Conventions in geometry and pragmatic reconstruction in Poincaré : a problematic reception in logical empiricism

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Les Preprints de la MSH Lorraine

N° 13
(2010)

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Axe 4, Dialogues, Rationalités, Formalismes (Diarafor)

Workshop : *Logic and rational Reconstruction of Science from a Pragmatic point of view*, 25-26 mars 2010, MSH Lorraine, Nancy Université

http://www.msh-lorraine.fr/fileadmin/images/ppmshl13-2010-06-axe4-heinzmann.pdf

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0. — The **main thesis** of this paper is that Poincaré conventionalism must be ranked among these sources of the logical empiricists, that seem to be at the same time a source of Quine's criticism of the two dogmas of the logical empiricism and survives consequently Quine's criticism. However, the ambition to locate logical empiricism in the inheritance of Poincaré does not go without saying:

There exist two traditions of interpreting Poincaré's work: one, which endorses his intuitionist tendency and at the same time his polemics against logicism or formalism[1], and the other, which opts for the conventional and linguistic aspects of his work. Given the fact that Poincaré didn't give a systematic presentation of his philosophy, one has to introduce interpretative hypotheses in order to reconstruct Poincaré's philosophy. My own interpretation is based on **the interpretative hypothesis**, that Poincaré's most important philosophical work is his article *On the Foundations of Geometry*, an important paper of 43 pages, written in English and published in the Monist of October 1898. Accordingly both usual interpretations of Poincaré are not only partial but also obstruct the understanding of Poincaré's thought. Poincaré is not sometimes Kantian, sometimes conventionalist, sometimes predicativist. He always defends, is my **interpretative presupposition**, the same philosophy, i.e. a pragmatic reconstruction of the process of understanding scientific theories where the construction of scientific objects is simultaneously conceived with the construction of language, or more exactly, where the empirical basis is the *occasion* of the process of language learning [cf. Heinzmann (2010)]. To call Poincaré's philosophy "conventionalism", actually turns out to be a misnomer, if it signifies, as normally, just the rational consent with respect of the object of convention and a liberty of mind. Jules Vuillemin (1970, 12) has understood this correctly and uses the term 'occasionalism', suggesting other philosophical connotations. In order not to lay myself open to additional problems of a terminological nature, I shall abide by the term 'convention'.

Instead to focus on conventions in mechanics and physics, I want draw the attention on epistemological questions raised at least by Carnap and Schlick in the context of their reading of Poincaré. In this context, I incline to follow Suzanne Wright's judgment and quote her thesis as my **final claim**:

"However reluctantly, Poincaré prepared the way for later anti-inductivist philosophies of science, for which theories are not seen as induced in slow stages from phenomena but immediately as whole conceptual systems, invented in a pragmatic

1 The conjecture that Wittgenstein could have been influenced by Poincaré in his criticism of the actual infinite or of set theory, for example, when he refuses in Tractatus [4.1273] the definition numbers with the aid of the hereditary property, is among the standard results of the Wittgenstein-interpretation [cf. Glock (1996), 265]. In literature, Poincaré and Wittgenstein even are often linked under the common features of intuitionism. And yet, neither Wittgenstein nor Poincaré are intuitionists [cf. Heinzmann/Nabonnand 2008].

procedure which is irreducible to a combination of clearly distinguished parts of observation and logic” [cf. Wright (1976), 295].

On the surface, at any rate, this interpretation is not evident, beneath the surface, however, things are clearer if one studies his reconstruction of geometry where the pragmatic elements ring together both mentioned interpretations of Poincaré’s work, represented by geometry and arithmetic[2].

The path I intended to follow is this: first to give an overview of some connections between Poincaré’s conventionalism in geometry and its “extension” by logical empiricists; secondly, to present a reconstruction of Poincaré’s “conventionalism” in geometry and to evaluate in these lights his position with respect to logical empiricism.

I. — Conventionalism was a main topic in the Vienna Circle, the Polish group (Ajdukiewicz) and even in Louis Rougier’s work. Papers of Poincaré, Duhem and Enriques were read and discussed in the Circles and there is not only a direct exchange between Duhem and Mach but also an indirect exchange between Mach and Poincaré about the genesis of geometry[3]. Philippe Frank planned so early as 1907, to establish a synthesis of Mach’s "economic descriptions of observed facts" and Poincaré’s " *free* creations from human mind", which, together, he thought "was at the origin of what was later called logical empiricism". The manifesto of the Vienna circle (1929) confirms these systematic and historical relations by echoing Poincaré’s conventionalism [Soulez (1985), p. 112]. Rougier and Frank underline again these French relations in their inauguration addresses of the Congress Descartes (Paris1935): Poincaré exercised a big influence on the groups of Vienna and of Prague where doctrines of Bergson, Meyerson and Boutroux were rejected (perhaps too quickly in respect with Boutroux). Louis Rougier was one of the rare *French* philosophers who did not limit the *positive* reception of Poincaré’s work to its

2 In arithmetic, Poincaré uses neither hypotheses nor conventions. Nevertheless, I have argued in [Heinzmann (2009)] that the elaboration of *apparent hypotheses* in geometry will be analogous to the “genesis” of the *synthetic a priori* induction principle.

