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TESTING UNIT ROOTS AND LONG RANGE DEPENDENCE OF FOREIGN EXCHANGE

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Key Words: foreign exchange rate; long memory processes; Monte Carlo simulation; non-stationary; test.

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1

ABSTRACT

Foreign exchange rate plays an important role in international finance. This paper examines unit roots and the long range dependence of 23 foreign exchange rates using Robinson's (1994) test, which is one of the most efficient tests when testing fractional orders of seasonal/cyclical long memory processes. Monte Carlo simulations are carried out to explore the accuracy of the test before implementing the empirical applications.

1 Introduction

In recent years, there have been many changes in financial system. The perennial BOP (Balance of Payment) problem forced the countries to adopt the deregulation of interest as well as foreign exchange. The adaption of the flexible exchange rate regime and the accelerated integration of financial markets with globalization make the behavior of exchange rate more complicated to understand, thus attracting a lot of attention of the policy makers. In fact, exchange rate affects all countries small or big. For example, large exchange rate fluctuations in an environment of increased international capital mobility affect the level of inflation predictability and the pricing of financial assets, while it may be desirable to promote competitive depreciation to suit domestic economic interests. Therefore, it is interesting and meaningful to study and predict behavior of the foreign exchange rates.

One popular technique to analyze the financial data is time series analysis methodology. The study of asset prices has begun with the findings of its stochastic nature, modeling the exchange rate by random walk, Brownian motion, etc. But there still exist many advancements concerning the modeling technique, for instance, long memory processes. Since the introduction of fractionally integrated process by Granger and Joyeux (1980) and Hosking (1981), many extensions of long memory processes have been made. For example, Gray et al. (1989) and Giraitis and Leipus (1995) introduced the Gegenbauer process, or seasonal long memory process, which takes into account the seasonal persistence. More generally,

2

Robinson (1994) proposed the seasonal/cyclical long memory process as follows:

$$F(B)X_t = (I - B)^{d_0+\theta_0} \prod_{i=1}^{k-1}(I - 2\nu_i B + B^2)^{d_i+\theta_i}(I + B)^{d_k+\theta_k} X_t = \varepsilon_t, \qquad (1)$$

where B is the backshift operator. For $i = 1, \cdots, k-1$, $\nu_i = cos\lambda_i$, λ_i being any frequency between 0 and π. For $i = 0, 1, \cdots k$, θ_i belongs to $[-1, 1]$, d_i is such that: $|d_i| < 1/2$ and $(\varepsilon_t)_t$ is an innovation process to be specified. In Robinson (1994), a parametric Lagrange Multiplier procedure is proposed to test whether the data stemmed from a stationary or a non-stationary process under uncorrelated and weakly-correlated innovations. This method has been proved to be the most efficient one when directed against the appropriate alternatives. In particular, the Robinson's (1994) test can also deduce a parametric estimation method since under the null hypothesis, the test chooses the best long memory parameter which corresponds to the greatest $p-$value of the Chi-squared test. In the literature, we can find many applications of this test. For example, Gil-Alana (2000, 2001), Depenya and Gil-Alana (2006), Gil-Alana and Robinson (1997, 2001), etc. They have made the applications on the fourteen US macroeconomic variables, Spanish Stock Market prices, UK and Japanese consumption and income series, etc.

However, before empirical application, we need to know the accuracy of the Robinson's (1994) test. One interesting work is that Ferarra et al. (2010) have investigated the finite sample behavior of Robinson's (1994) test for several stationary long memory processes. They study and assess the rate of convergence of the estimators, which provides a useful reference for empirical practitioners. Whereas, we can often observe non-stationarity in the foreign exchange data series. Many authors handle this problem by executing some transformations to obtain the stationarity. To name just a few, Granger and Joyeux (1980), Gray et al. (1989), Hsu and Tsai (2008). However, in practice, series can not always be made stationary even by transformation or sometimes it has no sense to make the data sets stationary. Therefore, some authors model the series directly by non-stationary processes. In this paper, we will study and assess the rate of convergence of the estimator derived

3

by Robinson's (1994) test in the non-stationary setting for long memory models, aiming at exploring the accuracy of the test in the non-stationary setting. Then we will carry out the empirical applications on the 23 exchange rate series. In fact, the presence of long memory dynamics in exchange rates violates the weak form of Efficient Market Hypothesis (EMH) as it implies nonlinear dependence in the first moment of the distribution and hence a potentially predictable component in series dynamics.

