



**HAL**  
open science

# Eliciting preferences on multiattribute societies with a Choquet integral

Patrick Meyer, Grégory Ponthière

► **To cite this version:**

Patrick Meyer, Grégory Ponthière. Eliciting preferences on multiattribute societies with a Choquet integral. 2008. halshs-00586244

**HAL Id: halshs-00586244**

**<https://shs.hal.science/halshs-00586244>**

Preprint submitted on 15 Apr 2011

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



**PARIS SCHOOL OF ECONOMICS**  
ÉCOLE D'ÉCONOMIE DE PARIS

**WORKING PAPER N° 2008 - 47**

**Eliciting preferences on multiattribute societies  
with a Choquet integral**

**Patrick Meyer**

**Grégory Ponthière**

**JEL Codes: C44, D69, I31**

**Keywords: Standards of living, indicators, measurement,  
multiattribute value theory, Choquet integral**



**PARIS-JOURDAN SCIENCES ÉCONOMIQUES**  
**LABORATOIRE D'ÉCONOMIE APPLIQUÉE - INRA**



48, Bd JOURDAN – E.N.S. – 75014 PARIS  
TÉL. : 33(0) 1 43 13 63 00 – FAX : 33 (0) 1 43 13 63 10  
[www.pse.ens.fr](http://www.pse.ens.fr)

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE – ÉCOLE DES HAUTES ÉTUDES EN SCIENCES SOCIALES  
ÉCOLE NATIONALE DES PONTS ET CHAUSSÉES – ÉCOLE NORMALE SUPÉRIEURE

# Eliciting preferences on multiattribute societies with a Choquet integral

Patrick Meyer

Institut TELECOM, TELECOM Bretagne  
UMR CNRS 3192 Lab-STICC  
Technopôle Brest Iroise CS 83818  
29238 Brest Cedex 3  
Université européenne de Bretagne, France  
patrick.meyer@telecom-bretagne.eu

Grégory Ponthière

PSE and Ecole Normale Supérieure (Paris)  
Boulevard Jourdan, 48  
75014 Paris, France  
gregory.ponthiere@ens.fr

September 22, 2008

## Abstract

This paper aims at reexamining the construction of indicators of standards of living, by focussing on the challenges raised by the subjectivity and the multidimensionality of living conditions. For that purpose, we propose to apply Choquet integral-based multiattribute value theory to the elicitation, from rankings of multiattribute hypothetical societies, of individual's preferences on different dimensions of living conditions. A simple application of the proposed approach highlights the existence of complementarities and redundancies between different dimensions of standards of living, and reveals a strong heterogeneity of individual preferences on hypothetical societies. We explore also how elicited preferences can be used to cast a new light on the ranking of actual multiattribute societies.

**Keywords:** Standards of living, indicators, measurement, multiattribute value theory, Choquet integral.

JEL CODES: C44, D69, I31.

## 1 Introduction

Are all human societies equally good? Or are standards of living better in some societies than in others? Economists have, since the early stages of the discipline, paid a particular attention to those questions and to the difficulties raised by the measurement of standards of living.<sup>1</sup>

---

<sup>1</sup>Given that the concept of wealth was, under classical political economy, including all that makes a life abundant and enjoyable, the measurement of standards of living in nations consisted firstly in measuring the wealth of nations (see [28]).

Actually, any measurement exercise involves, by definition, the assignment, in an empirical and objective way, of numbers to properties of objects in such a way as to describe these (see [9]). However, the major problem raised by the measurement of standards of living lies in two characteristics of the object to be measured: its subjective nature and its multidimensionality.

As far as the subjective nature of standards of living is concerned, there can be no doubt that some living conditions, although objectively defined, are likely to be valued by individuals in significantly different ways. Those differences reflect the specificities of the subjects of valuations, such as the conception of the good life to which they adhere.<sup>2</sup> But this heterogeneity in how individuals value living conditions is not unproblematic for the measurement of standards of living. This tends to question the relevancy of indicators based on weights that are *not* grounded on people's preferences.<sup>3</sup> Moreover, the heterogeneity of preferences tends also to qualify the extra-value brought by indicators based on some *average* preference parameters.<sup>4</sup>

Regarding the multidimensionality of standards of living, it is widely acknowledged that living conditions are made of various components, including, among other things, the enjoyment of a high purchasing power, of a vast amount of leisure time, and of a long and healthy life. Hence, the measurement of what North [25] calls the economic performance of a society - its capacity to overcome the fundamental problems of scarcity - raises inevitably the question of the weighting of the various things whose scarcity must be overcome. However, that weighting task is more complex than it seems at first glance, as there may exist interactions - i.e. complementarities or redundancies - between different dimensions of living conditions. This possibility of interactions between various components of standards of living casts some doubts on the validity of composite indicators based on a weighted sum of achievements on several dimensions.<sup>5</sup>

The problems raised by the subjectivity and the multidimensionality of living conditions can be well illustrated in the light of the most widely used indicator of standards of living: the United Nations' Human Development Index (HDI)[33], which is constructed by aggregating a life expectancy index, an education index and a GDP index by means of equal weights. Table 1, which shows the ranking of the first 10 countries in the light of the HDI indicator for 2007, exemplifies the major difficulties raised by the measure-

---

<sup>2</sup>The large heterogeneity in the conceptions of the good life may explain the difficulty to estimate, on the basis of people's choices, a unique set of preference parameters compatible with all observed choices (see [4]).

<sup>3</sup>Indicators of that kind include the United Nations' Human Development Index - the HDI - [33], and the Index of Economic Well-Being [26, 27].

<sup>4</sup>Indicators of standards of living based on average preference parameters include 'adjusted' income measures, inclusive of some aspects of living conditions (see [35, 36], [24], and [2]).

<sup>5</sup>Examples of such indicators include HDI [33], the Index of Economic Well-Being [26, 27].

ment of standards of living.<sup>6</sup>

countries	life expectancy index	education index	GDP index	HDI
Iceland	0.941	0.978	0.985	0.968
Norway	0.913	0.991	1.000	0.968
Australia	0.931	0.993	0.962	0.962
Canada	0.921	0.991	0.970	0.961
Ireland	0.890	0.993	0.994	0.959
Sweden	0.925	0.978	0.965	0.956
Switzerland	0.938	0.946	0.981	0.955
Japan	0.954	0.946	0.959	0.953
Netherlands	0.904	0.988	0.966	0.953
France	0.919	0.982	0.954	0.952

Table 1: International ranking, Human Development Index, 2007

Firstly, as this was stressed by Dasgupta [7], the picture provided by the HDI suffers from the arbitrariness of the selected weights. For instance, it is not obvious to see why a weight of  $1/3$  should be assigned to the longevity performance, as this is done in the HDI. Why not a weight of  $1/2$  or  $1/4$ ? Note that this criticism concerns a crucial aspect of the construction of a composite indicator, as the questioning of its weights tends also to question the plausibility of the index as a whole. That criticism is reinforced by the fact that, as shown by Table 1, a small change in the weights - while keeping the same three dimensions - would have a significant impact on the ranking, as countries are clustered in groups of close members.

Secondly, it is not obvious to see why the HDI relies on a simple weighted mean of the three indexes. One may have doubts about the compatibility of that formula with how people value different aspects of living conditions, on the grounds that a weighted mean does not do justice to the existence of complementarities and redundancies between different dimensions of living conditions. For instance, one can question that a country that scores highly on income, satisfactorily on education and badly on longevity is, in total, regarded as exactly as good as a country scoring satisfactorily on all dimensions. It may be the case that there exists a strong positive interaction between purchasing power and longevity, so that one ranks the latter country before the former. But taking those interactions into account would require to depart from weighted sum indicators.

The goal of this paper is to re-examine the construction of measures of standards of living, by paying a particular attention to the challenges raised by the subjective and multidimensional natures of the object to be measured. For that purpose, we propose to use multiattribute value theory (MAVT)[12], in order to extract, from an empirical basis, plausible weights to be assigned to various dimensions of standards of living. Those weights shall be here derived by means of a Choquet integral aggregator [6], which

<sup>6</sup>Sources: *Human Development Report 2007/2008* [23].

is a natural extension of the weighted mean, allowing the possibility of interactions between different dimensions of living conditions.

The relevancy of MAVT for the construction of an indicator of standards of living can be justified as follows. Given that the extra-value brought by a composite indicator of standards of living lies precisely in its capacity to aggregate various dimensions into a single one in a *non-arbitrary* way, the set of weights used in its construction must be based on nothing else than individual preferences. Actually, the proposed methodology allows us to derive, from individual preference orderings over societies characterised by various standards of living, a numerical representation of those preferences, where the weights assigned to the various attributes of societies reflect the intensity of people's subjective concern for those attributes.

The reason why we shall rely on a Choquet integral aggregator comes from its intuitive attractiveness as a simple generalisation of the - most often used - weighted mean. While it is tempting, for simplicity, to represent preferences over multiattribute societies by means of a classical weighted sum (as in the HDI), such an additive representation is likely to be inadequate for the purpose at hand, because this requires individual preferences over multiattribute societies to satisfy the postulate of *mutual preferential independence* among the attributes of societies. Given that such a strong postulate is likely to be violated by individual preferences on multiattribute societies, it makes sense to allow *a priori* the possibility of interactions between the various dimensions or attributes of societies. A natural way to take into account not only the importance of each attribute, but, also, of each subset of attributes, is to consider the representation of individual preferences by means of the Choquet integral, which can be regarded as an intuitive extension of the weighted arithmetic mean.

The rest of this paper is organised as follows. Section 2 presents Choquet integral-based MAVT. Section 3 examines the issues raised by its practical implementation for the elicitation of preferences over multiattribute societies and describes a simple experiment. Section 4 presents the results of this experiment. Conclusions are drawn in the last section.

## 2 Choquet integral-based multiattribute value theory

In this section we briefly describe the general foundations of Choquet integral-based MAVT and the capacity identification problem.<sup>7</sup>

---

<sup>7</sup>The interested reader can refer to [11] for a detailed study on these subjects.

## 2.1 Multiattribute value theory

Let  $X \subseteq X_1 \times \dots \times X_n$ ,  $n \geq 2$ , be a set of objects of interest described by a set  $N := \{1, \dots, n\}$  of decision attributes. MAVT [12] aims at modelling the preferences of a decision maker (DM), represented by a binary relation  $\succeq$  on  $X$ , by means of an *overall value function*  $U : X \rightarrow \mathbb{R}$  such that,

$$x \succeq y \iff U(x) \geq U(y), \quad \forall x, y \in X.$$

In order to situate this discourse in the present context of multiattribute societies, let us note that the decision objects consist here of (possibly hypothetical) societies, whereas the attributes under study represent the different dimensions of standards of living (e.g. consumption, health, environmental quality).

In practice, the overall value function  $U$  is determined through an interactive and incremental process requiring from the DM that he/she expresses his/her preferences over a small subset of selected objects. It is then possible to consider the resulting function  $U$  as a *numerical representation* of the preference relation  $\succeq$  on  $X$ , which can be used in applications as a *model* of the preferences of the DM.

In this study we consider the general *transitive decomposable model* of Krantz et al. [15, 3] in which  $U$  is defined by

$$U(x) := F(u_1(x_1), \dots, u_n(x_n)), \quad \forall x = (x_1, \dots, x_n) \in X, \quad (1)$$

where the functions  $u_i : X_i \rightarrow \mathbb{R}$  are called the *marginal value functions* and  $F : \mathbb{R}^n \rightarrow \mathbb{R}$ , non-decreasing in its arguments, is called the *aggregation function*. As far as the value functions are concerned, for any  $x \in X$ , the quantity  $u_i(x_i)$  can be interpreted, in short, as a measure of the ‘satisfaction’ of the value  $x_i$  for the DM.