3 Ernst Mach differentiated physiological and geometric properties of space since the first edition of his *The Analysis of Sensations*, appeared in 1886 under title *Beiträge zur Analyse der Empfindungen*. The physiological properties are for him the starting point of any geometry in the sense that they give "probably the first motivation to undertake geometric researches" [Mach (1886), 87, 99]. Nine years later, Poincaré introduces in his paper *L'espace et la géométrie* ([Poincaré (1891); 1902 modified reproduction in chapter IV of [Poincaré (SH)]) the same distinction which will be at the root of its 1898 essay *Foundations of Geometry* [Poincaré (FG]. In 1905, Mach’s *Erkenntnis und Irrtum* appears. This book includes three chapters on the genesis of geometry (chap. 20, 21, 22) among which the both last are an explicit resumption of articles appeared also in the Monist in 1902 and 1903 [cf. Mach (1905), 353, 389, notes 1]. On the advice of Mach himself the translator of *Erkenntnis and Irrtum* abolishes, in 1908, these two chapters in the French version, because they made "almost double job with what Mr Poincaré wrote on the question" [cf. Paty (1993), 247, notes 4]. On the contrary, chapter 20, entitled *Der physiologische Raum im Gegensatz zum metrischen* is translated. Its topic is the object of an article published in 1901, always in *The Monist*, and the first note of which explains us why it is maintained: "This article, which rests on researches begun almost forty years ago, are at variance with the views on sensible space that Prof. Poincaré advanced in the first part of his paper, "On the Foundations of Geometry", *The Monist*, Vol. IX. page 1 et seq. But my disagreement on this point has no bearing on the subsequent discussions of Professor Poincaré." [Mach (1901), 321]. See even below note 6.

psychologistic and intuitionistic part but linked conventionalism and logic. In his 1913 paper entitled “Poincaré et la mort des vérités necessaries” [Rougier (1913)] he uses, as do later Schlick and [Carnap (1966), 144] Poincaré’s famous Flatland model of the Lobachevskian geometry by supposing a world endowed with a peculiar temperature field in order to deduce the conventional character of physical geometry with respect to contingent circumstances of our milieu (i.e. suggested by experience [Rougier (1920)]. Further, he sees this conventional character increased for Hilbert-type formalism of all sorts. This extension of Poincaré’s geometrical conventionalism to physics and the largely unhistorical interpretation was common to much of the logical empiricists, especially to Schlick and Carnap. However, Michael Friedmann is right that one becomes no insights in Poincaré’s conventionalism if its main point is seen in Carnap’s and Schlick’s arguments: both deduce from observational equivalence of Euclidean and non-Euclidean theories of space the conventional character of theories [Friedman (1996), 333; Carnap (1966), 151]. What is nevertheless correct is that Poincaré’s analysis contributed to an “awareness of the conventional decisions entering into observation, generalization, experiment and the formulation of theoretical propositions” [Wright (1976), 295]. I will pursue this line of argument.

Before doing so, I should add, that it is obvious and was never contested that Poincaré himself was strongly influenced by and integrated in a philosophical movement related to a mixture of empiricism and Neo-Kantism [Marie Jo Nye (1979)]. According to Poincaré, mathematics requires intuition not only in the context of discovery but also in the context of justification, especially in arithmetic and logic. So it is not surprising that Poincaré was even introduced in German philosophical circles by the Kantian tradition: examples are Schlick, Riehl and his follow Ilse Schneider. However, the arithmetical “pure intuition” Poincaré introduces is intellectual in character and Poincaré does not at all solve the problem of the unity of spontaneity and receptivity by the introduction of a pure sensibility. Rather, he changes the terms of the Kantian opposition: what is important for him is the balance between exactness and objectivity; the latter concerns a consensus with respect to natural relations. He expresses the lost balance in a formula well known by its popularization under Einstein’s pen: “what they [mathematics] have gained in rigor, they have lost in objectivity. It is by distancing themselves from reality that they acquired this perfect purity” [Poincaré (SM), p. 131]. This is why Poincaré didn’t confine himself with the perfect purity. Alberto Coffa noted that Schlick’s quandary sounds very analogous: “Explicit definition from given primitives gives representations that are linked to reality, but it can guarantee no more intersubjectivity than is available in its starting point. Since its starting point always consists of the subjective target of ostension (singular representatiçn), it preserves the link with the reality at the price to give a solution [to the problem how to explain the rigorous character of scientific knowledge]. In contrast, implicite definition achieves sharpness, but the price is a complete lack of relation to the world” [Coffa (1991), 176].

