Environmental Policy and Public Debt Stabilization

Mouez Fodha, Thomas Seegmuller

GREQAM

**Groupement de Recherche en Economie
Quantitative d'Aix-Marseille - UMR-CNRS 6579
Ecole des Hautes Etudes en Sciences Sociales
Universités d'Aix-Marseille II et III**

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Thomas SEEGMULLER**

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Mouez Fodha

Centre d'Economie de la Sorbonne and Paris School of Economics.
University Panthéon-Sorbonne.
Maison des Sciences Economiques, 106-112 Bld de l'Hôpital,
75013 Paris Cedex, France.
Tel.: (33) 1 44 07 82 21. Fax.: (33) 1 44 07 82 31.
(fodha@univ-paris1.fr)

Thomas Seegmuller

CNRS and GREQAM.
Centre de la Vieille Charité,
2 rue de la Charité,
13236 Marseille Cedex 02, France.
Tel: (33) 4 91 14 07 89. Fax: (33) 4 91 90 02 27.
(thomas.seegmuller@univmed.fr.)

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Corresponding Author:

Mouez Fodha

Address: Centre d'Economie de la Sorbonne, University Paris 1 Panthéon-Sorbonne. Maison des Sciences Economiques, 106-112 Bld de l'Hôpital, 75013 Paris Cedex, France.

Telephone: +33 (0) 1 44 07 82 21.

Fax: +33 (0) 1 44 07 82 31.

Mailing address: fodha@univ-paris1.fr

Abstract

This article analyzes the consequences of environmental tax policy under public debt stabilization constraint. A public sector of pollution abatement is financed by a tax on pollutant emissions and/or by public debt. In the same time, households can also invest in private pollution abatement activities. We show that the economy may be characterized by an environmental-poverty trap if debt is too large or public abatement is not sufficiently efficient with respect to the private one. However, there exists a level of public abatement and debt for a stable steady state to be optimal.

1 Introduction

The growing environmental concerns have forced several countries to adapt their tax structure by introducing new taxes on pollutants. France, following some Scandinavian countries like Sweden, has planned to adopt a carbon tax on energy use in the next few years. The revenues of these green taxes are used to limit the economic distortions of the reform by reducing other taxes, or alternatively, are allocated to pollution abatement programs. In France, the main environmental protection agency (ADEME) is entirely financed[1] by revenues of taxes on pollutants, called General Tax on Polluting Activities. However, whatever the government's decision about distribution of the environmental tax revenues, public engagements in the environmental protection are often constrained by long-term fiscal objectives which impose to control public deficits and public debt evolution. The aim of this paper is to analyze the economic consequences of environmental policy under a debt stabilization constraint.

Alongside the rise of the public expenditures for environmental protection concerns, households have also massively increase their environmental spending. Households' environmental actions were initially limited to the purely personal, e.g. protection of homes against noise pollution, lawn and garden maintenance, individual wastewater purification of homes not connected to a collective sewage system. But these actions are nowadays widened to the entire environmental field: combating biodiversity loss, sorting and recycling waste, fighting air, water, and ground pollution. This increasing engagement of individuals in environmental protection corresponds to an increasing demand for more and more environmental requirements in the OECD countries. In France, over the period 1990-2003, households' spending for environmental protection has increased on average by 3.5% per year while public spending has increased by nearly 7%

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[1] Its budget is about 1% of the french GDP.

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per year. Thus, the growth of total expenditure in environmental protection is between 3% and 6%, and even approached 10% in the years 1993-1995. At the same time, GDP only grew by an annual average rate of 2%.[2]

Based on these observations, our objective is to study the consequences of environmental tax policy on capital accumulation, environmental quality and welfare, when households also invest in pollution abatement. Does the increase of the public environmental engagement lead to a green crowding out effect, penalizing hence private involvements in the environmental protection? When the environmental policy is characterized by environmental taxation and public pollution abatement sector, does it still respect the debt stabilization constraints? We extend the model developed by John and Pecchenino (1994)[3] to take into account public abatement. This paper has introduced pollution externalities in an overlapping generations model à la Diamond (1965), where the behavior of selfish individuals generates intergenerational inefficiencies and inequities.[4] Since we intend to study the interactions between voluntary commitments of individuals and government's intervention in environmental protection, we focus on an economy where households are involved in environmental maintenance, through a trade-off between polluting consumption and environmental quality, alongside public actions to protect the environment. We further assume that government spending is financed through a tax on polluting consumption on one side, and public debt on the other side. This means that a share of public environmental abatement is financed by future generations. Basically, we assume that generations who will benefit from the public environmental protection should pay for it. This assumption corresponds to a beneficiary-payer principle, enhancing the willingness to implement the environmental policy. Indeed, one of the results

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[2] See Ifen (2006).

[3] Ono and Maeda (2001) introduce uncertain lifetime in such a framework to study the consequences of distribution of ages among agents on the environmental quality.