The exact form of the overall value function  $U$  depends on the particular choice problem at hand. If *mutual preferential independence* (see e.g. [37]) among the attributes can be assumed, it is frequent to consider that the function  $F$  is additive and takes the form of a weighted sum. If interaction phenomena among attributes have to be taken into account, it has been proposed to substitute a monotone set function on  $N$ , called *capacity* [6] or *fuzzy measure* [31], to the weight vector involved in the calculation of weighted sums. Intuitively, this allows to take into account not only the importance of each attribute for the DM, but, also, the importance of each subset of attribute. In such a context, a natural extension of the weighted arithmetic mean is the *Choquet integral* with respect to (w.r.t.) the defined capacity.<sup>8 9</sup>

<sup>8</sup>For an axiomatic derivation of the Choquet integral, see [18]

<sup>9</sup>Note that this specification of the DM’s preferences requires that the marginal value functions are *commensurable*, i.e.,  $u_i(x_i) = u_j(x_j)$  if and only if for the DM, the object  $x$  of  $X$  is satisfied to the same extent on attributes  $i$  and  $j$ .

## 2.2 The Choquet integral as an aggregation operator

As already mentioned in Section 2.1, *capacities* [6] can be regarded as generalisations of weighting vectors involved in the calculation of weighted sums.

**Definition 2.1.** *A capacity on  $N$  is a set function  $\mu : \mathcal{P}(N) \rightarrow [0, 1]$  satisfying the following conditions<sup>10</sup>:*

- (i)  $\mu(\emptyset) = 0, \mu(N) = 1,$
- (ii) *for any  $S, T \subseteq N, S \subseteq T \Rightarrow \mu(S) \leq \mu(T).$*

In particular, a capacity  $\mu$  on  $N$  is said to be *additive* if  $\mu(S \cup T) = \mu(S) + \mu(T)$  for all disjoint subsets  $S, T \subseteq N$ . In the framework of aggregation, for each subset of attributes  $S \subseteq N$ , the number  $\mu(S)$  can be interpreted as the *weight* or the *importance* of  $S$ .

Having defined a capacity  $\mu$  on  $N$ , we can now define the Choquet integral w.r.t.  $\mu$  in the context of MAVT.

**Definition 2.2.** *The Choquet integral of an alternative  $x$  represented by the vector of partial values  $u(x) := (u_1(x_1), \dots, u_n(x_n))$  w.r.t. a capacity  $\mu$  on  $N$  is defined by*

$$C_\mu(u(x)) := \sum_{i=1}^n u_{\sigma(i)}(x_{\sigma(i)}) [\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i+1)})],$$

where  $\sigma$  is a permutation on  $N$  such that  $u_{\sigma(1)}(x_{\sigma(1)}) \leq \dots \leq u_{\sigma(n)}(x_{\sigma(n)})$ . Also,  $A_{\sigma(i)} := \{\sigma(i), \dots, \sigma(n)\}$ , for all  $i \in \{1, \dots, n\}$ , and  $A_{\sigma(n+1)} := \emptyset$ .

As an aggregation operator, the Choquet integral w.r.t.  $\mu$  can be considered as taking into account interaction phenomena among attributes, that is, *complementarity* or *substitutivity* among elements of  $N$  modeled by  $\mu$  [18].

Note that the Choquet integral generalises the weighted arithmetic mean in the sense that, as soon as the capacity is additive, which intuitively coincides with the independence of the attributes, it collapses into a weighted arithmetic mean.

Let us now consider an equivalent representation of the Choquet integral which allows us to introduce the concept of  $k$ -additivity, directly linked to the complexity of the model. The *Möbius transform* of a capacity  $\mu$  is a set function  $m_\mu : \mathcal{P}(N) \rightarrow \mathbb{R}$  defined by

$$m_\mu(S) = \sum_{T \subseteq S} (-1)^{s-t} \mu(T), \quad \forall S \subseteq N.$$

---

<sup>10</sup>In the sequel, in order to avoid a heavy notation, we will omit braces for singletons and pairs, e.g., by writing  $\mu(i)$ ,  $N \setminus ij$  instead of  $\mu(\{i\})$ ,  $N \setminus \{i, j\}$ . Furthermore, cardinalities of subsets  $S, T, \dots$ , will be denoted by the corresponding lower case letters  $s, t, \dots$ . Also, as classically done, the asymmetric part of a binary relation  $\succeq$  will be denoted by  $\succ$  and its symmetric part by  $\sim$ . Finally, the power set of  $N$  will be denoted by  $\mathcal{P}(N)$ .



In terms of the Möbius representation of a capacity  $\mu$  on  $N$ , for any  $u(x) = (u_1(x_1), \dots, u_n(x_n)) \in \mathbb{R}^n$ , the Choquet integral of an alternative  $x$  w.r.t.  $\mu$  is given by

$$C_{m_\mu}(x) = \sum_{T \subseteq N} m_\mu(T) \bigwedge_{i \in T} x_i$$

where the symbol  $\wedge$  denotes the minimum operator and  $m_\mu$  respects constraints derived from the boundary and monotonicity conditions of  $\mu$  [5].<sup>11</sup>

A capacity  $\mu$  on  $N$  is completely defined by the knowledge of  $2^n - 2$  coefficients. Such a complexity may be prohibitive in certain applications. The fundamental notion of *k-additivity* proposed by Grabisch [10] enables to find a trade-off between the complexity of the capacity and its modeling ability.

**Definition 2.3.** *Let  $k \in \{1, \dots, n\}$ . A capacity  $\mu$  on  $N$  is said to be  $k$ -additive if its Möbius representation satisfies  $m_\mu(T) = 0$  for all  $T \subseteq N$  such that  $t > k$  and there exists at least one subset  $T$  of cardinality  $k$  such that  $m_\mu(T) \neq 0$ .*

It is easy to check that the notion of 1-additivity coincides with that of additivity. It is also straightforward that a  $k$ -additive capacity ( $k < n$ ) is completely defined by the knowledge of  $\sum_{l=1}^k \binom{n}{l}$  coefficients.

### 2.3 Analysis of the aggregation

For a better comprehension of the interaction phenomena modeled by the underlying capacity, several numerical indices can be computed [19, 20]. Here, two of them are presented.

**Importance index** The overall importance of an attribute  $i \in N$  can be measured by means of its Shapley value [29], which is defined by

$$\phi_\mu(i) := \sum_{T \subseteq N \setminus i} \frac{(n-t-1)!t!}{n!} [\mu(T \cup i) - \mu(T)].$$

Recall that for each subset of attributes  $S \subseteq N$ ,  $\mu(S)$  can be interpreted as the *importance* of  $S$  in the decision problem. Consequently, the Shapley value of  $i$  can be thought of as an average value of the *marginal contribution*  $\mu(T \cup i) - \mu(T)$  of attribute  $i$  to a subset  $T$  not containing it.

**Interaction index** In order to intuitively approach the concept of interaction, consider two attributes  $i$  and  $j$  such that  $\mu(ij) > \mu(i) + \mu(j)$ . Clearly, the previous inequality models a *complementary* effect between  $i$

---

<sup>11</sup>The notation  $C_{m_\mu}$ , which is equivalent to the notation  $C_\mu$ , is used to emphasize the fact that the Choquet integral is here computed w.r.t. the Möbius transform of  $\mu$ .

and  $j$ . Similarly, the inequality  $\mu(ij) < \mu(i) + \mu(j)$  suggests that  $i$  and  $j$  interact in a *redundant* or *substitutive* way. Finally, if  $\mu(ij) = \mu(i) + \mu(j)$ , it seems natural to consider that attribute  $i$  and  $j$  do not interact, i.e., that they have *independent* roles.

Murofushi and Soneda [22] suggest to measure the average interaction between two attributes  $i$  and  $j$  by means of the following *interaction index*:

$$I_\mu(ij) := \sum_{T \subseteq N \setminus ij} \frac{(n-t-2)!t!}{(n-1)!} [\mu(T \cup ij) - \mu(T \cup i) - \mu(T \cup j) + \mu(T)].$$

The quantity  $I_\mu(ij)$  can be interpreted as a measure of the *average* marginal interaction between  $i$  and  $j$ . An important property is that  $I_\mu(ij) \in [-1, 1]$  for all  $ij \subseteq N$ , the value 1 (resp. -1) corresponding to maximum complementarity (resp. substitutivity) between  $i$  and  $j$  [10].

## 2.4 The capacity identification problem

As the capacity involved in the Choquet integral is defined (in general) by  $2^n - 2$  coefficients, it is hard to imagine that the DM is able to provide these parameters and therefore this complexity requires that the capacity is identified from some learning data. In this section we briefly discuss this *identification problem*.

For that purpose, we shall assume here that the marginal value functions have been determined beforehand. Hence, the next step consists in identifying a capacity, if it exists, such that the Choquet integral w.r.t. this capacity numerically represents the preferences of the DM (see Eq. (1)).

We suppose here that the DM is able to express such a piece of preferential information, in particular on a finite and usually small subset  $O$  of the set  $X$  of objects of interest. The set  $O$  is usually composed either of available objects or of selected, potentially fictitious objects.

These *initial preferences*, from which the capacity is to be determined, can take the form of:

- a partial weak order  $\succeq_O$  over  $O$  (ranking of the available objects);
- a partial weak order  $\succeq_N$  over  $N$  (ranking of the importance of the attributes);
- a partial weak order  $\succeq_P$  on the set of pairs of attributes (ranking of interactions);
- etc.

As shown in [11], most of the identification methods proposed in the literature can be stated under the form of an optimisation problem (where the constraints are derived from the initial preferences of the DM). Note

that the output of the different identification methods, as well as the resulting numerical representations of the DM's preferences, differ w.r.t. to some objective functions. A solution to these problems is a general capacity defined by  $2^n - 1$  coefficients. For large problems, both for computational and simplicity reasons, it may be preferable to restrict the set of possible solutions to  $k$ -additive capacities,  $k \in \{1, \dots, n\}$ , typically  $k = 2$  or  $3$ .

Of course, the optimisation problem may be infeasible if the constraints derived from the DM's preferences are inconsistent or if they are incompatible with some basic assumptions on the Choquet integral as an aggregation operator (see e.g. [38] concerning comonotonic contradictory tradeoffs).

## 2.5 A geometric illustration: preferences on two-dimensional societies

To illustrate the diversity of preferences that can be represented on the basis of Choquet integral-based MAVT, we conclude this section by showing geometrical representations of some - *a priori* possible - preferences patterns on two-dimensional societies. For that purpose, we consider societies characterised by their performance in terms of consumption (measured in euros per month) and longevity (measured in terms of life expectancy at age 65).

Throughout this illustration, we focus on three DMs, named  $A$ ,  $B$ , and  $C$ , whose preferences, in terms of capacities, on two-dimensional societies in the consumption/longevity space (con/lon) are summarized by Table 2. For simplicity, marginal value functions are here assumed to take the same *s-shaped* form for the three DMs<sup>12</sup>

DM	value function	$\mu(\emptyset)$	$\mu(\text{con})$	$\mu(\text{lon})$	$\mu(\text{con}, \text{lon})$
$A$	s-shaped	0	0.1	0.2	1
$B$	s-shaped	0	0.2	0.8	1
$C$	s-shaped	0	0.8	0.9	1

Table 2: Description of preferences of DM  $A$ ,  $B$  and  $C$

Note that the preferences of the three DMs differ regarding the existence and form of interactions between consumption and longevity. While  $A$  treats consumption and longevity as complements (i.e.  $\mu(\text{con}, \text{lon}) > \mu(\text{con}) + \mu(\text{lon})$ ),  $B$  perceives no interaction between the two dimensions (i.e.  $\mu(\text{con}, \text{lon}) = \mu(\text{con}) + \mu(\text{lon})$ ), whereas  $C$  regards consumption and

<sup>12</sup>By *s-shaped* marginal value function, we mean a continuous piecewise linear function that is convex for an achievement level that lies *below* some (intermediate) reference level, and concave for an achievement level lying *beyond* the reference level on the attribute under study. Reference levels are here fixed to 1300 euros per month for consumption, and a life expectancy at age 65 of 19 years. See Section 3.1.4 for a more detailed discussion on these functions.

longevity as redundant  
(i.e.  $\mu(\text{con}, \text{lon}) < \mu(\text{con}) + \mu(\text{lon})$ ).

The graphs of Figure 1 show the indifference maps of, respectively, DMs A, B and C, under the preferences of Table 2 (the vertical axis indicates the overall value of a society by means of a Choquet integral). While a higher consumption (resp. longevity) leads, for a fixed longevity (resp. consumption), to a higher utility level on *all* graphs, it should be stressed, nonetheless, that the indifference maps of the three DMs are quite different.

