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‘Upping the Ante’: how to design efficient auctions with entry?*

Laurent Lamy†

Abstract

In the symmetric independent private value model, we revisit auctions with entry by adding two additional ingredients: difficulties to commit to the announced mechanism, in particular not to update the reserve price after bidders took their entry decisions, and seller’s ex ante uncertainty on her reservation value which calls for flexibility. Shill bidding or ex post rights to cancel the sale may provide some valuable flexibility in second price auctions. However, both fail to be efficient since the seller may keep the good while it would be efficient to allocate it to the highest bidder. The English auction with jump bids and cancelation rights is shown to implement the first best in large environments. On the positive side, special emphasis is put on the equilibrium analysis of auctions with shill bidding and on a variety of associated new insights including counterintuitive comparative statics and a comparison with posted-prices.

Keywords: Auctions, auctions with entry, shill bidding, commitment failure, hold-up, posted-price, cancelation rights, jump bids, bilateral asymmetric information

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1 Introduction

> The exercise of rights to bid by or on behalf of the seller presents difficult problems. [...] So well established are these practices that their existence and effects appear to be accepted without demur, not only by the auctions world but also by judges. It is suggested, therefore, that specific external regulation is required if the practice of puffing is to be outlawed as has been recommended. (Frank Meisel [31], The Modern Law Review, 1996)

A major discrepancy between auction theory in economics[1] and auction practice is the tactical use of seller's bids, labeled henceforth as shill bidding, a pervasive phenomenon whose real-life importance is first suggested by the linguistic profusion to describe more and less the same activity: among them lift-lining, by-bidding, trotting, running, puffing, phantom bids, dummy bids, phony bids, fake bidders, sham bidders, cappers, decoy ducks, white bonnets, barkers, fictitious bids. In Cassady [9]'s book that describes how real-life auctions work and that had a major influence among the pioneering auction theorists, the topic appears in various chapters. In auctions law, the topic is also of central importance and is sometimes a source of litigations between sellers and bidders.[2] If the seller advertises the sale as being 'without reserve', the seller is supposed to propose a contract whose acceptance is by bidding: she commits to sell the object to the highest bona fide bidder whatever the price can be, i.e. for any price above the opening bid.[3] In an auction without reserve, the seller is neither allowed to withdraw the object from sale once the auction has started nor can she submit a bid as any other bidder. On the whole, shill bidding has a bad reputation among jurists where it is perceived as a breach of contract that is commonly observed and then tolerated as an usage in the auctions world. Shill bidding is mainly viewed either as an hold-up on the time and expenses in preparing and attending the auction or as a simulation fostering a false appearance of genuine competition that distorts bidders' representation of the value of the good. The latter deceitful aspect has been formalized in Lamy [24] in a model with interdependent valuations while the former hold-up issue is the central aspect of our present model with purely private valuations. While there is a large literature on hold-up problems (Che and Sakovics [10]), the present paper is, to the best of my knowledge, the first in the economic literature to analyze this

[1] See Krishna [22] and Milgrom [33] for recent textbooks.

[2] See Harvey and Meisel [17] for U.K. and Mauger-Vielpeau [29] for France for general textbooks on auctions law and [1], [31] and [32] for specific papers on seller's bids. We emphasize that this is a controversial topic in law such that no consensus has been reached so far, as testified by the quotation above from the conclusion in Meisel [31].

[3] The terminology 'auction without reserve' is sometimes used only for the case where the opening bid is also set to zero.

hold-up issue that preys on auctions. Furthermore, the literature in law has been powerless to formalize the possible benefits from shill bidding: if it allows to prevent a sale at an undervalue, the only legitimate object met by some judges, then it is argued that it could have been substituted by an open reserve price or at least by the right to withdraw the good from sale at any time. The argument is actually true in the simplest models where the seller knows her reservation value ex ante but becomes fallacious when this reservation value is only partially known, e.g. because the seller refines her valuation in the course of the auction itself, such that shill bidding may offer some valuable flexibility.

Shill bidding continuously attracts the attention of the media all over the world. In 2007, Eftis Paraskevaides, one of Britain's top eBay traders who used to sell more than £1.4m worth of antiquities a year on eBay, got caught by a journalist of the Sunday Times.[4] He has then been banned for life by eBay once the investigation found that his ex-wife was bidding up the price of goods he was selling without a reserve price. The aim of this strategy was to boost participation according to his own allegations. Snapnames is the largest reseller of Web site names that held above 1,000,000 domain name auctions over the years. The Washington Post[5] reports that the company admits the allegation that a former top executive regularly boosted the price on some auctions since 2005 through a bid account that has been explicitly created for this fraudulent usage. Heritage Auction Galleries advertises itself as the world's largest collectibles auctioneer and the third largest auction house, with over $700 million in annual sales and 450,000 online bidder-members. Several lawsuits have been filed against the firm for shill bidding. The president of Heritage acknowledges that the company regularly bids in her own auctions but argues that it is perfectly legal for her to do so according to its listed terms and conditions.[6] The recent attention of the media mainly covers online auctions partly because it affects consumers that were unaware of such unethical if not forbidden manipulations, contrary to audiences with professional traders where everybody knows that shill bidding is an integral part of the game such that nobody is actually deceived as argued by Meisel [31], but also because shill bidding can take "industrial" proportions on the net with the use of dozen of shill bidders for a single auction as in the manipulation reported in Lamy [24] or with the creation

[4]See the series of articles: Make me an offer: the eBay bid scam - dealers fix online auctions with a little help from their friends. The Sunday Times, January 28 2007; Revealed: how eBay sellers fix auctions. The Sunday Times, January 28 2007; Top trader banned for life over fixed eBay auctions. The Sunday Times, February 4 2007.

[5]B. Krebs, Snapnames: Former exec. bid up domain prices, The Washington Post, November 4 2009.

[6]See S. Duin, The auction of whiz $\sharp$ 17, The Oregonian, October 27 2009 and S. Merten, Lawsuit claims heritage auction galleries uses fake bidder to manipulate auctions, The Dallas Observer, September 10 2009 for details. Such a shill bidding activity is prohibited for Christie's and Sotheby's that are regulated by the New York City Department of Consumer Affairs. Heritage has also been accused of using a shill bidder that may withdraw his bids if he becomes the winner, a manipulation that is unambiguously prohibited and that the firm has strongly denied.

of a commercial company providing a service that automates the process of shill bidding (Ockenfels et al. [37]). We emphasize that shill bidding is not confined to online auctions as illustrated by the next example. In traditional auctions, shill bidding looks more home-made and sellers need to be more discreet in order not to be caught. However, in traditional auction houses, many agents offer their services to shill bid.[7] Shill bidding is thus pervasive in real estate auctions in Australia and New Zealand, two countries where private houses are commonly sold through auctions. In 2004, a new legislation has been brought in to change the way auctions were conducted in New South Wales and Victoria. However, it is not clear whether shill biding prohibition can actually be enforced if prosecutions are never engaged if suspected shill bidders deny the allegations.[8] The Australian experience where auctions are not confined to professional traders or to small amounts leads to a new profession: buyers' advocates. Pioneering in this activity is the former real-estate agent David Morrell who does not solely explain how to spot shill bidding in Morrell [34] but whose company offers to bid for a 1% fee in case of success. In other words, the client finds the property he is interested to bid on and then go to a buyer's advocate to which he reports his valuation and who will bid in the auction on the client's behalf. If real-life English auctions were fitting with their theoretical counterparts, then there will be few room for such an activity. David Morrell argues that a key element of their technical expertise is their ability to unmask the shill bidders throughout the auction process such that they switch to a bargaining phase with the seller once there is no real bidder to bid against. They also propose to raise some fictitious bids on their own insofar as one of their tricks is to use numerous identities in the auction, a strategy whose aim is precisely to influence the seller's beliefs and thus either her shill bidding activity or the way she exercises possible rights to cancel the sale.[9]

In auctions with entry, i.e. where potential bidders have to incur a sunk cost in order to be able to learn their valuation and participate to the auction, the optimal designs correspond to the ones that maximize the welfare as shown by Levin and Smith [26]: in an ex ante perspective, bidders have no informational rents such that the seller's expected payoff and the expected welfare coincide. In the symmetric independent private value (IPV) model, standard auctions where the reserve price is set to the seller's reservation value and without entry fees implement the first best: they are ex post efficient while implementing

[7] One of the French largest auction houses, the Hôtel Drouot, has been smeared by a theft scandal among his corporation of packers in 2009. As a by-product, it sheds some light on some questionable services that those packers informally offer and in particular shill bidding due to their special acquaintances with the sellers which result from their official handling responsibilities. In December 2009, they were deprived from their ability to take bidding orders and bid on their own in the auctions. See e.g. Les Mystères de l'Hôtel Drouot, Le Monde, February 02 2010.

[8] T. Giles and G. Mitchell. Dodgy bids clamp fails. Herald Sun, July 30 2005

[9] The theoretical foundations for such an activity are discussed in more details in remark 7.2.

the efficient level of entry. Levin and Smith [26]'s analysis relies on two crucial assumptions: first, the seller is able to commit fully to the auction mechanism before potential bidders make their entry decisions; second, the seller does not revise her reservation value in the course of the game, either from exogenous information or from endogenous information that comes from the auction itself as the observation of the number of entrants and possibly their bids. The purpose of this paper is to revisit auctions with entry à la Levin and Smith [26] while relaxing those two assumptions. We emphasize that we chose Levin and Smith [26]'s framework for tractability reasons but that the hold-up issue and the related insights would extend more generally to models with pre-participation investments where ex ante and ex post optimal mechanisms do not coincide.[10]

In a first step, we adopt a positive perspective: we revisit second price/English auctions with a strategic seller deciding whether to incur a cost to hire secretly a shill bidder being able to bid in the same way as any real bidder in the auction. With exogenous entry, i.e. after the potential bidders made their entry decisions, it is well-known from Riley and Samuelson [40] for standard auctions and Myerson [36] for general mechanisms that there is a discrepancy between revenue and welfare maximization: the seller has an incentive to reduce trade in order to minimize bidders' informational rents. If the cost to hire a shill bidder is small enough, then the seller can not credibly commit to ex post efficiency by setting the reserve price to her reservation value insofar as she would benefit from hiring a shill bidder that would bid up to the optimal reserve price à la Myerson [36]. Anticipating the shill bidding activity, the equilibrium probability of entry will then shrink such that the ex ante welfare (which still coincides with the seller's expected payoff if entry is mixed) does not correspond to the first best: inefficiency results from both ex post allocative inefficiencies and inefficient entry since bidders' payoffs are then smaller than their contribution to the welfare. The analysis of auctions with entry is thus utterly modified by shill bidding and a variety of new insights emerges. First, for some reserve prices, there is a multiplicity of bidder-symmetric equilibria. This results from a strategic complementarity between the seller's shill bidding activity and bidders' participation decisions: if the seller shill bids more, then bidders participate less which enhances the incentives to shill bid. Second, participation may rise with the announced reserve price since announced reserve prices may not coincide with "effective" reserve prices that should also take into account the shill bidding activity: contrary to reserve prices that are close enough to the optimal reserve price à la Myerson [36], reserve prices that are too low may not be "credible" insofar as the seller would strictly benefit from hiring a shill bidder for any candidate to be an equilibrium

[10] See Shi [41] for a setup where pre-participation investments correspond to information acquisition.

probability of entry. On the whole, raising the announced reserve price may then lower the effective reserve price and thus raise entry. Third, if the shill bidding constraint is binding insofar as the seller can not credibly commit not to shill bid when setting the reserve price at her reservation value, then the optimal reserve price lies strictly above her reservation value. Fourth, if the hold-up issue is a source of major inefficiencies, then posted-prices may outperform standard auctions since those latter do not suffer from any commitment failure. This occurs in particular if participation costs are high enough ceteris paribus. Fifth, models with shill bidding involve some surprising comparative statics: the seller's expected payoff may not increase in her reservation value such that she may strictly benefit from 'burning her ships'; the optimal announced reserve price is increasing in the entry costs which contrasts with optimal auctions without shill bidding where it is independent of the entry costs and which also contrasts with the optimal posted-price which is decreasing in the entry costs. Sixth, while the equilibrium probability of entry is efficient under free entry either under optimal auctions without shill bidding or under optimal posted-prices, i.e. the seller would not use entry fees/subsidies if this instrument were available, we show that the seller would strictly benefit from subsidizing entry in the optimal second-price auction with shill bidding when the shill bidding constraint is binding.

In a second step, we adopt a normative perspective. The literature in law notes that surprisingly it is not uncommon that courts show clemency toward shill bidding. "The consideration which moves the few dissenters from the rule against puffing appears to be a desire to shield the seller of property from an unjust price" ([1], p.432). Bennett [7] and Meisel [31] also emphasize how courts of equity recognized the right of the seller to participate in the auction to prevent a sale at a sacrifice. We consider then a larger framework where the seller may receive additional information about her reservation value only at the interim stage, i.e. after the choice of the selling mechanism and bidders' entry decisions. In such a setup, the design of an efficient auction with entry is no longer straightforward: in particular, an auction without shill bidding is no longer an appropriate solution. There is a need for leaving some role to the seller in order to let her express her reservation value at the interim stage. Otherwise inefficiency may occur either because she's selling the good though she has the highest valuation or she's keeping the good though the highest valuation among the bidders is above hers. Auctions with shill bidding provide some valuable flexibility but at some costs since the hold-up issue prevails, such that they fail to implement the first best. Flexibility can be introduced by an alternative instrument: the right to cancel the sale after the price has been fixed. The second price auction without reserve but with cancelation rights seems a good candidate for ex post efficiency. On the one hand, it prevents the seller to sell at undervalue and thus seems a perfect substitute to the legitimate motive for shill bidding to prevent a sacrifice. On the other hand, cancelation rights do not entail the undesirable effect associated with shill bidding labeled as “puffing the sale”. In jurists’ minds, this expression covers both the possible beliefs’ manipulation effect (absent in the present paper) and the “unfair” rent extraction that consists in making the winner pay a strictly higher price than the maximum of the seller’s reservation value and the highest valuation among his opponents. With cancelation rights, the winning price is always equal to the second highest valuation among the entrants which should be larger than the seller’s reservation value since the sale would have been canceled otherwise. Nevertheless, second price auctions with cancelation rights create a new source of inefficiency that never occurs with shill bidding: it may occur that the seller cancels the sale because the winning price is lower than her reservation value while ex post efficiency would dictate to allocate the good to the highest bidder. In other words, inefficiency occurs when the seller’s reservation value lies between the first and the second highest bids. In the English button auction (i.e. without jump bids), a format which is, in our pure private value model, strategically equivalent to the second price auction, we would like to allow the seller to “puff” the sale up to her reservation value to avoid such inefficiencies. The issue is that, once she’s allowed to bid freely, the seller would optimally shill bid up to her ex post optimal reserve price that lies strictly above her reservation value. If the seller’s reservation value were common knowledge at the interim stage, then a simple mechanism can avoid the aforementioned various pitfalls that preclude the implementation of the first best. It is the English auction with jump bids and with cancelation rights: if the second highest valuation lies below the seller’s reservation value, then the remaining bidder has the opportunity to make a jump bid up to the seller’s reservation value. Our analysis provides thus a completely new argument in favor of this popular auction format. Furthermore, our analysis sheds also some light on the role of auction houses.

Shill bidding has received few attention in the economic literature. First, in models with interdependent values, Lamy [24] shows how it can create ex-post regret and establishes that this commitment failure can reverse the well-known revenue-ranking between standard auction formats while Vincent [42] shows in an example how it gives more opportunity to observe the bidding of others when there is uncertainty on the seller’s reservation value, which may raise the expected revenue from a “Linkage Principle”’s channel, a channel that also supports the related use of ex post rights to cancel the sale in Horstmann and LaCasse [19]’s analysis of repeated sales with interdependent values. Second, in pure private value models, shill bidding offers some flexibility by allowing the seller to adapt her reserve price policy as a function of the bidding history, which can be good not solely in term of revenue but also in term of welfare: Graham et al. [16] consider asymmetric bidders with independent values while Bag et al. [5] consider symmetric bidders with conditionally independent values. In the same 'flexibility effect' vein, Lopomo [27] establishes more generally that the English button auction with a non-anonymous shill bidder is optimal among a large class of mechanisms when the seller does not have private information that is relevant to bidders and entry is exogenous.

