Benchmarking local ecological performance using ecological footprint accounts

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Abstract

Ecological footprint is good at telling us how much we weight relative to global ecological ressources. However it tells us nothing on their geographical origin and makes no distinction between ecological ressources used from distant or local land. Moreover it does not measure the intensity of exploitation of soils or other sustainability considerations like biodiversity loss. Therefore the results provided by the footprint calculation are barely suitable for planning and policy design, as information on real use of local land is lost in the process of calculation. This issues are well known and are part of the research agenda set by the footprint research community and recommendations has been made to deal with them. The research community already use various metrics to compute the ecological footprint: global hectares, actual hectares and disturbed hectares. The increasing use of input-output technics has also enhanced the localization of the footprint. In this paper all the metrics are used in conjunction with input-output techniques to produce four ecological performance indicators of a local economy taking in account: its global weight on the planet, its degree of dependence on distant ecological ressources, the sustainability of its farming practices, and the quantity of local fertile land not yet used in a bioproductive way. This four performance indicators are then plotted together giving an easy visualizing tool to compare various alternative scenarios.

1 Introduction

1.1 The Concept of Ecological Footprint

In one of the seminal papers of Rees and Wackernagel (1996) the ecological footprint of cities was pedagogically presented with a mental experiment in two steps. First imagine that the modern city could be enclosed in a glass or plastic hemisphere completely closed to material flows, meaning that the city would depend only on its own ecosystem inside the hemisphere. It is clear that all inhabitants of the city would perish in a few days, both from starving and suffocating from its wastes. In the second step imagine that the city is surrounded by croplands, pastures forest and watersheds all represented in proportion to their actual abondance on the Earth. And let's imagine that the glass enclosure is elastically expandable. The question becomes: how large would the hemisphere have to grow before the city as its center could sustain itself indefinitely and exclusively on the land and water ecosystems ? Answering this question provides an estimate of the defacto ecological footprint of the city. Hence the definition of the ecological footprint:

The ecological footprint is the total area of productive land and water required continuously to produce all the ressources consumed and to assimilate all the wastes produced, by a defined population, wherever on Earth that land is located.(Rees and Wackernagel (1996))

Applying their method to Vancouver they found that the city required a biological productive surface 180 times larger than its political surface, or if expressed in radius of concentric circles, the outside circle containing the bioproductive land has a radius 8 times the radius of the inner circle containing the city.

The remarkable fact of this introduction to ecological footprint is the necessity to do a mental experiment to imagine that the hinterland of the city is in its countryside meaning that in reality this is absolutely not the case. Indeed if it was possible to look at all the lands providing its ressources we would find them scattered all around the world. In practice we have only a very partial knowledge on where these pieces of land are localized, and very few if any indications on how sustainably these land are effectively used.

Of course this is one of the main characteristic of modern cities compared with cities one hundred year ago. At that time we would not have needed a mental experiment to conceptualize ecological footprint but we would have looked directly the countryside around the city. Even though trade existed it did not amount to anything like today. Most of the biological ressources necessary to sustain city were coming from its surrounding, even though it could extend quite far (but connected) for the very big cities like Paris(Billen et al. (2008)).

It is the profound originality of the ecological footprint concept to have reintroduced this notion of hinterland to describe this intrinsic bound of our society with the land. Indeed it is not only that our dependence on land was removed from eyesight and hidden and embodied in tradeable manufactured goods and services but also that this conscience of dependence was removed from our psychology and our way of thinking the economy. As it should be obvious that we live in a constrained world with finit limits our economy evolves and grows as if there was no such constraints. Although critics of our modern economy has been done before and sometimes successfully(Meadows et al. (1972)), the ecological footprint has been very successful in communicating this notion of a finite planet.

1.2   The Accounting Framework of Ecological Footprint

The definition given today by the Global Footprint Network is:

The Ecological Footprint measure humanity's demand on nature. It measures how much land and water area a human population requires to produce the ressource it consumes and to absorb its wastes, using prevailing technology.

Given this somewhat generic definition, the science of footprint provides us with different methodologies to calculate footprint of entities like individual, city, business, nation, or all of humanity. The results of ecological footprint are presented inside an accounting framework. On the demand size is the footprint. On the supply side is the available biologically productive area, shortly called biocapacity. If demands exceeds supply this indicates an ecological deficit. Note that this accounting framework makes sense for nation or for the whole planet but need to be adapted for other entities. For individual the supply side is set to the average world biocapacity available per capita. It is through the use of this accounting framework for individual that we hear narrative about "how many planet a person need to sustain his way of life". For example if everybody lived like an American we would need five planets. This just means that within the ecological footprint account the calculated footprint of an American is five times the average world biocapacity per capita.

The calculation of Ecological Footprint follows strictly the principle of consumer responsibility (a term introduced in the context of discussions on greenhouse gas accounting Munksgaard and Pedersen (2001)) meaning that footprint associated with products imported from foreign country are added to the domestic consumers footprint account. Hence countries effectively import and export footprint.

The footprint is divided in six major types of surfaces, pasture, forest, built land, crop land, energy land and aquatic surfaces. It excludes unproductive area like desert or deep oceans[1].

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[1]Compared to the surface of the planet, 50 billions of hectares from which 15 bha of land, the bioproductive surface is

Among the six surface types, five corresponds to real surfaces whereas energy land is notional and correspond to the surface of forest necessary to absorb the carbon dioxide emitted in the atmosphere. This measured surface does not correspond to a recommended solution for global warming but is a mean to express the carbon footprint in hectares. Almost half of the global measured footprint is due to this notional forest land.

The computation of ecological footprint is based on bioproductivity. As such it departs from other consideration on sustainability. First it deemphasizes the importance of non-renewable ressources like minerals or hydrocarbures. The main argument being that we can live without minerals or hydrocarbures but not without photosynthesis. The first are not life supporting whereas the second is. Hence among the possible ecological limits to consider, the availability of bioproductive surface is the prevalent one. On the other hand the focus on biological productivity seems at odd with other sustainability consideration like land cover disturbance, soil degradation and biodiversity. As pointed in Lenzen et al. (2007) biodiversity and ecosystem health might be more limiting factor than bioproductivity and there is no correlations between the two. There are examples of augmenting bioproductivity which threaten soils or biodiversity.

1.3 World Hectares and Actual Hectares

One of the peculiarity of the standard footprint account is its use of world hectare obtained by converting tonnes of primary product in hectares by using world average yields instead of actual yields. The first consequence is that all footprint calculation done at a scale which is not the whole planet give results which do not correspond to real hectares. The mean advantage of this metric is that it allows comparability and avoid to conclude wrongly that people living on low productive land have higher footprint than people living on high productive land. Moreover it reflects better the main concern of its conceptual root which is to measure human appropriation of bioproductive ressources[2] from which the hectares of bioproductive land and sea are just a proxy.

However as there are other coefficients other than yield involved in the calculation of footprint notably conversion factor to go from secondary product to primary product (for example embodied CO2 in manufactured goods), a methodological consistency would require that global average are also used for this factor. However as explained in Wiedmann and Lenzen (2006) it is not always the case in the standard calculation of footprint where for example the conversion factor between energy and CO2 emission use local factor. In this paper they recommended to use either only global factors to calculate in world hectares or to use only local factors when calculating in actual hectares.

However from a practical point of view there may be some cases where it may be legitimate to mix actual and world hectares. Indeed consider a local territory for which the ecological footprint is calculated. There will be two components for this footprint a local one for which the local yield is used and is expressed in actual hectares (which can be either measured by land survey, or calculated using local yield) and the footprint of the importations. If there is no information on the localization of production for these imports then world hectares will be used as they are the best estimate of import footprint in actual hectares. This gives another justification for the use of world hectares and fits well with the problematic of a globalized world with delocalization of production as a core characteristic.

