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Keywords: quality choices, differentiation by attributes, product innovation, product imitation

Product Innovation and Imitation in a Duopoly with Differentiation By Attributes

Reynald-Alexandre Laurent*

Abstract

This paper considers a probabilistic duopoly in which products are described by their specific attributes, this form of differentiation embodying the horizontal and vertical dimensions. Consumers make discrete choices and follow a random decision rule based on these attributes. A three-stage game is studied in which firms develop new attributes for their products (innovation), then may imitate the attributes of the competing product and finally compete in price. At the equilibrium, the firm selling the less appreciated product is generally incited to imitate its rival. Confronted to a threat of imitation, the benchmark firm sometimes decreases strategically its attribute index in order to diminish its unit cost of innovation and the differentiation on the market, deterring the imitation in this way. This strategy is efficient when imitation costs are sufficiently concave. In the opposite case, it is preferable for the benchmark firm to accept the imitation. Thus, according to the shape of imitation costs, equilibria with "deterrence" or with "accommodation" occur, completing the current typology of strategic responses to a threat of imitation.

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1 Introduction

Endowing a product with new attributes or copying the attributes from a competing product are very widespread practices on differentiated products markets. The firm selling a benchmark product is frequently the first target of copycats, as in the following examples. At the beginning of the 17th century, Venetian cloths were reputed for their very high quality. Hampered by a technological lag, North European producers decided to imitate these clothes in a very cunning way: "The chief stratagem of cloth smugglers was to imitate typical Venetian signs and marks on the head and the selvage of the bolt. These signs were supposed to guarantee the quality and origin of the cloth. (...) Another modus operandi was for foreign manufacturers to weave Venetian-style woolens of inferior or mixed wools." (Rapp, 1975, p 508). Such a

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behaviour can also be observed more recently. The Caterpillar company, a heavy earth-moving equipment manufacturer selling benchmark products on its market, faced similar difficulties at the beginning of the 1980's: "The powerful Komatsu, once an upstart Japanese heavy equipment manufacturer, eventually overtook rival Caterpillar by benchmarking its machines and finding ways to produce equivalent quality at a fraction of the cost" (Miller, 1990, p 201). These examples show that the growth of rising firms is often triggered by a partial imitation of a benchmark rival, forcing this incumbent to carry more attention to its costs. The threat of imitation also affects firms' choices of product innovation: for example, the incumbent may limit its R & D expenses, which seem to it unproductive.

This paper proposes a new framework for the analysis of interactions between product innovation and imitation. Consumer's beahviour is described by a probabilistic discrete choice model based on product attributes and consistent with a Selection By Aspects (Restle, 1961) or an Elimination By Aspects (Tversky, 1972). In this duopoly framework, the differentiation by attributes (or "DBA") embodies the standard horizontal and vertical forms of differentiation. Innovation is represented by the addition of new specific attributes to a given product and imitation by the cancellation of some specific attributes belonging to a competing product. Existence and properties of a perfect equilibrium in pure strategies are analyzed in a three-stage game with innovation, imitation and price competition.

Whereas initial works of Gilbert and Newberry (1982) and Benoit (1985) were devoted to process imitations, the combined analysis of product innovation and imitation with price competition is more recent. Such a work is carried out by Pepall and Richards (1994) and Pepall (1997) in a duopoly with differentiation by qualities (Gabszewicz and Thisse, 1979). But this framework is not very well fitted to deal with imitation choices. Indeed, when the market is covered, the two firms' profits increase with the quality gap between products, which represents the degree of differentiation. Thus, the high-quality firm is always incited to increase its quality. But even a costless quality improvement, which is a form of imitation, is not profitable for the low-quality firm because products become closer substitutes.

The two papers previously mentioned employ slightly modified structures in order to avoid these restrictions. Pepall and Richards (1994) study a Stackelberg duopoly with sequential pre-determined entry and introduce the possibility of an exogenous imitation. In the model, the entrant has the opportunity to pay a fixed cost (depending on the quality gap) to make a perfect copy of the innovator's product. The authors highlight an interesting strategic effect: the incumbent may increase its innovation level in order to rise the imitation fixed cost beard by the entrant, thus deterring the imitation. Pepall (1997) considers a similar model with sequential entry but the entrant can now determine the imitation size endogenously. However, it is also assumed that a quality increase performed by the low-quality firm *reduces* its fixed cost.[1] At the equilibrium, the entrant decides either not to imitate the rival's product or to achieve a partial imitation.

Using a DBA duopoly provides additional insights to analyze interactions between innovation and imitation. On the one hand, when vertical and horizontal differentiations are taken into account, each firm can innovate and imitate the competing product. The imitation level is chosen in an endogenous way

[1] This assumption seems stronger than the simple assertion according to which the imitation cost is lower than the innovation cost for a given quality.

and firms can adjust their innovation effort to deter an imitation. On the other hand, strategic incentives can be analyzed with endogenous unit costs whereas the current literature generally considers fixed costs.

The study of the three-stage game with innovation, imitation and price competition leads to the following new results. At first, the imitation can improve the attractiveness of the recipient product but also reduce differentiation on the market. In the DBA duopoly, an imitation is only profitable for the firm selling the less appreciated product if is possesses a relative cost advantage, as in the examples mentioned at the very beginning. Then, equilibrium properties depend on the shape of the imitation cost. When this cost is either null, or positive and concave, the firm selling the most appreciated product deters completely the imitation by reducing strategically its attribute index. This setting describes a "deterrence" equilibrium. When the imitation cost is strongly convex, a small imitation is realized and it is more profitable for the suffering firm not to react at all: this result characterizes an "accommodation" equilibrium. Finally, when the imitation cost is weakly convex, a deterrence or a mixed equilibrium (in which imitation is partially accepted and partially deterred) may be observed.

This paper is organized in the following way. The demand system of the DBA duopoly and forms of differentiation it embodies are presented in Section 2. Section 3 recalls the existence conditions for a perfect Nash equilibrium in the duopoly with innovative choice of attributes and price competition. Section 4 studies firms' incentives to imitate and demonstrates the existence of a perfect equilibrium in the three-stage game with innovation, imitation and price competition. Properties highlighted at the equilibrium are detailed in Section 5, leading to the taxonomy of "deterrence" and "accommodation". Conclusions are presented in Section 6 and proofs of several propositions are relegated in Appendixes.

2 Demand system of the DBA model

This section presents the choice probabilities of the DBA model. Then, forms of differentiation described by the corresponding demand system are analyzed.

2.1 Choice probabilities

Consider a market in which two differentiated products are available, each consumer purchasing exactly one unit of one product. It is supposed that consumers follow a probabilistic reasoning based on products' attributes, such that their behaviour can be described by a random decision rule model. The model employed here is called "Differentiation By Attributes" because products are differentiated according to the specific attributes they possess, an approach remembering that of Lancaster (1966). This model is formally equivalent to the structures with Selection By Aspects (Restle, 1961) and with Elimination By Aspects (Tversky, 1972), these two models being themselves identical when only two options are taken into account.

According to the conclusions of Payne and Bettman (2001), using such decision rules is time-saving for consumers because the decision making does not require a complete listening of all the attributes. As shown by Fader and McAlister (1990), this discrete choice model is more relevant than a random utility structure to describe small purchase decisions locking up a low amount of income (see also Batsell and Seetharaman, 2005). Indeed, it is simply unlikely that consumers make a complete examination of all the characteristics and of all the products in such a context. It is especially true for supermarket shopping because consumers make several small decisions in a reduced time. The DBA model is also relevant when a particular attribute triggers the purchase. The brand name seems to play such a role in the examples mentioned in introduction.

