_K < \bar{A}_K$. We then come back to the previous set of parameters (A_K, p, or η) such that $A_K > \bar{A}_K$, and we simulate the growth trajectory from the initial steady state. The simulated trajectory can thus be interpreted as the growth path resulting from positive shocks on technology or survival probabilities (A_K, p, or η).

5.3   Demographic transitions

We simulate here the transition from a Malthusian steady state to the ultimate growth regime. We examine two possible sources to get out of the Malthusian regime: a technological shock and an increase in survival probabilities. In each case, we generate the corresponding demographic transition.

5.3.1   Technological shock

We first examine a technological shock on A_K from $A_K = 10$ to $A_K = 50$. This permanent shock arises in one period. Nevertheless, it induces dynamics that last six periods on demographic and macroeconomic variables. If one assumes that a period is thirty years, this lasts 180 years. This increase in productivity allows a temporary increase in the fertility rate that leads to a higher population level in the long run. Starting from a steady state with no growth, the technology shock allows a positive GDP growth rate. As the fertility rate falls to the replacement rate in the long run, the GDP per head growth rate catches up with the GDP growth rate.

The shock occurs at date 5. It has a very small effect on fertility and production at date 5 because the capital stock is fixed at its level of steady state before the shock. Thus, it cannot be seen on the figures. The number of children per adult experiences a dramatic increase at date 6, that leads to an increase of the number of young households N at date 7. As the total population incorporates children, it increases right from date 6.

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Ak

m

Population

Total population — — Number of young agents N

GDP growth rate (%)

Total GDP — — GDP per capita

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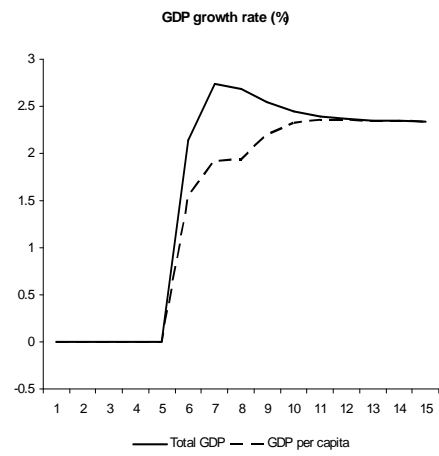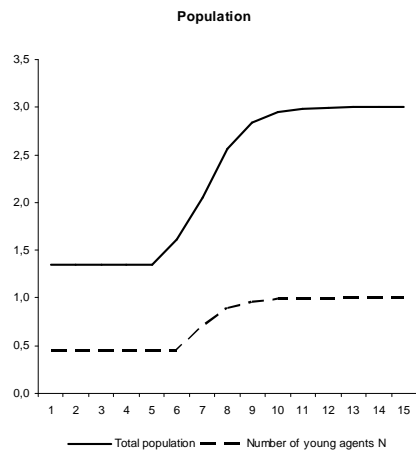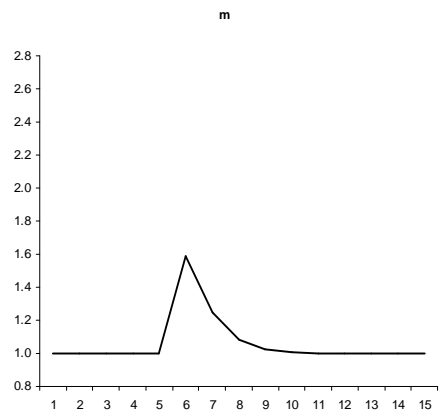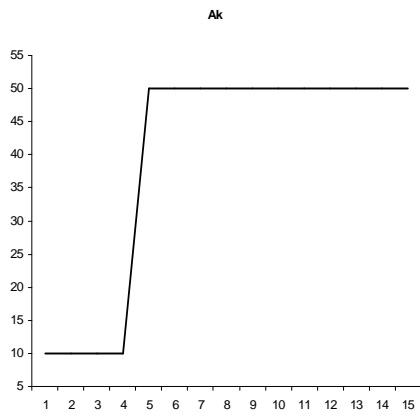
5.3.2 Increase in survival probabilities

We examine progressive positive shocks on the adult survival probability p_t (from $p_t = 0.15$ to $p_t = 1$) and on the child survival probability η_t (from $\eta_t = 0.70$ to $\eta_t = 1$).