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composed of 2,4 bha productive aquatic surface(corresponding to freshwater fishing zone in and oceans), and 8,8 billions productive land split in:

- 3,7 bha of forest;
- 3,4 bha of pasture
- 1,5 bha of crop lands
- and 0,2 bha of built land

[2]Coming from its notarious predecessor HANPP, the "human appropriation of net primary production" developed by Vitousek et al. (1986)

In the ecological footprint methodology a distinction is made between world and global hectares. The passage from one to the other is done by weighting the different type of land and sea with their relative productivities. It thus allows to express all this type of surface related footprint in the same common metric and to add them up to obtain one single footprint. In this paper this transformation wont be done and the different types of surfaces will be considered of equal productivity. Indeed as it is just a model it would just add complexity in the calculations without adding analytical understanding. Hence the adjectives global or world will be used indifferently in this paper.

1.4 Actual Hectares and Disturbed Hectares

Another research subject deals with the sustainability use of actual hectares. Can the same weight be given to hectares used sustainably and to hectares which are not ? It was proposed (Lenzen and Murray (2001) and Lenzen and Murray (2003)) that the hectares be given a weight of disturbance to take in account the fact the footprint might be higher due to unsustainable exploitation. However the way to find this sustainability weight is a matter for discussion, and is not easy to fit in the calculation framework of ecological footprint. However for a local territory where this kind of information is available it seems of interest to use this type of footprints side by side with other type of footprints as will be shown in the proposed methodology.

1.5 Localization of the Footprint with Input-Output Analyses

In Rees and Wackernagel (1996) en example of calculation is provided for the productive forest area required for paper production. A Canadian consumes about 244 kilograms of paper products each year and the production of each metric ton of paper in Canada requires $1,8m^3$ of wood. For ecological footprint analysis an average wood productivity of $2.3m^3/ha/yr$ is assumed. Therefore the average Canadian requires

$$\frac{0.244[\text{t/cap/yr}] * 1.8[\text{m}^3/\text{t}]}{2.3m^3/ha/yr} = 0.19[\text{ha/capita}] \text{ of forest in continuous production of paper}$$

If we imagine that the Canadian consumers want to diminish their paper footprint. The first thing they would realize is that they do not consume directly themselves 244 kilograms of paper products each year but a tiny fraction of it. In fact most of it is consumed in the industry through intermediate consumption. So if the consumer can not act directly what can he do ? Well he can find out which industries are most consuming of paper and reduce their consumption of products coming from these industries. But they might realize again that they do not consume that much product from these industries. In fact these industries are provider of products to other industries. And so and so and the consumer realize that he is trapped in the impossible task of tracking every single use of paper to decide where its reduced consumption would be most beneficial to reduce his consumption of paper.

That is exactly what the input-output analyses do. They allow to compute the embodied paper used for the production of every product. Of course it does it at a very coarse level, the economic sectors, but it does it in an exact manner. This identification of flows of footprint from economic sectors to the others allows by proceeding backward to first associate our consumption footprint with the relevant primary industries and second to localize them. This allows mapping between consumption location and the distant production footprints.

1.6 Ecological Performances Indicators from Local Footprint Accounts

Even though they are not the only relevant aspect of sustainability, understanding the flows of bioproductive material is crucial. The concept of ecological footprint was precisely developed to measure this

dimension of sustainability. We will propose now a framework to extract various useful and readable characteristics of this metabolism using the various concepts presented.

2   Proposed Framework

This framework try to capture in as synthetic way various aspect of sustainability. It gives a coherent multi-dimensional view of sustainability with global weight, intensity of exploitation of local hectares, degree of dependence and local surplus of biocapacity.

It starts with the Ecological Footprint of a territory which can be measured in three different type of hectares: Footprint in World hectares obtained by dividing production by world yield, Footprint in Actual hectares obtained by dividing production by local yield, Footprint in Sustainable hectares obtained by weighting actual hectares with sustainability indicators. The consumption of a territory being composed from import and local production there is also an Import Footprint measured in world hectares.

Each type of footprint convey its own useful information which can synthesized in four ecological performance indicators: Global Fair Share: Indicates if the territory consumes more or less of its share of the world biocapacity. Local Independence: Indicates the degree of biocapacity independence. Linked with the imported biocapacity. Local Sustainability: Indicator of the sustainable use of the local biocapacity. Local Biocapacity Surplus: Indicator of the amount of unexploited biocapacity.

The Global Fair Share is the primary focus of the common use of ecological footprint notably with the communication goal of conveying the notion of a finite planet. It is calculated from biocapacity fair share which is the total world biocapacity divided by the world population multiplied by the local population. In a very equalitarian point of view this is what the local territory could claim for its population if the world biocapacity was equally allocated to each individual. It gives information on the fairness of a given footprint allocation. If less footprint is used than the biocapacity fair share then the indicator is superior to one if the footprint is above then it is inferior to one. Mathematically it is the ratio of the biocapacity fair share and the total footprint (local+import) all expressed in world hectares:

$$\text{Fair Share} = \frac{B^{fs}}{(F'^l + F'^{w)}}$$

Local Independence informs on the degree of biocapacity self-reliance of the local territory. If the local territory imports, its independence is inferior to one. If it does not import and it has surplus then it is superior to one. Indeed among the recommended way to make the cities more sustainable is to reduce their dependence on external flows. For this urban regions may choose to develop explicit policies to invest in rehabilitating their own natural capital stocks and to promote the sustainable use of local fisheries, forests, agricultural land, etc. This would increase regional independence thus creating a hedge against rising international demand, global ecological change and potentially reduce productivity elsewhere(Rees and Wackernagel (1996)). The independence is again a ratio. At the numerator the maximum between local biocapacity and local footprint is taken to allow more than 100% independent if there are some surplus. At the denominator is the total footprint. All are expressed in world hectares.

$$\text{Independence} = \frac{B'^l}{F'^l + F'^w}$$

Surplus informs us on the local biocapacity not used. Hence if there is some idle biocapacity the value of this indicator will be superior to one. It is all used productively it is equal to one. If all the biocapacity is used and some stocks of previous year are consumed (Forests being exploited faster than

they regenerate for example) then the surplus is inferior to one. This surplus is the ration between the local biocapacity and local footprint all in actual hectares

$$\text{Surplus} = \frac{B^l}{F^l}$$

The last indicator is **Sustainability** which informs us on the sustainable usage of the biocapacity. If actual yield are superior to sustainable yield then the indicator is below one. It it is inferior it is above one. It is the ratio between local footprint and sustainable footprint.

$$\text{Sustainability} = \frac{F^l}{F^s}.$$

To calculate the four indicators all types of footprints were used. Indeed each carry its own useful information which can be translated in meaningful indicators.

	Biocapacity			Footprints				Formula
	Actual B^l (ha)	World B'^l (wha)	Fair Share B^{fs} (wha)	Actual F^l (ha)	World F'^l (wha)	Sustainable F^s (sha)	Import F'^w (wha)	
Fair Share			x		x		x	$\frac{B^{fs}}{(F'^l+F'^w)}$
Independence		x			x		x	$\frac{B'^l}{F'^l+F'^w}$
Surplus	x			x				$\frac{B^l}{F^l}$
Sustainability				x		x		$\frac{F^l}{F^s}$

The next step is to synthesize these information so we can compare very quickly the results of various scenarios. For this purpose we put plot the indicators in a diagram like this one:
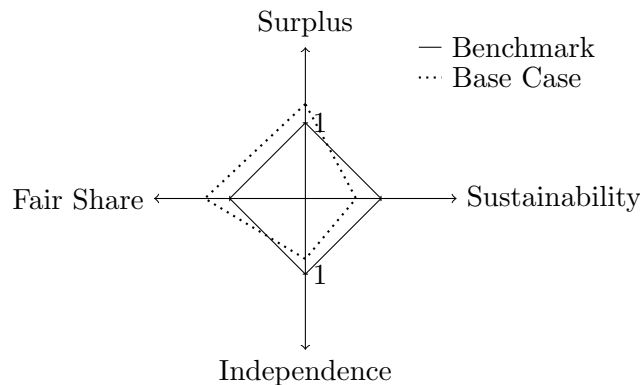