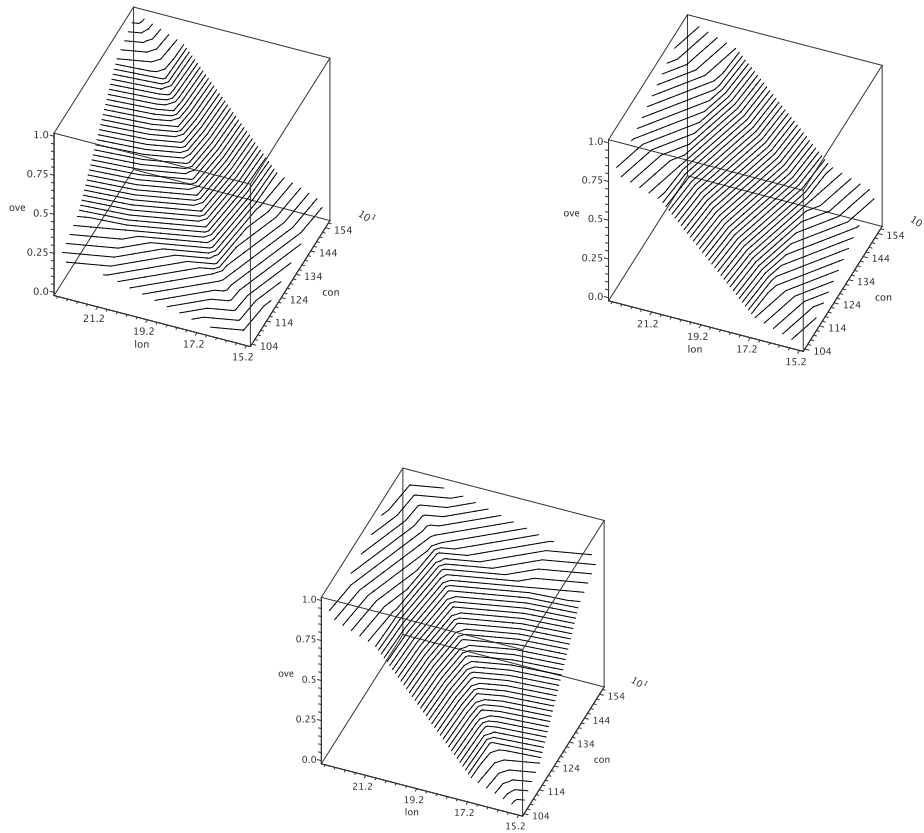


Figure 1: Indifference maps for the 3 hypothetical DM's

The first graph of Figure 1 exemplifies the existence of *complementarities* between consumption and longevity for DM A, in the sense that a high utility level can be achieved by A only if *both* consumption and longevity are sufficiently large. For A, there is little way in which some consumption can be used to ‘compensate’ life-years. Similarly, a low consumption can hardly be compensated by additional life-years. Compensation is hard, as

the two dimensions of standards of living complement each others: a good life requires to score well on both consumption and longevity.

However, as shown on the second graph of Figure 1, that observation does not hold for DM  $B$ , for whom there exists no interaction between consumption and longevity. According to  $B$ , it is generally possible to compensate some low consumption level by more life-years, in such a way as to reach a high welfare level despite a low consumption. The opposite compensation is also, in theory, possible, even though the additional consumption required to compensate a low longevity must be here significantly large (given the larger weight given to longevity by DM  $B$ ).

Finally, the third graph of Figure 1, which shows the indifference map of DM  $C$ , illustrates the second kind of interactions between dimensions of living conditions: *redundancies*. Consumption and longevity are here redundant in the sense that it is better to score extremely highly on *only one* of these, rather than to score satisfactorily on *both* of these. Starting from the middle of this graph, converting some life-years into consumption (or some consumption into life-years), is, for DM  $C$ , not only possible, but, also, largely welfare-improving, as what roughly matters is the best score *among* the two dimensions, rather than the scores on both of these.

### 3 A simple application

Let us now illustrate, by means of a simple application, how Choquet integral-based MAVT can cast a new light on the measurement of standards of living. For that purpose, we develop here a simple classroom survey entitled ‘*Making the World Better: a simple classroom experiment*’, which consists of submitting to a group of respondents a small size standardised questionnaire asking them to rank hypothetical multidimensional societies. Hence, from the answers of that survey, we elicitate individual preferences via a Choquet integral-based MAVT model. Note that in that context, each respondent plays the role of the DM of Section 2.

#### 3.1 Description of the application

This section aims at describing how one can deal with the various implementation problems raised by the attempt to construct a preferences-based indicator of standards of living.

##### 3.1.1 Dimensions

A first, central problem concerns the *selection* of a set of dimensions relevant for the measurement of standards of living. Given the large number of determinants of living conditions, the task of selecting a particular set of them is far from straightforward. However, that difficulty is here reinforced

by the constraint imposed by the necessity to obtain *well-thought* answers, that is, well-thought orderings over multiattribute societies. As it is well-known among psychologists, human-beings can handle at most seven plus or minus two items at the same time (see e.g. [21]), so that only a limited number of attributes can be selected for the purpose at hand.

In order to simplify the task of respondents, the hypothetical societies to be ranked here are merely 5-dimensional. The selected dimensions are: *consumption*, *environmental quality*, *health*, *leisure time* and *longevity*. This exclusive focus on those five dimensions involves a strong simplification of what real living conditions consist of. However, this choice can be justified on the ground that a significant achievement on each of those attributes is required to realise anyone's life plan.<sup>13</sup>

### 3.1.2 Attributes

For each of those 5 dimensions, a particular indicator has to be chosen. Here again, various options are open. However, it seems plausible to require that the selection of indicators should satisfy two rules. First, indicators should be sufficiently *general*, in order to capture the dimension under study, whatever the precise conception of a good life to which one adheres. Second, indicators should also be sufficiently *widespread* and *understood*, so that the DM can easily express preferences on objects described by them.

In the light of those two rules, we selected the following indicators for each aspect of standards of living. Consumption is measured here in terms of euros per month, environmental quality is expressed in terms of CO<sub>2</sub> tonnes per capita, health is measured by healthy life expectancy at age 65 (in years), leisure time is measured in terms of working time per week (in hours), while longevity is measured in terms of life expectancy at age 65 (in years). Clearly, other choices could have been made, but the selected indicators have the virtue of being simple and convenient.

At this stage, it cannot be overemphasized that the selection of a particular set of indicators might significantly influence the shape of an individual's preferences on multiattribute societies, through the existence of well-known framing effects [32]. However, the possibility of such effects can hardly be avoided, so that the potential existence of framing effects is a price to pay for this kind of application.<sup>14</sup>

---

<sup>13</sup>For instance, whatever the individual conception of a *good* life is, some minimal level of health seems necessary. The same can be said to some extent for the remaining four dimensions.

<sup>14</sup>Note that the best we could do would be to compare, in a second stage, the outcome of the preferences elicitation exercise under the selected indicators with the one under alternative indicators for each dimension under study. Given that such an exercise goes beyond the goal of this simple application, we shall not consider that problem here.

### 3.1.3 Societies

Regarding the construction of multidimensional societies to be ranked, let us first notice that two distinct options are *a priori* open. On the one hand, one may ask respondents to rank *actual* societies, i.e. societies whose attributes coincide with the ones exhibited by some countries in the real world; on the other hand, one may ask respondents to rank *hypothetical* societies, whose performances on each dimension under study do not necessarily coincide with any observed real economy.

Throughout this classroom survey, we choose to avoid submitting well identified actual societies, on the grounds that we would like the expressed preferences to tell us something about how respondents care about the various dimensions of standards of living under study, and not reflect some omitted variables, which may make them prefer one stereotypical society over another.<sup>15</sup>

Given that there exists a large number of hypothetical multidimensional societies to be ranked, an important task consists of restricting the set of decision objects to a subset of relatively plausible societies, i.e. ones that are not ‘too different’ from existing societies. The intuition behind that restriction goes as follows: on the one hand, preferences on too utopic societies could hardly be well-thought; on the other hand, preferences on societies that cannot be reached have little relevancy.

Hence, hypothetical societies are here constructed by departing from a *society of reference*, whose attributes take the levels that are roughly prevailing nowadays in the area of the world under study (contemporary Western Europe). On each attribute, four additional levels of achievements are introduced. The level ‘bad’ (resp. ‘very bad’) amounts to an achievement of 90 (resp. 80) percents of the standard achievement, whereas the level ‘good’ (resp. ‘very good’) amounts to an achievement of 110 (resp. 120) percents of the standard achievement. Our concern for societies lying in a *plus* or *minus* 20 percents interval around the reference society can be justified on the grounds that we would like to focus on societies that are, at least in the medium run, plausibly reachable.<sup>16</sup> The outcome of that construction of hypothetical societies is shown in Table 3.

### 3.1.4 Marginal value functions

After having defined the scale on each attribute describing societies, the following task consists of determining, on an empirical basis, the marginal

---

<sup>15</sup>The use of hypothetical societies has also the virtue to allow us to avoid difficult selection problems among existing societies, which would arise, for instance, if we submitted a subset of existing societies without indicating their names.

<sup>16</sup>Note that our construction of hypothetical societies exhibits another virtue: the relative variations on each dimension are here of *equal* sizes, which consists of some kind of clarifying framework helping the DM in the - inherently complex - ranking task.

Attributes	'very bad' society (-20 %)	'bad' society (-10 %)	society of reference	'good' society (+ 10 %)	'very good' society (+ 20 %)
<b>Consumption</b> (€ per month)	1040 €	1170 €	1300 €	1430 €	1560 €
<b>Environment</b> (CO <sub>2</sub> tonnes per capita)	12 tonnes	11 tonnes	10 tonnes	9 tonnes	8 tonnes
<b>Health</b> (healthy life expectancy at age 65)	12 years	13.5 years	15 years	16.5 years	18 years
<b>Labour time</b> (working hours per week)	45.6 hours	41.8 hours	38 hours	34.2 hours	30.4 hours
<b>Longevity</b> (life expectancy at age 65)	15.2 years	17.1 years	19 years	20.9 years	22.8 years

Table 3: Construction of hypothetical societies

value function on each attribute. Recall that in the context of Choquet integral-based MAVT these functions need to be commensurable (see Section 2.1). Such value functions can be determined by using the extension of the MACBETH methodology [1] proposed in [17]. Nevertheless, this task is not trivial and can take a large percentage of the time dedicated to the preference elicitation procedure. Moreover, such a construction may not be compatible with a classroom experiment or any large scale survey, both generally strongly constrained by time. Therefore, we decided to concentrate here on a particular form of marginal value functions, which we call the s-shaped form.

The s-shaped marginal value function is a continuous piecewise linear function which has the specificity to be convex for an achievement level that lies *below* the reference level, but concave for an achievement level lying *beyond* the reference level on the attribute under study. Thus, s-shaped marginal value functions satisfy Gossen's First Law (i.e. the law of the decreasing marginal utility) for gains w.r.t. reference achievements, and exhibit decreasing marginal welfare losses under a deterioration of achievements, in the sense that small departures from the reference society lead to marginal welfare losses of decreasing sizes. Although those properties are plausible, assuming s-shaped marginal value function for all attributes under study is not a weak assumption, so that we shall, in Section 4.3, examine the sensitivity of our results to the postulated marginal value functions. Table 4 summarises the initial, connection and terminal points of the four segments constituting this value function. <sup>17</sup>

<sup>17</sup>Note that via this construction, the society of reference has a satisfaction degree of 0.5 on each of the attributes.



value function	'very bad'	'bad'	'reference'	'good'	'very good'
s-shape	0	0.125	0.5	0.875	1

Table 4: S-shape value function

### 3.1.5 The model

As mentioned in Section 2.4, the determination of the parameters of the Choquet integral-based MAVT model requires the selection of a particular identification method. There exist various methods to determine the capacity underlying a Choquet integral in such a way as to be compatible with the preferences expressed by the respondents (see [11]). In the rest of this paper, we concentrate on a capacity determined by an identification method that minimises the variance of the capacity (see [13] for further details). The reason why we shall adhere to this identification method is the following.

