Licensing weak patents

David Encaoua, Yassine Lefouili

Licensing "Weak" Patents*

David Encaoua†and Yassine Lefouili‡

This Version: May 2009

Abstract

In this paper, we revisit the issue of licensing "weak" patents under the shadow of litigation. Departing from the seminal paper by Farrell and Shapiro (2008), we consider innovations of any size and not only "small" innovations, and we allow the number of licensees to be less than the number of firms in the downstream industry. It is shown that the optimal two-part tariff license from the patent holder's perspective may either deter or trigger litigation and conditions under which each case arises are provided. We also reexamine the claim that the licensing revenues from "weak" patents overcompensate the patent holder relative to what a natural benchmark would command. Finally we suggest two policy levers that may alleviate the harm raised by the licensing of "weak" patents.

Keywords: Licensing Schemes, Probabilistic Rights, Patent Litigation.
JEL classification: D45, L10, O32, O34.

1 Introduction

The focus of this paper is on licensing cost-reducing innovations covered by patents which may be invalidated by a court if challenged. Licensing usually occurs *before* any patent validity reexamination. Since it is now widely recognized that the quality of many granted patents raises serious doubts, the question whether the holder of a "weak" patent - i.e. a patent that has a high probability of being invalidated if challenged - can nevertheless get relatively high licensing revenues arises. While the literature on licensing patents has extensively examined the case where patents give their owners perfect protection[1], the so-called iron-clad patents, the economic literature devoted to licensing uncertain patents is more scarce despite the empirical evidence on the issuance and enforcement aspects showing that patents do not give their owners perfect protection. In a recent and pioneering work, Farrell and Shapiro (2008, FS hereafter) analyze the licensing properties of a cost-reducing technology covered by a patent whose validity is uncertain. They consider a situation where an upstream agent holding a "probabilistic patent"[2] uses a two-part tariff licensing scheme to sell licenses to a set of competing firms in an oligopolistic industry. The patent validity being uncertain, licensing occurs under the "shadow of litigation": if a downstream firm rejects the license offer and infringes the patent, the patentee sues the potential violator in court.[3] If the patent is held invalid by the court, all downstream firms use the cost-reducing technology free of charge, whereas if the patent is ruled valid, any licensing contracts already signed remain in force, and the unsuccessful challenger is constrained to use the backstop technology. A litigation is thus avoided only if the patent holder licenses the patent at a tariff that no potential licensee refuses. One of the main results in FS is that patents that have a high probability to be invalidated by a court if challenged (the so-called "weak" patents) are "overcompensated" relative to their true strength: the per-unit royalty rate accepted by all firms and the licensing revenue are higher than respectively the expected royalty and the expected revenue if patent validity were assessed prior to licensing. In other words, according to the expression used by Rockett (2008), *weak patents punch above their weight.* FS show that this result prevails when licenses are sold according to a two-part tariff, either when the fixed fee is unconstrained or when it is constrained to be non-negative. At one extreme, when negative up-front fees are

---

[1]See Arrow (1962), Katz and Shapiro (1985, 1987), Kamien and Tauman (1984, 1986), Kamien *et al.* (1992). The survey by Kamien (1992) summarizes some major results, especially by comparing the patent holder's profits under different licensing schemes. More recent works include Sen (2005) and Sen and Tauman (2007). Aoki and Hu (1999) examines how the choice between strategic licensing and litigating is affected by the levels of the litigation costs and their allocation between the plaintiff and the defendant.

[2]The term has been coined by Ayres and Klemperer (1999) and used by Lemley and Shapiro (2005, 2007). This uncertainty has many effects, summarized in the recent survey by Rockett (2008).

[3]Another possibility, more or less equivalent to the previous one, is that a downstream firm which refuses the licensing contract acts as a plaintiff against the patent holder, claiming that the patent is not valid in order to benefit freely from the new technology. An illustration of that point is the introduction by the Institut Curie of an opposition procedure in october 2001 against the patent granted to Myriad Genetics for a method for diagnosing a predisposition for breast and ovarian cancer associated with the BRCA1 gene.

allowed, a weak patent is licensed at a per-unit royalty which is the same as if the patent were iron-clad, and the corresponding maximal per-unit royalty rate is compensated by a lump-sum transfer from the licensor to the licensee. At the other extreme, when negative fixed fees are not allowed, the overcompensation result still holds, but in a weaker form: the per-unit royalty rate for a "weak" patent is still larger than the expected royalty but is lower than the corresponding value for an iron-clad patent.

Licensing a weak patent would not be harmful if the market impact of a weak patent were proportional to its strength. Indeed, in that case, the existence of "weak" patents would not call for any patent re-examination reform since the "rational ignorance principle" invoked by Lemley (2001) plays in favor of maintaining the current unsatisfactory examination procedure at the patent office (the majority of them being without significant commercial value) while letting the few commercially important patents be litigated. However, as long as the proportionality rule does not hold, FS call for an enhanced reexamination at the patent office level, with a special emphasis on patents useful for multiple downstream firms that compete against each other. A reinforcement of the standards of the examination procedure or an appropriate re-examination made prior to licensing, would lessen the *ex post* deadweight loss and improve the *ex ante* incentives towards more innovative projects.

Two points deserve a close attention when thinking on this policy recommendation. First, as it heavily rests on the overcompensation result, according to which per-unit royalties for weak patents are boosted over a "natural" benchmark, examining the robustness of this result is crucial. Second, improving the patent examination procedure by focussing on patents covering innovations which are likely to be used by multiple competitors supposes that the patent office has relevant information about the posterior use of patents. As this information is in general missing at the patent office, the proposed policy seems at least uneasy to implement.

Consider the first point: is the overcompensation result in FS robust? We argue that two assumptions under which FS obtain this result are restrictive.

1. Their analysis is restricted to innovations of *small size*, i.e. innovations involving a small cost reduction. By setting this assumption, FS implicitly identify the notion of a "weak" patent with a patent covering a marginal innovation. This identification is abusive for many reasons[4]. One of them is related to the notion of patentable subject matter.[5] For instance, software has been patentable in the US since the *Diamond v. Diehr* 1981 decision[6], but

---

[4]Even if we agree that many patents that should not have been granted by a patent office are weak because they are either not novel or obvious in the light of prior art, one has to take into account that prior art must include knowledge that is not necessarily in patent data bases. This explains why a patent examiner may fail to identify some unpatented prior art (during the limited time devoted to each patent application), particularly in new technological areas. See Guellec and van Pottelsberghe (2007).

[5]We thank an anonymous referee for suggesting a discussion of this point.

[6]At that time, the patent office rejected the patent application and the Supreme Court disagreed. While the rejection was based on the grounds that the only new aspect of the invention was the computer program, supposed to be a non-patentable subject matter, the Supreme Court argued that the invention was an improved

in Europe, patentability of a computer program is still explicitly excluded by the European Patent Convention. In biotechnology, despite the fact that genes and proteins are apparently already present in nature, the novelty criterion has been solved by denying their natural character because their isolation rests on a process that connects genomics with chemical engineering. These examples convey the same conclusion: a paten's validity may be denied by a court, even if the patent covers a large and valuable innovation. Many empirical studies show that the small proportion of granted patents that are litigated, and the still fewer which continue litigation until a trial, appear to be the high value patents and those drawn from a subset of particularly litigious technology areas (Lanjouw and Schankerman, 2001 and 2004, Scotchmer, 2004). Since all the results in FS analysis are obtained under the assumption of a small cost reduction magnitude, we think it is important to revisit them when this assumption is relaxed.

2. FS restrict their analysis to the situation where the license offer made by the patent holder is such that the whole set of firms in the downstream industry accept it. In other words, they implicitly assume that it is never in the interest of the patent holder to set a licensing contract that induces a number of licensees less than the number of downstream firms. We show that this restriction affects the results. If one begins the analysis by defining a demand function for licenses of uncertain patents, from which the two-part tariff that maximizes the licensing revenue is derived, the case where the patent holder prefers to choose a licensing contract that will be accepted only by a subset of the downstream firms, hence inducing litigation, cannot be excluded.[7] We show that such an outcome is possible. The two questions of how many firms will accept a licensing contract for a probabilistic patent and what is the best offer of the patent holder are far from being trivial and we devote a large attention to them in the paper.

Our paper examines the robustness of the results established by FS when the previous restrictions are relaxed. We depart from FS in several ways. First, in our model, the plaintiff role is played by a potential licensee who refuses the license offer and decides to challenge the patent validity. Second, insofar as we do not assimilate a "weak" patent with a small step innovation, we investigate the consequences of licensing a "weak" patent whatever the size of the innovation it covers. Third, we do not exclude the possibility that an unsuccessful challenger constrained to use the backstop technology is no more viable when competitors have access to the improved technology. Fourth, we relax the assumption that the patent holder

---

process of making rubber goods that happened to use a computer program (Hunt, 2001).

[7]FS (footnote 16) explain that even if in the iron-clad patent framework the patent holder may choose to exclude some firms, restricting the number of licenses offered does not work as a licensing strategy for a probabilistic patent since firms that do not receive licenses will infringe the patent, and the patent holder will sue them. However, FS compare a situation where all the firms accept the licensing contract under the shadow of litigation to a situation where *all* the firms are licensed after litigation in case the patent's validity is upheld. This means they implicitly assume that, even in the framework of an iron-clad patent, the patent holder would find it optimal to license all firms.

licenses every firm in the industry, which allows us to endogeneize the number of licensees.

On the policy side, our analysis suggests some remedies to the problem raised by licensing "weak" patents. The policy recommended by FS is to improve the examination process at the patent office, at least for patents usable by many competitors. However, this proposal raises obvious implementation issues due to the patent office's informational constraints, let alone the cost of a thorough assessment procedure. We rather propose policies aimed at lowering the patent holder's ability to extract a high royalty rate when licensing takes place before patent validity is determined, by making the litigation alternative more atractive to potential licensees.

Our main results are as follows. The optimal structure for licensing a weak patent depends on the level of the per-unit royalty that deters litigation. When this level is above a defined threshold, the optimal licensing scheme is a pure per-unit royalty that deters litigation, and this confirms FS results. However, when this level is below the threshold, the patent owner prefers to sell its license to a subset of firms, at the optimal two-part tariff that triggers litigation. It is precisely when the royalty rate acceptable by all the firms in the downstream industry is too low that the holder of a weak patent may prefer to sell a license at a higher royalty rate, opening the way to a possible patent validity challenge. We also show that the overcompensation result, while occuring under some conditions, may not hold under other circumstances. It appears that the number of licensees induced by the licensing contract proposed by the patent holder plays an important role in reaching these results. In particular, when it is optimal for an iron-clad patent holder to offer a contract accepted by the whole set of firms, three situations arise for a weak patent according to the value of the maximal royalty rate deterring litigation: i/ the patent holder is undercompensated if this royalty rate is low enough; ii/ the compensation is proportional to the patent strength for intermediate values of this royalty rate; iii/ there is overcompensation if this royalty rate is high enough (Proposition 7). Thus, the overcompensation result seems to be much less general than suggested by FS analysis. Finally we show how our results are affected under two alternative assumptions: i/ a patent holder cannot refuse to license an unsucessful challenger; ii/ downstream firms collectively decide whether to accept the license offer or challenge the patent's validity. We derive from those extensions the effects of two policy levers in alleviating the harm caused by licensing "weak" patents. Our first suggestion hinges on the main mechanism behind a possible overcompensation: individual incentives to challenge a patent's validity are low due to the positive externality on competitors. Therefore, encouraging collective challenges may help solve the problem. Our second suggestion explores the idea that, when the cost reduction magnitude is high or when the competition in the downstream market is tough, an unsuccessful challenger may be seriously harmed and even evicted from the market if deprived from the new technology. One way to solve the problem is to prevent a refusal to sell a license to an unsuccessful challenger, in the same way as injunctions that make an alleged infringer shut

down have been questioned in the current patent debate in the US.

The remainder of the paper is organized as follows. Section 2 presents the model and describes the timing of the three-stage licensing game. Section 3 derives the demand function for licenses for any two-part tariff and any patent strength. Section 4 is devoted to the determination of the optimal two-part tariff by the patent holder for licensing a "weak" patent. In section 5, we examine two policy levers that reduce the range of the overcompensation result. Section 6 concludes.