Now, today the Kantian heritage of Logical empiricism is even widely known. As for neo-Kantian and Poincaré, the Kantian a priori-synthetic concept has given rise to *aporiae* which logical empiricisme meant to overcome by postulating what Quine called the two dogmas: the dogma of the separation of observational and theoretical language and the dogma of the possible reduction of empirical meaning to experience. It is equally known that, among those who were considered as “forerunners” of the logical empiricists, Duhem distinguishes himself by the fact that he seems to be at the same time a forerunner of Quine's criticism. The Duhem-Quine holistic thesis concerns the calling in question of the separation of theoretical language from observational language. It is less known that Poincaré too must be ranked among these forerunners who survive Quine's criticism; not primarily for his holistic thesis but for his conception of geometrical conventions as a kind of bicephalous selection of analytical but non-logical propositions, "guided" at the same time by experience. It has but rarely been attempted to ascribe to the phrase "guided by experience" another meaning than that of a metaphor and it is probably not evident to do so without having first read the second Wittgenstein. Nevertheless, Carus’ remark that Poincaré’s conventions in geometry were “rather analytic, but not quite freely stipulated, rather dictated or tightly constrained, in an almost Wittgensteinian way” [Carus (2007), 117] deserve some precision. I will do this in the next chapter.

There are at least three reasons for misunderstanding Poincaré's conventionalistic standpoint[4]:

(a) the current association of observational equivalence and conventionalism, which is often combined by
(b) the tendency to associate holism with conventionalism.
(c) the identification of conventions with implicit definitions.

The arguments of David Stump and Michael Friedman, concluding that Poincaré’s conventionalism does not at all *reduce* to Duhem’s thesis, are quite correct: they mentioned the first of a threefold possible argument:

- If conventionalism in geometry is nothing more than holism, the specific role that has, according to Poincaré, geometry with respect to other sciences could not be understood. Poincaré wishes to distinguish a sense in which geometry is

4 Friedrich [Stadler (2007), 18, 19] conjectures that Poincaré’s conventionalism helped to bridge a rift between modern empiricism and symbolic logic. Indeed, he doesn’t comment if his conjecture contradicts or not Warren Goldfarb's claim that Poincaré and the logicians (and especially the logicists) are trying to achieve different objectives. Goldfarb finds that even Poincaré's "more serious antilogicist arguments" reveal a continued dependence on a psychologistic conception" [Goldfarb (1988), 64]. So, at first sight, it doesn’t seem easy to connect symbolic logic to Poincaré’s conventionalism. Actually, I think that Goldfarb’s argument needs some corrections. However these corrections do not allow returning directly to Stadler’s thesis. Nevertheless, it is right that Poincaré introduced in mathematical reasoning Mach’s principle of the economy of thought; but I don’t know if this fact is sufficient to say that he bridged the rift between empiricism and Logic.
Some years ago, Georg Kreisel asked me if the wittgensteinienne sentence, used by Kreisel as title of an article: "der unheilvolle Einbruch der Logik in die Mathematik" ([Wittgenstein (1956), 145]; [Kreisel (1976)]) was not finally a quotation of Poincaré. No, it is not a quotation of Poincaré: for him difficulties met in logic concerned at first sight not mathematics and he had a deep understanding of Cantor’s “Punktmannigfaltigkeiten” since he was involved in translation in French of this memo published in Acta mathematica. Still more, when he was, in 1885, secretary of the mathematical Society of France, he grabbed opportunity to offer Cantor as member of this society [Newsletter 13, 87, Gray (1991), 6]. I conjecture that the expression "der unheilvolle Einbruch der Logik in die Mathematik" is rather an adaptation of Wittgenstein to a sentence of Frege in his review of Husserl’s Philosophy of arithmetics (1894, 332) where Frege regrets ravagages caused by the "Einbruch der Psychologie in die Logik". But, it is true, Russell and others reproach Poincaré for merging psychology and logic. Again, I think things are not so simple.

conventional from the sense of 'conventional' on which all our theories are conventional. This problem was central in Grünbaum's Poincaré-interpretation.

- Holism can be accompanied by an empirical realism, which has absolutely nothing to do with Poincaré's approach. One can indeed maintain the dichotomy between experience and theory, between what is given and what is said, simply by observing that one has made a quantitative error: the adepts of such a position refuse to confront a singular hypothesis with the facts drawn from experience but deem this confrontation possible in relation with the totality of the pertinent theoretical propositions.
- Poincaré knows even conventions that do not only concern the choice between theories; rather they come into play as classifications with respect to the interpretation of the basic concepts of each theory. These classifications, as mediating elements between sensation and observation, have a character of decision-making. Perhaps the best way to see this is to follow Poincaré's distinction between the brute fact and the scientific fact with the help of a gradual differentiation which he inroduces in his book *La valeur de la science*. He broaches here both the problematic of protocol sentences and the problematic of the transition from common language to scientific language. Explained by means of an example, the terms "brute fact" and "scientific fact" stand, at a first level, respectively for "the [individual] impression of a [relative] darkness felt by the witness of an eclipse, and the statement: it is dark, which this impression draws from him" [Poincaré (VS), 157]. The latter statement presupposes, as a "convenient classification", a symbolic competence which represents the price of an objective articulation of nature. This sorte of convention are very different from Duhem's thesis.