In this model, specific non-price attributes of product i, with $i = \{1, 2\}$, provide consumers with a utility u_i. These agents do not take into account the attributes shared by the products because they are useless for the decision-making. As models with differentiation by attributes were primarily concerned with discrete attributes, the introduction of a continuous variable (for example the price p_i) remained an open question until the paper of Rotondo (1986). This author suggests representing a price difference as a specific attribute of the least expensive product. Consequently, the probability P_i of selecting product i rather than product j depends on the price hierarchy:

- if $p_i \geq p_j$,

$$P_i^{\overline{p}} = \frac{u_i}{u_i + u_j + p_i - p_j} \qquad P_j^{\underline{p}} = \frac{u_j + p_i - p_j}{u_i + u_j + p_i - p_j} \tag{2.1}$$

- if $p_j \geq p_i$,

$$P_i^{\underline{p}} = \frac{u_i + p_j - p_i}{u_i + u_j + p_j - p_i} \qquad P_j^{\overline{p}} = \frac{u_j}{u_i + u_j + p_j - p_i} \tag{2.2}$$

The numerator of the probabilities represents all the specific attributes of the product considered. For example, if $p_i \geq p_j$, good i has only non-price specific attributes providing the utility u_i, but if $p_j \geq p_i$, good i also possesses a price attribute, the price gap $p_j - p_i$. The denominator represents all the specific attributes of the two products. In a set of homogeneous goods ($u_i = u_j = 0$), the least expensive good is always selected. When prices are identical, choice probabilities simply equal the utility index ratio, as in the Luce model (1959).

These probabilities can be interpreted in two different ways. In the spirit of Restle, P_i is the probability of choosing product i because it possesses a specific attribute. For example, a specific brand name may be chosen immediately by a consumer without considering other attributes. Following Tversky, P_i may also represent the probability of eliminating j after having selected a specific attribute of i as the elimination criterion.[2] For example, all the products not possessing a "fair trade" label may be eliminated by an ethical consumer.

2.2 Forms of differentiation

Consider now a market in which N consumers follow a probabilistic reasoning based on attributes: N is supposed sufficiently high so that the expected demand functions are simply taken as effective by risk-neutral firms. Observation of choice probabilities shows that demand functions $X_i = NP_i$ and $X_j = NP_j$

[2]For a complete presentation of the Elimination By Aspects model, see Batsell *et al.* (2003).

are piecewise defined. Note that demand functions possess a kink in $p_i = p_j$ when $u_i \neq u_j$ but this kink vanishes when $u_i = u_j$. Henceforth, it is supposed that $p_1 \geq p_2$ and the following demand functions are used[3]:

$$X_1 = \frac{N u_1}{u_1 + u_2 + p_1 - p_2} \tag{2.3}$$

$$X_2 = \frac{N(u_2 + p_1 - p_2)}{u_1 + u_2 + p_1 - p_2} \tag{2.4}$$

Properties of the demand system are studied with more details by Laurent (2007a). In particular, it is shown that the form of differentiation on the market depends on the values of utility variables.

Horizontal differentiation. Product differentiation is said purely horizontal in a duopoly framework when market shares equal $P_i = P_j = 1/2$ for differentiated products sold at identical prices. This setting is observed in the DBA model when $u_1 = u_2 > 0$: the specific attributes of the two products are appreciated in the same way into the population of consumers and each agent chooses its preferred variety. This particular case of the DBA model is formally equivalent to a Quadratic Address Model (d'Aspremont *et al.*, 1979) in which consumers' density of preferences into the space of varieties is not uniform.[4]

Vertical differentiation. Differentiation is said purely vertical in the duopoly when market shares equal $P_i = 1$ and $P_j = 0$ for identical prices. In this framework, there exists a preference hierarchy between products. These proportions are obtained in the DBA model when $u_1 > 0$ and $u_2 = 0$. In this case, one of the products is strictly preferred by the consumers because it possesses all the specific attributes on the market. The existence of heterogeneous qualities can be integrated in the DBA model as a particular form of vertical differentiation. The method used for prices integration is also applied for qualities: the quality difference is simply a specific attribute of the high-quality good. When products are differentiated by their only qualities, utility variables become $u_1 = q_1 - q_2$ and $u_2 = 0$ (the minimum level of quality q_2 is shared by products 1 and 2 and is not taken into account during the choice process). In this setting, choice probabilities become:

$$P_1 = \frac{q_1 - q_2}{q_1 - q_2 + p_1 - p_2} \tag{2.5}$$

$$P_2 = \frac{p_1 - p_2}{q_1 - q_2 + p_1 - p_2} \tag{2.6}$$

This particular case of the DBA model is formally equivalent to a duopoly with differentiation by qualities (Tirole, 1988) when a specific non-uniform distribution of consumers' preference intensity for quality is

[3] It will be demonstrated in Proposition 1 that equilibrium prices actually satisfy the condition $p_1 \geq p_2$.

[4] More precisely, the density linking the two structures is symmetric and reaches its maximum at the middle of the segment of varieties: a greater number of consumers prefer the central variety.

considered.[5]

Horizontal and vertical differentiation. When product differentiation is horizontal and vertical, the market share of the most appreciated product belongs to the interval $]1/2; 1[$ and that of its competitor to $]0; 1/2[$. For $u_1 > u_2 > 0$, choice probabilities are given by $P_1 = u_1/(u_1 + u_2) > 1/2$ and $P_2 = u_1/(u_1 + u_2) < 1/2$ when prices are identical. Differentiation is horizontal up to the level u_2, the specific attributes of the two products providing the same utility, and vertical for a level $u_1 - u_2$, product 1 also offering additional attributes. In the DBA model, utility indices integrate many characteristics of differentiation in a simple and endogenous way. This framework is not identical to deterministic multi-dimensional structures (Neven and Thisse, 1990; Economides, 1993; Irmen and Thisse, 1998): these demand systems are often more complex and only embody a limited number of characteristics, given *a priori*. In these structures, the equilibrium analysis can not be carried out when no characteristic is "dominant" whereas the DBA model is not affected by this limitation. The DBA duopoly is also not equivalent to a binomial logit with heterogeneous qualities: indeed, the logit cannot convey the case of pure vertical differentiation (the DBA model is more general in this sense).

3 Perfect equilibrium with innovation and price competition

This section recalls existence conditions for a perfect equilibrium in a two-stage game with choice of new attributes and price competition.

3.1 Nash price equilibrium

Existence and uniqueness of a Nash price equilibrium in pure strategies is studied in the DBA model. Each firm i bears a unit cost c_i, a fixed cost F_i and its profit is given by $\Pi_i = (p_i - c_i)X_i - F_i$.

Proposition 1 (Laurent, 2007a, p 15) *There exists a Nash price equilibrium verifying $p_i \geq p_j$, with $i, j \in \{1, 2\}$ and $i \neq j$, if and only if:*

$$u_i \geq u_j \tag{3.1}$$

and

$$c_i - c_j \geq \sqrt{u_i u_j} - u_i \tag{3.2}$$

Moreover, this equilibrium is unique.

When the firm selling the most appreciated product has the highest unit cost, existence of the equilibrium is guaranteed. Equilibrium prices in $p_1 \geq p_2$ are given by:

[5] This density is a decreasing function of preference intensity for quality, which looks like the observed income distribution into the population of consumers.

$$p_1^* = \frac{u_1 + \sqrt{\Delta}}{2} + c_1 \tag{3.3}$$

$$p_2^* = u_1 + u_2 + c_1 \tag{3.4}$$

where $\Delta = u_1^2 + 4u_1(u_1 + u_2 + c_1 - c_2)$. At the equilibrium, firm 1, which sells the most appreciated product, selects a higher price than its rival. This Nash equilibrium is weak in the sense that firm 1 could make the same profit by choosing another price belonging to the interval $[p_2; +\infty[$. The study of best response functions reveals why p_2^* increases with c_1 but does not depend on c_2. At the equilibrium, if firm 1 modifies its price, the local best response of firm 2 is to modify its own price in the same direction. However, at the equilibrium, firm 1 is locally insensitive to a small price variation of firm 2. This observation recalls practices of pricing imitation like those described by Lazer (1957 p. 130-131), and particularly the case in which the firm selling the highest-quality good sets a *reference price* on the market. In this context, each other firm sets the reference price minus a certain amount, which depends on the quality gap with the benchmark firm. It will be shown afterwards that this property affects the attribute choices in the DBA model.