From this four-dimensional view, various alternative scenarios can easily be compared making this framework a possible decision tool. To fully understand this framework some numerical examples are presented in the next section

3   Numerical Examples

The methodology will be presented step by step through four numerical examples of growing complexity. The full analytical framework with all mathematical definition and demonstration will be relegated to the appendix. All our examples are based on a very simple stylistic representation of the world represented below. The world is divided in two parts, a local territory and the rest of the world. The economy can be divided in economic sectors. As an assumption each of the economic sector has its own associated biocapacity. For example in the example presented with two sectors, agriculture has biocapacity associated with arable land and industry has biocapacity associated with forest land

to absorb CO2. In our example we do not use equivalent factor as they just unnecessarily complicate the calculation without adding understanding to the analysis. To fix numerical values the world has a surface of 1000 hectare and the local territory has a surface of 100 hectare. In all our base case the world population is 10000 and the local one 1000. The world yield is 20t/ha/year for agriculture and 10t/ha/year for the forest associated with industry biocapacity. In our base case the yield for the local territory will be half the world one, i.e 10t/ha/year for agriculture and 5t/ha/year for the forest. This means that our local territory is rather poor in biocapacity compared to the rest of the world.
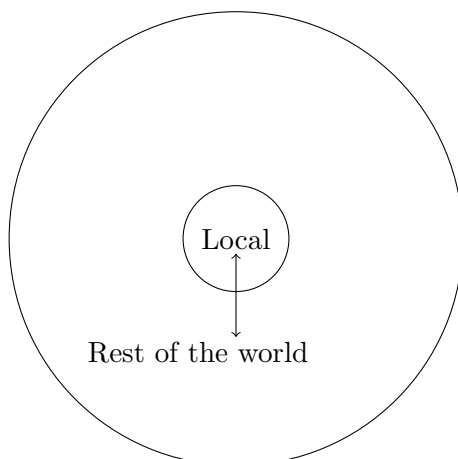
Let's introduce the four examples. The simplest one has only one economic sector, agriculture. From this very simple example, it will be possible to introduce most of our concept, i.e the various footprints and the four indicators we want to use to characterize our local economy. In the next example another economic sector is added, industry. The various footprints, and the indicators will be calculated separately for each economic sectors. In the next example there will be an interdependency between the two economic sectors which will break the one to one relationship between economic sectors and their footprints. Indeed two kinds of footprints can be computed for each economic sector, the production footprint and the consumption footprint depending if we consider the sector for their final usage, or for all their usage, final consumption and industrial consumption. The last example will add a level of complexity with the interdependencies extended to the economic sectors of the rest the world.

3.1 One industry

Let's add an additional hypothesis that the local population consume 1000kg of agricultural product (let's say wheat) per year. The first step is to compute the biocapacity of our local territory:

Biocapacity Calculation

	Real Biocap. (ha) B	Sustainable Yield(t/?ha/an)	Pot. Production (t/an)	Global Biocap. (gha) B'	Population P	Biocap. Fair Share (gha)
Local	B^l	y(t/sha/an)		B'^l	$P^l =1000$	B'^{fs}
Agriculture	100	10	1000	50	-	100
World	B^w	y'(t/gha/an)		B'^w	$P^w =10000$	
Agriculture	1000	20	20000	1000	-	-

Let's explain the various part of the table. The rows are split in two blocks world and local. For each block the biocapacity is calculated. The first column is the actual biocapacity which is just the area devoted to agriculture of the local territory and the world. Note that the local territory is part of the world in this table so that the 100 hundred hectares are part of the 1000 hectares of the world. The

second column is the maximum sustainable yield. It does not mean the actual yield but the higher sustainable yield which do not preclude future use of the soil with the same yield[3]. The third column is the potential production associated with the biocapacity and the yield. It is here as indicative and it is important to note that this production is potential not real. There can be idle biocapacity and different yield as already pointed out. The next column is global biocapacity. For the world it is the same as local biocapacity. However for the local territory it is equal to the surface land with world productivity which would produce the same amount as our local territory with current yield. Since the world productivity is twice the local one then the biocapacity is half the actual biocapacity. The last column is the biocapacity fair share, which is the total world biocapacity divided by the world population multiplied by the local population. In a very equalitarian point of view this is what the local territory could claim for its population if the world biocapacity was equally allocated to each individual.

With this table various notations are introduced which will be used all along. We use the letter **B** for biocapacity, **y** for yield. Following Wiedmann and Lenzen (2006), global and actual biocapacity are distinguished (it will be the same for footprint) with a superscript $'$ to designate global quantity, global yield, global biocapacity, global footprint... The superscript ^{s} is used to designate sustainable quantities. To differentiate between local quantities and world one superscript ^{l} and ^{w} are used. And the Biocapacity Fair Share is written B'^{fs}. Several superscripts can be used together ^{l} and $'$ gives $^{l'}$ meaning local quantities expressed in global hectares. Various mathematical relation exist between our various quantities

$$B'^l = \frac{B^l * y}{y'}$$

$$B'^{fs} = \frac{B^w * P^l}{P^w}$$

All these variables described can be called the state variable of the system as they form together a description of our system. Let's introduce another set which are the control variables. These are the quantities on which it is possible to have some control, meaning we can change their values. In the model these are the local population, the consumption levels, the importation and the actual yield of the soils. Each set of these variables are grouped to form a scenario. Let's take an example of such scenario:

Scenario (Pop.,Demand, Import, Yield)

Local Population	1000
Cons. per capita(t/an)	1,5
Import Agriculture	300
Yield Agriculture	15

The local economy is importing 300 t of agricultural product each year and the yield of the arable land is 15t/year above the sustainable yield of 10t/year. With this data it is possible to calculate various footprints.

The first footprint(the original one as introduced in Wackernagel and Rees (1996)) in global hectares:

Footprint in world hectares

	Demand (t/an)	Prod (t/an)	Import (t/an)	Yield (t/gha/an)	Local Footprint	Import Footprint
	f	x	i	y'	F'^l	F'^w
Agriculture	1500	1200	300	20	60	15

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[3]This usage is closed to the actual one in the footprint calculation where the yield which can be obtained by prevailing technology and ressource management schemes. This definitions however does not preclude unsustainability if this prevailing technology are unsustainable.

To calculate the footprint in global hectares, two steps were followed: split demand between production and import, then divide by the global yield. Mathematically, this gives:

$$x = f - i$$
$$F'^l = \frac{x}{y'}$$
$$F'^w = \frac{i}{y'}$$

Let's now calculate the footprints in actual hectares:

Footprint in actual hectares

	Demand (t/an) f	Prod (t/an) x	Import (t/an) i	Yield (t/ha/an) y	Local Footprint F^l
Agriculture	1500	1200	300	15	80

As before the first step was to calculate the production from the final demand. In the second step the production data are divided by the actual yield. However we can not do the same for import data since the actual yield is not known for import. The best thing is to use the average yield as it was done above. Mathematically:

$$x = f - i$$
$$F^l = \frac{x}{y}$$

The actual footprint is different from the global footprint as expected. It is bigger since the local yield is lower than the global one. It will be seen in the moment what is the use of calculating various footprints.

Finally let's compute the footprint in sustainable hectares:

Footprint in sustainable hectares

	Demand (t/an) f	Prod (t/an) x	Import (t/an) i	Yield (t/sha/an) y^s	Local Footprint F^{sl}
Agriculture	1500	1200	300	10	120

As before the same steps are involved, just the yield used is different. As expected the sustainable footprint is different from the other as the sustainable yield is different.

