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Dynamic factor analysis of carbon allowances prices: From classic Arbitrage Pricing Theory to Switching Regimes

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Abstract

The aim of this paper is to identify the fundamental factors that drive the allowances market and to built an APT-like model in order to provide accurate forecasts for CO2. We show that historic dependency patterns emphasis energy, natural gas, oil, coal and equity indexes as major factors driving the carbon allowances prices. There is strong evidence that model residuals are heavily tailed and asymmetric, thereby generalized hyperbolic distribution provides with the best fit results. Introducing dynamics inside the parameters of the APT model via a Hidden Markov Chain Model outperforms the results obtained with a static approach. Empirical results clearly indicate that this model could be used for price forecasting, that it is effective in and out of sample producing consistent results in allowances futures price prediction.

Keywords: Carbon, EUA, Energy, Arbitrage Pricing Theory, Switching regimes, Hidden Markov Chain Model, Forecast.

1 Introduction

At the dusk of the post-subprime crisis investors are searching for new yield sources less dependent of classic economic factors. Carbon allowances market appeared as an attractive option due to its original framework and to its row model in environmental investment. The carbon emission permits market raised in early 2005 as a key solution in the fight against the global warming.

Human activities, in particular the population growth and the development of industry over the last 200 years, have caused an increase in the emission and atmospheric concentration of certain gases, called "greenhouse gases" - primarily carbon dioxide and methane. These gases intensify the natural greenhouse effect that occurs on Earth, which in itself allows life to exist. The man-induced enhanced greenhouse effect leads to an increase in the average temperature

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of the planet that, would potentially cause increasingly severe and perhaps even more extreme disruptions to the Earth's climate, and consequently human activity. As a consequence several governments, firms and individuals have taken steps to reduce their greenhouse gas (GHG) emissions either voluntarily, or, increasingly, because of current or expected regulatory constraints. According to its provisions, the industrialized countries have to reduce in the period 2008-2012 the greenhouse gas emissions by 5 percent with respect to the 1990 year levels. The protocol dictates the trading of emission allowances as one of the primary mechanisms through which greenhouse gas emission reduction should be achieved. Thus, the right to pollute is considered to be a tradable asset, with its price determined by the market forces of supply and demand.

The carbon market encompasses both project-based emission reduction transactions and trading of emission permits. The first one concerns the purchase of carbon emission reductions from a project that reduces greenhouse gases emissions compared with what would have happened otherwise. The second one concerns the allowances that are allocated under existing or upcoming cap-and-trade regimes.

In this paper, we define carbon transactions as contracts whereby one party pays another party in exchange for a given quantity of GHG emissions permits that the buyer can use to meet its objectives vis-à-vis climate mitigation. Carbon transactions can be grouped in two main categories:

- Trades of emission allowances, such as, for example, Assigned Amount Units (AAUs) under the Kyoto Protocol, or allowances under the EU Trading Scheme (EUAs). These allowances are created and allocated by a regulator, usually under a cap-and-trade regime;
- Project-based transactions, that is, transactions in which the buyer participates in the financing of a project which reduces GHG emissions compared which what would have happened otherwise, and get emission credits in return. Unlike allowance trading, a project-based transactions can occur even in the absence of a regulatory regime: an agreement between a buyer and a seller is sufficient.

In some recent works, few authors including Paolella and Taschini [2006], Ulrih-Homburg and Wagner [2007], Benz and Truk [2008], Daskalakis, Psychoyios and Markellos [2008] and Frunza and Guégan(2009) focused on the econometric modelling of the emission allowances prices, underlining the particularities of this market like the non-Gaussian behavior, the auto-regressive phenomena and the presence of the convenience yield. They focus mainly on continuous time modelling and Extreme value approach. Most of their works are based on data concentrated on the period 2005 - 2007.

On the other hand other researches as Alberola et al. [2008], Bataller et al. [2009] and Keppler et al. [2009] showed the influence of different factors as oil, coal and gas in carbon market behavior. Thus Bataller et al. [2009] emphasize that CO_2 volatility is directly and indirectly (through the covariance) affected by the oil and natural gas volatility. Additionally, they show that shocks originated in the CO_2 and oil markets have an impact on CO_2 volatility and the behavior of oil volatility is similar to CO_2 volatility in what concerns volatility transmission. On the same topic Keppler et al. [2009] showed, using causality problematic that during Phase I coal and gas prices, through the clean dark and spark spread, impacted CO_2 futures prices, which in return were correlated to electricity prices. Furthermore it underlines that during the first year of the Phase II, electricity prices influenced CO_2 prices. In order to have a complete picture Alberola et al. [2008] showed that CO_2 spot prices react not only to energy markets but also to temperatures and to economic activity within the main sectors covered by the EU ETS such as proxied by sectoral production indices.

The objectives of the present paper are both: to enrich the actual econometric and financial literature on the carbon emission market, and to built a factorial model in order to explain the CO_2 behavior. After a deep analysis to retain the more pertinent factors which can explain the behavior of the CO2, we built a static APT model in the way described by Ross [1976], and we extend it in a dynamic way in order to be close to the real behavior of the CO_2 and its factors. Then, we compare the capability of these two models in terms of forecasting. Their capacity and accuracy to forecast are considered with appropriate criteria.

Our approach does not focus on volatility behavior of carbon and in that sense differs of Battaler et al. [2009] work. On the other hand we do not follow Keppler [2009] work whose approach is based on causality concept, and we focus on the detection of the most important fundamentals for CO_2, and their capacity to provide with robust forecasts. In order to identify the relevant drivers for CO_2 forecasts, we rely on two techniques, namely the PCA and the Pearson-correlation analysis between the different factors. Finally our work retains as main factors oil, gas, coal, power, dark spread, spark spread and stocks to explain the evolution of carbon prices. After this identification step, we introduce the APT model and its extension to explain and forecast the behavior of the CO_2.

Our results are based on EUA prices. Nevertheless, the EUA market is linked to the CER market which is source mainly by China and the other Asian countries. On one hand recent initiatives are made to develop carbon exchanges in Asian countries, mainly in Hong Kong; in the other hand, the European cap and trade framework is supposed to be deployed in the future in United States under the supervision of the actual American government. Thus, the present work could be relevant to have an apprehension of the future behavior in these different countries.

After describing in Section two the main features of the emission allowances market and the main factors (oil, dark spread, clean spread and equities) that influence its behaviors, we introduce the APT static modelling using several classes of distributions for residuals, in Section three. In Section four we extend our study introducing a new APT modelling with switches in the parameters modelled with a Markov chain. Section five emphasizes the applications of the previous sections on the price forecasting of the CO_2 based on the prediction of the other underlying factors. We benchmark the different approaches in order to identify the strengths and the weaknesses of each modelling. Section six concludes.