As discussed in [14], among all the feasible Choquet integrals compatible with the rankings provided by a given subject, the Choquet integral w.r.t. the minimum variance of the capacity is the one that will exploit the most, on average, its arguments. Note that choosing the Choquet integral w.r.t. this capacity amounts also to choosing the Choquet integral that will be the closest to the simple arithmetic mean. Hence, Choquet integrals estimated according to that method can be regarded as providing the benchmark case w.r.t. which the plausibility of indexes of standards of living based on equal weights (e.g. HDI) can be best assessed. Finally, note that the objective function of this identification method is strictly convex, which leads to a unique solution, if any.

We will choose numerical representations with the lowest possible level of  $k$ -additivity (see Section 2.2). This allows first to determine if an additive model exists (1-additivity), and second to have the simplest possible representation, which should be as general as possible.

## 3.2 Description of the questionnaire

The questionnaire submitted to the respondents has a quite simple structure.<sup>18</sup> The first section presents the goal of the survey (i.e. the exploration of the subject's preferences on multiattribute societies), while its second section explains what is required from respondents (i.e. ranking the proposed societies according to their own preferences).

Section 3 of the questionnaire describes the multiattribute societies to be compared, and localises these w.r.t. the society of reference (see Table 3). Section 3 ends with a simple example of what is expected from respondents.

<sup>18</sup>See the Appendix for a copy of an original blank questionnaire

Section 4, which includes the hypothetical societies to be ranked, is divided in two parts. The first part of Section 4 presents ten groups of six societies to be ranked. Each group presents societies with two attributes fixed at their reference levels and three attributes taking various values. For convenience, each group is labelled by the names of the varying attributes. Note that the structure of the marginal improvements and deteriorations w.r.t. the society of reference is identical across the ten groups. The second part of Section 4 asks each respondent to rewrite, in a new table, the ten first-ranked societies from the previous part, and to rank the top five ones. It is also asked to do the same with the worst societies of the first part.

The last section of the questionnaire includes a small set of control questions including, among other things, an estimation of one's degree of reliance in the provided answers.

### 3.3 Description of the group of respondents

The group of respondents consists of 14 students of the Master in Environmental Sciences and Management of the University of Liège (Belgium) during the academic year 2007-2008. Some characteristics are summarised in Tables 5 and 6.<sup>19</sup>

resp.	gender	age	nationality	degree of confidence in answers
01	female	22	Belgian	satisfactory
02	female	22	Belgian	satisfactory
03	male	23	Belgian	high
04	male	22	Lebanese	high
05	female	21	Belgian	low
06	male	38	Congolese	NA
07	male	34	Burundese	satisfactory
08	female	24	Luxembourgish	satisfactory
09	male	22	Belgian	satisfactory
10	male	NA	Cameroon	NA
11	male	33	Beninese	NA
12	male	30	Peruvian	satisfactory
13	male	30	Ivory Coast	satisfactory
14	male	28	Luxembourgish	high

Table 5: Basic characteristics of respondents

As shown in Table 5, our group of respondents includes people who differ not only on their gender, age and nationality, but, also, in their degree of confidence in the answers provided.<sup>20</sup> While 10 subjects out of 14 report a high or satisfactory degree of confidence in the rankings provided, only 3

<sup>19</sup>Both tables are based on answers given by respondents in the control questionnaire.

<sup>20</sup>The last column of Table 5 reports the degree of confidence of respondents (high / satisfactory / low), as assessed by themselves when answering the control questionnaire after having ranked hypothetical societies. Note that the entry NA corresponds to non-available information.

respondents report a high degree of confidence. This suggests that ranking hypothetical multidimensional societies is far from an easy activity, as trade-offs between alternatives are sometimes quite hard. Note, however, that only one respondent (respondent 05) reports an unsatisfactory confidence in her answers. Given that such a self-evaluation of one's answers reveals a significant dissatisfaction of that respondent with her answer, combined with the fact that no Choquet integral-based numerical representation of her preferences could be found, we shall, in the presentation of the results, exclude that element of the group.<sup>21</sup>

Table 6 shows how each respondent assesses his/her own actual standards of living. Here again, the group, although small, exhibits a strong heterogeneity: respondents tend to assess their own current living conditions in quite distinct manners. It should be stressed here that how respondents regard their own living conditions may, of course, have influenced the precise way in which they ranked hypothetical societies. For instance, it might be the case that respondents pay more attention to aspects of life that are currently regarded as bad. Hence, the pieces of information contained in Table 6 can be useful for the interpretation, at the individual level, of preferences on multiattribute societies.

resp.	con.	env.	hea.	lab.	lon.
01	bad	bad	satisfactory	bad	good
02	satisfactory	bad	good	bad	good
03	satisfactory	satisfactory	good	satisfactory	satisfactory
04	satisfactory	satisfactory	good	satisfactory	good
05	NA	satisfactory	good	NA	bad
06	satisfactory	good	NA	satisfactory	satisfactory
07	satisfactory	satisfactory	good	good	satisfactory
08	good	satisfactory	satisfactory	satisfactory	satisfactory
09	satisfactory	good	good	satisfactory	good
10	NA	NA	NA	NA	NA
11	good	satisfactory	good	good	good
12	satisfactory	good	satisfactory	satisfactory	satisfactory
13	satisfactory	satisfactory	satisfactory	satisfactory	bad
14	bad	good	good	bad	satisfactory

Table 6: Respondents' subjective valuations of their current standards of living

<sup>21</sup>Actually, examining respondent 05's questionnaire reveals that the non existence of a representation of respondent 05's preferences by means of a Choquet integral is caused by the mere fact that some of respondent 05's answers, both in the first and the second part of the questionnaire, violate a basic *dominance condition* .

## 4 Results

Let us now show how Choquet integral-based MAVT developed in Section 2 can be used to elicitate the preferences of our respondents on the hypothetical societies proposed in the questionnaire described in Section 3.

For that purpose, we shall, in a first stage, concentrate on the second part of the questionnaire, and leave its first part – and the issue of the compatibility between the two parts of the questionnaire – for Section 4.4. The reason why we proceed in that way is that we would like to concentrate first on the rankings of hypothetical societies with which respondents are (the most) familiar, that is, on decision objects that respondents have been (more) used to manipulate. Given that the rankings provided in the second part of the questionnaire concern hypothetical societies that were ranked as best or worst ‘of their group’ in the first part of the questionnaire, it is expected that respondents have a good idea of what those societies are, and how they value these.

Let us now present the results of our classroom experiment. For each of the 13 considered respondents, a Choquet integral-based numerical representation of the preferences expressed in Part 2 of the questionnaire can be found, via the ‘minimum variance’ capacity identification method. Table 7 summarises the minimal level of  $k$ -additivity required for each respondent.

01	02	03	04	06	07	08	09	10	11	12	13	14
2	3	2	3	2	2	4	3	2	2	2	4	2

Table 7: Levels of  $k$ -additivity for the 13 respondents

Table 7 clearly shows that the additive model has to be rejected for each of the 13 respondents, and that for some of them, even a 4-additive model has to be used.

### 4.1 Relative weights of the different dimensions

Let us now have a brief look at the relative importance of each attribute of the hypothetical societies under comparison, for each respondent. For that purpose, Table 8 presents the values of the Shapley indexes for each respondent. Recall that the Shapley value of an attribute, as defined in Section 2.3, indicates the average value of the marginal contribution of that attribute to a subset of attributes not containing it.

Table 8 invites two main observations. First, the Shapley indexes of distinct attributes take, generally, significantly different values. Hence, indicators of standards of living assigning equal weights to all dimensions of living conditions (e.g. HDI) seem, in the light of our group of respondents,

resp.	con.	env.	hea.	lab.	lon.	resp.	con.	env.	hea.	lab.	lon.
<b>01</b>	0.26	0.24	0.15	0.20	0.15	<b>09</b>	0.19	0.24	0.17	0.22	0.18
<b>02</b>	0.21	0.12	0.16	0.31	0.20	<b>10</b>	0.20	0.22	0.21	0.23	0.15
<b>03</b>	0.19	0.19	0.20	0.23	0.19	<b>11</b>	0.23	0.19	0.21	0.20	0.16
<b>04</b>	0.29	0.16	0.26	0.15	0.15	<b>12</b>	0.20	0.16	0.19	0.18	0.27
<b>06</b>	0.20	0.21	0.18	0.24	0.18	<b>13</b>	0.22	0.17	0.15	0.24	0.21
<b>07</b>	0.20	0.22	0.18	0.20	0.20	<b>14</b>	0.22	0.21	0.13	0.20	0.24
<b>08</b>	0.23	0.23	0.18	0.18	0.18						

Table 8: The values of the Shapley indexes for the 13 respondents

to misrepresent the complexity of human preferences on multiattribute societies. That observation is reinforced by the fact that capacities are here estimated in such a way as to minimise their variance. Given that significant differentials exist across Shapley indexes despite the ‘minimum variance’ identification method, there can be no doubt that the equal weights assumption must be, on the basis of our results, rejected.

Second, Table 8 shows also that there exists a significant heterogeneity across respondents. For instance, respondent 01 assigns a large weight to consumption (0.26) and a low weight to longevity (0.15), whereas respondent 12 does the opposite, and assigns a quite large weight to longevity (0.27) and a lower weight to consumption (0.20). Moreover, respondents differ also regarding the variance of the weights assigned to different attributes of societies. While respondent 07 is the one whose preferences are the most compatible with an ‘equal weight’ index of standards of living, respondent 04’s answers clearly reveal that his preferences on multiattribute societies cannot be represented by such an index.

Another way to look at the heterogeneity of preferences is to consider how respondents differ regarding the ranking of the importance of the different attributes of societies. For that purpose, Table 9 shows the ranks of the Shapley indexes of attributes constructed on the basis of Table 8.<sup>22</sup>

resp.	con.	env.	hea.	lab.	lon.	resp.	con.	env.	hea.	lab.	lon.
<b>01</b>	5.0	4.0	1.5	3.0	1.5	<b>09</b>	3.0	5.0	1.0	4.0	2.0
<b>02</b>	4.0	1.0	2.0	5.0	3.0	<b>10</b>	2.0	4.0	3.0	5.0	1.0
<b>03</b>	2.0	2.0	4.0	5.0	2.0	<b>11</b>	5.0	2.0	4.0	3.0	1.0
<b>04</b>	5.0	3.0	4.0	1.5	1.5	<b>12</b>	4.0	1.0	3.0	2.0	5.0
<b>06</b>	3.0	4.0	1.5	5.0	1.5	<b>13</b>	4.0	2.0	1.0	5.0	3.0
<b>07</b>	3.0	5.0	1.0	3.0	3.0	<b>14</b>	4.0	3.0	1.0	2.0	5.0
<b>08</b>	4.5	4.5	2.0	2.0	2.0						

Table 9: Fractional ranking of the Shapley indexes for the 13 respondents

<sup>22</sup>Rank 5 is assigned to the most important attribute, whereas rank 1 is assigned to the least important one. Items that are tied receive the same ranking number, which is the mean of what they would have under ordinal rankings.

Table 9 suffices to illustrate the large variety of preferences across individuals. For some respondents (i.e. respondents 02, 03, 06, 10 and 13), the most important attribute of a society is the amount of leisure time, whereas, for others (i.e. respondents 01, 04, 08 and 11), this is the consumption level, or the quality of the natural environment (i.e. respondents 07, 08 and 09), or even longevity (i.e. respondents 12 and 13). The same observations could be made regarding the attribute that seems to be the least important. There is thus little homogeneity of preferences across individuals.

Nevertheless, one can observe that the longevity and health dimensions have, quite often, lower ranks than the other ones. This could be explained by the fact that respondents are quite young (see Table 5), so that matters of longevity and health are not (yet) their main concerns.

## 4.2 Interactions of the different dimensions

Let us now consider the issue of the existence of interactions between the different attributes.