Therefore, the issue of non-stationarity and long range dependence is not merely a statistical curiosity but has several important implications for the modeling of exchange rates. In this paper, we try to investigate the presence of non-stationarity and fractional dynamics of the following foreign exchange rates, Brazilian Reals to one U.S. Dollar, Canadian Dollar to one U.S. Dollar, Chinese Yuan to one U.S. Dollar, Danish Kroner to one U.S. Dollar, Hong Kong Dollar to one U.S. Dollar, Indian Rupees to one U.S. Dollar, Japanese Yen to U.S. Dollar, South Korean Won to one U.S. Dollar, Malaysian Ringgit to one U.S. Dollar, Mexican New Pesos to one U.S. Dollar, Norwegian Kroner to one U.S. Dollar, Swedish Kroner to one U.S. Dollar, South African Rand to one U.S. Dollar, Singapore Dollar to one U.S. Dollar, Sri Lankan Rupees to one U.S. Dollar, Swiss Francs to one U.S. Dollar, New Taiwan Dollar to one U.S. Dollar, Thai Baht to one U.S. Dollar, U.S. Dollar to one Australian Dollar, U.S. Dollar to one Euro, U.S. Dollar to one New Zealand Dollar, U.S. Dollar to one British Pound, Venezuelan Bolivares to one U.S. Dollar.

The paper is organized as follows: in Section 2, we present our methodology and the application of Robinson's (1994) test on seasonal/cyclical long memory processes in the non-stationary setting. Section 3 is the result of Monte Carlo simulations. Section 4 presents the empirical application of foreign exchange rates using Robinson's (1994) test. Section 5 concludes.

<div align="center">4</div>

2 Modeling and Test

According to stationarity, stochastic processes can be classified into stationary processes and non-stationary processes. Ferrara et al. (2010) have explored the finite sample behavior of the Robinson's (1994) test for the stationary seasonal/cyclical long-memory processes. Here, we focus on the application of the Robinson's (1994) test on non-stationary long memory processes with constant parameter. We will explore the performance of the test and assess the rate of convergence of the estimator obtained from the test.

2.1 Some SCLM Models

We specify now the non-stationary long memory models we derived from model (1), which are often used for macroeconomic and financial data.

- Non-stationary FI(d) (Fractionally Integrated) process:

$$(I - B)^d X_t = \varepsilon_t, \tag{2}$$

  where $(\varepsilon_t)_t$ is a white noise and $d > 1/2$. This class of non-stationary models permits to take into account the existence of an infinite cycle, namely an explosion for the very low frequencies.

- Non-stationary models with a fixed seasonal periodicity s:

$$(I - B^s)^d X_t = \varepsilon_t. \tag{3}$$

  where $d > 1/2$ and $s = 4$ or $s = 12$. This representation is useful for quarterly data sets using $s = 4$ and for monthly data using $s = 12$. This filter allows to model non-stationary fractional seasonalities. The spectral density of the process (3) is unbounded at the three seasonal frequencies $\{0, \pi/2, \pi\}$ on the interval $[0, \pi]$.

- Non-stationary models with an infinite cycle and another seasonality:

$$(I - B)^{d_1}(I - B^s)^{d_2} X_t = \varepsilon_t. \tag{4}$$

5

where $d > 1/2$, $d_2 > 1/2$, $s = 4$ or $s = 12$. The parameter d_1 corresponds to the persistence associated to the infinite cycle and d_2 to the persistence associated to the fixed seasonality. The spectral density of the process (4) is unbounded at three frequencies $\{0, \pi/2, \pi\}$ on the interval $[0, \pi]$.

- Non-stationary models with k persistent periodicities:

$$\prod_{i=1}^{k}(I - 2\nu_i B + B^2)^{d_i} X_t = \varepsilon_t, \tag{5}$$

with, for $i = 1, \ldots, k$, $\nu_i = cos(\lambda_i)$, the frequencies $\lambda_i = \cos^{-1}(\nu_i)$ being the Gegenbauer frequencies or the G-*frequencies* and $d_i > 1/2$. The spectral density of the process (5) is unbounded at the k Gegenbauer frequencies.

2.2 Robinson's (1994) test

Now we briefly describe Robinson's (1994) test which is a Lagrange Multiplier test for testing unit roots and other fractionally hypotheses when the roots are located at any frequency on the interval $[0, \pi]$. The test is derived via the score principle and its asymptotic critical values obey the Chi-squared distribution. Let $(Y_t)_t$ be a stochastic process such that:

$$Y_t = \beta' Z_t + X_t, \tag{6}$$

where $(Z_t)_t$ is a $k \times 1$ observable vector, β an unknown $k \times 1$ vector and $(X_t)_t$ a process which follows Equation (1). Since we focus on the test of the long memory parameters in model (1), in the rest of the paper, we assume a priori that $\beta = 0$ in (6) and $(\varepsilon_t)_t$ is either a strong white noise or a weekly autocorrelated noise.