2 Data sets and factors

Our dataset contains daily closing prices for the EUA 2009 (EUA09) future contract, between 2006 and 2009, as quoted on Intercontinental Exchange (ICE)[1]. On Exhibit 1, EUA 2008 and 2009 historical prices exhibit high variability regimes and discontinuities in offer/demand equilibrium. Actually the EUA market is very liquid, with a good depth and a significant open interest. Even so for a certain number of trading days[2] the exchanged volumes of contracts are very small or even zero. In those particular days the prices are marked by the auction trading systems, therefore in our present work we adjust this bias with a moving window average. Looking at the sample autocorrelation function (ACF) based on the most recent 990 daily negative log return data of EUA09, we observe small correlation on the prices (Exhibit 2), while the ACF of squared return series does show evidence of serial dependence. On Exhibit 3 we give the distribution function of the EUA09: it shows negative skewness and fat tails also revealed by the QQplot diagram. Thus, This preliminary statistical study rejects the normality hypothesis of EUA09 daily returns.

It is now well admitted that the main factors followed by carbon traders are commodities like oil, gas, coal, power, dark spread, spark spread, economical activity (equities, index) and political news (UN and EU announcements), and also weather forecasts. We provide now a statistical study which permits to identify the "'actual"' main fundamentals for CO_2 prices. We paid a special attention to Dark Spread and Spark Spread as they seem to explain some particularities of emission prices. In fact in periods of high demands of electricity the consumption surplus is covered by the fossil power plants turning mainly on coal and gas, hence influencing both the gas and coal price on one hand and power price on the other hand. As the Spreads show the difference between energy and fossiles prices we consider that spreads are more efficient fundamental factors than pure fossile prices form the economic point of view.

In order to provide a robust approach in the choice of the factors, we focus on contracts available on the same period as our CO_2 data set, that are liquid enough, that are collected with the same frequencies and are relevant for Europe. First we consider oil[3], gas[4], coal[5],

[1] We used the 2009 EUA futures given by the Reuters ticker *CFI2Z9*

[2] We took in account the fact that the market dealt with low volumes at its very beginning in 2006 and that the exchange (ICE) is closed during holidays

[3] Brent crude future given by the Reuters ticker *LCOc1*; the contract is denominated in US dollars and adjusted in terms of euros

[4] Natural gas given by the Reuters ticker *NGLNMc1*; the contract is denominated in British pounds and adjusted in terms of euros

[5] Coal forward given by the Reuters ticker *RTRAP2Mc1*; the contract is denominated in US dollars and adjusted in terms of euros

power [6], dark spread[7], spark spread[8] and equities [9]. To distinguish between all these factors we use a Principal Component Analysis (PCA) approach as described by Chamberlain and Rothschild [1983] and Connor and Korajczyk [1985].

After doing a decomposition in principal component analysis using the previous mentioned factors, we observed that oil, dark spread, spark spread and equities explain the most part of CO_2 returns variance. Moreover in the following graph, we observe that these four factors are concentrated on the first axis corresponding to the first eigen-value and the gas and the spark spread have a different behavior explained by the second factor.

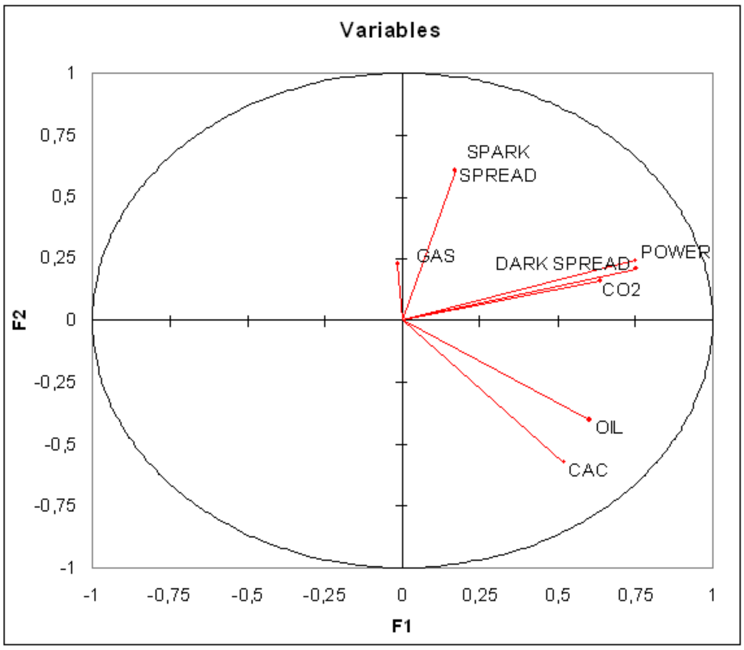

Figure 1: Results of Principal Components Analysis

Therefore, our analysis provided in the previous figure confirms that dark spread, spark spread, oil and equity pear as major drivers of CO_2 over the last two years. Thus, in the following we decide to work with these four factors.

3 Static APT modelling

In order to show the impact of the mentioned factors on the CO_2 prices we choose to use an extended APT modeling generated by more flexible noises than the Gaussian and also through a dynamic on beta parameters. We make this calibration using EUA prices on the period 2006 - 2009. Based on the historical time series we consider some models based on

[6] German calendar baseload power contract given by the Reuters ticker *BY1DE-1Y*

[7] German dark spread given by the Reuters ticker *DB1DE-1Y*

[8] German spark spread given by the Reuters ticker *SB1DE-1Y*

[9] We considered the equities market through the French index CAC40, given by the Reuters ticker *FCHI*. Actually CAC40 provided with better results than other possible competitors as MSCI Europe or FTSE

residuals that go from the classical Brownian diffusion to more sophisticated models based on generalized hyperbolic distributions. Our aim is not to find the "'true"' model that would explain the behavior of the carbon market but to detect models permitting to provide accurate forecasts.

The model supposes that a risky asset return follows a factor structure and has the following representation:

$$\tilde{r} = E(r) + \beta_1 F_1 + \beta_2 F_2 + \ldots + \beta_k F_k + \epsilon \tag{1}$$

where

- $E(r)$: id the expected return of Carbon Allowance
- F_k :is a systematic factor (assumed to have mean zero)
- β_i : is the sensitivity of the asset to factor i
- ϵ : is the idiosyncratic component
- $\mathbb{E}(F_i) = 0 \quad \forall i$
- $\mathbb{E}(\epsilon) = 0$

The APT states that if asset returns follow a factor structure as described then the following dependence exists between expected returns and the factor sensitivities:

$$E(r) = r_f + \beta_1(E(F_1) - r_f) + \beta_2(E(F_2) - r_f) + \ldots + \beta_k(E(F_k) - r_f) + \epsilon \tag{2}$$

where

- r_f is the risk free rate
- $E(F_k) - r_f$ is the risk premium of the factor k.

