Higher education, employers' monopsony power and the labour share in OECD countries

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JOB SEARCH WITH UBIQUITY AND THE WAGE DISTRIBUTION

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Job search with ubiquity and the wage distribution[1]

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Abstract: We propose a search equilibrium model in which homogenous firms post wages along with a vacancy to attract job-seekers, while homogenous unemployed workers invest in costly search. The key innovation relates to the organisation of the search market and the search behaviour of the job-seekers. The search market is segmented by wage level, and unlike the rest of the literature individuals are ubiquitous in the sense they can choose the amount of search effort spent on *each* (sub-)market. We show that there exists a non-degenerate equilibrium wage distribution. Remarkably, the density of this wage distribution is hump-shaped, and it can be right-skewed. Our results are illustrated by an example originating a Beta wage distribution.

Keywords: Search effort; Segmented markets; Monopsony; Wage dispersion

J.E.L. classification: D83; J31; J41; J64

1 Introduction

Since at least Stigler (1961), it is widely acknowledged that search frictions reduce workers' ability to generate wage competition between potential employers to attach labour services. This should result in employers' monopsony power, and wage dispersion for homogenous workers. Given the strength of this idea, the magnitude of residual wage dispersion found in most empirical studies[1], and the huge amount of progress achieved in job search theory, one would expect the theory to be now able to offer equilibrium wage distributions for homogenous workers that can replicate empirical regularities. In particular, the density of such wage distributions should be hump-shaped/single-peaked. And indeed, many structural estimates have been performed on the grounds of search equilibrium models. Surprisingly, many such estimates are based on equilibrium wage distributions which do not have good empirical properties. On top of that, the basic search models are generally unable to generate wage dispersion.

The main problem has been identified by Diamond (1971). If there is random search (or undirected search), firms have no reason to offer a wage above the common reservation wage, because it does not alter their rate of contact. In equilibrium, the wage thus collapses to unemployment income. Directed search models first introduced by Hosios (1990), Montgomery (1991), and Moen (1997), aim to solve this paradox. The search market is segmented by wage. Workers can choose which jobs they send their application to – or, alternatively, which market they prospect –, while the probability to get a job is a decreasing function of the length of the job queue. Wage competition thus takes place at the time of wage/market choice. In equilibrium, all wage offers must yield the same utility: if not, the jobs would not be prospected. This implies that the employment (recruitment) probability is a decreasing (an increasing) function of the wage. Typically, there is a unique wage offer balancing workers' marginal cost derived from searching higher wage firms (a lower employment probability) and their marginal benefit (a better wage once employed). Thus, workers receive more than their reservation wage, but there is still a unique wage offer. Put differently, the wage distribution is degenerate.

Given the inability of the basic models to generate wage dispersion, papers which aim to offer microfoundations to the wage distribution have been led to dramatically alter the seminal assumptions. Three groups of papers can be distinguished. First, there are papers which introduce on-the-job search. If workers search on-the-job, reservation wages are heterogeneous. Burdett and Mortensen (1998) show that this heterogeneity in reservation wages sustain the existence of a non-degenerate wage distribution with random search. However, the density of the wage distribution is counterfactually strictly increasing. De la Croix and Shi (forthcoming) consider a directed search version. They show that the density of the wage offer distribution is strictly decreasing. In addition, at given initial wage, all workers prospect the same jobs, which means they all receive the same wage in case of hiring. Second, there are papers which introduce firms' heterogeneity. Van den Berg and Ridder (1998) do so while estimating the Burdett-Mortensen model. Mortensen (2000) endogenizes productivity choices in the same model. Postel-Vinay and Robin (2002a) also endogenize firms' heterogeneity, but in a model where employers can react to other firms approaching their workers by making a counteroffer (see also Postel-Vinay and Robin, 2002b). Those papers manage to generate a hump-shaped wage distribution, but

[1] Autor and Katz (1999) show that simple regressions of the log of the wage on individual characteristics like education, age/experience, gender and location leave unexplained the two thirds of the variance. Of course, part of the residual wage dispersion resorts to unobserved heterogeneity.

of course at the cost of assuming ex-ante heterogeneity[2]. In a close framework, but with only ex-post heterogeneity (i.e. when the quality of the match is revealed), Moscarini (2005) shows that it is possible to arrive at a wage distribution with good empirical properties (unimodal, skewed, with a Paretian right tail) with a simple Gaussian output noise. Third, there are papers which modifiy the matching technology so that a worker can receive multiple offers at the same time. Thus, wage competition takes place at the time of choosing between different job offers. This way to analyze the search process is very close to Stigler's original insight. In Acemoglu and Shimer (2000), firms post wages and workers choose the number of costly offers they receive. Acemoglu and Shimer show that there is equilibrium wage dispersion. However, the density of the wage offer distribution is strictly decreasing with a mass point at its upper bound. In a similar vein, Galenianos and Kircher (2005) consider the directed search model of Albrecht, Gautier and Vroman (2005) in which firms post wages and the workers send multiple applications. Unlike Albrecht et al, Galenianos and Kircher assume that firms commit to pay the posted wage irrespective of the number of job offers received by the applicant[3]. They also obtain a strictly decreasing density of the wage distribution.

Our paper follows a different route. It remains in the tradition of wage-posting models with directed search such as Moen (1997). However, it modifies one of the crucial assumptions: workers are not attached to a single market. They rationally prospect jobs offering different wages, and, therefore, firms offer different wages in equilibrium. They do so because market-specific return to search is strictly decreasing. In standard directed search models, search efforts for different wages are perfectly substitutable. It means that the market-specific marginal return to search investment is independent on the level of search investment. That is why workers send all their applications to the same market, i.e. to jobs paying the same wage. Put differently, the specification of the search technology implies that workers are bound to search jobs on one and only one market. In our paper, the marginal return to search investment is strictly decreasing on each market. Hence, workers can offset a lower return at given search intensity by a lower search intensity. It leads them to participate to a continuum of markets: they are, in this respect, *ubiquitous*.

In this setting, we obtain (i) a non-degenerate wage distribution, and (ii) a single-peaked density of the wage offer distribution as natural outcomes.

The main results can be explained as follows. The path of market-specific search investment reflects the path of market-specific return to search. As the return to search first increases with wage, and then decreases, search investment evolves non-monotonously with the wage. It reaches its maximum at the only wage that would exist in the standard directed search model. In turn, the number of vacancies on each market reacts to two effects. First, it tends to decrease with the wage, as paying higher wages must be compensated by lower search costs, and thus longer job queues. Second, it tends to increase with the number of effective job-seekers, because recruitment rates depend on the ratio of vacancies to effective number of job-seekers. As a result, the number of vacancies tends to adopt the path of market-specific search investment. The combination of these two effects implies that the number of vacancies is first increasing, and then decreasing in wage. It follows that the density of the wage offer distribution is hump-

[2] Of course, this may be the correct story: there can be wage dispersion among homogenous workers because firms are heterogenous, no matter whether firms' heterogeneity is endogenous or not. Our purpose, however, is to generate wage dispersion with homogenous agents on both sides of the market.

[3] Put otherwise, firms cannot react to other offers by increasing their initial wage in Galenianos and Kircher, while they can in Albrecht et al.

shaped. Finally, the actual wage distribution can be deduced from the wage offer distribution and the knowledge of search investments. Its density is also single-peaked.

The wage distributions are also consistent with another property of empirical wage distributions: right-skewness. More precisely, the density of the wage offer distribution is always right-skewed, while the density of the actual wage distribution may or may not be right-skewed. All these properties are illustrated by an example, in which the matching technology is Cobb-Douglas, and the efficiency of search effort is isoelastic. We show that both the wage offer distribution and the actual wage offer distribution follow a Beta distribution.

Our paper matches two distinct ideas that have been previoulsy investigated in the literature on search unemployment. First, the search market is segmented by wage, and individuals choose which wage/job to prospect. We discussed below the importance of such an idea in the directed search literature. Second, individuals simultaneously participate to different segments of the search market. This idea is increasingly popular in models interested in two-sided heterogeneity. The search market is segmented by job type, and workers choose the subset of sub-markets they participate. In models on overeducation, educated workers seek both complex and simple positions while uneducated workers only search simple jobs (see Gautier, 2002). In models with multidimensional skills, workers have a bundle of skills and participate to sub-markets on the basis of comparative advantage (see Moscarini, 2001), or on the basis of their ability to perform on the underlying technologies (see Charlot, Decreuse and Granier, 2005).

Wage segmentation is a natural extension of job segmentation: different jobs are usually associated to different wages, so that individuals actually perceive job segmentation as wage segmentation. It is worth discussing the impact of this assumption in terms of congestion externalities. In our model, vacancies offering different wages do not create congestion effects to each other. An additional offer at 40,000 euro a year does not reduce the probability to fill a position offering 30,000 euro a year at given search intensities. However, the former offer raises the welfare of the unemployed. In response, the unemployed reduce their effort to get the latter wage offer. As a consequence, the probability to fill the latter position is lower. This argument may seem a bit more difficult to accept when one imagines the case of a 40,000 euro a year position versus a 39,999 euro a year position. Continuous segmentation is an assumption made for simplicity. We believe that accounting for discrete segmentation would not alter our main results[4].

Wage dispersion reflects employers' monopsony power in our setting[5]. However, it also ensures the segmentation of the search place into a continuum of sub-markets. Wages, therefore, have two roles. On the one hand, they have the traditional function to allocate the economywide resources between consumption (workers' income) and investment (vacancy costs, which absorb firms' profits given that there is free entry on the search market). On the other hand, wages shape the technological structure of the search market, that is the number of places that can be visited simultaneously by the job-seekers. These two different roles assigned to a unique instrument suggest that the decentralized outcome is inefficient, a result that deeply differs from

[4] Accounting for discrete segmentation would raise an important issue: how could one endogenize the different wage thresholds delimiting the different market segments? Is this a problem of information (it may be necessary to save on search costs to aggregate 30,000 euro to 35,000 euro positions), or preference (30,000 euro to 35,000 euro positions offer similar standards of living, while a 40,000 euro position may change the life of the family). Interestingly, models of segmentation by job type implicitly face similar problems: what is the difference between simple jobs and complex jobs?

[5] Of course, the wage distribution collapses when matching frictions disappear.

Moen (1997) where workers are attached to a single market.

The rest of the paper proceeds as follows. Section 2 presents a model in which workers can choose their search intensity, but are bound to choose one and only one search market. We call the associated equilibrium concept the localized search equilibrium, which is basically the equilibrium of a standard directed search model with search intensity. In Section 3, workers are no longer obliged to choose a particular market. We call the associated equilibrium concept the ubiquitous search equilibrium. Section 4 is devoted to the study of the equilibrium wage distributions. Section 5 studies the efficiency of the decentralized outcome. Section 6 concludes.

