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Abstract

This paper investigates the combined impact of a first-mover advantage and of firms' limited mobility on the equilibrium outcomes of a continuous-time model adapted from by Boyer, Lasserre, and Moreaux (2007). Two firms face market development uncertainty and may enter by investing in lumpy capacity units. With perfect mobility, when the first entrant plays as a Stackelberg leader a Markov perfect preemption equilibrium obtains in which the leader invests earlier, and the follower later, than in the Cournot benchmark scenario. There is rent equalization, and the two firms' equilibrium value is lower. This result is not robust to the introduction of firm-specific limited mobility constraints. If one firm is sufficiently less able than its rival to mobilize resources at early stages of the market development process, there is less rent dissipation, and no equalization, in a constrained preemption equilibrium. The first-mover advantage on the product market then results in more value for the less constrained firm, and in less value for the follower than when they play *à la* Cournot with perfect mobility. The leading firm maximizes value by entering immediately before its constrained rival, though later than made possible by its superior mobility. Greater uncertainty reduces the value differential to the benefit of the follower. It also increases the distance between the firms' respective investment triggers. The specifications and results are discussed in light of recent developments in the market for music downloads.

JEL classification: C73; D43; D92; L13. *Keywords:* Real options; Preemption; First-mover advantage; Mobility

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"The first mover and only the first mover, the company that acts while others dither,

has a true opportunity to gain time over its competitors – and time advantage, in

this business, is the surest way to gain market share."

Andrew Grove, Chairman & CEO, Intel Corp.[1]

1   Introduction

Business circumstances in which rival firms contemplate entry on an emerging or growing market are prevalent. In most cases, the installation of production and distribution facilities is needed as a first step, before operations may start. Investments are typically lumpy, and hardly recoverable. When demand is fluctuating, and changes are uncertain, the timing of entry impacts the expected value of operations. Early entry is risky, as demand may remain relatively low for a long time. On the other hand, a firm forgoes operating profits if it postpones entry to a distant period.

Firms also interact strategically, at two levels. Indeed their long-run investment choices are not made in isolation. When the sequence of entry is not established *a priori*, firms will compete for the lead position if it pays starting operations before others. This occurs if early entry results in monopoly profits for a while, before demand reaches a sufficiently high level that encourages new entrants. Firms also interact strategically in the short-run. As soon as more than one firm have entered, the formation of prices is the non-cooperative outcome of interdependent choices. Each firm's value thus depends on the nature of competition on the product market.

Only a few contributions to the theoretical literature have used a strategic real options approach to study investment strategies, when firms are not only confronted with a stochastic environment, but also with reacting competitors, both in a long-run investment game and in a short-run market subgame (see Boyer, Gravel, and Lasserre (2009) for a recent and comprehensive survey). The benchmark case, in what follows, appears in a recent paper by Boyer, Lasserre, and Moreaux (2007, BLM hereafter). In a continuous-time model, two rival firms face market development uncertainty. At each point in time from the very first moment, they may invest in a discrete number of capacity units, which have no resale value, in order to supply a homogeneous good. They cannot commit *ex ante* to a sequence of investments.

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[1]Andrew S. Grove, 1996, *Only the Paranoid Survive: How to Exploit the Crisis Points that Challenge Every Company and Career*, New York: Currency/Doubleday, p. 51.

In the early phase of development, if only one firm invests it benefits from a monopolistic position until the market grows to a threshold level that triggers entry by the other firm. On the product market, if the first entrant does not benefit from a first-mover advantage, and both firms hold production capacities, they compete by choosing quantities simultaneously *à la* Cournot. The quantity choices can be capacity constrained.[2] When firms have no existing capacity and contemplate entry, a unique Markov perfect preemption equilibrium always obtains, in which firms invest at different market development thresholds. Each firm may enter first with the same probability. Competition for the benefit of leading the investment game, hence to be the only supplier for some period of time, leads to rent equalization and partial dissipation.