The most important reason for misunderstanding Poincaré's approach is the ambiguous interpretation of logical empiricists that the propositionalist reading of geometrical axioms are inadequate. It may consist in attributing to Poincaré and Hilbert the shift in foundational purposes from what the axioms of geometry *say* to a syntactic feature of the axiomatic sentence [Coffa (1991), 309]. Even Alberto Coffa endorses this view and speaks from Hilbert's *and* Poincaré's revolution [cf. ibid.], echoed by Wittgenstein in his *Philosophische Bemerkungen* by "the axioms of Euclidean geometry are rules of syntax in disguise"[5]. Indeed, this is an one-sided interpretation doing not justice to Poincaré. Though the geometrical sentences, which correspond to the syntactic rules of the group theory, are exact, they require, according to Poincaré, also the objectivity of these rules. In fact, Poincaré doesn't speak of uninterpreted geometries. His conventionalism cannot be the result of a model-theoretical or semantic point of view where the physical geometry is just an interpretation of an abstract calculus. On the contrary, Poincaré does not even distinguish between mathematical and physical theory, a disturbing fact for many interpreters. According to Poincaré there exists only one single type of geometries, which is the result of a psycho-physiological genesis using theoretical and observational language as two different aspects of the constitution of the geometrical objects. In fact, since a long time ago, Hans Freudenthal has shown that these who interpret the geometric axioms in Poincaré as implicit definitions expose themselves

5 [Coffa (1995), 39] and [Wittgenstein (1964), 216]: "Die Axiome — z.B. — der euklidischen Geometrie sind verkappte Regeln einer Syntax. Das wird sehr klar, wenn man zusieht, was ihnen in der analytischen Geometrie entspricht".

to an anachronistic misunderstanding that wrongly considers Hilbert's views as an adequate instrument for the interpretation of Poincaré's book Science and Hypothesis. Even though this latter was not published until 1902, therefore three years after Hilbert's *The Foundations of Geometry*, it is in reality a collection that assembles articles written in the 1890s. Yet, in these articles, Poincaré does not anticipate Hilbert's view, according to which the axioms of a formal system are nothing other than the forms of statements and, for this reason, devoid of truth-values. It speaks for the critical understanding of Walter [Dubislav (1930/31)] that he doesn't mention Poincaré in his article "Wissenschaftstheorie der Geometrie" where he adopted a Hilbertian standpoint of geometry as an uninterpreted mathematical calculus.

Now, there are at most two subject area in which Poincaré's conventionalism is of some importance for the Logical Empiricists: the first was especially singled out by Friedman: Logical Empiricists "viewed the combination of Poincaré's geometrical conventionalism and Einstein's theory of relativity as a single unified whole." [Friedman (1996), 333]. Indeed, I will not speak about Poincaré's doctrine of Physical Space. My colleague Scott Walter at best clarified this part (see [Walter (2008) and (2009)]). Independently of the question if Emile Borel is right that the range and border of geometry as abstract science were definitely fixed by the criticism of Poincaré [Borel (1921), 417], Poincaré's considerations on geometry are in every case important for an already mentioned problem that I will now closer examine. It can be characterized by a twofold question:

1) Can two people *experience* (erleben) intersubjectively the same?
2) Can two people *know* intersubjectively the same?