[4] See Solow (1974, 1986) for seminal contributions.

of the previous literature is to show that environmental taxation implies such a welfare loss for present generations that its implementation cannot be wished: one of the generation who would decide it would also bear the heaviest burden. Finally, we also consider a debt stabilizing constraint that imposes a constant level of debt (per capita)[5] and allows us to treat debt as a policy parameter.[6]

Using this framework, we show that if public debt is low enough and public abatement sufficiently efficient, the economy monotonically converges to a long-run steady state. On the contrary, when public debt is not too low and efficiency of private maintenance is sufficiently high with respect to the public one, an environmental-poverty trap can emerge. Indeed, the capital stock may decrease because a large share of saving is devoted to maintain debt constant and households increase their labor income share dedicated to 'green' investment, through private abatement. This a priori negative effect of public policy is mitigated by the welfare analysis. Indeed, we show that the optimal stationary allocation can be decentralized choosing appropriately the two policy instruments, public abatement and debt. They both affect the consumption tax rate, which is endogenously determined by the intertemporal budget constraint of the government. However, government spending has also a direct impact on environmental quality as it corresponds to public abatement, while debt is linked to productive capital since it captures a share of saving.

Previous papers have analyzed the consequences of some environmental policy on environmental quality, growth and welfare. Nevertheless, in all these studies, public and private choices for abatement are always exclusive.[7] In particular, they have focused on the consequences of environmental taxes whose revenues are redistributed to households. In John et al. (1995), the optimal

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[5] There is no population growth.

[6] See Diamond (1965) for a seminal model without any environmental externalities.

[7] The distinction between households and (short-lived) governments is not precisely defined in this literature.

policy relies on a tax scheme composed of taxes on labor and capital incomes. In the same manner, Ono (1996) shows that the optimal allocation can also be achieved by a combination of taxes on consumption and capital income. In these papers, the authors develop the economic policy tools to reach the optimal capital stock and environmental quality simultaneously. The efficient policy rules have to influence the saving-abatement private arbitrage in that way. Therefore, the economy experiences a sustainable path. As in our article, we consider both private and public abatement technologies, we allow for interactions between these two environmental involvements, which may even generate an environmental-poverty trap. Nevertheless, public debt is considered as an economic policy tool which could help to reach economic efficiency. Therefore, in contrast to John et al. (1995) and Ono (1996), optimality is obtained with a unique tax rate. Another difference with our paper is the absence of public debt to finance pollution abatement sector. However, debt has already been introduced in dynamic models with environmental concerns (Bovenberg and Heijdra (1998), Heijdra, Kooiman and Ligthart (2006)), but these contributions focus on a different issue than ours. Instead of using debt to finance a share of public maintenance, debt policy makes possible to redistribute welfare gains from future to existing generations. In our model, the role of the public debt is twofold: as usual, it redistributes welfare among existing and future generations, but first of all, it also finances the public pollution abatement sector. Hence, the redistribution properties of the public debt are limited by the environmental engagement of the government.

In the next section, we present the model. The intertemporal equilibrium is defined in Section 3. The fourth section looks at the dynamics, while Section 5 is devoted to the long-run welfare analysis. The last section concludes. Several technical details are relegated to the Appendix.

2 A simple overlapping generations model

We consider an overlapping generations model with discrete time, $t = 0, 1, ..., +\infty$, and three types of agents: consumers, firms and a government.

2.1 Consumers

Consumers live for two periods and the population size of each generation is constant and normalized to one. Preferences of an household born at period t are represented by a simple log-linear utility function defined over future consumption c_{t+1} and environmental quality E_{t+1}:

$$ln(c_{t+1}) + \epsilon ln(E_{t+1}) \tag{1}$$

where $\epsilon \geq 0$ is a measure of the degree of "green" preferences.

At the first period of life, an household born at period t supplies inelastically one unit of labor, remunerated at the competitive real wage w_t, and shares his labor income between saving e_t, through available assets, and positive environmental abatement $m_t \geq 0$. At the second period of life, saving, remunerated at the real interest factor r_{t+1},[8] is used to consume the final good and pay consumption taxes, at a rate $\tau_{t+1} \geq 0$. Hence, a consumer faces the two following budget constraints:

$$w_t = e_t + m_t \tag{2}$$

$$(1 + \tau_{t+1})\, c_{t+1} = r_{t+1} e_t \tag{3}$$

We further assume that private consumption degrades environmental quality, while private environmental abatement and public spending, i.e. public environmental abatement, $G_t \geq 0$ can improve it. Assuming linear relationships, environmental quality follows the motion:

$$E_{t+1} = \gamma_1 m_t + \gamma_2 G_t - \alpha c_t \tag{4}$$

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[8] We assume complete depreciation of capital after one period of use. Therefore, r_{t+1} also denotes the real interest rate.

where $\alpha > 0$ represents the rate of pollution coming from private activities, while $\gamma_1 > 0$ and $\gamma_2 > 0$ are measures of the efficiency of private and public abatement, respectively. Note that, as soon as $\gamma_1 \neq \gamma_2$, private and public abatements have distinct effects on environmental quality. This representation of distinct techniques and/or of distinct productivities for the environmental protection[9] may be illustrated by many examples. Indeed, households spending is geared primarily toward curative actions (soundproofing, wastewater treatment, waste separation) while public spending is more preventive (air purity protection, protection of biodiversity, species conservation). Considering distinct consequences of public (γ_2) and private (γ_1) abatement may refer, for example, to the case of urban pollution and environmental performance of cities: without specific maintenance, the quality of the urban environment will degrade. Users would then derive less value from public parks, planted pathways, flower gardens, bicycle tracks, etc. Therefore, the municipality supports the preservation of the environment by replacing and maintaining trees, grass and flowers in public areas, while private agents intervene directly themselves (lower quantities of waste by collection and sorting, reduction of gaseous pollutants through investment in power saving appliances and fuel efficient cars).