For that purpose, Table 10 presents, for the 13 respondents, the number of strictly positive and strictly negative interactions between attributes. Concretely, a positive interaction between two attributes  $i$  and  $j$  reveals the existence of a complementarity between these, in the sense that the welfare level when there are high achievements on both attributes  $i$  and  $j$  exceeds the sum of the welfare level from achieving highly on attribute  $i$  only plus the welfare level from achieving highly on attribute  $j$  only. On the contrary, a negative interaction between two attributes  $i$  and  $j$  reveals the existence of some redundancy between these, in the sense that achieving highly on both attributes  $i$  and  $j$  brings a welfare level that is inferior to the sum of the welfare level when achieving highly on attribute  $i$  only plus the welfare level from achieving highly on attribute  $j$  only.

	complementarity ( $> 0$ )				redundancy ( $< 0$ )			
	env.	hea.	lab.	lon.	env.	hea.	lab.	lon.
con.	6	4	6	7	4	8	6	5
env.		6	7	2		6	3	7
hea.			6	0			5	10
lab.				6				7

Table 10: Number of strictly positive and strictly negative interactions

Although there exists, here again, a significant heterogeneity across subjects, Table 10 can be used to make some general observations on the structure of individual preferences.<sup>23</sup> Actually, Table 10 reveals the existence of a large number of interactions between attributes, interactions which can

<sup>23</sup>Full interaction matrices for each respondent are presented in the Appendix.

be either strictly positive or strictly negative. The existence of those interactions suggests, again, that preferences of respondents on hypothetical societies cannot be represented by a weighted sum index.

Let us now have a closer look at the interactions between attributes. Regarding positive interactions, the most largely observed complementarities are, on the one hand, between consumption and longevity, and, on the other hand, between leisure and environmental quality (each of these being observed on 7 respondents). Those complementarities can be interpreted as follows: for 7 respondents, scoring highly on *both* consumption and longevity brings, *ceteris paribus*, a welfare level that exceeds the sum of the welfare levels reached by scoring highly on *only one* of those dimensions. In other words, there exists a significant welfare *premium* from scoring highly on *both* consumption and longevity. Similarly, leisure time and environmental quality are, for 7 respondents, complementing each others as determinants of standards of living.

Besides positive interactions, there exist also some negative interactions, revealing the existence of some redundancy between attributes. The most observed redundancy is here the one between health and longevity (observed for 10 respondents). This means that, far from being perceived as complements, health and longevity are here regarded, to some extent, as ‘twice the same thing’. Put in another way, whereas there is, *ceteris paribus*, a welfare gain from scoring better on each of those dimensions *separately*, there is something like a negative welfare premium from scoring highly on *both* of these dimensions, as each of these tend to make achievements on the other dimension less valuable.

While the interpretation of those various complementarities and redundancies would require a separate paper on its own, it is worth underlining here that their mere existence suffices to reject the possibility to represent preferences on multiattribute societies with a weighted sum index, as this is nonetheless widely done in the existing literature on the measurement of standards of living (e.g. the HDI indicator).

### 4.3 Sensitivity to the postulated marginal value functions

Although the postulate of s-shaped marginal value functions for all attributes can be considered to be a plausible one, it may be worth exploring the sensitivity of our results to that particular postulate. For that purpose, this subsection proposes to assess the robustness of our estimates, by contrasting the results of the previous subsection (relying on s-shaped marginal value functions) with the ones obtained under numerical representations based on 3 other continuous piecewise linear marginal value functions summarised in Table 11. We will name ‘pessimistic’ (resp. ‘optimistic’) the convex (resp. ‘concave’) value function which requires at least level ‘good’ (resp. ‘bad’) to *satisfy* the respondent to at least 50%.

value function	'very bad'	'bad'	'reference'	'good'	'very good'
linear	0	0.25	0.5	0.75	1
pessimistic	0	0.1	0.3	0.5	1
optimistic	0	0.5	0.7	0.9	1

Table 11: S-shape value function

### 4.3.1 Sensitivity of the relative weights of the different dimensions

To analyse the sensitivity of the relative weights for the different selected marginal value functions, let us consider Table 12 which shows, for each of the 13 respondents, the values taken by the Kendall tau rank correlation coefficient between the rankings of the Shapley indexes under different assumptions on the marginal value functions.<sup>24</sup>

	s	pess.	opt.		s	pess.	opt.
	<b>01</b>				<b>09</b>		
lin.	0.95	0.60	1.00	0.95	0.74	0.95	
s		0.74	0.95		0.60	0.80	
pess.			0.60			0.80	
	<b>02</b>				<b>10</b>		
lin.	0.20	0.40	0.74	0.95	0.74	0.88	
s		0.80	0.11		0.80	0.84	
pess.			0.32			0.60	
	<b>03</b>				<b>11</b>		
lin.	0.43	0.88	0.88	0.84	0.76	0.88	
s		0.63	0.13		0.32	0.95	
pess.			0.56			0.44	
	<b>04</b>				<b>12</b>		
lin.	0.67	0.44	0.95	1.00	0.80	1.00	
s		0.67	0.53		0.80	1.0	
pess.			0.32			0.80	
	<b>06</b>				<b>13</b>		
lin.	0.89	0.95	1.00	0.95	NA	0.74	
s		0.95	0.89		NA	0.60	
pess.			0.95			NA	
	<b>07</b>				<b>14</b>		
lin.	0.71	0.71	0.60	0.84	0.00	0.63	
s		0.43	0.60		0.11	0.32	
pess.			0.12			-0.22	
	<b>08</b>						
lin.	0.82	NA	0.74				
s		NA	0.77				
pess.			NA				

Table 12: Kendall's taus between the rankings of the Shapley indexes, for each respondent and the different scenarios

<sup>24</sup>In Table 12, 'lin.' refers to linear marginal value functions, while 'pess.' and 'opt.' refer to, respectively, pessimistic and optimistic marginal value functions.



The Kendall tau coefficient is a well-known non-parametric statistic used to measure the degree of correspondence between two rankings. In this particular case, we use a variant of that statistic, which allows us to take into account ties in the rankings. Note that a Kendall Tau equal to 1 means that there is a perfect correspondence between the compared rankings, a Kendall Tau equal to 0 means that the two compared rankings are not correlated, whereas a Kendall Tau of -1 means that the compared rankings are exactly inverted.<sup>25</sup>

As shown by Table 12, the values of the Kendall tau coefficient between the s-shape based rankings of the importances and the other rankings are generally positive and large, suggesting that the reliance on s-shaped marginal value functions does not bias the numerical representation chosen in Sections 4.1 and 4.2. Actually, while it is true that there is in general no perfect equality between the s-shape based rankings and the other rankings (illustrated by values of the Kendall tau coefficient generally lower than 1), the values shown in Table 12 suffice to show that there is good stability of the rankings of the importances across the different tested marginal value functions (except for the pessimistic shape, which yields in general lower Kendall tau coefficients).

Having emphasized the overall robustness of the choice of the s-shaped marginal value functions, it remains true that the extent to which changing the postulates on marginal value functions affects the obtained rankings varies significantly across individuals.

For instance, whereas respondent 12's rankings under s-shaped, linear and optimistic marginal value functions are exactly the same (i.e. the Kendall tau coefficients are equal to 1), the robustness of the rankings of the importances is less sizeable for respondent 03 (for whom the Kendall tau coefficients are significantly lower).

Finally, it should also be stressed here that Table 12 does not have the pretension to provide an exhaustive study of the robustness of the present approach to the selection of marginal value functions on each attribute. Actually, a complete study of that robustness would require to consider *all* possible combinaisons of marginal value functions on all attributes. While such a complete study of the robustness of our results goes far beyond the scope of this simple application, it remains true that our focus on only some combinaisons of marginal value functions qualifies the generality of our conclusions in terms of robustness.

---

<sup>25</sup>Note that, for respondents 08 and 13, 'pessimistic' marginal value functions make the Choquet integral an inadequate representation of preferences on multiattribute societies, so that rankings are not available.

### 4.3.2 Sensitivity of the interactions of the different dimensions

After our study of the sensitivity of the respondents' rankings of the importances to the postulated marginal value functions, let us now examine the robustness of our conclusions regarding the existence of interactions between the different attributes of hypothetical societies. For that purpose, Table 13 shows the number of strictly positive and strictly negative interactions between attributes under different scenarios regarding marginal value functions: linear ('l'), s-shaped ('s'), pessimistic ('p') and optimistic ('o').

complementarity ( $> 0$ )																
env.				hea.				lab.				lon.				
	l	s	p	o	l	s	p	o	l	s	p	o	l	s	p	o
con.	8	6	6	9	4	4	5	5	8	6	6	6	7	7	7	7
env.					6	6	4	7	8	7	5	8	2	2	3	4
hea.									6	6	7	7	2	0	4	4
lab.													3	6	5	6
redundancy ( $< 0$ )																
env.				hea.				lab.				lon.				
	l	s	p	o	l	s	p	o	l	s	p	o	l	s	p	o
con.	4	4	4	3	7	8	5	8	4	6	4	6	4	5	2	4
env.					6	6	6	4	4	3	5	5	9	7	6	9
hea.									6	3	3	3	9	10	5	8
lab.													10	7	6	7

Table 13: Number of strictly positive and strictly negative interactions, for each scenario

Here again, the existence of a large number of positive and negative interactions between attributes is globally robust to the postulated marginal value functions. Hence, the rejection of the weighted mean indicators of standards of living does not seem to depend on a particular assumption on marginal value functions. While the existence of complementarities and redundancies between different attributes of societies is globally robust to the postulated marginal value functions, it should be stressed, nonetheless, that changing the form of marginal value functions does not leave the picture completely unchanged. Actually, although strictly positive interactions between longevity and consumption are observed for 7 individuals whatever the marginal value functions are, the same cannot be said for all interactions observed under s-shaped marginal value functions. For instance, shifting from s-shaped to 'optimistic' marginal value functions would raise the number of positive interactions between longevity and health from 0 to 4, and reduce the number of negative interactions between these from 10 to 8. While such a change does not appear to be sizeable, it remains, however, that the study, at a larger scale, of complementarities and redundancies between two specific dimensions of standards of living may still vary significantly depending on the postulated marginal value functions.

Moreover, whereas Table 13 seems to support some robustness of the

observed complementarities/redundancies to the selection of marginal value functions, one should remind that this table only provides some global counting of interactions. Hence, Table 13 may tend to underestimate the size of changes across scenarios, some changes of interaction for a particular respondent being somewhat 'compensated' by an inverse change for another respondent.

Finally, it should also be stressed here that, as Table 12, Table 13, by concentrating on scenarios where marginal value functions are the same on *all* attributes, does not provide an exhaustive study of the robustness of complementarities and redundancies, which would require to consider all combinaisons of such marginal value functions.

#### 4.4 Compatibility with the first part of the questionnaire

While the results presented so far are based on the second part of the questionnaire, it is worth exploring the issue of the compatibility of those results with the rankings of societies provided by respondents in the first part of the questionnaire. Actually, even if respondents were probably more familiar with the societies ranked in Part 2 of the questionnaire – on the grounds that these were preliminary ranked in Part 1, and, then, ranked again – this does not justify a neglect of the informational basis contained in Part 1 of the questionnaire.

This subsection aims at exploring the compatibility of the rankings given by respondents in the first part of the questionnaire with the rankings obtained from the estimation of a numerical representation (with s-shaped marginal value functions) on the basis of the second part of the questionnaire.

Note here that no numerical representation based on Choquet integral-based MAVT could be found for the preferential information of the first part of the questionnaire for any of the 13 respondents. This signifies that the rankings of the respondents, under the hypothesis of s-shaped marginal value functions, are incompatible with some basic assumptions on the Choquet integral as an aggregation operator (see e.g. [38] concerning comonotonic contradictory tradeoffs).

In order to evaluate the compatibility of the rankings given by the respondents in the first part of the questionnaire and the preferences elicited on the basis of the second part of the questionnaire, Table 14 shows, for each respondent, the value of the Kendall tau coefficient between the respondent's ranking within each of the 10 groups of societies (Part 1), and the model-based rankings (based on Part 2's rankings alone).