Let's see how these three different footprints can be used [4] and for this purpose four indicators are calculated from our three different footprints:

Performance Indicators

	Surplus $\frac{B^l}{F^l}$	Fair Share $\frac{B^{fs}}{(F^l+F'^w)}$	Independence $\frac{B'^l}{F'^l+F'^w}$	Sustainability $\frac{F^l}{F^s}$
Agriculture	1,25	1,33	0,8	0,67

To calculate the four indicators all types of footprints were used. Indeed each carry its own useful information which can be translated in meaningful indicators. The next step is to synthesize these information so we can compare very quickly the results of various scenarios. For this purpose we put plot the indicators in a diagram like this one:

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[4]Note that these three types of footprints exists in the literature as we have seen in the introduction
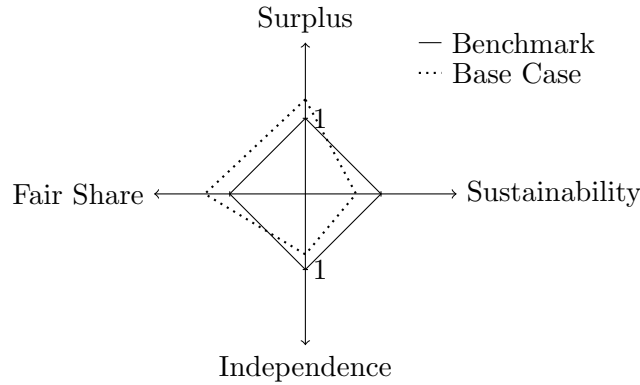
The plain line represent our benchmark in our stylized models. This means exactly global fair share, use of all biocapacity, no imports and sustainable yield. Of course this is a little arbitrary but it is a start to validate our methodology. The way to look at the diagram is to see how it departs from the benchmark. Hence an indicator below one will be inside the reference square and outside for value above one. What you expect in general is some kind of quadrilateral which crosses at several point the square. Indeed apart of some extreme case there should not be the four indicator above or below one. It it was the case it would probably indicate that the indicators and the benchmark are not well chosen. What we expect is that there is some tradeoff to do between the indicators and that when you try to improve one you will probably loose on another one. That is exactly what is shown next by running and plotting several scenarios.

Here are the summary of all our scenarios in terms of footprints and benchmarks:

Footprints

Scenario	Pop.,Demand, Import, Yield	Local Footprints Global(gha) F'^l	Real(ha) F^l	Sustainable(sha) F^{sl}	World Footprints Global(gha) F'^w
Base case	1000,1.4,300,15	55	73,33	110	15
Independent	1000,1.4,0,15	70	93,33	140	0
Sustainable	1000,1.4,300,15	55	110	110	15
Fair Share	1000,2,300,15	85	113,33	170	15
Surplus	1000,1.4,0,14	70	100	140	0

Performance Indicators

Scenario	Surplus $\frac{B^l}{F^l}$	Fair Share $\frac{B^{fs}}{(F^l+F'^w)}$	Independence $\frac{B'^l}{F'^l+F'^w}$	Sustainability $\frac{F^l}{F^s}$
Base case	1,36	1,43	0,79	0,67
Independent	1,07	1,43	1	0,67
Sustainable	0,91	1,43	0,79	1
Fair Share	0,88	1	0,85	0,67
Surplus	1	1,43	1	0,71

The first scenario is the base case we just presented above. From this base case we build other alternative scenarios. The first alternative consist to move toward more independence. So imports are set to 0 and replaced by local production. As expected it generates more local footprint and there is almost no more surplus of local biocapacity (1,07 instead of 1,36) whereas the independence is reestablished to one as it was our objective. We observe here a first possible trade off between surplus and independence which is not really surprising. The next alternative scenario is to move back to sustainability by reducing the yield. Again it has as a consequence to reduce the surplus. In our case this one is below one meaning in fact that we are consuming stocks of biocapacity. In the third scenario the consumption per capita is raised so that the local territory consumes its fair share of

biocapacity. Since there several possibilities , raising import, raising yield or lowering surplus it is the third case which is shown here. The last alternative scenario is to use entirely the local biocapacity (set surplus to one) and diminishing import. We can plot all these different scenarios in one single plot:
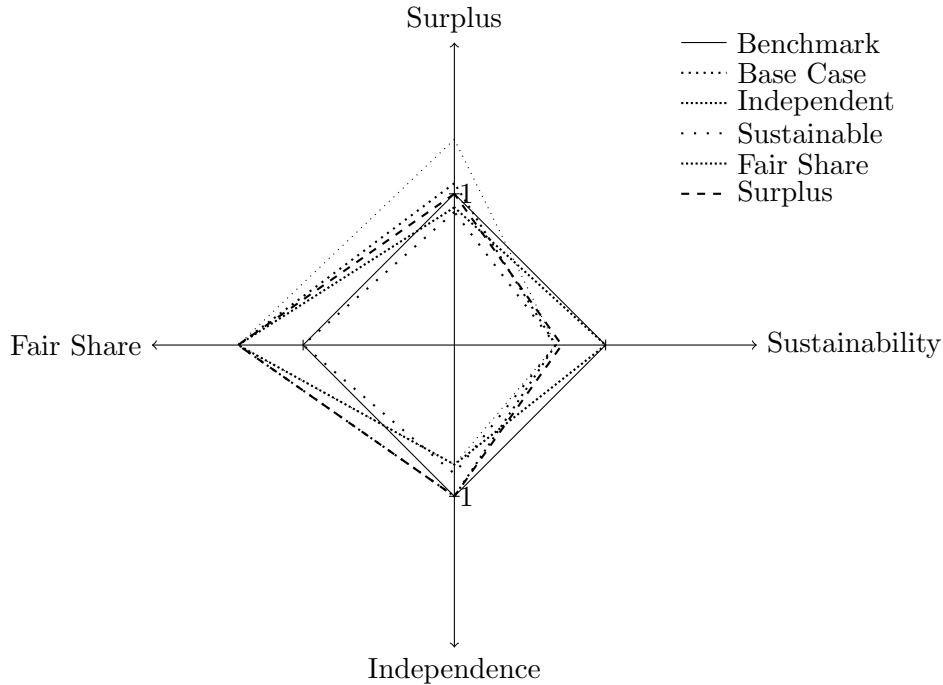
3.2   Two independent industries

In this example there is a second economic sector, called industry. Whereas in he previous example biocapacity and footprint were analyzed globally, it is possible now to calculate them for each sector separately. Note that the tonnes associated with industry are not the tonnes of products but the tonnes of C02 emitted as by process of production of industrial goods.

As above the first thing to do is to compute the biocapacity of our economy, the difference with above being that we can break up the biocapacity per economic sector. Let's recall that the assumption was made that each sector has its own biocapacity. In this particular case the biocapacity associated with agriculture is arable land and for industry it is forest land. The biocapacity of the world is the same as above so we do not present it here again.

Biocapacity

	Local Biocap (ha) B	Sustainable Yield(t/?ha/an)	Pot. Production (t/an)	Global Biocap. (gha) B'	Population	Biocap Fair Share (gha)
Local	B^l	y(t/sha/an)		B'^l	$P^l =2000$	B'^{fs}
Agriculture	50	10	500	25	-	100
Industry	50	5	250	12,5	-	100
Total	100	-	-	37,5	-	200

The main difference with the previous example is that each economic sectors has its own performance. Let's build a scenario which presents such discrepancy. There will be imports for agricultural products whereas industry products are entirely locally produced.

Local Population	2000
C per cap(t/an) Agri.	0,5
C per cap(t/an) Ind.	0,25
Import Agriculture	500
Import Industry	0
Yield Agriculture	8
Yield Industry	4

As above the three different footprints are computed with their respective yields. We present only the results here, and not the detail of calculation:

Footprints

	Global(gha)	Real(ha)	Sustainable(sha)	Import(gha)
	F'^l	F^l	F^{sl}	F'^w
Agriculture	25	62,5	50	25
Industry	50	125	100	0
Total	75	187,5	150	25

From which the ecological indicators are calculated:

Performance Indicators

	Surplus	Fair Share	Independence	Sustainability
	$\frac{B^l}{F^l}$	$\frac{B^{fs}}{(F^l+F'^w)}$	$\frac{B'^l}{F'^l+F'^w}$	$\frac{F^l}{F^s}$
Agriculture	1	2	1	0,5
Industry	0,5	2	1	1
Total	0,67	2	1	0,75

The performance of agriculture and industry are different. Whereas agriculture is not sustainable but do not use stock of biocapacity it is the opposite for industry which uses stock of biocapacity (in this particular case this means concentration of CO2 increasing in the atmosphere). This point to different policy responses depending of the economic sector.