All the proofs are in the appendix.

2 Localized search

What we call a localized search equilibrium is a version of Moen's (1997) competitive search equilibrium in which search effort is made endogenous.

2.1 Search technology

Our model follows the lines of the wage posting search model developed, among others, by Moen (1997) and Acemoglu and Shimer (1999). In this framework, firms post vacancies with non-negotiable wages. The workers, knowing all the posted wages, choose the amount of effort they will spend to search for a job. While making this choice, they are aware that if they decide to search for a job offering a wage w, they will compete with other workers seeking for the same wage. Symmetrically, if a firm posts a vacancy associated with the wage w, it will compete to attract workers with the other firms offering the same wage. In other words, for each wage w, workers face a specific queue length and vacant jobs have a specific probability to be filled. For Acemoglu and Shimer (1999), such a representation of the search process recognizes that the labor market is segmented by wages and that search frictions exist within each particular sub-market (or island).

As in Moen (1997), we assume that an unemployed person is bound to search for a job on one and *only* one island. This is the reason why we use the term "localized" to label our equilibrium concept. Thus, the search process consists in two stages: 1) firms post wages, 2) each unemployed chooses on which sub-market she will search for a job and decides upon the amount of search effort she will spend on the previously chosen sub-market.

An unemployed person whose search effort is s bears the cost $c(s)$, but increasing effort provides more contacts with potential employers. In such a perspective, we will distinguish the amount of search effort from the *efficiency* of such an effort. When a job-seeker invests an amount of effort s, the efficiency of this effort is measured by the function $x(s)$. In the sequel, we make the following assumption on the functions $c(s)$ and $x(s)$ influencing the search process.

Assumption A1 *The cost of effort function* $c : [0, +\infty) \to [0, +\infty)$ *is strictly increasing, convex, twice differentiable, and satisfies* $c(0) = 0$*,* $c'(0) = c_0 > 0$*, and* $c'(+\infty) = +\infty$*.*

The efficiency of effort function $x : [0, +\infty) \to [0, +\infty)$ *is strictly increasing, strictly concave, twice differentiable, and satisfies* $x(0) = 0$*,* $x'(0) = +\infty$*, and* $x'(+\infty) = 0$*.*

It is worth to notice that the assumption made on the cost of effort function authorized this function to be linear, i.e. of the form $c_0 \cdot s$. The marginal cost of search effort is increasing, while the marginal productivity of search effort is strictly decreasing. The difference between the search effort and its efficiency can be illustrated by the following example. Imagine that the search effort is measured by the time s spent in acquiring information on available vacancies and writing letters that are sent to potential employers. Then $x(s)$ represents the number of letters written during a time interval of length s. Assumption A1 states that the technology of information acquisition exhibits marginal decreasing returns. This assumption reflects the fact that it is more and more difficult to find additional vacancies on a particular market.

It is important to realize that this technology is (sub)market-specific, while the cost of search depends on the overall search effort. This distinction between market-specific and overall search effort is not useful in the localized search model, where workers must choose one and only one market before searching a job. However, the fact that there is a search technology with decreasing marginal returns on each market will be crucial in the ubiquitous search model of section 3.

A sub-market may be either closed – when no one enters this sub-market – or opened. Which sub-markets are closed and which sub-markets are opened is an outcome of the model. When there are $u(w)$ unemployed persons searching on the sub-market offering the wage w, and if the search effort of an individual on this sub-market reaches $s(w)$, the overall efficient search effort on this sub-market amounts to $\overline{x}(w)u(w)$ where $\overline{x}(w) \equiv \overline{x[s(w)]}$ is the market-specific mean efficient search intensity. With $v(w)$ vacancies offering the wage w, the flow number of matches on island w is equal to $M[\overline{x}(w)u(w), v(w)]$, where the matching function satisfies the following (standard) assumption:

Assumption A2 *The technology* $M : [0,+\infty) \times [0,+\infty) \to [0,+\infty)$ *is twice continuously differentiable, strictly increasing in each of its arguments, strictly concave and linearly homogenous. It satisfies the boundary conditions* $M(U,0) = M(0,V) = 0$*, and* $\lim_{U\to+\infty} M(U,V) = \lim_{V\to+\infty} M(U,V) = +\infty$.

Let $m(\theta) \equiv M(1/\theta, 1)$. The flow probability for a vacant job offering the wage w to meet a job-seeker is:

$$m[\theta(w)] \equiv \frac{M[\overline{x}(w)u(w), v(w)]}{v(w)},$$

while the flow probability for a job seeker to meet a vacant job *per efficient unit of search* is:

$$\theta(w)\, m[\theta(w)] \equiv \frac{M[\overline{x}(w)u(w), v(w)]}{\overline{x}(w)u(w)}.$$

In these formulas, $\theta(w) \equiv v(w)/\overline{x}(w)\, u(w)$ represents the market-specific tightness, that is the tightness specific to the island w.

2.2 Agents' behaviour

We study the steady state of a continuous time economy. There are a continuum of identical and infinitely-lived firms and workers. Each firm is associated with only one job. The measure of workers is normalized to one, while the measure of firms is endogenously determined through entry. Both are risk neutral and discount time at instantaneous rate r. Jobs can be either filled or vacant, while workers can be either employed or unemployed. A pair of worker/firm produces a flow output y until (exogenous) separation at rate q. Unemployed workers enjoy unemployment income z, $0 \leq z < y$, while firms endowed with a vacancy bear the flow cost h.

Following Moen (1997) and Acemoglu and Shimer (1999), we assume that workers observe all posted wage w and corresponding market tightness $\theta(w)$. Workers decide which sub-market to enter on the basis of this knowledge. Let $V_u(w)$ and $V_e(w)$ denote respectively, the value of unemployment and the value of employment on the sub-market offering the wage w. The asset value equations for $V_e(w)$ and $V_u(w)$ are given by:

$$rV_e(w) = w + q\left[V_u(w) - V_e(w)\right] \tag{1}$$

$$rV_u(w) = \max_s \left\{z - c(s) + x(s)\,\theta(w)\,m\left[\theta(w)\right]\left[V_e(w) - V_u(w)\right]\right\} \tag{2}$$

Let us denote by $R(w) \equiv rV_u(w)$ the flow gain of an unemployed. The optimal search investment $s(w)$ responds to:

$$c'\left[s(w)\right] = x'\left[s(w)\right]\theta(w)\,m\left[\theta(w)\right]\frac{w - R(w)}{r+q} \tag{3}$$

$$R(w) = z - c\left[s(w)\right] + x\left[s(w)\right]\theta(w)\,m\left[\theta(w)\right]\frac{w - R(w)}{r+q} \tag{4}$$

The asset values of a vacancy advertised at wage w, denoted $\Pi_v(w)$, and of a filled job paying w, denoted $\Pi_e(w)$, satisfy the arbitrage equations:

$$r\Pi_v(w) = -h + m\left[\theta(w)\right]\left[\Pi_e(w) - \Pi_v(w)\right], \qquad r\Pi_e(w) = y - w + q\left[\Pi_v(w) - \Pi_e(w)\right] \tag{5}$$

Consequently, when an entrepreneur decides to post the wage w, her expected gain is given by:

$$r\Pi_v(w) = \frac{-h(r+q) + m\left[\theta(w)\right](y-w)}{r+q+m\left[\theta(w)\right]} \tag{6}$$

The main consequence of the assumption of localized search, according to which an unemployed is constrained to search for a job on one and only one sub-market, is that competition between employers to attract workers oblige firms to offer the same expected utility for the unemployed on each opened sub-market. Let us denote by V_u this common value, and by $R = rV_u = \max_w R(w)$ the associated reservation wage. This has two implications.

First, the search effort is the same on all opened sub-markets. Formally, let $\Omega \in [R, y]$ be the set of opened sub-markets. For all $w \in \Omega$, equations (3) and (4) imply that

$$R = z - c\left[s(w)\right] + c'\left[s(w)\right]\frac{x\left[s(w)\right]}{x'\left[s(w)\right]} \tag{7}$$

Assumption A1 implies that, for a given $R \geq z$, there exists a unique search effort $s(w)$ which is solution of equation (7). This optimal search effort does not depend on w, and we will denote it by $\sigma(R)$. It is easy to check that $\sigma'(R) > 0$.

Second, the fact that a sub-market is opened in equilibrium means its tightness must have some optimality property from workers' perspective. Such tightness is monotonously related to workers' common reservation wage. Formally, for all $w \in \Omega$, equations (3) and (7) give:

$$c'\left[\sigma(R)\right] = x'\left[\sigma(R)\right]\theta(w)\,m\left[\theta(w)\right]\frac{w-R}{r+q} \tag{8}$$

This equality implicitly defines tightness as a function $\theta(w, R)$ of the workers' reservation wage. It is easy to check that $\theta_R(w, R) > 0$.

2.3 Localized search equilibrium

For each sub-market, firms must make expectations concerning the associated market tightness. Given that only a subset of potential sub-markets will be opened in equilibrium, such expectations concern both equilibrium and out-of-equilibrium outcomes. We shall denote by $\widetilde{\theta}(w)$ firms' common expectation on the pattern by wage of market tightness. In the sequel, we will restrain ourselves to the following hypothesis.

Assumption A3 *Let $R \geq 0$ be given. Firms' expectations are given by*

$$\widetilde{\theta}(w) = \begin{cases} \theta(w, R) & \textit{if } w \in [R, y] \\ 0 & \textit{elsewhere} \end{cases}.$$

The idea is the following. Firms have no reason to post a wage on a sub-market that will not be prospected by the job-seekers. Yet, they must assign a value to the tightness variable when evaluating the opportunity to post a wage on a particular sub-market. They rationally expect that if this sub-market were opened, it would be consistent with workers' maximisation process. Consequently, when an entrepreneur chooses to post a wage equal to w, she considers, for a given reservation wage R, that the corresponding effort $\sigma(R)$ and market tightness $\theta(w)$ must satisfy the system of equations (7) and (8).

