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A price uncertainty principle and the existence of sequential equilibrium: (I) a numerical example

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Abstract

In three related papers, we consider a pure exchange financial economy, where agents may observe private information signals, form private anticipations and face an 'exogenous uncertainty', on the future state, and an 'endogenous uncertainty', on the future prices. At a sequential equilibrium, all agents expect the 'true' price as a possible outcome, and elect optimal strategies, which clear on all markets at every time period. This concept differs from both traditional ones of temporary equilibrium and sequential equilibrium with perfect foresight. The first paper, developed hereafter, illustrates, on a heuristic example, why changing anticipations may alter equilibrium prices and allocations, explain bubbles or crashes on markets at equilibrium, or preclude any perfect price foresight. The second paper shows that correct anticipations need always embed a set of 'minimum uncertainty', depending on observed prices and the fundamental characteristics of the economy, and studies the properties of this set. The third paper proves, in the complete model, that the existence of a sequential equilibrium is still characterized by the no-arbitrage condition.

Key words: sequential equilibrium, temporary equilbrium, anticipations, endogenous uncertainty, incomplete markets, asymmetric information, arbitrage, existence.

JEL Classification: D52

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1 Introduction

The traditional approach to sequential equilibrium, relies on Radner's (1972), assumption that agents have '*perfect foresight*', that is, they anticipate with certainty exactly one price for each commodity (or asset) in each random state, which turns out to be the true price, if that state prevails tomorrow. A convenient outcome of perfect foresight is that equilibrium unfolds sequentially: ex post, agents would never face bankruptcy, or need retrade to optimize their welfare, while all markets clear at agents' ex ante optimal decisions. This outcome, however, does not require perfect foresight. Moreover, the standard, but restrictive, assumption of perfect foresight precludes the study of how erroneous beliefs by (uncertain) agents today may affect the value of prices and allocations tomorrow, in the sequential equilibrium model. It also restricts, in our view unduely, the scope of this model.

The alternative classical approach to equilibrium, called '*temporary equilibrium*', introduced by Hicks (1939) and developed by Grandmont (1977, 1982), Green (1973), Hammond (1983), Balasko (2003), among others, deals with exogenous expectations (and a possible uncertainty) about future prices. With exogenous beliefs on the endogenous price variable, the question arises whether and by what means tomorrow's equilibrium price needs be in every agent's expectations today. If not, the equilibrium may not be sequential, but temporary, that is, only current markets need clear at agent's ex ante optimal decisions, while some agents, observing unexpected prices on markets, may face bankruptcy or choose to retrade ex post.

Our model with private anticipations explores a route in between the two classical ones. We show (in the third paper) that a sequential equilibrium always exists in a standard economy with arbitrage-free markets, under a mild rationality assumption on agents, whose anticipations are prone to a minimum uncertainty. We show there may exist an incompressible set of plausible prices (which depends on observed prices and fundamental characteristics of the economy), any of which may obtain as a clearing market price tomorrow (depending on all agents' private anticipations).

This first paper introduces the basic concepts and results of our model on a heuristic economy with two time-periods, two agents and two markets, a commodity market and a financial market. To simplify presentation, we let agents face no 'exogenous uncertainty', on the state of nature, but (possibly) an 'endogenous uncertainty' on the future spot price, upon which each agent forms private anticipations. At a sequential equilibrium of this economy, the two agents expect the 'true' price as a possible outcome tomorrow, and elect optimal strategies, which clear on all markets at both periods. Throughout, the fundamentals of the economy (endowments, preferences, assets) are fixed and we let anticipations vary.

The paper is organized as follows.

Section 2 presents the heuristic economy.

On a first numerical application, Section 3 displays a continuum of sequential equilibrium allocations, depending on both agents' anticipations. It illustrates how speculation, bubbles or crashes on markets may be explained by anticipations, as a sequential equilibrium outcome, in the absence of any 'real' shock on the economy.