The first question, so Michael [Heidelberger (1985), 152], concerns the intersubjective comparability of the experiences, the second the intersubjective transferability of knowledge. Now it's well known that for both, Carnap and Schlick, question (1) has no sense: individual experiences are incomparable [Carnap (1928), §66; Schlick (1926)]. But the solution of both, i.e. to consider structural properties of data is one of Poincaré's heritages and is recognized as such of both [Schlick (1925), 91ff et Carnap (1928), 16]. First, I quote Carnap's Aufbau, which seems much more vague than Schlick with respect to Poincaré: "Considerations similar to the preceding ones have sometimes led to the standpoint that not the given itself (viz., sensations), but 'only the relations between the sensations have an objective value' (Poincaré (Wert) 198). This obviously is a move in the right direction, but does not go far enough. From the relations, we must go on to the structures of relations if we want to reach totally formalized entities. Relations themselves, in their qualitative peculiarity, are not intersubjectively communicable." Poincaré's relations are in this quotation of Carnap not yet connected with conventions, as Schlick did already three years before: In the second edition of his *General Theory of Knowledge*, he added as a result of his discussions with Reichenbach about coordinative definitions a new section, § 11, entitled " *Definitions, Conventions, and Empirical Judgments."* In doing so, he introduced, as Friedman observes, conventions as a third type of definition between the axiomatic or implicit and concrete or ostensive ones. These conventions attributed to Poincaré "are crucial for an understanding of how we achieve a coordination between concepts and empirical reality in the mathematical exact sciences” [Friedman (2007), 100]. I quote Schlick: “To define a concept implicitly is to determine it by means of its relations to other concepts. But to apply such a concept to reality is to choose, out of the infinite wealth of relations in the world, a certain group or complex and to embrace as a unit by designating it with a name. By suitable choice it is always possible under certain circumstances to obtain an unambiguous designation of the real by means of the concept. Conceptual definitions and coordinations that come into being in this fashion we call conventions (using this term in the narrower sense, because in the broader sense, of course, all definition are agreements). It was Henri Poincaré who introduced the term Convention in this narrower sense into natural philosophy; and one of the most important tasks of that discipline is to investigate the nature and meaning of the various conventions found in natural science.” Now, in order to solve their twofold problem through the consideration of relations between “Erlebnisse”, Carnap has invented the process of “Intersubjektivierung” and Schlick the “process of elimination of qualities” [Heidelberger (1985), 153] by means of the consideration of structural properties. It was Poincaré who first linked the intersubjectivity and the elimination of qualities by a structural account: “What is objective must be common to many minds and consequently transmissible from one to the other, and as this transmission can only come about by the ‘discourse’ […] we are even forced to conclude: no discourse, no objectivity” [Poincaré (VS), 347/348] and further: “Sensations are intransmissible, or rather all that is pure quality in them is intransmissible and forever impenetrable. But it is not the same with relations between these sensations. From this point of view all that is objective is devoid of all qualities and is only pure relation […] Science, in other words, is a system of relations” [Poincaré (VS), 348/349]. Clearly, Poincaré, Schlick and Carnap felt that on the one hand pure empiricism is not enough and that on the other hand Kantianism has failed. In this sense, conventions are the successor of the synthetic *a priori*.

We shall now study

a) in which sense Poincaré’s version of the “structural” account of knowledge can be read as a conventional construction combining conceptual definitions and coordinations;
b) if he contributes something to the answer of our second question: Can two people know intersubjectively the same?

Poincaré’s methodological approach has roots in what he calls a semi-skeptical attitude, capable of addressing both the absolute doubt of rationalism (Descartes) and the prejudice of naive realism. Poincaré is even very conscious of Helmholtz’s difficulty in attempting to deduce from psycho-physiological observations hypotheses concerning geometry without succumbing to a vicious circle in defining the rigid body (see [Carrier (1994)] and [Heinzmann (2001)]. Geometrical empiricism, which Poincaré rejected as having no rational meaning [SH, 101(86)] was either the view that space has an intrinsic metric (this is Adolf Grünbaum’s interpretation of what Poincaré meant by geometrical empiricism) or the view that logic and experience (experiments) determine unambiguously i. e. without leaving any choice, theoretical conclusions including a unique theory of space. One of the most prominent opponents of Grünbaum’s Poincaré interpretation was Jersey Giedymin. With him (letter received in 1991) I question Grünbaum’s claim that Poincaré’s geometrical conventionalism was nothing but an epistemological elaboration of Riemann's views on the amorphousness of space.

Such a view ignores the following components of Poincaré's conventionalism:

a) the influence of his mathematical background, especially Charles Hermite and Sophus Lie; similarly the ideas of transformations groups, invariants relative to a group etc.
b) the view that reality is knowable only up to its relational structure
c) the interrelation of conventionality and invariance of the empirical content
d) the influence of Darwin's theory of biological evolution.

Like many philosophers at the end of the 19th century "who saw the main aim of philosophy more in the epistemological analysis of science than in the pursuit of metaphysics" [Giedymin (1991), 4], Poincaré emphasizes the genetic point of view. What distinguishes Poincaré from other authors is his systematic — but non-historical — interpretation of the genesis and his anti-psychologistic tenet in the sense that it differs from the attempt to understand how mental knowledge develops over time in individuals; resembling what the logical empiricists called a (re)construction program..

The external justification of beliefs and propositions has its origin in humain practice. Carnap and Hempel suggested this thesis, called empirical pragmatism,[6] and Poincaré even did so by limiting geometry on such of constant curvatures. Indeed, to pass from the external to the internal perspective means for Poincaré to justify the results of empirical pragmatism by pragmaticism. We will now have a look on Poincaré's understanding of the pragmatistic reconstruction program of geometry in order to understand what is the epistemological significance of Poincaré's geometric conventions.

II. — Geometry

Poincaré compares in his first article including philosophical reflexions "The fundamental hypotheses of geometry" (1887), the choice between Euclidean geometry and that of Lobatschetvski to the choice between coordinate systems and observes that the hypotheses in question are neither "experimental facts nor analytic or synthetic judgments a priori" but rather the result of a choice guided by observation of certain physical phenomena [Poincaré (1887), 90–91]. The choice is therefore not at all arbitrary, and he equally specifies that the adoption of this conception is starting "to become commonplace". Indeed, we observe conventionalist tendencies in the works, more or less contemporary, of Ernst Mach, mentioned in the manifesto of the Vienna Circle, and Emile Boutroux who seemed to be rejected by the logical empiricists. But conventionalistic tendencies that strongly resemble Poincaré's conventionalism in physics are even found already in Carl Gustav Jacob Jacobi's lectures on analytic mechanics held in 1847/48 [see [Pulte (2000)].