On the basis of this expectation $\widetilde{\theta}(w)$, each entrepreneur maximizes her expected gain $\Pi_v(w)$ given by (6). Differentiating $\Pi_v(w)$ with respect to w and setting this derivative to zero yields:

$$\begin{aligned}&\left\{m'\left[\widetilde{\theta}(w)\right]\widetilde{\theta}'(w)(y-w) - m\left[\widetilde{\theta}(w)\right]\right\}\left\{r+q+m\left[\widetilde{\theta}(w)\right]\right\}\\ &+m'\left[\widetilde{\theta}(w)\right]\widetilde{\theta}'(w)\left\{-h(r+q)+m\left[\widetilde{\theta}(w)\right](y-w)\right\} = 0\end{aligned}$$

The free-entry condition, $\Pi_v(w) = 0$, implies that the last term between brackets vanishes and the optimal market wage is characterized by the following equation:

$$\alpha\left(\widetilde{\theta}(w)\right)\frac{\widetilde{\theta}'(w)}{\widetilde{\theta}(w)} = -\frac{1}{y-w} \tag{9}$$

where $\alpha(\theta) = -\theta m'(\theta)/m(\theta) \in (0,1)$ is the elasticity of the recruitment rate with respect to the market tightness.

On the other hand, differentiating relation (8) with respect to w, one gets:

$$\frac{\widetilde{\theta}'(w)}{\widetilde{\theta}(w)} = \frac{\theta_w(w,R)}{\theta(w,R)} = -\frac{1}{[1-\alpha(\theta(w,R))](w-R)}$$

Substituting this expression of $\widetilde{\theta}'(w)/\widetilde{\theta}(w)$ into (9) gives the optimal market wage as a function of the reservation wage R and the market tightness $\widetilde{\theta}(w)$:

$$w = \alpha\left(\widetilde{\theta}(w)\right)y + \left[1-\alpha\left(\widetilde{\theta}(w)\right)\right]R \tag{10}$$

In equilibrium, free entry implies the exhaustion of all rents and $\Pi_v(w) = 0$. Equation (6) implies that the market tightness $\theta(w)$ must satisfy:

$$m[\theta(w)] = \frac{h(r+q)}{y-w} \tag{11}$$

The consistency of expectations implies that $\widetilde{\theta}(w) = \theta(w)$ in equilibrium. This yields

$$w = \alpha(\theta(w))\, y + [1 - \alpha(\theta(w))]\, R \tag{12}$$

Interestingly, the following Lemma shows that this equation defines a unique wage for a given R.

Lemma 1 *Let $\phi : [0, y] \times \mathbb{R} \to \mathbb{R}$ be such that*

$$\phi(w, R) \equiv \alpha(\theta(w))\, y + [1 - \alpha(\theta(w))]\, R - w \quad \text{with} \quad \theta(w) = m^{-1}\left[\frac{h(r+q)}{y - w}\right],$$

For all $R \in [0, y]$, $\phi(w, R) = 0$ has a unique root in w.

The properties of the localized search equilibrium are summarized in the following proposition.

Proposition 1 THE LOCALIZED SEARCH EQUILIBRIUM
Under Assumptions A1, A2 and A3,

(i) A localized search equilibrium is characterized by a triplet $(\theta^*, s^*, w^*, R^*)$ corresponding to the equilibrium value of the wage, the flow gain of an unemployed and the labor market tightness that satisfies:

$$m(\theta) = \frac{h(r+q)}{y - w} \tag{13}$$

$$c'(s) = x'(s)\, \theta m(\theta) \frac{w - R}{r + q} \tag{14}$$

$$R = z - c(s) + x(s)\, \theta m(\theta) \frac{w - R}{r + q} \tag{15}$$

$$w = \alpha(\theta)\, y + [1 - \alpha(\theta)]\, R \tag{16}$$

(ii) There exists a unique localized search equilibrium.

Using equations (13) to (16), it is possible to show that s^* and θ^* are the solutions of the following system:

$$\frac{h}{m(\theta^*)} = \frac{[1 - \alpha(\theta^*)]\,[y - z + c(s^*)]}{r + q + x(s^*)\alpha(\theta^*)\, \theta^* m(\theta^*)}, \quad c'(s^*) = \frac{\alpha(\theta^*)}{1 - \alpha(\theta^*)} h\theta^* x'(s^*) \tag{17}$$

These two last equations will prove useful while studying the efficiency properties of the localized search equilibrium (see Proposition 7 in Section 5).

To end characterizing the localized search equilibrium, it remains to define the unemployment rate. As the equilibrium wage is unique, there is a unique opened (sub-)market. On this market, the job finding rate, denoted by λ^*, is given by:

$$\lambda^* = x\,[\sigma(R^*)]\, \theta^* m(\theta^*),$$

and the equilibrium unemployment rate, denoted by u^*, stems from the equality between the flows in and out of employment, i.e. $q(1-u^*) = \lambda^* u^*$. Finally, one has:

$$u^* = \frac{q}{q+\lambda^*}$$

For our purpose, the important result is that there is a unique equilibrium wage when the job search process is localized. We now consider the labour market equilibrium when we relax the assumption of localized search according to which there is indivisibility of search investments between sub-markets.

3 Ubiquitous search

3.1 Agents' behaviour with ubiquity

As in the previous section, the search market is segmented by wage. The behavior of the entrepreneurs remains unchanged, i.e. firms post vacancies with associated non-negotiable wages. However, we now assume that an unemployed person is able to search simultaneously on *every* existing sub-market. Hence, there is ubiquity on the search market: a job-seeker is not bound to search on only one sub-market. Importantly, ubiquity means that the worker has to decide the search investment on *every* existing sub-market.

If there are u unemployed persons in the economy, the overall search effort on the sub-market offering the wage w now amounts to $\overline{x}(w)u$ where $\overline{x}(w)$ still denotes the market-specific mean efficiency of search efforts. With $v(w)$ vacancies offering the wage w, the flow number of matches on sub-market w is equal to $M\left[\overline{x}(w)u, v(w)\right]$, where the matching function still satisfies Assumption A2. Consequently, the flow probability for a vacant job to meet a job-seeker and the flow probability for a job-seeker to meet a vacant job per efficient unit of search on the sub-market offering the wage w are given by:

$$m\left[\theta\left(w\right)\right] \equiv \frac{M\left[\overline{x}(w)u, v(w)\right]}{v(w)}, \quad \theta\left(w\right) m\left[\theta\left(w\right)\right] \equiv \frac{M\left[\overline{x}(w)u, v(w)\right]}{\overline{x}(w)u},$$

where market-specific tightness is now defined by $\theta\left(w\right) \equiv v\left(w\right) / \left(\overline{x}\left(w\right) u\right)$.

Let us denote by V_u the expected lifetime utility of an unemployed individual. If this person takes a job paying w, she obtains the lifetime utility $V_e(w)$ described by the arbitrage equation:

$$rV_e(w) = w + q\left[V_u - V_e(w)\right]$$

It follows that the reservation wage, R, is always such that $R = rV_u$. A priori, the set of possible wages – equivalently, the set of islands – covers the entire interval $[z, y]$, but the set of islands that will be visited belongs to the interval $[R, y]$. Like in the previous section, each job-seeker observes the posted wage w and the corresponding labor market tightness $\theta\left(w\right)$ on each sub-market.

But now, an unemployed person has to choose the set $\{s(w)\}$ of search efforts that she will *simultaneously* exert on *each* sub-market w. Let us denote by $S = \int_R^y s\left(w\right) dw$ the total search effort, the expected gain of a job seeker reads:

$$rV_u = \max_{s(.)} \left\{ z - c\left(S\right) + \int_R^y x\left[s\left(w\right)\right] \theta\left(w\right) m\left[\theta\left(w\right)\right] \left[V_e(w) - V_u\right] dw \right\} \tag{18}$$

On sub-market w, the optimal search effort of an unemployed person is thus characterized by the first-order condition:

$$c'(S) = x'[s(w)]\,\theta(w)\,m(\theta(w))\,\frac{w-R}{r+q}, \quad \text{for all } w \in [R, y] \tag{19}$$

And the equation (18) defining V_u becomes:

$$R = z - c(S) + \int_R^y x[s(w)]\,\theta(w)\,m(\theta(w))\,\frac{w-R}{r+q}dw \tag{20}$$

The first-order condition (19) states that, in every sub-market, the marginal cost of searching for a job must be equal to the marginal gain of this activity. This relation highlights the fact that the search technology $x(s)$ must exhibit marginal decreasing returns to obtain a definite search effort associated with each sub-market. Indeed, under constant marginal returns to search, a job-seeker would allocate her whole search investment to the sub-market that yields the largest reward. Such a sub-market offers the best combination of wage and employment probability, i.e. it maximizes the product $\theta(w)\,m(\theta(w))\,(w-R)$. Hence, despite workers would be allowed to search on several sub-markets at a time, they would not use this possibility and only one market would be opened. This result does not hold anymore with marginal decreasing returns in the search technology. A worker can then compensate a lower reward by investing less, which raises the marginal productivity of search effort and leaves the marginal benefit to search unchanged.

Consequently, there is no longer a unique value of the search effort: the search investment varies on each prospected sub-market. Moreover, equations (19) and (20) imply that the reservation wage R and the collection of search efforts $\{s(w)\}$ are linked by

$$R = z - c(S) + c'(S)\int_R^y \frac{x[s(w)]}{x'[s(w)]}dw \tag{21}$$

$$S = \int_R^y s(w)\,dw \tag{22}$$

As in the localized search case, the fact that the worker spends $s(w)$ on a particular sub-market means that the associated market tightness must have some optimality property which induces such a choice from the worker. When ubiquitous search is possible, it means that tightness on each sub-market must satisfy equation (19). This equation defines tightness as a function $\theta(w, R, \{s(w)\})$ in which the reservation wage R and the set of search efforts $\{s(w)\}$ are linked by (21) and (22).

3.2 Ubiquitous search equilibrium

As in the localized search case, for each sub-market, firms must make expectations concerning the associated tightness. We shall still denote by $\widetilde{\theta}(w)$ firms' common expectation on the pattern by wage of market tightness. We consider the following assumption.

Assumption A3' *Let $R \geq z$ and $\{s(w)\}_{w \geq R}$ such that (21) and (22) hold. Firms' expectations are given by*

$$\widetilde{\theta}(w) = \begin{cases} \theta(w, R, \{s(w)\}) & \text{if } w \in [R, y] \\ 0 & \text{elsewhere} \end{cases}.$$

Assumption A3' is a mere adaptation of Assumption A3 in the context of ubiquitous search. First, firms have no reason to post a wage on a sub-market that will not be prospected by the job-seekers. Second, they must assign a value to the tightness variable when evaluating the opportunity to post a wage on a particular sub-market. They rationally expect that if this sub-market were opened, it would be consistent with workers' maximisation process. Consequently, when an entrepreneur chooses to post a wage equal to w, she considers that, for a given reservation wage R and a given collection $\{s(w)\}_{w \geq R}$ of search efforts satisfying (21) and (22), the market tigthness must satisfy the equation (19).