As shown by Table 14, the extent to which the model estimated on the basis of the second part yields a ranking of societies equal to the actual one varies significantly across respondents and groups. For instance, whereas the ranking of societies provided by respondent 01 in the group 06 of Part

grp.	<b>01</b>	<b>02</b>	<b>03</b>	<b>04</b>	<b>06</b>	<b>07</b>	<b>08</b>
01	0.73	0.87	1.00	0.73	0.33	-0.20	0.60
02	0.73	0.60	0.87	1.00	0.73	-0.07	0.73
03	0.60	0.73	0.87	0.87	0.73	0.73	0.60
04	0.60	0.60	0.73	0.87	0.47	0.47	0.47
05	0.47	0.47	0.33	0.47	0.47	0.87	0.47
06	1.00	0.47	0.33	0.73	0.60	0.87	0.73
07	0.60	0.73	0.73	0.87	0.87	0.47	0.73
08	0.47	0.60	0.87	0.47	0.87	0.60	0.47
09	0.87	0.60	0.87	0.73	0.87	0.60	0.73
10	0.60	0.73	0.87	0.87	0.87	0.87	0.60
min.	0.47	0.47	0.33	0.47	0.33	-0.20	0.47
mean	0.67	0.64	0.75	0.76	0.68	0.52	0.61
max.	1.00	0.87	1.00	1.00	0.87	0.87	0.73
grp.	<b>09</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	
01	0.73	0.47	0.33	0.73	0.60	0.33	
02	1.00	0.87	0.73	0.33	0.73	0.87	
03	0.47	0.60	0.60	0.87	0.47	0.73	
04	0.60	0.87	0.87	-0.33	0.47	0.60	
05	0.60	0.60	0.33	-0.60	0.60	0.73	
06	0.60	0.73	0.60	-0.33	0.87	0.60	
07	0.60	0.87	0.73	-0.47	0.47	0.33	
08	0.33	0.33	0.47	-0.60	0.47	0.60	
09	0.33	0.60	0.73	0.47	0.47	0.47	
10	0.47	0.87	0.87	-0.73	0.33	0.47	
min.	0.33	0.33	0.33	-0.73	0.33	0.33	
mean	0.57	0.68	0.63	-0.07	0.55	0.57	
max.	1.00	0.87	0.87	0.87	0.87	0.87	

Table 14: Kendall’s taus between the obtained rankings and the ones given in Part 1 of the questionnaire

1 coincides perfectly with the ranking derived from preference elicitation process – as reflected by a Kendall Tau equal to 1 – the same cannot be said for the same respondent’s rankings in other groups, which coincide only imperfectly with the estimated model.

The precise extent to which the estimated model is compatible with Part 1-based rankings varies also significantly across respondents, as this is suggested by the varying mean Kendall tau coefficient (see bottom of Table 14). For instance, while the average Kendall tau coefficient equals 0.76 for respondent 04, it is as low as -0.07 for respondent 12.<sup>26</sup> Thus, while the model-based rankings of societies is strongly correlated with the actual, Part 1-based rankings for some respondents, there seems to be some kind of independence between those rankings for others. This suggests that the ability of the estimated model to replicate the observed rankings of societies in Part 1 is, although globally large, not large for all respondents.

Table 14 suggests that there exist some tensions between the rankings

<sup>26</sup>Note that the negative values for the Kendall tau coefficients of respondent 12 are due to rankings in Part 1 which do not respect the minimal preferential requirement of dominance. The same observation can be made for respondent 07.

obtained by the numerical representation based on the preferences expressed in Part 2 and the preferences of Part 1 of the questionnaire. The existence of those tensions is a result as such: these tensions reveal nothing else than the difficulty to rationalise all rankings provided by a given respondent by means of a mere Choquet integral.

#### 4.5 Back to actual societies

After having elicited the preferences of our group of respondents on *hypothetical* multiattribute societies, let us now show how the output of that elicitation exercise can be used for the ranking of *actual* multiattribute societies.

For that purpose, we selected a small group of actual economies – Austria, Denmark, Germany, Greece, Italy, the Netherlands and Spain – whose performance on the 5 dimensions under study is summarized in Table 15 (the year under study is 2004).<sup>27</sup>

Country	Con	Env	Hea	Lab	Lon
Austria	1557	8	15.95	39.9	18.60
Denmark	1428	10	15.70	35.6	17.40
Germany	1459	10	15.50	36	18.45
Greece	1281	9	15.30	43	17.95
Italy	1305	8	15.65	38.80	19.15
The Netherlands	1492	9	16.35	30.80	18.00
Spain	1255	8	14.15	39.60	19.00

Table 15: Performances of the 7 actual societies

It should be stressed that no actual society dominates the others. Whereas Austria ranks first on consumption, Italy and Spain are first in terms of environmental quality. Moreover, while the Netherlands ranks first on health and leisure time, Italy is dominant as far as longevity is concerned. Given that no actual society ranks first on all attributes, one could thus hardly say a priori which society is, in total, the best.

To answer that question, one needs to know more about the importance and the interactions of the different attributes, which are likely to vary across individuals. Given that this section contributed to elicit the preferences of 13 respondents on hypothetical multiattribute societies, we can now use that information to explore how those same respondents would rank the actual societies under comparison.

In order to carry out that task, it is first necessary to translate the description of those 7 actual societies in terms of the marginal value func-

<sup>27</sup>Sources: consumption statistics are from the OECD ([30]); CO2 emissions statistics are from the United Nations ([34]); healthy life expectancies and life expectancies are from the EHEMU ([8]); leisure statistics are from the OECD ([16]).

tions of Section 3.1. Table 16 summarises these values, under the s-shaped marginal value functions postulated earlier.

Country	Con	Env	Hea	Lab	Lon
Austria	0.997	1.000	0.738	0.313	0.421
Denmark	0.869	0.500	0.675	0.737	0.184
Germany	0.903	0.500	0.625	0.697	0.391
Greece	0.445	0.875	0.575	0.086	0.293
Italy	0.514	1.000	0.663	0.421	0.530
The Netherlands	0.935	0.875	0.838	0.987	0.303
Spain	0.370	1.000	0.288	0.342	0.500

Table 16: Marginal values of the 7 actual societies

It is then possible to extrapolate, on the basis of the capacities estimated earlier, how each of the 13 respondents would rank the actual societies under study. The results of that simple extrapolation exercise are shown in Table 17, where the highest (resp. lowest) rank is assigned to the best (resp. worst) ranked society.

Country	01	02	03	04	06	07	08	09	10	11	12	13	14
Austria	6	6	6	6	6	6	6	6	6	6	6	6	6
Denmark	3	4	4	4	3	3	3	3	5	3	3	3	3
Germany	5	5	5	5	5	4	5	4	4	5	4	5	4
Greece	1	2	1	2	1	1	1	1	1	1	1	1	1
Italy	4	3	3	3	4	5	4	5	3	4	5	4	5
Netherlands	7	7	7	7	7	7	7	7	7	7	7	7	7
Spain	2	1	2	1	2	2	2	2	2	2	2	2	2

Table 17: Ranks of the 7 actual societies for each respondent

As shown by Table 17, all respondents, without exception, would, on the basis of the preferences elicited in Sections 4.1 and 4.2, rank the Netherlands as the society where standards of living are the best. Moreover, there would be also a unanimity among respondents on the second position of Austria. Although those results could not be anticipated on the mere basis of the actual performance of the societies under study (Table 15), the preferences elicitation exercise carried out earlier can help us to understand why the Netherlands and Austria are unanimously regarded as exhibiting the best standards of living. Actually, Section 4.1 highlighted the large weights assigned by respondents to consumption, leisure time and environmental quality – as well as the existence of strong complementarities between these –, so that this does not come as a surprise that the Netherlands and Austria, which perform very well on those three dimensions, have higher overall values than the other societies under study.

However, one should notice that there exist some disagreements among respondents as far as the rest of the ranking is concerned. For instance,

whereas respondent 4 would, on the basis of the estimated model, rank Germany third, Denmark fourth and Italy fifth, respondent 12 would rank Italy third, Germany fourth and Denmark fifth. Such disagreements reflect the mere fact that those societies are much *closer* to each others than to the Netherlands and Austria, as reflected by Table 15, so that some small differences in individual preferences suffice to generate different rankings.

For completeness, Table 18 shows also the overall value that each respondent would, on the basis of the estimated model, assign to the 7 actual societies under comparison. While Table 18 confirms the unanimity on the positions of the Netherlands and Austria, it reveals, however, that respondents differ on the extent to which societies are regarded by them as more or less close, in terms of value, to each others. For instance, whereas the best and the worse societies are, in value terms, quite distant for respondent 04, the same is not true for respondent 12, where the various ranked societies are regarded as not so different from each others. Those disagreements reflect nothing else than differences in the structure of individual preferences that were revealed by the ranking of hypothetical societies in Sections 4.1 and 4.2.

Country	01	02	03	04	06	07	08
Austria	0.755	0.794	0.675	0.809	0.681	0.700	0.739
Denmark	0.620	0.695	0.611	0.777	0.601	0.602	0.609
Germany	0.643	0.703	0.632	0.781	0.627	0.625	0.632
Greece	0.474	0.452	0.428	0.504	0.442	0.449	0.483
Italy	0.631	0.586	0.605	0.632	0.622	0.628	0.631
Netherlands	0.819	0.911	0.813	0.912	0.802	0.802	0.820
Spain	0.524	0.432	0.473	0.482	0.504	0.521	0.515
Country	09	10	11	12	13	14	
Austria	0.712	0.678	0.725	0.673	0.664	0.695	
Denmark	0.585	0.643	0.608	0.555	0.587	0.563	
Germany	0.616	0.637	0.637	0.594	0.619	0.587	
Greece	0.459	0.453	0.460	0.458	0.411	0.474	
Italy	0.628	0.630	0.619	0.636	0.591	0.638	
Netherlands	0.801	0.865	0.795	0.746	0.792	0.785	
Spain	0.487	0.508	0.490	0.523	0.476	0.522	

Table 18: Overall values of the 7 actual societies for each respondent

Thus, despite the small size of our group of respondents, there exist significant disagreements on how actual societies should be ranked. Naturally, we shall not try, in this simple illustrative application, to solve the difficult problem of aggregation of individual preferences on hypothetical or actual societies. Nevertheless, we tend to think that the elicitation of individual preferences constitutes a first, necessary stage in the process of constructing, on the basis of individual indexes of standards of living, a *social* indicator of standards of living based on some *empirical* foundations.

## 5 Concluding remarks and perspectives

Although the measurement of standards of living is an old issue, the complexity of the task – due to the subjectivity and the multidimensionality of the object to be measured – keeps on inviting improvements of existing methods and indicators. The goal of this paper was to cast a new light on the measurement of standards of living, by developing an MAVT-based approach, whose specificity is to do justice to the subjectivity and multidimensionality of standards of living.

For that purpose, we firstly presented the Choquet integral, a simple aggregator, which has the virtue to allow the existence of complementarities or redundancies between different dimensions of the objects to be evaluated (Section 2). Then, we showed, by means of a classroom application, how Choquet integral-based MAVT can be applied for the measurement of standards of living (Sections 3 and 4). That application consisted in submitting a short standardised questionnaire, where respondents are asked to rank multidimensionnal hypothetical societies, which differ on their consumption, environmental quality, health, leisure time and longevity.

In the light of that simple application, it appears that individual preferences on hypothetical 5-dimensional societies are far from easy to express, and do not take a simple form. The different dimensions of standards of living under study are shown to matter to *unequal* extents. Moreover, significant *complementarities* and *redundancies* arise between the dimensions under study, so that a mere weighted sum is inadequate at representing preferences on hypothetical societies. Furthermore, there exists also a large *heterogeneity* of preferences among respondents, so that a set of weights representing the preferences of some ‘representative agent’ would have little foundation. But besides illustrating the richness and diversity of preferences on multiattribute societies, our application allowed us also to show how those preferences, once elicited, can then be used as a basis for constructing an indicator of standards of living aggregating, by means of preferences-based weights and interactions, various dimensions of living conditions in *actual* societies.

Note that this simple application, which was presented here as a mere illustration of the potential fruitfulness of the approach developed here, allows us also to point out to some crucial problems inviting further research.

Firstly, whereas our application was made on the basis of strong postulates on marginal value functions – which, as this was shown, may influence results significantly – it is clear that those functions should ideally be also derived *empirically* (on the basis of a separate section of the questionnaire). True, this would increase the duration of the survey, from about 1 hour and a half here to much more, but this would also provide results that represent preferences more accurately. Thus, a particular attention should be paid to this in the future.