At a first approach Poincaré's geometrical conventionalism consists of three theses:

6 According to [Carnap, Rudolf (1968), *Introduction to Symbolic Logic*, New York : Dover Publ., 79] empirical pragmatics includes "historical, sociological and psychological relations within the language community".

1° Experience does not relate to space but to empirical bodies. Geometry deals with ideal bodies and so it can be neither proved nor disproved by experiences. Since the propositions of geometry cannot be also analytical there may be conventions.
2° The choice between conventions, particularly between different geometries, is guided by experience [SH, 75].
3° Euclidean geometry has nothing to fear from experience, for it is the most advantageous and convenient one [SH, 95].

Very often, the conventions of the first thesis are interpreted linguistically in the sense of the holism thesis, the second point is considered as a metaphor and the third point as a confirmation of Poincaré's conservative conventionalism. In order to see how this common interpretation of Poincaré's conventionalism stands on wooden legs, i.e. asks complementary commentaries, let us now pass to a more technical description of Poincaré's position.

There are two main steps of the construction of space:

- its psycho-physiological genesis (intervention of a natural hypothesis concerning the presupposition of sensible space and an artificial convention in the sense of a classification).
- its mathematical formulation (intervention of apparent hypotheses or disguised definitions).

First we must repeat that, in contrast to Hilbert, Poincaré ties his research into the foundations of geometry to the concept of a group, which, since 1880, has been considered as the invariant form of different geometries [Gray/Walter (1997), 76]. Geometry is nothing other than the study of a group. And "since the existence of a group is not incompatible with that of another group", we cannot say that one geometry is true and that another is false: the choice between different geometries is comparable to that of a coordinate system [Poincaré (1887), 90 ff.].

An action can be conceived as well in respect to its result as in respect to the action-process. Accordingly, the localisation of the object in a space could be interpreted as the result of a tactile, motor or visual action or as a reflexion on the action-process intending to reach an object. In both cases, objects are represented in relation to our activities and to our body, but only the last case does not presuppose a given form of sensibility. Reflexions on intention materialized by actions-sequences allow us to choose, so to speak, different geodesics [SH, 68]. Rejecting an *a priori* form of sensibility, Poincaré adopts, without saying, this approach: he conceives the localisation of an object as a *reflection* on the sequence of actions needed to reach this object, that is, *reflecting* on non spatial impressions of muscular *sequences* of a motor-process[7]. Representation of an object in the sensible space means nothing

7 [Poincaré (FG), 1]) presupposes explicitly in his construction the space-less character of single sensations. Qua construction (he uses only *imagined* sequences of sensations) it is not important for Poincaré's argument if sensations are really space-less or not. This is an important difference to Mach's approach. According to Mach, the properties of physiological space are adapted to biological conditions and "this natural and ingenuous view [the biological adaption] leads directly to the theory advanced by Prof. William James, according to which *every* sensation is in part spatial in character; [...] In support of his hypothesis James frequently refers to Hering. This conception is, in fact, almost universally accepted for optical, tactile and organic sensations. Many years ago, I myself characterized the relationship of tones of different pitch as spatial, or rather as analogous to spatial." [Mach (1901), 324].

else than the deliberate and conscious *reproduction* of muscular sensations *thought necessary* to reach the object [SH 56sq. (82)]:

> "Our representations are only the reproduction of our sensations. They can therefore only be arranged in the same framework — that is to say, representative space [....]. When it is said, [...] that we 'localize' an object in a point of space, what does it mean? It simply means that we represent to ourselves these movements that must take place to reach that object [...]. When I say that we represent to ourselves these movements, I mean only that we represent to ourselves the muscular sensations which accompany them" [SH 56 sq.; transl. G.H][8].

To classify these sensations representations, Poincaré introduces the essentially vague category of sensible space [Poincaré (FG), 7 ff.]: there is neither measure nor the possibility of speaking of constant axes with respect to our body, but thanks to it, we can compare sensations of the same kind and observe the proximity of two objects. In themselves, all the sensations are different, since they are accompanied, for example, by "various olfactory or auditory sensations" [Poincaré (DP), 142]. Their indistinguishability is a consequence of our abstractive classification. We maintain that the representative space is not formed by a classification starting from motor sensations, but on the contrary that it is the necessary condition for a classification of representations of motor sensations. It is a form of our understanding and not of our sensibility, since an individual sensation can exist without it [FG, 3].