The asset values $\Pi_v(w)$ and $\Pi_e(w)$ of a vacancy posting a wage w and of a filled job paying this wage are still defined by the relations (5). Each entrepreneur can then maximize her expected gain $\Pi_v(w)$ given by (6). Formally, the entrepreneur's problem is the same as in the case with localized search. Thus, when the free-entry condition $\Pi_v(w) = 0$ is satisfied, the equilibrium value of the market tightness function $\theta(w)$ is still given by equation (11) for any wage in the interval $[R, y]$.

Proposition 2 THE UBIQUITOUS SEARCH EQUILIBRIUM

Under Assumptions A1, A2 and A3',

(i) An ubiquitous search equilibrium is characterized by a quadruplet $(\theta^*(w), s^*(w), S^*, R^*)$ *corresponding to the equilibrium value of the labor market tightness function, the search effort function, the global effort and the flow gain of an unemployed, that satisfies:*

$$m[\theta(w)] = \frac{h(r+q)}{y-w}, \ \forall w \in [R, y] \tag{23}$$

$$c'(S) = x'[s(w)]\,\theta(w)\, m[\theta(w)]\,\frac{w-R}{r+q}, \forall w \in [R, y] \tag{24}$$

$$R = z - c(S) + \int_R^y x[s(w)]\,\theta(w)\, m(\theta(w))\,\frac{w-R}{r+q}dw \tag{25}$$

$$S = \int_R^y s(w)\, dw \tag{26}$$

(ii) There exists a unique ubiquitous search equilibrium.

To end characterizing the ubiquitous search equilibrium, it remains to define the unemployment rate. The job-finding rate is worth

$$\lambda = \int_R^y x[s(w)]\,\theta(w)\, m[\theta(w)]\, dw,$$

that is the sum of the different rates of contact over the different markets the job-seekers prospect. Unemployment can then be computed from the equality between flows in and out of unemployment: $q(1-u) = \lambda u$. The unemployment rate is worth $u = q/(q+\lambda)$.

We now turn to the properties of the equilibrium.

3.3 Market-specific search investment

Differentiating the logarithm of both sides of equation (24) with respect to w, we get:

$$\frac{x''[s(w)]}{x'[s(w)]}s'(w)+\frac{\theta'(w)}{\theta(w)}\left[1-\alpha\left(\theta\left(w\right)\right)\right]+\frac{1}{w-R}=0, \tag{27}$$

while differentiating (23) with respect to w still gives (9). Then, eliminating $\theta'(w)/\theta(w)$ between (27) and (9) one obtains:

$$s'\left(w\right)\equiv-\frac{x'[s(w)]}{x''[s(w)]}\frac{\alpha\left(\theta\left(w\right)\right)y+\left[1-\alpha\left(\theta\left(w\right)\right)\right]R-w}{\alpha\left(\theta\left(w\right)\right)\left(y-w\right)\left(w-R\right)} \tag{28}$$

The wage has two conflicting effects on search effort. On the one hand, there is a positive direct effect. At given market tightness, a higher wage raises the return to search, thereby motivating search investment. On the other hand, there is a negative indirect effect. Indeed, market tightness decreases with the wage. A higher wage deteriorates the search prospects, thereby reducing search investment. Therefore the sign of $s'\left(w\right)$ seems ambiguous. We can go further by noticing that the function $\phi(w,R)$ defined in Lemma 1 appears at the right side of equation (28). More precisely, one can see that $s'\left(w\right)$ has the same sign as $\phi(w,R)$. This remark enables us to state the following proposition.

Proposition 3 THE PATTERN OF SEARCH INVESTMENT
Under Assumptions A1 to A3, the effort function $s:[R,y]\rightarrow[0,+\infty)$ *is* $\cap$*-shaped and satisfies* $s\left(R\right)=s\left(y\right)=0$

The key finding of Proposition 3 is the non-monotonicity of the relationship between wage and search investment depicted by Figure 1. The pattern of search investment reflects the pattern of marginal reward to search. Remind such reward consists in a peculiar combination of wage and employment probability. Search investment is thus very small at both low and high wages. In the former case, employment probability is large, but it is not worth to invest a lot as the wage is close to the reservation wage. In the latter case, the wage may be very good, but job opportunities collapse. More generally, the direct positive effect of the wage on search investment dominates at low wages, while the indirect negative effect due to lower tightness dominates at higher wages. The search investment then reaches a maximum on the market where the reward is the highest.

Let us denote by $w_1(R)$ the root of equation $\phi\left(w,R\right)=0$ that gives the largest search investment (see Figure 1). This wage is defined by $w_1(R)=\alpha\left[\theta\left(w_1(R)\right)\right]y+\left\{1-\alpha\left[\theta\left(w_1(R)\right)\right]\right\}R$. For R given, this wage is actually the only wage offer in the localized search equilibrium.

4 Wage distributions with ubiquity

The purpose of this section is to analyse the shape of the wage distribution that is implied by our model. We proceed in three steps. First, we focus on the equilibrium wage offer distribution. Second, we analyse the wage distribution among employed workers. Third, we consider an example.

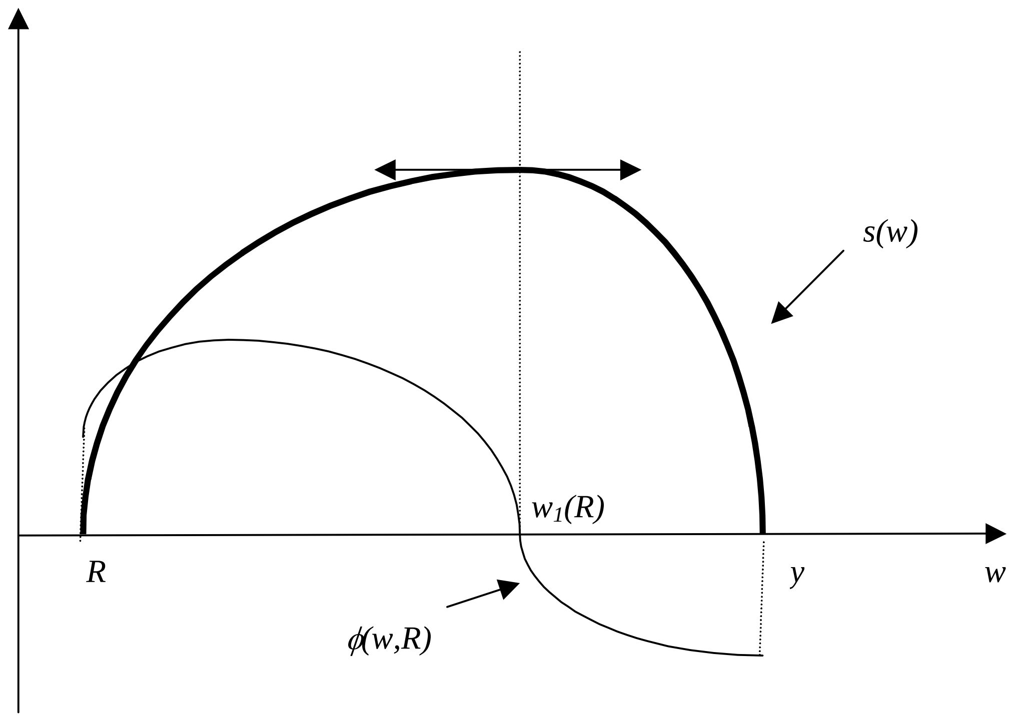

Figure 1: Search investment by wage

4.1 The wage offer distribution

The number of vacancies advertised at wage w is worth $v(w) = \theta(w) x[s(w)] u$, where u represents the unemployment rate. The total number of vacancies is thus $v = \int_R^y v(w) dw$. The cdf and the pdf of the wage offer distribution are then defined by

$$F(w) = \frac{\int_R^w v(\xi) d\xi}{v}, \quad F'(w) = \frac{v(w)}{v} = \theta(w) x[s(w)] \frac{u}{v} \tag{29}$$

How does the density change with the wage? Taking the second derivative of F yields

$$\frac{F''(w)}{F'(w)} \equiv \frac{\theta'(w)}{\theta(w)} + \frac{x'[s(w)] s'(w)}{x[s(w)]} \tag{30}$$

The density of the wage offer distribution is the result of two main factors: the pattern of market tightness by wage on the one hand, and the pattern of search investment by wage on the other hand. Hence, the right-hand side of equation (30) is composed of two terms. The first term is negative and reflects the fact that tightness is strictly decreasing in wage. Due to this term, the density of the wage offer distribution tends to decrease with the wage as the number of job offers per unit of search declines when the wage raises. The second term depicts the influence of search investments. It is non-monotonous, reflecting the non-monotonicity of $s(w)$. More precisely, it is positive at wages close to the lower bound of the support $[R, y]$ of the wage offer distribution, while it becomes negative at wages close to the upper bound.

With the help of (9) and (28), we have:

$$\frac{F''(w)}{F'(w)} = \frac{\gamma(w)\{\alpha(\theta(w)) y + [1 - \alpha(\theta(w))] R - w\} - (w - R)}{\alpha(\theta(w))(y - w)(w - R)} \tag{31}$$

with $\gamma(w) = -\frac{x'^2[s(w)]}{x[s(w)]x''[s(w)]} > 0$. It appears that $F''(w)$ has the same sign as the function $\psi(w,R) \equiv \gamma(w)\phi(w,R) - (w-R)$. Hence, the properties of the wage offer distribution will depend on the number of roots of the equation $\psi(w,R) = 0$. The following assumption will be useful to obtain more precise results.

Assumption A4 *Let $\psi : [0,y] \times \mathbb{R} \rightarrow \mathbb{R}$ be such that*

$$\psi(w,R) = \gamma(w)\phi(w,R) - (w-R)$$

For all $R \in [0,y]$, $\psi(w,R) = 0$ has a unique root in w.

One can check that this assumption is satisfied with a Cobb-Douglas matching function (α is then a constant) and with an iso-elastic efficiency search function (γ is then a constant).

Proposition 4 PROPERTIES OF THE WAGE OFFER DISTRIBUTION

Under Assumptions A1 to A3,

(i) Non-monotonicity. The wage offer distribution $F : [R,y] \rightarrow [0,1]$ is non-monotonous and satisfies $F'(R) = F'(y) = 0$.