Notice that $-E_{t+1}$ can be interpreted as pollution. Assuming that E_{t+1} does not depend on the current level of environmental quality E_t means that pollution is a flow or a stock with full regeneration after one period. Regarding the main pollutants, like sulphur dioxide, suspended particulate matter, carbon monoxide, and for some sorts of river pollution, although all these pollutants are stock pollutants, they all have short lifetimes and can therefore be considered as flow pollutants from a long-run point of view (IPCC (1996), Liu and Liptak (2000) and Lieb (2004)). In the atmosphere the lifetime of sulphur dioxide is

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[9]In the following, we do not distinguish between pollution abatement and protection or maintenance of the environment.

no more than four days and that of carbon monoxide is about three months. Suspended particulate matter is washed out by rain and thus has only a short lifetime. Since rivers are flowing, the concentrations of water pollutants would quickly decline if emissions stopped. So river pollutants are short-lived. Thus they can also be considered as flow pollutants. Since we consider an overlapping generation model with two-period lived agents i.e. the length of period is quite large, this does not seem to be too restrictive to consider that E_{t+1} does not depend on E_t.[10]

A consumer maximizes his utility function (1) under the constraints (2)-(4) and $m_t \geq 0$. One obtains:

$$\frac{r_{t+1}}{1 + \tau_{t+1}} E_{t+1} \geq \gamma_1 \epsilon c_{t+1}, \tag{5}$$

with equality when $m_t > 0$.

2.2 Firms

Taking into account that one unit of labor is inelastically supplied at each period, the production is given by $y_t = k_t^s$, where k_t indifferently denotes the capital stock or the capital-labor ratio, and $s \in (0, 1)$ the capital share in total income. From profit maximization, we get:

$$r_t = s k_t^{s-1} \equiv r(k_t) \tag{6}$$

$$w_t = (1 - s) k_t^s \equiv w(k_t) \tag{7}$$

2.3 Public sector

The aim of the government is to improve environmental quality, using public spending G_t to provide public environmental abatement. To finance these expenditures, as seen above, the government levies a tax on private consumption,

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[10]Seegmuller and Verchère (2007) use a similar assumption.

at the rate $\tau_t \geq 0$, or can use debt B_t. The intertemporal budget constraint of the government can be written:

$$B_t = r_t B_{t-1} + G_t - \tau_t c_t \tag{8}$$

with $B_{-1} \geq 0$ given.

In this paper, we focus on equilibria with constant debt or constant debt per capita, i.e. $B_t = B > 0$ for all $t \geq 0$.[11] This avoids explosive debt paths. This condition also ensures the long-term credibility of the environmental policy, and allows to use debt as an economic policy instrument. We further consider that public spending, which corresponds to the level of public abatement, is an exogenous instrument for environmental policy, i.e. $G_t = G \geq 0$ for all $t \geq 0$. Therefore, the budget constraint of the government (8) can be rewritten:

$$\tau_t \frac{r(k_t)e_{t-1}}{1 + \tau_t} = (r(k_t) - 1)B + G \tag{9}$$

3   Intertemporal equilibrium

In this paper, we are interested in the effect of public debt on the dynamics, but also on the respective roles of private versus public abatement. This explains that we focus on equilibria with strictly positive abatement $m_t > 0$. Besides, in contrast to many papers,[12] an explicit condition is derived below such that inequality $m_t > 0$ holds along the whole dynamic path. We note also that, as emphasized in the Introduction, positive private and public environmental abatements are supported by an empirical evidence.

Equilibrium on the asset market is ensured by:

$$e_t = k_{t+1} + B \tag{10}$$

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[11] See the seminal paper by Diamond (1965), which however ignores environmental issues.
[12] See for instance Ono (1996) or Zhang (1999).

Therefore, the budget constraint of the government (9) can be rewritten:

$$\tau_t \frac{r(k_t)(k_t + B)}{1 + \tau_t} = (r(k_t) - 1)B + G \tag{11}$$

It defines the consumption tax rate as a function of capital:

$$\tau_t = \frac{B(r(k_t) - 1) + G}{r(k_t)k_t + B - G} \equiv \tau(k_t) \tag{12}$$

We are interested in an economy where public debt is larger than public abatement:

Assumption 1 $B \geq G \geq 0$.

Hence, $\tau_t \geq 0$ is satisfied if $r(k_t) \geq 1 - G/B$. This requires $k_t \leq \overline{k}$, with:

$$\overline{k} \equiv r^{-1}(1 - G/B) = \left(\frac{sB}{B - G}\right)^{1/(1-s)} \tag{13}$$

Using (6) and (12), we notice that $\tau(k)$ is strictly decreasing $(\tau'(k) < 0)$, i.e. the tax rate is counter-cyclical.