This paper is organized as follows. Section 2 introduces the basic auction model with shill bidding when the seller's reservation value is common knowledge ex ante and lays the foundation of our equilibrium analysis. Under a mild additional assumption, section 3 derives the whole set of bidder-symmetric equilibria in the second price auction with shill bidding. Section 4 moves to the characterization of the optimal reserve price and derives related comparative statics. The possible use of entry fees is also discussed. Section 5 focus on the influence of the seller's reservation value on her expected payoff and sheds some light on her incentives to 'burn her ships'. Section 6 is devoted to the analysis of posted-price mechanisms and their relative performance with respect to auctions. Section 7 introduces the second main ingredient of our analysis: uncertainty on the seller's reservation value such that standard auctions without shill bidding are no longer efficient and which thus asks for flexible mechanisms that leave some active role to the seller after bidders made their entry decisions. We consider then the problem of designing efficient auctions and consider in particular cancelation rights and jump bids. Some simulations are reported in section 8. Section 9 concludes on the consequences for empirical works. The proofs are all relegated in Appendices A-M.

2 The Model

Consider a setting with $N \geq 2$ symmetric risk-neutral potential bidders and a risk-neutral seller who wants to sell an indivisible object for which her valuation (or reservation value) is $X_s \geq 0$. If they incur a fixed sunk cost $c_e > 0$, then potential bidders are learning their valuations and become eligible to bid in the subsequent auction. Bidders' valuations are private information and are independently distributed with a common CDF $F(.)$ that has a continuously strictly positive density $f(.)$ on its support $[\underline{x}, \overline{x}]$. The hazard rate function $x \to \frac{1-F(x)}{f(x)}$ is assumed to be a strictly decreasing function on $[\underline{x}, \overline{x}]$. For $k \leq n$, let $F^{(k:n)}$ denote the CDF of the kth order statistic among n i.i.d variables distributed according to the CDF $F(.)$.

The timing of the auction games with shill bidding is as follows. First, the seller announces a reserve price (or opening bid) $r \in \mathbb{R}_+$ and decides secretly whether to enroll a shill bidder. Hiring a shill bidder costs $c_{shill} \geq 0$. Second, each potential bidder decides whether or not to incur the sunk cost c_e in order to enter the auction game: entrants are then privately informed about their valuations. Third, the number of entrants is observed by the seller who gives instructions to the shill bidder on how to bid in the forthcoming auction. Fourth, the entrants and the shill bidder are playing a second price auction or, equivalently, an English button auction.

Variations on the auction game are introduced later in the course of the paper. Without further precisions, the default auction game is the one described above. Our analysis is restricted to so-called **bidder-symmetric equilibria** where potential bidders are using the same strategy at the entry stage, as in Levin and Smith [26]. Let $q \in [0, 1]$ denote the corresponding probability to enter the auction. We assume furthermore all over the paper that bidders are bidding their valuation at the auction stage, i.e. are playing their standard weakly dominant strategy:

Assumption A 1 *Entrants that have a valuation greater than r are bidding their value.*

If the seller can commit not to shill bid or equivalently if the shill bidding cost c_{shill} is prohibitively costly, then the model corresponds to the private value model in Levin and Smith [26] and the optimal reserve price is to set $r = X_s$. Once the entry decisions have been taken, the seller would like to update the reserve price up to the optimal reserve price with exogenous participation, denoted by r_M, which is uniquely characterized by the implicit equation:

$$X_s = r_M - \frac{1 - F(r_M)}{f(r_M)} \tag{1}$$

if $\underline{x} - \frac{1}{f(\underline{x})} < 0$ and $r_M = \underline{x}$ otherwise. As a corollary, we obtain in equilibrium that if the seller enrolls a shill bidder, then she will instruct him to bid r_M. In equilibrium, note that the strategy of the shill bidder is thus pure on the contrary to the equilibria in Lamy [24] where it is always mixed. Nevertheless, the decision to hire a shill bidder may be mixed, but is not necessarily so as detailed below. Note also that the instruction to the shill bidder does not depend on the number of entrants in the auction, a well-known insight that comes from the i.i.d. assumption of the model.

Remark 2.1 In a first price auction, there seems to be no room for shill bidding if the announced reserve price r is larger than X_S: as in Lamy [24], a shill bidder is powerless to raise the winning price while it may only cancel profitable sales. This argument is valid only if the shill bidding activity is secrete or if bid submissions are simultaneous, i.e. if the seller can not use a shill bidder that submits a bid and the amount of his bid with ostentation such that the real bidders are facing then a game that corresponds to a first auction with the reserve price set to the shill bidder's open bid. Furthermore, if we replace the shill bidding activity by the stronger commitment failure of being unable to commit not to update the announced reserve price after potential entrants decide whether to enter the auction, then other standard formats will suffer from the hold-up issue analyzed in the present paper. On the whole, this discussion suggests that our analysis may also be relevant for first price auctions.

An equilibrium is thus fully characterized by two variables: potential bidders' probability to enter the auction $q \in [0,1]$ and the probability, denoted by $p \in [0,1]$, that the seller enrolls a shill bidder.

From the mechanism design approach (see Myerson [36]), bidders' expected gross payoffs (i.e. after the participation costs have been sunk) can be computed from the way the good is assigned to the bidders according to their valuation profile. More precisely, the expected gross payoff of a bidder with valuation x and facing n entrants is given by:

$$\vartheta_n(\underline{x}) + \int_{\underline{x}}^{x} Prob_n^{win}(s)ds, \tag{2}$$

where $Prob_n^{win}(s)$ corresponds to the probability of winning the good with valuation s and $\vartheta_n(\underline{x})$ to the expected gross payoff of a bidder having the lowest valuation $\underline{x}$. This formula holds for general selling mechanisms, thus not solely for standard auctions without shill bidding (where $Prob_n^{win}(s) = F^n(s)$ if $s \geq r$ and 0 otherwise), but also for auctions with shill bidding as applied below, for posted-prices as in section 6 and auctions with cancelation rights as in section 7.

If a given potential bidder faces n entrants and if there is no shill bidding activity, then the integration of (2) over the bidder's valuation CDF leads to the following ex ante expected profit in the auction: $\int_r^{\overline{x}} (\int_r^u F^n(s)ds)dF(u)$. If the seller enrolls a shill bidder with probability p, then the way the good is assigned corresponds to the following one: with probability $(1-p)$, the good is assigned exactly in the same way as in the auction without shill bidding and with probability p in the same way as in an auction without shill bidding where the reserve price would have been set to r_M. The expected gross payoff is now given by $(1-p) \cdot \int_r^{\overline{x}} (\int_r^u F^n(s)ds)dF(u) + p \cdot \int_{r_M}^{\overline{x}} (\int_{r_M}^u F^n(s)ds)dF(u)$.

Let $B(q,r,p)$ denote the expected gross payoff (once the entry costs have been sunk) of a given entrant in the auction if the reserve price is r and the strategies of his opponents are characterized by q and p.

$$B(q,r,p)=\sum_{n=0}^{N-1}\binom{N-1}{n}q^n(1-q)^{N-1-n}[(1-p)\cdot\int_r^{\overline{x}}(\int_r^u F^n(s)ds)dF(u)+p\cdot\int_{r_M}^{\overline{x}}(\int_{r_M}^u F^n(s)ds)dF(u)]. \tag{3}$$

A straightforward calculation leads to:

$$B(q,r,p)=(1-p)\cdot\int_r^{\overline{x}}(\int_r^u [qF(s)+(1-q)]^{N-1}ds)dF(u)+p\cdot\int_{r_M}^{\overline{x}}(\int_{r_M}^u [qF(s)+(1-q)]^{N-1}ds)dF(u). \tag{4}$$

Note that $B(q,r,p)$ is thus strictly decreasing in q on $[0,1]$ if $r<\overline{x}$ and strictly decreasing in p if $r<r_M$. We make the following two assumptions in order to guarantee that the equilibria of the game involve mixed entry, i.e. $q\in(0,1)$, for any announced reserve price $r\leq r_M$.[11]

Assumption A 2 *The entry cost is large enough such that full entry is never an equilibrium:*

$$c_e>\int_0^{\overline{x}}(\int_0^u F^{(N-1)}(s)ds)dF(u)\equiv B(1,0,0). \tag{5}$$

Assumption A 3 *The entry cost is small enough such that no entry is not an equilibrium with the reserve price r_M:*

$$c_e<\int_{r_M}^{\overline{x}}(u-r_M)dF(u)\equiv B(0,r_M,0). \tag{6}$$

Inequality (5) means that entry is not individually rational if all the remaining potential bidders decide to enter the auction even if the reserve price is null and without any shill bidding activity. A fortiori, entry can not be individually rational if all potential bidders decide to enter the auction for general reserve prices and with possibly some shill bidding activity, i.e. $B(1,r,p)<c_e$, since such equilibria would leave smaller informational rents $(B(1,r,p)\leq B(1,0,0))$. Inequality (6) means that entry is individually rational if all the remaining potential bidders decide not to enter the auction if the reserve price is r_M and thus a fortiori for any reserve price below r_M and with possibly some shill bidding activity.

Under assumptions A2-A3, entry is thus mixed in any equilibrium of an auction with $r\leq r_M$. From now on, we actually assume $r\leq r_M$.[12] The potential bidders' equilibrium equation is given by:

[11] If N is large enough, then $\int_0^{\overline{x}}(\int_0^u F^{(N-1)}(s)ds)dF(u)<\int_{r_M}^{\overline{x}}(u-r_M)dF(u)$ such that there exists some entry costs such that both conditions (5) and (6) are satisfied.

[12] Auctions with $r>r_M$ are always outperformed by the auction with $r=r_M$.

$$B(q,r,p) = c_e. \tag{7}$$

For a given p and r, we have $B(0,r,p) > c_e$ while $B(1,r,p) < c_e$ such that (7) a unique solution on $(0,1)$, denoted by $q^*(r,p)$. For $p = 0$, let $q^*(r)$ denote the solution. By integrating by parts (4), we obtain equivalently:

$$B(q^*(r),r,0) = \int_r^{\overline{x}} (1-F(u))[q^*(r)F(u) + (1-q^*(r))]^{N-1} du = c_e. \tag{8}$$

The differentiation of (8) leads to:

$$\frac{\partial q^*}{\partial r} = -\frac{(1-F(r))[q^*(r)F(r) + (1-q^*(r))]^{N-1}}{(N-1)\int_r^{\overline{x}} (1-F(u))^2 [q^*(r)F(u) + (1-q^*(r))]^{N-2} du}. \tag{9}$$

We obtain thus that $q^*(.)$ is a strictly decreasing function for any $r \leq r_M$. If $p = 1$, the solution of the equilibrium equation (7) equals $q^*(r_M)$. Furthermore, since $B(q,r,p)$ is strictly decreasing in q on $[0,1]$ and decreasing in p on $[0,1]$, then any probability of entry q that satisfies (7) should then satisfy $q \in [q^*(r_M), q^*(r)]$ for any $p \in [0,1]$.

We now move to the seller's side. Let $\Pi_n(r)$ denote the expected payoff of the seller with a reserve price r when there are n entrants. For $n = 0$, we have $\Pi_n(r) = X_s$. For $n \geq 1$, we have $\Pi_n(r) = F^{(1:n)}(r) \cdot X_s + [F^{(2:n)}(r) - F^{(1:n)}(r)] \cdot r + \int_r^{\overline{x}} u dF^{(2:n)}(u)$. A straightforward calculation leads to:[13]

$$\Pi_n(r) - \Pi_n(0) = \int_0^r [X_s + \frac{1-F(u)}{f(u)} - u] d[F^{(1:n)}(u)]. \tag{10}$$

Let $R_A(X_S,r,q)$ and $W_A(X_s,r,q)$ denote respectively the seller's expected payoff and the expected welfare at the ex ante stage when the reserve price equals r and the probability of entry q and if she does not hire a shill bidder.

$$R_A(X_S,r,q) = \sum_{n=0}^{N} \binom{N}{n} q^n (1-q)^{N-n} \Pi_n(r) \tag{11}$$

A straightforward calculation leads to:

$$R_A(X_S,r,q) = [\sum_{n=0}^{N} \binom{N}{n} q^n (1-q)^{N-n} \Pi_n(0)] + \int_0^r [X_s + \frac{1-F(u)}{f(u)} - u] d[(qF(u) + (1-q))^N] \tag{12}$$

From (11) and since $\Pi_n(r)$ is strictly increasing in the number of entrants n (for any

[13] The expression (10) is slightly awkward outside the range $[\underline{x}, \overline{x}]$ where $f(u) = 0$. All over the paper, we should read $[X_s + \frac{1-F(u)}{f(u)} - u] \cdot f(u)$ as $(1-F(u))$ when $f(u) = 0$.

$r < \overline{x}$), we obtain that $R_A(X_S, r, q)$ is strictly increasing in q. From (12), $r \to R_A(X_S, r, q)$ is strictly increasing on $[0, r_M]$ for any $q > 0$.

A similar calculation as for R_A leads also to:

$$W_A(X_s, r, q) = (q \cdot F(r) + (1-q))^N \cdot X_S + \int_r^{\overline{x}} ud[(qF(u) + (1-q))^N] - qN \cdot c_e. \quad (13)$$

From Levin and Smith [26], we know that in an equilibrium where potential bidders strictly mix then the seller's expected payoff coincides with the expected welfare, i.e. $W_A(X_s, r, q^*(r)) = R_A(X_s, r, q^*(r))$.

Let $H(q, r)$ denote the expected benefit in the auction of the shill bidding activity before the number of entrants has been revealed and after the cost to enroll a shill bidder has been sunk, i.e. the difference between the expected payoff in the auction when the seller has hired a shill bidder (that bids then r_M) and the expected payoff in the auction when the seller has not hired a shill bidder: $H(q, r) := R_A(X_S, r_M, q) - R_A(X_S, r, q)$. To alleviate the notation, we have dropped the dependence on the variable X_S. From (12), we obtain:

$$H(q, r) = \int_r^{r_M} [X_s + \frac{1 - F(u)}{f(u)} - u] d[(qF(u) + (1-q))^N]. \quad (14)$$

For any equilibrium where the seller strictly mix, i.e. if $0 < p < 1$, the seller's equilibrium equation is characterized by the indifference between enrolling or not a shill bidder. The seller's indifference equation is then given by:

$$H(q, r) = c_{shill}. \quad (15)$$

For an equilibrium where the seller does not enroll a shill bidder, we have $q = q^*(r)$ and the following inequality needs to be satisfied:

$$H(q^*(r), r) \leq c_{shill}. \quad (16)$$

For an equilibrium where the seller always enrolls a shill bidder, we have $q = q^*(r_M)$ and the following inequality needs to be satisfied:

$$H(q^*(r_M), r) \geq c_{shill}. \quad (17)$$

Under assumptions A2-A3, we have thus three kinds of candidates to be an equilibrium:

- Type 1: a unique equilibrium without any shill bidding activity and where the entry probability equals $q^*(r)$,

- Type 2: a unique equilibrium where the seller always enrolls a shill bidder and where the entry probability equals $q^*(r_M)$,
- Type 3: equilibria where the seller strictly mix and that are characterized as the solutions of the equations (7) and (15) and where the entry probability necessarily belongs to the interval $(q^*(r_M), q^*(r))$ for $r < r_M$.

The first two kinds of equilibria are called "pure strategy equilibria" in the following since the seller uses a pure strategy contrary to the third kind of equilibria where the seller hires a shill bidder with a probability $p \in (0,1)$ which are called "mixed strategy equilibria". I emphasize that the terminology pure/mixed refers to the seller's behavior and not to the potential participants which always use a mixed strategy under A2-A3. The following proposition establishes equilibrium existence.