(ii) Single-peak. If in addition A4 holds, the wage offer distribution is $\cap$-shaped

(iii) Right-skewness. If $\lim_{\theta \to 0} \alpha(\theta) > 0$ and $\lim_{w \to y} \gamma(w) < \infty$, $F''(y) = 0$

We obtain three results. First, the density of the wage offer distribution is non-monotonous. This is in sharp contrast with the literature discussed in the introduction, which predicts either increasing or decreasing density of the wage offer distribution. Actually, the result is induced by the non-monotonicity of the pattern of search investment by wage level. If search investment could not vary with the wage, the density of the wage offer distribution would be strictly decreasing, only reflecting the decreasing pattern of tightness with respect to the wage. Note that $\psi(w_1(R), R) < 0$: the peak of the wage offer distribution corresponds to a lower wage than the peak of the search investment function. This reflects the fact that tightness is strictly decreasing in wage. Second, the density is single-peaked provided some additional (yet not too demanding) restrictions on the matching technology and the efficiency of effort function hold. Third, the wage offer distribution generally has a flat tail at its upper bound.

4.2 The actual wage distribution

As search intensity varies with the wage level, the *actual* wage distribution (i.e. the distribution of wages among the employees, which coincides with the wage distribution among newly employed workers) departs from the wage offer distribution. Let $G(w)$ be the cdf of the actual wage distribution among the employees. It can be deduced from a standard flow equilibrium reasoning. For each wage $w \in [R,y]$, the outflow from the pool of those employed who earn less than w equals the inflow from the pool of unemployed. This reads:

$$q(1-u)G(w) = u \int_R^w x\left[s(\xi\right]\theta(\xi)m\left[\theta(\xi)\right]d\xi$$

Since $q(1-u) = \lambda u$, remembering that $v\left(w\right) = vF'\left(w\right) = x\left[s\left(w\right)\right]\theta(w)u$ it comes:

$$G(w) = \frac{v}{\lambda u}\int_R^w F'(\xi)m\left[\theta(\xi)\right]d\xi$$

Thus one has:

$$G'(w) = \frac{v}{\lambda u} F'(w) m\left[\theta(w)\right] \tag{32}$$

Differentiating this latter equality with respect to w and taking into account (9) gives:

$$\frac{G''(w)}{G'(w)} = \frac{F''(w)}{F'(w)} - \alpha\left(\theta(w)\right)\frac{\theta'(w)}{\theta(w)} = \frac{F''(w)}{F'(w)} + \frac{1}{y-w} \tag{33}$$

Using relations (31) that defines $F''(w)$, one arrives at:

$$\frac{G''(w)}{G'(w)} = \frac{\psi(w,R) + \alpha\left(\theta(w)\right)(w-R)}{\alpha\left(\theta(w)\right)(y-w)(w-R)} \tag{34}$$

It appears that $G''(w)$ has the same sign as the function $\chi(w,R) \equiv \psi(w,R) + \alpha\left(\theta(w)\right)(w-R)$. Hence, the properties of the actual wage distribution will depend on the number of roots of the equation $\chi(w,R) = 0$. The following assumption will be useful to obtain more precise results.

Assumption A5 *Let* $\psi : [0,y] \times \mathbb{R} \to \mathbb{R}$ *be such that*

$$\chi(w,R) = \psi(w,R) + \alpha\left(\theta(w)\right)(w-R)$$

For all $R \in [0,y]$, $\chi(w,R) = 0$ *has a unique root in* w.

One can check that this assumption is satisfied with a Cobb-Douglas matching function (α is then a constant) and with a iso-elastic efficiency search function (γ is then a constant).

Proposition 5 Properties of the actual wage distribution

Under Assumptions A1 to A3,

(i) Non-monotonicity. The actual wage distribution $G : [R,y] \to [0,1]$ *is non-monotonous and satisfies* $G'(R) = G'(y) = 0$.

(ii) Stochastic dominance. $G(w) < F(w)$ *for all* $w \in (R,y)$

(iii) Single-peak. If in addition A5 holds, the actual wage distribution is $\cap$*-shaped*

Like the wage offer distribution, the actual wage distribution features properties that are remarkably consistent with the facts: non-monotonous and generally single-peaked. However, unlike the wage offer distribution, skewness, though a possible output, is not a systematic property of the actual wage distribution. Note that the wage offer distribution first-order stochastically dominates the actual wage distribution. It means that individuals confronted with both distributions would unambiguously choose the latter. Such result is not very surprising: the job-seekers observe the wage offer distribution, and alter the wage they will be paid later by modulating their search investment on each sub-market. This optimization process makes the actual wage distribution looks better than the wage offer distribution.

4.3 A Cobb-Douglas example

We end up this section by considering usual explicit forms for the matching function and the efficiency of effort function. In the sequel, we will refer to this particular case as the Cobb-Douglas example. It appears that with such specifications, the wage offer distribution and the actual wage distribution are strongly linked with a well-known statistical distribution, the Beta distribution.

Proposition 6 THE COBB-DOUGLAS EXAMPLE

Assume that $m(\theta) = M_0\theta^{-\alpha}$, $M_0 > 0$, $\alpha \in (0,1)$ *and* $x(s) = s^{\frac{\gamma}{1+\gamma}}$, $\gamma > 0$. *Let* $\omega = (w-R)/(y-R)$ *be the normalized wage, and let also* H_F *be the cdf of the normalized wage offer distribution, while* H_G *is the cdf of the actual normalized wage distribution. Then,*

(i) H_F' *is the density of a* $\beta\left(\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha} + 1, \gamma + 1\right)$ *distribution, that is*

$$H_F'(\omega) = \frac{(1-\omega)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}}\omega^{\gamma}}{B\left(\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha} + 1, \gamma + 1\right)},\ \forall\omega \in [0,1]$$

(ii) H_G' *is the density of a* $\beta\left(\frac{1-\alpha}{\alpha}(\gamma+1) + 1, \gamma + 1\right)$ *distribution, that is*

$$H_G'(\omega) = \frac{(1-\omega)^{\frac{1-\alpha}{\alpha}(\gamma+1)}\omega^{\gamma}}{B\left(\frac{1-\alpha}{\alpha}(\gamma+1) + 1, \gamma + 1\right)},\ \forall\omega \in [0,1]$$

where B *is the Beta function such that*

$$B(t_1+1, t_2+1) = \int_0^1 (1-\xi)^{t_1}\xi^{t_2}d\xi$$

The Cobb-Douglas example displays several appealing features. First, we can find a normalization of the wage such that the offer distribution and the actual distribution of such normalized wage follow simple Beta distributions. Second, the parameters of the Beta distributions only involve the elasticity of the matching function and the elasticity of effort function. We do not need to solve the model to find the shape of the different wage distributions. Third, the wage distributions are both single-peaked. Fourth, the wage offer distribution has a flat right tail. Moreover,

$$F''(R) = \gamma\frac{(y-R)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}}}{\int_R^y (y-\xi)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}}(\xi - R)^{\gamma}d\xi}\lim_{w\to R}(w-R)^{\gamma-1}$$

Thus $F''(R)$ can either be nil or infinite depending on whether γ is larger or lower than one. Fifth, we can highlight the parameter circumstances under which the actual wage distribution has a flat right tail. Indeed, $G"(y) = 0$ if $\gamma > \frac{2\alpha-1}{1-\alpha}$ and $G"(y) = -\infty$ if $\gamma < \frac{2\alpha-1}{1-\alpha}$. Thus, the actual wage distribution is right-skewed when the parameters of the matching function and the search function satisfy $\gamma > \frac{2\alpha-1}{1-\alpha}$. Right-skewness is not a systematic property but can exist for a wide range of parameters of the model. Similarly, we can show that $G''(R) = 0$ if $\gamma > 1$ and $G''(R) = \infty$ if $\gamma < 1$. Therefore, the Cobb-Douglas case is consistent with an actual wage distribution characterized by a single peak, a flat right tail, and no left tail. It is so when $\frac{2\alpha-1}{1-\alpha} < \gamma < 1$. The following figure depicts the pdf of the wage offer and actual wage distributions in such a case.

Finally, note the role played by the parameter γ of the efficiency of effort function. When γ tends to 0, the search intensity is the same in each sub-market. The shape of the wage distribution only reflects the pattern of market tightness by wage. The density of the wage offer distribution as well as the density of the actual wage distribution are then strictly decreasing in wage. Conversely, when γ tends to infinity, the efficiency of effort function has constant marginal returns. As a result, workers concentrate their search investment in the sub-market where the

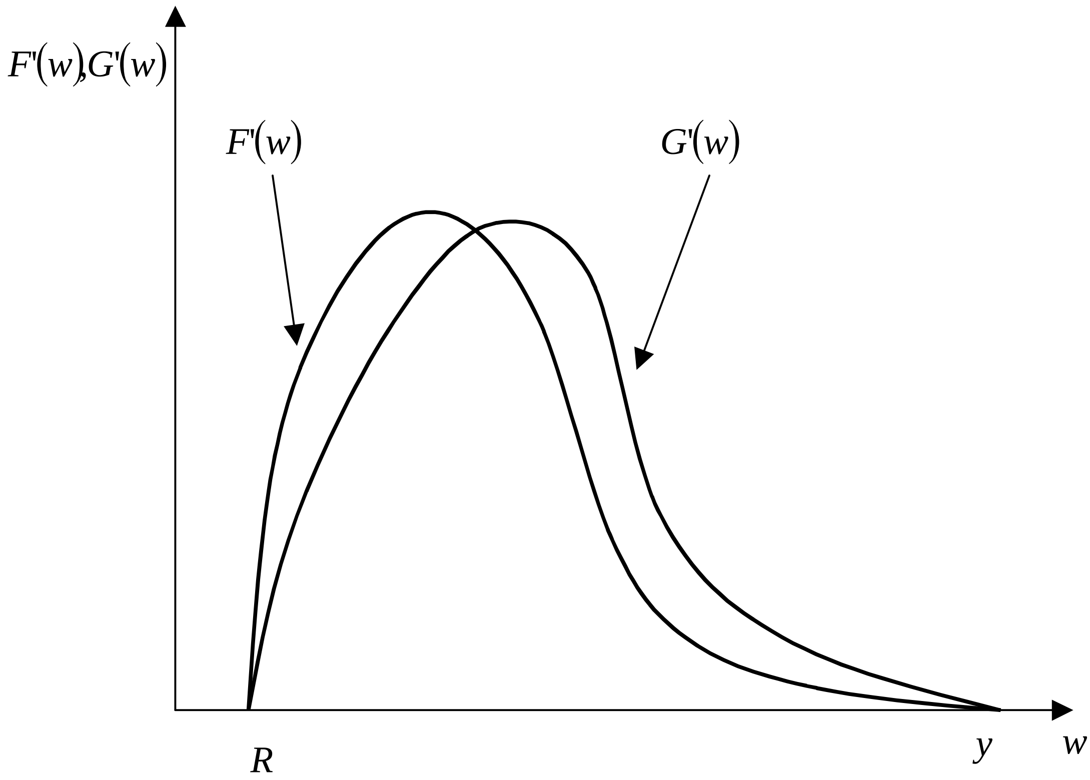

Figure 2: Wage offer and actual wage distributions - case $\frac{2\alpha-1}{1-\alpha} < \gamma < 1$

returns are the highest. Both the wage offer distribution and the actual wage distribution collapse to a single wage, the only wage offer of the localized search equilibrium.