First, we consider only an increase in adult longevity p_t from $p_t = 0.15$ to $p_t = 1$ in seven periods. This corresponds to an increase in longevity of 25.5 years in 210 years. For $p_t = 0.15$, the economy is initially in a stationary state with no growth.
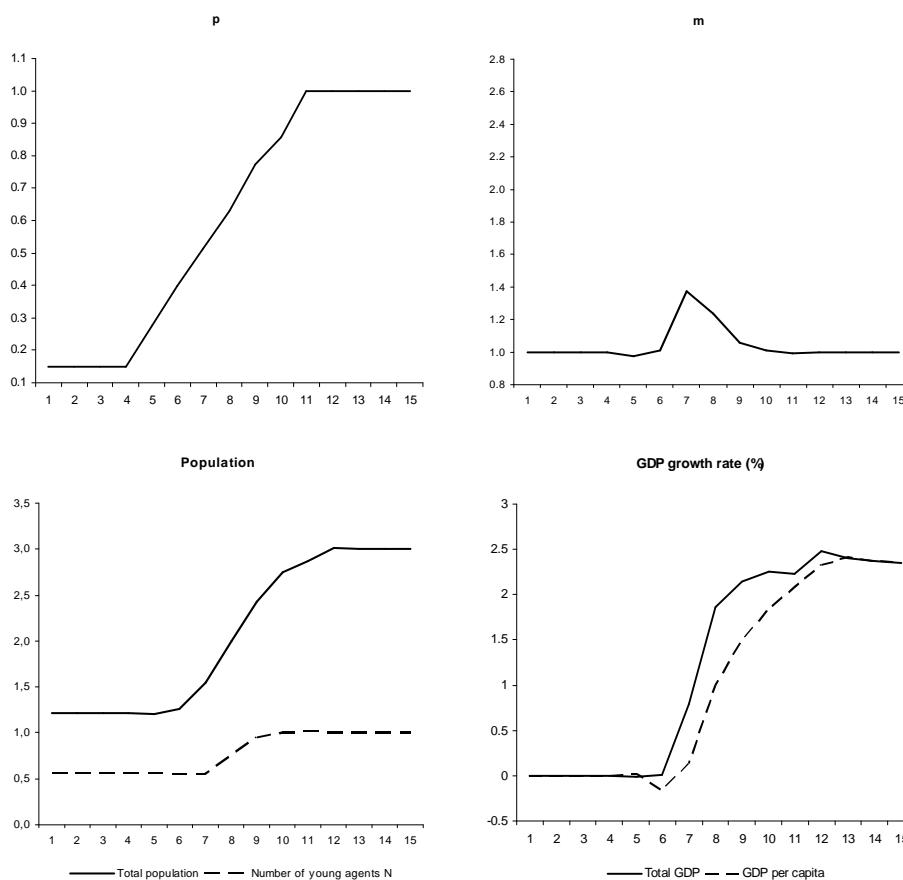
The increase in life expectancy incites agents to increase savings which induces long run growth. As technological progress increases, land is less used in production and can be used by households. This leads to a temporary rise in the birth rate and to a higher population level. The initial fall in fertility at the date of the first increase in longevity is due to the increase of the propensity to save γ_2 and the corresponding fall in the propensity to make children γ_3, which both depend on the adult survival probability p.

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Secondly, we examine a progressive shock on p_t (from $p_t = 0.15$ to $p_t = 1$) and on η_t (from $\eta_t = 0.70$ to $\eta_t = 1$). The increase in η_t from 0.70 to 1 in seven periods corresponds to a decline in mortality at birth from 30% to *zero* in 210 years, which is roughly in line with what has been observed historically for developed countries (in France this rate was of 296/1000 in 1740, see Maddison, 2005). The combination of the two shocks doubles the impact on gross fertility and has a noticeable impact on net fertility. Indeed, the decline in infant mortality can be viewed as a reduction in the surviving child costs ϕ. This increases the number of children. This increase is so fast, that it even induces an overshooting of the size of total population above its long term stationary level.