Proposition 2.1 *There exists a bidder-symmetric equilibrium in the second price auction with shill bidding. Bidder-symmetric equilibria are of type 1,2 or 3.*

If $\max_{q\in[q^*(r_M),q^*(r)]} H(q,r) < c_{shill}$, then the seller would strictly prefer not to hire a shill bidder for any equilibrium candidate: there is then a unique equilibrium and it is of type 1. Since $H(q,r)$ is strictly decreasing in r on the range $[0, r_M]$ for any $q > 0$ and since the interval $[q^*(r_M), q^*(r)]$ shrinks with r, then the function $r \to \max_{q\in[q^*(r_M),q^*(r)]} H(q,r)$ is strictly decreasing on $[0, r_M]$ while the expression equals zero at r_M. If $c_{shill} > 0$, there is then a threshold $\widehat{r} < r_M$ such that if the reserve price r is larger than $\widehat{r}$ then there is a unique equilibrium and it involves no shill bidding. If $\min_{q\in[q^*(r_M),q^*(r)]} H(q,r) > c_{shill}$, then the seller would strictly prefer to hire a shill bidder for any equilibrium candidate: there is then a unique equilibrium and it is of type 2. If c_{shill} is small enough, then there is a threshold $\widetilde{r} < \widehat{r}$ such that if the reserve price r is smaller than $\widetilde{r}$ then there is a unique equilibrium and the seller always hires a shill bidder. When we move to a reserve price below $\widetilde{r}$ to one $r \in (\widehat{r}, r_M)$, then the participation in equilibrium rises from $q^*(r_M)$ to $q^*(r)$. On the whole, we obtain thus the surprising insight that **participation may be enhanced by higher reserve prices**. The intuition is that the incentives to hire a shill bidder shrinks with the reserve price. Naturally, for reserve prices in $(\widehat{r}, r_M)$ then shill bidding does not arise in equilibrium such that entry decreases with the reserve price, i.e. the standard comparative statics holds. Outside of the range $[\widetilde{r}, \widehat{r}]$, we have thus obtained equilibrium uniqueness and the equilibrium is in pure strategy. On the contrary, on the range $[\widetilde{r}, \widehat{r}]$, equilibrium multiplicity may arise as developed in next section.

3 Characterization of the equilibrium set [14]

This section is devoted to the characterization of the equilibrium set when all the primitives of the model are fixed and making vary the shill bidding cost. We still consider $r < r_M$. For this section and this section only, we introduce an additional assumption that will guarantee that the incentives to hire a shill bidder are decreasing in the probability of entry for any probability of entry that is compatible with an equilibrium candidate.

Assumption A 4 $F(r_M) < 1 - \frac{1}{Nq^*(r_M)}$.[15]

Lemma 3.1 *Under A4, the gain from hiring a shill bidder decreases with the probability of entry ceteris paribus: $q \to H(q, r)$ is strictly decreasing in q on the range $[q^*(r_M), 1]$ for any $r < r_M$.*

For a given reserve price, some shill bidding activity would reduce potential bidders' gains from participation compared to the equilibrium where the seller could commit not to shill bid. Thus it decreases the probability of entry to some $q \geq q^*(r_M)$ which makes the shill bidding activity even more profitable according to lemma 3.1. **Due to this amplification effect, multiple equilibria may arise**: an equilibrium without shill bidding may coexist with one involving shill bidding with probability one and also with an equilibrium where the seller strictly mix. From lemma 3.1, such mixed equilibria are unique under A4 since (15) has then a unique solution for r and c_{shill} fixed.

Let $c^*_{shill}(r) := H(q^*(r), r)$ and $c^{**}_{shill}(r) := H(q^*(r_M), r)$. Lemma 3.1 guarantees that $c^{**}_{shill}(r) > c^*_{shill}(r)$. From (16), an equilibrium without shill bidding exists if and only if $c_{shill} \leq c^*_{shill}(r)$. From (17), an equilibrium where the seller hires a shill bidder with probability one exists if and only if $c_{shill} \geq c^{**}_{shill}(r)$. A strictly mixed equilibrium exists if (15) has a solution on the range $q \in (0, 1)$ or equivalently (still using lemma 3.1) if and only if $c^*_{shill}(r) > c_{shill} > c^{**}_{shill}(r)$. The previous discussion is summarized in Figure 1, where the equilibrium set is depicted, and in the following proposition.

Proposition 3.2 *Consider a reserve price $r < r_M$ and under A4. There is a unique equilibrium, which involves no shill bidding, if the shill bidding costs are high enough, $c_{shill} > c^{**}_{shill}(r)$, and a unique equilibrium, where the seller enrolls a shill bidder with probability 1, if the shill bidding costs are small enough, $c_{shill} < c^*_{shill}(r)$. For intermediate shill bidding*

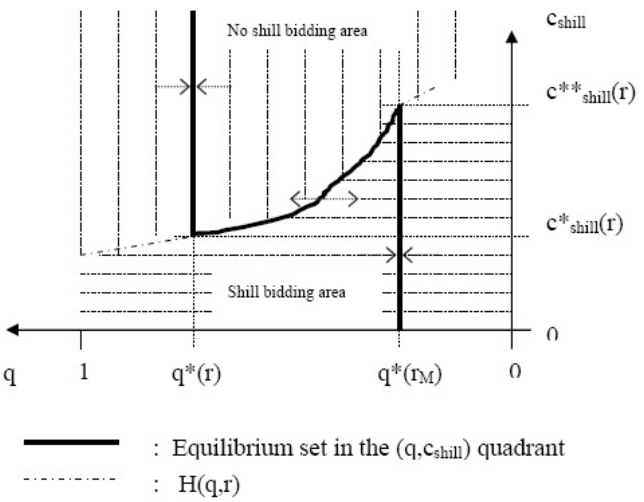
[14]A reader eager to get our main economic insights may skip this section at first reading.

[15]The assumption holds if the expected number of entrants ($Nq^*(r_M)$) in the auction with the reserve price r_M is large enough. If $F(r_M) < 1 - \frac{1}{N}$, then it holds if the entry costs, ceteris paribus, are sufficiently small.

costs in the interval $(c^*_{shill}(r), c^{**}_{shill}(r))$, there are three equilibria: one without shill bidding, one with shill bidding with probability 1 and one in strictly mixed strategy.[16]

For a given shill bidding cost, a pure strategy equilibrium always exists under A4. Furthermore, multiplicity arises for intermediate shill bidding costs on the range $(c^*_{shill}(r), c^{**}_{shill}(r))$ and multiplicity necessarily involves the coexistence of both pure strategy equilibria.

Figure 1: Equilibrium set in auctions with entry



Let us discuss informally the stability properties of the various equilibria, properties that are illustrated in Figure 1 with dotted arrows. We adopt an alternating-best-responses dynamics' perspective with respect to the two parameters (q, p) in the similar way as in Cournot adjustments (Fudenberg and Levine [15]). The pure strategy equilibria where $H(q, r) \neq c_{shill}$ are "stable" insofar as if the probability of entry moves locally then it does not modify the seller's incentives to hire a shill bidder such that the probability of entry should stabilize at its equilibrium level q, where $q = q^*(r)$ and $q = q^*(r_M)$ if the equilibrium respectively involves no shill bidding or involves shill bidding with probability 1. On the contrary, strictly mixed equilibria can be viewed as unstable since any local perturbation of the probability of entry would induce a destabilizing dynamic: if the probability of entry is slightly lower than q then it raises the incentives to hire a shill bidder and if the shill bidding activity raises then it lowers again the incentives to participates and so on. Similarly, if the probability of entry is slightly higher than q then it lowers the incentives to hire a shill bidder and if the shill bidding activity shrinks then it raises again the incentives to participates and so on. The two degenerate mixed strategy equilibria that lie on the indifference curve $H(q, r) = c_{shill}$ are stable for one direction of the perturbation of the

[16]For the degenerate cases, $c_{shill} = c^*_{shill}(r)$ or $c_{shill} = c^{**}_{shill}(r)$, there are only two equilibria. The mixed strategy equilibrium collapses with one in pure strategy.

probability of entry and unstable in the other one. Remark that it is the corresponding equilibrium without shill bidding that will emerge in next sections under an optimal reserve price.

Potential bidders are indifferent between all equilibria with $q \in (0,1)$ (as guaranteed by A2-A3) since their expected payoffs are then null. Let us discuss the seller's preferences over the equilibrium set for a given reserve price and making vary the shill bidding cost. If the seller does not hire a shill bidder with some positive probability then her expected payoff is fixed and equals $W_A(X_S, r, q^*(r)) = R_A(X_S, r, q^*(r))$. The seller's expected payoff for the equilibria on the curve $H(q,r) = c_{shill}$ is given by $R_A(X_S, r, q^*(r,p))$. This latter expression is strictly decreasing in p. Consequently, the seller prefers equilibria with the lowest shill bidding activity. In Figure 1, its means that the seller's expected payoff shrinks when we move from $(q^*(r), \infty)$ to $(q^*(r_M), c^{**}_{shill}(r))$ on the equilibrium set curve. However, when we move from $(q^*(r_M), c^{**}_{shill}(r))$ to $(q^*(r_M), 0)$, then the seller's expected payoff raises: on this portion of the curve, she always hires a shill bidder in equilibrium such that she's better off with lower shill bidding costs. In other words, for a given reserve price, if the shill bidding cost is too low such that the seller can not prevent an equilibrium which involves shill bidding with probability 1 to be played, then she is better off if shill bidding becomes frictionless. However, if she can choose the reserve price strategically, this insight is no longer true as shown in corollary 6.1. Note that the seller's preferred equilibrium which involves shill bidding with probability 1, i.e. for $c_{shill} = 0$, is equivalent to the equilibrium without shill bidding where he would have set the reserve price r_M. If $r \geq X_S$, then we obtain that the seller's preferred equilibria in the equilibrium set are those without shill bidding since the inequality $W_A(X_S, r, q^*(r)) > W_A(X_S, r_M, q^*(r_M))$ would hold (if $r \geq X_S$, then $u \to W_A(X_S, u, q^*(u))$ is strictly decreasing on $[r, r_M]$ as established in the proof of proposition 4.3).

4 Optimal reserve price policy

We now consider the strategic choice of the announced reserve price by the seller: we characterize the optimal reserve price that maximizes her expected payoff. We assume implicitly that the seller can select her most preferred equilibrium, which is actually (weakly) Pareto dominant in the equilibrium set as noted previously. The following lemma shows that for any equilibrium which involves some shill bidding activity then we can pick a reserve price and an equilibrium that raises a strictly higher expected payoff than the former one.

Lemma 4.1 *Suppose that $c_{shill} > 0$. If there is a reserve price r such that the probability of entry q and the shill bidding probability $p > 0$ is an equilibrium profile, then there is a reserve price $r' \in [r, r_M]$ such that the same probability of entry q and no shill bidding activity is an equilibrium profile if the announced reserve price is r'. Furthermore, this latter equilibrium raises a strictly higher payoff than the former one.*

As a corollary to lemma 4.1, there is no loss of generality to restrict the analysis to equilibria without shill bids when we are looking for an optimal reserve price, i.e. to seek for reserve prices such that $H(q^*(r), r) \leq c_{shill}$. Those reserve prices are now called "implementable reserve prices" according to the following definition.

Definition 1 *A reserve price is implementable if there exists an equilibrium without any shill bidding activity when this reserve price is announced.*

Reserve prices strictly above r_M are obviously implementable since the incentives to hire a shill bidder are then null ($H(q^*(r), r) = 0$ for $r \geq r_M$). Nevertheless, they are suboptimal since they are outperformed by the reserve price r_M. Below r_M, lowering the reserve price has two impacts on the incentives to hire a shill bidder: on the one hand, the standard channel through the variation of the reserve price ceteris paribus makes shill bidding more profitable; on the other hand, lowering the reserve price raises the probability of entry[17] which may negatively impact the incentives to shill bid. Next lemma shows that the first channel always dominates the second one under a mild additional assumption.

Lemma 4.2 *If $x \to (X_S - x)\frac{f(x)}{1-F(x)}$ is decreasing on $[0, \overline{x})$[18], then the gain from hiring a shill bidder decreases with the announced reserve price if bidders participate as if there were no shill bidding activity: $r \to H(q^*(r), r)$ is decreasing in r.*

We obtain thus that the set of implementable reserve prices is an interval, denoted by $[r^*, \infty)$. If $H(q^*(0), 0) < c_{shill}$, then any reserve price is implementable, i.e. $r^* = 0$. Otherwise, r^* is the lowest solution in $\mathbb{R}_+$ of the equation

$$H(q^*(r), r) = c_{shill}. \tag{18}$$

For the rest of the paper, we only need to assume that the set of implementable reserve prices is an interval as guaranteed by the monotonicity assumption in lemma 4.2.[19] Note

[17] From lemma 3.1, it does surely under the supplementary assumption A4.

[18] Note that the assumption on the hazard rate guarantees that $x \to (X_S - x)\frac{f(x)}{1-F(x)}$ is decreasing on $[X_S, \overline{x})$. The monotonicity holds also on the range $[0, \underline{x})$ since the function is constant on this range. On the whole potential failures of this additional assumption may arise only on the range $[\underline{x}, X_S]$ if $X_S > \underline{x}$. Otherwise, if $X_S \leq \underline{x}$, it is necessarily true.

[19] This assumption is far from being necessary and has been chosen actually for simplicity. It is left to the reader to check that a completely different assumption that

that this is always true if $c_{shill} = 0$ where $H(q^*(r), r) > 0$ if and only if $r < r_M$ such that $r^* = r_M$ independently of any monotonicity property for $r \to H(q^*(r), r)$.

As a corollary to lemma 4.1, the seller's maximization program reduces to:

$$\max_{r \geq r^*} W_A(X_s, r, q^*(r)). \tag{19}$$

The constraint $r \geq r^*$ is called the 'shill bidding constraint'.

Proposition 4.3 *The seller maximizes her expected payoff by setting the lowest implementable reserve price above her reservation value, i.e.* $r_{opt} = \max\{X_s, r^*\}$, *while the equilibrium without shill bidding is played.*

If $r^* > X_s$ (resp. $r^* \leq X_S$), then we say that shill bidding is binding (resp. not binding). When shill bidding is binding, then the seller's expected payoff is strictly below the one that would prevail in an auction without shill bidding. It is the case in particular if $c_{shill} = 0$ and more generally if the shill bidding cost is small enough. On the contrary, if c_{shill} is large enough, then $r^* = 0$ such that hiring a shill bidder is never profitable if potential entrants base their entry decisions on a no-shill bidding activity assumption and we are back to models without shill bidding.

Proposition 4.4 (Comparative statics) *The seller's optimal reserve price and the corresponding probability not to enter the auction are:*

- *increasing in the entry cost and in the seller's reservation value,*
- *decreasing in the shill bidding cost.*

Proposition 4.4 stands in contrast with the corresponding result for auctions without shill bidding (as in Levin and Smith [26]) where the optimal reserve price depends solely on X_S since it is precisely equal to X_S. Proposition 4.4 can then be used to analyze the impact of the shill bidding cost, the entry cost and the seller's reservation value on the seller's expected payoff, or equivalently the welfare, at the optimal reserve price.

$$\frac{dW_A}{dX_S}(X_S, q^*(r_{opt}), r_{opt}) = \frac{dW_A}{dr}(X_S, q^*(r_{opt}), r_{opt}) \cdot \frac{\partial r_{opt}}{\partial X_S} + \frac{\partial W_A}{\partial X_S}(X_S, q^*(r_{opt}), r_{opt})$$
$$\frac{dW_A}{dc_{shill}}(X_S, q^*(r_{opt}), r_{opt}) = \frac{dW_A}{dr}(X_S, q^*(r_{opt}), r_{opt}) \cdot \frac{\partial r_{opt}}{\partial c_{shill}}$$
$$\frac{dW_A}{dc_e}(X_S, q^*(r_{opt}), r_{opt}) = \underbrace{\frac{dW_A}{dr}(X_S, q^*(r_{opt}), r_{opt}) \cdot \frac{\partial r_{opt}}{\partial c_e}}_{\text{impact from the shill bidding constraint}} \underbrace{+\frac{\partial W_A}{\partial q}(X_S, q^*(r_{opt}), r_{opt}) \cdot \frac{\partial q^*}{\partial c_e} + \frac{\partial W_A}{\partial c_e}(X_S, q^*(r_{opt}), r_{opt})}_{\text{standard impact}}$$

For the shill bidding cost, only the impact through the shill bidding constraint matters: raising c_{shill} relieves the constraint and thus raises the seller's expected payoff. For the

would guarantee lemma 4.2 is: $\int_{r_M}^{\overline{x}} (1 - F(u))^2 du[q^*(r_M)F(u) + (1 - q^*(r_M))]^{N-2} \geq \int_0^{X_S} (1 - F(u))^2[1 + (X_S - u)\frac{f(u)}{1-F(u)}][q^*(0)F(u) + (1 - q^*(0))]^{N-2} du$.

entry cost, the channel through the shill bidding constraint reinforces the standard channel: lowering c_e raises the seller's expected payoff through a rise of the probability of entry which jointly relieves the shill bidding constraint. On the contrary, the two impacts are oppositive for the seller's reservation value, lowering X_S intrinsically deteriorates the seller's expected payoff on the one hand but also relieves the shill bidding constraint on the other hand such that the joint impact is undetermined. The resulting comparative statics are summarized in corollary 6.1 while a deeper analysis of the impact of the seller's reservation value is left for section 5.