From an empirical perspective, the Beta distribution should be rejected by the data because it does not feature the paretian tail typical of empirical wage distributions. Yet, two points should be made. On the one hand, such Beta distribution is obtained for homogenous firms and workers. Introducing some heterogeneity on the firm/worker side should make the Cobb-Douglas example compatible with a paretian-tailed aggregate wage distribution. On the other hand, the main objective of the Cobb-Douglas example is to illustrate our main results. Other parametrizations of the effort function can generate wage distributions that fit the empirical wage distributions better. But, of course, at the cost of losing the simplicity of the Cobb-Douglas example.

5 Efficiency

In this section, we compare the decentralized outcome to the efficient allocation. This comparison is made under the two cases highlighted so far, i.e. when search is localized and when workers are ubiquitous. We show that the localized search equilibrium is efficient, while the ubiquitous search equilibrium is not. We proceed in two steps. First, we compute the efficient allocations at given number of matching places. Second, we endogenize the number of matching places.

5.1 Efficient allocations at given number of matching places

The main conceptual difficulty associated to the efficient allocation relies to the segmentation of the search market. In the decentralized economy, the search market is segmented by wage: each wage is associated to an autonomous sub-market. It means that market segmentation requires wage dispersion. For the planner's problem, we shall assume that the search place is segmented i.e., in this sub-section, we suppose as Moen (1997) that the mass-number of matching places (islands, for short) is given.

Let I be the measure of islands, and $i \in [0, I]$ be their index. Under localized search, the unemployed are bound to search a job on a single matching place. The benevolent planner chooses the number of unemployed $u(i)$ and the number of vacancies $v(i)$ assigned to island i. The overall unemployment rate is then given by $u = \int_0^I u(i)\, di$. Under ubiquitous search there is no restriction on the number of prospected places. Therefore on each island all the unemployed, whose number is still denoted by u, are able to search. In both cases, the planner sets the search effort $s(i)$ of workers seeking a job on island i. As a consequence, the total cost of search investment is defined by $\int_0^I c(s(i))\, u(i)\, di$ when search is localized and by $uc\left(\int_0^I s(i)\, di\right)$ when search is ubiquitous. The tightness specific to island i is given by $\theta(i) = v(i) / [x(s(i))\, u]$ in case of ubiquitous search and by $\theta(i) = v(i) / [x(s(i))\, u(i)]$ in case of localized search. In both cases, the job-finding rate specific to island i is equal to $x(s(i))\theta(i)m[\theta(i)]$. When search is ubiquitous, the dynamic of unemployment is:

$$\dot{u} = q(1-u) - u \int_0^I x(s(i))\theta(i)m(\theta(i))\, di \tag{35}$$

While when search is localized, we have:

$$\dot{u}(i) = q[1-u(i)] - u(i)x(s(i))\theta(i)m(\theta(i))\, di, \text{ for all } i \in [0, I] \tag{36}$$

The instantaneous net social products in case of ubiquitous search and in case of localized search are respectively given by:

$$\omega = y\,(1-u) + uz - uc\left(\int_0^I s\,(i)\,di\right) - hu\int_0^I \theta(i)x(s(i))di$$

$$\omega = y\left(1-\int_0^I u\,(i)\,di\right) + z\int_0^I u\,(i)\,di - c\left(\int_0^I s\,(i)\,u(i)di\right) - h\int_0^I \theta(i)x(s(i))u(i)di$$

The planner's problem is to maximize the discounted social product $\int_0^{+\infty} \omega e^{-rt}dt$ with respect to the relevant variables $s(i), \theta(i), u\,(i)$ or u, and subject to the relevant law of motion, that is (35) or (36).

The following result describes the stationary solutions of this maximization program for each search environment.

Proposition 7 The efficient allocations

Under Assumptions A1 and A2, for any given number I of search places

(i) In the localized search case, there is a unique stationary efficient allocation which is such that $s\,(i) = s^l$ and $\theta\,(i) = \theta^l$ for all $i \in [0, I]$, with

$$\frac{h}{m\left(\theta^l\right)} = \frac{\left[1-\alpha\left(\theta^l\right)\right]\left[y-z+c\left(s^l\right)\right]}{r+q+x(s^l)\alpha\left(\theta^l\right)\theta^l m\left(\theta^l\right)} \tag{37}$$

$$c'\left(s^l\right) = \frac{\alpha\left(\theta^l\right)}{1-\alpha\left(\theta^l\right)} h\theta^l x'\left(s^l\right) \tag{38}$$

(ii) In the ubiquitous search case, there is a unique stationary efficient allocation which is such that $s\,(i) = s^u$ and $\theta\,(i) = \theta^u$, for all $i \in [0, I]$, with

$$\frac{h}{m\left(\theta^u\right)} = \frac{\left[1-\alpha\left(\theta^u\right)\right]\left[y-z+c\left(Is^u\right)\right]}{r+q+x(s^u)\alpha\left(\theta^u\right)\theta^u m\left(\theta^u\right)I} \tag{39}$$

$$c'\left(Is^u\right) = \frac{\alpha\left(\theta^u\right)}{1-\alpha\left(\theta^u\right)} h\theta^u x'(s^u) \tag{40}$$

Comparing (37) and (38) with their decentralized counterpart (17) shows that the localized search equilibrium is efficient. This result is very similar to Moen (1997). Wage-posting can thus decentralize the efficient allocation.

When search is ubiquitous, part (ii) of the proposition shows that the planner sets the same tightness and the same search intensity for all individuals in each island of the interval $[0, I]$. Both tightness and search intensity are decreasing in the measure I of islands. It follows that the ubiquitous search equilibrium is inefficient. Consider for instance the case where $I = y - R$, with R the equilibrium reservation wage. In this case, the number of opened matching places is the same in the social optimum and in the decentralized economy. However, the search investment varies from a sub-market to another in the decentralized economy, while it is constant on each island at the social optimum.

The reason of inefficiency is simple. Prices serve two purposes in the decentralized economy: an allocative objective, and an informational objective. According to the allocative objective, wages must achieve the optimal sharing of output between consumption and vacancy costs (investment). According to the informational objective, wages must ensure the segmentation of the search market into a continuum of sub-markets. The number of instruments is thus lower than the number of objectives and inefficiency results.

5.2 On the number of matching places

In this sub-section, we discuss the optimal number of matching places that would be chosen by the social planner.

Let us begin with the localized search case. Proposition 7 shows that at given number of matching places, all allocations featuring a search intensity s^l and a tightness θ^l are efficient. Owing to constant returns to scale in the matching technology and due to the fact that each worker must be assigned to a single market place, the stationary social product does not depend on the number of matching places. As a result, the efficient number of matching places is indeterminate under localized search. It also means that if there were a fixed cost associated to each market creation, the planner would only create a single matching place.

Now, we turn to the ubiquitous search environment. To simplify the exposition, consider the case where the discount rate r tends to 0. Then, the efficient allocation maximizes the stationary social product. The optimal number of market places results from:

$$\max_{I\geq 0}\left\{\omega\left(I\right)=\left(1-u\left(I\right)\right)y+u\left(I\right)\left[z-c\left(Is^{u}\left(I\right)\right)\right]-hu\left(I\right)I\theta^{u}\left(I\right)x^{u}\left(I\right)\right\} \tag{41}$$

where θ^u and s^u are defined by Proposition 7, and $u\left(I\right)=q/\left[q+\theta^{u}\left(I\right)m\left(\theta^{u}\left(I\right)\right)x\left(s^{u}\left(I\right)\right)I\right]$. The derivative of the objective with respect to I is:

$$\omega'\left(I\right)=-\frac{\partial u\left(I\right)}{\partial I}\left[y-z+c\left(Is^{u}\left(I\right)\right)+hI\theta^{u}\left(I\right)x^{u}\left(I\right)\right]-u\left(I\right)s^{u}\left(I\right)c'\left(Is^{u}\left(I\right)\right)-hu\left(I\right)\theta^{u}\left(I\right)x\left(s^{u}\left(I\right)\right) \tag{42}$$

Using equations (39) and (40), we obtain:

$$\omega'\left(I\right)=\frac{\alpha}{1-\alpha}hu\left(I\right)\theta^{u}\left(I\right)x\left(s^{u}\left(I\right)\right)\left[1-s^{u}\left(I\right)x'\left(s^{u}\left(I\right)\right)/x\left(s^{u}\left(I\right)\right)\right] \tag{43}$$

which has the sign of the term between brackets. This term is positive for all I, given that s^u tends to 0 as I tends to infinity and x is strictly concave. It follows that the optimal number of matching places is infinite. Indeed, the planner exploits the decreasing marginal returns to search investment: it opens an infinite number of matching places and sets an arbitrarily small search intensity in each place. Similarly tightness tends to 0. This result illustrates the inefficiency of the decentralized economy, in which the mass-number of sub-markets is finite. However, it also highlights the asymmetry between the centralized and the decentralized mechanisms. Indeed, the planner can achieve the segmentation of the search place without any instrument, while market segmentation in the decentralized economy requires equilibrium wage dispersion.

To obtain a finite mass-number of islands at the social optimum, we can marginally modify the technological side of the model. The point is that for the planner, it is always worth decreasing the search intensity, while simultaneously increasing the number of matching places. Therefore, we need to alter the search technology, and in particular the efficiency of effort function. For instance, suppose that $x(s) > 0$ if and only if $s > s_0$. Or, alternatively, suppose that x is strictly convex, and then concave. Then, the optimal number of matching places is finite, and the search intensity is positive in each matching place. The latter simply responds to the following condition familiar to specialists of the efficiency wage literature:

$$s^u(I)\, x'(s^u(I)) / x(s^u(I)) = 1 \tag{44}$$

Similarly, one can modify the search cost function so that it directly depends on the number of prospected places. This assumption would indeed limit the optimal number of matching places. Of course, the decentralized economy would also be affected by such assumptions. The main difference would be that both the lower bound and the upper bound of the wage distributions would become endogenous. Indeed, sub-markets offering wages such that $V^e(w)$ is close to V^u (i.e. w close to R), or such that $\theta(w)$ is close to 0 (i.e. w close to y) would not be opened in equilibrium. However, the density of the wage offer distribution, as well as the density of the actual wage distribution would still be hump-shaped[6].