Assuming $m_t > 0$, the consumer trade-off (5) writes:

$$E_{t+1} = \gamma_1 \epsilon(B + k_{t+1}) \tag{14}$$

Substituting this expression in (4) and using (2), (3), (6), (7) and (12), we obtain:

$$k_{t+1} = \frac{1}{\gamma_1(1 + \epsilon)}[(\gamma_1(1 - s) - \alpha s)k_t^s + X] \equiv H(k_t) \tag{15}$$

with

$$X \equiv (\alpha + \gamma_2)G - (\alpha + \gamma_1(1 + \epsilon))B \tag{16}$$

We notice now that, using (2) and (10), $m_t > 0$ is equivalent to $w(k_t) - B - k_{t+1} > 0$. This is satisfied if $(1 - s)k_t^s - B > H(k_t)$ holds, i.e. $k_t > \underline{k}$, with:

$$\underline{k} \equiv \left[\frac{(\alpha + \gamma_2)G - \alpha B}{\alpha s + \gamma_1 \epsilon(1 - s)}\right]^{1/s} \tag{17}$$

Under the following assumption:

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Assumption 2 $\gamma_2 < \frac{\alpha s + \gamma_1 \epsilon (1-s)}{G} \left(\frac{sB}{B-G} \right)^{s/(1-s)} + \alpha \frac{B-G}{G} \equiv \overline{\gamma}_2$

$\underline{k}$ is strictly lower than $\overline{k}$. This assumption simply indicates that for a sufficiently large level of public abatement productivity ($\gamma_2 \geqslant \overline{\gamma}_2$), private abatement falls to zero. In that case, there is obviously no need for private protection, the public one being highly efficient.

We are now able to define an equilibrium:

Definition 1 *Under Assumptions 1-2, an intertemporal equilibrium with strictly positive private abatement ($m_t > 0$) is a sequence $k_t \in (\underline{k}, \overline{k}]$, $t = 0, 1, ..., +\infty$, such that equation (15) is satisfied, given $k_0 \in (\underline{k}, \overline{k}]$.*

Therefore, the dynamics are driven by a one-dimensional dynamic equation, where k_t is a predetermined variable. Since $E_t = \gamma_1 \epsilon (B + k_t)$, this also determines the evolution of environmental quality.

4   Dynamics and environmental trap

We are now able to analyze the role of debt, public spending and the effectiveness of private and public environmental abatements on dynamics. By direct inspection of (15), we immediately see that two main cases may emerge, depending on the sign of $\alpha s - \gamma_1 (1-s)$. When $\alpha s > \gamma_1 (1-s)$, it is possible to show that, under Assumptions 1-2, there is no steady state belonging to $(\underline{k}, \overline{k}]$ and no persistent trajectory staying in this interval. Therefore, we exclude this case and we will focus on the configuration where:

Assumption 3 $\alpha s < \gamma_1 (1-s)$.

For further reference, since $H(0) = X/[\gamma_1(1+\epsilon)]$, it is useful to note that $X \geq 0$ if and only if $\gamma_2 \geq \gamma_{20}$, with:

$$\gamma_{20} \equiv (\alpha + \gamma_1(1+\epsilon))B/G - \alpha \tag{18}$$

and $X < 0$ otherwise, where $\gamma_{20} < \overline{\gamma}_2$ if B and G are not too far.[13] Since $\gamma_{20} > \gamma_1$, positive values of X requires $\gamma_2 > \gamma_1$. This means that if public and private abatements have an identical efficiency ($\gamma_2 = \gamma_1$) or public abatement is the less efficient ($\gamma_2 < \gamma_1$), we have $X < 0$.

The following proposition examines cases where the economy converges to a unique long-run steady state (see Fig. 1):

Proposition 1 *Let*

$$\widetilde{\gamma}_2 \equiv \frac{\alpha s + \gamma_1 \epsilon (1-s)}{G} \left(\frac{\gamma_1(1-s) - \alpha s}{\gamma_1(1+\epsilon)} \right)^{s/(1-s)} + \alpha \frac{B-G}{G}$$

Under Assumptions 1-3,

1. *When $\gamma_2 = \gamma_{20} < \widetilde{\gamma}_2$, there is one stable steady state given by $k_1 = \left[\frac{\gamma_1(1-s)-\alpha s}{\gamma_1(1+\epsilon)} \right]^{1/(1-s)}$ if $\frac{B-G}{B} < s\frac{\gamma_1(1+\epsilon)}{\gamma_1(1-s)-\alpha s}$;*

2. *When $\widetilde{\gamma}_2 > \gamma_2 > \gamma_{20}$, there is one stable steady state $k_1 > \left[\frac{\gamma_1(1-s)-\alpha s}{\gamma_1(1+\epsilon)} \right]^{1/(1-s)}$ if B and G are not too far.*

Note that $\widetilde{\gamma}_2 > \gamma_{20}$ requires a not too large B. Otherwise, k_1 becomes smaller than $\underline{k}$. In this case, k_t decreases until it reaches its lower bound $\underline{k}$. Finally, if B is too far from G, k_1 may be larger than $\overline{k}$. Then, k_t grows until it reaches its upper bound $\overline{k}$.

Proof. See the Appendix. ∎

This proposition shows that, when $X \geq 0$, there is at most one stable steady state. Obviously, this occurs without government intervention ($B = G = 0$).[14] This proposition shows that this is still relevant if public debt is not too large

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[13]Indeed, this requires $B \left(\frac{B-G}{B} \right)^{s/(1-s)} < \frac{\alpha s + \gamma_1(1-s)\epsilon}{\gamma_1(1+\epsilon)} s^{s/(1-s)}$.