Secondly, while this application only observed the existence of some heterogeneity among the preferences of respondents, it did not attempt to construct, on the basis of all elicited preferences, some *social* indicator of standards of living. That complex interpersonal aggregation exercise – which requires some ethical foundations – goes far beyond the scope of the present paper, and is thus left for future research.

Hence, much work remains to be done, in the future, to be able to derive preferences-based indicators of standards of living measuring the precise extent to which different societies are good at fighting against all forms of scarcity.

## References

- [1] C.A. Bana e Costa and J.C. Vansnick. Preference relations in MCDM. In T. Gal and T. Hanne T. Steward, editors, *MultiCriteria Decision Making: Advances in MCDM models, algorithms, theory and applications*. Kluwer, 1999.
- [2] G.S. Becker, T.J. Philipson, and R.R. Soares. The quantity and quality of life and the evolution of world inequality. *American Economic Review*, 95(1):277–291, 2005.
- [3] D. Bouyssou and M. Pirlot. Preferences for multi-attributed alternatives: Traces, dominance, and numerical representations. *J. of Mathematical Psychology*, 48:167–185, 2004.
- [4] M. Browning, L.P. Hansen, and J.J. Heckman. Micro data and general equilibrium models. In T. Gal, T. Steward, and T. Hanne, editors, *Handbook of Macroeconomics*, volume 1A. Elsevier Science, 1999.
- [5] A. Chateauneuf and J-Y. Jaffray. Some characterizations of lower probabilities and other monotone capacities through the use of Möbius inversion. *Mathematical Social Sciences*, 17(3):263–283, 1989.
- [6] G. Choquet. Theory of capacities. *Annales de l’Institut Fourier*, 5:131–295, 1953.
- [7] P. Dasgupta. *An Inquiry into Well-Being and Destitution*. Oxford University Press, 1993.
- [8] European Health Expectancy Monitoring Unit (EHEMU). Estimations of health expectancy at age 65 in European Union countries in 2004, 2007. Table 4.
- [9] L. Finkelstein. Theory and philosophy of measurement. In P.H. Sydenham, editor, *Handbook of Measurement Science*, volume 1. Wiley and Sons, 1982.

- [10] M. Grabisch.  $k$ -order additive discrete fuzzy measure and their representation. *Fuzzy Sets and Systems*, 92:131–295, 1997.
- [11] M. Grabisch, I. Kojadinovic, and P. Meyer. A review of capacity identification methods for Choquet integral based multi-attribute utility theory: Applications of the Kappalab R package. *European Journal of Operational Research*, 2008. in press.
- [12] R. L. Keeney and H. Raiffa. *Decision with multiple objectives*. Wiley, New-York, 1976.
- [13] I. Kojadinovic. Minimum variance capacity identification. *European Journal of Operational Research*, 177(1):498–514, 2007.
- [14] I. Kojadinovic, J-L. Marichal, and M. Roubens. An axiomatic approach to the definition of the entropy of a discrete Choquet capacity. *Information Sciences*, 172:131–153, 2005.
- [15] D.H. Krantz, R.D. Luce, P. Suppes, and A. Tversky. *Foundations of measurement, volume 1: Additive and polynomial representations*. Academic Press, 1971.
- [16] OECD Labour Statistics. Average usual weekly hours worked on the main job, 2007. [http://stats.oecd.org/wbos/default.aspx?DatasetCode=AVE\\_HRS](http://stats.oecd.org/wbos/default.aspx?DatasetCode=AVE_HRS).
- [17] C. Labreuche and M. Grabisch. The Choquet integral for the aggregation of interval scales in multicriteria decision making. *Fuzzy Sets and Systems*, 137:11–16, 2003.
- [18] J.-L. Marichal. An axiomatic approach of the discrete Choquet integral as a tool to aggregate interacting criteria. *IEEE Transactions on Fuzzy Systems*, 8(6):800–807, 2000.
- [19] J.-L. Marichal. Behavioral analysis of aggregation in multicriteria decision aid. In J. Fodor, B. De Baets, and P. Perny, editors, *Preferences and Decisions under Incomplete Knowledge*, pages 153–178. Physica-Verlag, 2000.
- [20] J-L. Marichal. Tolerant or intolerant character of interacting criteria in aggregation by the Choquet integral. *European Journal of Operational Research*, 155(3):771–791, 2004.
- [21] G.A. Miller. The magical number seven, plus or minus two: Some limits on our capacity for processing information. *The Psychological Review*, 63:81–97, 1956.

- [22] T. Murofushi and S. Soneda. Techniques for reading fuzzy measures (iii): Interaction index. In *9th Fuzzy System Symposium*, pages 693–696, Sapporo, Japan, 1993.
- [23] United Nations. United Nations Millennium Development Goals Indicators, Goal 7 (Sustainable Development), 2007. <http://mdgs.un.org/unsd/mdg/Data.aspx>.
- [24] W. D. Nordhaus. The health of nations: the contribution of improved health to living standards. In K.M. Murphy and R.H. Topel, editors, *Measuring the Gains from Medical Research*. University of Chicago Press, 2003.
- [25] D. C. North. Economic performance through time. *American Economic Review*, 84:359–368, 1994.
- [26] L. Osberg and A. Sharpe. An index of economic well-being for selected oecd countries. *Review of Income and Wealth*, 48(3):291–316, 2002.
- [27] L. Osberg and A. Sharpe. How should we measure the economic aspects of well-being? *Review of Income and Wealth*, 51(2):311–336, 2005.
- [28] W. Petty. Political arithmetick. In Charles Henry Hull, editor, *The Economic Writings of Sir William Petty*, number 1, pages 233–313. Cambridge University Press, 1676.
- [29] L. S. Shapley. A value for  $n$ -person games. In *Contributions to the theory of games, vol. 2*, Annals of Mathematics Studies, no. 28, pages 307–317. Princeton University Press, Princeton, N. J., 1953.
- [30] OECD National Accounts Statistics. Actual individual consumption per head, euros, 2007. <http://stats.oecd.org/wbos/default.aspx?datasetcode=SNA>.
- [31] M. Sugeno. *Theory of fuzzy integrals and its applications*. PhD thesis, Tokyo Institute of Technology, Tokyo, Japan, 1974.
- [32] A. Tversky and D. Kahneman. The framing of decisions and the psychology of choice. *Science*, 211:453–458, 1981.
- [33] United Nations Development Program (UNDP). *Human Development Report*. Oxford University Press, New York, 1990.
- [34] United Nations Development Program (UNDP). *Human Development Report 2007/2008*. Oxford University Press, New York, 2008.
- [35] D Usher. An imputation to the measure of economic growth for changes in life expectancy. In M. Moss, editor, *The measurement of economic*

*and social performance*, volume 38 of *NBER Studies in Income and Wealth*. 1973.

- [36] D. Usher. *The Measurement of Economic Growth*. Columbia University Press, 1980.
- [37] P. Vincke. *Multicriteria Decision-aid*. Wiley, 1992.
- [38] P.P. Wakker. *Additive Representations of Preferences: A new Foundation of Decision Analysis*. Kluwer Academic Publishers, Dordrecht, Boston, London, 1989.

# Appendices

## A Interaction indexes for all respondents for s-shaped marginal value functions

### A.1 Respondent 01

	Con	Env	Hea	Lab	Lon
Con	NA	0.03	0.04	-0.03	-0.03
Env	0.03	NA	-0.03	0.00	0.00
Hea	0.04	-0.03	NA	0.00	-0.02
Lab	-0.03	0.00	0.00	NA	0.04
Lon	-0.03	0.00	-0.02	0.04	NA

### A.2 Respondent 02

	Con	Env	Hea	Lab	Lon
Con	NA	-0.03	-0.03	-0.16	-0.07
Env	-0.03	NA	0.04	0.04	-0.04
Hea	-0.03	0.04	NA	-0.11	-0.11
Lab	-0.16	0.04	-0.11	NA	-0.15
Lon	-0.07	-0.04	-0.11	-0.15	NA

### A.3 Respondent 03

	Con	Env	Hea	Lab	Lon
Con	NA	0.03	-0.03	0.04	-0.03
Env	0.03	NA	0.01	0.00	0.02
Hea	-0.03	0.01	NA	-0.02	0.00
Lab	0.04	0.00	-0.02	NA	-0.02
Lon	-0.03	0.02	0.00	-0.02	NA

### A.4 Respondent 04

	Con	Env	Hea	Lab	Lon
Con	NA	-0.11	-0.25	-0.03	-0.16
Env	-0.11	NA	-0.07	0.11	0.04
Hea	-0.25	-0.07	NA	-0.04	-0.13
Lab	-0.03	0.11	-0.04	NA	-0.02
Lon	-0.16	0.04	-0.13	-0.02	NA

### A.5 Respondent 06

	Con	Env	Hea	Lab	Lon
Con	NA	0.00	-0.01	0.00	0.01
Env	0.00	NA	0.00	0.00	0.00
Hea	-0.01	0.00	NA	0.01	0.00
Lab	0.00	0.00	0.01	NA	-0.01
Lon	0.01	0.00	0.00	-0.01	NA

## A.6 Respondent 07

	Con	Env	Hea	Lab	Lon
Con	NA	0.03	-0.04	0.03	-0.02
Env	0.03	NA	-0.03	0.01	-0.01
Hea	-0.04	-0.03	NA	0.07	-0.05
Lab	0.03	0.01	0.07	NA	0.02
Lon	-0.02	-0.01	-0.05	0.02	NA

## A.7 Respondent 08

	Con	Env	Hea	Lab	Lon
Con	NA	0.02	-0.03	0.06	0.00
Env	0.02	NA	0.03	-0.03	0.00
Hea	-0.03	0.03	NA	0.00	-0.02
Lab	0.06	-0.03	0.00	NA	-0.03
Lon	0.00	0.00	-0.02	-0.03	NA

## A.8 Respondent 09

	Con	Env	Hea	Lab	Lon
Con	NA	0.02	-0.06	-0.03	0.04
Env	0.02	NA	0.08	0.03	-0.05
Hea	-0.06	0.08	NA	-0.02	-0.01
Lab	-0.03	0.03	-0.02	NA	0.01
Lon	0.04	-0.05	-0.01	0.01	NA

## A.9 Respondent 10

	Con	Env	Hea	Lab	Lon
Con	NA	0.00	0.01	0.03	0.06
Env	0.00	NA	0.03	0.02	-0.05
Hea	0.01	0.03	NA	0.09	-0.07
Lab	0.03	0.02	0.09	NA	-0.11
Lon	0.06	-0.05	-0.07	-0.11	NA

## A.10 Respondent 11

	Con	Env	Hea	Lab	Lon
Con	NA	0.00	0.05	-0.09	0.02
Env	0.00	NA	-0.03	0.02	0.00
Hea	0.05	-0.03	NA	0.02	-0.01
Lab	-0.09	0.02	0.02	NA	0.03
Lon	0.02	0.00	-0.01	0.03	NA

## A.11 Respondent 12

	Con	Env	Hea	Lab	Lon
Con	NA	-0.01	0.02	0.04	0.05
Env	-0.01	NA	-0.03	-0.02	-0.09
Hea	0.02	-0.03	NA	0.01	-0.02
Lab	0.04	-0.02	0.01	NA	0.03
Lon	0.05	-0.09	-0.02	0.03	NA

### A.12 Respondent 13

	Con	Env	Hea	Lab	Lon
Con	NA	-0.01	0.00	-0.04	0.06
Env	-0.01	NA	-0.01	0.02	-0.06
Hea	0.00	-0.01	NA	0.02	0.00
Lab	-0.04	0.02	0.02	NA	0.02
Lon	0.06	-0.06	0.00	0.02	NA

### A.13 Respondent 14

	Con	Env	Hea	Lab	Lon
Con	NA	0.13	-0.03	0.10	0.15
Env	0.13	NA	0.03	-0.13	-0.01
Hea	-0.03	0.03	NA	-0.03	-0.14
Lab	0.10	-0.13	-0.03	NA	-0.06
Lon	0.15	-0.01	-0.14	-0.06	NA

## B Empty questionnaire

*See following page.*

# **‘Making the world better’**

## **A simple classroom experiment**

**Patrick Meyer<sup>\*</sup> and Gregory Ponthiere<sup>\*\*</sup>**

**Date:** Friday 21 December 2007.

**Timetable:** 9.30 to 11.05

**Location:** University of Liege, Campus of Arlon, Belgium.

### **Program:**

- 1. Reading of Section 1 (purpose of the experiment), Section 2 (requirements) and Section 3 (description of the experiment and examples) by the experimenter. Answer to questions (length: 30 minutes).**
- 2. Conduct of the experiment: ranking of societies in Section 4 by the subjects (maximum length: 60 minutes).**
- 3. Subjects complete the feedback questionnaire (5 minutes).**

---

<sup>\*</sup> Department of Applied Mathematics, University of Luxembourg.