Described by Ross [1976] and based on the underlying hypothesis that the markets are efficient the APT model assumes a Gaussian distribution for the residuals. Given the atypical nature of the CO_2 , the assumptions of the APT model are in some cases broken. Hence the residuals do not follow a Normal distribution and the dependencies are not stationary over the time. In Exhibit 4 the QQ plot of the APT residuals shows clearly that the residuals of the multiple regression are asymmetric and fat tailed. In this paper, in order to overpass this issue we use different distributions to replace the classic Gaussian modeling for residuals. The candidate functions t-Student, GED and Normal Inverse Gaussian (NIG) retained our attention for their capacity to take in account heavy tails. The NIG distribution (detailed in Annexe) is part of the generalized hyperbolic distributions, which show also asymmetry and are able to integrate the skewness.

The Table 3 synthesize the results of our static calibration over the considered dataset using a weekly timestep, for different residual distributions. The discriminator element is the log likelihood: the higher it is, the best is the modeling. It appears that the level of dependencies of CO_2 price are generally close for the different models. But the degree of fitness depends highly of the chosen model. Hence the NIG distribution for residuals captures better the behavior of the residuals.

The Ljung-Box test of residuals autocorrelation show no presence of persistence at 99 percent of significance for all the distributions. As already noticed the NIG-based regression provides globally with a better fitting also with the t-Student distribution. One more interesting features of both NIG and t-Student model is the relevance of calibrated beta parameters from the angle of trust intervals. Hence we observe that for NIG residuals all four fundamentals factors are accepted but for classical Normal and GED distributions some factors are rejected. It appears clearly that NIG's capacity of taking into account skewness and heavy tails represents a real leverage factor for the model. Seen from another angle NIG distribution represents a Levy process that includes jumps, so in order to improve our forecasting model we oriented our work on a dynamic modeling searching for switching regimes proofs.

	Gaussian	GED	T-Student	NIG
Oil	0.131	0.103	0.122	0.116
	[0.055 0.205]	[-0.149 0.352]	[0.070 0.174]	[0.063 0.169]
Dark Spread	0.262	0.2115	0.261	0.260
	[0.231 0.349]	[0.014 0.468]	[0.221 0.304]	[0.221 0.302]
Spark Spread	0.002	0.0020	0.002	0.002
	[-0.001 0.003]	[-0.009 0.033]	[0.000 0.004]	[0.001 0.004]
Equity	0.12	0.157	0.160	0.168
	[0.017 0.242]	[-0.213 0.543]	[0.088 0.247]	[0.097 0.250]
R^2	0.24	0.24	0.23	0.24
Log Likelihood	2054	2087	2092	2094

Table 3: Modelling results

4 Dynamic APT modelling

The particularities of the CO_2 market described in the first part, also as the strong regulatory influence suggest that different regimes govern the behavior of allowances. In a previous work (Frunza et Guégan [2009]) upon the econometrics of the CO_2 prices the authors underlined the existence of switching regimes in the CO_2 yields and the presence of jumps. Using this guideline we introduce a dynamic factor analysis for the allowances prices based on the findings of the previous section.
In order to sustain the switching regime hypothesis we estimated the data process described in the previous section on moving windows with the intention to underline the non-stationarity in dependency structure. We calibrate the APT models from the previous section on moving temporal windows of 90 trading days. The purpose is to emphasize that the dependencies are varying over the time. Exhibits 5 and 6 shows the evolutions of the beta parameters p-values

for each factor described in the previous sections. We apprehend that the values of the beta parameters face important variations over times and these parameters become non relevant over some periods for the carbon price. As a conclusion not only the level of dependencies is non-stationary but also the drivers of the CO_2 prices. We observe that at different moments equities, oil and gas are offset from the model. The presence of power as a factor is more homogeneous and yet there are periods when this factor switched off from the model.

In order to take in account this dynamic behavior two solutions can be studied:

- Dynamic Factor Calibration of the APT model over a shorter horizon of 90 trading days taking in account only factors that are relevant at that time;
- Switching Regimes Calibration of a Hidden Markov Switching model that has few states and that allows the flip-flops of factors.

If the first solution is pretty common given the fact that the multi-regression has to be repeated on a regular basis, we test this alternative for forecasting purpose in the next section. The second alternative is more delicate and needs a more laborious econometric work. The idea behind consists to affirm that there are several states in the CO_2 price that switch following a transition matrix which is determined by a hidden factor. This "hidden" factor could be determined by regulatory announces, legislations or interventions of new dealers on the market. Nevertheless as the purpose of this paper stays mainly around prices modeling, we do not study the relevance of exogenous regulatory-like factors on the switching regimes, and we consider the existence of a Markov chain to explain these switches.

Following Hamilton [1990] we assume that a 2-regime switching model explain the CO_2 allowances behavior. Using the same model as introduced in (1), we assume now that the parameter β_i associated to each factor f_i, $i = 1, \cdots, 4$ has the following dynamics: $\beta_i = \beta_i^1(S_t) + \beta_i^2(1 - S_t)$ where S_t follows a 2 state Markov chain as shown in Figure 1, and $S_t = 1$ if S_t is in State 1 and $S_t = 0$ if S_t is in State 2.

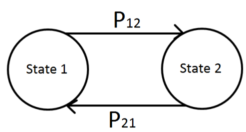

Figure 2: Switch Markov Chain

To get the results of the calibration of the 2-states Markov Switching model of r_t provided in Table 4, we use respectively a Normal, t-Student and GED distribution for the residuals. The log-likelihoods (LL) and the Transition probabilities (TP) form State 1 to State2 are also given. We observe also that the R^2 for the switching model is higher for static models than for dynamic ones and the use of t-Student and GED distributions provide with the better fits for residuals. Finally we exhibit the dynamic of the switching regimes in Exhibit 7.