Survival probabilities

Fertility

Population

GDP growth rate (%)
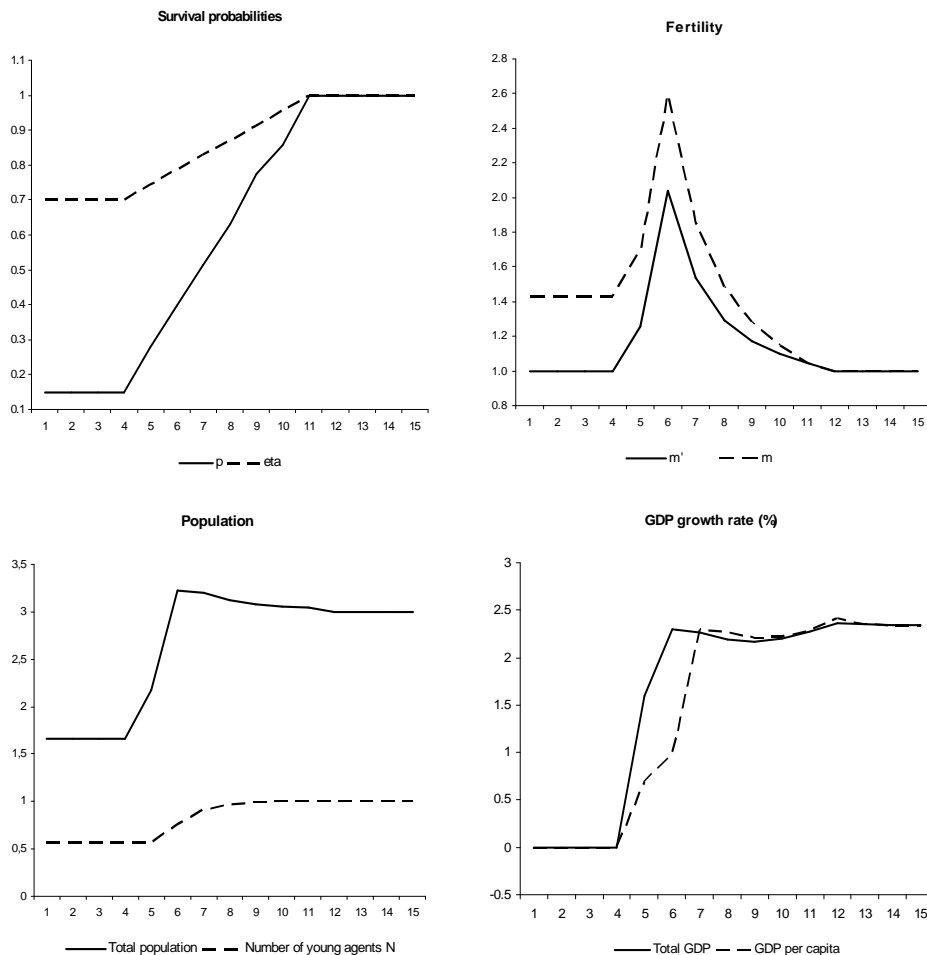
6   Conclusions

This paper has developed a new explanation for the evolution of economic and population growth with a long run perspective, emphasizing the role of land in the development process. Starting from a pre-industrialization state called the "Malthusian regime", land and labor are the main production factors. The size of population is limited by the quantity of land available for households and by incomes. Technical progress driven by a "Boserupian effect" may push the economy toward a take-off regime. In this regime, capital accumulation begins and a "learning-by-doing" effect in production takes over from the "Boserupian effect". If this effect is strong enough, the economy can reach an "ultimate growth regime". In the different phases, land plays a crucial role.

Our analysis could be extended in different directions. First, as mortality rates depend on economic development, they could be endogenized in introducing explicit investments in health. This assumption could lead to multiple long run equilibria. The economy could be trapped in underdevelopment resulting from a low survival probability that itself results from underinvestment in health. Secondly, the model could be extended to replicate historical facts on land price, population density, and growth.