The present paper investigates the impact on equilibrium outcomes of two natural extensions to the benchmark model, separately and in combination. It first focuses on situations in which, if a firm leads the entry process, it benefits from a first-mover advantage when the other firm invests also and starts supplying.[3] The endogenous sequence of investments, and *ex ante* identical firms' value, are compared with the outcomes of the benchmark case in which early entry does not induce a favorable position in the market stage. The paper also examines the consequences of assuming that firms may not be symmetric in their ability to mobilize the resources they need to enter a new market. This is done by introducing limited mobility constraints, defined as firm-specific technological specifications. They capture real-world circumstances in which firms become aware of profitable market opportunities only when the level of demand is sufficiently high, or require some non negligible amount of time to install new productive assets.

Does the first-mover advantage impact the entry sequence of firms? Does it preserve the rent equalization and dissipation properties? Or can it generate a rent for the leader? How can adjustment costs, or limited mobility constraints, modify equilibrium outcomes? What is the consequence of an increase in growth or uncertainty, with limited mobility, and

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[2]The main virtues of the model appear clearly when compared with a reference contribution to the management science literature by Kulatilaka and Perotti (1998). In the latter paper, the real option approach is used with two firms that may also interact strategically both in the investment stage (see the extension in Section 3, pp. 1027-8) and on the product market (Cournot competition). However, in the two-period setup firms may invest a given amount only once, at the same time 0, before competing in quantities on the market at time 1. Uncertainty is resolved between the two periods. In BLM (2007), firms face a continuously changing and uncertain demand with an infinite horizon, successive investments are possible, and can constraint quantity choices. The timing of investment decisions, and their discrete levels, are endogenous.

[3]For recent real-options models that consider the case of a second-mover advantage, see Hoppe (2000), Thijssen, Huisman, and Kort (2006), and Mason and Weeds (2008).

with a first-mover advantage? These questions find only incomplete answers in the empirical literature that explicitly refers to a real options framework to investigate the impact of first-mover advantages on entry decisions. In a research note, Folta and Miller (2002) conjecture that first-mover advantages accelerates entry. Folta and O'Brien (2004) use data from a broad array of industries to find support to their hypothesis that the choice by a given firm to enter a new activity is positively related to a measure of early mover advantages. In their study, entry – the dependent variable – is defined as observed activity by an existing firm in an industry in which the same firm had not reported involvement in the previous two years. This cannot fully capture the strategic dimension (in the game theoretic sense) that characterizes the entry choice by several firms on the same market. In particular, whether two firms enter almost simultaneously (the same year), or one after another over a large period of time (in two different years), is not considered in the econometric model. Therefore it is not clear whether a first-mover advantage on the product market, as gained by an early entrant, will impact the timing of decisions, and firms' value, in a given industry. In another recent empirical study, Driver, Temple, and Urga (2008) exploit the observed heterogeneity across forty industries, as measured in a quarterly survey over two decades. Despite considerable heterogeneity across industries, they find evidence that indicators of first-mover advantages (specifically, the R&D and advertising intensity) contribute to a positive relationship of investment to uncertainty. At a finer level of analysis, however, it remains uneasy to disentangle the impact of industry wide characteristics from strategic decisions taken by interdependent firms.

In the theoretical analysis that follows, by assuming that a firm, if it enters first, will behave as a Stackelberg leader should the other firm enter also, and when limited mobility constraints are absent or weak, one obtains a unique preemption equilibrium again, as in the original setting with Cournot behavior. However, the leader enters earlier, the follower enters later, than without a first-mover advantage. This means that, although leadership in the short-run market subgame expands the range of market development levels for which the first entrant is the unique supplier, it only delays for a while the other firm's entry, without blockading it. The delay monotonically increases, while firms' value decreases, when the difference between the equilibrium profits of the leader and the follower in the market subgame increases. The leader obtains no rent from the first-mover advantage. The rent equalization property is preserved, and the two firms' equal value is lower than in the benchmark setup. This is because competition for the benefit of being the first supplier is even stronger in the Stackelberg scenario, leading to more dissipation than in the Cournot scenario. By contrast, sufficiently strong limited mobility constraints result in less dissipation and no equalization.