Equilibrium profits are given by:

$$\Pi_1^* = Nu_1 - F_1 \text{ and } \Pi_2^* = \frac{N(u_1 + u_2 + c_1 - c_2)(\sqrt{\Delta} - u_1)}{\sqrt{\Delta} + u_1} - F_2 \tag{3.5}$$

When fixed costs are identical $F_1 = F_2$, comparison of profits leads to the following condition:

$$\Pi_1^* \geq \Pi_2^* \Leftrightarrow c_1 - c_2 < u_1 - u_2$$

Thus, the firm selling the most appreciated product $(u_1 > u_2)$ makes the highest profit only if the differentiation gap is higher than the cost gap.[6]

3.2 Perfect equilibrium in the two-stage game

Choices of product specification are studied by Laurent (2007b) in a two-stage game with new attributes selection and price competition. In this framework, product imitation is assumed impossible, for example because the fabrication process of specific attributes is secret.

In the *first stage*, each firm i selects specific attributes for its product, this choice being described by a synthetic index a_i, a positive and continuous variable $a_i \in [0; +\infty[$. This index is nothing else but a simplified representation of the positioning chosen for the product in order to maximize its producer's profit. However, this index provides no information on the precise nature of attributes leading to such a positioning (a quality difference, an improvement of brand image, an additional accessory, *etc.*). The

[6] This property is also highlighted by Anderson and Renault (2006) in a framework with pure vertical differentiation.

attribute index selected by firm i affects its unit costs of innovation and the utility obtained by consumers of product i.

The utility function $u : a \rightarrow \mathbb{R}^+$ is assumed satisfying $u'(a_i) > 0$ for finite attribute indices, $u''(a_i) \leq 0$ and Inada conditions $\lim_{a_i \rightarrow +\infty} u'(a_i) = 0$ and $\lim_{a_i \rightarrow 0} u'(a_i) \rightarrow +\infty$. The assumption $u'(a_i) > 0$ means that an increase in specific attributes provides with the consumer an additional utility. The marginal utility of attributes decreases when the attribute index increases implying that $u''(a_i) \leq 0$. Moreover, offering new attributes generates a unit cost $c : a \rightarrow \mathbb{R}^+$, which is constant with the quantity produced but increasing and convex with the attribute index: $c(0) = 0$, $c'(a_i) > 0$ and $c''(a_i) \geq 0$. This standard assumption (see for example Motta, 1993) means that it is more costly to improve a product which already possesses many specific attributes. Exogenous fixed costs are assumed equal $F_1 = F_2$.

In the *second stage*, firms compete in prices. There exists a price Nash equilibrium if conditions (3.1) and (3.2) are verified. Equilibrium prices are given by (3.3) and (3.4) and profits by (3.5).

The game is solved by backward induction. Existence and uniqueness of a perfect Nash equilibrium are studied. The following proposition presents equilibrium attribute choices:

Proposition 2 (Laurent, 2007b, p 8) *When unit costs are endogenous, there exists two perfect Nash equilibria verifying $p_i \geq p_j$ and differing only by the identity of firms. Firm i chooses the highest possible index such that $u'(a_i) = 0$. Firm j chooses the index equating the marginal utility of the consumer and the marginal cost of production: $u'(a_j) = c'(a_j)$.*

Although its unit cost is attribute dependent, the firm selling the most appreciated product (henceforth "firm 1") selects the highest possible index of attributes. This finding is a consequence of the weak nature of the price Nash equilibrium, an effect previously named as "reference price". Indeed, firm 1 being the benchmark on the market, any increase in its unit cost increases its price, which also rises that of its rival in the same proportion: market position of firm 1 is not weakened. Finally, firm 2 chooses a lowest innovative attribute index. This result can be linked with firms' practices and in particular refers to the "product supremacy" strategy carried out by Venetian tradesman and by the Caterpillar company (see Rapp, 1975, p 507-508 and Miller, 1990, p 22). As shown in the next section, introducing an imitation does not keep these findings unchanged.

Note that the equilibrium embodies the two dimensions of differentiation, which is horizontal for a level a_2^* and vertical for a level $a_1^* - a_2^*$. The existence of several forms of differentiation at the equilibrium is a new result in itself, not yet observed in multi-dimensional models. Indeed, in these structures, product differentiation is maximal for one dominant characteristic but remains null for all the other dominated characteristics.

4 Product improvement by innovation and by imitation

This section shows that an imitation may be profitable for a low cost firm which offers few specific attributes. Our assumptions on utility and costs are presented and the existence of equilibrium is demonstrated in a three-stage game.

4.1 Profitability of imitation and nature of costs

In a framework of differentiation by attributes, a firm i can improve the relative attractiveness of its product by an *innovation* or by an *imitation* of its competitor j. As established previously, an innovation is represented by the development of new specific attributes, this action increasing a_i. But each firm can also provide its product with some attributes being so far exclusively offered by a competitor. An imitation by firm i reduces a_j, the imitated attributes of j being now shared by the products (and thus ignored by consumers during their choice process). But this action does not modify a_i, which depends only on the specific attributes of product i.

On markets with differentiated products, an imitation could however be unprofitable, even if this imitation is costless. When attribute indices vary in the DBA model, comparative statics properties of equilibrium profits can be explained by the combination of a "differentiation effect" and an "attractiveness effect".[7] On the one hand, increasing one of the attribute indices strengthens market differentiation (reduces product substitutability), this effect rising firms' profits. On the other hand, increasing the specific attribute index of product i improves its relative attractiveness: profit of firm i is increased and that of firm j decreased. An imitation from firm i reduces a_j and thus implies a positive attractiveness effect and a negative differentiation effect on Π_i, the net effect being ambiguous. The relative weight of these effects depends both on products' positioning and on the type of imitation costs.

When costs are exogenous, proposition 3 provides a condition of profitability for an imitation :

Proposition 3 *Suppose that firms' attribute indices before imitation satisfy $a_i^* \geq a_j^*$. Then, the imitation is never profitable for firm i. Imitation is profitable for firm j if:*

$$5u(a_i^*) - u(a_j^*) < c_i - c_j \tag{4.1}$$

Proof: For firm i, imitation is profitable if Π_i increases when u_j decreases. Suppose that firm 1 sells the most appreciated product ($u_1 > u_2$). In this case, firm 1 never increases its quality by imitation because $\partial\Pi_1^*/\partial u_2 = 0$. For firm 2, profit derivative is:

$$\frac{\partial \Pi_2^*}{\partial u_1} = \frac{4Nu_1 x(u_1 - x + \sqrt{\Delta})}{\sqrt{\Delta}(\sqrt{\Delta} + u_1)^2}$$

with $x = u_1 + u_2 + c_1 - c_2$. This derivative equals zero for $5u_1 = u_2 + c_1 - c_2$, leading to condition 4.1. Second order condition is:

[7] For more details, see Laurent (2007a, p 18).

$$\left.\frac{\partial^2 \Pi_2^*}{\partial u_1^2}\right|_{\frac{\partial \Pi_2^*}{\partial u_2}=0} = \frac{4Nu_1 x(3u_1 + 2x)}{(u_1^2 + 4u_1 x)(\sqrt{\Delta} + u_1)^2} \geq 0$$

The profit of firm 2 is thus quasi-convex in u_1.■

This proposition shows that *the imitation is never profitable for the firm selling the most appreciated product* (henceforth firm 1) because the differentiation effect is always dominant. Moreover, condition (4.1) means that firm 2 is incited to imitation if its unit cost advantage is higher than a threshold, which depends on its attributes disadvantage. When condition (4.1) holds, the attractiveness effect of imitation is more significant than the differentiation effect for firm 2. Consequently, the imitation is performed by a low quality and low cost firm (note also that firm 2 makes a higher profit than firm 1 before imitation). This finding seems empirically relevant because firms of developing countries frequently imitate their rivals belonging to developed countries.

These effects are not highlighted in a classical framework of differentiation by qualities. Indeed, the only differentiation effect matters and imitation remains unprofitable for the low-quality firm.[8] The DBA model embodies an additional horizontal differentiation which strengthens the incentive to imitation.