On a second example, Section 4, displays a continuum of equilibrium prices, related to a continuum of anticipations. It illustrates why a correct forecast of future prices may require some uncertainty, or caution in anticipation, which precludes perfect foresight, when agents' forecasts are private. In the example, the continuum of equilibrium prices tomorrow is associated to a common equilibrium price at the first period, so agents can infer no information from observing current prices.

2 The basic model

We consider a pure exchange economy with two periods, or dates, $t \in \{0, 1\}$, with two agents, $i \in \{0, 1\}$, L consumption goods, $l \in \{1, ..., L\}$, and one asset, traded at $t = 0$ and paying one unit of account at $t = 1$.

Each agent $i \in \{0, 1\}$ receives a bundle of commodities, $e_i(0) \in \mathbb{R}^L_+$, at $t = 0$, and $e_i(1) \in \mathbb{R}^L_+$, at $t = 1$, and we let $e_i := (e_i(0), e_i(1)) \in \mathbb{R}^{2L}_+$ be her total endowment. The commodity price, $(p(0), p(1)) \in \mathbb{R}^{2L}_+$, embeds the spot prices $p(0) := (p^l(0)) \in \mathbb{R}^L_+$, that agents observe at $t = 0$, and $p(1) := (p^l(1)) \in \mathbb{R}^L_+$, they will observe at $t = 1$. The asset price, denoted by $q \in \mathbb{R}_+$, is observed by agents at $t = 0$.

The only uncertainty the generic agent $i \in \{1, 2\}$ may face at $t = 0$ is an endogenous uncertainty on the spot price $p(1)$, represented by her anticipation probability law, π_i, on $\mathbb{R}^L_+$, whose support is denoted by $P(\pi_i)$, and, for simplicity, assumed to be finite. A given pair of discrete probabilities (π_1, π_2) on $\mathbb{R}^L_+$, whose supports are finite subsets of $\mathbb{R}^L_{++}$, will be called a (price) structure and we will denote their set by Π.

Let $(\pi_1, \pi_2) \in \Pi$, and, for each $i \in \{1, 2\}$, let $P(\pi_i) := \{p \in \mathbb{R}_+ : \pi_i(p) > 0\}$ and $n(\pi_i) := \#P(\pi_i)$ be given. The generic i^{th} agent has $X(\pi_i) := \mathbb{R}^{(1+n(\pi_i))L}_+$ for consumption set. Namely, a consumption plan $x \in X(\pi_i)$, embeds a quantity bundle $x(0) := (x^l(0)) \in \mathbb{R}^L_+$ of commodities, that the i^{th} agent decides consume at $t = 0$, and, for each expected price, $p_i \in P(\pi_i)$, a conditional bundle of goods, $x(p_i) := (x^l_i(p_i)) \in \mathbb{R}^L_+$, that she plans to consume at $t = 1$, if price p_i is observed on the spot market.

Consumption plans are ordered. Namely, at $t = 0$, the generic i^{th} agent has preferences over consumption plans, represented by a V.N.M. utility function, $u^{\pi_i}_i : x \in \mathbb{R}^{(1+n(\pi_i))L}_+ \mapsto \sum_{p_i \in P(\pi_i)} \pi_i(p_i) v_i(x(0), x(p_i))$, where $v_i : (x, y) \in \mathbb{R}^{2L}_+ \to \mathbb{R}$ defines her ex post utility function over consumptions.

At $t = 0$, the generic i^{th} agent elects a strategy, $(x, z) \in X(\pi_i) \times Z$, consisting of a consumption plan $x \in X(\pi_i) := \mathbb{R}_+^{(1+n(\pi_i))L}$, as defined above, and a portfolio $z \in \mathbb{R}$ (or quantity of asset), which she decides to buy (if $z > 0$), or sell (if $z < 0$), on the financial market. Thus, for every price $(p(0), q) \in \mathbb{R}_+^{L+1}$, she may observe at $t = 0$, her budget set is defined as follows:

$$B_i(p(0), q, \pi_i) := \left\{ (x, z) \in \mathbb{R}_+^{(1+n(\pi_i))L} \times \mathbb{R} : \left[\begin{array}{l} p(0) \cdot (x(0) - e_i(0)) \leqslant -qz \\ p_i \cdot (x(p_i) - e_i(1)) \leqslant z, \ \forall \ p_i \in P(\pi_i) \end{array} \right. \right\}.$$

It represents the set of strategies, which are affordable at all price the i^{th} agent observes or expects. The agent's behavior will be to elect a strategy at $t = 0$, which maximizes the utility function in the budget set.