		Gaussian	t-Student	GED	NIG
Oil	State 1	0.0961	0.1524	0.1056	0.075
		[0.0369 0.1553]	[0.0774 0.2274]	[-0.085 0.2962]	[0.035 0.105]
	State 2	0.2869	0.0763	0.1056	-0.519
		[-0.1265 0.7003]	[-0.0711 0.2237]	[0.0018 0.0022]	[-0.619 -0.419]
Dark Spread	State 1	0.3107	0.298	0.1337	0.220
		[-0.1265 0.7003]	[0.239 0.357]	[0.0555 0.2119]	[0.201 0.239]
	State 2	0.256	0.1697	0	-0.504
		[0.2595 0.3619]	[0.0259 0.3135]	[-0.0006 0.0006]	[-0.526 -0.482]
Spark Spread	State 1	-0.0001	-0.0002	0.1727	0.004
		[-0.0015 0.0013]	[-0.0026 0.0022]	[-0.1353 0.4807]	[-0.0026 0.0030]
	State 2	0.0019	0.0022	0.3738	0.314
		[-0.0033 0.0071]	[0.0002 0.0042]	[-0.1932 0.9408]	[-0.190 0.808]
Equity Spread	State 1	0.1838	0.1582	0.2967	0.2708
		[0.0946 0.273]	[0.0532 0.2632]	[0.2431 0.3503]	[0.240 0.301]
	State 2	-0.195	0.2374	0.1393	0.048
		[-0.9054 0.5154]	[-0.0042 0.479	[0.0723 0.2063]	[0.028 0.068]
LL		686	686	688	690
R^2		0.25	0.25	0.25	0.25
TP		0.04	0.04	0.04	0.05

Table 4: Calibration results of the Hidden Markov Model

5 Discussion

The final purpose of the present work is to use the previous models in order to provide robust forecasts for the CO_2 prices using the foreseeable variations of the underling factors. Considering the markets are efficient, and taking the equation (1) under a risk neutral framework we could write

$$E(r^*) = r_f + \beta_1(E(F_1^*) - r_f) + \beta_2(E(F_2^*) - r_f) + \ldots + \beta_k(E(F_k^*) - r_f) + \epsilon \tag{3}$$

where

- $E(r^*)$ is the forecasted expected return
- $E(F_1^*) - r_f$ is the forecasted risk premium of the factor k.

In order to validate the pertinence of this application we first consider the true variations of the factors as input for the model and we compare the forecast price of the CO_2 allowances with the realized price of the next period. We use weekly data and our predictions are over one week horizon. We break the dataset in two parts in-sample and out-of-sample. We calibrate the model over an in-sample period and we use it to predict the prices for an out-sample period. As we backtest the prediction power of the model over the past history our in-sample and out-of-sample breakpoint moves over the whole dataset. As an example we calibrate the model over the first 6 months of the time series and we predict the value of the next week, then we include the in-sample data with one week and we predict the value again until we attend the end of the dataset. This technique of moving in-sample window is more adapted to dynamic factor analysis as it allows to take in account the variation of the factors.

In order to compare the quality of each model we use two metrics: the prediction capacity which represents the proportion of good predictions of the moving sense of the market (up or down) and the forecast accuracy which represents a distance between the predicted prices and the realized ones. The Table 5 shows the results for the two categories of dynamic models: Static Factor Calibration (SFC), Dynamic Factor Calibration (DFC) and Switching Regimes Calibration (SR).
We benchmarked our results with a simple forecasting model based on a technical momentum approach. Hence we estimate that momentum is given by a 20 days moving average and if the actual price is superior to the average th market is bullish.

$$Prediction\ capacity = \frac{Number\ of\ good\ predictions\ of\ variation}{Total\ number\ of\ predictions} \tag{4}$$

$$Forecast\ accuracy = \sqrt{\frac{\sum_i (Forecast\ price(i) - Realized\ price(i))^2}{Total\ of\ forecasts}}, \tag{5}$$

where i is the number of forecasts.

Distribution	Framework	Prediction capacity(%)	Forecast accuracy
Gaussian	SFC[10]	50	0.232
	DFC[11]	65	0.241
	SRC[12]	68	0.261
t-Student	SFC	50	0.223
	DFC	65	0.237
	SRC	69	0.271
GED	SFC	52	0.252
	DFC	65	0.235
	SRC	68	0.283
NIG	SFC	53	0.238
	DFC	64	0.235
	SRC	70	0.361
Technical Momentum	-	53	0.376

Table 5: Forecast model benchmark

Generally speaking for all the models the prediction power is bigger than 50 % which confirms that the model has a discriminative capacity. Compared to the benchmark approach the dynamic and swithcing model provides with better prediction capacities. The dynamic models outperform the static models in term of prediction capacity, because the static approach fails to capture the non-stationarity of the dependency pattern. It appears that switching regimes with NIG innovation have a better prediction power, but are less accurate. The NIG modeling is less robust due the fact that a lot of information is kept in the residuals and less explained by the factors. Accross this work we observe that there is a trade off between the information modelled by the dependencies and the information kept in the residuals, as the factors seems not to be exhaustive for describing CO_2 behaviour. The dynamic models with t-Student innovation offer the best compromise between prediction capacity and accuracy (Exhibit 8).

6 Conclusions

Understanding the emission allowances market goes beyond the classic stochastic apprehension of the financial assets like commodities and enters in a more subjective area of the behavioral finance. The present paper might be completed by considering the influence of temperature and of the Gross Domestic Product (GDP). These topics will constitute the focus of a future paper.

Thus, the main topic of this paper is to search for an extended APT or APT modelling that could fit the best the historical time series, using the likelihood function and R^2 as a discriminant factor to rank models relevance. The CO_2 allowances prices show a pronounced non-Gaussian behavior with fat tails and negative skewness. The NIG distribution outperforms the classic Brownian models in terms of quantity of information, but lacks to give accurate forecasts. The main reason is the ability of the GH models to be customized in same time to different skews and tails forms. In our case the carbon market is far from being Gaussian. It appears clearly that dynamic factors are a necessary hypothesis for an accurate modeling of the CO_2 prices.

In terms of forecast both proposed models Dynamic Factor Calibration and Switching Regimes Calibration provide with a good prediction capacity, hence making the model eligible for trading and management strategies. But more simple Dynamic Factor models fail to give an accurate forecast for the CO_2 prices and only the Markov Chain Modeling allows for a reasonable forecast.

Indeed, in the perspective of this work, natural further developments will include Markov switching models with Bilinear terms and memory effects in the model calibration Diongue, Guégan and Wolff (2009), and also the econometric study of the EUA - CER spread.