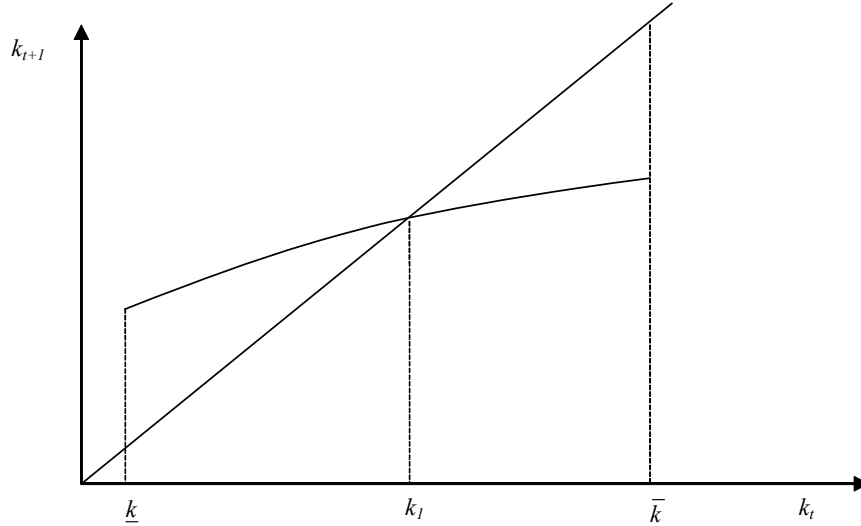[14]In this case, one converges to the steady state for all $k > 0$.

Figure 1: Illustration of Proposition 1.

with respect to public abatement or, as underlined above, public environmental abatement is sufficiently efficient with respect to the private one ($\gamma_2 > \gamma_1$).

Indeed, when the level of debt is not too great and private environmental abatement not too efficient, a large share of labor income is devoted to capital accumulation, which fosters convergence.

Proposition 1 is also useful to deduce the impact of a (slight) increase of debt or public abatement on the long run stable steady state. Since X is increasing in G, but decreasing in B, public abatement and debt have opposite effects on the stationary capital stock. The first one promotes capital accumulation, reducing private abatement in favor of productive saving. In contrast, the second one lowers capital at the steady state, because of a crowding out effect reducing the share of saving through capital.

When $X < 0$ ($\gamma_2 < \gamma_{20}$), two steady states, a stable one $k_1 > 0$ and an unstable one k_2 ($< k_1$), may coexist. However, when X (γ_2) is sufficiently close to 0 (γ_{20}), k_2 is lower than $\underline{k}$. In this case, Proposition 1 still applies. On the contrary, when X is negative enough or γ_2 sufficiently lower than γ_{20}, we show that an environmental trap may emerge. To examine this possibility, we assume that B and G satisfy the following inequality:

Assumption 4 $[\gamma_1(1-s)-\alpha s]^{s/(1-s)}[\alpha s+\gamma_1\epsilon(1-s)] > [\gamma_1(1+\epsilon)]^{1/(1-s)}s^{-s/(1-s)}B$ $-(1-s)[\gamma_1(1-s)-\alpha s]^{1/(1-s)}G.$

This allows us to show the following proposition (see Fig. 2):

Proposition 2 *Let*

$$\gamma_{2sn} \equiv [\alpha+\gamma_1(1+\epsilon)]\frac{B}{G} - \alpha - (1-s)[\gamma_1(1-s)-\alpha s]^{1/(1-s)}\left[\frac{s}{\gamma_1(1+\epsilon)}\right]^{s/(1-s)}$$

$$\widehat{\gamma}_2 \equiv \alpha\frac{B-G}{G} + \frac{\alpha s+\gamma_1\epsilon(1-s)}{G}\left[s\frac{\gamma_1(1-s)-\alpha s}{\gamma_1(1+\epsilon)}\right]^{s/(1-s)}$$

Suppose that Assumptions 1-4 are satisfied. When $\gamma_{2sn} < \gamma_2 < \widehat{\gamma}_2$ with γ_2 close enough to γ_{2sn}, there exists two steady states, one is unstable ($k_2 > \underline{k}$) and the other one is stable ($\overline{k} \geq k_1 > k_2$). A saddle-node bifurcation occurs for $\gamma_2 = \gamma_{2sn}$ ($k_2 = k_1$) and there is no steady state for $\gamma_2 < \gamma_{2sn}$. In this last case, $k_{t+1} = H(k_t) < k_t$ for all k_t, and k_t reaches the lower bound $\underline{k}$ after a finite number of periods.