We denote the above economy by $\mathcal{E}$ and define its equilibrium notions as follows.

Definition 1 *The collection of prices* $(p(0), p(1)) \in \mathbb{R}_+^{2L}$*, on the spot markets, and* $q \in \mathbb{R}_+$*, on the financial market, of a price structure* $(\pi_1, \pi_2) \in \Pi$*, and of strategies,* $(x_1, z_1) \in B_1(p(0), q, \pi_1)$ *&* $(x_2, z_2) \in B_2(p(0), q, \pi_2)$*, is a sequential equilibrium (respectively, a temporary equililibrium) of the economy* $\mathcal{E}$*, or 'correct foresight equilibrium' (CFE), if the following Conditions* (a)*-*(b)*-*(c)*-*(d) *(resp., Conditions* (b)*-*(c)*-*(d)*) hold:*

(a) $p(1) \in P(\pi_1) \cap P(\pi_2)$;

(b) $(x_1, z_1) \in \arg\max_{(x,z) \in B_1(p(0),q,\pi_1)} u_1^{\pi_1}(x)$ *and* $(x_2, z_2) \in \arg\max_{(x,z) \in B_2(p(0),q,\pi_2)} u_2^{\pi_2}(x)$;

(c) $x_1(0) + x_2(0) = e_1(0) + e_2(0)$ *and* $x_1(p(1)) + x_2(p(1)) = e_1(1) + e_2(1)$*, if* $p(1) \in P(\pi_1) \cap P(\pi_2)$;

(d) $z_1 + z_2 = 0$.

Thus, at a CFE, all agents have the true price in their anticipations and elect optimal strategies, at $t = 0$, given their budget constraints, which clear on all markets at both periods. Ex post, equilibrium consumption decisions remain optimal (given agents' endowments and portfolios), so no agent needs retrade. It is not so, in general, at a temporary equilibrium.

In the economy $\mathcal{E}$, the endowments, (e_1, e_2), financial structure and preference relations, (v_1, v_2), are the fundamentals, and the price structure, $(\pi_1, \pi_2) \in \Pi$, is exogenous. In the following Sections, we set the fundamentals fixed and let (π_1, π_2) vary in Π, and we study the corresponding CFE on numerical applications.

3 A first application: speculation and crashes

The classical property of local uniqueness of sequential equilibrium stems from the single price expectation assumption (implied by perfect foresight). Dropping that assumption, we present a heuristic economy, where sequential equilibria vary continuously with expectations. The example illustrates how anticipations can explain market speculation or crashes, when no shock on fundamentals takes place.

Throughout, we let, for simpler calculations, $L = 1$, normalize the spot prices to one (i.e., $p(0) = p(1) = 1$), and set as given endowments and preferences as follows:

$e_1 := (1, 3)$ and $e_2 := (3, 1)$;

$v_1 : (x, y) \in \mathbb{R}^2_+ \mapsto x + y$ and $v_2 : (x, y) \in \mathbb{R}^2_+ \mapsto \sqrt{xy}$.

We consider the following cases of price structures ($(\pi_1, \pi_2) \in \Pi$) and corresponding sequential equilibria, where the second agent always has perfect foresight.

3.1 Stable beliefs & perfect foresight

If both agents observe the asset price $q = 1$ at $t = 0$, and anticipate with certainty the true price, $p(1) = 1$, for the next period, the reader will readily check (on colinear gradients) there exists exactly one corresponding equilibrium with perfect foresight, which is symmetric and defined by:

$p(0) = p(1) = 1$, $q = 1$, $x_1 = x_2 = (2, 2)$ and $z_2 = -z_1 = 1$.