Robinson (1994) works with the general model (1) for a fixed d and tests the assumption

$$H_0 : \theta = (\theta_0, \cdots, \theta_k)' = 0,$$

against the alternative:

$$H_a : \theta \neq 0.$$

6

The test statistic is defined by:

$$\tilde{R} = \frac{T}{\tilde{\sigma}^4} \frac{\tilde{a}^2}{\tilde{A}}, \tag{7}$$

where T is the length of the raw time series and

$$\tilde{\sigma}^2 = \frac{2\pi}{T} \sum_{j}^{*} I_{\tilde{\varepsilon}}(\lambda_j).$$

$I_{\tilde{\varepsilon}}(\lambda_j)$ is the periodogram of $\tilde{\varepsilon}_t$ with $\tilde{\varepsilon}_t = F(B)Y_t$, $F(B)$ being given in Equation (1) under the null. Moreover, we get:

$$\tilde{A} = \frac{2}{T} \sum_{j}^{*} \psi(\lambda_j) \cdot \psi(\lambda_j)',$$

and

$$\tilde{a}^2 = \frac{-2\pi}{T} \sum_{j}^{*} \psi(\lambda_j) I(\lambda_j),$$

where $\sum_{j}^{*}$ is the sum over $\lambda_j = \frac{2\pi j}{T} \in M = \{\lambda : -\pi < \lambda < \pi, \lambda \notin (\rho_l - \eta, \rho_l + \eta)\}$ such that ρ_l are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$, η is a given positive constant. Finally, we get:

$$\psi(\lambda_j) = (\psi_l(\lambda_j)),$$

with

$$\psi_l(\lambda_j) = \delta_{0l} \log |2sin\frac{1}{2}\lambda_j| + \delta_{kl} \log(2cos\frac{1}{2}\lambda_j) + \sum_{i=1}^{k} \delta_{il} \log(|2(cos\lambda_j - \cos\omega_i)|),$$

for $l = 0, 1, \cdots, k$, where $\delta_{il} = 1$ if $i = l$ and 0 otherwise.

Under certain regularity conditions, Robinson (1994) established that:

$$\tilde{R} \to_d \chi^2_{k+1},$$

where $k + 1 = dim(\theta)$. If χ^2_{k+1} represents the χ^2 distribution with $k + 1$ degrees of freedom then $\chi^2_{k+1,\alpha}$ represents a quantile for a given level α. As soon as $\tilde{R} > \chi^2_{k+1,\alpha}$, we reject H_0, with a risk α.

7

2.3 Estimation

In fact, the Robinson's (1994) test can work as a parameter estimation method for the SCLM models both in the stationary and in the non-stationary settings, since under the null, the test chooses the best long memory parameter which corresponds to the greatest p-value of the Chi-squared test. We accept the null hypotheses if the $p-$value is greater than the significant level and we reject it if the p-value is smaller than or equal to the significance level. Thus, using the grid-search method, we can obtain the best estimation of the fractional parameter in the models.

3 Monte Carlo Experiment

In this section, we carry out the Monte Carlo experiments for the non-stationary models derived from model (1) using different sample sizes with replications in order to assess the accuracy of the Robinson's (1994) test for various sample sizes. Under (H_0), we simulate different models using a strong Gaussian white noise $(\varepsilon_t)_t$ with zero mean and unit variance.

We consider nine various models: one model (2), two models (3), two models (4) and four models (5). When we have only one factor in the model, the true value of the long memory parameter is $d = 0.8$; in presence of two factors, we use $d_1 = 0.8$ and $d_2 = 0.9$; in presence of three factors, we use $d_1 = 0.7$, $d_2 = 0.8$ and $d_3 = 0.9$.

We consider several sample sizes T from 100 to 3000. We do not give the results up to 3000. In all cases, we use three sizes of replication, $TT = 100, 1000, 5000$. We only present the results for $TT = 5000$, because the results are quite similar with the other two cases. The results are available upon request.

We carry out the code on the computer Dell Optiplex, written in language R. The random numbers are generated by the command "rnorm()" as the pseudo random numbers. In the

8

tables the notation $\hat{d}$ represents the estimated mean of the TT realizations $(\hat{d}_1, \ldots, \hat{d}_{TT})$ possessing the greatest $p-$value of the test. In Table 1, ..., Table 9, n represents the percentage of times that we get the true value for all the long memory parameters involved into the models.