6 Conclusion

This paper offers a search equilibrium model in which firms post wages, and there are homogenous firms and workers. The main originality of the model relies on the working of the search market. We assume that the search market is segmented by wage, and workers choose the amount of search effort they spend on each (sub-)market. Workers are thus ubiquitous in the sense they are not bound to choose one and only one market, but can visit the whole set of markets opened in equilibrium. The main result is that a non-degenerate equilibrium wage distribution exists (despite there is no on-the-job search) and can replicate two important properties of empirical wage distributions, e.g. the distribution can be single-peaked and right-skewed. All the results are illustrated by a Cobb-Douglas example, in which the wage distribution is a Beta distribution.

A key feature of our model relies on its simplicity. Its main goal is to show that a rather natural extension of the usual directed search assumptions (precisely the consideration of ubiquity in market participation) leads to an equilibrium distribution of wages displaying empirically convincing properties. But, as it stands, our model cannot directly pretend to fit actual wage distributions. In this respect, we believe that it can be extended in a number of directions that will make it more relevant from an empirical perspective. The next step is to introduce heterogeneity. This can be done by abandoning the assumption according to which the productivity of a worker is constant once matched to a firm. One could rather assume that there exists a non-degenerate distribution of output reflecting firm heterogeneity, worker heterogeneity, or both.

Another area of study concerns the profile of wages. A well-documented property of individual wage profile is that it is increasing with tenure. Such a property does not appear in our model but happens in models with on-the-job search. Thus, another possible step is to add in our model of ubiquitous search the possibility of searching while employed. Some policy issues also remain open. Unlike the standard directed search model, the ubiquitous search model ends up with an equilibrium that is not efficient because, in this context, the wage as too many roles to play. Hence, other instruments are needed to achieve efficiency.

[6] The model would become more difficult to solve, and thus would lose some of its appealing features.

APPENDIX

A Proof of Lemma 1

We have $\phi(R,R) \geq 0$, $\phi(y,R) \leq 0$. Then, note that according to Assumption A2, the matching function is strictly concave. This implies that $[\theta m(\theta)]' > 0$ and $[\theta m(\theta)]" < 0$. As $[\theta m(\theta)]' = m(\theta)[1-\alpha(\theta)]$, it comes

$$\begin{aligned} [\theta m(\theta)]" &= m'(\theta)(1-\alpha) - \alpha'(\theta) m(\theta) \\ &= m(\theta)\left[-\frac{\alpha(\theta)}{\theta}(1-\alpha(\theta)) - \alpha'(\theta)\right] \end{aligned}$$

It follows that:

$$\alpha'(\theta) > -\frac{\alpha(\theta)}{\theta}(1-\alpha(\theta)) \tag{45}$$

Now, consider the derivative of function ϕ with respect to w:

$$\phi_w(w,R) = \alpha'(\theta(w))\,\theta'(w)(y-R) - 1$$

with

$$\alpha(\theta(w))\frac{\theta'(w)}{\theta(w)} = -\frac{1}{y-w} \tag{46}$$

Equations (45) and (46) imply that

$$\phi_w(w,R) < \frac{1-\alpha(\theta(w))}{y-w}(y-R) - 1 = \frac{-\phi(w,R)}{y-w}$$

This relationship implies that $\phi_w(w,R) < 0$ whenever $\phi(w,R) > 0$.

B Proof of Proposition 1

Part *(i)*. The formulas appearing in *(i)* simply replicate relations (8), (11) and (12).

Part *(ii)*. As a preliminary step, let us examine the property of the function $\sigma : [z,y] \rightarrow \mathbb{R}$ given by equation (7). Assumption A1 implies that $\sigma(z) = 0$ and $\sigma'(R) > 0$. The solving reduces to find (w^*, R^*) such that

$$\begin{aligned} c'[\sigma(R)] &= x'[\sigma(R)]\,\theta(w)\,m(\theta(w))\,\frac{w-R}{r+q} && (47) \\ w &= \alpha(\theta(w))\,y + [1-\alpha(\theta(w))]\,R && (48) \end{aligned}$$

>From Lemma 1, equation (48) implicitly defines a unique $w_1 = w_1(R)$. It is strictly increasing in R, with $z < w_1(z) < w_1(y) = y$. Now, consider the following function:

$$J(w,R) = x'[\sigma(R)]\,\theta(w) m(\theta(w))\,\frac{w-R}{r+q} - c'[\sigma(R)]$$

An equilibrium solves $K(R) = J(w_1(R), R) = 0$. Assumptions A1 and A2 together with the properties of the function σ established below imply that

$$\begin{aligned} K(z) &= \theta(w_1(z))m(\theta(w_1(z)))\frac{w_1(z)-z}{r+q}\lim_{R\to z} x'(\sigma(R)) > 0 \\ K(y) &= -c'(\sigma(y)) < 0 \end{aligned}$$

Therefore, it is sufficient to show the function K is strictly decreasing. But,

$$K'(R) = w_1'(R)J_w(w_1(R), R) + J_R(w_1(R), R)$$

Since $J_w(w_1(R), R) = 0$ for all R, we have

$$K'(R) = J_R(w_1(R), R) = \left[x"\theta m(\theta)\frac{w_1(R)-R}{r+q} - c"\right]\sigma'(R) - x'\theta m(\theta)\frac{1}{r+q} < 0$$

It follows that R^* is unique. Then, $w^* = w_1(R^*)$ and $\theta^* = m^{-1}\left(\frac{h(r+q)}{y-w}\right)$ are uniquely defined.

C Proof of Proposition 2

The formulas appearing in Proposition 2 have been established in the text. It remains to show that they define a unique equilibrium. To do this, one can remark that thanks to Assumption A2 equation (23) defines a unique specific market tightness function $\theta(w)$. Then, equation (24) can be solved in s as a function of w, S and R. Let $e(w, R, S)$ be this unique solution. From Assumption A1 and the implicit function theorem, the partial derivatives of $e(w, R, S)$ are such that:

$$e_S = \frac{c"(S)}{c'(S)}\frac{x'(e)}{x"(e)} < 0, \quad e_R = \frac{1}{w-R}\frac{x'(e)}{x"(e)} < 0$$

In addition, $\lim_{S\to 0} e(w, R, S) < \infty$, while $\lim_{S\to\infty} e(w, R, S) = 0$.

Now, substitute $e(w, R, S)$ for $s(w)$ in equation (25), and consider the function ψ such that

$$\psi(R, S) = R - z + c(S) - \int_R^y x[e(w, R, S)]\theta(w)m[\theta(w)]\frac{w-R}{r+q}dw \tag{49}$$

The properties of the function ψ are as follows:

$$\begin{aligned} \lim_{S\to 0}\psi(R, S) &= R - z - \int_R^y x[e(w, R, 0)]\theta(w)m[\theta(w)]\frac{w-R}{r+q}dw \\ \lim_{S\to\infty}\psi(R, S) &= +\infty \\ \psi_S(R, S) &= c'(S)\left(1 - \int_R^y e_S dw\right) > 0 \end{aligned}$$

It follows that there exists a unique $S_1 \equiv S_1(R)$ such that $\psi(R, S_1) = 0$ iff $\lim_{S\to 0}\psi(R, S) \leq 0$. But,

$$\begin{aligned} \lim_{R\to z}\psi(R, S) &= c(S) - \int_z^y x[e(w, z, S)]\theta(w)m[\theta(w)]\frac{w-R}{r+q}dw \\ \lim_{R\to y}\psi(R, S) &= y - z > 0 \\ \psi_R(R, S) &= 1 + \int_R^y \frac{x[s(w)]\theta(w)m(\theta(w))}{r+q}dw - c'(S)\int_R^y e_R dw > 0 \end{aligned} \tag{50}$$

Therefore, there exists a unique $\widetilde{R} \in (z, y)$ such that $\lim_{S\to 0} \psi(R, S) \leq 0$ if and only if $R \leq \widetilde{R}$.

To summarize, equation (25) implicitly defines $S_1(R)$ for all $R \in [z, y]$, with

$$S_1\left(\widetilde{R}\right) = 0 \text{ and } S_1'(R) < \frac{\int_R^y e_R dw}{1 - \int_R^y e_S dw} < 0 \tag{51}$$

Moreover, when $e(w, R, S)$ is substituted for $s(w)$ in equation (26), we obtain another equation defining a unique S as a function of R. We call this function $S_2(R)$. Differentiating this latter equation with respect to R gives:

$$\left(1 - \int_R^y e_S dw\right) \frac{dS}{dR} = -e(R, R, S) + \int_R^y e_R dw$$

Assumption A2 and equation (24) imply that $e(R, R, S) = 0$. Consequently

$$S_2'(R) = \frac{\int_R^y e_R dw}{1 - \int_R^y e_S dw} < 0 \tag{52}$$

So far, we have shown that $S_1'(R) < S_2'(R) < 0$. In addition, $S_2(y) = 0$, which implies that $S_2\left(\widetilde{R}\right) > S_1\left(\widetilde{R}\right) = 0$. Lastly, $S_1(z)$ and $S_2(z)$ are given by:

$$S_2(z) = \int_z^y e\left[w, z, S_2(z)\right] dw$$

$$c\left[S_1(z)\right] = c'\left[S_1(z)\right] \int_z^y \frac{x\left[e\left(w, z, S_1(z)\right)\right]}{x'\left[e\left(w, z, S_1(z)\right)\right]} dw \tag{53}$$

Assumption A1 implies that $x(e)/x'(e) > e$ and $c(s)/c'(s) < s$, therefore (53) gives

$$S_1(z) > \frac{c\left[S_1(z)\right]}{c'\left[S_1(z)\right]} > \int_z^y e\left[w, z, S_1(z)\right] dw$$

which proves that $S_1(z) > S_2(z)$. All these properties of the functions $S_1(R)$ and $S_2(R)$ entail that they cross once at a point such that $R^* \in (z, y)$. Thus, the equilibrium values of R and S are unique. It follows that the equilibrium functions $\theta^*(w)$ and $s^*(w)$ given respectively by (23) and (24) are also unique.

D Proof of Proposition 3

Relation (23) and Assumption A2 imply that $\theta(y) = 0$. Therefore, relation (24) and Assumption A1 imply that $s(y) = 0$. Furthermore, (23) shows that $\theta(R)$ is finite and (24) and Assumption A1 then entails that $s(R) = 0$. Since s is continuous and $s(w) > 0$ for all $w \in (R, y)$, the function s is not monotonous. Then, $s'(w)$ is continuous and has the sign of $\phi(w, R)$. The result follows from Lemma 1.