7   References

Adjemian S., Bastani H., Juillard M., Mihoubi F., Perendia G., Ratto M., Villemot S., 2010, "*Dynare Manual*", mimeo, www.dynare.org

Aghion P., Howitt P., Murtin F., 2010, "The relationship between health and growth: when Lucas meets Nelson-Phelps", *NBER working paper* N°15813, March

Boserup E., 1965, "*The conditions of Agricultural Growth: The Economics of Agrarian Change under Population Pressure*", London: Allen & Unwin

Brunt L., Garcia-Penalosa C., 2010, "Urbanization and the onset of modern economic growth", mimeo, April

Diamond P., 1965, "Nominal Debt in a Neoclassical Growth Model", *American Economic Review* 55(5), pp. 1126-50.

Galor O., Moav O., 2004, "From Physical to Human Capital Accumulation: Inequality and the Process of Development", *Review of Economic Studies* 71, pp. 1001-1026.

Galor O., Moav O., Vollrath D., 2009, "Inequality in Land Ownership, the Emergence of Human Capital: Promoting Institutions and the Great Divergence", *Review of Economic Studies* 76 (1), pp. 143-179.

Galor O., Weil D. N., 2000, "Population, Technology and Growth : From the Malthusian Regime to the Demographic Transition", *American Economic Review* 90 (4), pp. 806-828.

Glaude M., Moutardier M., 1991, "Une évaluation du coût direct de l'enfant de 1979 à 1989", *Economie et Statistique*, n°248, Novembre.

Hansen G., Prescott E., 2002, "Malthus to Solow", *American Economic Review* 92, No. 4, pp. 1205-1217.

Kongsamut P., Rebelo S., Xie D., 2001, "Beyond Balanced Growth", *Review of Economic Studies* 68, pp. 869-882.

Kremer M., 1993, "Population Growth and Technological Change: one million B.C. to 1990", *The Quarterly Journal of Economics* 108, pp. 681-716.

Maddison A., 2005, *The World Economy: A Millennial Perspective,* OECD

Malthus T., 1798, *An essay on the principle of population,* Oxford University Press (1993 printing).

Romer, P., 1986, "Increasing Returns and Long Run Growth", *Journal of Political Economy* 98, 1002-37.

Rostow W. W., 1959, "The Stages of Economic Growth", *Economic History Review* 12 (1), pp. 1-16.

8 Appendix

8.1 Appendix 1: Proof of Lemma 1

Assume that there exists a steady state with $k > 0$ and $x > 0$ for (44) not true. At a steady state, $m' = 1$. From (16), we obtain:

$$\frac{w}{\pi} = \frac{\xi}{\gamma_3 - \phi} \tag{64}$$

and from (17),

$$v = \xi + \frac{\gamma_4 \xi}{\gamma_3 - \phi} \tag{65}$$

As land is used both by the consumer and the firm, $\chi = \pi$, and equations (4), (5) and (6) become:

$$
\begin{aligned}
w &= (1 - \alpha)\left[\lambda(Ak) + (1 - \lambda)x\right]^\alpha \\
R &= \alpha \lambda A \left[\lambda(Ak) + (1 - \lambda)x\right]^{\alpha - 1} \\
\pi &= \alpha(1 - \lambda)\left[\lambda(Ak) + (1 - \lambda)x\right]^{\alpha - 1}
\end{aligned}
$$

From (64), the ratio w/π is known, which gives:

$$\frac{w}{\pi} = \frac{1 - \alpha}{\alpha(1 - \lambda)}\left[\lambda(Ak) + (1 - \lambda)x\right] = \frac{\xi}{\gamma_3 - \phi}$$

Defining

$$\Gamma = \frac{\xi \alpha (1 - \lambda)}{(\gamma_3 - \phi)(1 - \alpha)}$$

we obtain:

$$\lambda(Ak) + (1 - \lambda)x = \Gamma \tag{66}$$

w, R and π can be written:

$$
\begin{aligned}
w &= (1 - \alpha)\Gamma^\alpha & (67) \\
R &= \alpha \lambda A \Gamma^{\alpha - 1} & (68) \\
\pi &= \alpha(1 - \lambda)\Gamma^{\alpha - 1} & (69)
\end{aligned}
$$