Proof. See the Appendix. ∎

This proposition establishes that, for $\gamma_{2sn} < \gamma_2 < \widehat{\gamma}_2$, there is a poverty trap for all $\underline{k} < k_t < k_2$. Since $E_t = \gamma_1\epsilon(B + k_t)$, this also corresponds to an environmental trap, where environmental quality degrades. Furthermore, it is interesting to notice that since $X < 0$, Proposition 2 applies for $\gamma_2 \leq \gamma_1$, i.e.
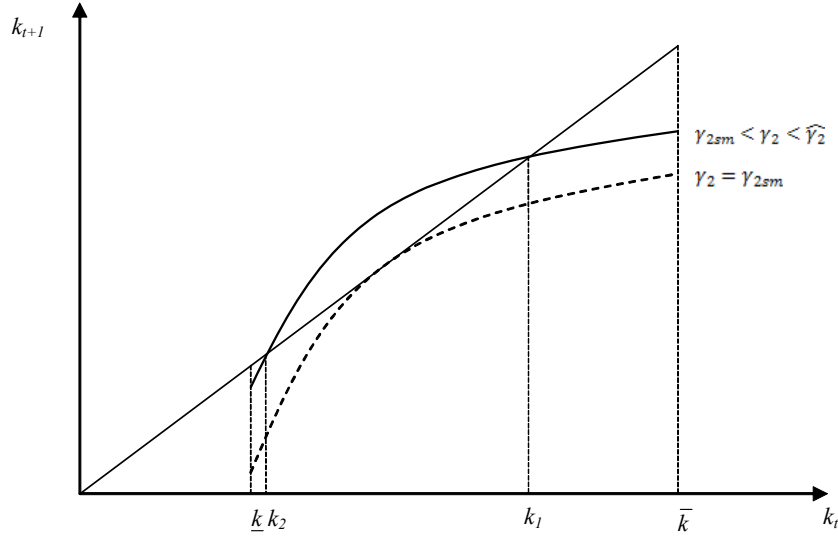
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Figure 2: Illustration of Proposition 2.

public and private abatements have the same effectiveness or public abatement is the less efficient. Notice also that inequality $X < 0$ is strengthened by a high level of public debt.

If public debt is sufficiently large, a low share of income is devoted to productive saving. In this case, capital accumulation may decrease. This effect is reinforced by a great efficiency of private environmental abatement, because in this case, households invest a large part of income in private abatement. However, environmental quality degrades because of the consumer trade-off between consumption and environmental quality, which stipulates that this last one evolves in the same direction than capital accumulation.

Therefore, being a source of environmental-poverty trap, it could seem that the policy introduced in this paper should not be recommended. Analyzing welfare at the steady state, we will see, in the next section, that this conclusion

may be mitigated.

5 Welfare analysis

As it is well-known, an overlapping generation economy may be characterized by over or under-accumulation of capital. As it is emphasized by John and Pecchenino (1994), the same happens for environmental quality: one may have over or under-maintenance. We re-examine this issue, determining first the optimal stationary allocation. Then, we will see that this allocation can be decentralized by an appropriate choice of our two policy parameters B and G.

We start by solving the planner problem. Using the two resource constraints:

$$c + k + m = k^s - G \tag{19}$$

$$E = \gamma_1 m - \alpha c + \gamma_2 G \tag{20}$$

we get:[15]

$$c = \frac{\gamma_1(k^s - k) - E + (\gamma_2 - \gamma_1)G}{\alpha + \gamma_1} \tag{21}$$

Substituting this expression in the utility function $\ln c + \epsilon \ln E$, the planner solves:

$$\max_{k,E} \ln \left[\frac{\gamma_1(k^s - k) - E + (\gamma_2 - \gamma_1)G}{\alpha + \gamma_1} \right] + \epsilon \ln E \tag{22}$$

taking $G \geq 0$ as given. Using the first order conditions:

$$\frac{1}{c} \frac{\gamma_1}{\alpha + \gamma_1}(sk^{s-1} - 1) = 0 \tag{23}$$

$$\frac{\epsilon}{E} - \frac{1}{(\alpha + \gamma_1)c} = 0 \tag{24}$$

we deduce that the optimal stationary allocation $(\widetilde{k}, \widetilde{E})$ is given by:

$$s\widetilde{k}^{s-1} = 1 \quad \Leftrightarrow \quad \widetilde{k} = s^{\frac{1}{1-s}} \tag{25}$$

$$\widetilde{E} = \frac{\epsilon}{1+\epsilon}[\gamma_1(\widetilde{k}^s - \widetilde{k}) + (\gamma_2 - \gamma_1)G]$$

$$= \frac{\epsilon}{1+\epsilon}[\gamma_1 s^{\frac{s}{1-s}}(1 - s) + (\gamma_2 - \gamma_1)G] \tag{26}$$

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[15] We can also deduce the value of private abatement, given by $m = [\alpha(k^s - k) - (\alpha + \gamma_2)G + E]/(\alpha + \gamma_1)$.

where $\widetilde{k} < \overline{k}$ corresponds to the standard golden rule. We also notice that $\widetilde{k} > \underline{k}$ for $\gamma_2 < \widetilde{\gamma}_2$, with:

$$\widetilde{\gamma}_2 \equiv \frac{\alpha s + \gamma_1 \epsilon(1-s)}{G} s^{s/(1-s)} + \alpha \frac{B-G}{G} < \overline{\gamma}_2 \tag{27}$$

Then, the optimal allocation corresponds to a stationary solution with strictly positive tax rate $(\tau > 0)$ and private abatement $(m > 0)$.

We are now able to evaluate whether a steady state can be optimal. A stationary solution is defined by $k = H(k)$. Hence, the level of capital is optimal if $\widetilde{k} = H(\widetilde{k})$, which is equivalent to $X = \widetilde{X}$, with:

$$\widetilde{X} = s^{\frac{s}{1-s}}[\alpha s - \gamma_1(1 - s(2 + \epsilon))] \tag{28}$$

There is a unique level of X, determined by a combination of the policy parameters G and B, such that the stationary level of capital is optimal. However, since the inequality $H'(\widetilde{k}) < 1$ always holds under Assumption 3, a steady state monotonically unstable, k_2, can never be optimal. In contrast, a monotonically stable steady state, k_1, can be optimal, for an appropriate choice of B and G.