E Proof of Proposition 4

Part *(i)*. According to proposition 3, one has $s(R) = s(y) = 0$. Remembering that $\theta(y) = 0$ and that $\theta(R)$ is finite, relation (29) arrives at $F'(R) = F'(y) = 0$. The result follows from the facts that F' is continuous and $F'(w) > 0$ for all $w \in (R, y)$.

Part *(ii)*. Note that $F''(w)$ has the sign of $\psi(w, R)$. Hence, each root of the equation $\psi(w, R) = 0$ corresponds to a point where $F''(w) = 0$. If Assumption A4 holds, the equation $\psi(w, R) = 0$ has a unique root and the wage offer distribution is $\cap$-shaped.

Part *(iii)*. Equation (31) shows that:

$$F\text{"}(y) = \lim_{w \to y} \frac{1}{\alpha(\theta(w))} \frac{F'(w)}{y - w} \{[\alpha(\theta(w)) - 1] \gamma(w) - 1\}$$

Using (29) and (23), one arrives at:

$$F\text{"}(y) = \frac{u}{vh(r+q)} \lim_{w \to y} \frac{\theta(w) m(\theta(w)) x(s(w))}{\alpha(\theta(w))} \{[\alpha(\theta(w)) - 1] \gamma(w) - 1\}$$

The result follows from the facts that $\theta(w) m(\theta(w))$ and $x[s(w)]$ are equal to zero when $w \to y$.

F Proof of Proposition 5

Part *(i)*. As $G'(w) = vF'(w)m[\theta(w)] / \lambda u$, part *(i)* of Proposition 4 implies that $G'(R) = G'(y) = 0$. The result follows from the fact that $G'(w) > 0$ for all $w \in (R, y)$ and the continuity of G'.

Part *(ii)*. As $m[\theta(w)] = \frac{h(r+q)}{y-w}$ – see (23) –, relation (32) becomes:

$$G'(w) = \frac{v}{\lambda u} \frac{h(r+q)}{y - w} F'(w)$$

Let us denote by w_0 the unique wage such that $\frac{v}{\lambda u} \frac{h(r+q)}{y - w_0} = 1$. A priori, w_0 can be greater or smaller than R. Let us suppose first that $w_0 \geq R$. One has $G'(w) < F'(w)$ for $w < w_0$ and $G'(w) > F'(w)$ for $w > w_0$. Therefore, when $w < w_0$ one has:

$$G(w) = \int_R^w G'(\xi) d\xi < \int_R^w F'(\xi) d\xi = F(w)$$

While, when $w > w_0$ one has:

$$1 - G(w) = \int_w^y G'(\xi) d\xi > \int_w^y F'(\xi) d\xi = 1 - F(w) \tag{54}$$

Hence, one always has $G(w) < F(w)$ when $w_0 \geq R$.

Now, let us assume that $w_0 < R$. Then, one has $G'(w) > F'(w)$ for all $w \geq R$, and (54) holds for all $w \geq R$. Consequently, $G(w) < F(w)$ when $w_0 < R$.

Part *(iii)*. $G''(w)$ has the sign of $\chi(w, R)$. Hence, each root of the equation $\chi(w, R) = 0$ corresponds to a point where $F''(w) = 0$. If Assumption A5 holds, the equation $\chi(w, R) = 0$ has a unique root and the actual wage distribution is $\cap$-shaped.

G Proof of Proposition 6

Part *(i)*. In the Cobb-Douglas case, equation (31) becomes:

$$\frac{F"(w)}{F'(w)} = \frac{\gamma\left[\alpha y + (1-\alpha)R - w\right]}{\alpha(y-w)(w-R)} - \frac{1}{\alpha(y-w)}$$

Integrating this equation with the condition $\int_R^y F'(w)dw = 1$ yields:

$$F'(w) = \frac{(y-w)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}}(w-R)^{\gamma}}{\int_R^y (y-\xi)^{\frac{1-\alpha}{\alpha}\gamma + \frac{1}{\alpha}}(\xi - R)^{\gamma} d\xi}, \forall w \in [0, y]$$

The cdf of the normalized wage satisfies

$$H_F(\omega) = \Pr\{\frac{w-R}{y-R} \leq \omega\} = \Pr\{w \leq R + \omega(y-R)\} = F\left[R + \omega(y-R)\right]$$

Therefore, $H_F'(\omega) = (y-R)F'\left[R + \omega(y-R)\right]$, and the result follows.

Part *(ii)*. Using the definitions of the functions $\phi(w, R)$ and $\psi(w, R)$, equation (34) becomes:

$$\frac{G''(w)}{G'(w)} = \frac{\gamma(y-R)}{(y-w)(w-R)} - \frac{1+\gamma-\alpha}{\alpha(y-w)}$$

Integrating this equation with the condition $\int_R^y G'(w)dw = 1$ yields

$$G'(w) = \frac{(y-w)^{\frac{(1+\gamma)(1-\alpha)}{\alpha}}(w-R)^{\gamma}}{\int_R^y (y-\xi)^{\frac{(1+\gamma)(1-\alpha)}{\alpha}}(\xi - R)^{\gamma} d\xi} \tag{55}$$

The cdf of the actual normalized wage is such that

$$H_G(\omega) = \Pr\{\frac{w-R}{y-R} \leq \omega\} = \Pr\{w \leq R + \omega(y-R)\} = G\left[R + \omega(y-R)\right]$$

Therefore one gets $H_G'(\omega) = (y-R)G'\left[R + \omega(y-R)\right]$, and the result follows.

H Proof of Proposition 7

Part *(i)*. Let $\rho(i)$ denote the co-state variable related to (36). The current-valued Hamiltonian of the problem is:

$$\mathcal{H} = \omega e^{-rt} + \int_0^I \rho(i)\left\{q\left[1-u(i)\right] - u(i)x(s(i))\theta(i)m\theta(i)\right\} di$$

The first-order conditions are:

$$\frac{\partial \mathcal{H}}{\partial s(i)} = \frac{\partial \mathcal{H}}{\partial \theta(i)} = 0, \ \frac{\partial \mathcal{H}}{\partial u(i)} = -\dot{\rho}(i)$$

The condition $\partial\mathcal{H}/\partial s(i) = 0$ gives

$$\left[c'(s(i)) + h\theta(i)x'(s(i))\right] e^{-rt} + \rho(i)x'(s(i))\theta(i)m\left(\theta(i)\right) = 0, \quad \forall i[0, I] \tag{56}$$

Similarly, the condition $\partial\mathcal{H}/\partial\theta(i) = 0$ gives

$$he^{-rt} = -\rho(i)m\left(\theta(i)\right)\left[1 - \alpha\left(\theta(i)\right)\right], \quad \forall i[0, I] \tag{57}$$

The Euler equations read as:

$$\left[-y + z - c(s(i)) - h\theta(i)x(s(i))s(i)\right] e^{-rt} - \rho(i)\left[q + x(s(i))\theta(i)m\left(\theta(i)\right)\right] = -\dot{\rho}(i) \tag{58}$$

Differentiating (57) with respect to time, and taking the result at the stationary state (where $\dot{\theta}(i) = 0$), entails $\dot{\rho}(i) = -r\rho(i)$. Equations (58) and 57) then give

$$\frac{h}{m\left(\theta(i)\right)} = \frac{\left[1 - \alpha\left(\theta(i)\right)\right]\left[y - z + c\left(s(i)\right)\right]}{r + q + x(s(i))\alpha\left(\theta(i)\right)\theta(i)m\left(\theta(i)\right)} \tag{59}$$

Similarly, equations (56) and (57) give

$$c'\left(s(i)\right) = \frac{\alpha\left(\theta(i)\right)}{1 - \alpha\left(\theta(i)\right)} h\theta(i)x'(s(i)) \tag{60}$$

Equations (59) and (60) form a system of two equations with two unknowns $s(i)$ and $\theta(i)$. These conditions are similar to the localized search equilibrium given by the system (17). Therefore a unique solution results and $s(i) = s^l$, while $\theta(i) = \theta^l$, for all $i[0, I]$. Finally, note that the Euler conditions are satisfied for all $u\left(i\right) \geq 0$, so that if I were also a control variable, the number of opened matching places would be indeterminate.

Part *(ii)*. Let us denote by ρ the costate variable related to (35), the Hamiltonian of the problem is:

$$\mathcal{H} = \omega e^{-rt} + \rho\left[q(1 - u) - u\int_0^I x(s(i))\theta(i)m\left(\theta(i)\right)di\right] \tag{61}$$

The first-order conditions are

$$\frac{\partial\mathcal{H}}{\partial s(i)} = \frac{\partial\mathcal{H}}{\partial\theta(i)} = 0,\ \forall i[0, I],\ \frac{\partial\mathcal{H}}{\partial u} = -\dot{\rho} \tag{62}$$

The f.o.c. with respect to $s\left(i\right)$ implies

$$\left[c'\left(\int_0^I s(i)di\right) + h\theta(i)x'(s(i))\right] e^{-rt} + \rho x'(s(i))\theta(i)m\left(\theta(i)\right) = 0, \quad \forall i[0, I] \tag{63}$$

Similarly, the f.o.c. with respect to $\theta\left(i\right)$ implies

$$he^{-rt} = -\rho m\left(\theta(i)\right)\left[1 - \alpha\left(\theta(i)\right)\right], \quad \forall i[0, I] \tag{64}$$

The Euler equation reads

$$\left[-y+z-c\left(\int_0^I s(i)di\right)-h\int_0^I \theta(i)x(s(i))di\right]e^{-rt}-\rho\left[q+\int_0^I x(s(i))\theta(i)m\left(\theta(i)\right)di\right]=-\dot{\rho} \tag{65}$$

Equation (64) shows that $\theta(i)$ does not depend upon i, hence $\theta(i)=\theta^u$ for all $i\in[0,I]$. Then, eliminating ρ between (63) and (64) arrives at:

$$c'\left(\int_0^I s(i)di\right)=\frac{\alpha(\theta)}{1-\alpha(\theta)}h\theta x'(s(i)) \tag{66}$$

This equation shows that $s(i)$ does not depend on i, hence $s(i)=s^u$, for all $i\in[0,I]$. This last equation gives

$$c'(Is^u)=\frac{\alpha(\theta^u)}{1-\alpha(\theta^u)}h\theta^u x'(s^u) \tag{67}$$

Differentiating (64) with respect to time, and taking the result at the stationary state (where $\dot{\theta}=0$), entails $\dot{\rho}=-r\rho$. The Euler equation (65) combined with (64) then gives

$$\frac{h}{m(\theta^u)}=(1-\alpha(\theta^u))\frac{y-z+c(Is^u)}{r+q+Ix(s^u)\alpha(\theta^u)\theta^u m(\theta^u)} \tag{68}$$