4.2 Three-stage game with imitation of attributes

Suppose now that innovation and imitation costs are attribute-dependent. In this case, introducing a product imitation may affect the attribute choices highlighted in the two-stage game of section 3 because condition (4.1) may be satisfied at the equilibrium with endogenous unit costs. That is why a three-stage game with innovation, imitation and price competition is studied.

In the *first stage*, each firm i selects a specific attribute index $a_i \in [0; +\infty[$. In the DBA model, imitation choices are neither modified by the number of consumers nor by fixed costs values (see condition 4.1): it is assumed thereafter that $N = 1$ and that $F_1 = F_2$. However, the unit cost of firm i depends on the attribute index:

Assumption 1: *The unit cost function of innovation has a quadratic form:* $c_i = a_i^2$.

This cost function is also used by Anderson, de Palma and Thisse (1992) and by Motta (1993). At the end of the stage, the attribute index is noted a_i^*.

In the *second stage*, each firm observes its competing product and may decide to imitate specific attributes belonging to it. We suppose the absence of patent protection. This assumption can be true in several contexts: protection is not efficient (counterfeit), the patent costs are too high, attributes are already patented by a provider, *etc.* An imitation by firm i decreases the attribute index of product j above a_j^*. The attribute index of product j *after* imitation by firm i is noted $\widehat{a_j}(a_i^*, a_j^*)$: this index verifies

[8] However, two similar effects appear when imitation is cost reducing, as discussed by Pepall (1997).

$\widehat{a_j}(a_i^*, a_j^*) \leq a_j^*$. Thus, the level of imitation performed by firm i is given by $a_j^* - \widehat{a_j}(a_i^*, a_j^*)$ (in the absence of imitation, we have $\widehat{a_j}(a_i^*, a_j^*) = a_j^*$).

Assumption 2: *An imitation does not reverse the hierarchy of attribute indices:* $\widehat{a_j} \in [a_i^*, a_j^*]$ *if* $a_j^* > a_i^*$ *and* $\widehat{a_j} \in [0, a_j^*]$ *if* $a_j^* \leq a_i^*$.

This simplifying assumption is reasonable when some specific attributes of the firm selling the most appreciated product cannot be imitated. Indeed, it is very infrequent to observe an imitation making disappear all the comparative advantages of the benchmark firm.

Assumption 3: *An imitation of firm j by firm i generates a unit cost depending on the imitation level:* $\widehat{c_i} = \alpha(a_j^* - \widehat{a_j})^\beta$ *with* $0 \leq \alpha \leq 1$ *and* $0 < \beta \leq 2$.

Intuitively, the imitation affects the only cost of the imitating firm. This cost increases with the gap between the index of j before imitation (first stage) and after imitation by i (second stage).[9] Imitation is costless when $\alpha = 0$. For these intervals of α and β, it is generally less costly to improve a product by imitation than by innovation (see Assumption 1), which seems plausible: the average ratio of imitation to innovation costs is estimated at 0.65 by Mansfield, Schwartz and Wagner (1981). At the end of the second stage, product i provides consumers with the following utility:

Assumption 4: *The utility is a linear function of the attribute index after imitation:* $u_i = \widehat{a_i}$.

In the *third stage*, firms compete in prices and there exists a price Nash equilibrium if conditions (3.1) and (3.2) are satisfied.

4.3 Existence of the equilibrium with innovation and imitation

As it is inferred from proposition 3, the benchmark firm never imitates its rival in the second stage: this preliminary result greatly simplifies the resolution of the three-stage game previously exposed. The classical backward induction method is employed. For a given couple of attribute indices $(\widehat{a_1}; \widehat{a_2})$ in the third stage, the price subgame is solved by the values $(p_1^*(\widehat{a_1}; \widehat{a_2}); p_2^*(\widehat{a_1}; \widehat{a_2}))$ such that any deviation is unprofitable. In the second stage, the imitation subgame is solved by the attribute indices after imitation $(\widehat{a_1}(a_1^*; a_2^*); \widehat{a_2}(a_1^*; a_2^*))$ maximizing firms' local profits, knowing the equilibrium index chosen in the first stage. These expressions define the local perfect Nash equilibrium for all the settings (a_1, a_2). This equilibrium concept is relevant in this framework of complex interactions on product attributes because the most radical deviations cannot reasonably be investigated by firms.[10] The existence of the equilibrim is established in the following proposition (the firm choosing the highest index in the first stage is called firm 1):

[9] Note that Pepall (1997) uses a quadratic cost function which depends on the quality gap.

[10] This concept is also used by Bonanno (1988), Bonanno and Zeeman (1985), Gary-Bobo (1989) and Irmen and Thisse (1998).

Proposition 4 *Under assumptions 1 to 4, there exists a local perfect Nash equilibrium in the three-stage game with innovation, imitation and price competition. In the first stage, firm 2 selects* $a_2^* = 0.5$ *and firm 1 a level* $a_1^* > a_2^*$*. In the second stage, firm 1 never imitates its rival whereas firm 2 may imitate its competitor leading to* $\widehat{a_1} \in [a_2^*, a_1^*]$.

Proof: This proof is provided in Appendix 1, section 8.

According to the parameters of the imitation cost, different properties may be highlighted at the equilibrium. These properties depend on the existence of a strategic reaction from the benchmark firm in the first stage and on the realization of an imitation (or not) in the second stage. By analyzing potential forms of equilibrium, the following classification can be established[11]:

a) *Deterrence Equilibrium.* Firm 1 modifies in a strategic way its attribute index a_1^* in the first stage, this action deterring completely the imitation by firm 2. No imitation is realized and attribute indices are not modified in the second stage ($\widehat{a_1^*} = a_1^*$).

b) *Accommodation Equilibrium.* An imitation is performed by firm 2 in the second stage ($\widehat{a_1^*} \in [a_2^*, a_1^*[$). It is preferable for firm 1 to accept the imitation in the first stage instead of deterring it: consequently, its attribute index a_1^* is kept unchanged.

c) *Mixed equilibrium.* A strategic behaviour of firm 1 in the first stage deters partially the imitation but not completely. Consequently, a partial imitation is performed by firm 2 in the second stage, leading to $\widehat{a_1^*} \in]a_2^*, a_1^*[$.

5 Properties of the equilibrium and nature of imitation costs

In this section, properties of the equilibrium in the three-stage game are studied under different cost assumptions.

5.1 Equilibrium with costless imitation

This case with costless imitation ($\alpha = 0$) provides a reference point for the study of interactions between innovation and imitation. In the first stage, firm 1 selects strategically its attribute index a_1^* in order to deter the imitation. Indeed, the profit of firm 1 at the end of the second stage is given by $\Pi_1^* = \widehat{a_1}$: its objective is thus to choose in the first stage the value a_1^* maximizing the level $\widehat{a_1^*} \in [a_2^*, a_1^*]$ which will be selected by firm 2 in the second stage, knowing that firm 2 chooses $a_2^* = 0.5$ in the first stage (Proposition 4). Equilibrium properties are given by the following proposition:

Proposition 5 *When* $\alpha = 0$*, firm 1 selects* $a_1^* \approx 2.45542$ *and its rival* $a_2^* = 0.5$ *in the first stage. In the second stage, firm 2 does not imitate firm 1. A* deterrence equilibrium *is obtained.*

[11] This classification can be linked with that of Fudenberg and Tirole (1984). These authors study the strategic choice of investment realized by an incumbent to deter the entry of a rival or to soften the competition with it.

Proof: This proof is provided in Appendix 2, section 9.1.

In the second stage, the nature of the equilibrium depends on the shape of $\Pi_2(\widehat{a_1})$ in the interval $[a_2^*, a_1]$. Here, when $\widehat{a_1}$ increases, Π_2 is strictly increasing for low values of a_1, strictly quasi-convex for intermediate values of a_1 and strictly decreasing for high values of a_1. Consequently, firm 1 selects the value $a_1^* \approx 2.45542$ such that $\Pi_2(\widehat{a_1})$ is quasi-convex and such that $\Pi_2(a_1^*) = \Pi_2(a_2^*)$. Thus, firm 1 *deters the imitation by decreasing its attribute index* compared to the two-stage game, leading to a deterrence equilibrium.