2   The model

We consider an industry consisting of $n \geq 2$ symmetric risk-neutral firms producing at a marginal cost c (fixed production costs are assumed to be zero). A firm P outside the industry holds a patent covering a technology that allows each firm of the industry to reduce its marginal cost from c to $c - \epsilon$. The patent is uncertain in the sense that it could be invalidated by a court if litigated: it is only with a probability θ that the patent is upheld. The parameter θ measures the patent's strength. We examine the following three-stage game:

First stage: The patent holder P proposes a two-part tariff licensing contract (r, F) whereby a licensee can use the patented technology against the payment of a per-unit royalty rate r and a fixed fee F.[8]

Second stage: The n firms simultaneously and independently decide whether to purchase a license (r, F). If a firm does not accept the license offer, it can challenge the patent validity before a court.[9] If the patent is upheld then a firm that does not purchase the license uses the old technology,[10] thus producing at marginal cost c, whereas those who accepted the license offer use the new technology and pay the royalty rate r to the patent holder, having thus an effective marginal cost equal to $c - \epsilon + r$ and an effective fixed cost equal to F. If the patent is invalidated, all the firms, including those who accepted the offer, can use for free the new technology and their common marginal cost is $c - \epsilon$. [11]

---

[8]We assume as in FS, and consistently with intellectual property law in the US, that the royalty rate r cannot exceed the innovation size ϵ.

[9]In the US, a firm can seek a declaratory judgement against the validity of a patent if it has a "reasonable apprehension" of being sued for infringement by the patentholder. A firm that is planning to use a patented technology, or is currently using it without a license can reasonably fear to be sued for infringement.

[10]This assumption may seem quite strong but recall that IP laws do not compel patentholders to license others, particularly those who challenge the validity of a patent or sue the patent holder for infringement of their own patents. To illustrate, when Intergraph, a company producing graphic work stations, sued Intel for infringement of its Central Processing Unit patent, Intel countered by removing Intergraph from the list of customers and threatened to discontinue the sale of Intel microprocessors to Intergraph (See Encaoua and Hollander, 2002). We relax this assumption in Section 6 by introducing renegotiation between the unsuccessful challenger and the patentholder.

[11]Note that, in a setting without litigation costs, as in our model and FS, who the plaintiff/defendant is does not matter. What matters in both models is that a trial in which patent validity is examined by a court will occur whenever at least one firm does not accept the licensing contract. In FS, the patent holder always finds it

6

Third stage: The n firms produce under the cost structure inherited from stage 2. The kind of competition that occurs is not specified. It is only assumed that there exists a unique Nash equilibrium in the competition game between the members of the oligopoly for any cost structure of the firms. Considering an industry of n firms out of which k firms ($k < n$) - called "efficient" firms - produce at the marginal cost $x < c$ and the remaining $n - k$ firms - called "inefficient" firms - produce at the marginal cost c, we denote by $\pi^e(k, x)$ (respectively $\pi^i(k, x)$) the equilibrium profit function - gross of a potential fixed cost corresponding to the up-front fee - of an efficient firm (respectively an inefficient firm). In the case where all firms produce at the same marginal cost $x \leq c$, we denote a firm's profit indifferently by $\pi^e(n, x)$ or $\pi^i(n, x)$ since all firms are equally efficient.

We set the following general assumptions that hold for a large class of economic environments including Cournot competition and differentiated Bertrand competition both with linear demand:

A0. If all the downstream firms produce with the old technology, they make positive profits: $\pi^e(n, c) > 0$.

A1. An efficient (respectively inefficient) firm's equilibrium profit $\pi^e(k, x)$ (respectively $\pi^i(k, x)$) is continuous in x over $[0, c]$ and twice differentiable in x over the subset of $[0, c]$ in which $\pi^i(x, k) > 0$.

A2. An inefficient firm's equilibrium profit is increasing in the efficient firms' marginal cost: If $\pi^i(k, x) > 0$ then $\pi_2^i(k, x) \equiv \frac{\partial \pi^i(k, x)}{\partial x} > 0$ and if $\pi^i(k, x) = 0$ then $\pi^i(k, x') = 0$ for any $x' > x$.

A3. In a symmetric oligopoly, an identical drop in all firms' costs raises each firm's equilibrium profit: $\pi_2^e(n, x) \equiv \frac{\partial \pi^e(n, x)}{\partial x} < 0$.

A4. A firm's profit is decreasing in the number of efficient firms in the industry: $\pi^e(k, x) > \pi^e(k + 1, x)$ and $\pi^i(k, x) \geq \pi^i(k + 1, x)$ for any $x < c$ and any $k < n$.

A5. The incremental profit from getting efficient decreases with the number of efficient firms: for any $x < c$, $\pi^e(k, x) - \pi^i(k - 1, x)$ is decreasing in k.

Note that assumption A3 holds if own cost effects dominate rival's cost effects. This assumption, while being fulfilled in a wide range of competitive settings may not be satisfied under Cournot competition when the demand is "very convex" (see Kimmel 1992, Février and Linnemer 2004). The other assumptions are quite usual in oligopoly theory (see for instance Amir and Wooders, 2000).

---

optimal to sue a firm that uses its technology without a license and the alleged infringer challenges the patent validity as a defense strategy. In our model a firm that refuses the licensing contract always finds it optimal to challenge the patent validity. For an analysis of the tradeoff between litigation and private settlement when litigation costs are introduced, see Crampes and Langinier (2002).

3 The demand function for licenses

We consider the case of two-part tariff licenses (r, F). To determine the demand function for licenses, we start with a preliminary observation: when only $k < n$ firms accept the license offer (r, F), a situation where none of the remaining $n - k$ firms challenges the patent validity cannot be a Nash equilibrium of stage 2 whenever $\theta < 1$.[12]

The following proposition fully characterizes the equilibria of stage 2 according to the two-part tariff offer (r, F)[13]:

Proposition 1 *Denote* $F_k(r) = \pi^e(k, c - \epsilon + r) - \pi^i(k - 1, c - \epsilon + r)$, $k = 1, ..., n$, *and* $\Psi_n(r, \theta) = F_n(r) - (1 - \theta)\left[\pi^e(n, c - \epsilon) - \pi^i(n - 1, c - \epsilon + r)\right]$

- If $F \leq \Psi_n(r, \theta)$ then all the firms purchasing a license is the unique equilibrium of stage 2.

- If $F_n(r, \theta) < F \leq F_{n-1}(r)$ then the Nash equilibria of stage 2 are the situations where only $n - 1$ firms buy a license.

- If $F_k(r) < F \leq F_{k-1}(r)$ where $2 \leq k \leq n - 1$ then the Nash equilibria of stage 2 are the situations where only $k - 1$ firms buy a license

- If $F > F_1(r)$ then the unique equilibrium of stage 2 is the situation where all the firms refuse the license offer.

Proof. See Appendix A. ∎

This proposition shows that for any pair (r, F) there exists an integer $k(r, F)$ such that all the equilibria of stage 2 involve the number $k(r, F)$ of licenses. This allows to interpret $k(r, F)$ as the demand function for licenses. The intuition behind the proposition follows from two conditions that must be satisfied at a Nash equilibrium: i/ a licensee has no incentive to deviate unilaterally by refusing the contract; ii/ a non-licensee has no incentive to deviate and become a licensee. These two conditions respectively define an upper bound and a lower bound for F.

Note that no restriction has been put on the fixed fee F up to now. In particular, in proposition 1, we allow F to be negative, that is, we do not discard the possibility of a transfer from the patent holder to the licensee. Note also that a necessary and sufficient

---

[12]Indeed, if one of these firms challenges the patent validity it gets an expected profit of $\theta\pi^i(k, c - \epsilon + r) + (1 - \theta)\pi^e(n, c - \epsilon)$ whereas it gets a profit equal to $\pi^i(k, c - \epsilon + r)$ if no firm challenges the patent validity. From A3 and A4, it follows that:

$$\pi^i(k, c - \epsilon + r) < \pi^i(n, c - \epsilon + r) = \pi^e(n, c - \epsilon + r) \leq \pi^e(n, c - \epsilon)$$

which yields:

$$\theta\pi^i(k, c - \epsilon + r) + (1 - \theta)\pi^e(n, c - \epsilon) > \pi^i(k, c - \epsilon + r)$$

whenever $\theta < 1$. This means that if not all firms accept the license offer, there is necessarily litigation in equilibrium.

[13]We assume that a firm which is indifferent between accepting the license offer and not, purchases a license.

condition to avoid any litigation when the two-part tariff for a patent of strength θ is (r, F) is that $F \leq \Psi_n(r, \theta)$. If $\Psi_n(r, \theta) < 0$, then the contract must involve a reverse payment from the licensor to the licensee at least equal to $\mid \Psi_n(r, \theta) \mid$ to induce every firm to accept it.

It is easy to derive from proposition 1 a demand function for pure per-unit royalty licenses as this merely amounts to imposing the restriction $F = 0$. We do so because we get a quite remarkable result on the number of licensees in this case. Moreover, the pure per-unit royalty licensing scheme will turn to be optimal in the class of constrained two-part tariff licenses for "weak" patents as we will see later.

Corollary 1 *Consider the class of licenses involving a pure per-unit royalty $r \leq \epsilon$. Only two possibilities arise at Nash equilibrium:*

- If $\Psi_n(r, \theta) \geq 0$ there exists a unique equilibrium of stage 2: the n firms purchase a license;

- If $\Psi_n(r, \theta) < 0$ then the Nash equilibria of stage 2 are the situations where $n - 1$ firms buy a license.

Proof. See Appendix A. ∎

Licensing an uncertain patent under a pure per-unit royalty scheme may only lead to two types of equilibria: either each firm accepts the licensing contract or all firms but one accept the contract. Note that the latter case occurs if and only if $\Psi_n(r, \theta) < 0$ which is equivalent to:

$$\pi^e(n, c - \epsilon + r) < \theta \pi^i(n - 1, c - \epsilon + r) + (1 - \theta)\pi^e(n, c - \epsilon)$$

This inequality means that when confronted to $n - 1$ firms that accept the license at a royalty r, the remaining firm prefers to challenge the patent's validity rather than accept the license. The intuition behind corollary 1 is that when the licensing scheme does not involve any fixed fee, a firm is always better off accepting to pay a royalty rate $r \leq \epsilon$ if it anticipates that litigation will be initiated by one of its rivals, which rules the possibility of a Nash equilibrium with less than $n - 1$ firms.

We now return to the class of two-part tariffs and we assume that negative fixed fees are not allowed, i.e. $F \geq 0$. Under this assumption, all firms accept the licensing contract (r, F) if and only if:

$$\Psi_n(r, \theta) \geq 0 \tag{1}$$

and

$$0 \leq F \leq \Psi_n(r, \theta) \tag{2}$$

Inequality 1[14] can be rewritten as:

---

[14]The role of inequality 1 here is to provide a necessary condition on r for inequality 2 to hold over a non-empty range of fixed fee values F.

$$\pi^e(n, c - \epsilon + r) \geq \theta\pi^i(n - 1, c - \epsilon + r) + (1 - \theta)\pi^e(n, c - \epsilon) \tag{3}$$

It is important to note that the royalty rate r affects both sides of inequality (3). Due to assumption A3, the LHS, which represents a firm's gross profit when all firms accept the license is decreasing in r. Due to assumption A2, the RHS, which represents the expected profit of a challenger when all other firms accept the license offer, is (weakly) increasing in r. Thus, for a potential licensee, a lower royalty rate r makes the license option more attractive than the outside option, namely the challenge option, for two reasons:

   - It increases the payoff from the license option: $\pi^e(n, c - \epsilon + r)$ increases with r (direct effect)

   - It decreases the payoff from the outside option: $\theta\pi^i(n - 1, c - \epsilon + r) + (1 - \theta)\pi^e(n, c - \epsilon)$ decreases with r (indirect effect).

Note that the indirect effect arises only if $\pi^i(n - 1, c - \epsilon + r) > 0$. However, it may happen that the extent of the cost asymmetry between the licensees and an unsuccessful challenger result in zero profit for the latter, that is, $\pi^i(n - 1, c - \epsilon + r) = 0$. In this case the indirect effect does not appear. We therefore distinguish between two cases according to whether such royalty rate values exist or not.

Case 1: $\pi^i(n - 1, c - \epsilon) = 0$

This case, absent from the analysis in FS, may occur for a sufficiently large innovation (high value of ϵ) or a sufficiently intense competition (e.g. large number n of firms, price competition with high substitutability between the products).

Using assumptions A0 and A2, one easily shows that there exists a threshold $\hat{r} \in [0, \epsilon]$ such that $\pi^i(n - 1, c - \epsilon + r) = 0$ if $r \leq \hat{r}$ and $\pi^i(n - 1, c - \epsilon + r) > 0$ if $r > \hat{r}$. In other words, an unsuccessful challenger will not be viable if the royalty rate is below some threshold $\hat{r}$, and will make positive profit if the royalty rate is above the threshold $\hat{r}$.

The next two lemmas define a threshold function, in each of the subcases $r \leq \hat{r}$ and $r > \hat{r}$, that will be shown to be the maximal per-unit royalty acceptable by all firms.