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By dampening competition for the lead position, they transform the first-mover advantage on the product market, in the Stackelberg scenario, into an additional rent to the leader, and a loss for the follower, in comparison to firms' respective values in the Cournot setup, all other things remaining equal. Greater uncertainty reduces the value differential. It always benefits the less mobile firm, in that its constrained equilibrium expected value increases. It also increases the endogenous distance between the two firms' investment triggers.

The results are of interest because many business situations illustrate the proposed scenario of cumulative leadership, in which the firm that detects in advance a new market opportunity, and is able to mobilize resources promptly, preempts the lead position in the capacity development process, and thereby benefits from a first-mover advantage in the product market stage. The market for legal music downloads offers a recent example: demand was inexistent a few years ago, it is growing at an uncertain rate, large firms compete by distributing goods (songs) available from very similar sources (catalogs of titles), and variable costs are negligible relative to investment levels (in digital rights, technological diffusion capacities, or advertising campaigns), which are – at least partly – irreversible. In this industry, Apple first invested in 2001 to install iTunes Music Store, when the market was burgeoning. It was an early move, in that sales remained very limited for almost three years before accelerating sharply.[4] Interestingly, Apple CEO Steve Jobs is frequently portrayed as one who "*has the phenomenal quality of figuring out where the next industry movement would be*" (http://www.iipm.edu, Sept. 23, 2007). This does not apply to the management of Microsoft, a more recent participant in the market for music downloads. It is well accepted that "[*t*]*hroughout its history, Microsoft has been slow to grasp some of the computer industry's biggest technology shifts and business changes.* (. . .) *It also was late coming to market with its own music player, and despite a push, remains far behind Apple*" (http://online.wsj.com, July 30, 2007). In fact Microsoft waited until late 2006 to launch its own player Zune, and then MSN Music Store, when demand had reached a much higher level. At that time, the consensus among observers was that Apple's sales would not be strongly impacted by the new entrant. The president of Microsoft's entertainment division acknowledged that "*analysts don't expect the early effort to make a serious dent in Apple's market share.* (. . .). *Apple's obviously still going to be the leader. I think that's fair.* (. . .) *While Microsoft is a great brand name it's, you know, not the first word that comes to your mind when someone*

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[4]According to the Recording Industry Association of America, in 2001 digital downloads represented only 0.2% of total sales (see: http://www.riaa.com/keystatistics.php). It rose to 0.5% in 2002, and 1.3% in 2003, then fell to 0.9%, before jumping to 5.7% in 2005. It kept increasing since then.

says, Hey, music!" (http://www.businessweek.com, Nov. 13, 2006). Another business expert was even more explicit, as it claimed that Apple *"has something that the Zune certainly lacks: first mover advantage. This advantage is primarily kept where people have bought their music via the iTunes store. The amount of effort to get your music in a format that is playable on the Zune (or any other player) is just too much.*" (http://www.zdnet.com, Sept. 20, 2006). The first-mover advantage is likely to be a long lasting phenomenon when it is supported by a combination of brand loyalty, a technical device or a proprietary format, and use habits, at the expense of late comers. Microsoft's market share is only a small portion of Apple's position – that is more than 70% of worldwide online digital music sales in mid 2008 – more than one year after Zune was introduced. In this example, a new consumer pays for a hardware (a portable media player) before getting access to an online music store. In addition, a customer gets used to specific routines (to search the catalog content, to download files, to pay) that render costly a shift to an alternative supplier. When satisfied with the first supplier, one is likely to keep consuming from it. This is emblematic of the many circumstances captured by the new model specification introduced here.

The remain of the paper is organized as follows. In section 2, the benchmark model is presented, together with the first-mover advantage specification we introduce, for the analysis to be self contained. In section 3, the equilibrium outcomes of the Cournot and Stackelberg scenarios are compared and characterized. In section 4, the impact on equilibrium outcomes of limited mobility constraints are examined. In section 5, comparative statics results describe the consequences of a change in growth and volatility. Final remarks appear in section 6.

2   The Model

This section presents briefly BLM's model in the same notation, augmented by a few additional specifications that introduce the Stackelberg scenario. For a more detailed description and intuitive comments please refer to the original paper.