A decrease in a_1^* diminishes product differentiation before imitation and improves the cost competitiveness of firm 1, these two effects reducing the profitability of an imitation for firm 2 in the second stage. Such an imitation would improve the attractiveness of product 2 (increasing Π_2) but it would also reduce product differentiation on the market (decreasing Π_2). The strength of these two effects depends on the degree of differentiation before imitation: when a_1 diminishes, the differentiation effect becomes more essential. Moreover, as shown by condition (4.1), imitation is profitable for firm 2 if and only if it possesses a unit cost advantage: by reducing a_1, firm 1 also diminishes c_1, making the imitation less profitable.

5.2 Equilibrium with a concave imitation cost

In this section, it is supposed that an imitation entails a positive cost with a concave shape ($\alpha > 0$ and $0 \leq \beta \leq 1$) depending on the imitation size. This assumption may induce firm 1 to select a higher attribute index in the first stage so as to deter the imitation by increasing the imitation cost beard by its rival. At the equilibrium, the following property is highlighted:

Proposition 6 *When $\alpha > 0$ and $0 \leq \beta \leq 1$, firm* 1 *selects a level a_1^* in the first stage such that firm* 2 *never imitates in the second stage. A* deterrence equilibrium *is realized.*

Proof: This proof is provided in Appendix 2, section 9.2.

As previously, the type of equilibrium depends on the shape of $\Pi_2(\widehat{a_1})$ in the second stage, which itself depends on a_1. Figure 1 provides an illustration of $\Pi_2(\widehat{a_1})$ in the interval $[a_2^*, a_1^*]$ (remember that $a_2^* = 0.5$) for several representative values of a_1.

The same properties are observed for other values of $\alpha > 0$ and $0 \leq \beta \leq 1$. *The equilibrium computed here is similar to that obtained when imitation is costless*: in the first stage, firm 1 selects a level a_1^* such that Π_2 is strictly quasi-convex and whose exact value is determined by the implicit equation $\Pi_2(a_1^*) = \Pi_2(a_2^*)$. Thus a_1^* is the maximum level of a_1 inciting firm 2 not to imitate. This level depends on α and β and can not be computed explicitly.[12]

[12] However, a numerical approximation reveals that a_1^* increases with α: the effort required to deter the imitation is lower when the imitation cost increases (for a given β). For example, when $\alpha = \beta = 0.5$, we have $a_1^* \simeq 3.7289$.

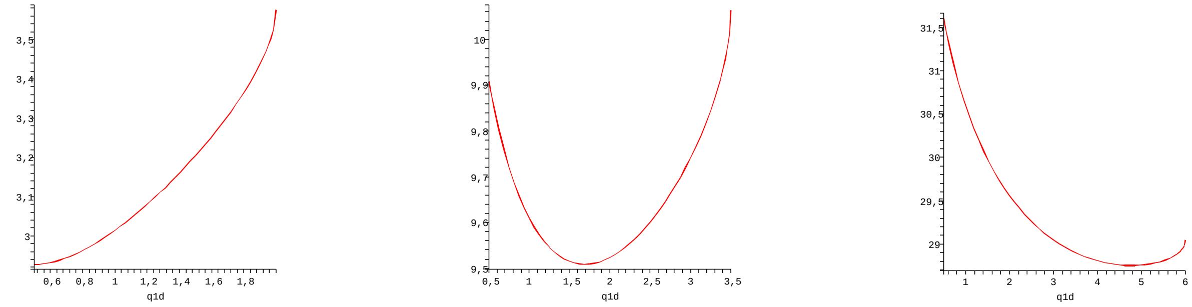

Figure 1: Evolution of Π_2 with $\widehat{a_1}$ when $\alpha = 0.5$ and $\beta = 0.5$ for $a_1 = 2$, $a_1 = 3.5$ and $a_1 = 6$.

The concavity of imitation costs plays an important role in the determination of the equilibrium properties. For such a function, it is very costly for firm 2 to set off an imitation of the very first attributes of product 1 because the marginal cost is high. However, after this "starting cost" paid, it is less and less costly to imitate the other attributes. In this framework, it is more profitable for firm 1 to reduce its own attribute index in order to deter the imitation at its beginning.

5.3 Equilibrium with a convex imitation cost

It is assumed now that the imitation cost is convex ($\alpha > 0$ and $1 < \beta \leq 2$). The following lemma identifies the properties of $\Pi_2(\widehat{a_1})$ and the potential forms of equilibrium:

Lemma 7 *When $\alpha > 0$ and $1 < \beta \leq 2$, $\Pi_2(\widehat{a_1})$ has at most two extreme points in the interval $]a_2^*, a_1^*[$ according to the value of a_1^*. If there are two extreme points, the first one is a minimum and the second one a maximum. If there is one extreme point, it may be a maximum or a minimum. The three types of equilibrium may occur: "deterrence", "accommodation" and "mixed".*

Proof: This proof is provided in Appendix 2, section 9.3.

A simple nested numerical computation is used to determine which equilibrium is effectively obtained. A grid of size 10^{-3} representing every combination of parameters $0 < \alpha \leq 1$ and $1 < \beta \leq 2$ is considered. For each combination, the maximum of $\Pi_2(\widehat{a_1})$ is identified in the interval $[0.5, a_1]$ for a continuum of values of a_1. It is assumed that a_1 varies into $]0.5; 10^8[$ in the first stage (the number 10^8 provides a finite approximation of the theoretical upper bound $+\infty$). For a pair of parameters (α,β), the value of a_1 (selected by firm 1) leading to the highest $\widehat{a_1^*}$ (selected by firm 2) determines the type of equilibrium.[13] As shown in the following figure, three distinct areas are identified[14]:

[13] If $\widehat{a_1^*} = a_1^* < 10^8$, we have a deterrence equilibrium. If $\widehat{a_1^*} < a_1^* = 10^8$, we have an accommodation equilibrium. If $\widehat{a_1^*} < \widehat{a_1^*} < 10^8$, we have a mixed equilibrium.

[14] Note that a minor variation of the upper bound 10^8 could modify the exact frontiers of areas in the figure but not the qualitative properties highlighted.

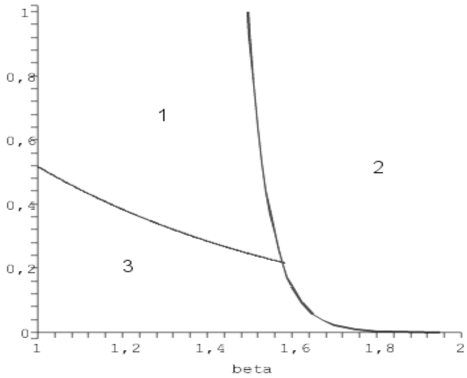

Figure 2: Typology of equilibria with α on axis Y and β on axis X. Area 1: "mixed". Area 2: "accommodation". Area 3: "deterrence".

These findings are summarized in the following proposition:

Proposition 8 *Suppose that the imitation cost is convex: $\alpha > 0$ and $1 < \beta \leq 2$. When β is high, the equilibrium is characterized by an accommodation of firm 1. When β is low and α low, the equilibrium is characterized by an imitation deterrence of firm 2 by firm 1. When β is low and α high, a mixed equilibrium occurs.*

The three possible settings are now interpreted more precisely by representing the evolution of $\Pi_2(\widehat{a_1})$ for different values of a_1. The special case of $\alpha = 0.4$ is taken as an example because the three types of equilibrium are observed according to the threshold β selected (we consider $\beta = 1.1$, $\beta = 1.45$ and $\beta = 1.6$).