Poincaré is conscious of the fact, that sensible reproduction can only be arranged in sensible space, and that we do not represent geometrical figures, but only *reason* about the sensation-representations thought to reach them [SH, 56]. How do we pass from sensible space to geometric space?[9]

The construction of geometric space now proceeds from the *observable* fact that a set of impressions can be modified in two distinct ways: on the one hand without our feeling muscular sensations, and on the other, by a voluntary motor action accompanied by muscular sensations. So, similarly to Carnap's Aufbau, the starting point is here the definition (guided by experience) of two two-place relations satisfying certain *minimal* empirical conditions: un *external chance* α (with '$x\ \alpha\ y$' for '*x changes in y **without** muscular sensation*') and an *internal change S* (with '$x\ S\ y$' for '*x changes in y **accompanied by** muscular sensations*').

This observation suggests a conventional classification of external changes: among external changes some can be compensated by an internal change, others cannot. The first are called **changes of position**, the second **changes of state** or **alterations.**

We have the following situation.

1) Changes of position := locomotions [Torretti (1978)]

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8 There is a serious error in the translation of the second sentence of this passage: the sentence "elles ne peuvent donc se ranger que dans le même cadre qu'elles, c'est à dire dans l'espace représentatif" [SH, 82, French edition] is translated as "They cannot therefore [!!!] be arranged [….]".

9 The essential properties of geometric space are: "1st, it is continuous; 2nd, it is infinite; 3rd, it is of three dimensions; 4th, it is homogeneous—that is to say, all its points are identical one with another, 5th, it is isotropic. Compare this now with the framework of our representations and sensations, which I may call representative space" [SH, 52]. In fact, "it is neither homogeneous nor isotropic; we cannot even say that it is three dimensional" [SH, 56].

2) Two internal changes have to be considered identical iff they have induced the same muscular sensations [VS 79 : trop voisins l'un de l'autre].

3)

$$\left.\begin{array}{l} \alpha, \beta \ \text{external} \\ \hline S \ \ \text{internal} \end{array}\right\} \text{changes}$$

$$\alpha \sim \beta \quad \text{iff} \quad \exists S \ (S\alpha \doteq S\beta \doteq I)$$

that means : two external changes are equivalent because they possess a common character (i.e. to be canceled by S), and in spite of their not possessing it exactly.

4) $S \approx S' \ \text{iff} \ \exists \alpha \ (\alpha S \doteq \alpha S' \doteq I)$

5) If ~, ≈ are equivalent relations, then the equivalence class of the locomotions is a *displacement*. So we can recognize that two displacements are identical.

We have to mention that Poincaré often uses the same word "displacement" as well for the equivalence class as for its elements (locomotions).

6) ~ and ≈ are not transitive.

$$\alpha \sim \beta \ \text{iff} \ \exists S \ (S\alpha \doteq S\beta \doteq I)$$

$$\beta \sim \gamma \ \text{iff} \ \exists S' \ (S'\beta \doteq S'\gamma \doteq I)$$

$$\alpha \sim \gamma \ \text{iff} \ \exists Q \ (Q\alpha \doteq Q\gamma \doteq I)$$

7) in VS 51 (72; 1903), Poincaré gives the following additional law:

$$\alpha \neq \beta \qquad \left.\begin{array}{l} S\alpha \doteq I \\ S\beta \doteq I \\ S'\beta \doteq I \end{array}\right\} \Rightarrow \ S'\alpha \doteq I$$

This law gives us the transitivity. What is the character of this law? Poincaré asserts that it is an empirical fact : "If it were not verified, at least approximately, there would be no geometry, there would be no space" [VS 72]. [Torretti (1978), 343] has pointed out that Poincaré is here mistaken in his explanation. He gives the following example:

external change	internal change
□	S (step of 2 yards)
□	S' (step of 2 yards k inches)
□	S" (step of 2 yards 2k inches)

I may be that

$$S\alpha \doteq I$$
$$S\beta \doteq I$$
$$S''\beta \doteq I \quad \text{but} \quad S''\alpha \neq I$$

This is surely correct, but it is not a refutation of Poincaré, because Poincaré is not an empiricist: experience can only teach "that the compensation has approximately been effected", it gives only the "mind's […] occasion to perform this operation", but "the classification is not a crude datum of experience" [FG, 9]. Poincaré is very precise about the vagueness having in mind at this level of reconstruction:

"When experience teaches us that a certain phenomenon does not correspond *at all* to these laws, we strike it from the list of displacements. When it teaches us that a certain change obeys them only approximately, we consider the change, *by an artificial convention*, as the resultant of two other component changes. The first is regarded as a displacement *rigorously* satisfying the laws [...], while the second component, which is small, is regarded as a qualitative alteration" [FG, 11].

Poincaré is also well aware that the validity of the group properties in question does not result from an a priori condition, but that it seems on the contrary exposed to the danger of an empirical refutation. And yet, "geometry is sheltered from any revision" [FG, 19]. To solve this paradox, Poincaré introduces some conventions, beginning with first steps of his construction: in effect, changes of position can never be exactly realized since, in general, the starting position does not exactly coincide with the ending position. When the observation of our sense-representations, provoked by an internal change, does not correspond to the expected compensation, it is either struck out or replaced by a new "artificial convention".