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Exhibits

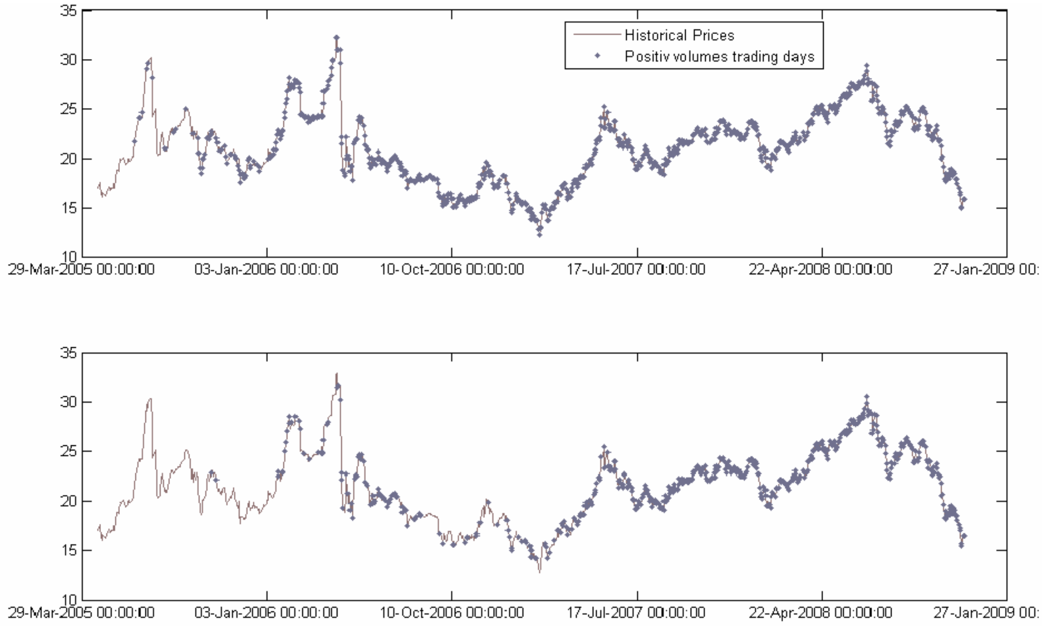

Exhibit 1: EUA08 and EUA09 prices history

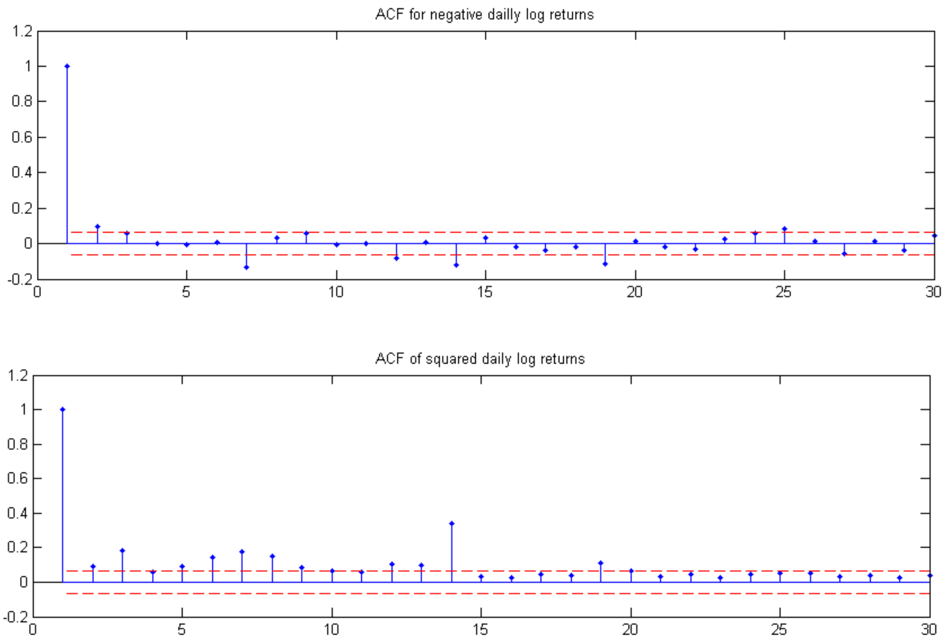

Exhibit 2: Autocorrelation for EUA09 negative daily returns

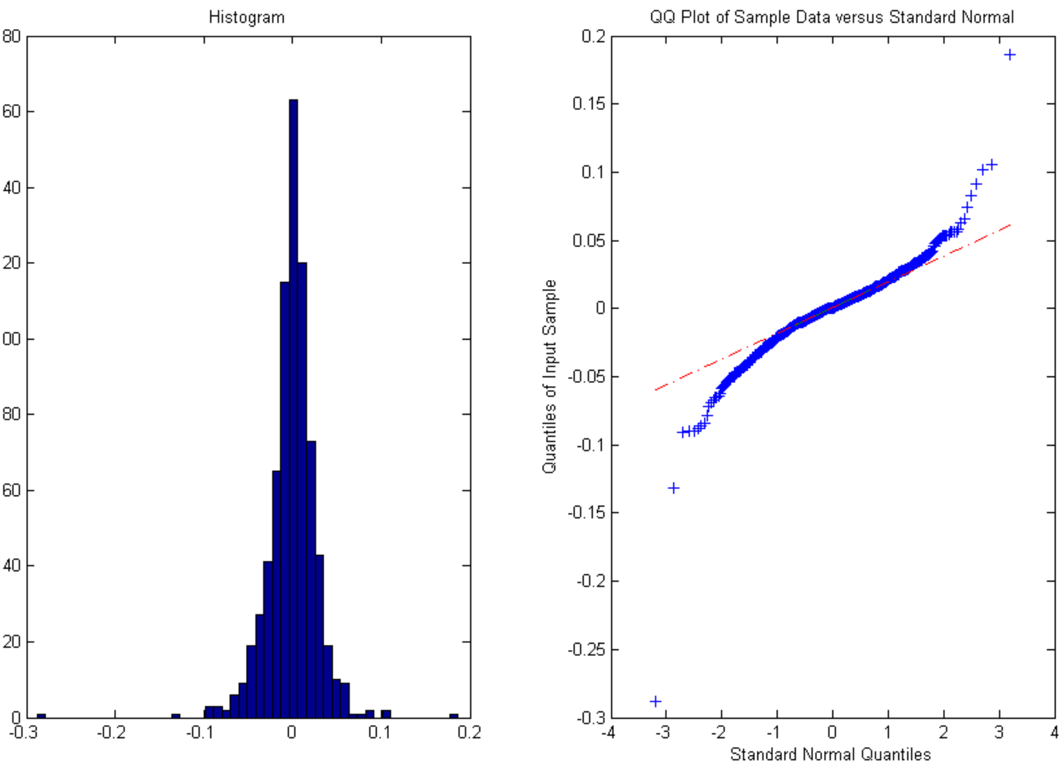

Exhibit 3: Distributions of EUA09 daily yields and QQ Plots

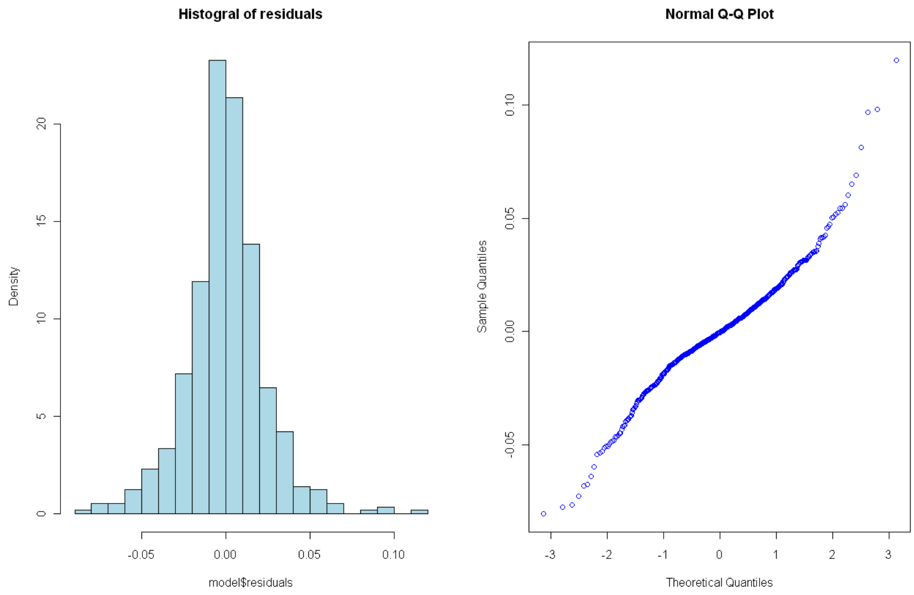

Exhibit 4: Residuals of Gaussian APT

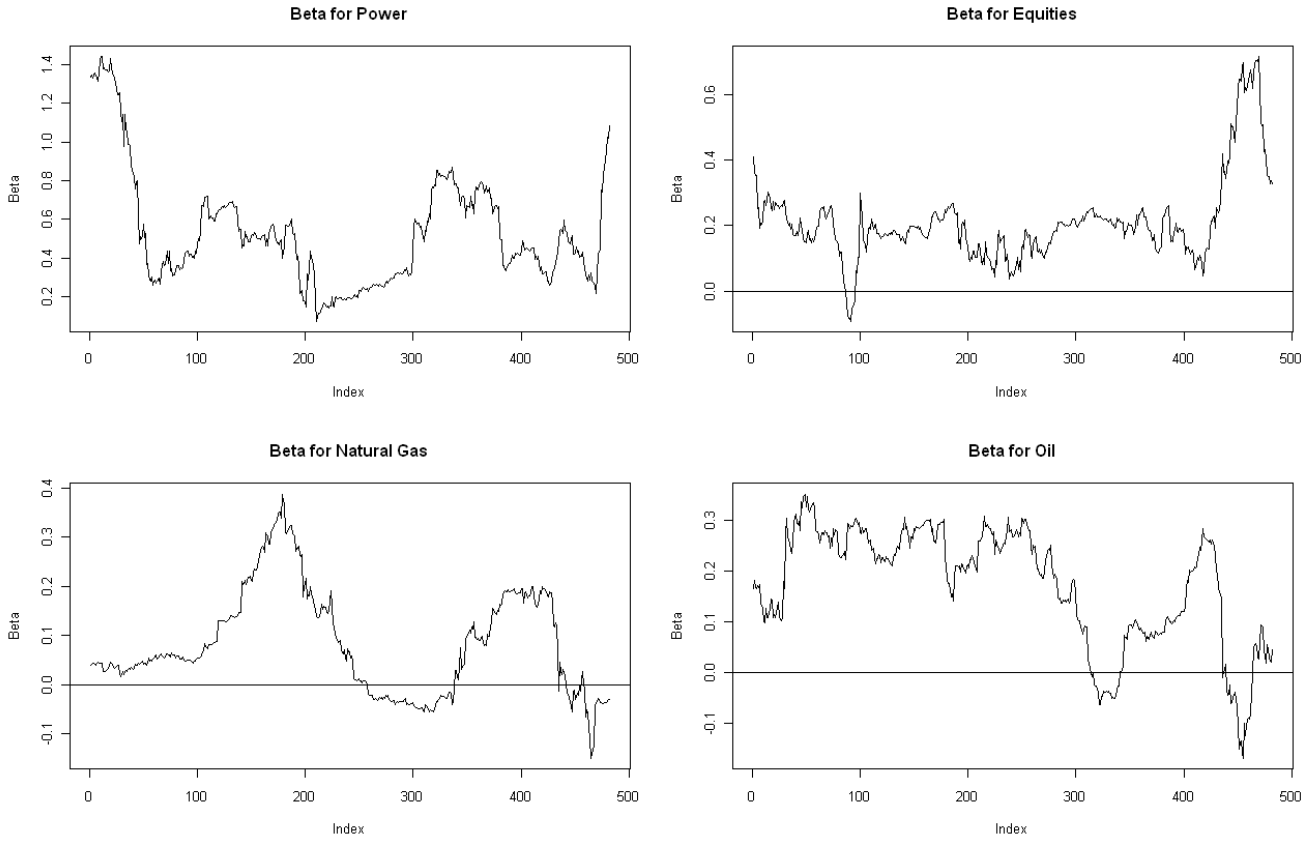

Exhibit 5: Evolution of β

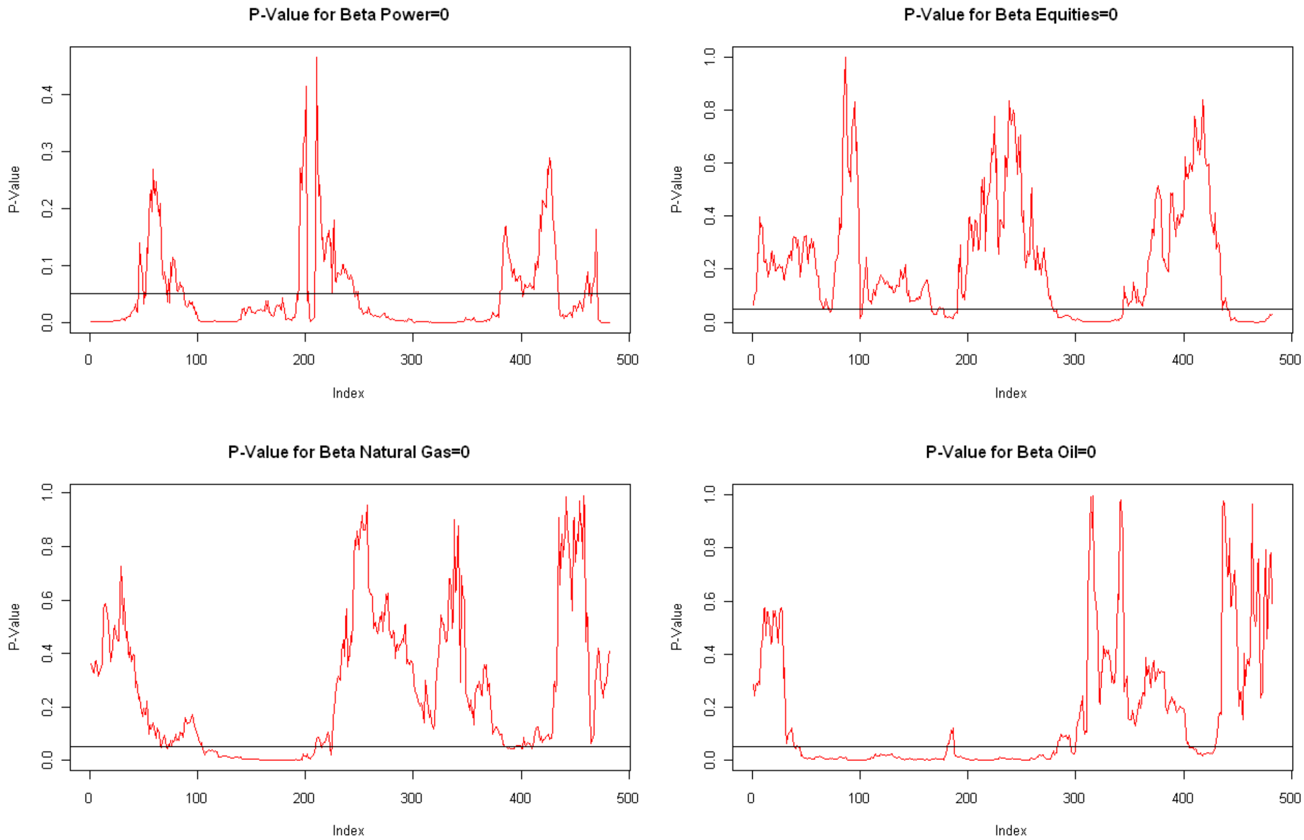

Exhibit 6: Evolution of β p-value

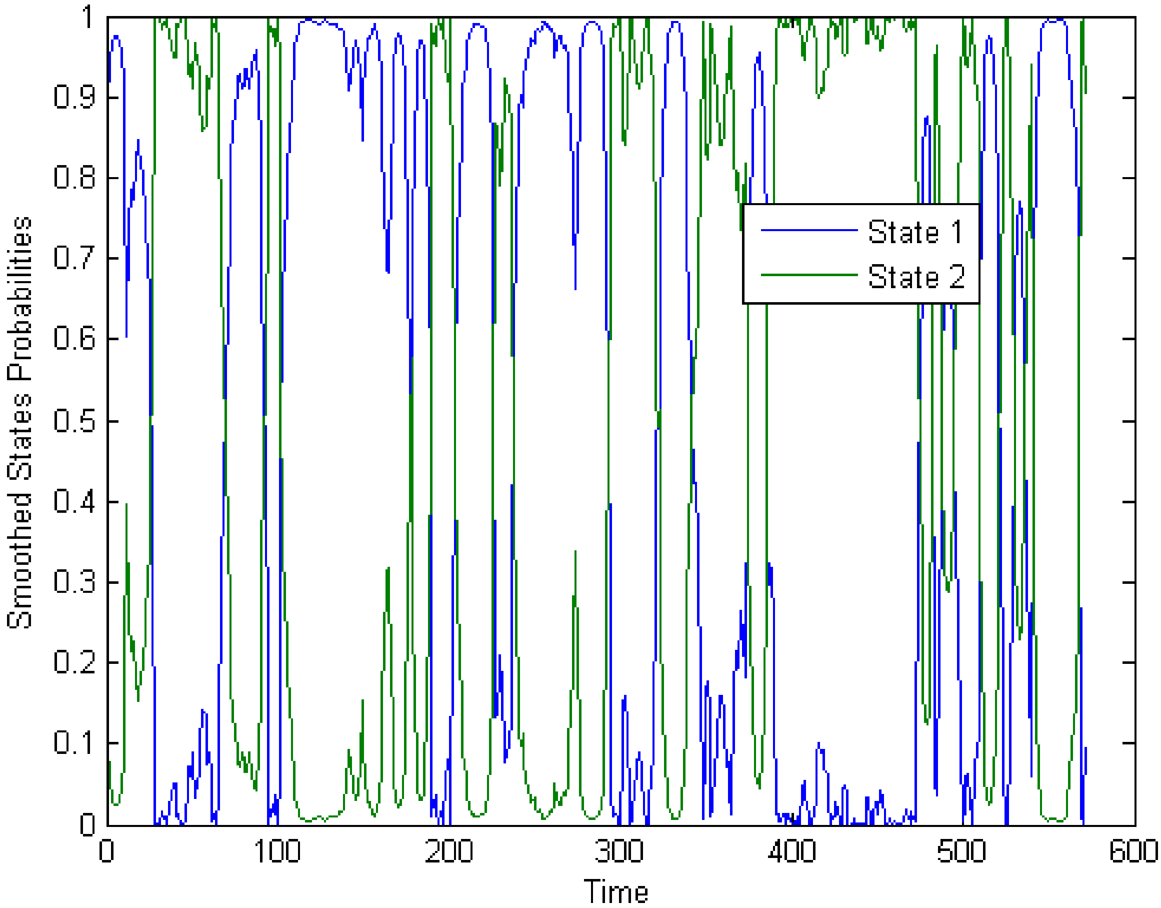

Exhibit 7: Evolution of β p-value

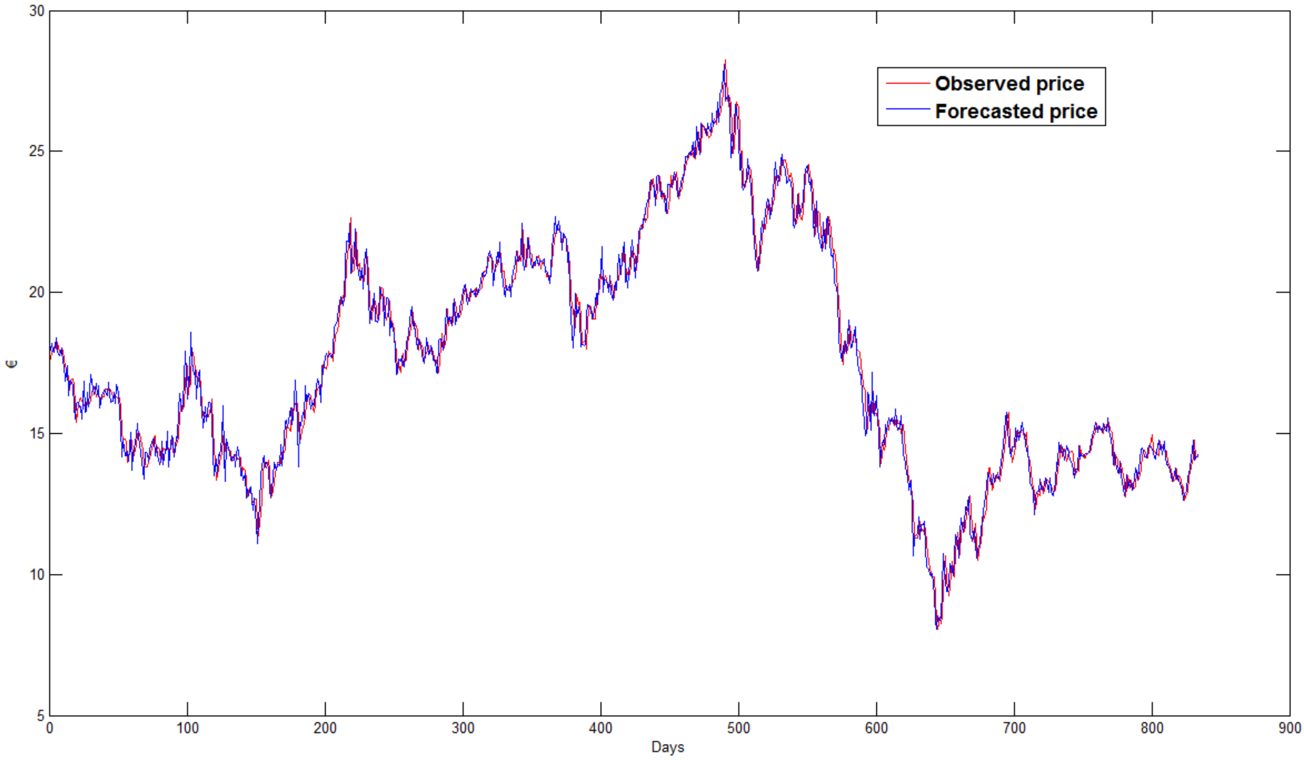

Exhibit 8: Forecast backtesting with a t-Student based dynamic model

Annexe 1 : Distributions

T-Student Distribution

Probability density function :

$$f_t(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}(\frac{\nu}{2})}(1+\frac{x^2}{\nu})^{\frac{\nu+1}{2}} \tag{6}$$