Consider first a two-part tariff (r, F) involving a royalty rate $r \leq \hat{r}$. In this case, condition (3) can be rewritten as:

$$\pi^e(n, c - \epsilon + r) \geq (1 - \theta)\pi^e(n, c - \epsilon) \tag{4}$$

Let $\hat{\theta} \in [0, 1]$ be the unique solution in θ to the equation $\pi^e(n, c - \epsilon + \hat{r}) = (1 - \theta)\pi^e(n, c - \epsilon)$.

Lemma 1 *Assume that $\pi^i(n - 1, c - \epsilon) = 0$. The equation $\pi^e(n, c - \epsilon + r) = (1 - \theta)\pi^e(n, c - \epsilon)$ has a unique solution in r over $[0, \hat{r}]$ for any $\theta \in \left[0, \hat{\theta}\right]$. This solution, denoted $r_1(\theta)$, satisfies*

the following properties: i/ $r_1(\theta)$ is differentiable and increasing in θ over $\left[0,\hat{\theta}\right]$, ii/ $r_1(0) = 0$ and $r_1(\hat{\theta}) = \hat{r}$.

Proof. See Appendix A. ∎

Consider now a two-part tariff (r, F) involving a royalty rate $r > \hat{r}$. It will be accepted by all firms if and only if the conditions (1) and (2) hold.

Lemma 2 *Assume that $\pi^i(n-1, c-\epsilon) = 0$. The equation $\pi^e(n, c-\epsilon+r) = \theta\pi^i(n-1, c-\epsilon+r) + (1-\theta)\pi^e(n, c-\epsilon)$ has a unique solution in r over $[\hat{r}, \epsilon]$ for any $\theta \in \left[\hat{\theta}, 1\right]$. This solution, denoted $r_2(\theta)$, satisfies the following properties: i/ $r_2(\theta)$ is differentiable and increasing in θ over $\left[\hat{\theta}, 1\right]$, ii/ $r_2(\hat{\theta}) = \hat{r}$ and $r_2(1) = \epsilon$.*

Proof. See Appendix A. ∎

We can now characterize the set of two-part tariff licenses (r, F) that are accepted by all firms whenever $\pi^i(n-1, c-\epsilon) = 0$.

Proposition 2 *If $\pi^i(n-1, c-\epsilon) = 0$ then all firms accepting the two-part tariff license (r, F) is a Nash equilibrium if and only the following conditions hold:*

i/ $r \leq r(\theta)$ where:

$$r(\theta) = \begin{cases} r_1(\theta) & if \quad \theta \in \left[0,\hat{\theta}\right] \\ r_2(\theta) & if \quad \theta \in \left]\hat{\theta}, 1\right] \end{cases}$$

ii/ $0 \leq F \leq \Psi_n(r, \theta)$

Proof. See Appendix A. ∎

To sum-up, when the innovation size is sufficiently large or the intensity of competition sufficiently high, proposition 2 shows that the firms' incentives to accept a given licensing contract crucially depend on whether the patent is relatively "weak" (i.e. $\theta \leq \hat{\theta}$) or relatively "strong" (i.e. $\theta > \hat{\theta}$). When the patent is "strong", the positive effect of a higher royalty rate on the outside option profit (i.e. a challenger's profit) plays a role in constraining the royalty rates acceptable by all firms: $\pi^i(n-1, c-\epsilon+r(\theta)) > 0$ because $r(\theta) > \hat{r}$ for all $\theta > \hat{\theta}$. However, when the patent is "weak", this indirect effect does not play a role since $\pi^i(n-1, c-\epsilon+r(\theta)) = 0$, due to $r(\theta) \leq \hat{r}$ for all $\theta < \hat{\theta}$. In this sense, a firm has an additional incentive not to accept a licensing contract when the patent is strong enough.[15]

---

[15]One can get to the same interpretation using a more formal argument: defining the threshold $r_1(\theta)$ not only for $\theta \in \left[0,\hat{\theta}\right]$ but for all $\theta \in [0, 1[$ as the unique solution to the equality derived from inequality (4), we can show that $r_1(\theta) < r_2(\theta)$ for all $\theta \in \left]\hat{\theta}, 1\right[$.

Remark : From lemmas 1 and 2, it is clear that the maximal royalty rate $r(\theta)$ acceptable by all firms is increasing and continuous over $[0,1]$. Moreover, it is differentiable over $\left[0,\hat{\theta}\right]$ and $\left[\hat{\theta},1\right]$ but its left-sided derivative is different from its right-sided derivative at point $\theta = \hat{\theta}$. One can show that the former is greater than the latter (see figure 1) which is in line with our previous observation that an extra force (stemming from the indirect effect we pointed out) constrains the royalty rates acceptable by all firms when $\theta > \hat{\theta}$.

INSERT FIGURE 1 HERE

Case 2: $\pi^i(n-1,c-\epsilon) > 0$

In this case, whatever the royalty rate $r \geq 0$ proposed by the patent holder, the profit of an unsuccessful challenger remains positive even when all other firms purchase a license: $\pi^i(n-1,c-\epsilon+r) \geq \pi^i(n-1,c-\epsilon) > 0$. Therefore, in this case, we use the same notation $r_2(\theta)$ for the unique solution in r to the equation $\pi^e(n,c-\epsilon+r) = \theta\pi^i(n-1,c-\epsilon+r)+(1-\theta)\pi^e(n,c-\epsilon)$ for all $\theta \in [0,1]$.[16] The existence, uniqueness and properties of $r_2(\theta)$ can be established as under case 1. These are stated in the following lemma:

Lemma 3 *Assume that $\pi^i(n-1,c-\epsilon) > 0$. The equation $\pi^e(n,c-\epsilon+r) = \theta\pi^i(n-1,c-\epsilon+r) + (1-\theta)\pi^e(n,c-\epsilon)$ has a unique solution in r over $[0,\epsilon]$ for any $\theta \in [0,1]$. This solution, denoted $r_2(\theta)$, satisfies the following properties: i/ $r_2(\theta)$ is differentiable and increasing in θ over $[0,1]$, ii/ $r_2(0) = 0$ and $r_2(1) = \epsilon$.*

Proof. See Appendix A. ∎

The next proposition characterizes the set of licenses accepted by all firms whenever $\pi^i(n-1,c-\epsilon) > 0$.

Proposition 3 *If $\pi^i(n-1,c-\epsilon) > 0$ then, for any $\theta \in [0,1]$, all firms accepting the two-part tariff license (r,F) is a Nash equilibrium if and only if the following two conditions hold:*
    i/ $r \leq r(\theta) = r_2(\theta)$.
    ii/ $0 \leq F \leq \Psi_n(r,\theta)$

Proof. See Appendix A. ∎

Note that the indirect effect that captures the positive externality of a higher royalty rate on a challenger's expected profit is always at work in constraining the royalty rates acceptable

---

[16]The threshold $r_2(\theta)$ that could be denoted $r_2(\theta,\epsilon)$ to explicitly display its dependence upon ϵ, has been previously defined for the values of ϵ such that $\pi^i(n-1,c-\epsilon) = 0$, and for patent strength values $\theta \in \left[\hat{\theta},1\right]$ (see lemma 5). Here, this threshold is defined for the values of ϵ that satisfy $\pi^i(n-1,c-\epsilon) > 0$ and for all patent strength values $\theta \in [0,1]$.

by all firms when the innovation size is sufficiently small or/and the competition intensity is sufficiently low to permit an unsuccessful challenger to be maintained in the market. This is the case on which FS focus their analysis.

4   The patent holder's optimal license offer

The question is to know whether the optimal two-part tariff from the patent holder's perspective induces the whole set of firms or only a subset of them to become licensees. For a patent strength θ, we consider four classes of two-part tariffs:

- $L = \{(r, F) / r \in [0, \epsilon], F \geq 0\}$ is the general set of two-part tariffs with a non-negative fixed fee;

- $L_n = \{(r, F) / r \in [0, r(\theta)], 0 \leq F \leq \Psi_n(r, \theta)\}$ is the subset of two-part tariffs that induce the whole set of n firms to accept the licensing contract, hence deterring litigation;

- $L_{-n} = \{(r, F) / r \in]r(\theta), \epsilon]$ or $F > \Psi_n(r, \theta)\}$ is the subset of two-part tariffs that induce less than n firms to accept the licensing contract, hence triggering litigation;

- $\bar{L}_n = \{(r, F) / r \in [0, \epsilon], F \leq \Psi_n(r, \theta)\}$ is the set of two-part tariffs, with no constraint on the sign of the fixed fee, that induce the whole set of firms to accept the licensing contract.

The optimal two-part tariffs in each of these classes are respectively denoted $(r^*(\theta), F^*(\theta))$, $(r_n^*(\theta), F_n^*(\theta))$, (r_{-n}^*, F_{-n}^*) and $(\bar{r}_n(\theta), \overline{F}_n(\theta))$. Note first that in the class L_{-n}, the optimal license (r_{-n}^*, F_{-n}^*) does not depend on the patent strength θ as can be easily derived from the demand function for licenses.

Define now $P^*(\theta)$, $P_n^*(\theta)$ and $P_{-n}^*(\theta) = \theta P_{-n}^*(1)$ as the patent holder's (expected) licensing revenues corresponding to the first three optimal two-part tariffs.

We need an additional assumption to ensure the existence and uniqueness of those licenses. Defining $q^e(n, x)$ as a firm's equilibrium output when all firms produce at the marginal cost x, we introduce the following technical assumption that holds for instance in the case of Cournot competition with linear demand

A6. The licensing revenue (per license) function $rq^e(n, c - \epsilon + r) + \Psi_n(r, \theta)$ is strictly concave in r.[17]

We first examine the optimal two-part tariff which deters litigation.

---

[17]This assumption is quite reasonable since a higher royalty rate increases the revenue per unit of ouput but is likely to have a negative effect on the demand adressed to each licensee, which would make the licensing revenues subject to two opposite effects, possibly resulting in a concave shape for those revenues.

4.1 The optimal two-part tariff deterring litigation

Under the restriction $F \geq 0$, the optimal two-part tariff $(r_n^*(\theta), F_n^*(\theta))$ deterring litigation for a patent of strength θ is such that:

$$
\begin{aligned}
r_n^*(\theta) &= \arg\max_{0 \leq r \leq r(\theta)} \left[rq^e(n, c - \epsilon + r) + \Psi_n(r, \theta) \right] \\
&= \arg\max_{0 \leq r \leq r(\theta)} \left[\underbrace{rq^e(n, c - \epsilon + r) + F_n(r)}_{\text{objective function under } \theta = 1} + \underbrace{(1 - \theta)\left[\pi^i(n - 1, c - \epsilon + r) - \pi^e(n, c - \epsilon) \right]}_{\text{increasing in } r \text{ and decreasing in } \theta} \right]
\end{aligned}
$$

and

$$
F_n^*(\theta) = \Psi_n(r_n^*(\theta), \theta)
$$

The fact that $r_n^*(\theta) \leq r(\theta)$ ensures that the optimal fixed fee $F_n^*(\theta) = \Psi_n(r_n^*(\theta), \theta)$ is indeed non-negative. If we were not restricting to licenses involving a non-negative fixed fee, the optimal royalty rate would be given by the maximum of the same objective function over the larger set of royalty rates $[0, \epsilon]$:

$$
\bar{r}_n(\theta) = \arg\max_{0 \leq r \leq \epsilon} \left[rq^e(n, c - \epsilon + r) + \Psi_n(r, \theta) \right]
$$

In order to know how the optimal two-part tariff $(r_n^*(\theta), F_n^*(\theta))$ is affected by the patent strength θ, we first need to know how $\bar{r}_n(\theta)$ varies with θ. We get the following result which is in line with figure 3 in FS.[18]

Lemma 4 *In the class of two-part tariff licenses $\bar{L}_n$, the optimal royalty rate $\bar{r}_n(\theta)$ that induces n licensees is (weakly) decreasing in θ*

Proof. See Appendix A. ∎

Using the previous lemma, we can now characterize the optimal license in the class L_n of licenses that deter litigation under the restriction $F \geq 0$:

Proposition 4 *There exists a threshold $\bar{\theta} \in]0, 1]$, such that*

$$
(r_n^*(\theta), F_n^*(\theta)) = \begin{cases} (r(\theta), 0) & if \quad \theta \leq \bar{\theta} \\ (\bar{r}_n(\theta), \Psi_n(\bar{r}_n(\theta), \theta)) & if \quad \theta > \bar{\theta} \end{cases}
$$

Proof. See Appendix A. ∎

This proposition states that in the set of two-part tariffs L_n that are accepted by all firms, the optimal licensing scheme for "weak" patents, i.e. $\theta \leq \bar{\theta}$, is a pure per-unit royalty

---

[18]Note however, that in the general setting we consider, there is no reason that $\bar{r}_n(0) = \epsilon$ as in FS. Moreover, FS show that $\bar{r}_n(\theta) = \epsilon$ for θ sufficiently small but do not formally establish that $\bar{r}_n(\theta)$ is (weakly) decreasing over the interval $[0, 1]$ as we do.