<sup>\*\*</sup> FRS-FNRS, CREPP, University of Liege.



### **Section 1: Goal of the survey/experiment:**

Given the multi-dimensionality of welfare, there exist many ways to make the world better. The goal of this survey is to explore the extent to which improvements on some dimensions of living conditions are more or less valuable than others. For that purpose, we would like to ask you to rank different hypothetical multi-dimensional societies proposed in our questionnaire, in order to have an idea of how the existing living conditions could be best improved.

Note that this questionnaire has a purely scientific purpose. The authors of this survey have no political affiliation or any intention to use the results of this survey for any non-scientific, political, or commercial purpose. Moreover, the anonymity of answers is fully guaranteed.

### **Section 2: Requirements:**

It is crucial to stress that there exists no “good” and no “bad” answers to the questions asked in this survey: you are free to rank societies in the way you like. The only requirement is that the rankings you provide reflect your tastes, and are not dictated by any other concern. In no way will a value judgement – either positive or negative – be expressed when interpreting your rankings of the hypothetical multi-dimensional societies proposed in this survey.

### **Section 3: Description of the experiment and examples:**

**The 5 dimensions of living conditions considered here are:**

**Consumption** (measured in euros per month)

**Environment** (measured in annual tonnes of CO<sub>2</sub> emissions per capita)

**Health status** (measured in healthy life expectancy at age 65, years)

**Labour time** (measured in hours of work per week)

**Longevity** (measured in life expectancy at age 65, years)

**To simplify, we shall define the ‘society of reference’ as involving the following values:**

**Consumption** = 1300 euros per month

**Environment** = 10 tonnes of CO<sub>2</sub> emissions per capita

**Health status** = 15 expected disability-free years at age 65

**Labour time** = 38 hours of work per week

**Longevity** = 19 expected years of life at age 65

Dimensions	'very bad' society (-20 %)	'bad' society (-10 %)	Society of reference	'good' society (+ 10 %)	'very good' society (+ 20%)
Consumption (€ per month)	1040 €	1170 €	<b>1300 €</b>	1430 €	1560 €
Environment (CO2 tonnes per capita)	12 tonnes	11 tonnes	<b>10 tonnes</b>	9 tonnes	8 tonnes
Health (healthy life expectancy at age 65)	12 years	13.5 years	<b>15 years</b>	16.5 years	18 years
Labour time (working hours per week)	45.6 hours	41.8 hours	<b>38 hours</b>	34.2 hours	30.4 hours
Longevity (life expectancy at age 65)	15.2 years	17.1 years	<b>19 years</b>	20.9 years	22.8 years

### Example of questions:

You may be asked to rank the two societies ES001 and ES002. ES001 corresponds to the current living conditions, while ES002 is the hypothetical society where consumption is increased by 10 %, while leisure time is reduced by 10 %.

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	RANK
ES001	1300 €	10 t	15 y	38 h	19 y	
ES002	1430€	11 t	15 y	38 h	19 y	

If you prefer ES001 to ES002, you can inform us about this by ranking it *before* society ES001. On the contrary, if you prefer ES002 to ES001, then you can inform us about this by ranking it *after* society ES002.

### Form of the questions:

Throughout this questionnaire, we shall ask you to rank societies that appear **in groups of six societies**.

Hence, your preferences can be expressed by assigning:

- rank 1 to the best society from your point of view,
- rank 2 to the second-best society,
- rank 3 to the third-best society,
- ...
- rank 6 to the society that is the least desirable among the group of hypothetical societies.

### For instance:

When facing the following group of six societies:

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	<b>RANK</b>
ES003	1430 €	8 t	15 y	38 h	22.8 y	
ES004	1560 €	8 t	15 y	38 h	20.9 y	
ES005	1560 €	9 t	15 y	38 h	22.8 y	
ES006	1170 €	11 t	15 y	38 h	15.2 y	
ES007	1170 €	12 t	15 y	38 h	17.1 y	
ES008	1040 €	11 t	15 y	38 h	17.1 y	

If you rank the society ES006 first among this list, the society ES003 second, the society ES008 third, the society ES005 fourth, the society ES007 fifth, and the society ES004 last of the group, this can be reflected by the following table:

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	<b>RANK</b>
ES003	1430 €	8 t	15 y	38 h	22.8 y	2
ES004	1560 €	8 t	15 y	38 h	20.9 y	6
ES005	1560 €	9 t	15 y	38 h	22.8 y	4
ES006	1170 €	11 t	15 y	38 h	15.2 y	1
ES007	1170 €	12 t	15 y	38 h	17.1 y	5
ES008	1040 €	11 t	15 y	38 h	17.1 y	3

#### **Section 4: The hypothetical societies to be ranked:**

##### **Group 1: Consumption, health, labour**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	<b>RANK</b>
S001	1430 €	10 t	18 y	30.4 h	19 y	
S002	1560 €	10 t	16.5 y	30.4 h	19 y	
S003	1560 €	10 t	18 y	34.2 h	19 y	
S004	1170 €	10 t	12 y	41.8 h	19 y	
S005	1170 €	10 t	13.5 y	45.6 h	19 y	
S006	1040 €	10 t	13.5 y	41.8 h	19 y	

##### **Group 2: Consumption, labour, longevity**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	<b>RANK</b>
S007	1430 €	10 t	15 y	30.4 h	22.8 y	
S008	1560 €	10 t	15 y	30.4 h	20.9 y	
S009	1560 €	10 t	15 y	34.2 h	22.8 y	
S010	1170 €	10 t	15 y	41.8 h	15.2 y	
S011	1170 €	10 t	15 y	45.6 h	17.1 y	
S012	1040 €	10 t	15 y	41.8 h	17.1 y	

##### **Group 3: Consumption, environment, labour**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	<b>RANK</b>
S019	1430 €	8 t	15 y	30.4 h	19 y	
S020	1560 €	9 t	15 y	30.4 h	19 y	
S021	1560 €	8 t	15 y	34.2 h	19 y	
S022	1170 €	12 t	15 y	41.8 h	19 y	
S023	1170 €	11 t	15 y	45.6 h	19 y	
S024	1040 €	11 t	15 y	41.8 h	19 y	

**Group 4: Consumption, health, longevity**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	RANK
S025	1430 €	10 t	18 y	38 h	22.8 y	
S026	1560 €	10 t	18 y	38 h	20.9 y	
S027	1560 €	10 t	16.5 y	38 h	22.8 y	
S028	1170 €	10 t	13.5 y	38 h	15.2 y	
S029	1170 €	10 t	12 y	38 h	17.1 y	
S030	1040 €	10 t	13.5 y	38 h	17.1 y	

**Group 5: Consumption, environment, health**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	RANK
S037	1430 €	8 t	18 y	38 h	19 y	
S038	1560 €	9 t	18 y	38 h	19 y	
S039	1560 €	8 t	16.5 y	38 h	19 y	
S040	1170 €	12 t	13.5 y	38 h	19 y	
S041	1170 €	11 t	12 y	38 h	19 y	
S042	1040 €	11 t	13.5 y	38 h	19 y	

**Group 6: Consumption, environment, longevity**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	RANK
S049	1430 €	8 t	15 y	38 h	22.8 y	
S050	1560 €	9 t	15 y	38 h	22.8 y	
S051	1560 €	8 t	15 y	38 h	20.9 y	
S052	1170 €	12 t	15 y	38 h	17.1 y	
S053	1170 €	11 t	15 y	38 h	15.2 y	
S054	1040 €	11 t	15 y	38 h	17.1 y	

**Group 7: Health, labour, longevity**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	RANK
S061	1300 €	10 t	18 y	34.2 h	22.8 y	
S062	1300 €	10 t	18 y	30.4 h	20.9 y	
S063	1300 €	10 t	16.5 y	30.4 h	22.8 y	
S064	1300 €	10 t	13.5 y	41.8 h	15.2 y	
S065	1300 €	10 t	12 y	41.8 h	17.1 y	
S066	1300 €	10 t	13.5 y	45.6 h	17.1 y	

**Group 8: Environment, health, labour**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	RANK
S073	1300 €	8 t	18 y	34.2 h	19 y	
S074	1300 €	9 t	18 y	30.4 h	19 y	
S075	1300 €	8 t	16.5 y	30.4 h	19 y	
S076	1300 €	12 t	13.5 y	41.8 h	19 y	
S077	1300 €	11 t	12 y	41.8 h	19 y	
S078	1300 €	11 t	13.5 y	45.6 h	19 y	

**Group 9: Environment, labour, longevity**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	RANK
S085	1300 €	8 t	15 y	34.2 h	22.8 y	
S086	1300 €	9 t	15 y	30.4 h	22.8 y	
S087	1300 €	8 t	15 y	30.4 h	20.9 y	
S088	1300 €	12 t	15 y	41.8 h	17.1 y	
S089	1300 €	11 t	15 y	41.8 h	15.2 y	
S090	1300 €	11 t	15 y	45.6 h	17.1 y	

**Group 10: Environment, health, longevity**

Number	Consumption (per month)	Environment (annual tonnes of CO2 emissions per capita)	Health (healthy life expectancy at age 65)	Labour time (hours per week)	Longevity (life expectancy at age 65)	<b>RANK</b>
S097	1300 €	8 t	16.5 y	38 h	22.8 y	
S098	1300 €	9 t	18 y	38 h	22.8 y	
S099	1300 €	8 t	18 y	38 h	20.9 y	
S100	1300 €	12 t	13.5 y	38 h	17.1 y	
S101	1300 €	11 t	13.5 y	38 h	15.2 y	
S102	1300 €	11 t	12 y	38 h	17.1 y	

**(1) Rewrite the BEST SOCIETIES from EACH group,**

**(2) Select the TOP 5 out of these**

**(3) RANK these top 5 societies from rank 1 to 5**

Group number	Number of best society	Consumption	Environment	Health	Labour	Longevity	<b>RANK</b>
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

**(1) Rewrite the WORST SOCIETIES from EACH group,**

**(2) Select the 5 WORSE out of these (i.e. the FLOP-5)**

**(3) RANK these flop5 societies from rank 1 to 5**

Group number	Number of worse society	Consumption	Environment	Health	Labour	Longevity	<b>RANK</b>
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

## FEEDBACK QUESTIONNAIRE:

The anonymity of this survey is fully guaranteed. However, in order to construct a large data set on our respondents, and, more importantly, in order to be able to improve the quality of future experiments, we would like to ask you to answer the following questions.

**Respondent number:**

**Gender:**

**Age:**

**Nationality:**

**Current university:**

**Undergraduate studies:**

**Postgraduate studies:**

**Number of years of high education (excluding the current one):**

<b>A few questions (please tick in one case on each row):</b>	<b>good</b>	<b>satis- factory</b>	<b>bad</b>	<b>None of these</b>
<b>Do you regard your current purchasing power as</b>				
<b>Do you regard the natural environment where you live as</b>				
<b>Do you regard your current health status as</b>				
<b>Do you regard your current working time as</b>				
<b>Do you regard your (expected) future longevity as</b>				

**Would you characterize your confidence in the answers you provided in this survey as high, satisfactory or low (on average)? HIGH SATISFACTORY LOW  
(circle what best describes your feelings)**

**Please write any comment or suggestion you may have about the present experience, or about your participation to it.**

**We thank you very much for your collaboration.**