The moments (mean, variance, skewness and kurtosis) are respectively equal to:

$$\mathtt{E}(X) = 0 \text{ for } \nu > 1\text{undefined otherwise} \tag{7}$$

$$\mathtt{V}(X) = \frac{\nu}{\nu-2} \text{ for } \nu > 2, \quad +\infty \text{ for } \nu = 2\text{undefined otherwise} \tag{8}$$

$$\mathtt{S}(X) = 0 \text{ for } \nu > 3 \tag{9}$$

$$\mathtt{K}(X) = \frac{6}{\nu-4} \text{ for } \nu > 4 \tag{10}$$

$$\tag{11}$$

GED Distribution

Probability density function :

$$f_t(x) = \frac{\beta}{2\alpha\Gamma(\frac{1}{\beta})}e^{-\left(\frac{|x-\mu|}{\alpha}\right)^{\beta}} \tag{12}$$

The moments (mean, variance, skewness and kurtosis) are respectively equal to:

$$\mathtt{E}(X) = \mu \tag{13}$$

$$\mathtt{V}(X) = \frac{\alpha^2\Gamma(\frac{3}{\beta})}{\Gamma(\frac{1}{\beta})} \tag{14}$$

$$\mathtt{S}(X) = 0 \tag{15}$$

$$\mathtt{K}(X) = 3 + \frac{\Gamma(\frac{5}{\beta})\Gamma(\frac{1}{\beta})}{\Gamma(\frac{3}{\beta})^2} - 3 \tag{16}$$

$$\tag{17}$$

Generalized Hyperbolic Distribution