scheme. This result hinges on the constraint put on the up-front fee ($F \geq 0$). For "strong" patents, i.e. $\theta > \bar{\theta}$, the optimal licensing scheme is the unconstrained two-part tariff that maximizes the licensing revenue.[19]

4.2 The optimal two-part tariff

An important question, absent from the analysis in FS, is whether the optimal two-part tariff induces the whole set or only a subset of the downstream firms to become licensees. To address this question, we now compare the revenues from the license $(r_n^*(\theta), F_n^*(\theta))$ that deters litigation to the revenues $P_{-n}^*(\theta) = \theta P_{-n}^*(1)$ derived from the optimal license offer (r_{-n}^*, F_{-n}^*) inducing less than n licenses and hence triggering litigation. To make this comparison in the case of "weak" patents, i.e. $\theta \leq \hat{\theta}$, we consider the following equation:

$$nrq^e(n, c - \epsilon + r) = \theta P_{-n}^*(1) \tag{5}$$

The LHS of this equation corresponds to the licensing revenues from a license $(r, 0)$ accepted by all firms and the RHS is the highest expected licensing revenues the patent holder can get from a license not accepted by all firms. Denote $s(\theta)$ the solution to this equation in r over the interval $[0, \tilde{r}]$ where $\tilde{r} = \arg\max_{0 \leq r \leq \epsilon} [rq^e(n, c - \epsilon + r)]$. We show in the appendix that this solution exists and is unique. Further, denote $\tilde{\theta} = \min\left(\hat{\theta}, \frac{\tilde{r}q^e(n,c-\epsilon+\tilde{r})}{P_{-n}^*(1)}\right)$. The following proposition characterizes the optimal license offer for sufficiently weak patents:

Proposition 5 *For sufficiently weak patents, i.e. $\theta \leq \tilde{\theta}$, the optimal license offer is:*

$$(r^*(\theta), F^*(\theta)) = \begin{cases} (r(\theta), 0) & if \quad r(\theta) \geq s(\theta) \\ (r_{-n}^*, F_{-n}^*) & if \quad r(\theta) < s(\theta) \end{cases}$$

where $s(\theta)$ is the unique solution in r to the equation $nrq^e(n, c - \epsilon + r) = \theta P_{-n}^*(1)$.

Proof. See Appendix A. ∎

This result establishes that the optimal license offer depends on the level of maximal the per-unit royalty $r(\theta)$ that deters litigation. If this level is high, i.e. above the defined threshold $s(\theta)$, the optimal licensing scheme is the pure royalty rate $r(\theta)$, and it is accepted by all firms. But if this level is low, i.e. under the threshold $s(\theta)$, the patent owner prefers to sell its license to a subset of firms, at the optimal two-part tariff that triggers litigation. Recall that the latter tariff does not depend on the patent strength θ.

Proposition 5 calls for a comparison of $r(\theta)$ and $s(\theta)$ for sufficiently "weak" patents. This comparison, which is rather technical, is presented in Appendix B.

---

[19]The result in Proposition 4 is present in FS. However FS state that: "*If licenses cannot use such negative fixed fees, we **assume** that they will consist simply of a per-unit royalty rate*" (p.1350). This assumption is justified later in FS through a "heuristic" argument, but is not rigorously proven as in our paper.

4.3 Are "weak" patents always overcompensated?

Now that we have characterized the optimal license offer for "weak" patents, we can address one of the main questions raised in this paper: are "weak" patents always overcompensated?

4.3.1 Comparison of the equilibrium royalty with a benchmark

We consider the following *natural* benchmark for the royalty rate: the expected value of the maximal royalty rate accepted by all firms if licensing takes place after litigation, that we denote by $r^e(\theta)$. This benchmark can be easily computed: with probability θ the patent is upheld by the court, becoming thus an iron-clad right that can be licensed at a *maximal* per-unit royalty ϵ, and with probability $1 - \theta$ the patent is invalidated and the firms can use it for free, leaving the patent holder with zero royalty. Thus, the expected value of the maximal royalty rate when litigation precedes licensing is equal to $r^e(\theta) = \theta\epsilon$. In FS, this benchmark is interpreted as the *ex ante* value of the per-unit royalty rate that the owner of a process innovation reducing the cost by ϵ can expect when the patent has a probability θ of being granted by the patent office. Note that this benchmark is relevant only for non-drastic innovations, i.e. innovations such that $\pi^i(1, c - \epsilon) = 0$. For a drastic innovation, the holder of an iron-clad patent ($\theta = 1$) would not license its innovation at the maximal royalty rate ϵ but at a lower rate, i.e $r^*(1) < \epsilon$ (see Kamien *et al.* 1992). In this case, the natural benchmark would be $\theta r^*(1) < \theta\epsilon$. To make our results comparable to those of FS, we focus on the case where the relevant benchmark is $\theta\epsilon$, i.e. the case of non-drastic innovations.

The comparison of $r^*(\theta)$ and $\theta\epsilon$ when litigation is deterred is made in the following proposition:

Proposition 6 *Assume that litigation is deterred, i.e.* $r(\theta) \geq s(\theta)$. *Define* $\eta(\epsilon) = \frac{\varepsilon \left| \pi_2^e(n, c - \epsilon) \right|}{\pi^e(n, c - \epsilon) - \pi^i(n - 1, c - \epsilon)}$.
*   If $\eta(\epsilon) < 1$ then $r^*(\theta) = r(\theta) \geq \theta\epsilon$ for sufficiently "weak" patents*
*   If $\eta(\epsilon) > 1$ then $r^*(\theta) = r(\theta) \leq \theta\epsilon$ for sufficiently "weak" patents*

Proof. See Appendix A. ∎

Note that if an unsuccessful challenger is not viable[20], i.e. $\pi^i(n - 1, c - \epsilon) = 0$ (either because the innovation is sufficiently large or the competitive environment is tough), $\eta(\epsilon) = \frac{\varepsilon \left| \pi_2^e(n, c - \epsilon) \right|}{\pi^e(n, c - \epsilon)}$ which is the elasticity of a firm's profit with respect to cost reduction when all firms benefit from this reduction. Thus, the elasticity of a firm's profit (in a symmetric oligopoly) with respect to cost reduction plays a crucial role in the comparison of the optimal royalty rate $r^*(\theta)$ with the "fair" benchmark $r^e(\theta) = \theta\epsilon$.[21] The intuition behind the result is that a

---

[20] Note that the condition $\pi^i(n - 1, c - \epsilon) = 0$ which defines this case is strictly weaker than the condition $\pi^i(1, c - \epsilon) = 0$ which defines a drastic innovation whenever $n \geq 3$.

[21] In their theorem 8, FS introduce the oligopoly's relativity coefficient ρ while we use $\eta(\epsilon)$. Both parameters allow the comparison of the equilibrium per-unit royalty rate with the benchmark $\theta\epsilon$. Does there exist

low value of this elasticity entails a low (negative) effect of an increase in the royalty rate on the firms' profit when they all purchase a license. Under such conditions, the patent holder may be able to impose a high royalty rate. In particular, the level of the royalty rate may be greater than the benchmark level $r^e(\theta)$. However, if the elasticity of the profits with respect to cost reduction is high, the patent holder may not be able to overcharge the license with a royalty higher than $\theta\epsilon$ without triggering a challenge: such royalty could result in a relatively weak profit for the licensees hence making the challenge option more attractive for them. Thus, in a situation where the patent holder prefers to deter a challenge, the per-unit royalty must be less than $\theta\epsilon$ when the elasticity of a firm's profit with respect to cost reduction is greater than one.

However, in the frameworks of Cournot oligopoly and Bertrand differentiated oligopoly, both with linear demand, the inequality $\eta(\epsilon) < 1$ always holds for non-drastic innovations. This implies that the overcompensation result in FS, in terms of the per-unit royalty extracted by the patent holder, is robust for non-drastic, but possibly significant, innovations at least in those particular frameworks.[22]

4.3.2 Comparison of $P^*(\theta)$ with $\theta P^*(1)$

The overcompensation result has been examined in the latter paragraph through the comparison of the equilibrium royalty rate to a "fair" benchmark.[23] We argue that it is more relevant to assess the overcompensation result through the comparison of the patent holder' licensing revenues $P^*(\theta)$ under the shadow of litigation and the natural benchmark $\theta P^*(1)$ which corresponds to the expected licensing revenues if the patent validity were determined before licensing takes place. To make this comparison we need to define, for any $\theta \leq \tilde{\theta}$, a threshold $v(\theta)$ as the unique solution to the equation $nrq^e(n, c - \epsilon + r) = \theta P^*(1)$.[24] Note

---

a relationship between these two parameters? If ϵ is "sufficiently small" then $\frac{\pi^e(n,c-\epsilon)-\pi^i(n-1,c-\epsilon)}{\epsilon}$ can be approximated by the numerator of ρ (that is, $\pi_1(0,0)$ if we refer to the notations in FS). Hence $\eta(\epsilon)$ can be approximated by $\frac{1}{\rho}$ in this case. FS state that $\rho > 1$ in most competitive settings. However, their argument mainly rests on the special case of symmetric Cournot oligopoly with linear or iso-elastic demand. They also suggest that ρ is a measure of the strength of downstream competition. However, we argue that one cannot exclude a priori that $\eta(\epsilon)$, which is the "counterpart" of $\frac{1}{\rho}$ whenever we do not make any approximation, can actually be greater than 1. This means that the approximation under the "small size" assumption is far from being inocuous. In particular, it seems problematic to interpret the (non-approximated) coefficient $\frac{1}{\eta(\epsilon)}$ as a mesure of competition intensity, as suggested in FS. Indeed, under the assumption $\pi^i(n-1, c-\epsilon) = 0$, the parameter $\eta(\epsilon)$ does not depend on the competition intensity in many competitive environments. For instance, it can be shown that under Bertrand competition with differentiated products and linear demand, the elasticity $\eta(\epsilon)$ does not depend on the degree of substituability between the products and hence is independent of the intensity of competition. We can also show that $\eta(\epsilon)$ is not affected by switching from perfect collusion to Cournot competition under both linear and iso-elastic demand.

[22]We have not been able to find simple, analytically tractable, examples in which the inequality $\eta(\epsilon) > 1$ holds for non-drastic innovations.

[23]The choice of the benchmark $\theta\epsilon$ made in FS and used in the latter paragraph is quite arbitrary. What matters is the patent holder's (expected) profit which generally depends non-linearly on the royalty rate.

[24]The existence and uniqueness of $v(\theta)$ can be established in the same fashion as $s(\theta)$

that $v(\theta) = s(\theta)$ if and only $P^*(1) = P^*_{-n}(1)$, that is, if and only if the optimal license offer in the case of an iron-clad patent ($\theta = 1$) is not accepted by all firms in the industry. Otherwise, it holds that $P^*(1) > P^*_{-n}(1)$ which entails that $v(\theta) > s(\theta)$. The comparison of $P^*(\theta)$ and $\theta P^*(1)$, stated in the next proposition, offers a valuable assessment for the overcompensation result.

Proposition 7 *Consider sufficiently "weak" patents, i.e. $\theta \leq \tilde{\theta}$.*

1. If $P^*(1) = P^*_{-n}(1)$, i.e. it is optimal for an iron-clad patent holder to offer a contract which is not accepted by all the firms, then the following statements hold:

1.a. If $r(\theta) \leq s(\theta)$ then $P^*(\theta) = \theta P^*(1)$

1.b. If $r(\theta) > s(\theta)$ then $P^*(\theta) > \theta P^*(1)$

2. If $P^*(1) > P^*_{-n}(1)$, i.e. it is optimal for an iron-clad patent holder to offer a contract accepted by the whole set of firms, then the following statements hold:

2.a. If $r(\theta) < v(\theta)$ then $P^*(\theta) < \theta P^*(1)$

2.b. If $r(\theta) = v(\theta)$ then $P^*(\theta) = \theta P^*(1)$

2.c. If $r(\theta) > v(\theta)$ then $P^*(\theta) > \theta P^*(1)$

Proof. See Appendix A. ∎

This proposition shows that licensing "weak" patents may lead to overcompensation as well as undercompensation relative to the expected licensing revenues $\theta P^*(1)$ in case validity is determined prior to licensing . Two questions matter to determine the outcome of such a comparison: i/ Would it be optimal for the patent holder to license every firm or only a subset of them, if the patent were iron-clad? ii/ What is the level of the per-unit royalty $r(\theta)$ that deters litigation relative to the thresholds $s(\theta)$ and $v(\theta)$ derived from the situations where the patent holder is indifferent between deterring any litigation, and respectively licensing to less than n firms or obtaining the expected revenue under litigation? The answer to the first question has been investigated in the literature on licensing iron-clad patents (Sen and Tauman, 2007). The answer to the second question is the result of a comparison: it is only when $r(\theta)$ is above the thresholds that the licensing revenue from a weak patent punches above the benchmark revenue corresponding to the patent's weight. Note that undercompensation can occur at equilibrium even when litigation is deterred. This happens when $s(\theta) \leq r(\theta) < v(\theta)$, that is, when the licensing revenues from the optimal license deterring litigation is greater than the expected licensing revenues from litigation $\theta P^*_{-n}(1)$ but less than the benchmark licensing revenues $\theta P^*(1)$.