Suppose we have succeeded in abstracting pure notions from sensation-representations by building by conventions equivalence classes called :

external $D_\alpha = \{ x | \alpha \sim x \}$ and

internal $D_S = \{ y | S \approx y \}$ displacements,

then Poincaré's main result is that each set of displacement classes (external & internal) forms a group in the mathematical sense.

Poincaré goes on to generate a continuous group of transformations G and to distinguish among the displacements belonging to groups isomorphic to G (among which some may operate on simpler material than the representative space) those that conserve certain sensation representations: these are the subgroups of rotations. By taking these subgroups into considerations, he obtains a characterization of groups that correspond to geometries of constant curvature. Among these groups, he finally chooses the one that allows "the assertion of the existence of an invariant subgroup all of whose displacements are interchangeable and which is formed of all the translations" [FG, 34]. In other words, he chooses the group that corresponds to Euclidean geometry, because these subgroups are best suggested by experience. In principle, we could have fixed another convention.

I do not want to describe any more this process in detail — in fact it is circular[10] — but draw rather some philosophical conclusions. Like the category of representative

10 I owe the following remark to my colleague Philippe Nabonnand: to get groups allowing to choose between different geometries with constant curvature, Poincaré presupposes in an essential way

space, the general concept of a group is a form of our understanding. Thus, the genesis of Euclidean geometry is based on an epistemological process founded on previous classifications, carried out according to the norms of invariance and conventions. It follows that the axiom of Euclidean distance is not *exclusively* a conventional definition in the linguistic sense. This form of convention where exists a choice between different possibilities only becomes involved at the last stage.

It is for this reason that the claimed axiom of Euclidean distance is not a definition by convention in the proper sense: it is a *disguised definition* or an *apparent hypothesis*.

In fact, Poincaré uses the term ‘disguised definition’ already before 1899, the year of Hilbert's famous *Foundations of Geometry*, to express his belief that a language used in an apparently descriptive manner is not descriptive. The logical genesis of Euclidean distance outlined above generates the objectivity of a fact, which is only apparently described by an axiom. A disguised definition determines its object at the end of a very complex procedure of interdependence between theory and experience. Therefore a disguised definition, although opposed to an ordinary explicit definition, is truly explicit. First and foremost, however, it contradicts an intuitive view of facts. In other words, Poincaré’s conventions in the sense of apparent hypotheses could be called *disguised* definitions for two reasons: they are neither proper definitions, nor proper descriptions. They are distinguished by a mixed form, as suggested by Roger Pouivet (1995), who finds a community of spirit between Goodman’s reflective equilibrium principle and Poincaré’s conventions. We are very far from Hilbert's axiomatic schemata. In view of their non-propositional character they was interpreted, from Louis [Rougier (1920)] on, as conventions in the sense of language rules such that ‘disguised definitions’ turn into *implicit definitions* in Hilbert’s sense.

Poincaré begins his construction only apparently with sensations as ostentive contacts with the given[11]. In reality, he takes a structural position without disengaging completely meaning and knowledge from ostension: he introduces, similarly to Helmholtz’s conception of intuition as imagined sensible impressions, a representation of a two-places sensation relation, based on the *imagination* of single sensations[12]. They are the (last) content of the categories (forms) of sensible space and of groups. Poincaré’s conventions are the tool to close the gap between the exactness of forms and the objectivity of relations of sensations based on an imagined ostensive contact (*reflecting on sensations*). If this interpretation is right, then Poincaré’s project has a strong affinity with Carnap’s *Aufbau* and Schlick’s *General Theory of Knowledge*. Nevertheless, his concept of structure is not the new Hilbertian deriving from his axiomatization of Euclidean geometry, but the traditional algebraic one and concerns continuous groups. With respect to this mathematical but

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Lie’s classification of transformation-groups operating on ….. R^3, what is circular: no space has to be presupposed. Nevertheless, Poincaré noted his mistake introduced in his subsequent work the amorphous continuum of *Analysis situs*, with the consequence that Geometry is not further independent of any mathematical space. May be that Grünbaum’s interpretation fits very well with this second solution whereas Giedymin’s interpretation has the first one in mind.

11 “Our sensation cannot give us the notion of space. That notion is built up by the mind from elements which preexist in it, and external experience is simply the occasion for its exercising this power” [Poincaré (FG), 1].

12 [Torretti (1978), 340] observes rightly: “geometrical space […] arises *with*, though certainly not *from* it [experience].”

non logical structure and the basic relations α and S, Poincaré succeeds to define the geometrical objects as *objective and exact* positions of a structure. But contrary to the structuralists, he can speak *about* the structure by adopting his psycho-physiological genesis of *real* actions with *imagined* sensations. In this sense geometry is as such a whole conceptual system, invented in a *pragmatistic* procedure, which is irreducible to a combination of clearly distinguished parts of observation and logic. This was exactly my announced *final claim*.

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