Corollary 4.5 *Under the optimal reserve price, the seller's expected payoff, which equals the expected welfare, is decreasing in the entry cost and increasing in the shill bidding cost.*

4.1 Entry fees

Without shill bidding, efficient entry is guaranteed under free entry when the reserve price equals the seller's reservation value: a given bidder's net benefit from entry equals his incremental contribution to the welfare (See Theorem 6.1 in Milgrom [33]). On the contrary, there is some room for subsidizing entry with shill bidding when the shill bidding constraint is binding as established in proposition 4.6. The intuition is the following. For a given reserve price r, the probability of entry $q^*(r)$ under free entry would maximize the welfare if the seller's reservation value were exactly r. If the seller's reservation value X_S is actually strictly below r then the bidders do not fully internalize their impact on the welfare: in the case where a given bidder is the unique bidder with a valuation x above r this bidder's net profit equals $x - r - c$ while his incremental contribution to the welfare equals $x - X_S - c$. On the whole, participation is too low in the free entry equilibrium.

Additionally to the announced reserve price, we consider now that the seller may introduce an entry fee $t \in \mathbb{R}$ to entrants: $t < 0$ corresponds to an entry subsidy. Entry fees have no impact ceteris paribus on the welfare: $W_A(X_s, r, q, t) = W_A(X_s, r, q)$ since fees are just monetary transfers between the seller and the bidders if bidders' strategies are fixed. However, entry fees have an indirect impact since they modify the equilibrium level of participation which is now denoted as $q^*(r;t)$ for an equilibrium without an shill bidding activity. If entry is mixed, then $q^*(r;t)$ is uniquely characterized by:

$$B(q^*(r;t), r, 0) = c_e + t. \tag{20}$$

$B(q, r, p)$ is strictly decreasing in q and $q^*(r;t)$ is thus nonincreasing in the entry fee t and strictly decreasing at a point where $q^*(r;t) \in (0, 1)$. In particular, if the seller subsidies entry, i.e. $t < 0$, then $q^*(r;t) > q^*(r)$. By selecting appropriately t, $q^*(r;t)$ can thus take any value in $[0,1]$. Let $t(q,r) := B(q,r,0) - c_e$: the entry fee $t(q,r)$ guarantees the level of participation q if the reserve price equals r. All our previous equilibrium analysis without entry fees extends to the larger framework with entry fees: from the bidders' perspective it just corresponds to a shift in the participation costs, while the incentives to shill bid remain unchanged for a given level of participation q. In particular, lemma 4.1 and 4.2 still hold such that there is still no loss of generality to limit the analysis to implementable reserve prices, i.e. reserve prices in the interval $[r_t^*, \infty)$ where $r_t^* = 0$ if $H(q^*(0,t),0) < c_{shill}$ and r_t^* is the unique solution to the equation $H(q^*(r_t^*,t), r_t^*) = c_{shill}$ otherwise. Finally, the seller's maximization program with respect to the two instruments r and t and with the shill bidding constraint $r \geq r_t^*$ can be equivalently written as a maximization program with respect to r and q:

$$\max_{q \in [0,1], r \geq r^*_{t(q,r)}} W_A(X_s, r, q). \tag{21}$$

Proposition 4.6 *In the optimal second price auction with entry fees, if shill bidding is binding then the seller strictly subsidies entry with a fee $t < 0$ and sets the lowest implementable reserve price with respect to t, r_t^*, which satisfies $r_t^* > X_S$. Otherwise, if shill bidding is not binding, the optimal second price auction involves free entry and the reserve price X_S.*

In other words, if shill bidding is binding with free entry then strict subsidies are optimal. Note that this result stands in contrast with Levin and Smith [26] that have shown that it is rather positive entry fees that are optimal beyond the IPV model, e.g. with common values or with conditionally independent private values. Here there are two motives for subsidies. On the one hand and as already discussed above, potential bidders do not fully internalize the impact of their participation decisions to the welfare if the reserve price is strictly above the seller's reservation value such that strict subsidies realign bidders' benefits from participation with the seller's one. On the other hand, by increasing participation the seller extends the set of implementable reserve prices and thus reduces the shill bidding constraint. This second channel vanishes if the shill bidding cost is null since the set of implementable reserve prices is then invariably $[r_M, \infty)$.

In the rest of the paper, we limit our analysis to environments with $c_{shill} = 0$ such that $r_{opt} = r_M$ for any auctions, possibly with entry fees. We will also consider alternative selling mechanisms such that we will now always specify the mechanism in hand. Under the terminology 'auctions with shill bidding', we consider second price/English auctions where the announced reserve price is below r_M which are all strategically equivalent, while the terminology 'auctions without shill bidding' refers to the auction game where the seller is not allowed to hire a shill bidder anymore.

5 Some rationale for 'burning your ships'

In the auction literature, the seller's reservation value is typically normalized to zero since it plays no salient role for auction design. Furthermore, either in frameworks with exogenous or endogenous entry, if bidders' valuations do not depend on the seller's reservation value, then the seller's expected payoff is non-decreasing in her reservation value: by choosing the (possibly suboptimal) mechanism that was optimal with a lower reservation value, she is guaranteed to obtain at least the same payoff while strictly benefiting from her higher valuation in the case she keeps the good. Most of the auction design literature considers that the seller is able to commit to the rules of the announced mechanism and there is thus no rationale for a seller to 'burn her ships'. This general insight is no longer valid under imperfect commitment if the set of implementable mechanisms, i.e. the set of announced mechanisms she can credibly commit to, may depend on the seller's (known) reservation value and more precisely if this set may shrink with the seller's reservation value. We show that, **with shill bidding, the imperfect commitment channel may overwhelm the standard force in favor of a larger reservation value such that the seller may benefit from 'burning her ships'.**[20] Our simulations in section 8 provide numerical examples while next proposition establishes a simple sufficient condition that guarantees that the seller's expected payoff fails to be increasing in her reservation value.

Proposition 5.1 *If $\frac{1}{f(\underline{x})} \leq \underline{x}$ and $c_e \leq e^{-1} \cdot \int_{\underline{x}}^{\overline{x}} (1 - F(u))du$, then the seller's expected payoff in auctions with shill bidding is non monotonic with respect to her reservation value.*

Arozamena and Cantillon [2] analyzed a related issue: the benefits for a bidder to invest in order to upgrade the CDF from which his valuation is picked. In first price auctions, investment incentives are suboptimal contrary to second price auctions. Furthermore, it could also happen that a bidder losses from an upgrade or equivalently gains if his valuation is picked from a less favorable distribution. Here we consider the incentives for the seller to invest in order to upgrade her reservation value and we obtain a similar counterintuitive loss, a loss that may also occur with optimal entry fees as shown in the simulations reported in section 8.

[20]See Chapter 6 in Dixit and Nalebuff [12] for other examples where burning its ships can be profitable due to a commitment effect.

6 Auctions versus posted-price selling

The current evolution of eBay's business and the success of Amazon is a puzzle from a standard auction design perspective: 56% of worldwide sales on eBay are now concluded at a price predetermined by the seller (e.g. by means of the "Buy it Now" option in auctions) while eBay's new CIO made a U-turn to the company's motto by emphasizing that eBay's future lies in fixed-price sales. In early 2008, eBay has also modified her fee structure in favor of fixed-price selling mechanisms.[21] In this section, we analyze an alternative class of selling mechanisms, posted-price mechanisms, who do not suffer from the hold-up issue such that they may outperform auctions though they do suffer intrinsically from ex post inefficiencies across potential buyers.

The posted-price game corresponds to the previous auction game where the seller announces a posted-price, also denoted by $r \in \mathbb{R}_+$, instead of a reserve price while the entrants report simultaneously whether they are willing to buy the good at the price r in the fourth stage. If several entrants are demanding the good at that price then the winner is selected at random with equal probability. Obviously, only posted-prices with $r \geq X_s$ make sense and there is then no room for any shill bidding activity since it would only reduce the probability to sell the good while it can not manipulate the final price.[22]

In a standard auction design perspective, without any possibility to enroll a shill bidder, auctions obviously outperform posted-price selling. Posted-price selling creates a kind of inefficiency that unambiguously lowers the seller's expected payoff. In our framework with shill bidding, the picture is completely different: posted-price selling may outperform the second price auction with the optimal reserve price. The intuition is that the seller can credibly commit to any possible posted-prices above her reservation value and in particular the optimal one, while she can credibly commit only to a limited set of reserve price in an auction. Contrary to the literature on optimal search procedures as in Weitzman [43] or Crémer et al [11], the way we model a posted-price mechanism does not involve a sequential structure that would allow to save up entry costs. Our model abstracts thus from the gains that result from a coordination of the entry process. Indeed the optimal mechanisms in such environments would involve both posted-price and auction-like ingredients. Furthermore, auctions would also gain from coordinated entry, e.g. by means of a sequential procedure

[21] See C. Holahan, Auctions on eBay: a dying breed, BusinessWeek, June 3 2008 and A. Jamieson, eBay auctions overtaken by fixed price sales, The Telegraph, December 10 2009.

[22] In line with remark 2.1, under the worst possible commitment failure, the seller could update the posted-price after entry. Once the probability of entry is exogenously given by q, then the optimal posted-price r_P^* is solution of the equation $r - X_S = \frac{(1-F(r))}{f(r)} \cdot \frac{\sum_{k=0}^{N-1}[qF(r)+(1-q)]^{k-N+1}}{N}$. Since $\frac{\sum_{k=0}^{N-1}[qF(r)+(1-q)]^{k-N+1}}{N} > 1$, we obtain then that $r_P^* > r_M$. As a corollary, if posted-prices suffer also from such a strong commitment failure, then they will be outperformed by auctions with shill bidding.

as in Engelbrecht-Wiggans [13]. Our insights are robust to coordinated entry.

If a given potential buyer faces n entrants and if the posted-price equals r, then his probability to obtain the good if his valuation is above r is given by:

$$P(r,n) := \sum_{j=0}^{n} \binom{n}{j} F(r)^j (1-F(r))^{n-j} \cdot \frac{1}{n-j+1} = \frac{1}{n+1} \frac{(1-F^{n+1}(r))}{(1-F(r))}.$$

His expected profit if he incurs the entry cost is thus given by $\int_r^{\overline{x}} (u-r) \cdot P(r,n) dF(u)$. Let $B_P(q,r)$ denote the ex ante expected benefit of a given entrant under the posted-price r and if the probability of entry of his opponents equals q. A straightforward calculation leads to:

$$B_P(q,r) = \frac{[1-(qF(r)+(1-q))^N]}{Nq} \cdot \int_r^{\overline{x}} (u-r) \frac{dF(u)}{1-F(r)}. \tag{22}$$

We make the following additional assumption in order to guarantee that the equilibrium with optimal posted-prices involves mixed entry.

Assumption A 5 [23] *The entry cost is large enough such that full entry is never an equilibrium for any posted-price above X_S:*

$$c_e > \max_{r \geq X_S} \int_r^{\overline{x}} (u-r) \cdot P(r, N-1) dF(u) = \max_{r \geq X_S} B_P(1,r). \tag{23}$$

Assumption A5 guarantees that $q < 1$ for any reserve price above X_S. Since $B_P(0,r) = B(0,r,0)$, assumption A3 guarantees that participation is strictly positive for any posted-price below r_M. If q were equal to zero in the optimal posted-price mechanism, then the seller's maximal expected payoff equals X_S which would raise a contradiction since the seller's expected payoff would lie strictly above X_S for any posted-price in $(X_S, r_M]$ since entry and thus sale of X_S would occur with some positive probability. On the whole, assumptions A3 and A5 imply that entry is mixed for any candidate to be an optimal posted-price. From now on, only such posted-prices are considered and thus posted-prices r that satisfy $B_P(0,r) > c_e$ while $B_P(1,r) < c_e$. The entry equilibrium equation is thus given by:

$$B_P(q,r) = c_e. \tag{24}$$

Let $z := qF(r) + (1-q)$ and $\widetilde{B}_P(z,r) := B_P(q,r)$. The function $z \to \frac{1-z^N}{1-z} = \sum_{i=0}^{N-1} z^i$ is strictly increasing with respect to z on $[0,1]$. We have $\widetilde{B}_P(1,r) > c_e$ while $\widetilde{B}_P(F(r),r) < c_e$.

[23] If N is large enough, then $\max_{r \in [X_S, r_M]} \int_r^{\overline{x}} (u-r) \cdot P(r,N-1) dF(u) \leq \frac{1}{N(1-F(r_M))} \cdot \max_{r \in [X_S, r_M]} \int_r^{\overline{x}} (u-r) dF(u) < \int_{r_M}^{\overline{x}} (u-X_s) dF(u)$ such that there exists some entry costs such that both conditions (6) and (23) are satisfied.

There is thus a unique solution $z_P^*(r) \in (F(r), 1)$ such that $\widetilde{B}_P(z_P^*(r), r) = c_e$. Since $z \to \frac{1-z}{1-F(r)} \equiv q$ is strictly decreasing in z, we obtain equivalently that there is a unique solution $q \in (0, 1)$ such that (24) holds. Let $q_P^*(r)$ denote the solution.

Remark 6.1 Contrary to $q^*(r)$, $q_P^*(r)$ may not be non-increasing in r. The intuition is that raising the reserve price reduces the allocative inefficiency among the entrants which may overwhelm the standard effect from the reduction of the level of trade.

Let $W_P(X_s, r, q)$ denote the expected welfare if the posted-price equals r and the probability of entry equals q. A straightforward calculation leads to:

$$W_P(X_s, r, q) = (q\cdot F(r)+(1-q))^N \cdot X_S + (1-(q\cdot F(r)+(1-q))^N)\cdot \int_r^{\overline{x}} u \frac{dF(u)}{(1-F(r))} - qN\cdot c_e. \tag{25}$$

The seller's maximization program with posted-prices is then given by:

$$\max_{r \geq X_S} W_P(X_s, r, q_P^*(r)) \tag{26}$$

Contrary to auctions, there is no constraint on the set of implementable reserve prices. Since the solution r_P^* is necessarily interior, it satisfies the first order condition $\frac{dW_P(X_s,r,q_P^*(r))}{dr} = 0$. In the following, to alleviate the presentation, we assume that the maximization program (26) has a unique solution.

Proposition 6.1 (Comparative statics) *The seller's optimal posted-price and the corresponding seller's expected payoff, which corresponds also to the expected welfare, are:*

- *increasing in the seller's reservation value,*
- *decreasing in the entry cost,*
- *independent of the shill bidding cost.*

There is a notable difference between propositions 4.4 and 6.1: the comparative statics with respect to the entry cost are opposite. With low entry costs, the seller can raise her posted-price since she's facing higher demand. On the contrary, in auctions with shill bidding, higher demand translates into smaller incentives to hire a shill bidder and thus the possibility to credibly announce lower reserve prices and in particular to be able to get closer to the optimal reserve price X_S. Another difference is the absence of the "burning your ships" effect with posted prices, as expected since there is no commitment issue.