5   Policy levers for "weak" patents

In this section, we discuss two policy levers that can be used to alleviate the concerns raised by licensing "weak" patents. First it must be clear that the patent quality problem has several

dimensions related to the processes that occur in different phases, running from application by innovators, prosecution by patent examiners, private settlements between the concerned agents to avoid a trial, until legal enforcement by different courts to reach decisions on the private suits brought by different agents. All these phases are complex, evolve in time, differ among countries and largely depend on the intellectual property law adopted by legislative bodies. Thus, it is very difficult if not impossible to discuss all the relevant aspects raised by the "patent quality" problem. But remember what has been said in the introduction: "bad patents" or patents of "weak" quality are not only patents that cover non-novel or obvious inventions. They concern also inventions that may be invalidated by a court for other reasons such as the patentable subject matter, the utility criterion or any other ambiguities that exist in the patent law. Therefore, the objective of this section is to derive some policy suggestions to improve the performance of the licensing process, as long as the "weakness" of some patents and the low individual incentives to challenge them are acknowledged.

The first suggestion is to prevent a patent holder from refusing to license its right to an agent who would have tempted unsuccessfully to dispute the validity. We argue in the first sub-section that the effect of such prevention would be to reduce the level of the per-unit royalty acceptable by every firm, anticipating that such royalty should be renegotiated in case of an unsuccessful challenge. The second suggestion is to encourage a set of agents to dispute collectively the validity of a patent rather than restrict this possibility to each of them individually. We argue that, by ruling out the positive externality that an agent offers to competitors when he or she disputes alone the patent validity, a collective challenge rules out the possibility that a "weak" patent holder could impose a royalty rate higher than the benchmark. For the sake of exposition, we restrict attention in what follows to pure per-unit royalty schemes.

5.1 Preventing license refusal to an unsuccessful challenger

So far we have assumed that in case of litigation, an unsuccessful challenger produces with marginal cost c because the patent holder refuses to sell him or her a license. Whether such a commitment to refuse a license to an unsuccessful challenger is credible or not must be discussed. From the challenger's perspective this commitment is equivalent to an offer of a new licensing contract involving a royalty rate $\bar{r} = \epsilon$. However, from the patent holder's perspective, this equivalence does not hold. Moreover a situation where an unsuccessful challenger is offered a new licensing contract involving a royalty rate $\bar{r} < \epsilon$ may be preferred by the patent holder to a situation where it is offered a contract based on $\bar{r} = \epsilon$. Such an issue is important since a potential challenger will take the decision whether to accept the license or contest the patent validity, anticipating what would happen if the patent is validated. If patent law prevents the patent holder from refusing to license an unsuccessful challenger,

then the commitment of the former not to renegotiate with the latter after the challenge is undermined.

Formally if we allow for renegotiation when $(n-1)$ firms accept a licensing contract based on a royalty rate r and the remaining firm challenges the patent unsuccessfully, then the patent holder will offer to the challenger a contract involving a royalty rate $\bar{r} \in [0, \epsilon]$ that maximizes its licensing revenues denoted $P(r, \bar{r})$ and given by:

$$P(r, \bar{r}) = (n-1)\, r q^L\, (c - \epsilon + r, c - \epsilon + \bar{r}) + \bar{r} q^{NL}\, (c - \epsilon + r, c - \epsilon + \bar{r})$$

where[25] $q^L\, (c - \epsilon + r, c - \epsilon + \bar{r})$ denotes the equilibrium quantity produced by each of the $(n-1)$ firms that accepted initially the license offer r and $q^{NL}\, (c - \epsilon + r, c - \epsilon + \bar{r})$ is the equilibrium quantity produced by the unsuccessful challenger who produces at marginal cost $c - \epsilon + \bar{r}$. If $\bar{r}(r)$ is the royalty rate that maximizes $P(r, \bar{r})$ with respect to $\bar{r}$, a licensing contract involving a royalty rate r will be accepted by all the firms if and only if:

$$\pi(c - \epsilon + r, c - \epsilon + r) \geq \theta \pi(c - \epsilon + \bar{r}\,(r)\,, c - \epsilon + r) + (1 - \theta)\pi(c - \epsilon, c - \epsilon) \qquad (6)$$

Since $\bar{r}\,(r) \leq \epsilon$ we have $\pi(c - \epsilon + \bar{r}\,(r)\,, c - \epsilon + r) \geq \pi(c, c - \epsilon + r)$ which entails that constraint (6) is (weakly) more stringent than (3). More specifically, a royalty rate r could be accepted if the patent holder commits to refuse a license to a challenger or license him at $\bar{r} = \epsilon$, but not accepted if he cannot commit. This implies that the maximal royalty rate the patent holder can make the n firms pay is (weakly) smaller when renegotiation of a licensing contract (after patent validation) is introduced.

In the next proposition, we show that under Cournot competition with linear demand $Q = a - p$, the maximal royalty rate accepted by all firms if renegotiation is possible can indeed be below the relevant benchmark $\theta \epsilon$ for non-drastic innovations (i.e. $\epsilon < a - c$) whereas the maximal royalty rate if renegotiation is not possible is above $\theta \epsilon$.

Proposition 8 *Assume renegotiation is possible. In a Cournot model with homogeneous product and a linear demand $Q = a - p$, the maximal per-unit royalty rate that induces a perfect subgame equilibrium in which all firms choose to buy a license of a patented technology that reduces the marginal cost by $\epsilon \in \left]\frac{3}{5}\,(a - c)\,, a - c\right[$ is given by $r^p\,(\theta) = (a - c + \epsilon)\left(1 - \frac{\sqrt{4 - 3\theta}}{2}\right)$ for a patent strength θ smaller than a threshold $\bar{\theta} \in\,]0, 1[$. The royalty $r^p(\theta)$ is sustained by a renegotiated royalty $\bar{r}(r^p(\theta)) < \epsilon$, and is smaller than the benchmark $r^e\,(\theta) = \theta \epsilon$ if the patent is sufficiently weak.*

Proof. See Appendix A. ∎

---

[25] A notation different from the one used in previous sections is needed here since an unsuccessful challenger produces now at marginal cost $c - \epsilon + \bar{r}$ and not at marginal cost c.

5.2 Encouraging collective challenges

Suppose that at stage 2 the firms cooperatively agree on whether to buy the license or refuse it and challenge all together the patent validity.[26] In this case, the firms will cooperatively accept a licensing contract involving a royalty rate r if and only if:

$$\pi^e(n, c - \epsilon + r) \geq \theta \pi^e(n, c) + (1 - \theta) \pi^e(n, c - \epsilon)$$

The function w defined by $w(r) = \pi^e(n, c - \epsilon + r) - \theta \pi^e(n, c) - (1 - \theta) \pi^e(n, c - \epsilon)$ is continuous, strictly decreasing (by A3) and satisfies the conditions $w(0) \geq 0$ and $w(\epsilon) \leq 0$. Hence there exists a unique solution $r^c(\theta) \in [0, \epsilon]$ to the equation $w(r) = 0$, and the inequality $w(r) \geq 0$ is equivalent to $r \leq r^c(\theta)$. This means that all firms cooperatively accept to buy a license at a royalty rate r if and only if $r \leq r^c(\theta)$.

We establish in the next proposition that the maximal per-unit royalty deterring a collective challenge is lower than the maximal royalty that deters individual challenge, which is not surprising because the free-riding problem that arises when the decision to challenge is made non-cooperatively disappears when challenges are conducted collectively. The proposition gives also a condition under which the royalty rate deterring a collective challenge $r^c(\theta)$ is lower than the expected royalty rate in case of litigation $\theta \epsilon$.

Proposition 9 *The maximal royalty rate deterring a collective challenge is lower than the non-cooperatively royalty rate accepted by all firms : $r^c(\theta) \leq r(\theta)$ for all $\theta \in [0, 1]$. Moreover, the function $r^c(\theta)$ satisfies the following properties:*

i/ $r^c(\theta)$ is increasing over $[0, 1]$ and $r^c(0) = 0$, $r^c(1) = \epsilon$,

ii/ $r^c(\theta)$ is convex over $[0, 1]$ if (and only if) the function $x \to \pi^e(n, x)$ is convex over $[c - \epsilon, c]$ and in this case $r^c(\theta) \leq r^e(\theta) = \theta \epsilon$

Proof. See Appendix A. ∎

Note that the convexity of $x \to \pi^e(n, x)$ holds in a wide range of competitive environments including Cournot competition with linear or iso-elastic demand as well as differentiated Bertrand oligopoly with linear demand. Hence the fact that the equilibrium royalty rate may exceed the benchmark $\theta \epsilon$ is mainly due the free-riding problem. Getting rid of the latter by encouraging collective challenges may then be a solution to reduce the potentially high market power of "weak" patent holders.

---

[26]Firms are allowed to challenge collectively the validity of a patent, at least in the US. An example is the PanIP Group Defense Fund which is a coalition of fifteen e-retailers that has been created to invalidate a patent covering some key aspects of electronic commerce, hold by Pangea Intellectual Properties (US patent number 5.576.951). However such collective challenges seem to be quite rare in practice.

6 Conclusion

The consequences of licensing "weak" patents have been examined in this paper by addressing the following questions: 1. To what extent does licensing a patent that has a high probability to be invalidated by a court if challenged favor the patent holder when the license agreement occurs prior to the patent validity determination? 2. How may the concerns raised by licensing weak patents be alleviated? These questions were addressed by FS under the heading "How strong are weak patents?". In this paper we have tried to investigate the same issues in a more general framework where two assumptions made in FS are relaxed. First, we consider that weak patents need not cover small-sized innovations. We have argued that the restriction to small innovations is not justified insofar as the weakness of a patent may arise from other reasons than the standard novelty and non-obviousness criteria. Second, we allow the patent holder to choose a licensing contract that is not accepted by every firm. We have shown that the restriction to contracts accepted by all firms is not justified when the optimal licensing contract is derived from a well-defined demand function for licenses.

Removing these two restrictions and keeping the same two-part tariff structure for the licensing contract as in FS, we have reached a more subtle view of the effects of licensing a "weak" patent by giving a complete characterization of the three-stage game involving the patent holder and the potential users in a downstream industry, whatever the size of the protected invention and the number of licensees. Two results at least deserve a close attention. First, what matters in the optimal choice of the per-unit royalty rate made by the patent holder is the level of the maximal royalty rate that deters litigation. If this level is above a defined threshold, the optimal licensing scheme is a pure per-unit royalty that deters litigation which is in line with the findings in FS. But, if this level is below the threshold, the patent owner prefers to sell its license to a subset of firms, at the optimal two-part tariff that triggers litigation. Hence, the threat of a patent litigation may be sufficient to reduce the licensor's market power. Second, licensing a weak patent does not always lead to overcompensation. The expected maximal licensing revenue appears as being a more relevant benchmark than the expected maximal royalty rate in assessing the overcompensation result since it takes into account the endogeneous determination of the number of licensees. In particular, when it is optimal to offer a contract accepted by the whole set of firms if the patent validity were perfect, an undercompensation result may hold for a weak patent if the maximal royalty rate acceptable by every firm is sufficiently low.

Our analysis yields new policy perspectives. Since the patent system involves a two-tier process combining patent office examination and judicial challenge of the patent validity before a court, two approaches to the problem raised by licensing "weak" patents are possible. One of them, privileged by FS, argues in favor of an enhancement of the examination process, through an increase of the resources devoted to the patent office. Two objections can be raised

against this approach. First it is not clear whether so many weak patents are granted because patent examiners are "rationally ignorant" of the objective validity principles (Lemley, 2001) or because there exists a bias of policies and procedures, at the USPTO (Lei and Wright, 2009) as well as at the EPO (IDEI, 2006), in favor of applicants ("customers" in the patent office language). It is only if the former rational ignorance argument prevails (due to resource limitations) that the suggested policy in favor of a more thorough examination procedure, focused towards patents whose potential users are competitors, could be relevant. But, even in this case, the evident informational constraint is an obstacle in implementing such a targeted policy. Second, our model suggests that undercompensation as well as overcompensation may result from licensing a weak patent. However, even if this indicates that the problems from licensing weak patents may be less serious than suggested by FS, our theoretical results raise no less acute implementation problems than the FS suggestion. It is indeed clear that the patent office cannot discriminate between patents according to the level of the licensing royalty that deters litigation.