It is possible to plot these three different result on the same plot:
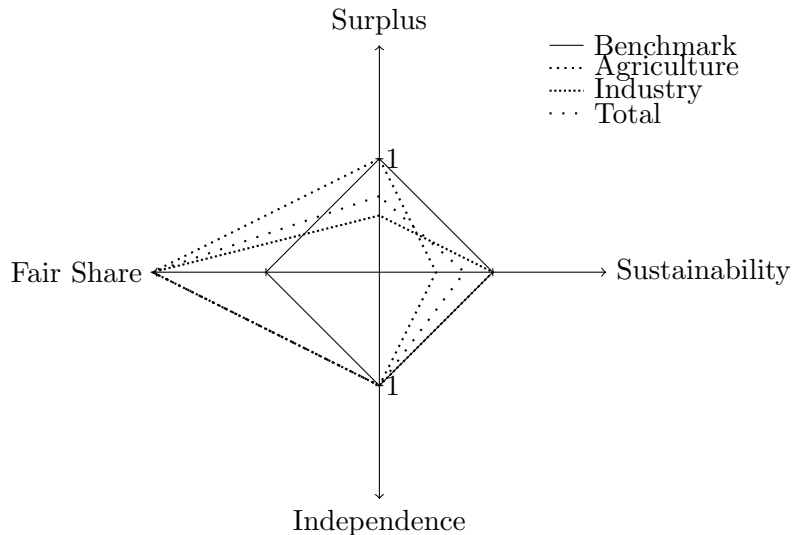
3.3 Two industries interdependent in one region

In this example we introduce an interdependency between the two sectors of our economy. This means that the production of one sector is not entirely used by final demand but also by the other economic

sector as a production output. This relation between sector can be described in technical coefficient matrix. Suppose that in our economy we have this matrix:

Technical Coefficients Matrix

From \ To	Intermediate consumption (t/an) Agriculture	(t/an) Industry	Available for Final Demand **f**	Total Output **x**
Agriculture	0,25	0,4	0,35	1
Industry	0,14	0,12	0,74	1

In the left column we have the economic sectors as producers and in row the same sectors are seen as industrial consumer. Hence we read in this table that agriculture use 0,25 units of its own output and 0,4 units of industry output to produce one unit. The industry uses 0,12 of its own output to produce one unit. Here the main difference between the previous example is that there is no more equality between the final demand and the corresponding production. And moreover consumption of agricultural product induces industry production. This new relations involves new calculation step to compute production values from final demand. For those familiar with input-output techniques it just involve some matrix inverse calculation. These are described in the appendix. Let's give it the result of this calculation and comment on them:

Requirement matrix $= (\mathbf{I} - \mathbf{A})^{-1}$

From \ To	Industrial consumption (t output/t demand/an) Agriculture	Industry	Final Demand
Agriculture	1,46	0,66	1
Industry	0,23	1,24	1

The reading of this table requires some caution as there is a correspondence between lines and columns. The first column correspond to the output of each sector necessary for a final demand of one unit of sector one products. The second column correspond to the output of each sector necessary for a final demand of one unit of sector two products. Hence a final demand of one unit of agricultural product requires the production of 1,46 unit of its own output and 0,66 from industry. And the final demand of one unit of industry products requires the production of 1,24 unit of its own output and 0,23 units from agriculture. Now if we consider the following scenario:

Scenario (Pop.,Demand, Import, Yield)

Local Population	1000
C per cap(t/an) Agri.	0,3
C per cap(t/an) Ind.	0
Import Agriculture	0
Import Industry	0
Yield Agriculture	8
Yield Industry	4

There is no consumption of industrial product but as we can be expect there will be indirect production. Let's calculate the production values. It is obtained by multiplying the requirement matrix by the final demand.

$$\text{Requirement table} = \begin{array}{ccc} 1,46 & 0,66 & 0,3 \\ 0,23 & 1,24 & 0 \end{array}$$

This gives:

$$\text{Requirement Table} = (\mathbf{I} - \mathbf{A})^{-1}(\widehat{\mathbf{f} - \mathbf{i}})$$

From \ To	Effective Production (t/an) Agriculture	(t/an) Industry	Final Demand (t/an) f	Total Output x	Import
Agriculture	437,09	0	300	437,09	0
Industry	69,54	0	0	69,54	0

The first steps of this calculation did not involve biocapacity nor footprint data. It is just an intermediate step to compute production from demand. However this new relations involves a different look at the footprint. Indeed in this framework the economic sectors has two faces as producing sectors in row and as consuming sectors in column. So the industry sector produce both for products of the industry sector but also indirectly for agricultural products. Now to the question what is the footprint of this sector there is now two possible answers depending if it is seen as producer or as consumer. Hence we introduce two different footprints for each sectors which we call production footprint and consumption footprints. Let's compute them to understand clearly what they means. As usual we start with the biocapacity:

Biocapacity

	Local Biocap (ha) B	Sustainable Yield(t/?ha/an)	Pot. Production (t/an)	Global Biocap. (gha) B'	Population	Biocap Fair Share (gha)
Local	B^l	y(t/sha/an)		B'^l	1000	B'^{fs}
Agriculture	50	10	500	25	-	50
Industry	50	5	250	12,5	-	50
Total	100	-	-	37,5	-	100

Then let's compute the global footprint:

Global Footprint Requirement Table $= \hat{\mathbf{y}}'^{-1}(\mathbf{I} - \mathbf{A})^{-1}\hat{\mathbf{f}}$ — **Global Footprint**

Global Yield y'(t/gha/an)	From \ To	Industrial consumption Agriculture	Industry	Production Footprint (gha) F'^l	Consumption Local Footprint	Import Footprint(gha)
20	Agriculture	21,85	0	21,85	28,81	0
10	Industry	6,95	0	6,95	0	0
	Cons. Footprint	28,81	0	28,81	28,81	0

As we had done before the transformation was done through the given yields applied to the requirement matrix. This table has the same structure it was just multiplied in rows by the yield. The novelty is that now we have for each sector a column and a row so the sum can be done in rows or in column. It so happens that they correspond to the production footprint and the consommation footprint we described above. Summing in column we obtain the consumption footprint and summing in row we obtain the production footprint. Note that this distinction disappear as soon as we look at the local economy as a whole, then the two are equal. The results shows that industry has a consumption footprint of 0 as expected but a production footprint positive. Indeed its productive capacity were used to provide the sector agriculture. That is the main point of this section to show this indirect effects. In this simple example where agriculture do not produce C02 directly but produce it indirectly through its use of the industry sector. This framework allows to measure this. The same calculations are done in actual hectares and sustainable hectares. Leaving the details out this gives:

Footprints

	Production Footprints Global(gha) F'^l_p	Real(ha) F^l_p	Sustainable(sha) F'^s_p	Consumption Footprints Global(gha) F'^l_c	Real(ha) F^l_c	Sustainable(sha) F'^s_c	Import Global(gha) F'^w
Agriculture	21,85	54,64	43,71	28,81	72,02	57,62	0
Industry	6,95	17,38	13,91	0	0	0	0
Total	28,81	72,02	57,62	28,81	72,02	57,62	0

The particularity of this table is the subdivision of footprint in two parts production and consumption.