Next proposition formalizes how the level of the participation costs influences the relative performance of posted-prices and auctions with shill bidding. Our results fit the basic intuition that posted-prices become more benefic in environments with high participation costs. Nevertheless, we emphasize that the result does not come from the standard channel where posted-prices are a way to save on participation costs due to miscoordination as in the literature mentioned above but from the shill bidding channel: for high participation costs, the hold-up issue precludes any incentives to participate in auctions with shill bidding while entry and subsequent profitable trades are possible with posted-prices.

Proposition 6.2 *Ceteris paribus, there is a threshold $\underline{c} > 0$ such that auctions with shill bidding strictly outperform the optimal posted-price if $c_e < \overline{c}$ and a threshold $\overline{c} > 0$ such that the optimal posted-price outperforms auctions with shill bidding and with entry fees if $c_e > \overline{c}$ while the difference is strict in the right neighborhood of $\overline{c}$.*

Proposition 6.2 emphasizes the role of participation costs in the comparison between auctions with shill bidding and posted-prices. Another crucial ingredient is the relative heterogeneity between bidders. In the limiting case where the difference $(\overline{x} - \underline{x})$ goes to zero, then the ex post inefficiencies inherent to posted-price selling vanish such that the performance of the optimal posted-price converges to the first best, i.e. the optimal auction without shill bidding.[24] On the contrary, auctions with shill bidding are remaining inefficient: when there is a unique entrant, he has to pay a price strictly greater than X_S such that the probability of entry is strictly smaller than the optimal one.
On the one hand, online markets seem to favor auctions against posted-prices since those markets are reducing participation costs compared to traditional brick and mortar markets. On the other hand, online markets homogenize buyers' valuation since opportunity costs are inherent to valuations and that online markets homogenize those costs, at least for goods that are frequently sold.

6.1 Entry fees

In the same way as for auctions, entry fees in a posted-price mechanism allow the seller to be able to adjust the probability of entry. If entry is mixed, then the equilibrium probability of entry $q_P^*(r;t)$ is uniquely characterized by:

$$B_P(q_P^*(r;t), r) = c_e + t. \tag{27}$$

[24]The level of participation will be approximately the same in the optimal posted-price and the optimal auction without shill bidding when $X_S < \underline{x}$ and $\overline{x} \to \underline{x}$. The difference lies in the price paid when there is at least one entrant: under the posted-price, it is equal to the posted-price while the price is equal either to X_S or $\overline{x}$, respectively in the case where there is a unique entrant or several entrants.

With entry fees, the seller's maximization program with posted-prices is then given by:

$$\max_{r \geq X_S, q \in [0,1]} W_P(X_s, r, q). \tag{28}$$

Surprisingly, next proposition establishes that the seller would not gain from using entry fees.

Proposition 6.3 *In the optimal posted-price selling mechanism with entry fees, the seller uses no entry fees.*

7 Efficient auctions with uncertain reservation values?

Our analysis in the previous sections has formalized the rigorous perspective in the literature in law. Committing to low reserve prices encourages participation. To be effective, the reserve price commitment should also include a banning of any shill bidding activity or of withdrawing the good from sale. The literature in law puts forward two kinds of motives for judges' clemency toward shill bidding. First, it seems legitimate in presence of a ring: Marshall et al. [16]'s early work on shill bidding in the economic literature do actually adopt this view, shill bidding allows the seller to adjust the reserve depending on the strength of the highest bidder, in particular whether she is facing a ring or not. Second, shill bidding may be legitimate if it is used to prevent a sacrifice, i.e. a sale at a price below the seller's reservation value, as courts of equity recognized at some points. The literature in law is actually embarrassed with both arguments. On the one hand, they argue that rings should not be fought with a fraudulent activity but with legal means. Bid ringing is a pervasive phenomenon (Porter and Zona [38]) and colluders seem to be rarely caught. This position may thus not be pragmatic. On the other hand, they emphasize that if the seller wants to secure herself against a sacrifice, the use of an appropriate reserve price is sufficient. This second argument does actually hold in the limited theoretical model considered so far where we have assumed that the seller knows her reservation value ex ante: standard auctions with free entry, with a reserve price equal to the seller's reservation value and where the seller would be able to commit not to shill bid are optimal mechanisms or equivalently welfare maximizing mechanisms.[25] This argument becomes fallacious in a larger framework when the seller is uncertain about her reservation value ex ante. It could seem awkward that the seller significantly revises her reservation value in the short period of time between

[25]If bidders' valuations were conditionally independently and identically distributed, such standard auctions are still ex post efficient but ex post efficiency is not enough to implement the first best, e.g. entry is excessive under the first price auction, however the second price or the English auction would do the job as shown in Levin and Smith [26].

the advertisement of the auction and the course of the auction. Nevertheless, a large part of the seller's reservation value comes from the good's "market value" (e.g. due to future re-auctioning opportunities) which may be largely unknown to the seller ex ante and where the auction itself -the number of participants, the bid history- could matter a lot.

From now on, we consider more general environments where the seller's reservation value is a random variable whose realization interim $\widetilde{X}_S$ may depend on an interim public signal $\widetilde{S}_P$, on an ex ante private signal $\widetilde{V}_A$ and on an interim private signal $\widetilde{V}_I$. The interim stage refers to the stage after potential bidders' entry decisions and before the selling mechanism starts. We assume that those signals are distributed independently of the number of entrants and their valuations.

$$\widetilde{X}_S = U(\widetilde{S}_P, \widetilde{V}_A, \widetilde{V}_I). \tag{29}$$

Remark 7.1 To simplify our analysis, we have implicitly considered that those additional signals are exogenous and disclosed before the auction starts. While it seems to contradict our initial motivations where we emphasize that the seller may refine her valuation through the course of the auction itself, most of our insights could be also obtained with models with endogenous information that comes from the bids themselves, e.g. from the prices at which the losing bidders have exited in the English button auction.

We use then the terminology 'complete information on the reservation value' for the cases where U depends solely on $\widetilde{S}_P$ while 'incomplete information on the reservation value' refers to more general environments where U may depend also on additional information that is private information of the seller. Among the latter environments, 'ex ante private information' refers to the environments where U does not depend on any interim private signal $\widetilde{V}_I$.

Definition 2 *For a given environment, we say that a mechanism implements* ***the first best*** *if, in equilibrium, it maximizes the ex ante welfare.*

The first best criterium in definition 2 does not take into account the incentive compatibility constraints that may limit what can be implemented in equilibrium with a (feasible) mechanism. Furthermore, as in Levin and Smith [26], we also implicitly assume that entry is uncoordinated: the welfare is then characterized by an entry probability (that can only depend on the information known at the ex ante stage) and an assignment rule among entrants.

Lemma 7.1 (Characterization of first best mechanisms) *A mechanism implements the first best if and only if it is almost surely ex post efficient, i.e. it allocates the good to the agent (including the seller) with the highest valuation with probability one, and the probability of entry is the same as the one that would prevail in a second price auction with free entry and where the reserve price would be set at the interim stage at the seller's reservation value* $\widetilde{X}_S$ *and if potential entrants knew the signal* $\widetilde{V}_A$.

7.1 Complete information on the reservation value

In the second price auction without shill bidding, ex post efficiency is reached only if the reserve price equals the seller's reservation value in the events where $\widetilde{X}_S \in (\underline{x}, \overline{x})$ and with at least one entrant. Otherwise, for some realizations of bidders' valuations, the good will stay in the seller's hand while it would be optimal to sell (if $r > \widetilde{X}_S$) or the good will be sold while it would be optimal that the seller keeps it (if $r < \widetilde{X}_S$). As a corollary, a unique reserve price announced ex ante is powerless to implement the first best with some uncertainty with respect to the seller's reservation value. Due to the hold-up issue, we know that shill bidding can not implement the first best too. A milder instrument that introduces flexibility and that receives attention in the literature in law is the possibility to cancel the auction appended to the auction game presented in section 2. We emphasize that what we call cancelation rights is a right to withdraw the good from sale after the price has been fixed, a much stronger device than the right to withdraw the good from sale before the auction has begun which is typically not precluded. We enter slightly into the details of the analysis of the second price auction with cancelation rights. To simplify the presentation we assume that there is no reserve price nor entry fees. The probability to win the good for a bidder with valuation u, with n opponents and the seller's valuation being equal to $\widetilde{X}_S$ is given by $[F^n(u) - F^n(\widetilde{X}_S)]$ for $u > \widetilde{X}_S$ and 0 otherwise. From (2), the expected benefit in the auction of a given entrant once the entry cost has been sunk is then given by:

$$B_c(q) = E_{\widetilde{X}_S}[\sum_{n=0}^{N-1} \binom{N-1}{n} q^n (1-q)^{N-1-n} \cdot \int_{\widetilde{X}_S}^{\overline{x}} (\int_{\widetilde{X}_S}^{u} [F^n(s) - F^n(\widetilde{X}_S)]ds)dF(u)]. \quad (30)$$

When entry is mixed, a straightforward calculation leads to the following equilibrium equation:

$$B_c(q) = E_{\widetilde{X}_S}[\int_{\widetilde{X}_S}^{\overline{x}} (1-F(u))([qF(u)+(1-q)]^{N-1} \underbrace{-[qF(\widetilde{X}_S)+(1-q)]^{N-1}}_{\text{inefficiency term}})du] = c_e. \tag{31}$$

The 'inefficiency term' is the additional term with respect to the first best: it corresponds to the event where the efficient allocation would assign the good to the highest bidder while the seller prefers to keep the good because the auction price is below her valuation. In the subsequent calculation, we will continue to trace this additional term that captures the departure from the first best. Note that $B_c(.)$ may not be strictly decreasing contrary to $B(., r, p)$. In particular, if q is small, it is rather the opposite: in particular in the limiting case where $q = 0$, an entrant never wins the good and his expected benefit is null. Therefore, the equilibrium equation (31) may have multiple interior solutions $q_c^* \in (0, 1)$.[26] For a given probability of entry, the expected welfare is then given by:

$$W_c^{sp}(q) = \underbrace{E_{\widetilde{X}_S}[W_A(\widetilde{X}_S, \widetilde{X}_S, q)]}_{\text{ex post efficient}} \underbrace{-E_{\widetilde{X}_S}[\int_{\widetilde{X}_S}^{\overline{x}} (u-\widetilde{X}_S)(qF(\widetilde{X}_S)+(1-q))^{N-1}d[F(u)]]}_{\text{inefficiency term}}. \tag{32}$$

The difference between the second price auction with cancelation rights and the first best depends on the probability of entry and on the distribution of the seller's reservation value. If the seller's reservation value is always smaller than the lower bound $\underline{x}$ of bidders' valuations, then inefficiency occurs solely in the case when there is a unique entrant (the inefficiency term equals $(1-q)^{N-1}E_{\widetilde{X}_S}[\int_{\underline{x}}^{\overline{x}} (u-\widetilde{X}_S)d[F(u)]]$). If $\widetilde{X}_S \leq \underline{x}$ almost surely, the second price auction with cancelation rights can actually implement the first best with an appropriate combination of an announced reserve price and entry fees.[27]

Let us summarize the different kinds of inefficiencies in standard auction games. In the discussion below, we consider general environments where we may have $r_M(\widetilde{X}_S) > \underline{x}$. Shill bidding creates ex post inefficiencies because the seller sets a reserve price that lies strictly above her reservation value and which is also binding. With a right to cancel the auction, the seller's optimal strategy is to cancel the auction if and only if the auction price lies below her reservation value. However, in a second price auction, cancelation rights are opening the door for a new source of ex post inefficiency: if only one bidder has a valuation

[26]The non interior solution $q = 0$ is always a solution in the second price auction game with cancelation rights and $r = 0$ if $\widetilde{X}_S > 0$ since the seller would always cancel the sale if there is just one participant.

[27]Environments with $\widetilde{X}_S \leq \underline{x}$ do not call for flexibility. More generally, if $r_M(\widetilde{X}_S) \leq \underline{x}$ for almost any realization of the seller's reservation value, then all the standard formats considered so far can implement the first best with appropriate entry fees.

above the seller's reservation value, then the final price would lie strictly below the seller's reservation value such that the seller will keep the good while it would be ex post efficient to allocate it to the provisionally winning bidder. Note that the latter would be prepared to pay the seller's reservation value. Nevertheless, the second price auction does not leave him the opportunity to make such an offer to the seller contrary to the English auction where he could top his own winning bid by submitting a jump bid up to the seller's reservation value since it is common knowledge interim under complete information. From lemma 7.1, we obtain thus that, with the opportunity to submit jump bids, the English auction with free entry implements the first best. On the whole, we have shown the following proposition.

Proposition 7.2 *Second price/ English button auctions (with or without shill bidding) may not implement the first best if the seller is uncertain about her reservation value ex ante. The English auction with cancelation rights, free entry, no reserve price and the possibility to submit jump bids always implements the first best under complete information on the seller's reservation value, while the second price auction with cancelation rights does not.*

In pure private value environments, second price and English auctions are typically viewed as strategically equivalent: proposition 7.2 provides a new argument in favor of the latter in presence of cancelation rights and the opportunity to use jump bids. Jump bids have received some attention in the literature where they have been perceived as offering bidders an opportunity to signal their strength in the auction process (see Hörner and Sahuguet [18] for a recent contribution). Here the motives for submitting a jump bid are radically different: while they typically occur at the beginning of the bidding process in previous works as in Fishman [14] where a firm aims to discourage information acquisition from her opponents, here a given bidder may benefit from topping his own bid at the end of the sale when he is the unique remaining active bidder. The motives are thus seller-oriented.

Remark 7.2 If entrants can submit multiple bids in the second price auction (e.g. by using shill bidders themselves), then the second price auction can implement the first best: each participants would submit two bids, one up to his valuation and the other up to the seller's valuation. However, with regards to remark 7.1, the solution is awkward: it would not be robust to learning from the course of the auction, i.e. motives to cancel the sale that come from the submitted bids themselves. On the contrary, in the English auction, once all bidders expect one have exited the auction, then the price below which the seller would cancel the sale would become common knowledge between the seller and the bidders. In other words, when only one bidder remains in the English auction, then the bulk of the uncertainty about the seller's reservation value is cleared up. General models where the seller would possibly refine her knowledge about her reservation value and about bidders' valuation distributions from the bidding history raise additional issues: bidders may have incentives to submit multiple bids to influence the seller.[28] A similar roundabout motive for bidders' shill bidding occurs in a case before the Court reported by Bennett [7] (p.16) as an example of bidder's fraud where a bidder employs a shill bidder known to have had connections with the seller in the past: the purpose of bidders' shill bidding was to influence the strategy of other potential real bidders and more precisely to discourage them from entering the auction because they thought that the seller will surely shill bid.

Proposition 7.2 has stated the inefficiency of three standard auction games: second price auctions with shill bidding, without shill bidding and finally with cancelation rights. In all three cases, the good may be inefficiently allocated such that entry fees are powerless to restore the first best. The way the good may be inefficiently allocated is actually different in the three cases such that there is no ranking between those standard formats as illustrated by the simulations in section 8.

7.2 Incomplete information on the reservation value

Consider first environments with ex ante private information. It can be easily checked that if the seller can communicate about her ex ante private information, but only on a pure cheap talk basis, then truthful revelation is typically not an equilibrium since the seller would always prefer to make believe to potential entrants that she has a lower valuation in order to boost participation.[29] Nevertheless, the first best can be implemented if the seller delegates the exercise of her cancelation rights to a third party.

Definition 3 *A delegated cancelation rights policy is a function $D : \mathcal{S}_P \to R_+$ that maps to any interim public signal S_P a threshold $D(S_P)$ such that the third party cancels the auction if and only if the final price is strictly below $D(S_P)$.*
The truthful delegated cancelation rights policy corresponds to the one where $D(.) := U(., V_A)$ if her private signal is V_A.

[28] In this vein, Levin and Smith [25] show with affiliated private values that the optimal reserve price (ex post or equivalently once entry costs have been sunk) is decreasing in the number of participants such that bidders would benefit from hiring shill bidders in order to lower the reserve price set by the seller, as argued informally by Morrell [34] from a practitioner's perspective.