Therefore, the second policy approach, privileged in this paper, focuses on the second component of the two-tier process. Since the existence of weak patents seems to be more or less unavoidable or too costly to be reduced at the patent office level, it might be easier and less costly to operate at the second tier level, namely the judicial level. This could be done by giving more resources to the judicial system and encouraging third parties to bring to courts pieces of evidence facilitating the possibility to challenge the validity of the presumably "weak" patents. Such encouragement is necessary since a firm's decision to challenge a patent's validity benefits all other downstream firms (Farrell and Merges, 2004, Lemley and Shapiro, 2005), and the corresponding free-riding argument is precisely the key to the potential overcompensation result. The post-grant opposition in Europe seems to play this role in a more appropriate way than the post-grant reexamination in the United States (Graham *et al.*, 2003). In the same vein, injunction against infringement seems to be a less desirable remedy than damages (Hylton, 2006). Therefore, giving potential licensees more incentives to challenge patent validity seems to be appropriate in this perspective. This is why we are confident that our policy suggestions, namely forbidding license refusals to unsuccessful challengers and encouraging a collective approach among potential licensees, would be effective in alleviating the problems resulting from the licensing of "weak" patents.

7 References

Amir, R. and J. Wooders, 2000, One-way spillovers, endogenous innovator/imitator roles, and research joint ventures, *Games and Economic Behavior*, 31, 1-25.

Aoki, R. and J-L. Hu, 1999, Licensing versus litigation: the effect of the legal system on incentives to innovate, *The Journal of Economics and Management Strategy*, 8, 133-160.

Arrow, K., 1962, Economic welfare and the allocation of resources for invention, in R. Nelson (Ed.), *The rate and direction of incentive activity*, Princeton, Princeton University Press.

Ayres, J. and P. Klemperer, 1999, Limiting patentees market power without reducing innovation incentives: the perverse benefits of uncertainty and non-injunctive remedies, *Michigan Law Review,* Vol. 97, 985-1033.

Crampes, C. and C. Langinier, 2002, Litigation and settlement in patent infringement cases, *Rand Journal of Economics*, 33(2), 258-274.

Encaoua, D. and A. Hollander, 2002, Competition policy and innovation, *Oxford Review of Economic Policy,* 18, 1, 63-79.

Farrell, J. and C. Shapiro, 2008, How strong are weak patents?, *American Economic Review,*98:4, 1347-1369

Février, P and L. Linnemer, 2004, Idiosyncratic shocks in a asymetric Cournot oligopoly, *International Journal of Industrial Organization*, 22, 6, 835-848.

Graham, S., B. Hall, D. Harhoff and D. Mowery, 2003, Post-issue patent "quality control": a comparative study of US patent re-examinations and European patent oppositions, in W. Cohen and S. Merrill, eds.: *Patents in the Knowledge-Based Economy,* The National academic Press, Washington, D.C.

Guellec, D. and B. van Pottelsberghe de la Potterie, 2007, *The Economics of the European Patent System*, Oxford University Press, Oxford.

Hunt, R., 2001, You can patent that? Are patents to computer programs and business methods good for the New Economy?, *Federal Reserve Bank of Philadelphia Business Review,* Quarter 1, 5-15

Hunt, R., 2008, Business method patents and US financial services, WP n° 08-10, Research Department, Federal Reserve Bank of Philadelphia

Hylton, K., 2006, Property rules and liability rules, once again, *Review of Law and Economics, 2,*137-191.

IDEI, 2006, *Objectives and Incentives at the European Patent Office*, Institut d'Economie Industrielle, Toulouse

Kamien, M., 1992, Patent licensing, In: Aumann, R.J., S. Hart (Eds.) *Handbook of Game Theory with Economic Applications*, Elsevier Science, North Holland, ch. 11, 331-354.

Kamien, M., S. Oren and Y. Tauman, 1992, Optimal licensing of cost-reducing innovation, *Journal of Mathematical Economics*, 21, 483-508.

Kamien, M., and Y. Tauman, 1984, The private value of a patent: a game-theoretic analysis, *Z. Nationalőkon, Journal of Economics,* 4 (Supplement), 93-118.

Kamien, M. and Y. Tauman, 1986, Fees versus royalties and the private value of a patent, *Quarterly Journal of Economics*, 101, 471-491.

Katz, M. and C. Shapiro, 1985, On the licensing of innovations, *Rand Journal of Economics*, 16, 504-520.

Katz, M. and C. Shapiro, 1987, R&D rivalry with licensing or imitation, *American Economic Review,* 77, 3, 402-420

Kimmel, S., 1992, Effects of cost changes on oligopolists' profits, *Journal of Industrial Economics*, 40, 4, 441-449.

Lanjouw, J. and M. Schankerman, 2001, Characteristics of patent litigation: A window on competition, *Rand Journal of Economics,* 32, 1, 129-151

Lanjouw, J. and M. Schankerman, 2004, Protecting intellectual property rights: Are small firms handicapped?, *Journal of Law and Economics*, 47, 1, 45-74

Lei, Z. and B. Wright, 2009, Why weak patents? Rational ignorance or pro-customer tilt? Department of Agricultural and Resource Economics, UC Berkeley

Lemley, M., 2001, Rational ignorance at the patent office, *Northwestern University Law Review*, 95, 4, 1497-1532

Lemley, M. and C. Shapiro, 2005, Probabilistic patents, *Journal of Economic Perspective*s, 19, 2, 75-98.

Lemley, M. and C. Shapiro, 2007, Patent hold-up and royalty stacking, *Texas Law Review*, 85, 1990-2049.

Rockett, K., 2008, Property rights and invention, DP n° 663, Department of Economics, University of Essex, to appear in *The Handbook of Economics and Technical Change*, Bronwyn Hall and Nathan Rosenberg, Editors.

Sen, D., 2005, Fee versus royalty reconsidered, *Games and Economic Behavior*, 53, 141-147.

Sen, D. and Y. Tauman, 2007, General licensing schemes for a cost-reducing innovation, *Games and Economic Behavior*, 59, 163-186.

Scotchmer, S., 2004, *Innovation and Incentives*, The MIT Press, Cambridge, Ma.

8 Appendix

8.1 Appendix A: Proofs

Proof of Proposition 1

The situation where the n firms accept the licensing contract F is a Nash equilibrium if and only if:

$$\pi^e(n, c - \epsilon + r) - F \geq \theta \pi^i(n - 1, c - \epsilon + r) + (1 - \theta)\pi^e(n, c - \epsilon)$$

which can be rewritten as:

$$F \leq \pi^e(n, c - \epsilon + r) - \theta \pi^i(n - 1, c - \epsilon + r) - (1 - \theta)\pi^e(n, c - \epsilon)$$

that is

$$F \leq \Psi_n(r, \theta)$$

A situation where $n-1$ firms accept the licensing contract and one firm does not is a Nash equilibrium (of stage 2) if and only if:

$$\theta \pi^i(n-1, c-\epsilon+r) + (1-\theta) \pi^e(n, c-\epsilon) \geq \pi^e(n, c-\epsilon+r) - F \qquad (7)$$

and

$$\theta[\pi^e(n-1, c-\epsilon+r) - F] + (1-\theta) \pi^e(n, c-\epsilon) \geq \theta \pi^i(n-2, c-\epsilon+r) + (1-\theta) \pi^e(n, c-\epsilon) \quad (8)$$

Condition (7) means that the one firm that does not accept the licensing contract and challenges the patent validity does not find it optimal to unilaterally deviate by accepting the licensing contract. Condition (8) means that none of the $n-1$ firms which accept the licensing contract find it optimal to unilaterally deviate by refusing the contract. When the number of firms accepting the contract is strictly less than n, litigation occurs which entails that the firms accepting the contract pay the fixed fee F and the running royalties only if the patent validity is upheld, which happens with probability θ. With the complementary probability $1 - \theta$, the patent is invalidated and all the firms get the same profit namely $\pi^e(n, c-\epsilon)$. It is straightforward to show that conditions (7) and (8) are equivalent to the following double inequality:

$$\pi^e(n, c-\epsilon+r) - \theta \pi^i(n-1, c-\epsilon+r) - (1-\theta)\pi^e(n, c-\epsilon) \leq F \leq \pi^e(n-1, c-\epsilon+r) - \pi^i(n-2, c-\epsilon+r)$$

that is:

$$\Psi_n(r, \theta) \leq F \leq F_{n-1}(r)$$

Note that for all $\theta \in [0, 1]$, we have $\Psi_n(r, \theta) \leq \Psi_n(r, 1) = F_n(r) \leq F_{n-1}(r)$ due to assumption A5, which ensures that the interval $[\Psi_n(r, \theta), F_{n-1}(r)]$ is not empty.

A situation where only $k \in \{1, 2, ..., n-2\}$ firms accept the licensing contract is a Nash equilibrium of stage 2 if and only if:

$$\theta \left(\pi^e(k, c-\epsilon+r) - F \right) + (1-\theta) \pi^e(n, c-\epsilon) \geq \theta \pi^i(k-1, c-\epsilon+r) + (1-\theta) \pi^e(n, c-\epsilon) \quad (9)$$

and

$$\theta \pi^i(k, c-\epsilon+r) + (1-\theta) \pi^e(n, c-\epsilon) \geq \theta \left(\pi^e(k+1, c-\epsilon+r) - F \right) + (1-\theta) \pi^e(n, c-\epsilon) \quad (10)$$

Condition (9) means that none of the k firms accepting the licensing contract finds it optimal to unilaterally deviate by refusing the contract and condition (10) means that none of the $n-k$ firms refusing the licensing contract finds it optimal to unilaterally deviate by accepting the contract. It is easy to see that conditions (9) and (10) can be combined into the following

26

double inequality that does not depend on θ:

$$\pi^e(k+1, c-\epsilon+r,) - \pi^i(k, c-\epsilon+r) \leq F \leq \pi^e(k, c-\epsilon+r) - \pi^i(k-1, c-\epsilon+r)$$

that is:

$$F_{k+1}(r) \leq F \leq F_k(r)$$

Finally, a situation where no firm accepts the licensing contract is a Nash equilibrium if and only if:

$$\theta\pi^i(0, c-\epsilon+r) + (1-\theta)\pi^e(n, c-\epsilon) \geq \theta\left(\pi^e(1, c-\epsilon+r) - F\right) + (1-\theta)\pi^e(n, c-\epsilon)$$

which can be rewritten as:

$$F \geq \pi^e(1, c-\epsilon+r) - \pi^i(0, c-\epsilon+r) = F_1(r)$$

Proof of Corollary 1

Combining Proposition 1 under the restriction $F = 0$ with the fact that $F_k(\epsilon) = 0$ for all $1 \leq k \leq n-1$ yields the result.

Proof of Lemma 1

Denote $h(r, \theta) = \pi^e(n, c-\epsilon+r) - (1-\theta)\pi^e(n, c-\epsilon)$ and consider, for a given $\theta \in \left[0, \hat{\theta}\right]$, the equation $h(r, \theta) = 0$. Note that $h(0, \theta) = \theta\pi^e(n, c-\epsilon) \geq 0$ and $h(\hat{r}, \theta) = \left(\theta - \hat{\theta}\right)\pi^e(n, c-\epsilon) \leq 0$ for any $\theta \in \left[0, \hat{\theta}\right]$. Since $h(., \theta)$ is continuous and strictly decreasing over $[0, \hat{r}]$ (due to A1 and A3), we can use the intermediate value theorem to state that the equation $h(r, \theta) = 0$ has a unique solution in r, which we denote $r_1(\theta)$, over $[0, \hat{r}]$. Moreover, assumption A1 implies that $h(., .)$ is continuously differentiable over $[0, \hat{r}] \times \left[0, \hat{\theta}\right]$, which allows to state (using the implicit function theorem for instance) that $r_1(\theta)$ is differentiable over $\left[0, \hat{\theta}\right]$ and

$$r_1'(\theta) = \frac{-\pi^e(n, c-\epsilon)}{\pi_2^e(n, c-\epsilon+r_1(\theta))}$$

This implies that $r_1'(\theta) > 0$ since $\pi_2^e(n, c-\epsilon+r_2(\theta)) < 0$ by A3. Therefore $r_1(\theta)$ increases in the patent strength θ over $\left[0, \hat{\theta}\right]$. Furthermore, it is obvious that $r_1(0) = 0$ and we derive from the definition of $\hat{\theta}$ that $r_1(\hat{\theta}) = \hat{r}$.