Performance Indicators

	Surplus $\frac{B^l}{F^l}$	Fair Share $\frac{B^{fs}}{(F^l+F'^w)}$	Independence $\frac{B'^l}{F'^l+F'^w}$	Sustainability $\frac{F^l}{F^s}$
Agriculture	0,92	2,29	1,14	1,25
Industry	2,88	7,19	1,8	1,25
Total	1,39	3,47	1,3	1,25
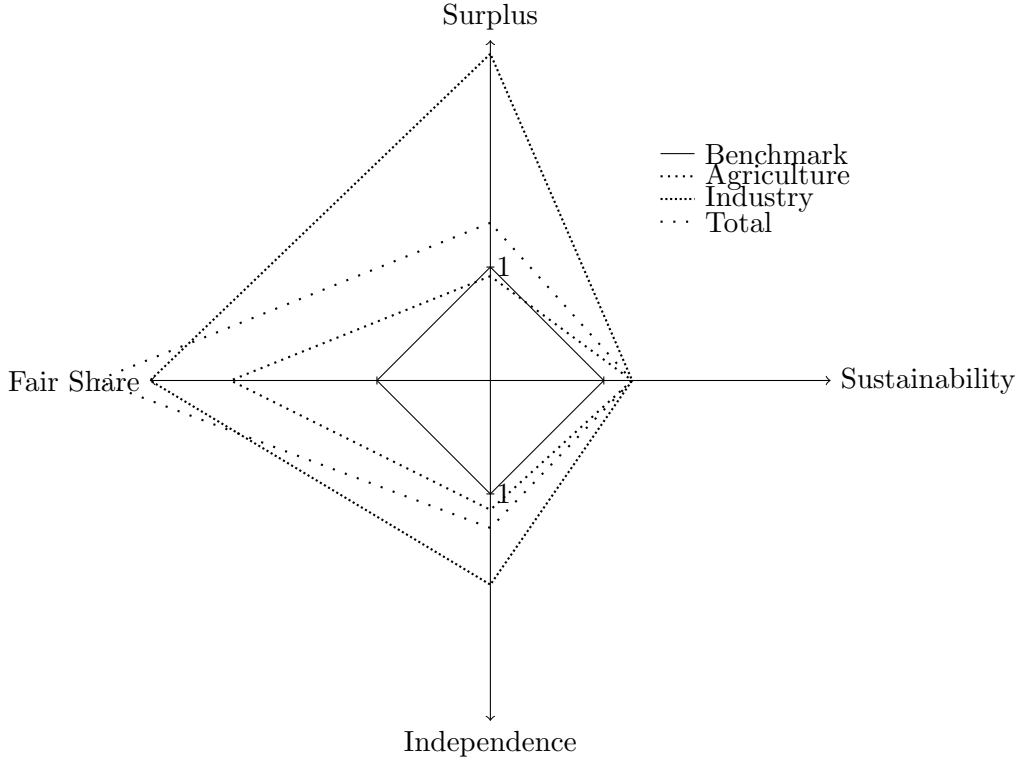
4 Discussion and Conclusion

The framework presented in this paper allows the representation of sustainability in a four dimensional view (surplus, global weight, sustainability, independence) calculated from local ecological footprint accounts. The presented methodology is highly stylistic and has only been tested on simple numerical examples. The next step is to test it on empirical study of a city or region.

Among the evolution the framework might integrate additional indicators. Notably the needs of population are not taken in account here. Hence there is the possibility of a starving population with very goods ecological indicators all above one.

More reflexion needs to be put on the proper scaling of the indicators. First which value should be the benchmark. Here it is 1 but it could be 0 with the possibility of negative values. Another idea would be to linearize the indicators, meaning that influence of different factors could be isolated and added to each other. Hence in our example the ecological performance of the local territory would be the addition of the agriculture performance and industry performance.

The disagregation of performance per economic sector is only sketched in this paper and need more reflexion. The distinction of consumer footprint and producer footprint seems crucial to allow this disagregation.

The indicator on sustainability is probably weak as yield is probably not the most relevant issue. Another type of weighting might be more adapted.

A   Analytic Framework to compute Footprint derived Indicators

In this basic model a local economy is embedded in the world economy and some exchanges occurs with the rest of the world(ROW).
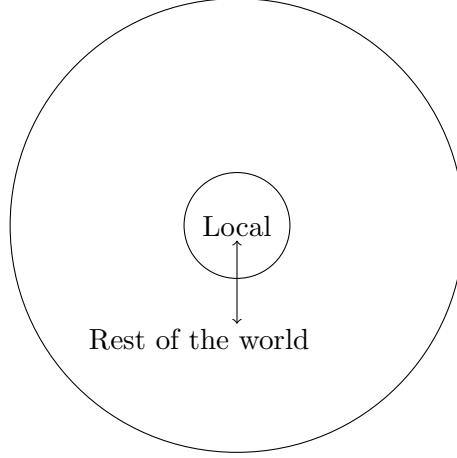
The economy is divided in n sectors and as a simplification each sector has its own biocapacity which can be 0 if no bioproductive ressources are involved in the production process. The first step is to compute the various biocapacity:

Biocapacity Calculation

	Real Biocap. (ha)	Sustainable Yield(t/ha/an)	Global Biocap. (gha)	Population	Biocap. Fair Share (gha)
Local					
Sector 1	B_1^l	y_1	$B_1'^l$	-	$B_1'^{fs} = \frac{B_1^w * P^l}{P^w}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sector n	B_n^l	y_n	$B_n'^l$	-	$B_n'^{fs} = \frac{B_n^w * P^l}{P^w}$
Total	B^l	y	B'^l	P^l	$B'^{fs} = \frac{B^w * P^l}{P^w}$
World					
Sector 1	B_1^w	y_1'	-	-	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sector n	B_n^w	y_n'	-	-	-
Total	B^w	y'	-	P^w	

The first column is the actual biocapacity which is just the area devoted to the economic sectors of the local territory and the world. The global biocapacity is equal to the surface land with world productivity which would produce the same amount as our local territory with current yield. The biocapacity fair share is the total world biocapacity divided by the world population multiplied by the local population. In a very equalitarian point of view this is what the local territory could claim for its population if the world biocapacity was equally allocated to each individual.

x_i^l is the total production of local sector i. z_{ij}^l and z_{ij}^w are respectively the amount of industrial transaction between local sector i as provider and local or ROW sector j as purchaser. f_i^l and f_i^w are respectively local and ROW final demand with export and import netted out. The basic relation between production, final consumption and intermediate consumption is:

$$\underbrace{x_i^l}_{\substack{\text{Local} \\ \text{Production}}} = \underbrace{z_{i1}^{ll} + ... + z_{in}^{ll}}_{\substack{\text{Local} \\ \text{Intermediate} \\ \text{Consumption}}} + \underbrace{z_{i1}^{lw} + ... + z_{in}^{lw}}_{\substack{\text{ROW} \\ \text{Intermediate} \\ \text{Consumption}}} + \underbrace{f_i^l}_{\substack{\text{Local} \\ \text{Consumption}}}$$

Symmetrically for rest of the world sector i:

$$\underbrace{x_i^w}_{\substack{\text{ROW} \\ \text{Production}}} = \underbrace{z_{i1}^{wl} + ... + z_{in}^{wl}}_{\substack{\text{Local} \\ \text{Intermediate} \\ \text{Consumption}}} + \underbrace{z_{i1}^{ww} + ... + z_{in}^{ww}}_{\substack{\text{ROW} \\ \text{Intermediate} \\ \text{Consumption}}} + \underbrace{f_i^w}_{\substack{\text{ROW} \\ \text{Consumption}}}$$

Which can be presented in an accounting table. The left part of this table is called the input-output table.