3.2 Pessimism and crash

We now assume that the second agent only has a perfect foresight of $p := p(1) = 1$. With above notations, her utility function and (satiated) budget constraints are:

$$x_2 \in \mathbb{R}^2_+ \mapsto u_2(x_2) := \sqrt{x_2(0)x_2(p)} \text{ and } \begin{cases} x_2(0) = 3 - qz_2 \\ x_2(p) = 1 + z_2 \end{cases}.$$

On the other hand, agent $i = 1$ is now 'pessimistic' by anticipating, at $t = 0$, both the true price, $p := p(1) = 1$, and $p_1 = \frac{1}{3}$ with probability $\frac{1}{2}$, for the second period. Her (ex ante) utility function and (satiated) budget constraints are then, respectively:

$$x_1 \in \mathbb{R}^3_+ \mapsto u_1(x_1) := x_1(0) + \tfrac{1}{2}(x_1(p) + x_1(p_1)) \text{ and } \begin{cases} x_1(0) = 1 - qz_1 \\ x_1(p) = 3 + z_1 \\ x_1(p_1) = 3 + 3z_1 \end{cases}.$$

We let the reader check, from utility functions, and first order and market clearing conditions, that there exists exactly one corresponding CFE, defined by:

$$p(0) = p(1) = 1,\ q = 2,\ x_1 = (\tfrac{3}{2}, \tfrac{11}{4}, \tfrac{9}{4}),\ x_2 = (\tfrac{5}{2}, \tfrac{5}{4}) \text{ and } z_2 = -z_1 = \tfrac{1}{4}.$$

Ex post, the second agent looses in welfare (as compared with the above perfect foresight equilibrium), whereas the first agent achieves a small welfare increase (since $v_1(x_1(0), x_1(p)) > 4$), owing to better terms of trade on the asset. The cost of credit doubles ($q = 2$), or, similarly, the relative price of the commodity falls. Trade shrinks by three fourth, while the economy had suffered no 'real' shock (i.e., on the fundamentals). Only (the first agent's) beliefs are responsible for the trade and credit crunch and the change in the equilibrium allocation and relative prices.

We now examine what happens when the first agent's 'pessimism' varies, while the second remains perspicuous. Assume that the first agent anticipates $p_1 := \frac{1}{3}$, with an arbitrary probability $\varepsilon < 1$, and $p := p(1) = 1$, with probability $1-\varepsilon$, and the second agent has perfect foresight of $p := p(1) = 1$. The second agent's utility function and budget constraints are as above, while the first agent's are as follows:

$$x_1 \in \mathbb{R}^3_+ \mapsto u_1(x_1) := x_1(0) + (1-\varepsilon)x_1(p) + \varepsilon x_1(p_1)) \text{ and } \begin{cases} x_1(0) = 1 - qz_1 \\ x_1(p) = 3 + z_1 \\ x_1(p_1) = 3 + 3z_1 \end{cases}.$$

By the same token, we let the reader check, from the first order and market clearance conditions and from the utility functions, that there exists one single CFE corresponding to the above spot prices and anticipations, defined by:

$$p(0) = p(1) = 1,\ q^\varepsilon = 1{+}2\varepsilon,\ x_1^\varepsilon = (2\text{-}\varepsilon, \tfrac{2+7\varepsilon}{1+2\varepsilon}, \tfrac{9\varepsilon}{1+2\varepsilon}),\ x_2^\varepsilon = (2{+}\varepsilon, \tfrac{2+\varepsilon}{1+2\varepsilon}) \text{ and } z_2^\varepsilon = -z_1^\varepsilon = \tfrac{1-\varepsilon}{1+2\varepsilon}.$$

We notice that, when ε inreases continuously in $\varepsilon \in [0,1[$, that is, when certainty about the true price decreases, there exists a continuum of associated equilibria, which vary from the perfect foresight equilibrium of sub-Section 3.1 to vanishing-trade equilibria. At equilibrium, the relative price of the commodity and trade decrease continuously with $\varepsilon \in [0,1[$, while the asset price increases (from 1 to 3).