Proof of Lemma 2

Consider, for a given $\theta \in \left[\hat{\theta}, 1\right]$, the equation $\Psi_n(r, \theta) = 0$ where $\Psi_n(r, \theta) = \pi^e(n, c-\epsilon+r) - \theta\pi^i(n-1, c-\epsilon+r) - (1-\theta)\pi^e(n, c-\epsilon)$ is continuous and strictly decreasing in r over $[0, \epsilon]$ due to assumptions A1, A2 and A3. Moreover $\Psi_n(0, \theta) = \left(\theta - \hat{\theta}\right)\pi(c-\epsilon, c-\epsilon) \geq 0$ for any

$\theta \in \left[\hat{\theta}, 1\right]$ and $\Psi_n(\epsilon, \theta) = (1 - \theta) \left[\pi^e(n, c) - \pi^e(n, c - \epsilon)\right] \leq 0$. Using the intermediate value theorem, we can then state that the equation $\Psi_n(r, \theta) = 0$ has a unique solution in r over the interval $[\hat{r}, \epsilon]$, that we denote by $r_2(\theta)$. Further, $\Psi_n(r, \theta) = \pi(c - \epsilon + r, c - \epsilon + r) - \theta\pi(c, c - \epsilon + r) - (1 - \theta)\pi(c - \epsilon, c - \epsilon)$. Note that $g(\theta, \epsilon) = (1 - \theta) \left[\pi(c, c) - \pi(c - \epsilon, c - \epsilon)\right] \leq 0$ for any $\theta \in \left[\hat{\theta}, 1\right]$ (by A4). Moreover, the function $\Psi_n(., \theta)$ is continuous and strictly increasing over $[\hat{r}, \epsilon]$. Then, using the intermediate value theorem, we state that the equation $\Psi_n(r, \theta) = 0$ has a unique solution in r for any $\theta \in \left[\hat{\theta}, 1\right]$, which we denote by $r_1(\theta)$. Furthermore, assumption A1 ensures that $\Psi_n(., .)$ is continuously differentiable over $[\hat{r}, \epsilon] \times \left[\hat{\theta}, 1\right]$, which allows to state that $r_2(\theta)$ is differentiable over $\left[\hat{\theta}, 1\right]$ and:

$$r_2'(\theta) = \frac{\pi^i \left(n - 1, c - \epsilon + r_2(\theta)\right) - \pi^e(n, c - \epsilon)}{\pi_2^e(n, c - \epsilon + r_2(\theta)) - \theta\pi_2^i \left(n - 1, c - \epsilon + r_2(\theta)\right)} \tag{11}$$

The denominator is negative due to A2 and A3. The numerator is negative as well because $\pi_2^i \left(n - 1, c - \epsilon + r_2(\theta)\right) \leq \pi_2^i \left(n, c - \epsilon + r_2(\theta)\right) = \pi_2^e \left(n, c - \epsilon + r_2(\theta)\right) < \pi^e(n, c - \epsilon)$. The first inequality follows from $r_2(\theta) \leq \epsilon$ and the second one from A3. Thus, $r_2'(\theta) > 0$, that is $r_2(\theta)$ is strictly increasing in the patent strength θ over $\left[\hat{\theta}, 1\right]$. Furthermore, it is obvious that $r_2(\epsilon) = 1$ and we derive from the definition of $\hat{\theta}$ that $r_2(\hat{\theta}) = \hat{r}$.

Proof of Proposition 2

We distinguish two cases:

Case 1: $\theta \in \left[0, \hat{\theta}\right]$.

Consider a royalty rate $r \leq \hat{r}$. In this case, inequality (1) is equivalent to $h(r, \theta) \geq 0$ where h has been defined in the proof of lemma 1. Since $h(r, \theta)$ is decreasing in r, $h(r, \theta) \geq 0$ if and only if $r \leq r_1(\theta)$ where $r_1(\theta)$ is defined in lemma 1.

Consider now $r > \hat{r}$. Since $\Psi_n(r, \theta)$ and $h(r, \theta)$ are decreasing in r and $r_1(\theta) \leq \hat{r}$ for $\theta \in \left[0, \hat{\theta}\right]$ then for any $r > \hat{r}$, it holds that $\Psi_n(r, \theta) \leq \Psi_n(\hat{r}, \theta) = h(\hat{r}, \theta) < h(r_1(\theta), \theta) = 0$ which shows that such $r > \hat{r}$ will not be accepted by all firms. Hence, for any $\theta \in \left[0, \hat{\theta}\right]$, inequality (1) holds if and only if $r \leq \min(\hat{r}, r_1(\theta)) = r_1(\theta)$.

Case 2: $\theta \in \left[\hat{\theta}, 1\right]$.

Consider a royalty rate $r \leq \hat{r}$. In this case, inequality (1) is equivalent to $h(r, \theta) \geq 0$. Since $\Psi_n(r, \theta)$ and $h(r, \theta)$ are decreasing in r, it holds that $h(r, \theta) \geq h(\hat{r}, \theta) = \Psi_n(\hat{r}, \theta) \geq \Psi_n(r_1(\theta), \theta) = 0$.

Consider now a royalty rate $r > \hat{r}$. Since the function $\Psi_n(r, \theta)$ is decreasing in r, $\Psi_n(r, \theta) \geq 0$ if and only if $r \leq r_2(\theta)$.

Hence, for any $\theta \in \left[\hat{\theta}, 1\right]$, inequality (1) holds if and only if $r \leq \max(\hat{r}, r_1(\theta)) = r_1(\theta)$.

Proof of Lemma 3

The existence and unicity can be proven as in lemma 1. However, a difference with lemma 1

is that we do not need to restrict to $\theta \in \left[\hat{\theta}, 1\right]$ to get the differentiability property. Indeed, as $r \to \pi^i (n-1, c - \epsilon + r)$ remains strictly positive for any $r \geq 0$, it is differentiable over $[0, \epsilon]$ due to A1. This ensures the differentiability of $\Psi_n(r, \theta)$ over $[0, \epsilon]$ and allows to state that $r_2(\theta)$ is differentiable over $[0, 1]$ and $r_2'(\theta)$ has the expression given by (11), which ensures the increasingness of $r_2(\theta)$. The equalities $r_2(0) = 0$ and $r_2(1) = \epsilon$ are straightforward.

Proof of Proposition 3

Let $\theta \in [0, 1]$. A two-part tariff (r, F) is accepted by all firms if and only if:

1. $\Psi_n(r, \theta) \geq 0 = \Psi_n(r_2(\theta), \theta)$ which is equivalent to $r \leq r_2(\theta)$ because $\Psi_n(r, \theta)$ is decreasing in r.

2. $0 \leq F \leq \Psi_n(r, \theta)$.

Proof of Lemma 4

$$
\begin{aligned}
\bar{r}_n(\theta) &= \underset{0 \leq r \leq \epsilon}{\arg\max} P(r, \theta) = rq^e(n, c - \epsilon + r) + \Psi_n(r, \theta) \\[2ex]
&= \underset{0 \leq r \leq \epsilon}{\arg\max} \left[\underbrace{rq^e(n, c - \epsilon + r) + F_n(r)}_{\text{objective function under } \theta = 1} + \underbrace{(1 - \theta)\left[\pi^i(n-1, c - \epsilon + r) - \pi^e(n, c - \epsilon)\right]}_{\text{increasing in } r \text{ and decreasing in } \theta} \right]
\end{aligned}
$$

If $\bar{r}_n(\theta) \in {]}0, \epsilon{[}$ then the FOC $\frac{\partial P}{\partial r}(\bar{r}_n(\theta), \theta) = 0$ holds and differentiating it with respect to θ, we get that:

$$
\begin{aligned}
\frac{d}{d\theta}\bar{r}_n(\theta) &= \frac{-\frac{\partial P}{\partial \theta \partial r}(\bar{r}_n(\theta), \theta)}{\frac{\partial^2 P}{\partial r^2}(\bar{r}_n(\theta), \theta)} \\[2ex]
&= \frac{\frac{\partial}{\partial r}\pi^i(n-1, c - \epsilon + r)\,|_{r = \bar{r}_n(\theta)}}{\frac{\partial^2 P}{\partial r^2}(\bar{r}_n(\theta), \theta)}
\end{aligned}
$$

Assumption A2 entails that the numerator $\frac{\partial}{\partial r}\pi^i(n-1, c - \epsilon + r)\,|_{r = \bar{r}_n(\theta)}$ is non-negative. Combining this with the denominator being negative (since P is concave in r), we obtain that $\frac{d}{d\theta}\bar{r}_n(\theta) \leq 0$.

To rigorously conclude that $\bar{r}_n(\theta)$ is (weakly) decreasing in θ, it remains to show that if it happens that $\bar{r}_n(\theta) = 0$ for some θ then $\bar{r}_n(\theta') = 0$ for any $\theta' \geq \theta$. Assume that $\bar{r}_n(\theta) = 0$ for some θ. Then, given the concavity of P in r, it must hold that $r \to P(r, \theta)$ is decreasing over $[0, \epsilon]$. Considering $\theta' \geq \theta$, we have: $P(r, \theta') = P(r, \theta) + (\theta - \theta')\pi^i(n-1, c - \epsilon + r)$. Using A2, we can then state that $(\theta - \theta')\pi^i(n-1, c - \epsilon + r)$ is (weakly) decreasing which yields that $P(r, \theta')$ is (weakly) decreasing and results in $\bar{r}_n(\theta') = 0$.

We can now state that $\bar{r}_n(\theta)$ is (weakly) decreasing in θ.

Proof of Proposition 4

Since $rq^e(n, c - \epsilon + r) + \Psi_n(r, \theta)$ is strictly concave in r then $r_n^*(\theta) = \min(r(\theta), \bar{r}(\theta))$.

Moreover we have already showed that $r(\theta)$ is strictly increasing over $[0,1]$ and $r(0) = 0$, $r(1) = \epsilon$ while $\bar{r}(\theta)$ is (weakly) decreasing in θ and $\bar{r}(1) \leq \epsilon$. This allows us to state that there exists $\bar{\theta} \in]0,1]$ such that:

$$r_n^*(\theta) = \begin{cases} r(\theta) & if \quad \theta \leq \bar{\theta} \\ \bar{r}_n(\theta) & if \quad \theta > \bar{\theta} \end{cases}$$

This yields the result because:

$$F_n^*(\theta) = \Psi_n(r_n^*(\theta), \theta) = \begin{cases} 0 & if \quad \theta \leq \bar{\theta} \\ \Psi_n(\bar{r}_n(\theta)) & if \quad \theta > \bar{\theta} \end{cases}$$

Proof of the existence and unicity of $s(\theta)$

Consider $\theta \leq \tilde{\theta}$. The existence and unicity of $s(\theta)$ is derived from the following three points: i/ the function $g : r \longrightarrow nrq^e(n, c - \epsilon + r)$ is continuous and strictly increasing over $[0, \tilde{r}]$ because $r \to P(r, \theta) = n[rq^e(n, c - \epsilon + r) + \Psi_n(r, \theta)]$ is strictly increasing over $[0, r(\theta)]$ for any $\theta \leq \bar{\theta}$ and $\Psi_n(r, \theta)$ is decreasing in r, ii/ $g(0) = 0 \leq \theta\tilde{P}(1)$, iii/ $g(\tilde{r}) \geq \theta\tilde{P}(1)$.

Proof of Proposition 5

The patent holder will prefer the licence $(r_n^*(\theta), F_n^*(\theta)) = (r(\theta), 0)$ if and only if

$$nr(\theta)q^e(n, c - \epsilon + r(\theta)) \geq \theta\tilde{P}(1) = ns(\theta)q^e(n, c - \epsilon + s(\theta))$$

Note first that for any $\theta \leq \tilde{\theta}$, it holds that $r(\theta) \leq \tilde{r}$ because $r \to rq^e(n, c - \epsilon + r)$ is also strictly increasing over $[0, r(\theta)]$. Given that we have also $s(\theta) \leq \tilde{r}$ and $r \to rq^e(n, c - \epsilon + r)$ is strictly increasing over $[0, \tilde{r}]$, we can state that the inequality $nr(\theta)q^e(n, c - \epsilon + r(\theta)) \geq ns(\theta)q^e(n, c - \epsilon + s(\theta))$ holds if and only if $r(\theta) \geq s(\theta)$, which means that $(r^*(\theta), F^*(\theta)) = (r_n^*(\theta), F_n^*(\theta)) = (r(\theta), 0)$ if and only if $r(\theta) \geq s(\theta)$. Otherwise, the patent holder prefers the license (r_{-n}^*, F_{-n}^*) even though it triggers litigation.

Proof of Proposition 6

Since we tackle the case of sufficiently weak patents we can derive a comparison of $r^*(\theta) = r(\theta)$ to $r^e(\theta)$ for θ small enough from the comparison of $r'(0)$ to ϵ. Indeed, if $r'(0) > \epsilon$ (resp. $r'(0) < \epsilon$) then for θ sufficiently small, but different from 0, we will have $r(\theta) > \theta\epsilon$ (resp. $r(\theta) < \theta\epsilon$).