The equilibrium on the rent market gives:

$$v + x = \overline{x} \tag{70}$$

The equilibrium on the capital market leads to:

$$\gamma_2 w = k + q\overline{x} \tag{71}$$

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The arbitrage condition can be written:

$$(R-1)q\overline{x} = \pi\overline{x} \tag{72}$$

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$$A = \left[\mu(\frac{A_N}{\overline{x}^{\beta}})^{1-\frac{1}{\nu}} + (1-\mu)\left(A_K^{\frac{1}{\alpha}}\left(\frac{k\bar{X}}{\overline{x}} \right)^{(\frac{1}{\alpha}-1)} \right)^{1-\frac{1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \tag{73}$$

Finally, the constraints $k \geq 0$ and $x \geq 0$ imply (from (66)):

$$x \leq \frac{\Gamma}{1-\lambda} \text{ and } Ak \leq \frac{\Gamma}{\lambda} \tag{74}$$

and equation (72) is valid only for $R > 1$, which implies (from (68):

$$\alpha\lambda A\Gamma^{\alpha-1} > 1 \tag{75}$$

Using (71) and (72), we obtain:

$$(R-1)\left(\gamma_2 w - k \right) = \pi\overline{x}$$

Replacing prices by expressions (67), (68) and (69), and using (70) and (66), we obtain:

$$(\alpha\lambda A\Gamma^{\alpha-1} - 1)\left(\gamma_2(1-\alpha)\Gamma^{\alpha} - k \right) = \alpha(1-\lambda)\Gamma^{\alpha-1}\left[v + \frac{(\Gamma - \lambda Ak)}{(1-\lambda)} \right]$$

Finally,

$$k = \alpha(1-\lambda)\Gamma^{\alpha-1}v + \alpha\Gamma^{\alpha} + \gamma_2(1-\alpha)\Gamma^{\alpha} - \gamma_2(1-\alpha)\Gamma^{\alpha}\alpha\lambda\Gamma^{\alpha-1}A \tag{76}$$

which allows us to express k as a simple function of A. All other parameters are known and constant. This expression implies that A must belong to an interval: $[0, A_{\max}]$ with:

$$A_{\max} = \frac{\alpha(1-\lambda)\Gamma^{\alpha-1}v + \alpha\Gamma^{\alpha} + \gamma_2(1-\alpha)\Gamma^{\alpha}}{\gamma_2(1-\alpha)\Gamma^{\alpha}\alpha\lambda\Gamma^{\alpha-1}}$$

From (73) and as $\nu > 1$, it is possible to write:

$$A \geq \mu^{\frac{\nu}{\nu-1}}A_N\overline{x}^{-\beta}$$

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and by (74) $\overline{x} = v + x \leq v + \frac{\Gamma}{1-\lambda}$. Thus:

$$A \geq \mu^{\frac{\nu}{\nu-1}} A_N \left(v + \frac{\Gamma}{1-\lambda} \right)^{-\beta}$$

To obtain the existence of a steady state with $k > 0$ and $x > 0$, it is necessary that

$$A_{\max} > \mu^{\frac{\nu}{\nu-1}} A_N \left(v + \frac{\Gamma}{1-\lambda} \right)^{-\beta}$$

or:

$$\frac{\alpha(1-\lambda)\Gamma^{\alpha-1}v + \alpha\Gamma^\alpha + \gamma_2(1-\alpha)\Gamma^\alpha}{\gamma_2(1-\alpha)\Gamma^\alpha \alpha\lambda\Gamma^{\alpha-1}} > \mu^{\frac{\nu}{\nu-1}} A_N \left(v + \frac{\Gamma}{1-\lambda} \right)^{-\beta}$$

Using the expressions of Γ and v, it is easy to show that this inequality gives (44), which contradicts the assumption. Therefore, there does not exist any steady state with $k > 0$ and $x > 0$ when (44) is not true.

8.2   Appendix 2: Proof of Lemma 2

Proof.   In the long run, land is not used any more in production ($x_t = 0$).
Equations (4) and (5) become

$$w_t = (1 - \alpha) [\lambda A_t k_t]^\alpha \tag{77}$$

$$R_t = \alpha \lambda A_t [\lambda A_t k_t]^{\alpha-1} \tag{78}$$

Population is stationary ($m'_t = 1$). From (16), one can deduce that

$$\frac{\pi_t}{w_t} = \frac{\gamma_3 - \phi}{\xi} \tag{79}$$

from (24) that $\bar{x}_\infty = v_\infty$, and from (17) that $_\infty = \xi \left(\frac{\gamma_3 + \gamma_4 - \phi}{\gamma_3 - \phi} \right)$ The size of
population is:

$$N_\infty = \frac{\bar{X}}{\bar{x}_\infty} = \frac{\bar{X}}{\xi} \frac{\gamma_3 - \phi}{\gamma_3 + \gamma_4 - \phi} \tag{80}$$