For the perspicuous agent, $i = 2$, the utility of equilibrium consumption is the same ex ante and ex post, namely, $u_2(x_2^\varepsilon) = v_2(x_2^\varepsilon) = \frac{(2+\varepsilon)}{\sqrt{1+2\varepsilon}}$, and decreases in $\varepsilon \in [0,1[$. Agent $i = 2$ always has an incentive to trade (so, agents exchange). But an increase in confidence by her partner (i.e., a fall in ε) boosts trade and increases her welfare.

For the uncertain agent, $i = 1$, the ex ante utility of equilibrium consumption is constant, namely, $u_1(x_1^\varepsilon) = 4$. But the ex post utility, $v_1(x_1^\varepsilon) := \frac{2(2+5\varepsilon-\varepsilon^2)}{1+2\varepsilon}$, increases with $\varepsilon \in [0, \frac{\sqrt{3}-1}{2}]$ and decreases for higher values. Thus, the the first agent's uncertainty is rewarding, via better terms of trade ($q^\varepsilon = 1 + 2\varepsilon$), up to an inflexion point. Moreover, the agent's welfare is the same (ex post) at both limits (whether perfectly informed or disinformed), that is: $\lim_{\varepsilon \to 1} v_1(x_1^\varepsilon) = \lim_{\varepsilon \to 0} v_1(x_1^\varepsilon) = 4$.

3.3 Volatility, speculation, bubbles

As previously, the second agent remains realistic and anticipates the true price $p := p(1) = 1$ with probability 1. Her utility function and budget constraints are the same as above. But the first agent is now optimistic about the value of her future endowment, and anticipates both prices $p = 1$ and $p_1 = 2$ with probability $\frac{1}{2}$. In that case, her (ex ante) utility function and (satiated) budget constraints become:

$$x_1 \in \mathbb{R}^3_+ \mapsto u_1(x_1) := x_1(0) + \tfrac{1}{2}(x_1(p) + x_1(p_1)) \text{ and } \begin{cases} x_1(0) = 1 - qz_1 \\ x_1(p) = 3 + z_1 \\ x_1(p_1) = 3 + \frac{z_1}{2} \end{cases} .$$

We let the reader check there exists exactly one related equilibrium, defined by:

$$p(0) = p(1) = 1,\ q = \tfrac{3}{4},\ x_1 = (\tfrac{17}{8}, \tfrac{3}{2}, \tfrac{9}{4}),\ x_2 = (\tfrac{15}{8}, \tfrac{5}{2}) \text{ and } z_2 = -z_1 = \tfrac{3}{2}.$$

We notice that the perspicuous agent ($i = 2$) now gains in welfare (compared with the perfect foresight's situation), whereas the optmistic agent ($i = 1$), keeping the same ex ante utility ($u_1(x_1) = 4$), looses in welfare ex post ($v_1(x_1) = \frac{29}{8} < 4$), due to worse terms of trade on the asset. The commodity price increases relative to the price of money. In that sense, anticipations are self-fulfilling, explaining speculation, the more so since the first agent's optimism now boosts trade. Again, beliefs affect equilibrium prices and allocations, in the absence of any 'real' shock.

Thus, the heuristic economy suggests how speculation, bubbles and crashes, as well as long run effects on equilibrium allocations, can stem from a simple change in anticipations (and no change in fundamentals). These phenomena appear at sequential equilibrium, when endogenous uncertainty prevails on competitive markets.

We now examine why a single price expectation by agents - as implied by the perfect foresight assumption - may lead to their incorrect forecast of future prices.