Two risk-neutral symmetric profit-maximizing firms, f and $-f$, contemplate entry on a market to sell a non-differentiated good in quantities x_t^f and x_t^{-f}, respectively, at each point in time t. Production requires an investment in a positive integer number of capacity units, which augment existing capacity stocks k_t^f and k_t^{-f}, with $k_0^f = k_0^{-f} = 0$. Each capacity unit allows a firm to supply up to *one* output unit. It does not depreciate, and has no resale value. The fixed cost of acquiring a capacity unit is I in current value.

At time $t \geq 0$, inverse demand is described by the function

$$P(t, X_t) = Y_t D^{-1}(X_t), \qquad (1)$$

where $X_t = x_t^f + x_t^{-f} \geq 0$ is the total output, $Y_t \geq 0$ is a random shock, and $D^{-1}(X_t) \geq 0$ is a non-stochastic term. Aggregate shocks $(Y_t)_{t \geq 0}$ follow a geometric Brownian motion

$$dY_t = \alpha Y_t dt + \sigma Y_t dZ_t, \qquad (2)$$

with $Y_0 > 0$, $\alpha > 0$ (growth rate), $\sigma > 0$ (volatility), and where $(Z_t)_{t \geq 0}$ is a standard Wiener process. The time-invariant function $D(.)$ is strictly decreasing, continuously differentiable, and integrable on R_+, with $D(0) = \lim_{p \downarrow 0} D(p) < \infty$; the mapping $X_t \longmapsto X_t D^{-1}(X_t)$ is strictly concave on $(0, D(0))$.

Within the interval $[t, t+\tau)$ the timing of the game is as follows: 1) given the realization of Y_t, and existing capital stocks, each firm f chooses to invest a number of capacity units ν_t^f; 2) Given capacity units, each firm f selects an output level under capacity constraint $x_t^f \leq k_t^f + \nu_t^f$; 3) given output levels, market price is determined according to (1).

The results presented in the next sections compare the Cournot and Stackelberg versions of the model. In the latter version, the first entrant is also a first-mover in the market subgame. In the former version of the model, the two firms choose quantities simultaneously. In both scenarios, market outcomes depend on each firm's installed capacity, and although Y_t is not bounded for above, an assumption guarantees that investments stop as soon as each firm holds one capacity unit (or more):

■ *Cournot* Firms choose quantities simultaneously. Let $k^c = \lceil x^c \rceil$ be the minimum capital stock (an integer) required to produce x^c, that is the unconstrained equilibrium quantity for each firm. For a benchmark, as in BLM assume that the market subgame admits a unique equilibrium (x^c, x^c), with $0 < x^c \leq 1$, so that $k^c = 1$. When firms both operate only one capacity unit each, they cannot be constrained.

■ *Stackelberg* Firms choose quantities sequentially. Let $k^F = \lceil x^F \rceil$ and $k^L = \lceil x^L \rceil$ be the minimum capital stocks required to produce x^F and x^L, the unconstrained equilibrium quantities of the follower and the leader, respectively. The comparison of this scenario with the benchmark commands again to assume that the market subgame admits a unique equilibrium (x^F, x^L), with $0 < x^F \leq x^L \leq 1$, which means that $k^F = k^L = 1$. No firm is constrained when both hold only one capacity unit.

Given capacities, in the Cournot and Stackelberg scenarios alike, it follows from (1) that the equilibrium of the market subgame does not depend on Y_t. When a firm holds k capacity units while its rival holds l capacity units, its instantaneous gross profit is $Y_t \pi_{kl}$, where π_{kl}, as computed in the market subgame, depends on installed capacity units only. There is no commitment by firms relative to the size of their investments, to the quantity they sell, nor to the sequence of entry, which is endogenous. Only one firm may choose to enter by investing in capacity units, or both firms may enter simultaneously, or sequentially. As no restriction on capacity is imposed in the investment game, firms are specified to stop investing when it is known with certainty that none of them will ever undertake any further investment, as it would only result in higher fixed costs.[5]