First we make a brief review of the Generalized Hyperbolic distribution functions focusing on the Normal Inverse Gaussian. The generic form of a Generalized Hyperbolic model is given by :

$$f(x;\lambda;\chi;\psi;\mu;\sigma;\gamma) = \frac{(\sqrt{\psi\chi})^{-\lambda}\psi^{\lambda}(\psi+\frac{\gamma^2}{\sigma^2})^{0.5-\lambda}}{\sqrt{2\pi}\sigma K_{\lambda}(\sqrt{\psi\chi})} \times \frac{K_{\lambda-0.5}(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})^{\lambda-0.5}},$$

where $K_\lambda(x)$ is the modified Bessel function of the third kind:

$$K_\lambda(x) = \frac{1}{2}\int_0^\infty y^{\lambda-1}e^{-\frac{x}{2}(y+y^{-1})}dy. \tag{18}$$

Among the Generalized Hyperbolic family, we will focus on the Normal Inverse Gaussian distribution obtained by setting $\lambda = -\frac{1}{2}$ in the previous equation. Thus:

$$f(x;-\frac{1}{2};\chi;\psi;\mu;\sigma;\gamma) = \frac{\chi^{\frac{1}{2}}(\psi+\frac{\gamma^2}{\sigma^2})}{\pi\sigma e^{\sqrt{-\psi\chi}}} \times \frac{K_1(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})}.$$