We find that for the models with only one term, like models (2) and (5) with $k = 1$, the test performs correctly for sample size greater than 1000. However, for the models (3), although there is only one parameter to test, the test does not present good performances. The performances become correct for sample sizes equal to 2000 and 3000. The same results are observed when we simulate models with several factors, like the models (4) and (5) with $k \geq 2$. The more the explosions are inside the spectral density, the worse is the test's performance. From a general point of view, as expected, the performance of the test improves with the sample size. These results are consistent with the simulation results for the stationary long memory processes realized by Ferrara et al. (2010).

In the presence of an infinite cycle, when we simulate the models (2) and (4), the comparison of the performances of the test shows that the convergence is slower. Sometimes the test does not converge at all. We observe also that the test does not converge when we use the model (3) with $s = 12$. In any case, the test's convergence is very slow for all the models we use. As soon as we have more than one explosion, we need to use almost 1000 data to be sure to attain in mean the correct estimated value. When we have more than one explosion inside the spectral density, it appears difficult to use the test for samples whose size is smaller than 3000.

Comparing our results with that of the stationary long memory processes using Robinson's (1994) test, we find that they are quite similar. The sample size is still crucial for the accuracy of the test, which can not be too small on order to apply Robinson's (1994) test. This is not surprising since they stemmed from the same test.

The main results are the following: assuming that the noise $(\varepsilon_t)_t$ is a strong white noise in

9

T	100	300	500	700	900	1000	2000	3000
n	35.98	67.8	81.08	88.96	94.22	95.74	99.94	99.48
$\hat{d}$	0.750	0.785	0.792	0.797	0.799	0.799	0.800	0.800

Table 1: Test for model $(1 - B)^{0.8}X_t = \varepsilon_t$ where $\varepsilon_t \sim$ N(0,1).

T	100	300	500	700	900	1000	2000	3000
n	6.24	36.06	58.24	73.28	83.88	87.44	98.62	99.88
$\hat{d}$	0.596	0.730	0.760	0.775	0.785	0.788	0.799	0.800

Table 2: Test for model $(1 - B^4)^{0.8}X_t = \varepsilon_t$ where $\varepsilon_t \sim$ N(0,1).

T	100	300	500	700	900	1000	2000	3000
n	0	1.94	1.94	7.36	19.34	23.4	76.74	95.88
$\hat{d}$	0.5	0.551	0.643	0.6929	0.7172	0.7223	0.7767	0.796

Table 3: Test for model $(1 - B^{12})^{0.8}X_t = \varepsilon_t$ where $\varepsilon_t \sim$ N(0,1).

T	100	300	500	700	900	1000	2000	3000
n	2.24	17.42	32.98	46.6	56.44	61.62	88.72	95.96
$\hat{d}_1$	0.785	0.804	0.803	0.804	0.802	0.803	0.801	0.800
$\hat{d}_2$	0.658	0.819	0.855	0.870 9	0.878	0.882	0.895	0.898

Table 4: Test for model $(1 - B)^{0.8}(1 - B^4)^{0.9}X_t = \varepsilon_t$ where $\varepsilon_t \sim$ N(0,1).

T	100	300	500	700	900	1000	2000	3000
n	0	0.78	3.74	10.9	21.04	24.04	65.92	87.94
$\hat{d}_1$	0.753	0.807	0.804	0.8044	0.8045	0.804	0.802	0.800
$\hat{d}_2$	0.501	0.629	0.737	0.7875	0.8186	0.823	0.870	0.889

Table 5: Test for model $(1 - B)^{0.8}(1 - B^{12})^{0.9}X_t = \varepsilon_t$ where $\varepsilon_t \sim$ N(0,1).

all the models:

- For the model (2), $\hat{d} = 0.8$ when T reaches 2000.

10

T	100	300	500	700	900	1000	2000	3000
n	36.38	69.28	81.5	89.5	94.1	94.54	99.54	99.98
$\hat{d}$	0.749	0.786	0.793	0.797	0.799	0.799	0.800	0.800

Table 6: Test for model $(1 - 2\nu B + B^2)^{0.4} X_t = (1 + B)^{0.8} = \varepsilon_t$ where $\varepsilon_t \sim$ N(0,1).

T	100	300	500	700	900	1000	2000	3000
n	16.06	39.24	54.94	61.54	69.9	71.82	89.8	95.82
$\hat{d}$	0.674	0.747	0.764	0.770	0.777	0.779	0.792	0.797

Table 7: Test for model $(1 - 2\nu B + B^2)^{0.8} X_t = \varepsilon_t$, $\nu = \cos\frac{\pi}{3}$ and $\varepsilon_t \sim$ N(0,1).