First, when α and β are low, the deterrence equilibrium computed is similar to that highlighted with concave or null imitation costs. In this case, the form of Π_2 is the following:

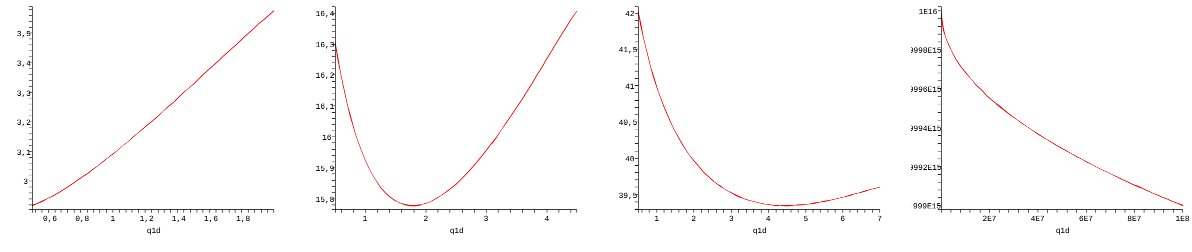

Figure 3: Evolution of Π_2 with $\widehat{a_1}$ when $\alpha = 0.4$ and $\beta = 1.1$ for $a_1 = 2$, $a_1 = 4.5$, $a_1 = 7$ and $a_1 = 10^8$.

In the second stage, firm 2 selects either a maximum imitation $\widehat{a_1^*} = a_2^* = 0.5$ (for "high" values of a_1) or no imitation $\widehat{a_1^*} = a_1$ (for "low" values of a_1). Thus, in the first stage, firm 1 selects the index a_1^* such that $\Pi_2(a_1^*) = \Pi_2(a_2^*)$ (in this example, $a_1^* \approx 4.652$), which is the highest index leading to a complete deterrence (no imitation). This strategic behavior remains possible because imitation costs are weakly convex.

Second, when β is high (imitation costs are strongly convex), the evolution of $\Pi_2(\widehat{a_1})$ is represented in the following figure:

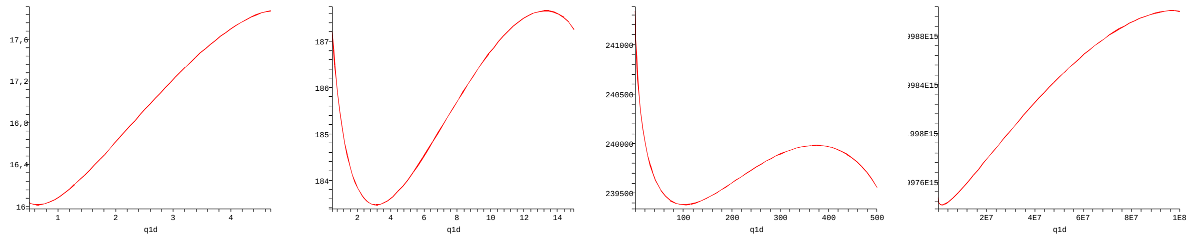

Figure 4: Evolution of Π_2 with $\widehat{a_1}$ when $\alpha = 0.4$ and $\beta = 1.6$ for $a_1 = 4.7$, $a_1 = 15$, $a_1 = 500$ and $a_1 = 10^8$

In the second stage, firm 2 may select a partial imitation $a_2^* < \widehat{a_1^*} < a_1^*$ (for "quite low " or "high" values of a_1, when the profit is maximum at the second extreme point), a maximum imitation $\widehat{a_1^*} = a_2^* = 0.5$ (for "intermediate" values of a_1) or no imitation $\widehat{a_1^*} = a_1$ (for "very low" values of a_1). In the first stage, firm 1 knows that the size of imitation chosen by firm 2 remains low, even if a high a_1 is selected. This choice being the most profitable, an accommodation equilibrium occurs in which $\widehat{a_1^*} < a_1^* = 10^8$. The existence of such an equilibrium can be linked with the shape of the imitation cost. When this cost is strongly convex, a small imitation is not very costly for firm 2 and a complete deterrence strategy requires a high profit cut for firm 1. However, increasing the size of imitation is more and more costly for firm 2 and its effective imitation remains always moderate, even if a_1 is very high. Consequently, it is more profitable for firm 1 to allow this imitation instead of deterring it.

Third, when β is low and α high, $\Pi_2(\widehat{a_1})$ varies in the following way according to a_1:

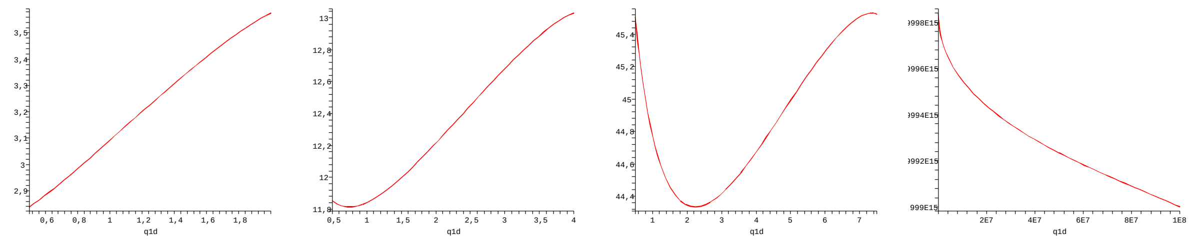

Figure 5: Evolution de Π_2 with $\widehat{a_1}$ when $\alpha = 0.4$ and $\beta = 1.45$ for $a_1 = 2$, $a_1 = 4$, $a_1 = 7.5$ and $a_1 = 10^8$.

In the second stage, firm 2 may select a maximum imitation $\widehat{a_1^*} = a_2^* = 0.5$ (for "high" values of a_1), a partial imitation $a_2^* < \widehat{a_1^*} < a_1^*$ (for "intermediate" values of a_1, when the profit is maximum at the second extreme point) or no imitation $\widehat{a_1^*} = a_1$ (for "low" values of a_1). In the first stage, the most profitable strategy for firm 1 is to select the highest level of a_1 avoiding the maximum imitation by its rival. A mixed equilibrium occurs $\widehat{a_1^*} < a_1^* < 10^8$ (in this example, $a_1^* \approx 7.535$). The complete deterrence cannot be an equilibrium: starting from such a setting, an increase in a_1 also increases $\widehat{a_1}$, even if a partial imitation is done by firm 2. This equilibrium can be explained by the form of the imitation cost: as the degree of cost convexity (β) is quite low, a deterrence strategy is useful for firm 1 in order to avoid a maximum imitation. However, as the absolute level of imitation cost (α) is high, a complete deterrence is not required and a partial imitation is accepted by firm 1.

5.4 Additional comments

The strategic decrease of attribute indices highlighted in the DBA model can be linked with observed behaviours of some benchmark firms. At the beginning of the 1980's, the systematic product improvement strategy carried out by the Caterpillar company strongly reduces its cost competitiveness. This weakness incited its rival Komatsu to make profitable product imitations while keeping moderate prices.[15] To avoid the bankruptcy, "CEO George Schaefer initiated a \$1.8 billion "Plant with a Future" program. He began to implement a speedy, flexible manufacturing system that is already cutting the costs of some operations by 20 percent. The firm also shut down inefficient plants and slashed payrolls by 30 percent" (Miller, 1990, p 226). Afterwards, Komatsu gave up progressively its imitation strategy and chooses to endow its product with new attributes and services.

Compared to the two-stage game studied in section 3.2, introducing an imitation phase reduces the vertical dimension of differentiation at the equilibrium. Equilibrium values are closer of those computed in papers studying quality choices (see for example Motta, 1993). It is also meaningful to study the evolution of attribute indices before and after imitation when the cost parameters α and β vary. When the level of imitation cost (α) increases, the strategic index reduction required to deter the imitation is lower for the leader. The index after imitation $\widehat{a_1}$ increases with α, in accordance with the intuition. Moreover, observe that no effective imitation is performed when imitation costs are null whereas it is not true when imitation costs are positive and convex. To explain this result, note that a complete deterrence is profitable for the benchmark firm only if the degree of convexity β of imitation cost is relatively low.