Differentiating the equation $k = H(k)$ with respect to k and X, we get:

$$\frac{dk}{dX} = \frac{1}{\gamma_1(1 + \epsilon)(1 - H'(k))} > 0 \text{ iff } H'(k) < 1$$

Therefore, a stable steady state (k_1) is characterized by over-accumulation if $X > \widetilde{X}$ and by under-accumulation if $X < \widetilde{X}$. In the first case, the optimal allocation can be reached by increasing debt and/or decreasing public abatement, while the opposite recommendation is relevant when there is under-accumulation. Indeed, as already emphasized, G and B have opposite effects on the stationary level of capital. Increasing public abatement, by reducing the incentive to provide private abatement, raises productive saving, while increasing debt lowers capital, because a smaller share of saving is devoted to capital holding.

At a steady state, the level of environmental quality is given by $E = \gamma_1 \epsilon (B + k)$, which is generically different to its optimal level $\widetilde{E}$. Hence, the question we address now is the following: by an appropriate choice of the two policy parameters B and G, can a decentralized steady state be optimal?

Proposition 3 *Let*

$$
\begin{aligned}
\gamma_2^a &\equiv [\alpha(2 + \epsilon) + \gamma_1(1 + \epsilon)]\gamma_1/\alpha \\
\gamma_2^b &\equiv [\alpha s(2 + \epsilon) - \gamma_1(1 - s(2 + \epsilon))]\gamma_1/(\alpha s)
\end{aligned}
$$

Under Assumptions 1-2, $\alpha s < \gamma_1(1 - 2s)$ and $\gamma_2^b \leq \gamma_2 < \min\{\gamma_2^a, \widetilde{\gamma}_2\}$, where $\widetilde{\gamma}_2$ is given by (27), there is a unique value of the policy parameters B and G, with $B \geq G > 0$, such that the stationary levels of capital and environmental quality are optimal.

Proof. See the Appendix. ∎

This proposition shows that there exists a unique choice of public environmental abatement and debt which allow to a steady state to be characterized by optimal levels of capital and environmental quality. This provides an argument in favor of the policy considered in this paper. We notice that this result arises even if, in contrast to John et al. (1995) and Ono (1996), we introduce a unique tax rate. However, there is still two policy parameters, B and G. In fact, G has an impact through two channels, the level of public abatement on environmental quality and the tax rate. The level of debt B affects the tax rate as well, but also capital accumulation and private abatement because it captures a share of saving.

We finally notice that $\widetilde{X} \geq 0$ if and only if $\epsilon \geq \widetilde{\epsilon}$, with:

$$
\widetilde{\epsilon} \equiv \frac{\gamma_1(1 - 2s) - \alpha s}{\gamma_1 s}
$$

In such a case, the optimal policy corresponds to a configuration when there is a unique stable steady state. However, when $\epsilon < \widetilde{\epsilon}$, multiplicity of steady states and a trap may not a priori be excluded.

6   Conclusion

Among several countries, non explosive public debt is a major constraint. Nevertheless, the growing concerns about the environmental degradation (biodiversity losses, climate change...) lead many Governments to fight against pollution and hence, to increase environmental spending. In many countries, the pollution mitigation induces the adoption of environmental taxes bearing on households, alongside with the increase of the individual environmental engagements.

In this paper, we show that, when the environmental tax is allocated to environmental protection, and in the same time, it aims to stabilize public debt, then the environmental public policy may lead the economy to a poverty-environmental trap. Indeed, if the public debt is high enough, the stabilization of the latter reduces households' share of income devoted to productive saving. This effect is reinforced by private abatement sufficiently productive with respect to the public one. Indeed, households are encouraged to protect environment instead of saving for future consumption. Nevertheless, welfare analysis shows that, whatever the initial economic conditions are, there exists a unique value of the policy parameters, namely the debt level and the public spending, such that the decentralized steady state is optimal. This allows us to recommend that policy-makers should carefully evaluate the efficiency of private versus public environmental abatements and the level of public debt before increasing their environmental engagement and introducing new taxes.

7 Appendix

Proof of Proposition 1

Under Assumption 3, the function $H(k)$ is strictly increasing $(H'(k) > 0)$ and concave $(H''(k) < 0)$. When $\gamma_2 = \gamma_{20}$, we further have $H(0) = 0$. In this case, there is a unique strictly positive steady state $k_1 = \left[\frac{\gamma_1(1-s)-\alpha s}{\gamma_1(1+\epsilon)}\right]^{1/(1-s)}$ solving the equation $k = H(k)$. Moreover, using (13) and (17), we deduce that k_1 is smaller than $\overline{k}$ if B and G are not too far, such that $\frac{B-G}{B} < s\frac{\gamma_1(1+\epsilon)}{\gamma_1(1-s)-\alpha s}$, and is larger than $\underline{k}$ if $\gamma_2 < \widetilde{\gamma}_2$, where $\widetilde{\gamma}_2 > \gamma_{20}$ is equivalent to:

$$B < \frac{\alpha s + \gamma_1 \epsilon (1-s)}{[\gamma_1(1+\epsilon)]^{1/(1-s)}}[\gamma_1(1-s) - \alpha s]^{s/(1-s)}$$

We immediately see that this last inequality is satisfied if B is not too large. We may also easily conclude that, since $H(k)$ is strictly increasing and concave, the steady state k_1 is stable.