Input-Output Table

		Buying sectors						Final demand	Total Production
		Local			Rest of the World				
Selling sectors		1	$\ldots$	n	1	$\ldots$	n		
Local	1	z_{11}^{ll}	$\ldots$	z_{1n}^{ll}	z_{11}^{lw}	$\ldots$	z_{1i}^{lw}	f_1^l	x_1^l
		$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
	n	z_{n1}^{ll}	$\ldots$	z_{nn}^{ll}	z_{n1}^{lw}	$\ldots$	z_{ni}^{lw}	f_n^l	x_n^l
Rest of the	1	z_{11}^{wl}	$\ldots$	z_{1n}^{wl}	z_{11}^{ww}	$\ldots$	z_{1i}^{ww}	f_1^w	x_1^w
world(ROW)		$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
	n	z_{n1}^{wr}	$\ldots$	z_{nn}^{wr}	z_{n1}^{ww}	$\ldots$	z_{nn}^{ww}	f_n^w	x_n^w

So in matrix notation $\begin{bmatrix} \mathbf{x^l} \\ \mathbf{x^w} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}^{ll} & \mathbf{Z}^{lw} \\ \mathbf{Z}^{wl} & \mathbf{Z}^{ww} \end{bmatrix} \mathbf{i} + \begin{bmatrix} \mathbf{f^l} \\ \mathbf{f^w} \end{bmatrix}$ where $\mathbf{i}$ is the unity column vector.

From the input-output table the technical coefficient $a_{ij} = \frac{z_{ij}}{x_j}$ are calculated. In matrix notation:

$$\mathbf{A} = [a_{ij}] = \mathbf{Z}\hat{\mathbf{x}}^{-1}$$

where the $\hat{}$ is used to denote a diagonalized vector: $\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \rightarrow \hat{\mathbf{x}} = \begin{bmatrix} x_1 & \cdots & 0 \\ \vdots & x_i & \vdots \\ 0 & \ldots & x_n \end{bmatrix}$.

$\mathbf{A}$ is a partitioned matrix $\mathbf{A} = \begin{bmatrix} \mathbf{A}^{ll} & \mathbf{A}^{lw} \\ \mathbf{A}^{wl} & \mathbf{A}^{ww} \end{bmatrix}$

with $\mathbf{A}^{ll} = \begin{bmatrix} a_{11}^{ll} & \ldots & a_{1n}^{ll} \\ \vdots & \ddots & \vdots \\ a_{n1}^{ll} & \ldots & a_{nn}^{ll} \end{bmatrix}$, $\mathbf{A}^{lw} = \begin{bmatrix} a_{11}^{lw} & \ldots & a_{1n}^{lw} \\ \vdots & \ddots & \vdots \\ a_{n1}^{lw} & \ldots & a_{nn}^{lw} \end{bmatrix}$, $\mathbf{A}^{wl} = \begin{bmatrix} a_{11}^{wl} & \ldots & a_{1n}^{wl} \\ \vdots & \ddots & \vdots \\ a_{n1}^{wl} & \ldots & a_{nn}^{wl} \end{bmatrix}$ and $\mathbf{A}^{wl} = \begin{bmatrix} a_{11}^{ww} & \ldots & a_{1n}^{ww} \\ \vdots & \ddots & \vdots \\ a_{n1}^{ww} & \ldots & a_{nn}^{ww} \end{bmatrix}$

The basic input output relations in the inter-regional context are:

$$\begin{bmatrix} \mathbf{x}^l \\ \mathbf{x}^w \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{ll} & \mathbf{A}^{lw} \\ \mathbf{A}^{wl} & \mathbf{A}^{ww} \end{bmatrix} \begin{bmatrix} \mathbf{x}^l \\ \mathbf{x}^w \end{bmatrix} + \begin{bmatrix} \mathbf{f}^l \\ \mathbf{f}^w \end{bmatrix}$$

Or

$$\begin{bmatrix} \mathbf{I} - \mathbf{A}^{ll} & \mathbf{A}^{lw} \\ \mathbf{A}^{wl} & \mathbf{I} - \mathbf{A}^{ww} \end{bmatrix} \begin{bmatrix} \mathbf{x}^l \\ \mathbf{x}^w \end{bmatrix} = \begin{bmatrix} \mathbf{f}^l \\ \mathbf{f}^w \end{bmatrix}$$

Which inverted gives :

$$\begin{bmatrix} \mathbf{x}^l \\ \mathbf{x}^w \end{bmatrix} = \begin{bmatrix} \mathbf{I} - \mathbf{A}^{ll} & \mathbf{A}^{lw} \\ \mathbf{A}^{wl} & \mathbf{I} - \mathbf{A}^{ww} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{f}^l \\ \mathbf{f}^w \end{bmatrix}$$

where $\mathbf{L} = \begin{bmatrix} \mathbf{L}^{ll} & \mathbf{L}^{lw} \\ \mathbf{L}^{wl} & \mathbf{L}^{ww} \end{bmatrix} = \begin{bmatrix} \mathbf{I} - \mathbf{A}^{ll} & \mathbf{A}^{lw} \\ \mathbf{A}^{wl} & \mathbf{I} - \mathbf{A}^{ww} \end{bmatrix}^{-1}$ is the Leontief Inverse. It is important to be aware $\mathbf{L}^{ll}$, $\mathbf{L}^{lw}$, $\mathbf{L}^{wl}$, $\mathbf{L}^{ww}$ are not the respective inverse of $\mathbf{A}^{ll}$, $\mathbf{A}^{lw}$, $\mathbf{A}^{wl}$, $\mathbf{A}^{ww}$.

Indeed for example:

$$\mathbf{L}^{ll} = ((\mathbf{I} - \mathbf{A}^{ll}) - \mathbf{A}^{lw}(\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{wl})^{-1}$$

As there are feedbacks terms which need to be taken in account in an input output inter-regional framework.

Since only local consumption is of interest to us we can lower the dimension of the problem. $\mathbf{f}^w$ is set to 0. The basic relation becomes

$$\begin{bmatrix} \mathbf{x}^l \\ \mathbf{x}^w \end{bmatrix} = \begin{bmatrix} L^{ll} \\ L^{wl} \end{bmatrix} \begin{bmatrix} \mathbf{f}^l \end{bmatrix}$$

The l_{ij} are the direct multipliers of the input-output framework. The notion of multipliers is an integral part of the input-output framework. They are based on the difference between the initial effect of an exogenous change et the total effect of this change. We modify our basic relation to take in account the relation between changes instead of absolute values. This gives

$$\Delta \begin{bmatrix} \mathbf{x}^l \\ \mathbf{x}^w \end{bmatrix} = \begin{bmatrix} L^{ll} \\ L^{wl} \end{bmatrix} \Delta \begin{bmatrix} \mathbf{f}^l \end{bmatrix}$$

If in Δf we set Δf_i^l to 1 and all other components to 0 (we note this $\Delta \mathbf{f^l}(i)$), we obtain $\Delta \mathbf{x}(i)$ which represents production of each sector necessary to provide one unity of production i. If we do the sum of all the $\Delta \mathbf{x_i}$ we obtain the first multipliers $m(o)_i^{ll} = i'\Delta x^l(i)$ and $m(o)_i^{wl} = i'\Delta x^w(i)$. It is not difficult to show that $m(o)_i^{ll} = \sum_{j=1}^n l_{ji}^l$ and $m(o)_i^{wl} = \sum_{j=1}^n l_{ji}^w$ There exist one multiplier for each sector, we group them in one vector:

$$\mathbf{m}(o)^{ll} = [m(o)_1^{ll}, \cdots, m(o)_n^{ll}] = \underbrace{\mathbf{i'} \overbrace{\mathbf{L}^{ll}}^{\substack{ratio \\ demand-per-sector \\ output-per-sector}}}_{\substack{ratio \\ demand-per-sector \\ Output-for-entire-economy}} \tag{1}$$

And similarly $\mathbf{m}(o)^{wl} = [m(o)_1^{wl}, \cdots, m(o)_n^{wl}] = \mathbf{i'L}^{wl}$

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Most multipliers are built on this model. We will modify the multiplier so that instead of the relation $(\Delta f_j^l = 1) \to (\Delta x_i)$ we have $(\Delta f_j^l = 1) \to$ (a function of Δx_i).