Differentiating the equation defining $r(\theta)$ and using the fact that $r(0) = 0$, we get:

$$r'(0) = \begin{cases} r_1'(0) = \frac{\pi^e(n, c - \epsilon)}{|\pi_2^e(n, c - \epsilon)|} & if \quad \pi^i(n - 1, c - \epsilon) = 0 \\ r_2'(0) = \frac{\pi^e(n, c - \epsilon) - \pi^i(n - 1, c - \epsilon)}{|\pi_2^e(n, c - \epsilon)|} & if \quad \pi^i(n - 1, c - \epsilon) > 0 \end{cases}$$

30

Note that in both cases, $r'(0)$ can be rewritten as:

$$r'(0) = \frac{\pi^e(n, c - \epsilon) - \pi^i(n - 1, c - \epsilon)}{|\pi^e_2(n, c - \epsilon)|}$$

Therefore,

$$r'(0) > \epsilon \Longleftrightarrow \frac{-\pi^e(n, c - \epsilon) + \pi^i(n - 1, c - \epsilon)}{\pi^e_2(n, c - \epsilon)} > 1$$

which yields

$$r'(0) > \epsilon \Longleftrightarrow \eta(\epsilon) < 1$$

Proof of Proposition 7

We derive from proposition 5 that $P^*(\theta) = \theta P^*_{-n}(1)$ if $r(\theta) \leq s(\theta)$ and $P^*(\theta) > \theta P^*_{-n}(1)$ if $r(\theta) > s(\theta)$. This directly yields the result under case 1, i.e. $P^*(1) = P^*_{-n}(1)$. If the latter equality does not hold, which means $P^*(1) > P^*_{-n}(1)$ (as we always have $P^*(1) \geq P^*_{-n}(1)$) then: i/ $P^*(\theta) = \theta P^*_{-n}(1) < \theta P^*(1)$ if $r(\theta) < s(\theta)$, ii/ $P^*(\theta) = nr(\theta) q^e(n, c - \epsilon + r(\theta)) < nv(\theta) q^e(n, c - \epsilon + v(\theta)) = \theta P^*(1)$ if $s(\theta) \leq r(\theta) < v(\theta)$, iii/$P^*(\theta) = \theta P^*(1)$ if $r(\theta) = v(\theta)$, iv/ $P^*(\theta) = nr(\theta) q^e(n, c - \epsilon + r(\theta)) > nv(\theta) q^e(n, c - \epsilon + v(\theta)) = \theta P^*(1)$ if $r(\theta) > v(\theta)$.

Proof of Proposition 8

Denote firm n the challenging firm and $\bar{r}$ the per-unit royalty rate at which a license is offered if the challenge fails. Cournot competition between $(n-1)$ firms (indexed by $i = 1, 2, ..., n-1$) whose marginal cost is $c - \epsilon + r$ and firm n whose marginal cost is $c - \epsilon + \bar{r}$ leads to the following equilibrium outputs:

$$q_i(r, \bar{r}) = \begin{cases} \frac{a - c + \epsilon - 2r + \bar{r}}{n+1} & \text{if} \quad i = 1, .., n - 1 \\ \frac{a - c + \epsilon - n\bar{r} + (n-1)r}{n+1} & \text{if} \quad i = n \end{cases}$$

For a given r, the value of the royalty rate $\bar{r}$ that maximizes the patentholder's licensing revenue is the solution to the following program:

$$\max_{\bar{r} \in [0, \epsilon]} P(r, \bar{r}) = (n - 1)r \frac{a - c + \epsilon - 2r + \bar{r}}{n + 1} + \bar{r} \frac{a - c + \epsilon - n\bar{r} + (n - 1)r}{n + 1}$$

Suppose that the innovation is non-drastic, i.e. $\epsilon < a - c$. The unique unconstrained maximum of the concave function $\bar{r} \longrightarrow P(r, \bar{r})$ is given by the FOC $\frac{\partial P(r, \bar{r})}{\partial \bar{r}} = \frac{a - c + \epsilon + 2(n-1)r - 2n\bar{r}}{n+1} = 0$. The maximum of the function $P(r, \bar{r})$ over the interval $\bar{r} \in [0, \epsilon]$ is reached at

$$\bar{r}(r) = \min\left(\epsilon, \frac{(n - 1)}{n}r + \frac{a - c + \epsilon}{2n}\right) = \min\left(\epsilon, r + \frac{1}{n}\left(\frac{a - c + \epsilon}{2} - r\right)\right)$$

Since $\epsilon < a - c$, we have $\frac{a - c + \epsilon}{2} > \epsilon$. Therefore, $r \in [0, \epsilon] \Longrightarrow \frac{a - c + \epsilon}{2} - r \geq 0$ and consequently $\bar{r}(r) \geq r$. Hence a firm which refuses a licensing contract and unsuccessfully challenges the

31

patent validity will get a new licensing offer with a higher royalty rate than the royalty paid by licensees that have accepted the initial licensing contract.

Moreover, the condition $\bar{r}(r) < \epsilon$ is fulfilled if and only if $r < (\frac{2n-1}{2(n-1)})\epsilon - \frac{a-c}{2(n-1)} \equiv \varphi$, which is positive whenever $\epsilon > \frac{a-c}{2n-1}$. For such a royalty rate r, we have $\pi(c - \epsilon + \bar{r}(r), c - \epsilon + r) = [\frac{a-c+\epsilon-n\bar{r}(r)+(n-1)r}{n+1}]^2$, and the condition expressing that all firms accept the licensing contract r is:

$$\pi(c - \epsilon + r, c - \epsilon + r) \geq \theta\pi(c - \epsilon + \bar{r}(r), c - \epsilon + r) + (1 - \theta)\pi(c - \epsilon, c - \epsilon)$$

Replacing $\bar{r}(r)$ by its value, one obtains:

$$\frac{(a - c + \epsilon - r)^2}{(n+1)^2} \geq \theta \left[\frac{a - c + \epsilon}{2(n+1)}\right]^2 + (1 - \theta)\left[\frac{a - c + \epsilon}{(n+1)}\right]^2 = \frac{4 - 3\theta}{4}\left[\frac{a - c + \epsilon}{(n+1)}\right]^2$$

This inequality is satisfied if and only if:

$$r \leq (a - c + \epsilon)\left(1 - \frac{\sqrt{4 - 3\theta}}{2}\right)$$

Hence a royalty rate $r < \varphi$ is accepted by all firms if and only if the previous inequality holds. Denoting $\bar{\theta}$ the unique solution in θ to the equation $(a - c + \epsilon)\left(1 - \frac{\sqrt{4-3\theta}}{2}\right) = \varphi$, we can then state that for $\theta \leq \bar{\theta}$, the maximal royalty rate accepted by all firms when post-trial license offer is possible is given by:

$$r^p(\theta) = (a - c + \epsilon)\left(1 - \frac{\sqrt{4 - 3\theta}}{2}\right)$$

Straightforward computations lead to $\frac{dr^p}{d\theta}(0) = \frac{3}{8}(a - c + \epsilon)$. It is easy to show that $\frac{dr^p}{d\theta}(0) < \epsilon$ for any $\epsilon \in \left]\frac{3}{5}(a - c), a - c\right[$. Consequently for such intermediate innovations, $r^p(\theta) < \theta\epsilon$ for sufficiently small values of θ. Note that for such innovations, the condition $\epsilon > \frac{a-c}{2n-1}$ is satisfied since $\frac{3}{5}(a - c) > \frac{a-c}{2n-1}$ for any $n \geq 2$.

Proof of Proposition 9

We have: $\pi^e(n, c) \geq \pi^i(n - 1, c - \epsilon + r^c(\theta))$ because $r^c(\theta) \leq \epsilon$. Since $\pi^e(n, c - \epsilon + r^c(\theta)) = \theta\pi^e(n, c) + (1 - \theta)\pi^e(n, c - \epsilon)$ we obtain that $\pi^e(n, c - \epsilon + r^c(\theta)) \geq \theta\pi^i(n - 1, c - \epsilon + r^c(\theta)) + (1 - \theta)\pi^e(n, c - \epsilon)$. The latter inequality implies that a royalty rate $r = r^c(\theta)$ will be non cooperatively accepted by all firms if proposed by the patent holder. Therefore $r^c(\theta) \leq r(\theta)$. Differentiating the equation $\pi^e(n, c - \epsilon + r^c(\theta)) = \theta\pi^e(n, c) + (1 - \theta)\pi^e(n, c - \epsilon)$ with respect to θ, we get $\frac{dr^c(\theta)}{d\theta} = \frac{\pi^e(n,c,c)-\pi^e(n,c-\epsilon)}{\pi^e_2(n,c-\epsilon+r^c(\theta))}$. Both the numerator and the denominator are negative which implies that $r^c(\theta)$ is increasing.

Since $\pi^e(n, c) - \pi^e(n, c - \epsilon) < 0$ (due to A3), the derivative $\frac{dr^c(\theta)}{d\theta}$ is increasing in θ over $[0, 1]$ (i.e. $r^c(\theta)$ is convex) if and only if $\pi^e_2(n, c - \epsilon + r^c(\theta))$ is increasing in θ over $[0, 1]$. Since $r^c(\theta)$ is continuous and strictly increasing from $r^c(0) = 0$ to $r^c(1) = \epsilon$, the latter condition

is equivalent to $\pi_2^e(n,x)$ is increasing in x over $[c - \epsilon, c]$, which means that $x \to \pi^e(n,x)$ is convex over $[c - \epsilon, c]$. In this case, $r^c(\theta) \geq \theta r^c(1) + (1 - \theta)r^c(0) = \theta\epsilon$.

8.2 Appendix B: Comparison of $r(\theta)$ with the threshold $s(\theta)$ for "weak" patents

This comparison can be easily made for θ sufficiently small since we just need to compare $r'(0)$ and $s'(0)$. Given that $s(0) = 0$, differentiating the equation

$$ns(\theta)q^e(n, c - \epsilon + s(\theta)) = \theta P_{-n}^*(1)$$

with respect to θ at the point $\theta = 0$, we get:

$$ns'(0)q^e(n, c - \epsilon) = P_{-n}^*(1)$$

which yields:

$$s'(0) = \frac{P_{-n}^*(1)}{nq^e(n, c - \epsilon)}$$

Since

$$r'(0) = \frac{\pi^e(n, c - \epsilon) - \pi^i(n - 1, c - \epsilon)}{|\pi_2^e(n, c - \epsilon)|} = \frac{F_n(r = 0)}{|\pi_2^e(n, c - \epsilon)|}$$

we get the following result:

Proposition 10 If $\frac{nF_n(r=0)}{P_{-n}^*(1)} > \frac{|\pi_2^e(n,c-\epsilon)|}{q^e(n,c-\epsilon)}$ then for sufficiently weak patents, the patent holder's optimal license offer is $(r(\theta), 0)$ and litigation over the patent validity is deterred at equilibrium.

If $\frac{nF_n(r=0)}{P_{-n}^*(1)} < \frac{|\pi_2^e(n,c-\epsilon)|}{q^e(n,c-\epsilon)}$ then for sufficiently "weak" patents, the patent holder's optimal license offer is (r_{-n}^*, F_{-n}^*) and litigation over the patent validity takes place at equilibrium.

The interpretation of the inequality in proposition 9 is not evident. However, note that the LHS is a ratio of licensing revenues: the numerator $nF_n(r = 0)$ is the highest licensing revenues that the patent holder can get from licensing an iron-clad patent to all firms through a pure fixed fee scheme $(r = 0)$, while the denominator $P_{-n}^*(1)$ is the highest licensing revenues generated by a two-part tariff license offer inducing less than n licensees. The RHS of the inequality can be rewritten as the product of two terms $\eta(\epsilon)\left[\frac{p^e(n,c-\varepsilon)-(c-\varepsilon)}{\varepsilon}\right]$ where $\frac{p^e(n,c-\varepsilon)-(c-\varepsilon)}{\varepsilon}$ is the margin per unit of cost reduction the firms get if all of them use the new technology royalty-free and $\eta(\epsilon) = \frac{\varepsilon|\pi_2^e(n,c-\epsilon)|}{\pi^e(n,c-\epsilon)}$ is the elasticity of a firm's profit with respect to cost reduction when all firms benefit from this reduction. Therefore, the inequality in the proposition includes all the ingredients that play a role in licensing "weak" patents: the revenues generated by various licensing schemes on the one hand and the determinants of the

33

downstream market competition on the other hand. One of the insights we can get from the latter proposition is that if the fixed fee mechanism is quite effective in extracting relatively high licensing revenues (as suggested by the literature on the comparison of licensing mechanisms in an iron-clad patent setting, see for instance Kamien 1992) then litigation deterrence at equilibrium is quite likely to hold. Furthermore, we show in proposition 6 that the elasticity $\eta\left(\epsilon\right)$ plays a crucial role in determining whether "weak" patents are overcompensated.