By changing the variables of the previous equation $c = \frac{1}{\sigma^2}$; $\beta = \frac{\gamma}{\sigma^2}$; $\delta = \sqrt{\frac{\chi}{c}}$; $\alpha = \sqrt{\frac{\psi}{\sigma^2}+\beta^2}$ we obtain a more popular representation, and the density of a random variable X following the $NIG(\alpha,\beta,\mu,\ \delta)$ distribution is equal to:

$$f_{NIG}(x;\alpha;\beta;\mu;\delta) = \frac{\delta\alpha\cdot exp(\delta\gamma+\beta(x-\mu))}{\pi\cdot\sqrt{\delta^2+(x-\mu)^2}}K_1(\alpha\sqrt{\delta^2+(x-\mu)^2}).$$

The moments (mean, variance, skewness and kurtosis) are respectively equal to:

$$\mathtt{E}(X) = \mu+\delta\frac{\beta}{\gamma} \tag{19}$$

$$\mathtt{V}(X) = \delta\frac{\alpha^2}{\gamma^3} \tag{20}$$

$$\mathtt{S}(X) = 3\frac{\beta}{\alpha\cdot\sqrt{\delta\gamma}} \tag{21}$$

$$\mathtt{K}(X) = 3+3(1+4(\frac{\beta}{\alpha})^2)\frac{1}{\delta\gamma}. \tag{22}$$

Thus, the NIG distribution allows behavior characterized by heavy tail and strong asymmetries, depending on the parameters α, β and δ.