[29] Making believe that she has a "lower" signal that she actually has has also an additional effect: in the event where only one bidder has a valuation above the seller's reservation value, then the seller will cancel the sale, while she would have sold the good if bidders knew her true ex ante private signal. However, this side effect is neutral from the perspective of the seller's payoff since this is an event where the seller's payoff would have been $\widetilde{X}_S$ if bidders knew her true signal, i.e. exactly the same payoff when she keeps the good.

Proposition 7.3 *The English auction without reserve, with the truthful delegated cancelation rights policy and the possibility to submit jump bids implements the first best if the seller is privately informed ex ante.*

We are not aware of any formal use of a delegated cancelation rights policy in real-world auctions. Nevertheless, the expertise role of traditional auction houses[30] could be viewed as playing a related role: the information they disclose reduces the informational asymmetry between the seller and the potential bidders, it can be partially viewed as an agent certifying the signal V_A. In any cases, the solution in proposition 7.3 is powerless to implement the first best in more general environments with private information at the interim stage. With bilateral asymmetric information at the interim stage, then we know from Myerson and Satterthwaite [35] that no budget-balanced mechanism is ex post efficient. As a corollary of lemma 7.1, no budget-balanced mechanism can implement the first best.[31]

Corollary 7.4 (General impossibility with bilateral asymmetric information) *No budget-balanced mechanism can implement the first best if the seller is privately informed at the interim stage.*

8 A numerical example

Consider that bidders' valuations are independently and uniformly distributed on the interval $[0, 1]$. The number of potential participants N is set to 10 and the shill bidding cost to zero. Our simulations consist in computing the probability of entry (the "participation" row), the welfare (the 'welfare/revenue' row) and possibly additional characteristics of some standard selling mechanisms while making vary two kinds of parameters: the participation cost and the distribution of the seller's reservation value. In a first step, we consider in Table 1 that the seller's reservation value X_S is known ex ante. We compare the optimal auction with shill bidding (with and without entry fees) and the optimal posted-price mechanism with the first best which is reached with the optimal auction without shill bidding. In a second step, we consider in Table 2 that the seller's reservation value is a random variable distributed uniformly on the interval $[0, \overline{X}_S]$ and revealed publicly at the interim stage. We consider then two additional mechanisms: the second price auction and the English auction without reserve but with cancelation rights.

[30] See Ashenfelter [3] and Ashenfelter and Graddy [4].

[31] We do not enter into the technical details. In particular, Myerson and Satterthwaite's impossibility result relies on the point that $\widetilde{X}_S$ may lie strictly above $\underline{x}$. Furthermore, this general impossibility result relies on the statistical independence that has been assumed both between bidders and also between the seller and the bidders. Correlated information would enable the seller to implement the first best as in McAfee and Reny [30].

The striking feature is that shill bidding may matter a lot: e.g. for $X_S = 0.2$ and $c_e = 8\%$ the seller's expected payoff shrinks from 0.47 to 0.20 from the first best to the optimal auction with shill bidding. With those parameters, moving from auctions with shill bidding to the optimal posted-price mechanism involves a one hundred percent increase of the seller's payoff. The 'burning your ships' effect is also significant: from $X_S = 0.3$ and $c_e = 6\%$, lowering the reservation value to 0 involves a 40% increase of the seller's payoff for an auction with shill bidding. Allowing for entry fees attenuates the 'burning your ships' effect which almost vanishes in our simulations but still occurs: for $c_e = 6\%$, the seller's payoff shifts from 0.499 to 0.501 when we move from $X_S = 0.3$ to $X_S = 0$. Furthermore, the appropriate reference point is not the null when we move from $X_S = 0$ to $X_S = 0.3$ but the associated gain in the first best mechanism where the seller's payoff increases from 0.534 to 0.542. We observe also this effect in Table 2 for $c_e = 1\%$ when we move from $\overline{X}_S = 0.75$ to $\overline{X}_S = 0.25$ not solely for auctions with shill bidding but also for the second price auction with cancelation rights where the seller's payoff goes from 0.783 to 0.809.

Our simulations illustrate that there is no general ranking between posted-price mechanisms and auctions with shill bidding, even if entry fees are allowed. However, it seems that entry subsidies can do a great deal to attenuate the deteriorating effects resulting from shill bidding such that it is only in the simulation with $X_S = 0.2$ and $c_e = 6\%$ that the optimal posted-price outperforms auctions with shill bidding and entry fees. There is also no general ranking between r_M and the optimal posted-price r_P^*. In our simulations, the inequality $r_P^* > r_M$ holds solely for the lowest participation cost $c_e = 2\%$. With entry fees, we can remark that the optimal fees (the 'entry subsidy' row) are lying between 30% and 60% of the entry costs in the simulations reported in Table 1. This ratio is the most important when participation is reduced a lot by the shill bidding activity as under the parameters $X_S = 0.4$ and $c_e = 6\%$ where the probability of entry moves from 0.31 in the optimal auction without shill bidding to 0 with shill bidding.

In Table 2, there is no ranking between the various suboptimal auction formats. The second price auction with cancelation rights strictly outperforms auctions without shill bidding for $c_e = 1\%$ and $\overline{X}_S = 1.75$ and auctions with shill bidding and optimal entry fees (and thus also auctions with shill bidding) for $c_e = 2\%$ and $\overline{X}_S = 0.25$. On the contrary, the second price auction with cancelation rights is strictly outperformed by the optimal auction without shill bidding for $c_e = 12\%$ and $\overline{X}_S = 0.25$ and by auctions with shill bidding (and thus also by the optimal auction with shill bidding and entry fees) for $c_e = 1\%$ and $\overline{X}_S = 1.25$. Consider now the comparison between auctions with shill bidding (with or without entry fees) and optimal auctions without shill bidding. With a known

X_S	$X_S = 0.2$					0	0.1	0.2	0.3	0.4
c_e	2 %	4 %	6 %	8 %	10 %	$c_e = 6\%$				
r_M	0.6					0.5	0.55	0.6	0.65	0.7
Optimal auction without shill bidding [first best]:										
participation	0.67	0.47	0.36	0.30	0.25	0.38	0.38	0.36	0.34	0.31
welfare/revenue	0.73	0.62	0.54	0.47	0.42	0.534	0.535	0.538	0.542	0.551
Auction with shill bidding:										
participation	0.59	0.29	0.12	0	0	0.25	0.19	0.12	0.010	0
welfare/revenue	0.71	0.54	0.37	0.2	0.2	0.436	0.406	0.367	0.312	0.40
Auction with shill bidding and optimal entry fees:										
entry subsidy	0.60%	1.8%	3.0 %	4.4 %	5.8%	2.6 %	2.8%	3.0%	3.3 %	3.5 %
participation	0.76	0.54	0.42	0.33	0.27	0.44	0.44	0.42	0.39	0.33
welfare/revenue	0.72	0.59	0.50	0.42	0.36	0.501	0.498	0.497	0.499	0.508
Optimal posted-price:										
posted-price r_P^*	0.69	0.59	0.52	0.47	0.43	0.49	0.51	0.52	0.54	0.56
participation	0.72	0.44	0.32	0.25	0.20	0.37	0.35	0.32	0.29	0.25
welfare/revenue	0.65	0.54	0.46	0.41	0.36	0.431	0.445	0.462	0.484	0.512

Table 1: the reservation value is known.

$\overline{X}_S$	0.25					0.5	0.75	1	1.25	1.5	1.75
c_e	12%	8%	4%	2%	1 %						
Optimal auction without shill bidding:											
reserve price	0.125					0.25	0.375	0.5	0.625	0.75	0.875
participation	0.225	0.309	0.471	0.674	0.953	0.953	0.953	0.950	0.922	0.773	0
welfare/revenue	0.362	0.468	0.619	0.730	0.809	0.809	0.809	0.809	0.810	0.816	0.875
Auction with shill bidding:											
participation	0	0.070	0.335	0.617	0.940	0.910	0.832	0.696	0.575	0.480	0.403
welfare/revenue	0.125	0.248	0.553	0.715	0.807	0.804	0.797	0.801	0.848	0.915	1.00
Auction with shill bidding and optimal entry fees:											
entry subsidy	6.9 %	4.2 %	1.6 %	0.53%	0.11%	0.15 %	0.24 %	0.34 %	0.38 %	0.4%	0.43%
participation	0.243	0.354	0.546	0.753	1	1	1	0.950	0.844	0.763	0.699
welfare/revenue	0.302	0.420	0.595	0.722	0.808	0.805	0.802	0.809	0.854	0.928	1.02
Auction without reserve and with cancelation rights - English with jump bids [first best]:											
participation	0.224	0.308	0.470	0.674	0.953	0.953	0.941	0.856	0.753	0.676	0.616
welfare/revenue	0.363	0.468	0.620	0.730	0.809	0.809	0.810	0.818	0.864	0.937	1.03
Auction without reserve and with cancelation rights - English without jump bids/second price:											
participation	0.179	0.179	0.441	0.671	0.953	0.951	0.911	0.788	0.678	0.593	0.522
welfare/revenue	0.166	0.238	0.590	0.729	0.809	0.808	0.783	0.715	0.769	0.848	0.941

Table 2: the reservation value is uniformly distributed on $[0, \overline{X}_S]$.

deterministic reservation value, we already know that the latter outperform the former. Such a result holds in our reported simulations for $\overline{X}_S = 0.25$, i.e. when the uncertainty on the seller's reservation value is small. On the contrary, the optimal auction with shill bidding outperforms auctions without shill bidding in our simulations with $\overline{X}_S \geq 1$.

9 Conclusion

Informal evidence suggests that shill bidding may be of first-order importance in real-world auctions contrary to the way auctions are usually modeled in economic theory. Shill bidding is a fundamentally secretive activity making it difficult to detect with standard auction data. However, Lamy [23] develops a general structural approach under anonymous data, a methodology which could be applied to propose tests for shill bidding without imposing any parametric assumptions on the distribution of bidders' valuations. In management and computer science, where shill bidding detection is a hot topic, methodologies relying on questionable patterns of bidding in multiple simultaneous dynamic auctions have been proposed.[32] On the contrary, empirical strategies that neglect shill bidding may be strongly biased. Furthermore, shill bidding may also distort field experiments: when the experimenter imposes exogenous variations on the reserve price[33], it is not clear whether he controls the beliefs of the bidders with respect to the shill bidding activity since this latter is closely mingled with the reserve price policy. As an example, our analysis emphasizes that participation may decrease in the reserve price at some point since the seller may not be credible not to hire a shill bidder if the announced reserve price is too low. In a similar perspective, shill bidding may also offer a parsimonious explanation for the puzzle analyzed recently by Brown and Morgan [8]: the difference between Yahoo! and eBay's auction results seems inconsistent with equilibrium behavior. In particular, the number of bidders and the seller's revenue are significantly larger on eBay. Those features would be consistent with equilibrium behavior on the bidder's side if potential bidders expect more shill bidding on Yahoo!'s auction site. This is precisely what would be predicted by our equilibrium theory with shill bidding according to the difference between both sites in term of fees paid by the seller: with much higher final value fees on eBay compared to Yahoo! (the final value fees up to $25 were respectively 5.75 and 2 percent on those sites at the time their data were collected), shill bidding becomes less profitable on eBay.[34] Additionally to the differences in term of listing fees, the shill bidding issue coupled with entry costs may reinforce the puzzling difference in term of transaction rates between both auction sites: for the summer 1999, this rate was equal to 16% and 54% for respectively Yahoo! and eBay according to Lucking-Reiley [28].

In a nutshell, we first ask for a more cautious interpretation of empirical data, including field experiments, as, e.g., in the works surveyed by Bajari and Hortaçsu [6] for online auctions. Second, the models we developed with shill bidding, in the present paper and in Lamy [24], can be useful to set about a structural approach that would include a seller's bidding maximization program.

[32]see e.g. Kauffman and Wood [21] and Xu et al. [44].

[33]See e.g. Reiley [39] and Katkar and Reiley [20].

[34]Brown and Morgan [8]'s field experiments suggest also that sellers are irrational and should switch from Yahoo! to eBay. However, in their data, it corresponds to the revenue raised by an hypothetical seller that does not shill bid which does not necessarily correspond to the expected revenue under equilibrium behavior.

References

[1] Agreements for fictitious bids at auctions. *Yale Law Journal*, 31(4):431–436, 1922.

[2] L. Arozamena and E. Cantillon. Investment incentives in procurement auctions. *Review of Economic Studies*, 71(1):1–18, 2004.

[3] O. Ashenfelter. How auctions work for wine and art. *Journal of Economic Perspectives*, 3(3):23–36, 1989.

[4] O. Ashenfelter and K. Graddy. Auctions and the price of art. *Journal of Economic Literature*, 41(3):763–787, 2003.

[5] P. K. Bag, E. M. Dinlersoz, and R. Wang. exposita notes : More on phantom bidding. *Economic Theory*, 15(3):701–707, 2000.

[6] P. Bajari and A. Hortacsu. Economic insights from internet auctions. *Journal of Economic Literature*, 42(2):457–486, 2004.

[7] E. Bennett. Auctions sales. *The American Law Register*, 31(1):1–21, 1883.

[8] J. Brown and J. Morgan. How much is a dollar worth? tipping versus equilibrium coexistence on competing online auction sites. *Journal of Political Economy*, 117(4):668–700, 2009.

[9] R. Cassady. *Auctions and Auctioneering.* University of California Press, 1967.

[10] Y.-K. Che and J. Sakovics. Hold-up problem. *The New Palgraves Dictionary of Economics, Second Edition.*

[11] J. Cremer, Y. Spiegel, and C. Z. Zheng. Optimal search auctions. *Journal of Economic Theory*, 134(1):226–248, 2007.

[12] A. Dixit and B. Nalebuff. *Thinking Strategically.* Norton, 1993.

[13] R. Engelbrecht-Wiggans. Optimal auctions revisited. *Games Econ. Behav.*, 5(2):227–239, 1993.

[14] M. J. Fishman. A theory of preemptive takeover bidding. *RAND Journal of Economics*, 19(1):88–101, 1988.

[15] D. Fudenberg and D. K. Levine. *The Theory of Learning in Games.* MIT Press Books. The MIT Press, 1998.

[16] D. A. Graham, R. C. Marshall, and J.-F. Richard. Phantom bidding against heterogenous bidders. *Econ. Letters*, 32:13–17, 1990.

[17] B. Harvey and F. Meisel. *Auctions Law and Practice.* Oxford University Press, 1995.

[18] J. Horner and N. Sahuguet. Costly signalling in auctions. *Review of Economic Studies*, 74(1):173–206, 2007.

[19] I. Horstmann and C. LaCasse. Secret reserve prices in a bidding model with a resale option. *Amer. Econ. Rev.*, 87(4):663–684, 1997.

[20] R. Katkar and D. Reiley. Public versus secret reserve prices in ebay auctions: Results from a pokémon field experiment. *Advances in Economic Analysis and Policy*, 6(2), 2006.

[21] R. J. Kauffman and C. A. Wood. *Running up the Bid: Modeling Seller Opportunism in Internet Auctions*, pages 929–935. In Proceedings of the Sixth Americas Conference on Information Systems (AMCIS 2000), 2000.

[22] V. Krishna. *Auction Theory.* Academic Press, 2002.

[23] L. Lamy. The econometrics of auctions under anonymous asymmetric bidders. working paper PSE, 2009.

[24] L. Lamy. The shill bidding effect versus the linkage principle. *Journal of Economic Theory*, 144:390–413, 2009.

[25] D. Levin and J. Smith. Optimal reservation prices in auctions. *Econ. J.*, 106(438):1271–1283, 1996.

[26] D. Levin and J. L. Smith. Equilibrium in auctions with entry. *Amer. Econ. Rev.*, 84(3):585–599, 1994.

[27] G. Lopomo. Optimality and robustness of the english auction. *Games Econ. Behav.*, 36:219–240, 2000.

[28] D. Lucking-Reiley. Auctions on the internet: What's being auctioned, and how? *Journal of Industrial Economics*, 48(3):227–52, 2000.