The specification that firms invest in a production capacity, and not in an abstract project or entry ticket, offers a very natural mechanism to compare the different instantaneous profit levels firms earn across the two versions of the model, namely π_{10} (monopoly profits), π_{11} (Cournot duopoly profits), π^F (Stackelberg follower's profits), and π^L (leader's profits). Suppose that $x^c < 1$, so that investing in one capacity unit is sufficient to supply the unconstrained Cournot output. Then standard demand specifications, say a linear $D(.)$, lead to the usual ranking $\pi^F < \pi_{11} < \pi^L < \pi_{10}$. Still the magnitude of profit differences depends on the status of other capacity constraints. In particular, π^L is relatively high if $x^L \leq 1$ (the slack case), and low otherwise, though *strictly* above π_{11}, which is constant across the two cases. A less usual ranking occurs when $x^c = 1 < x^m$, so that investing in one capacity unit is exactly sufficient to supply the unconstrained Cournot output, but not the unconstrained monopoly quantity. It follows that $\pi^F = \pi_{11} = \pi^L < \pi_{10}$. This obtains whenever best-reply functions have a negative slope in the quantity plane, hence for a very large class of demand expressions. There is no need to evoke a more sophisticated technological issue, nor a very specific functional form.[6] As for the equality $\pi_{11} = \pi_{10}$, or $\pi^L = \pi_{10}$ with a first-mover advantage, it is a limit case that can only be approached here (since demand is strictly decreasing), and will turn a useful reference for the comparison of investment triggers across the two versions

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[5]This is established rigorously in BLM (2007) as Proposition 1: for the investment game to be over, it is necessary that either 1) no capacity constraint binds in the short-run market subgame, or 2) the two firms' capacity constraints are binding in the short-run market subgame, and would remain so, should any firm invest in another unit. The first condition – which is satisfied in the Cournot and Stackelberg scenarios of the present model – is also sufficient.

[6]Absent capacity constraints, and without introducing an additional process of imitation, innovation, or externalities, a very specific condition on the inverse demand function would be needed for the Stackelberg market substage equilibrium to coincide with the Cournot equilibrium, as shown in Colombo and Labrecciosa (2008).

of the game. Note also that $k^c = k^F = k^L = 1$ makes possible that $\pi_{v0} > \pi_{10}$, with $v > 1$, in which case the first entrant is constrained with only one capacity unit and finds it profitable to invest more. This situation is left aside in this paper because it does not lead to other comparisons in instantaneous profit levels than already obtained with $v = 1$. Hereafter an investment in exactly one capacity unit simply coincides with the decision to enter.

The equilibrium concept is the Markov perfect equilibrium (MPE). This means that a firm's investment and output decisions at each point in time depend only on the current levels of capacity units and of the industry-wide shock. It follows that, given installed capacities, firms may not attempt to coordinate output decisions over time. At each date they play the unique equilibrium of the market subgame.[7] An MPE outcome of the game is an ordered sequence of investment triggers, with related instantaneous gross profit levels. The latter are denoted by y_{ij} (with $y_{ij} = y_{ji}$), where i and j refer to the firms' capacities immediately before Y_t reaches the level y_{ij} for the first time. Note that $y_{11} = \infty$ since the game stops when $(i,j) \geq (1,1)$.

In the next section, we first characterize the MPE in the Cournot scenario, when firms may invest with no delay at any point in time. Some formal developments are presented more extensively than in BLM, as required for the subsequent characterization of the MPE in the Stackelberg version. It is demonstrated that the properties of the equilibrium outcomes across the two versions are rooted in the relative levels of profits a firm may earn as a monopolist, as a Cournot player, or as a Stackelberg leader/follower in the short-run market subgame.

3   Cournot vs. Stackelberg

In the benchmark case, when both firms hold production capacities, they are assumed to choose individual quantities simultaneously, *à la* Cournot. The analysis investigates all situations for which

$$0 < \pi_{11} \leq \pi_{10}. \tag{3}$$

This simply says that, when both firms have entered, so that each of them produces the non-constrained quantity $x^c \leq 1$, they earn lower individual profits than a monopolist.

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[7]A more formal definition of Markov strategies and payoffs under uncertainty is given in Boyer *et al.* (2004, Appendix A).