6 Conclusion

When demand functions are described by the DBA duopoly, a three-stage game with innovation, imitation and price competition is studied in this work. Existence of a perfect Nash equilibrium is demonstrated when endogenous unit costs of innovation and imitation are considered. It is shown that a product imitation is profitable for the firm selling the less appreciated product if its unit cost is sufficiently low. On the contrary, an imitation is never profitable for the benchmark firm. Moreover, this paper highlights a new strategic behaviour of imitation deterrence: the benchmark firm decreases its attribute index to diminish product differentiation and improve its cost competitiveness, reducing the profitability

[15] "Cat's obsession with quality had boosted expenses to the point where it could no longer compete. Its production methods had become too inefficient to enable it to match Komatsu's prices." (Miller, 1990, p 226)

of imitation for its rival.

Three types of equilibrium are distinguished, according to the level and the degree of convexity of imitation costs. When imitation costs are null or concave, a deterrence equilibrium occurs: the strategic decrease of attribute index completely deters the imitation. When imitation costs are strongly convex, the imitation size remains so moderate that the benchmark firm accepts it without modifying its attribute index: an accommodation equilibrium occurs. Finally, when imitation costs are weakly convex (but sufficiently high), we observe a mixed equilibrium with partial deterrence and partial accommodation. The DBA framework thus provides new insights for the study of interactions between product innovation and imitation.

Such a deterrence action differs of that obtained in Pepall and Richards (1994): the leader firm is not incited to over-invest in innovation in order to increase the imitation cost of its competitor. These two approaches are complementary because different cost functions are employed (endogenous unit/fixed costs) but the differentiation structure used here also introduces new effects, which explain the different findings observed. However, our results can be linked with the literature on entry deterrence. Fudenberg and Tirole (1984) showed that an incumbent may under-invest in advertising in the current period to threat a rival of a future advertising war if it enters on the market (the "lean and hungry look").

The impact of imitation (effective or deterred) on welfare is *a priori* ambiguous and should be studied more precisely. A deterrence action reduces prices (lower costs and less differentiation on the market) but also the product innovation. An effective imitation increases the cost of the imitating firm but also turn specific attributes into shared attributes, a positive effect on consumer surplus. Finally, a global welfare study must include the impact of imitation on firms' profits. A more accurate analysis requires defining a consumer surplus function associated to the DBA model, what is currently not a settled question.

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8 Appendix 1: Existence of the perfect Nash equilibrium

This section provides the proof of proposition 4. The three-stage game is solved by backward induction.

In the *last stage*, equilibrium prices are given by (3.3) and (3.4).

In the *second stage*, the profit of firm 1 is independent of a_2 (see equation 3.5) and firm 1 never imitates firm 2. Consequently, the attribute index at the end of the second stage $\widehat{a_2}$ is always equal to that chosen by firm 2 in the first stage. As $\widehat{a_2} = a_2^*$, the profit of 2 only depends on a_2^* and on the attribute indices of firm 1 before imitation a_1^* and after imitation $\widehat{a_1}$. For the utility and unit costs functions employed, the profit of firm 2 in the second stage is:

$$\Pi_2(\widehat{a_1}) = \frac{(\sqrt{\Delta} - \widehat{a_1})x}{\sqrt{\Delta} + \widehat{a_1}} \tag{8.1}$$

with $\Delta = \widehat{a_1^2} + 4\widehat{a_1}x$ and $x = \widehat{a_1} + a_2^* + (a_1^*)^2 - (a_2^*)^2 - \widehat{c_2}(a_1^*, \widehat{a_1})$.

Firm 2 selects a level $\widehat{a_1}$ in the interval $[a_2^*, a_1^*]$: the shape of $\Pi_2(\widehat{a_1})$ in this interval depends on the value a_1^* chosen by firm 1 in the first stage and on the imitation cost parameters. The first derivative of the profit is given by:

$$\frac{\partial \Pi_2}{\partial \widehat{a_1}} = \frac{4x\widehat{a_1}[(\widehat{a_1} + \sqrt{\Delta})(1 - \widehat{c_2'}) - x]}{\sqrt{\Delta}(\sqrt{\Delta} + \widehat{a_1})} \tag{8.2}$$

with $\widehat{c_2'} = \dfrac{\partial \widehat{c_2}}{\partial \widehat{a_1}} = -\alpha\beta(a_1^* - \widehat{a_1})^{\beta-1}$

Thus, any extreme point of Π_2 is determined by the following first order condition:

$$(\widehat{a_1} + \sqrt{\Delta})(1 - \widehat{c_2'}) - x = 0 \tag{8.3}$$

Zero, one or several extreme points may exists in the interval $[a_2^*; a_1^*]$. The nature of an extreme point can be identified thanks to the following second order condition:

$$\left.\frac{\partial^2\Pi_2}{\partial\widehat{a_1}^2}\right|_{\frac{\partial\Pi_1}{\partial\widehat{a_1}}=0} = \frac{4x\widehat{a_1}[(2x+2\widehat{a_1}\widehat{c_2'}+3\widehat{a_1})(1-\widehat{c_2'})-\sqrt{\Delta}(\widehat{a_1}+\sqrt{\Delta})\widehat{c_2''}]}{\Delta(\sqrt{\Delta}+\widehat{a_1})^2} \tag{8.4}$$

with $\widehat{c_2''} = \dfrac{\partial^2\widehat{c_2}}{\partial\widehat{a_1^2}} = \alpha\beta(\beta-1)(a_1^*-\widehat{a_1})^{\beta-2}$

When $\widehat{c_2''} < 0$, any extreme point is necessarily a minimum but when $\widehat{c_2''} > 0$, the sign of the second derivative is unknown. However, as by definition $\widehat{a_1}(a_1^*, a_2^*)$ belongs to the constrained interval $[a_2^*, a_1^*]$, it is not required knowing the exact level of imitation to demonstrate the existence of a local perfect Nash equilibrium. A unique value $\widehat{a_1}$ maximizing $\Pi_2(\widehat{a_1})$ can always be computed. Consequently, the existence can be demonstrated simply by extending the proof used in the two-stage game with innovation and price competition (Laurent, 2007b, p 20).

In the *first stage*, firm 2 knows that firm 1 will not imitate its product in the second stage. Its attribute choice is thus identical to that highlighted in the two-stage game: $u'(a_2^c) = c'(a_2^c)$. For the particular functions employed here, equilibrium values are given by $a_2^* = 0.5$, $u_2 = 0.5$ and $c_2 = 0.25$.

When there is no imitation, the two-stage game analysis concludes that firm 1 selects the highest index $a_1^* \to +\infty$. But when the threat of imitation is anticipated, profit of firm 1 is given by $\Pi_1 = \widehat{a_1}(a_1^*, a_2^*)$. This profit could be maximized for a lower value of a_1 than in the two-stage game because of the deterrence imitation effect. As each attribute index before imitation a_1 leads to a unique index after imitation a_1^*, the equilibrium value in the first stage a_1^* is uniquely defined.

Consequently, the existence of unique perfect Nash equilibrium is established. ■

9 Appendix 2: equilibrium properties and nature of costs

9.1 Equilibrium with a costless imitation

As it is of common knowledge that $a_2^* = 0.5$, the study of condition (8.3) in the second stage shows that $\Pi_2(\widehat{a_1})$ has a unique extreme point given by equation $\widetilde{\widehat{a_1}} = (1/5)a_1^{*2} + 1/20$. The second order condition (8.4) reveals that this point is a minimum but it can be inside or outside the interval $[a_2^*, a_1^*]$. Following an increase in $\widehat{a_1}$, Π_2 strictly increases when $\widetilde{\widehat{a_1}} \leq a_2^* \Leftrightarrow a_1^* \leq 1.5$, strictly decreases when $\widetilde{\widehat{a_1}} \geq a_1^* \Leftrightarrow a_1^* \geq 2.5+\sqrt{6}$ and is quasi-convex when $1.5 \leq a_1^* \leq 2.5+\sqrt{6}$.