[29] L. Mauger-Vielpeau. *Les ventes aux enchères publiques.* Economica, 2002.

[30] R. P. McAfee and P. J. Reny. Correlated information and mechanism design. *Econometrica*, 60(2):395–421, 1992.

[31] F. Meisel. 'upping the ante': Market distortion in auction sales. *The Modern Law Review*, 59(3):398–413, 1996.

[32] F. Meisel. What price auctions without reserve. *The Modern Law Review*, 64(3):468–472, 2001.

[33] P. Milgrom. *Putting Auction Theory to Work.* Cambridge Univ. Press, Cambridge, 2004.

[34] D. Morrell. *Real Estate Myths exploded.* Lothian Books, 2004.

[35] R. Myerson and M. Satterthwaite. Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 29(2):265–281, 1983.

[36] R. B. Myerson. Optimal auction design. *Mathematics of Operation Research*, 6(1):58–73, 1981.

[37] A. Ockenfels, D. Reiley, and A. Sadrieh. *Online Auctions.* Handbook of Economics and Information Systems, forthcoming. Elsevier Science.

[38] R. H. Porter and J. D. Zona. Detection of bid rigging in procurement auctions. *Journal of Political Economy*, 101(3):518–38, 1993.

[39] D. Reiley. Field experiments on the effects of reserve prices in auctions: More magic on the internet. *RAND Journal of Economics*, 37(1):195–211, 2006.

[40] J. Riley and W. Samuelson. Optimal auctions. *Amer. Econ. Rev.*, 71(3):381–392, 1981.

[41] X. Shi. Optimal auctions with information acquisition. Working papers, University of Toronto, Department of Economics, Dec. 2007.

[42] D. Vincent. Bidding off the wall: Why reserve prices may be kept secret. *J. Econ. Theory*, 65:575–584, 1995.

[43] M. Weitzman. Optimal search for the best alternative. *Econometrica*, 47(3):641–54, 1979.

[44] H. Xu, C. K. Bates, and S. M. Shatz. Real-time model checking for shill detection in live online auctions. In *Software Engineering Research and Practice*, pages 134–140, 2009.

Appendix

A Proof of proposition 2.1

If there is no pure strategy equilibria, then both inequalities (16) and (17) fail to hold. It also implies that $r < r_M$. From the intermediate value theorem, there exists $\widehat{q} \in (q^*(r_M), q^*(r))$ such that $H(\widehat{q}, r) = c_{shill}$. Since $B(q, r, p)$ is strictly decreasing in q for any p, we obtain from $B(q^*(r), r, 0) = c_e$ and $B(q^*(r_M), r, 1) = c_e$ that $B(\widehat{q}, r, 0) < c_e$ and $B(\widehat{q}, r, 1) > c_e$. Another application of the intermediate value theorem guarantees the existence of $\widehat{p} \in (0, 1)$ such that $B(\widehat{q}, r, \widehat{p}) = c_e$. On the whole, $(\widehat{q}, \widehat{p})$ is an equilibrium of type 3.

B Some properties of $q \to W_A(X_s, r, q)$

Lemma B.1

- *$q \to W_A(X_s, r, q)$ is strictly concave on $[0, 1]$ if $r \geq X_S$.*
- $\frac{\partial W_A(r,r,q^*(r))}{\partial q} = 0$.
- *If $\overline{x} > r > X_S$ and $q \leq q^*(r)$, then $\frac{\partial W_A(X_s,r,q)}{\partial q} > 0$.*
- *If $q > q^*(r)$, then $\frac{\partial W_A(r,r,q)}{\partial q} < 0$.*

Proof Using the decomposition $W_A(X_s, r, q) = [W_A(X_s, r, q) - W_A(r, r, q)] + W_A(r, r, q)$ and an integration per parts of eq. (13) for $W_A(r, r, q)$ leads to:

$$W_A(X_s, r, q) = -[r - X_S] \cdot (q \cdot F(r) + (1 - q))^N + \overline{x} - \int_r^{\overline{x}} (q \cdot F(u) + (1 - q))^N du - qN \cdot c_e.$$

Since $\frac{d^2(q\cdot F(u)+(1-q))^N}{d^2q} > 0$ for $q \in (0,1)$ and $u < \overline{x}$, we obtain that $q \to W_A(X_s, r, q)$ is strictly concave on $(0,1)$. The second bullet $\frac{\partial W_A(r,r,q^*(r))}{\partial q} = 0$ has been shown previously by Levin and Smith [26]. Note that $\frac{\partial W_A(X_s,r,q)}{\partial q} = [r - X_S]\cdot N(1-F(r))(q\cdot F(r)+(1-q))^{N-1} + \frac{\partial W_A(r,r,q)}{\partial q}$. The first term is strictly positive for $\overline{x} > r > X_S$. Since W_A is concave with respect to q, we obtain finally that $\frac{\partial W_A(X_S,r,q)}{\partial q} > \frac{\partial W_A(r,r,q)}{\partial q} \geq 0$ for $q \leq q^*(r)$ and $\overline{x} > r \geq X_S$ and $\frac{\partial W_A(r,r,q)}{\partial q} < 0$ for $q > q^*(r)$. **Q.E.D.**

C Proof of lemma 3.1

From (14) and an integration by parts, we have

$$H(q,r) = -(qF(r)+(1-q))^N\cdot[X_s + \frac{1-F(r)}{f(r)} - r] - \int_r^{r_M}(qF(u)+(1-q))^N d[X_s + \frac{1-F(u)}{f(u)} - u]$$

The differential with respect to q can be written as:

$$\begin{aligned}\frac{\partial H(q,r)}{\partial q} = \quad & N(qF(r)+(1-q))^{N-1}(1-F(r))\cdot[X_s+\tfrac{1-F(r)}{f(r)}-r] \\ & +N\textstyle\int_r^{r_M}[(qF(u)+(1-q))^{N-1}](1-F(u))d[X_s+\tfrac{1-F(u)}{f(u)}-u]\end{aligned}$$

Assumption 4 implies that $F(r_M) < 1 - \frac{1}{Nq}$ for any $q \in [q^*(r_M),1]$ and thus $F(r) < 1-\frac{1}{Nq}$ for any $q \in [q^*(r_M),1]$. This last inequality is equivalent to $(N-1)q(1-F(r)) - (qF(r)+(1-q)) > 0$ for any $q \in [q^*(r_M),1]$. Finally we obtain that $[(qF(u)+(1-q))^{N-1}]\cdot(1-F(u))$ is decreasing in u on the range $[r, r_M]$ for any $q \in [q^*(r_M),1]$ and we have thus $[(qF(u)+(1-q))^{N-1}](1-F(u)) > [(qF(r)+(1-q))^{N-1}](1-F(r))$ for any $u \in [r, r_M)$. On the whole we obtain that $\frac{\partial H(q,r)}{\partial q} < 0$.

D Proof of lemma 4.1

Note first that if the reserve price r implements an equilibrium profile involving some shill bidding activity, then we must have $r < r_M$: otherwise the seller would never hire a costly shill bidder that could not raise her expected payoff. If $p = 1$, then the result is straightforward: take $r' = r_M$, the seller raises a higher payoff than in the former equilibrium since she saves up the shill bidding costs. Consider now $p \in (0,1)$. From (4) and since $r < r_M$, we obtain that $\frac{\partial B}{\partial p}(q,r,p') < 0$ for any $p' \in [0,1]$. Consequently, $B(q,r,p) = c_e$ implies that $B(q,r,0) > c_e$ and $B(q,r,1) < c_e$. Finally we obtain $B(q,r,0) > c_e$ and $B(q,r_M,0) = B(q,r,1) < c_e$. Since $\frac{\partial B}{\partial r}(q,r'',p) < 0$ for $r'' \in [r,r_M]$ if $p < 1$, there exists a (unique) $r' \in [r,r_M]$ such that $B(q,r',0) = c_e$ which guarantees that the bidders' equilibrium equation is satisfied. Furthermore, under the probability of entry q, not hiring a shill bidder is a seller's best response if the reserve price is r' since the gain from shill bidding $r'' \to H(q,r'')$ is decreasing in r'' while $H(q,r) = c_{shill}$.

E Proof of lemma 4.2

For $r \geq r_M$, we have $H(q^*(r),r) = 0$. Below we consider in our calculations that $r \leq r_M$.

$$\frac{dH(q^*(r),r)}{dr} = \frac{\partial H(q^*(r),r)}{\partial r} + \frac{\partial q^*(r)}{\partial r} \cdot \frac{\partial H(q^*(r),r)}{\partial q}$$

After differentiating (14) with respect to r and q, we obtain:

$$\begin{aligned}
&\tfrac{\partial H(q^*(r),r)}{\partial r} = -q^*(r)N[q^*(r)F(r) + (1-q^*(r))]^{N-1}[X_s + \tfrac{1-F(r)}{f(r)} - r]f(r) \\
&= \tfrac{\partial q^*(r)}{\partial r} N(N-1)q^*(r) \int_r^{\overline{x}} (1-F(u))^2[q^*(r)F(u) + (1-q^*(r))]^{N-2}du[X_s + \tfrac{1-F(r)}{f(r)} - r]\tfrac{f(r)}{(1-F(r))} \\
&\leq \tfrac{\partial q^*(r)}{\partial r} N(N-1)q^*(r) \int_r^{r_M} (1-F(u))^2[q^*(r)F(u) + (1-q^*(r))]^{N-2}du[X_s + \tfrac{1-F(r)}{f(r)} - r]\tfrac{f(r)}{(1-F(r))}
\end{aligned}$$

The last inequality comes from $\frac{\partial q^*(r)}{\partial r} < 0$ and $X_s + \frac{1-F(r)}{f(r)} - r > 0$.

$$\begin{aligned}
&\tfrac{\partial H(q^*(r),r)}{\partial q} = N\int_r^{r_M} [q^*(r)F(u) + (1-q^*(r))]^{N-1}[X_s + \tfrac{1-F(u)}{f(u)} - u]dF(u) \\
&-N(N-1)q^*(r)\int_r^{r_M} (1-F(u))[q^*(r)F(u) + (1-q^*(r))]^{N-2}[X_s + \tfrac{1-F(u)}{f(u)} - u]dF(u) \\
&\geq -N(N-1)q^*(r)\int_r^{r_M} (1-F(u))[q^*(r)F(u) + (1-q^*(r))]^{N-2}[X_s + \tfrac{1-F(u)}{f(u)} - u]dF(u)
\end{aligned}$$

We obtain then:

$$\begin{aligned}
&\tfrac{dH(q^*(r),r)}{dr} \leq -\tfrac{\partial q^*(r)}{\partial r} N(N-1)q^*(r)\int_r^{r_M} (1-F(u))[q^*(r)F(u) + (1-q^*(r))]^{N-2}[X_s + \tfrac{1-F(u)}{f(u)} - u]dF(u) \\
&+\tfrac{\partial q^*(r)}{\partial r} N(N-1)q^*(r)\int_r^{r_M} (1-F(u))^2[q^*(r)F(u) + (1-q^*(r))]^{N-2}du[X_s + \tfrac{1-F(r)}{f(r)} - r]\tfrac{f(r)}{(1-F(r))}
\end{aligned}$$

In order to obtain $\frac{dH(q^*(r),r)}{dr} \leq 0$, it is thus sufficient to show that:

$$\begin{aligned}
&\int_r^{r_M} (1-F(u))[q^*(r)F(u) + (1-q^*(r))]^{N-2}[X_s + \tfrac{1-F(u)}{f(u)} - u]f(u)du \\
&\leq \int_r^{r_M} (1-F(u))^2[q^*(r)F(u) + (1-q^*(r))]^{N-2}du[X_s + \tfrac{1-F(r)}{f(r)} - r]\tfrac{f(r)}{(1-F(r))}
\end{aligned}$$

This last inequality holds since $[X_s + \frac{1-F(u)}{f(u)} - u]\frac{f(u)}{(1-F(u))} \leq [X_s + \frac{1-F(r)}{f(r)} - r]\frac{f(r)}{(1-F(r))}$ for any $u \geq r$, an inequality which is equivalent to $[X_s - u]\frac{f(u)}{(1-F(u))} \leq [X_s - r]\frac{f(r)}{(1-F(r))}$ which holds since $x \to (X_S - x)\frac{f(x)}{1-F(x)}$ is assumed to be decreasing on $[0,\overline{x})$.

F Proof of proposition 4.3

Without the shill bidding constraint, Levin and Smith [26] have shown that the optimal reserve price equals X_S. Thus if $r^* \leq X_S$, the seller still maximizes her expected payoff by setting the implementable reserve price X_S. For $\overline{x} > r > X_S$, we have $\frac{dW_A}{dr}(X_s, r, q^*(r)) = \underbrace{\frac{\partial W}{\partial r}}_{<0} + \underbrace{\frac{\partial q^*(r)}{\partial r}}_{<0} \cdot \frac{\partial W}{\partial q}$. At $q = q^*(r)$, we have from lemma B.1 (Appendix B) that $\frac{\partial W}{\partial q} > 0$. On the whole, if $r^* > X_S$, we have $\frac{dW_A}{dr}(X_s, r, q^*(r)) < 0$ on the interval $[r^*, \overline{x})$ such that the optimal reserve price equals r^*.

G Proof of proposition 4.4

Consider first the comparative statics for the optimal reserve price r_{opt}. It is sufficient to derive the corresponding result for the lowest implementable reserve price r^*.

If $H(q^*(0), 0) < c_{shill}$, then r^* is locally a constant ($r^* = 0$) as a function of the parameters X_S, c_e and c_{shill}. Otherwise, r^* is the solution of (18) and we have thus:

$$\frac{dr^*}{dX_S} = -\frac{\frac{dH}{dX_S}(q^*(r^*), r^*)}{\frac{dH}{dr}(q^*(r^*), r^*)}; \quad \frac{dr^*}{dc_e} = -\frac{\frac{dH}{dc_e}(q^*(r^*), r^*)}{\frac{dH}{dr}(q^*(r^*), r^*)} \text{ and } \frac{dr^*}{dc_{shill}} = \frac{1}{\frac{dH}{dr}(q^*(r^*), r^*)}.$$

From lemma 4.2, we have $\frac{dH}{dr}(q^*(r^*), r^*) \leq 0$ and it is thus sufficient to show that $\frac{dH(q^*(r),r)}{dX_s} \geq 0$ and $\frac{dH(q^*(r),r)}{dc_e} \geq 0$ to obtain the desired comparative statics. From (14) and since $q^*(r)$ does not depend on X_S, we obtain immediately that $\frac{dH(q^*(r),r)}{dX_s} = \frac{\partial H(q^*(r),r)}{\partial X_s} \geq 0$. From (8), we have $\frac{\partial q^*(r)}{\partial c_e} = -\frac{1}{(N-1)\int_r^{\overline{x}} (1-F(u))^2 [q^*(r)F(u)+(1-q^*(r))]^{N-2} du} \leq 0$ while lemma 3.1 establishes $\frac{\partial H(q^*(r),r)}{\partial q} < 0$. On the whole, we obtain thus that $\frac{dH(q^*(r),r)}{dc_e} = \frac{\partial q^*(r)}{\partial c_e} \cdot \frac{\partial H(q^*(r),r)}{\partial q} \geq 0$.

Consider then the comparative statics for the probability not to enter the auction $(1 - q^*(r_{opt}))$. Since $\frac{\partial q^*}{\partial X_S}$ and $\frac{\partial q^*}{\partial c_{shill}}$ equals zero, we obtain then $\frac{dq^*}{dX_S} = \frac{\partial q^*}{\partial r} \cdot \frac{dr^*}{dX_S}$ and $\frac{dq^*}{dc_{shill}} = \frac{\partial q^*}{\partial r} \cdot \frac{dr^*}{dc_{shill}}$ such that the same comparative statics hold for $(1 - q^*(r_{opt}))$ and r_{opt}. Furthermore, we have $\frac{dq^*}{dc_e} = \frac{\partial q^*(r)}{\partial c_e} + \frac{\partial q^*}{\partial r} \cdot \frac{dr^*}{dc_e}$. Both terms are negative, which concludes the proof.