For notational parsimony, as in the original model define

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left[\left(\frac{\alpha}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2} \right]^{\frac{1}{2}}, \tag{4}$$

where r is a constant interest rate.[8] We have $\beta > 1$ for all $r > \alpha$ (the latter inequality must hold otherwise a firm's value is maximized by postponing investments for ever).[9]

When the two firms may invest with no delay at any point in time, from $t = 0$ onward, their current value depends on the date at which each of them chooses to enter. Suppose a firm enters when $Y_t = y$. Then the value of its competitor, if it enters later when Y_t reaches a higher level y_{0v}, is

$$F(y) = \left(\frac{y}{y_{0v}} \right)^\beta \left(\frac{\pi_{11}}{r - \alpha} y_{0v} - I \right), \tag{5}$$

all $v \geq 1$, where v is the number of units acquired by the opponent before the firm acquires its first (and single) unit.[10] The maximum $F^*(y)$ with respect to y_{0v} is obtained at

$$y_{0v}^* = y_{01}^* = \frac{r - \alpha}{\pi_{11}} I \frac{\beta}{\beta - 1}, \tag{6}$$

which does not depend on v.[11] When a firm invests in one capacity unit immediately, when $Y_t = y$, while the other one remains out of the market as long as Y_t has not hit y_{01}^*, its value is

$$L(y) = \frac{\pi_{10}}{r - \alpha} y - I + \left(\frac{y}{y_{01}^*} \right)^\beta \frac{\pi_{11} - \pi_{10}}{r - \alpha} y_{01}^*, \tag{7}$$

for all $y < y_{01}^*$ (recall from Proposition 1 in BLM, together with $0 < x^c \leq 1$ by assumption, that the game ends as soon as the other firm enters by investing 1 capacity unit, or more).

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[8]The expression of β is standard. See Dixit and Pindyck (1994, pp. 140-144) for a detailed exposition of the steps that lead to it.

[9]I thank Benoit Chevalier-Roignant for suggesting that, if r is higher than the risk-free rate (e.g., firms may discount profits at the capital market rate), then the condition $r > \alpha$ authorizes a relatively high level of α. This makes sense in the case of publicly held firms that contemplate entry on an emerging market. See Dixit and Pindyck (1994, pp. 147-150) for different interpretations of the discount rate in investment models.

[10]The expression of value functions, as proposed in BLM, follows Harrison (1985, chapter 3). The term $\left(\frac{y}{y_{0v}} \right)^\beta$ reads as the expected discounted value, measured when $Y_t = y$, of receiving 1 monetary unit when Y_t reaches y_{0v} for the first time.

[11]The expressions of firms' value are borrowed from BLM's Lemma 1. Note that $F(y)$ is concave in y_{0v} if and only if $y_{0v} < \left(1 + \frac{1}{\beta} \right) y_{01}^*$. This second-order condition is satisfied at $y_{0v} = y_{01}^*$ for all $\beta > 1$, all $v \geq 1$.

When the same firm does not invest immediately at the current level y, and chooses to postpone entry until Y_t reaches a higher level y_{00}, its value is

$$L_{y_{00}}(y) = \left(\frac{y}{y_{00}}\right)^{\beta}\left(\frac{\pi_{10}}{r-\alpha}y_{00} - I\right) + \left(\frac{y}{y_{01}^*}\right)^{\beta}\frac{\pi_{11} - \pi_{10}}{r-\alpha}y_{01}^*, \tag{8}$$

for all $y < y_{00}$. The maximum of $L_{y_{00}}(y)$ with respect to y_{00}, hereafter denoted by $L^*(y)$, is obtained at