In the first stage, firm 1 selects its index before imitation a_1^* in order to maximize the remaining level after imitation $\widehat{a_1}$. When $a_1^* \leq 1.5$, firm 2 does not imitate in the second stage, implying $\widehat{a_1} = a_1^*$. When $1.5 \leq a_1^* \leq 2.5+\sqrt{6}$, firm 2 does not imitate if $\Pi_2(a_2^*) \leq \Pi_2(a_1^*)$ and decides to imitate ($\widehat{a_1} = a_2^* = 0.5$) if $\Pi_2(a_2^*) > \Pi_2(a_1^*)$. When $a_1^* \geq 2.5+\sqrt{6}$, firm 2 always imitates, leading to $\widehat{a_1} = a_2^*$. Consequently, the value of a_1^* maximizing the index $\widehat{a_1}$ belongs to the interval $[1.5, 2.5+\sqrt{6}]$ and is such that $\Pi_2(a_1^*) = \Pi_2(a_2^*)$, leading to $a_1^* \approx 2.45542$.■

9.2 Equilibrium with a concave imitation cost

This section provides the proof of proposition 6, which is carried out in two steps. First, it is shown that $\Pi_2(\widehat{a_1})$ is either strictly increasing or strictly quasi-convex in the interval $[a_2^*, a_1^*]$ according to the value of a_1^*. Second, this property is used to eliminate the "mixed" and "accommodation" forms of potential equilibria.

The first order condition (8.3) can be rewritten as an equality $f(\widehat{a_1}) - g(\widehat{a_1}) = 0$ where functions f and g are defined by:

$$f(\widehat{a_1}) = \sqrt{\Delta}(1 - \widehat{c_2'}) - x \tag{9.1}$$

$$g(\widehat{a_1}) = \widehat{a_1}(\widehat{c_2'} - 1) - x \tag{9.2}$$

with $\Delta = \widehat{a_1^2} + 4\widehat{a_1}x$ and $x = \widehat{a_1} + a_2^* + (a_1^*)^2 - (a_2^*)^2 - \widehat{c_2}$.

By keeping in mind that $\widehat{c_2'} < 0$ and $\widehat{c_2''} < 0$, the study of functions f and g leads to the following results:

$$\frac{\partial g(\widehat{a_1})}{\partial \widehat{a_1}} = -1 + \widehat{c_2'} + \widehat{a_1}\widehat{c_2''} < 0 \tag{9.3}$$

$$\frac{\partial f(\widehat{a_1})}{\partial \widehat{a_1}} = \frac{(1 - \widehat{c_2'})[\widehat{a_1} + 2x + 2\widehat{a_1}(1 - \widehat{c_2'}) - \sqrt{\Delta}] - \widehat{a_1}(\widehat{a_1} + 4x)\widehat{c_2''}}{\sqrt{\Delta}} \tag{9.4}$$

The derivative in (9.4) is always positive because $\widehat{a_1} + 2x > \sqrt{\Delta}$ (it can be checked by squaring the two sides of the equality). Thus, when $\widehat{a_1}$ increases, function f also increases whereas function g decreases. Consequently, $\Pi_2(\widehat{a_1})$ possesses at most a unique extreme point in the interval $[a_2^*, a_1^*]$: the profit function may be strictly increasing, strictly decreasing or quasi-convex. But the slope of Π_2 is always positive when $\widehat{a_1} \to a_1$. Indeed, $\lim_{\widehat{a_1} \to a_1} \widehat{c_2'} \to -\infty$, implying $\lim_{\widehat{a_1} \to a_1} f(\widehat{a_1}) \to +\infty$ and $\lim_{\widehat{a_1} \to a_1} g(\widehat{a_1}) \to -\infty$. This property is true whatever the value of a_1. Consequently $\Pi_2(\widehat{a_1})$ is either strictly increasing or strictly quasi-convex in the interval.

According to a_1^*, firm 2 selects either $\widehat{a_1} = a_1^*$ (deterrence equilibrium with no imitation) or $\widehat{a_1} = a_2^*$ (accommodation equilibrium with maximum imitation) in the second stage, excluding the possibility of a mixed equilibrium. However, this accommodation equilibrium never occurs because firm 1 can always deter the imitation by reducing a_1 in the first stage. This affirmation is demonstrated by observing that Π_2 strictly increases with $\widehat{a_1}$ in $[0.5; a_1]$ when $a_1 \to 0.5$ (see the previous paragraph). Consequently, there exists at least a value $a_1^* > a_2^*$ leading to a deterrence equilibrium with $\widehat{a_1} = a_1^*$ (which is more profitable than an accommodation equilibrium for firm 1).■

9.3 Equilibrium with a convex imitation cost

This section provides the proof of lemma 7. When the imitation cost is convex, it is shown that $\Pi_2(\widehat{a_1})$ possesses zero, one or two extreme points in the interval $[a_2^*, a_1^*]$ according to the value of a_1^*.

As in section 9.2, the first order condition (8.3) is rewritten as an equality $f(\widehat{a_1}) - g(\widehat{a_1}) = 0$ where functions f and g are defined by equations (9.1) and (9.2). By keeping in mind that $\widehat{c_2'} < 0$ and $\widehat{c_2''} > 0$, signs of first derivatives (equations 9.3 and 9.4) are undetermined. That is why the second derivatives of f and g are studied. Their computation require to know the third derivative of the imitation cost:

$$\widehat{c_2'''} = \frac{\partial^3 \widehat{c_2}}{\partial \widehat{a_1^3}} = -\alpha\beta(\beta-1)(\beta-2)(a_1^* - \widehat{a_1})^{\beta-3} > 0$$

The second derivative of function f is given by:

$$\left.\frac{\partial^2 f(\widehat{a_1})}{\partial \widehat{a_1^2}}\right|_{\frac{\partial \Pi_1}{\partial \widehat{a_1}}=0} = \widehat{c_2''}\frac{\sqrt{\Delta} - 2(\widehat{a_1} + 2x + 2\widehat{a_1}(1-\widehat{c_2'}))}{\sqrt{\Delta}} - \sqrt{\Delta}\widehat{c_2'''} + (1-\widehat{c_2'})h(\widehat{a_1}) \tag{9.5}$$

with

$$h(\widehat{a_1}) = \frac{\widehat{a_1}(\widehat{a_1}+4x)[1+4(1-\widehat{c_2'}) - 2\widehat{a_1}\widehat{c_2''}] - [\widehat{a_1} + 2x + 2\widehat{a_1}(1-\widehat{c_2'})]^2}{\sqrt{\Delta^3}}$$

and again $\Delta = \widehat{a_1^2} + 4\widehat{a_1}x$ and $x = \widehat{a_1} + a_2^* + (a_1^*)^2 - (a_2^*)^2 - \widehat{c_2}$.

It can be proven that $f''(\widehat{a_1}) < 0$ if it is demonstrated that $h(\widehat{a_1}) \leq 0$. By simplifying function h, we find:

$$h(\widehat{a_1}) = \frac{-4[x - \widehat{a_1}(1-\widehat{c_2'})]^2}{\sqrt{\Delta^3}} - \frac{2\widehat{a_1}\widehat{c_2''}}{\sqrt{\Delta}} < 0$$

Consequently, function f is concave (monotonic or quasi-concave).

Moreover, the second derivative of g takes the following form:

$$\left.\frac{\partial^2 g(\widehat{a_1})}{\partial \widehat{a_1}^2}\right|_{\frac{\partial \Pi_1}{\partial \widehat{a_1}}=0} = 2\widehat{c_2''} + \widehat{a_1}\widehat{c_2'''} > 0$$

Thus, function g is convex (monotonic or quasi-convex). There are at most two intersections between the concave function f and the convex function g in the interval $]0.5, a_1^*[$ and Π_2 possesses at most two extreme points. If there is one extreme point, this point can be a maximum or a minimum. If there are two extreme points, the first one is necessarily a minimum and the second one a maximum. Finally, the maximum of $\Pi_2(\widehat{a_1})$ can be reached either at a bound of the interval ("deterrence" or "accommodation" equilibria) or inside the interval ("mixed" equilibrium) and the three types of equilibrium may occur.■