H Proof of proposition 4.6

If $r^* \leq X_S$, the result comes from Levin and Smith [26]. We now consider $r^* > X_S$. As a preliminary, note that the function $r \to W_A(X_s, r, q)$ is strictly increasing on $[\underline{x}, X_S]$ and then strictly decreasing on $[X_S, \overline{x}]$ such that the maximization program (21) leads to the optimal reserve price $\max\{X_S, r^*_{t(q,r)}\}$ for any given q.

We first show that the optimal entry fees should be such that $r_t^* > X_S$. Suppose that $r_t^* < X_S$, then the optimal second-price auction involves the reserve price X_S and $t < 0$. We have $\frac{dW_A(X_S, X_S, q^*(X_S;t))}{dt} = \frac{\partial W_A}{\partial q}(X_S, X_S, q^*(X_S;t)) \cdot \frac{\partial q^*}{\partial t}(X_S;t)$. From lemma B.1, we have $\frac{\partial W_A}{\partial q}(X_S, X_S, q^*(X_S;t)) < 0$ since $q^*(X_S;t) > q^*(X_S;0)$. We have furthermore $\frac{\partial q^*}{\partial t}(X_S;t) < 0$. Finally, we have $\frac{dW_A(X_S,X_S,q^*(X_S;t))}{dt} > 0$, which stands in contradiction with t optimal since the seller would raise her payoff by lowering the subsidy. We now show that $r_t^* \neq X_S$ since otherwise the seller would strictly benefit from lowering slightly the entry fee. Let $\widehat{t}$ such that $r_{\widehat{t}}^* = X_S$. We have $\frac{dW_A(X_S, r_t^*, q^*(X_S;t))}{dt} = \frac{\partial W_A}{\partial r}(X_S, r_t^*, q^*(X_S;t)) \cdot \frac{\partial r_t^*}{\partial t} + \frac{\partial W_A}{\partial q}(X_S, X_S, q^*(X_S;t)) \cdot \frac{\partial q^*}{\partial t}(X_S;t)$. At $t = \widehat{t}^+$, it reduces to $\frac{dW_A(X_S,r_t^*,q^*(X_S;t))}{dt} = \frac{\partial W_A}{\partial q}(X_S, X_S, q^*(X_S;t)) \cdot \frac{\partial q^*}{\partial t}(X_S;t) < 0$ and we have thus shown that $\widehat{t}$ can not be optimal.

It remains to show that the optimal entry fee t is a subsidy, i.e. it satisfies $t < 0$. If $\overline{x} > r > X_S$, we obtain as a corollary of lemma B.1 that $Arg\max_{q\in[0,1]} W_A(X_s, r, q) > q^*(r)$. For the optimal instruments r, t, it means that $q^*(r;t) > q^*(r)$. From (8) and (20), we obtain that $t = B(q^*(r;t), r, 0) - B(q^*(r), r, 0)$ and thus $t < 0$ since $B(q, r, p)$ is strictly decreasing in q.

I Proof of proposition 5.1

For $X_S \geq \overline{x}$, there is no trade and the seller's expected payoff equals her reservation value and is thus strictly increasing with respect to her reservation value. Let $\widehat{X}_S := \underline{x} - \frac{1}{f(\underline{x})}$. From the assumption $\frac{1}{f(\underline{x})} \leq \underline{x}$, we obtain that $\widehat{X}_S$ is a feasible reservation value, i.e. $\widehat{X}_S \geq 0$. We show below that the assumption $c_e \leq e^{-1}\int_{\underline{x}}^{\overline{x}}(1 - F(u))du$ guarantees that $W_A^*(X_s) := W_A(X_s, r_M, q^*(r_M))$ is strictly decreasing in the right neighborhood of $\widehat{X}_S$.

$$\frac{dW_A^*(X_s)}{dX_s} = \frac{\partial W_A}{\partial X_s} + \frac{\partial r_M}{\partial X_s} \cdot [\frac{\partial W_A}{\partial r} + \frac{\partial q^*(r_M)}{\partial r} \cdot \frac{\partial W_A}{\partial q}]$$

From (1), we have $\frac{\partial r_M}{\partial X_s} = [2 + \frac{f'(r_M)(1-F(r_M))}{(f(r_M))^2}]^{-1}$. The assumption on the hazard rate function guarantees that $\frac{\partial r_M}{\partial X_s} \geq 1$. In the proof of proposition 4.3, we have shown that the term $\frac{\partial q^*(r_M)}{\partial r} \cdot \frac{\partial W_A}{\partial q} \leq 0$ while $\frac{\partial W_A}{\partial r} \leq 0$ for $r \geq X_S$. We obtain thus:

$$\frac{dW_A^*(X_s)}{dX_s} \leq \frac{\partial W_A}{\partial X_s} + \frac{\partial W_A}{\partial r} = (q^*(r_M)F(r_M) + (1-q^*(r_M)))^N - (r_M - X_S)Nf(r_M)(q^*(r_M)F(r_M) + (1-q^*(r_M)))^{N-1}.$$

After the substitution $r_M(\widehat{X}_S) = \underline{x}$ such that $F(r_M(\widehat{X}_S)) = 0$, we obtain:

$$\frac{dW_A^*(X_s)}{dX_s}\,|_{X_s \leq \widehat{X}_S^+} \leq (1-q^*(\underline{x}))^N - Nq^*(\underline{x})(1-q^*(\underline{x}))^{(N-1)} = (1-q^*(\underline{x}))^{(N-1)}[1-(N+1)q^*(\underline{x})].$$

To conclude the proof, we show below that $q^*(\underline{x}) > \frac{1}{N+1}$ for any N. From (4), an integration per parts leads to $B(q,\underline{x},0) = \int_{\underline{x}}^{\overline{x}}(1-F(u))\cdot[qF(u)+(1-q)]^{(N-1)}du$. We obtain then:

$$B(\frac{1}{N+1},\underline{x},0) \geq \int_{\underline{x}}^{\overline{x}}(1-F(u))\cdot[1-\frac{1}{N+1}]^{(N-1)}du = \underbrace{[1-\frac{1}{N+1}]^{(N-1)}}_{>e^{-1}}\cdot\int_{\underline{x}}^{\overline{x}}(1-F(u))du.$$

The last inequality can be viewed as a corollary of the well-known inequality $ln(1-x) > x$ for any $x \in (0,1)$. From the assumption $c_e \leq e^{-1}\int_{\underline{x}}^{\overline{x}}(1-F(u))du$, we obtain thus that $B(\frac{1}{N+1},\underline{x},0) > c_e$ and finally that $q^*(\underline{x}) > \frac{1}{N+1}$ which concludes the proof.

J Proof of proposition 6.1

Let $\widetilde{W_P}(X_S,r,z) := W_P(X_S,r,q)$. We have then:

$$W_P(X_S,r,q_P^*(r)) = \widetilde{W_P}(X_S,r,z_P^*(r)) = [z_P^*(r)]^N\cdot X_S + (1-[z_P^*(r)]^N)\cdot r, \qquad (33)$$

where $z_P^*(r)$ is characterized by

$$\frac{1-[z_P^*(r)]^N}{1-[z_P^*(r)]} = \frac{Nc_e}{\int_r^{\overline{x}}(u-r)dF(u)}. \qquad (34)$$

The differentiation of (34) leads to

$$\frac{dz_P^*}{dr} = \frac{Nc_e(1-F(r))}{[\int_r^{\overline{x}}(u-r)dF(u)]^2}\cdot\frac{1}{\sum_{i=0}^{N-1}i(z_P^*)^{i-1}}, \qquad (35)$$

$$\frac{dz_P^*}{dc_e} = \frac{N}{\int_r^{\overline{x}}(u-r)dF(u)}\cdot\frac{1}{\sum_{i=0}^{N-1}i(z_P^*)^{i-1}} \quad \text{and} \qquad (36)$$

$$d^2z_P^*\sum_{i=0}^{N-1}i(z_P^*)^{i-1} + (dz_P^*)^2\sum_{i=0}^{N-1}i(i-1)(z_P^*)^{i-2} = \frac{N(1-F(r))}{[\int_r^{\overline{x}}(u-r)dF(u)]^2}\cdot dc_e dr. \qquad (37)$$

In (37), we replace $\frac{(dz_P^*)^2}{dc_e dr}$ by means of (35) and (36) which leads to:

$$\frac{d^2z_P^*}{dc_e dr} = \frac{\frac{N(1-F(r))}{[\int_r^{\overline{x}}(u-r)dF(u)]^2} - \frac{N(1-F(r))}{[\int_r^{\overline{x}}(u-r)dF(u)]^2}\cdot\frac{Nc_e\sum_{i=0}^{N-1}i(i-1)(z_P^*)^{i-2}}{\int_r^{\overline{x}}(u-r)dF(u)[\sum_{i=0}^{N-1}i(z_P^*)^{i-1}]^2}}{\sum_{i=0}^{N-1}i(z_P^*)^{i-1}} \quad \text{and then} \qquad (38)$$

$$\frac{d^2z_P^*}{dc_edr}=\frac{N(1-F(r))\sum_{i=0}^{N-1}(z_P^*)^i}{[\int_r^{\overline{x}}(u-r)dF(u)\cdot\sum_{i=0}^{N-1}i(z_P^*)^{i-1}]^2}\cdot[\frac{\sum_{i=0}^{N-1}i(z_P^*)^{i-1}}{\sum_{i=0}^{N-1}(z_P^*)^i}-\frac{\sum_{i=0}^{N-1}i(i-1)(z_P^*)^{i-2}}{\sum_{i=0}^{N-1}i(z_P^*)^{i-1}}] \tag{39}$$

The term in the bracket in (39) is strictly positive (for any $i \geq 1$ we have $\frac{iz^{i-1}}{z^i} > \frac{i(i-1)z^{i-2}}{iz^{i-1}}$). On the whole, we have shown that $\frac{dz_P^*}{dr}$, $\frac{dz_P^*}{dc_e}$ and $\frac{d^2z_P^*}{dc_edr} > 0$ for $r < \overline{x}$. The differentiation of (33) leads to:

$$\frac{d^2W_P(X_S,r,q_P^*(r))}{drdX_S}=N(z_P^*(r))^{N-1}\frac{dz_P^*}{dr}>0, \tag{40}$$

$$\frac{d^2W_P(X_S,r,q_P^*(r))}{drdc_e}=-N(z_P^*(r))^{N-1}\frac{dz_P^*}{dc_e}-N(N-1)(z_P^*(r))^{N-2}\frac{dz_P^*}{dr}[r-X_S]-N(z_P^*(r))^{N-1}\frac{d^2z_P^*}{drdc_e}[r-X_S]<0 \tag{41}$$

$$\text{and} \quad \frac{d^2W_P(X_S,r,q_P^*(r))}{drdc_{shill}}=0. \tag{42}$$

We obtain thus the desired comparative statics results for the solution of the equation $\frac{dW_P(X_S,r,q_P^*(r))}{dr}=0$ that corresponds to the optimal posted-price.

For the results on the seller's expected payoff, the result comes from the equality $\frac{\partial W_P(X_S,r,q_P^*(r))}{\partial X_S}=[z_P^*(r)]^N>0$, $\frac{\partial W_P(X_S,r,q_P^*(r))}{\partial c_e}=-q_P^*(r)N<0$, $\frac{\partial W_P(X_S,r,q_P^*(r))}{\partial c_{shill}}=0$ and the Envelope Theorem which guarantees the equality between derivatives and partial derivatives in this setup.

K Proof of proposition 6.2

If participation costs are small enough, then there is full participation in auctions with shill bidding and the model converges to one with exogenous entry such that the expected payoff of the seller converges to the optimal auction. On the contrary, the allocative inefficiency associated to posted-price mechanisms does not vanish which shows the first part of the result. Any choice larger and sufficiently close to $B(1,r_M,0)$ for $\underline{c}$ will work.
Let $\overline{c}:=\int_{r_M}^{\overline{x}}(u-X_S)dF(u)>0$: $\overline{c}$ corresponds to the smallest participation cost such that $Arg\max_{q\in[0,1]}W_A(X_S,r_M,q)=0$, i.e. such that the optimal auction with shill bidding and entry fees involves no trade with probability one (for $c_e=\overline{c}$, we have $\frac{\partial W_A}{\partial q}(X_S,r_M,0)=0$). In a posted-price mechanism, we have $B_P(0,r)=\int_r^{\overline{x}}(u-r)dF(u)$. If r is close enough to X_S, then $B_P(0,r)>\overline{c}$ uniformly. For any participation cost c_e in the right neighborhood of $\overline{c}$, there is thus a reserve price $r\in(X_S,r_M)$ such that $B_P(0,r)>c_e$ or equivalently such that there is some participation with the posted-price r and so such that the seller's expected payoff is strictly greater than X_S, i.e. greater than the revenue in auctions with shill bidding and entry fees, which concludes the proof.

L Proof of proposition 6.3

Any interior solution of the maximization program (28) satisfies the first order conditions: $\frac{\partial W_P(X_s,r,q)}{\partial r} = 0$ and $\frac{\partial W_P(X_s,r,q)}{\partial q} = 0$. From (25), those conditions imply respectively:

$$q \cdot N(q \cdot F(r) + (1-q))^{N-1} \cdot \left(\int_r^{\overline{x}} u \frac{dF(u)}{(1-F(r))} - X_S\right) = (1-(q \cdot F(r)+(1-q))^N) \cdot \int_r^{\overline{x}} (u-r) \frac{dF(u)}{(1-F(r))^2} \tag{43}$$

$$\text{and} \quad (q \cdot F(r) + (1-q))^{N-1} \cdot \left(\int_r^{\overline{x}} u dF(u) - (1-F(r))X_S\right) = c_e. \tag{44}$$

(43) and (44) imply $B_P(q,r) = c_e$ or equivalently from (27) that the entry fee is null. To conclude the proof, it remains to show that the solution is necessarily interior. From A3, both $q = 0$ or $r = X_S$ are suboptimal. Consider $r \geq X_S$, the optimal probability of entry is smaller than one since $\frac{1}{N}\frac{\partial W_P}{\partial q}(X_s, r, 1) = F(r)^{N-1} \cdot \int_r^{\overline{x}} (u - X_S) dF(u) - c_e \leq P(r, N-1) \cdot \int_{X_S}^{\overline{x}} (u - X_S) dF(u) - c_e < 0$ (the last inequality comes from A6).

M Proof of lemma 7.1

The ex ante welfare depends solely on the probability of entry and the allocation rule among the seller and the entrants. For a given probability of entry, the ex ante welfare is maximized if and only if the allocation rule gives the good to the agent with the highest valuation with probability one. In other words, the ex ante welfare is maximized if a second price auction (without shill bidding) where the reserve price $\widetilde{X}_S$ is used. At best, the probability of entry can be fine-tuned as a function of $\widetilde{V}_A$. For a given probability of entry function $q(\widetilde{V}_A)$, the ex ante expected welfare is then given by $E_{\widetilde{V}_A}[E_{\widetilde{X}_S|\widetilde{V}_A}[W_A(\widetilde{X}_S, \widetilde{X}_S, q(\widetilde{V}_A))]]$. For any $\widetilde{V}_A$, the function $q \to E_{\widetilde{X}_S|\widetilde{V}_A}[W_A(\widetilde{X}_S, \widetilde{X}_S, q)]$ is strictly concave on $[0,1]$ since $q \to W_A(\widetilde{X}_S, \widetilde{X}_S, q)$ is strictly concave on $[0,1]$ for any realization $\widetilde{X}_S$ (lemma B.1). There is thus a unique optimal solution $q^{opt}(\widetilde{V}_A)$ for any $\widetilde{V}_A$. It remains to show that this solution coincides with the one that would prevail in a second price auction with free entry where the reserve price would be equal to the seller's reservation value and if potential entrants knew the signal $\widetilde{V}_A$, a result that is a straightforward extension of Levin and Smith [26] to the case where there is some uncertainty on the seller's valuation. The right intuition for this result is exposed in Milgrom [33] (Theorem 6.1): in a second price auction, for any realization of the agents' valuations, the net profit of an entrant coincides with his incremental contribution to the welfare.