Finally, when $\gamma_2 > \gamma_{20}$, $H(0)$ becomes strictly positive. Since the steady state k_1 is stable, it becomes larger than $\left[\frac{\gamma_1(1-s)-\alpha s}{\gamma_1(1+\epsilon)}\right]^{1/(1-s)}$, but keep the same properties than under $\gamma_2 = \gamma_{20}$. In particular, we still have $\underline{k} < k_1 < \overline{k}$ if $\gamma_2 < \widetilde{\gamma}_2$ and B and G are not too far. ∎

Proof of Proposition 2

Using (15), we find that $H'(k) = 1$ is equivalent to $k = k_{sn}$, with:

$$k_{sn} \equiv \left[s\frac{\gamma_1(1-s)-\alpha s}{\gamma_1(1+\epsilon)}\right]^{1/(1-s)}$$

Moreover, we have $H(k_{sn}) \leq k_{sn}$ when $\gamma_2 \leq \gamma_{2sn}$. Since $H(k)$ is strictly increasing and concave, we deduce that $H(k_t) < k_t$ for $\gamma_2 < \gamma_{2sn}$. In this case, there is no steady state and $k_{t+1} < k_t$ for all $k_t \in (\underline{k}, \overline{k}]$.

Notice now that $k_{sn} < \overline{k}$ is always satisfied and $k_{sn} > \underline{k}$ if $\gamma_2 < \widehat{\gamma}_2$, with $\widehat{\gamma}_2 > \gamma_{2sn}$ under Assumption 4.

Therefore, when $\widehat{\gamma}_2 > \gamma_2 > \gamma_{2sn}$ with γ_2 sufficiently close to γ_{2sn}, there is a stable steady state k_1 and an unstable one k_2, such that $\underline{k} < k_2 < k_{sn} < k_1 < \overline{k}$.

When $\gamma_2 = \gamma_{2sn}$, we have $k_2 = k_{sn} = k_1$, which corresponds to the critical point where the saddle-node bifurcation occurs. ∎

Proof of Proposition 3

A steady state (k_i, E_i) is optimal if $k_i = \widetilde{k}$ and $E_i = \widetilde{E}$, i.e. $X = \widetilde{X}$ and $\widetilde{E} = \gamma_1 \epsilon (B + \widetilde{k})$. Using (16), (25), (26) and (28), this is equivalent to:

$$
\begin{pmatrix} 1 + \epsilon & 1 - \gamma_2/\gamma_1 \\ -(\alpha + \gamma_1(1 + \epsilon)) & \alpha + \gamma_2 \end{pmatrix} \begin{pmatrix} B \\ G \end{pmatrix} = s^{\frac{s}{1-s}} \begin{pmatrix} 1 - s(2 + \epsilon) \\ \alpha s - \gamma_1(1 - s(2 + \epsilon)) \end{pmatrix}
$$

Let $\Delta \equiv (\gamma_2^a - \gamma_2)\alpha/\gamma_1$. Since $\Delta > 0$, B and G are uniquely determined by:

$$
\begin{pmatrix} B \\ G \end{pmatrix} = s^{\frac{s}{1-s}} \Delta^{-1} \begin{pmatrix} \alpha + \gamma_2 & \gamma_2/\gamma_1 - 1 \\ \alpha + \gamma_1(1 + \epsilon) & 1 + \epsilon \end{pmatrix} \begin{pmatrix} 1 - s(2 + \epsilon) \\ \alpha s - \gamma_1(1 - s(2 + \epsilon)) \end{pmatrix}
$$

which is equivalent to:

$$
\begin{pmatrix} B \\ G \end{pmatrix} = s^{\frac{s}{1-s}} \Delta^{-1} \begin{pmatrix} \alpha[1 - s(3 + \epsilon)] + \gamma_1[1 - s(2 + \epsilon)] + \gamma_2 s\alpha/\gamma_1 \\ (1 - s)\alpha \end{pmatrix}
$$

Note that $G > 0$ for $\gamma_2 < \gamma_2^a$. Hence, Assumption 1 is satisfied if $B \geq G$. This is ensured by $\gamma_2 \geq \gamma_2^b$, where $\gamma_2^b < \gamma_2^a$.

Finally, we have seen that $\widetilde{k} < \overline{k}$, but $\widetilde{k} > \underline{k}$ requires $\gamma_2 < \widetilde{\gamma}_2$. Therefore, to complete this proof, we need to show that the interval $[\gamma_2^b, \min\{\gamma_2^a, \widetilde{\gamma}_2\})$ is non-empty. We have $\gamma_2^b < \gamma_2^a$. Taking $\gamma_2 = \gamma_2^b$, we obtain $\Delta = \gamma_1(1 - s)/s$ and $B = G$. In this case, the inequality $\gamma_2^b < \widetilde{\gamma}_2$ is satisfied when $\alpha s < \gamma_1(1 - 2s)$. ∎

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