4 A second application: correct, versus perfect, foresight

While the fundamentals are kept fixed, we now show that a continuum of spot prices and related sequential equilibria may result from various anticipations. We infer that agents who form their anticipations privately may not have perfect foresight. Throughout, we let $L = 2$ and set endowments and preferences as follows:

$e_1 := ((1,1),(1,5))$ and $e_2 := ((3,3),(1,1))$;

$v_1 : (x,y,z,t) \in \mathbb{R}^4_+ \mapsto x+y+z+t$ and $v_2 : (x,y,z,t) \in \mathbb{R}^4_+ \mapsto \sqrt{xyzt}$.

From first order and market clearance conditions, equilibrium prices and consumptions need be symmetric in $l \in \{1,2\}$ at $t = 0$. So, we let $p(0) = (1,1)$ be the spot price and $x_i(0) = (x_i^0, x_i^0) \in \mathbb{R}^2_+$ be the generic i^{th} agent's consumption, at $t = 0$.

4.1 A perfect foresight equilibrium

If both agents observe the market prices, $p(0) = (1,1)$ and $q = 1$, at $t = 0$, and anticipate with certainty that the spot price will not change at $t = 1$, the reader will check (on colinear gradients) there exists exactly one related perfect foresight equilibrium, defined by: $p(0) = p(1) = (1,1)$, $q = 1$, $x_1 = x_2 = (2,2,2,2)$, $z_2 = -z_1 = 2$.

4.2 Endogenous uncertainty

We now assume that the first agent's price expectations, namely, $p_1 = (1, \frac{1}{2})$, $p_2 = (1,2)$ and $p_\varepsilon := (p_\varepsilon^1, p_\varepsilon^2) := (\frac{4}{5}\text{-}\frac{\varepsilon}{7}, \frac{4}{5}\text{-}\varepsilon)$, are endowed with a probability distribution π_1^ε, such that $\pi_1^\varepsilon(p_1) = \frac{1}{20}\text{-}\varepsilon$, $\pi_1^\varepsilon(p_2) = \frac{3}{20}\text{+}2\varepsilon$ and $\pi_1^\varepsilon(p_\varepsilon) = \frac{4}{5}\text{-}\varepsilon$ (depending on a parameter ε, e.g., $\varepsilon \in [0, \frac{1}{100}]$). Her utility function and (satiated) budget constraints are:

$$x_1 \in \mathbb{R}^6_+ \mapsto u_1(x_1) := 2x_1^0 + \pi_1^\varepsilon(p_1)x_1^2(p_1) + \pi_1^\varepsilon(p_2)x_1^1(p_2) + \pi_1^\varepsilon(p_\varepsilon)x_1^2(p_\varepsilon)$$

$$\text{and} \begin{cases} 2(x_1^0 - 1) = -qz_1 \\ p_1 \cdot (x_1(p_1) - (1,5)) = z_1 \\ p_2 \cdot (x_1(p_2) - (1,5)) = z_1 \\ p_\varepsilon \cdot (x_1(p_\varepsilon) - (1,5)) = z_1 \end{cases} .$$

The second agent is assumed to anticipate $p_\varepsilon := (p_\varepsilon^1, p_\varepsilon^2)$ with a probability 1 and we let π_2^ε be the corresponding probability on $\mathbb{R}^2_+$. Her utility function and (satiated) budget constraints are the following:

$$x_2 \in \mathbb{R}^4_+ \mapsto u_2(x_2) := x_2^0\sqrt{x_2^1(p_\varepsilon)x_2^2(p_\varepsilon)} \text{ and } \begin{cases} 2(x_2^0 - 3) = -qz_2 \\ p_\varepsilon \cdot (x_2(p_\varepsilon) - (1,1)) = z_2 \end{cases} .$$

This defines an economy, $\mathcal{E}_\varepsilon$, of the second Section's type, having $(\pi_1^\varepsilon, \pi_2^\varepsilon)$ for price structure. We let the reader check (as tedious and little instructive), on utility functions, first order and clearing markets conditions, that, for every $\varepsilon \in [0, \frac{1}{100}]$, the economy $\mathcal{E}_\varepsilon$ admits a sequential equilibrium along Definition 1 with:

- $p(0) = (1,1)$ and $p_\varepsilon := (p_\varepsilon^1, p_\varepsilon^2) := (\frac{4}{5} - \frac{\varepsilon}{7}, \frac{4}{5} - \varepsilon)$ for spot prices (at $t = 0$ and $t = 1$);
- $q = \frac{5}{4}$ for asset price;
- $(\pi_1^\varepsilon, \pi_2^\varepsilon)$ for price structure;
- $z_2 = \frac{56+20\varepsilon}{35}$ and $z_1 = -\frac{56+20\varepsilon}{35}$ for agents' portfolios;
- $x_1^\varepsilon = ((\frac{28+5\varepsilon}{14}, \frac{28+5\varepsilon}{14}), (0, \frac{133-40\varepsilon}{35}), (\frac{329-20\varepsilon}{35}, 0), (0, \frac{112-200\varepsilon}{28-35\varepsilon}))$ for the first agent's plan;
- $x_2^\varepsilon := ((\frac{28-5\varepsilon}{14}, \frac{28-5\varepsilon}{14}), (2, \frac{56-10\varepsilon}{28-35\varepsilon}))$ for the second agent's consumption plan.

We now consider an economy, where the first agent's beliefs are volatile. They are always represented by a probability law of the type π_1^ε, but with a parameter,

$\varepsilon \in [0, \frac{1}{100}]$, which is prone to change (throughout $[0, \frac{1}{100}]$) during the first period, until the time when the agent elects and implements her strategy. We refer to this situation as agent $i = 1$ having '*endogenous uncertainty*', since the agent may hesitate and reaches no certainty in her forecasts of the equilibrium price, which (itself) depends endogenously on the forecasts (through $\varepsilon \in [0, \frac{1}{100}]$). To be more specific, we shall consider the parameter, ε, which agent $i = 1$ eventually selects, as a random variable, having a continuous probability, with support $[0, \frac{1}{100}]$.

In such an economy, it agent $i = 2$ cannot reasonably be assumed to have perfect foresight (as along π_2^ε), unless she is informed of her partner's beliefs at the time of trading (i.e., of $\varepsilon \in [0, \frac{1}{100}]$ and π_1^ε). Assume, by contaposition, that agent $i = 2$ has a single expectation, $p \in \mathbb{R}_+^2$, which is also the true price. Then, from above, p belongs to the set $\Delta := \{(\frac{4}{5}\text{-}\frac{\varepsilon}{7}, \frac{4}{5}\text{-}\varepsilon) : \varepsilon \in [0, \frac{1}{100}]\}$, whereas any price of that continuous set Δ may obtain as an equilibrium price (depending on ε). If agent $i = 2$ is unaware of the parameter ε, reprenting her partner's ultimate beliefs, her (single) anticipation, $p \in \Delta$, will, almost certainly, be mistaken (i.e., $p \neq p_\varepsilon$), and prevent a sequential equilibrium to obtain, as a decentralized solution to consumers' problems.

This heuristic economy illustrates why the single price expectation assumption (which is implied by perfect foresight) may preclude a '*correct forecast*' of future prices, hence, the occurence of sequential equilibrium, when some agents form their anticipations privately, with endogenous uncertainty. By making a '*correct forecast*', we mean expecting today the true price of tomorrow as a possible outcome. In the above example, the set Δ is one of incompressible uncertainty for the second agent, forecasting prices correctly, when she is unaware of her partner's beliefs ($\varepsilon \in [0, \frac{1}{100}]$).

In a general model, where agents' form private anticipations, our second companion paper will show the existence (and study the properties) of a minimum uncertainty set for each agent forecasting prices correctly. This set of minimum uncertainty, denoted by $\mathcal{P}(p(0), q)$, may depend on the spot price, $p(0)$, and the asset price, q, that agents observe on current markets (at $t = 0$). Our third companion paper will prove that, whenever agents embed the minimum uncertainty set into their expectations, a sequential equilibrium always exists in the standard conditions.

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