T	100	300	500	700	900	1000	2000	3000
n	1.56	19	37	52.38	66.76	70.54	93.82	98.82
$\hat{d_1}$	0.556	0.701	0.741	0.760	0.776	0.780	0.796	0.799
$\hat{d_2}$	0.713	0.830	0.858	0.871	0.882	0.885	0.897	0.900

Table 8: Test for model $(1 - 2\nu_1 B + B^2)^{0.8}(1 - 2\nu_2 B + B^2)^{0.9} X_t = \varepsilon_t$, $\nu_1 = \cos\frac{\pi}{3}, \nu_2 = \cos\frac{5\pi}{6}$ and $\varepsilon_t \sim$ N(0,1).

T	100	300	500	700	900	1000	2000	3000
n	0.28	12.46	33.74	56.04	73.2	79.02	98.56	99.96
$\hat{d_1}$	0.545	0.631	0.659	0.676	0.687	0.690	0.699	0.699
$\hat{d_2}$	0.665	0.746	0.769	0.781	0.788	0.792	0.800	0.800
$\hat{d_3}$	0.646	0.812	0.849	0.871	0.884	0.887	0.899	0.900

Table 9: Test for model $(1 - 2\nu_1 B + B^2)^{0.2}(1 - 2\nu_2 B + B^2)^{0.3}(1 - 2\nu_3 B + B^2)^{0.4} X_t = \varepsilon_t$, $\nu_1 = \cos\frac{\pi}{6}, \nu_2 = \cos\frac{\pi}{2}, \nu_3 = \cos\frac{2\pi}{3}$ and $\varepsilon_t \sim$ N(0,1).

- For the models (3), $\hat{d} = 0.8$, for $s = 4$ when T reaches 2000, and the test does not converge when we use $s = 12$.

- For the models (4), $\hat{d_1} = 0.800$, $\hat{d_2} = 0.898$, for $s = 4$ and $\hat{d_1} = 0.800$, $\hat{d_2} = 0.889$ for $s = 12$, when T reaches 3000.

11

- For the 1-factor model (5),

    1. If $\nu = -1$, $\hat{d} = 0.800$ when T reaches 2000.

    2. If $\nu = cos(\pi/3)$, $\hat{d} = 0.797$ when T reaches 3000.

- For a 2-factors model (5), $\hat{d}_1 = 0.799$ and $\hat{d}_2 = 0.900$ when T reaches 3000.

- For a 3-factors model (5), $\hat{d}_1 = 0.699$, $\hat{d}_2 = 0.800$ and $\hat{d}_3 = 0.900$ when T reaches 3000.

4 Empirical Application

In this section, we explore the 23 series of daily foreign exchange rates: Brazilian Reals to one U.S. Dollar, Canadian Dollar to one U.S. Dollar, Chinese Yuan to one U.S. Dollar, Danish Kroner to one U.S. Dollar, Hong Kong Dollar to one U.S. Dollar, Indian Rupees to one U.S. Dollar, Japanese Yen to U.S. Dollar, South Korean Won to one U.S. Dollar, Malaysian Ringgit to one U.S. Dollar, Mexican New Pesos to one U.S. Dollar, Norwegian Kroner to one U.S. Dollar, Swedish Kroner to one U.S. Dollar, South African Rand to one U.S. Dollar, Singapore Dollar to one U.S. Dollar, Sri Lankan Rupees to one U.S. Dollar, Swiss Francs to one U.S. Dollar, New Taiwan Dollar to one U.S. Dollar, Thai Baht to one U.S. Dollar, U.S. Dollar to one Australian Dollar, U.S. Dollar to one Euro, U.S. Dollar to one New Zealand Dollar, U.S. Dollar to one British Pound, Venezuelan Bolivares to one U.S. Dollar. The data are obtained from the Board of Governors of the Federal Reserve System. We include all the historical available data in this study from 1971/01/04 to 2010/05/14. Noon buying rates in New York City for cable transfers is payable in foreign currencies.

First of all, we explore the graphical characteristics, such as trajectory, autocorrelation function (ACF), spectrum and histogram of these 23 series, see Figure 1, ..., Figure 23.