Thus, the share of rents in wages is

$$\frac{\pi_t v_\infty}{w_t} = \gamma_3 + \gamma_4 - \phi \tag{81}$$

The price of land (q_∞) is computed from (27):

$$q_t \bar{x}_t = (q_{t+1} \bar{x}_{t+1} + \pi_{t+1} \bar{x}_{t+1}) \frac{m'_t}{R_{t+1}} \tag{82}$$

Rewriting (25),

$$\gamma_2 w_t - q_t \overline{x}_t = m'_t k_{t+1} \tag{83}$$

Dividing, (83) by (82), member by member, one gets:

$$\frac{\gamma_2 w_t}{q_t \overline{x}_t} - 1 = \frac{1}{\frac{q_{t+1} \bar{x}_{t+1}}{R_{t+1} k_{t+1}} + \frac{\pi_{t+1} \bar{x}_{t+1}}{R_{t+1} k_{t+1}}} \tag{84}$$

In the long run the value of land relative to wages is constant

$$\frac{q_t \overline{x}_t}{w_t} \equiv B \tag{85}$$

From (77) and (78),

$$\frac{q_{t+1} \bar{x}_{t+1}}{R_{t+1} k_{t+1}} = \frac{q_{t+1} \bar{x}_{t+1}}{w_{t+1} \frac{\alpha}{1-\alpha}} = B \frac{1 - \alpha}{\alpha}$$

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From (77), (78), and (79)

$$\frac{\pi_{t+1}\bar{x}_{t+1}}{R_{t+1}k_{t+1}} = \frac{\frac{\gamma_3-\phi}{\xi}w_{t+1}\bar{x}_\infty}{w_{t+1}\frac{\alpha}{1-\alpha}} = (\gamma_3 + \gamma_4 - \phi)\frac{1-\alpha}{\alpha}$$

Thus, (84) gives an equation for B:

$$\frac{\gamma_2}{B} - 1 = \frac{1}{B\frac{1-\alpha}{\alpha} + (\gamma_3 + \gamma_4 - \phi)\frac{1-\alpha}{\alpha}} \tag{86}$$

Thus, the positive solution of the corresponding second degree equation is:

$$\begin{aligned}
B = &\ \frac{1}{2}\left[-\left(\frac{\alpha}{1-\alpha} + \gamma_3 + \gamma_4 - \phi - \gamma_2\right)\right]\\
&+\frac{1}{2}\sqrt{\left(\frac{\alpha}{1-\alpha} + \gamma_3 + \gamma_4 - \phi - \gamma_2\right)^2 + 4\gamma_2(\gamma_3 + \gamma_4 - \phi)} \tag{87}
\end{aligned}$$

Thus, B gives the share of the value of land in wages. Using (85) in (25) allows to calculate the evolution of k_t:

$$k_{t+1} = (\gamma_2 - B)w_t$$

From (77)

$$k_{t+1} = (\gamma_2 - B)(1 - \alpha)\lambda^\alpha A_t^\alpha k_t^\alpha$$

With (8), one can get

$$k_{t+1} = (\gamma_2 - B)(1 - \alpha)\lambda^\alpha\left[\mu(\frac{A_N}{\bar{x}_\infty^\beta})^{1-\frac{1}{\nu}} + (1 - \mu)\left(A_K^{\frac{1}{\alpha}}N_\infty^{(\frac{1}{\alpha}-1)}k_t^{(\frac{1}{\alpha}-1)}\right)^{1-\frac{1}{\nu}}\right]^{\frac{\alpha\nu}{\nu-1}}k_t^\alpha$$

∎

8.3 Appendix 3: Simulations

We provide here the intermediate steps to derive the simulated dynamic system.