In the case of ecological footprint where in its most basic expression production is divided by a yield to obtain a footprint. With $\mathbf{y} = \begin{bmatrix} \mathbf{y^l} \\ \mathbf{y^w} \end{bmatrix}$ the vector of yield with $\mathbf{y}^l = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ and $\mathbf{y}^w = \begin{bmatrix} y_1^w \\ \vdots \\ y_n^w \end{bmatrix}$. The footprint multiplier will become:

$$\mathbf{m}(F) = \begin{bmatrix} \mathbf{m}(F)^{ll} \\ \mathbf{m}(F)^{wl} \end{bmatrix} = \begin{bmatrix} \mathbf{i}'(\hat{y^l})^{-1}\mathbf{L}^{ll} \\ \mathbf{i}'(\hat{y^w})^{-1}\mathbf{L}^{wl} \end{bmatrix} \text{ where } (\hat{\mathbf{y}}^l)^{-1}\mathbf{L}^{ll} = \begin{bmatrix} \frac{l_{11}^{ll}}{y_1} & \cdots & \frac{l_{1n}^{ll}}{y_1} \\ \vdots & \ddots & \vdots \\ \frac{l_{n1}^{ll}}{y_n} & \cdots & \frac{l_{nn}^{ll}}{y_n} \end{bmatrix} \text{ and } (\hat{\mathbf{y}}^w)^{-1}\mathbf{L}^{wl} = \begin{bmatrix} \frac{l_{11}^{wl}}{y_1^w} & \cdots & \frac{l_{1n}^{wl}}{y_1^w} \\ \vdots & \ddots & \vdots \\ \frac{l_{n1}^{wl}}{y_n^w} & \cdots & \frac{l_{nn}^{wl}}{y_n^w} \end{bmatrix}$$

Hence total footprint is:

$$F = \mathbf{m}(F)\hat{\mathbf{f}}^l = \begin{bmatrix} \mathbf{m}(F)^l \\ \mathbf{m}(F)^w \end{bmatrix} \hat{\mathbf{f}}^l = \begin{bmatrix} \mathbf{i}'(\hat{\mathbf{y}}^l)^{-1}\mathbf{L}^{ll} \\ \mathbf{i}'(\hat{\mathbf{y}}^w)^{-1}\mathbf{L}^{wl} \end{bmatrix} \hat{\mathbf{f}}^l = \begin{bmatrix} \mathbf{i}'\mathbf{F}^l \\ \mathbf{i}'\mathbf{F}^w \end{bmatrix}$$

$$\text{With } \mathbf{F}^l = \begin{bmatrix} F_{11}^l & \cdots & F_{1n}^l \\ \vdots & \ddots & \vdots \\ F_{n1}^l & \cdots & F_{nn}^l \end{bmatrix} \text{ and } \mathbf{F}^w = \begin{bmatrix} F_{11}^w & \cdots & F_{1n}^w \\ \vdots & \ddots & \vdots \\ F_{n1}^w & \cdots & F_{nn}^w \end{bmatrix}$$

F_{ij}^l reads footprint generated by the production of sector i required to satisfy f_j the demand of products from sector j. If you do the sum in line you obtain the production footprint of sector i so satisfy demand for products of all sector. If the sum is done in column, it is the consumption footprint caused by f_i in all production sectors. Since the consumption footprint is our main concern it is better to look at the transpose at the matrix to make the reading easier.

The calculation was done for three different type of yield, global, actual, sustainable. The footprint for the rest of the world can only be calculated in global hectares as we no information on the actual and sustainable yield. The superscript $'$ and ^{s} are used to designate global and sustainable footprints.

| | Demand | Ecological Footprint of producing sectors | | | | | |
| | | Local | | | Rest of the World | | |
		Sector 1$(_{1.})$	Sector n$(_{n.})$	Total	Sector 1	Sector n	Total
Footprint in Global hectares							
Sector 1$(._1)$	f_1	F'^l_{11}	F'^l_{n1}	$F'^l_{.1}$	F^w_{11}	F^w_{n1}	$F^w_{.1}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Sector n$(._n)$	f_n	F'^l_{1n}	F'^l_{nn}	$F'^l_{.n}$	F^w_{1n}	F^w_{nn}	$F^w_{.n}$
Total	$f = \sum f_i$	$F'^l_{1.}$	$F'^l_{n.}$	F'^l	$F^w_{1.}$	$F^w_{n.}$	F^w
Footprint in Real hectares							
Sector 1$(._1)$	f_1	F^l_{11}	F^l_{n1}	$F^l_{.1}$			
⋮	⋮	⋮	⋮	⋮			
Sector n$(._n)$	f_n	F^l_{1n}	F^l_{nn}	$F^l_{.n}$			
Total	$f = \sum f_i$	$F^l_{1.}$	$F^l_{n.}$	F^l			
Footprint in Sustainable hectares							
Sector 1$(._1)$	f_1	F^{sl}_{11}	F^{sl}_{n1}	$F^{sl}_{.1}$			
⋮	⋮	⋮	⋮	⋮			
Sector n$(._n)$	f_n	F^{sl}_{1n}	F^{sl}_{nn}	$F^{sl}_{.n}$			
Total	$f = \sum f_i$	$F^{sl}_{1.}$	$F^{sl}_{n.}$	F^{sl}			

This three type of footprints values and the biocapacity values are used to calculate some performance indicators:

Performance Indicators

	Surplus	Fair Share	Independence	Sustainability
Sector 1	$\dfrac{B^l_1}{F^l_{1.}}$	$\dfrac{B^{fs}_1}{(F^l_{1.}+F'^w_{1.})}$	$\dfrac{B'^l_1}{F'^l_{1.}+F'^w_{1.}}$	$\dfrac{F^l_1}{F^s_{1.}}$
	⋮	⋮	⋮	⋮
Sector n	$\dfrac{B^l_n}{F^l_{n.}}$	$\dfrac{B^{fs}_n}{(F^l_{n.}+F'^w_{n.})}$	$\dfrac{B'^l_n}{F'^l_{n.}+F'^w_{n.}}$	$\dfrac{F^l_{n.}}{F^s_{n.}}$
Total	$\dfrac{B^l}{F^l}$	$\dfrac{B^{fs}}{(F^l+F'^w)}$	$\dfrac{B'^l}{F'^l+F'^w}$	$\dfrac{F^l}{F^s}$

References

G. Billen, Sabine Barles, J. Garnier, Joséphine Rouillard, and P. Benoit. The food-print of paris : long term reconstruction of the nitrogen flows imported into the city from its rural hinterland. *Regional Environmental Change*, 9:13–24, 2008.

Manfred Lenzen and S.A. Murray. A modified ecological footprint method and its application to Australia. *Ecological Economics*, 37, 2001.

Manfred Lenzen and S.A. Murray. The ecological footprint - issues and trends. *ISA-Research Paper 01-03*, 2003. Site internet, http://www.isa.org.usyd.edu.au/publications/documents/Ecological Footprint Issues and Trends.pdf.

Manfred Lenzen, Carina Borgstrom Hansson, and Stuart Bond. On the bioproductivity and land-disturbance metrics of the Ecological Footprint. *Ecological Economics*, 61, 2007.

D. H. Meadows, J. Randers, D. L. Meadows, and W. W. Behrens. *The Limits to growth.* Universe Books, 1972.

J. Munksgaard and K.A Pedersen. C02 accounts for open economies: producer or consumer respons-
ability. *Energy Policy*, 2001.

William Rees and Mathis Wackernagel. Urban ecological footprints: why cities cannot be sustainable-
and why they are a key to sustainability. *Environmental Impact Assessment Review*, 16,4-6:223–248,
1996.

P. M. Vitousek, P. R. Ehrlich, A. H. Ehrlich, and P. A. Matson. Human appropriation of the products
of photosynthesis. *BioScience*, 36(6):363–373, 1986.

Mathis Wackernagel and William Rees. *Our Ecological Footprint: Reducing Human Impact on the
Earth*. New Society Publishers, Gabriola Island, BC, 1996.

Thomas Wiedmann and Manfred Lenzen. On the conversion between local and global hectares in
ecological footprint analysis. *Ecological Economics*, 60, 2006.