Therefore, we decide to model the series by the model

$$(I - B)^d = \varepsilon_t, \tag{8}$$

12
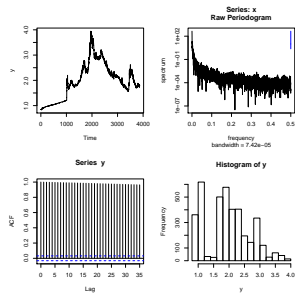
Figure 1: Trajectory, ACF, spectrum and histogram of Brazil/U.S. exchange rate.
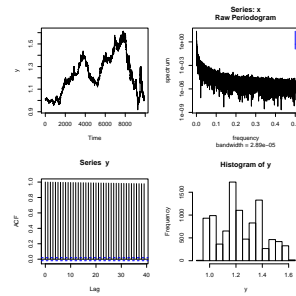
Figure 2: Trajectory, ACF, spectrum and histogram of Canada/U.S. exchange rate.
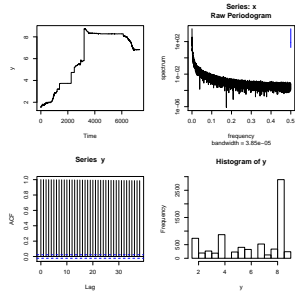
Figure 3: Trajectory, ACF, spectrum and histogram of China/U.S. exchange rate.
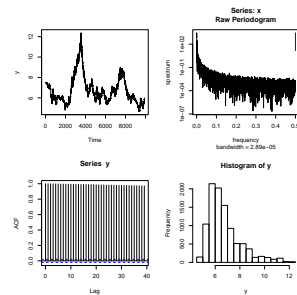
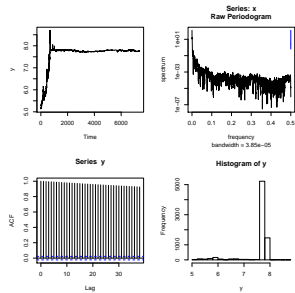Figure 4: Trajectory, ACF, spectrum and histogram of Denmark/U.S. exchange rate.

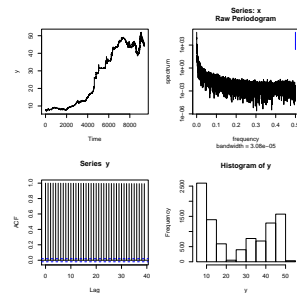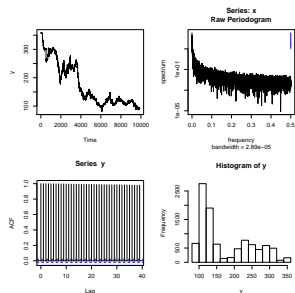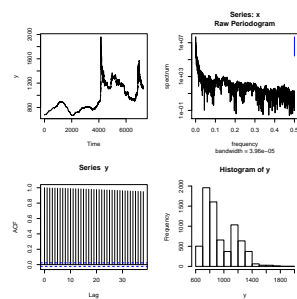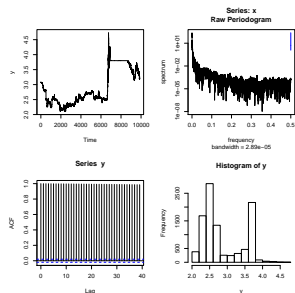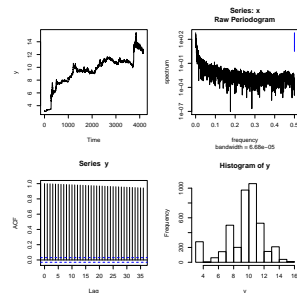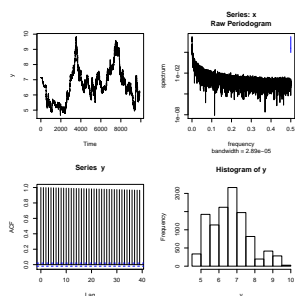Figure 5: Trajectory, ACF, spectrum and histogram of Hong Kong/U.S. exchange rate.

Figure 6: Trajectory, ACF, spectrum and histogram of India/U.S. exchange rate.

13

Figure 7: Trajectory, ACF, spectrum and histogram of Japan/U.S. exchange rate.

Figure 8: Trajectory, ACF, spectrum and histogram of South Korea/U.S. exchange rate.

Figure 9: Trajectory, ACF, spectrum and histogram of Malaysia/U.S. exchange rate.

Figure 10: Trajectory, ACF, spectrum and histogram of Mexico/U.S. exchange rate.