Assuming the same behaviors for consumers and the markets as in the theoretical model and the CES production function (3), the dynamics of the economy are fully characterized by the following set of 12 equations:

$$R_t = \alpha\lambda A_t^{(1-\frac{1}{\varepsilon})} k_t^{-\frac{1}{\varepsilon}} \left[\lambda(A_t k_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda)x_t^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}-1} \tag{88}$$

$$w_t = (1-\alpha) \left[\lambda(A_t k_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda)x_t^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} \tag{89}$$

$$\pi_t = \alpha(1-\lambda)x_t^{-\frac{1}{\varepsilon}} \left[\lambda(A_t k_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda)x_t^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}-1} \tag{90}$$

$$A_t = \left[\mu(\frac{A_N}{\overline{x}_t^{\beta}})^{1-\frac{1}{\nu}} + (1-\mu)\left(A_K^{\frac{1}{\alpha}} K_t^{(\frac{1}{\alpha}-1)}\right)^{1-\frac{1}{\nu}}\right]^{\frac{\nu}{\nu-1}} \tag{91}$$

$$m_t' = \frac{\gamma_3 w_t}{(\phi_t w_t + \xi\pi_t)} \tag{92}$$

$$v_t = \frac{\xi\gamma_3 w_t}{(\phi_t w_t + \xi\pi_t)} + \gamma_4\frac{w_t}{\pi_t} \tag{93}$$

$$N_{t+1} = m_t' N_t \tag{94}$$

$$\overline{x}_t = \frac{\overline{X}}{N_t} \tag{95}$$

$$v_t + x_t = \overline{x}_t \tag{96}$$

$$\gamma_2 w_t = m_t' k_{t+1} + q_t \overline{x}_t \tag{97}$$

$$R_{t+1} = \frac{q_{t+1} + \pi_{t+1}}{q_t} \tag{98}$$

$$K_t = N_t k_t \tag{99}$$

with twelve variables R_t, A_t, k_t, x_t, w_t, π_t, K_t, m_t', v_t, $\overline{x}_t$, q_t, N_t.

Rearranging the equations without K_t and N_t, one gets the simulated ten equation system:

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$$m'_t = \frac{\gamma_{3,t} w_t}{(\phi_t w_t + \xi \pi_t)} \tag{100}$$

$$v_t = \frac{\xi \gamma_{3,t} w_t}{(\phi_t w_t + \xi \pi_t)} + \gamma_{4,t} \frac{w_t}{\pi_t} \tag{101}$$

$$\overline{x}_t = x_t + v_t \tag{102}$$

$$\gamma_{2,t} w_t = m'_t k_{t+1} + q_t \overline{x}_t \tag{103}$$

$$R_{t+1} = \frac{q_{t+1} + \pi_{t+1}}{q_t} \tag{104}$$

$$\pi_t = \alpha(1-\lambda) x_t^{-\frac{1}{\varepsilon}} \left[\lambda(A_t k_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda) x_t^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon\alpha}{\varepsilon-1}-1} \tag{105}$$

$$w_t = (1-\alpha) \left[\lambda(A_t k_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda) x_t^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} \tag{106}$$

$$R_t = \alpha\lambda A_t^{(1-\frac{1}{\varepsilon})} k_t^{-\frac{1}{\varepsilon}} \left[\lambda(A_t k_t)^{1-\frac{1}{\varepsilon}} + (1-\lambda) x_t^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon\alpha}{\varepsilon-1}-1} \tag{107}$$

$$A_t = \left[\mu(\frac{A_N}{\overline{x}_t^\beta})^{1-\frac{1}{\nu}} + (1-\mu) \left(A_K^{\frac{1}{\alpha}} \left(k_t \frac{\overline{X}}{\overline{x}_t} \right)^{(\frac{1}{\alpha}-1)} \right)^{1-\frac{1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \tag{108}$$

$$\overline{x}_{t+1} = \frac{\overline{x}_t}{m'_t} \tag{109}$$

with ten variables R_t, A_t, k_t, x_t, w_t, π_t, m'_t, v_t, $\overline{x}_t$, q_t.

Deflating w_t, q_t, k_t, π_t by G^{*t} and A_t by $(G^{*t})^{\frac{1}{\alpha}-1}$, and substituting x_t by $x_t = y_t \left(\frac{1}{G^{*t}} \right)^{\frac{\varepsilon-1}{\alpha}}$. One reaches the simulated system.

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