Figure 11: Trajectory, ACF, spectrum and histogram of Norway/U.S. exchange rate.
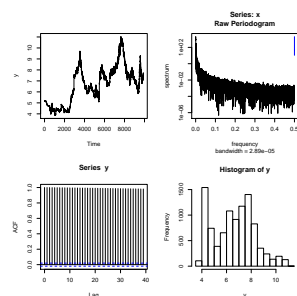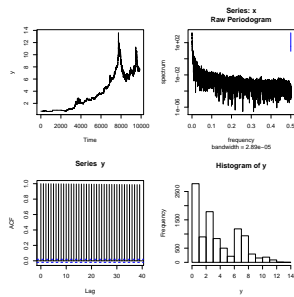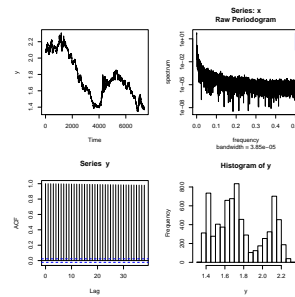

Figure 12: Trajectory, ACF, spectrum and histogram of Sweden/U.S. exchange rate.

14

Figure 13: Trajectory, ACF, spectrum and histogram of South Africa/U.S. exchange rate.

Figure 14: Trajectory, ACF, spectrum and histogram of Singapore/U.S. exchange rate.
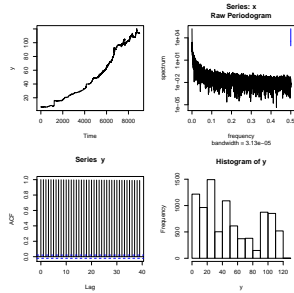

Figure 15: Trajectory, ACF, spectrum and histogram of Sri Lanka/U.S. exchange rate.
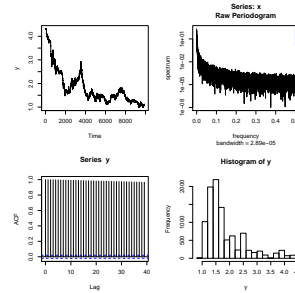

Figure 16: Trajectory, ACF, spectrum and histogram of Switzerland/U.S. exchange rate.
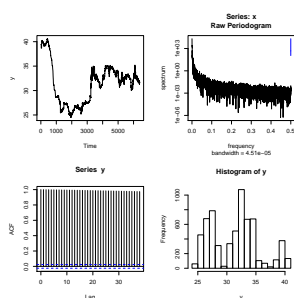

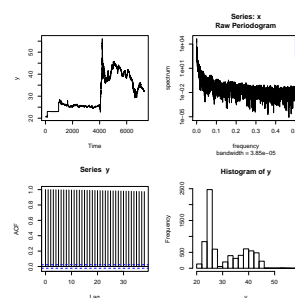Figure 17: Trajectory, ACF, spectrum and histogram of Taiwan/U.S. exchange rate.

Figure 18: Trajectory, ACF, spectrum and histogram of Thailand/U.S. exchange rate.
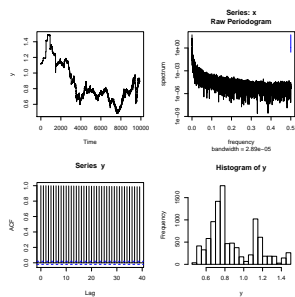
15

Figure 19: Trajectory, ACF, spectrum and histogram of U.S./Australia exchange rate.
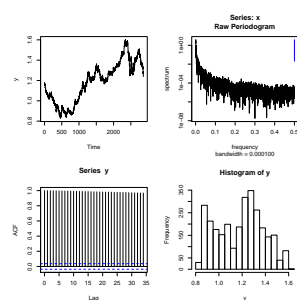

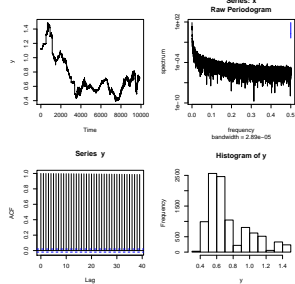Figure 20: Trajectory, ACF, spectrum and histogram of U.S./EU exchange rate.

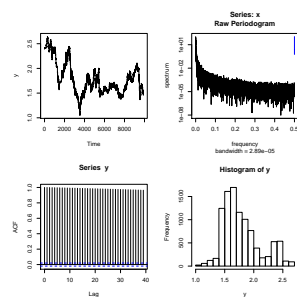Figure 21: Trajectory, ACF, spectrum and histogram of U.S./New Zealand exchange rate.

Figure 22: Trajectory, ACF, spectrum and histogram of U.S./UK exchange rate.
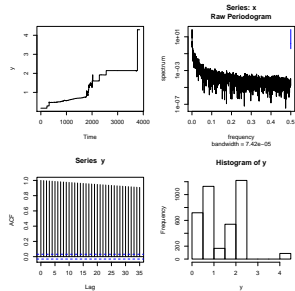

Figure 23: Trajectory, ACF, spectrum and histogram of Venezuela/U.S. exchange rate.

16

where d is a real number and ε_t is an innovation. We adopt Robinson's (1994) test as the estimation method. Since the sample sizes of these 23 series are all large enough, the accuracy of the estimation is ensured according to the Monte Carlo simulation results obtained in the previous section. The estimation results are presented in the Table 10.

Reading the table, we find that the estimates for the parameter d in model (2) are all close to 1, which indicates the existence of unit roots. More precisely, We can classify the series into three classes. There are nine exchange rate series possessing the parameters d which are greater than 1, i.e., Brazilian Reals/U.S. Dollar, Canadian Dollar/U.S. Dollar, Japanese Yen/U.S. Dollar, South Korean Won/U.S. Dollar, Malaysian Ringgit/U.S. Dollar, Swiss Francs/U.S. Dollar, New Taiwan Dollar/U.S. Dollar, Thai Baht/U.S. Dollar and U.S. Dollar/British Pound. Besides, there are thirteen series possessing the parameter d slightly smaller than 1, i.e., Danish Kroner/U.S. Dollar, Hong Kong Dollar/U.S. Dollar, Indian Rupees/U.S. Dollar, Mexican New Pesos/U.S. Dollar, Norwegian Kroner/U.S. Dollar, Swedish Kroner/U.S. Dollar, South African Rand/U.S. Dollar, Singapore Dollar/U.S. Dollar, U.S./Australian Dollar, U.S. Dollar/Euro, U.S. Dollar/New Zealand Dollar and Venezuelan Bolivares/U.S. Dollar. The third class is the Chinese Yuan/U.S. Dollar, whose parameter d equals strictly to 1. Thus, we can have a basic understanding of the difference and similarity between the exchange rate series of 23 countries and regions.

5   Conclusion

In this paper, we first evaluate the performances of the Robinson's (1994) test for several simulated SCLM models in the non-stationary setting. We show that the sample size is crucial for the accuracy of the test in the non-stationary setting. It appears that the use of this test is mainly recommended when we observe only one explosion in the spectral density if we have at least 1000 points. If more than one explosion are present inside the spectral density, this test does not provide accurate information if the sample size of the data set is

Exchange Rate Series	#Observation	estimated parameter $\hat{d}$	p-value
Brazilian Reals to one U.S. Dollar	3865	1.013	0.9892759
Canadian Dollar to one U.S. Dollar	9889	1.008	0.9787710
Chinese Yuan to one U.S. Dollar	7323	1.000	0.9782972
Danish Kroner to one U.S. Dollar	9882	0.976	0.9610163
Hong Kong Dollar to one U.S. Dollar	7383	0.967	0.9672684
Indian Rupees to one U.S. Dollar	9375	0.98	0.9831581
Japanese Yen to U.S. Dollar	9877	1.027	0.9823676
South Korean Won to one U.S. Dollar	7269	1.004	0.9610163
Malaysian Ringgit to one U.S. Dollar	9861	1.012	0.9714096
Mexican New Pesos to one U.S. Dollar	4151	0.957	0.9886739
Norwegian Kroner to one U.S. Dollar	9882	0.994	0.9718242
Swedish Kroner to one U.S. Dollar	9882	0.996	0.9909451
South African Rand to one U.S. Dollar	9856	0.986	0.9922439
Singapore Dollar to one U.S. Dollar	7382	0.965	0.9907880
Sri Lankan Rupees to one U.S. Dollar	9023	0.951	0.9726769
Swiss Francs to one U.S. Dollar	9883	1.010	0.9820905
New Taiwan Dollar to one U.S. Dollar	6396	1.009	0.9733031
Thai Baht to one U.S. Dollar	7302	1.002	0.9935269
U.S. Dollar to one Australian Dollar	9867	0.996	0.9754904
U.S. Dollar to one Euro	2862	0.970	0.9977455
U.S. Dollar to one New Zealand Dollar	9867	0.988	0.9972029
U.S. Dollar to one British Pound	9883	1.030	0.9978828
Venezuelan Bolivares to one U.S. Dollar	3859	0.996	0.9838211

Table 10: Estimation Results for the long memory parameter in model (8) using Robinson's (1994) test.

less than 3000. This latter result raises concern as regards the applications of Robinson's (1994) test to seasonal macroeconomics data.

The application of Robinson's (1994) test on the popular 23 daily foreign exchange rate series indicates the existence of unit roots and some kind of long rang dependence. According to the values of estimations of the parameters, we classified the series into three classes, which indicates the similar GDP behavior with respect to the long memory parameter in the same class.

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