$$y_{00}^L = \frac{r-\alpha}{\pi_{10}}I\frac{\beta}{\beta - 1}, \tag{9}$$

which is strictly lower than (or equal to) y_{01}^* since $\pi_{11} < (=)\pi_{10}$.[12] If, at $t = 0$, a firm can commit relative to the level of Y_t that triggers entry, and thereby be protected from preemption by its rival, it would enter at y_{00}^L. This describes a point of reference that will be useful in the following, and which corresponds to the stochastic time $\tau_{00}^L = \inf\{t \geq 0 : Y_t \geq y_{00}^L\}$. As y_{00}^L depends on short-run monopoly profits π_{10}, not on π_{11}, it will not be impacted by the level of profits earned in the short-run duopoly game when both firms have entered (in the Cournot and Stackelberg versions, indifferently). Observe from (5) and (7) that $F^*(0) = 0 > L(0) = -I$. Moreover, $F^*(y)$, which is obviously strictly increasing with y for all relevant parameter values, is also strictly convex, since

$$\frac{\mathrm{d}^2F^*(y)}{\mathrm{d}y^2} = \frac{I\beta}{y^2}\left(\frac{y}{y_{01}^*}\right)^{\beta}, \tag{10}$$

all $y > 0$, while $L(y)$ is strictly (weakly) concave if and only if $\pi_{11} < (=)\pi_{10}$, since

$$\frac{\mathrm{d}^2L(y)}{\mathrm{d}y^2} = \frac{\beta(\beta-1)}{y^2}\left(\frac{y}{y_{01}^*}\right)^{\beta}\frac{\pi_{11} - \pi_{10}}{r-\alpha}y_{01}^*, \tag{11}$$

all $y > 0$. It follows that $F^*(y) - L(y) = 0$ may admit up to two roots. Since y_{01}^* is a root, it is sufficient to observe that

$$\left.\frac{\mathrm{d}F^*(y)}{\mathrm{d}y}\right|_{y=y_{01}^*} = \frac{\pi_{11}}{r-\alpha},$$

and that

$$\left.\frac{\mathrm{d}L(y)}{\mathrm{d}y}\right|_{y=y_{01}^*} = \frac{\pi_{11}}{r-\alpha} - (\beta - 1)\frac{\pi_{10} - \pi_{11}}{r-\alpha},$$

to conclude that there exists an other positive root $y_{00}^P < y_{01}^*$ if $\pi_{11} < \pi_{10}$, and that $y_{00}^P = y_{01}^*$ otherwise. This, together with the comparison of $F^*(y_{00}^L)$ with $L(y_{00}^L)$, and the comparison of (6) with (9), lead to the following remark.

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[12] $L_{y_{00}}(y)$ is concave in y_{00} if and only if $y_{00} < \left(1 + \frac{1}{\beta}\right)y_{00}^L$. This second-order condition is satisfied at $y_{00} = y_{00}^L$ for all $\beta > 1$.

Remark 1 $y_{00}^p < (=)y_{00}^L < (=)y_{01}^*$ *if and only if* $\pi_{11} < (=)\pi_{10}$.

In words, when we let the profits π_{10} of a constrained monopolist, which has invested in one capacity unit, exactly match the individual profits π_{11} of unconstrained duopolists, $L(y)$ is a line which is tangent to the graph of F^* at $y_{00}^p = y_{00}^L = y_{01}^*$. In the latter limit case only, the Cournot scenario collapses to the benchmark commitment situation, since the value of the two firms exactly equates the level of a firm protected from preemption.

To compare, introduce now a modification in the benchmark model by assuming that firms play sequentially in the market subgame. The first entrant anticipates that the other firm is going to start supplying later, when Y_t reaches a given level, and accordingly adapts its behavior at that time, *à la* Stackelberg. To capture this in the largest possible class of functional specifications for the non-stochastic component of demand D, let the two firms earn asymmetric profits when they have both invested in capacity units. When the two firms are active, the one that entered first earns π^L, while the second entrant earns π^F, with $\pi^F \leq \pi^L$.[13] As in the Cournot version, when only the leader is active, it earns monopoly profits π_{10}. The counterpart to (3) now is

$$0 < \pi^F \leq \pi_{11} \leq \pi^L \leq \pi_{10}. \tag{12}$$

This specification is sufficiently general to capture a large class of circumstances.[14] Then substitute π^F for π_{11} in (5) to obtain $\bar{F}(y)$. The maximum $\bar{F}^*(y)$ with respect to y_{0v} is reached at

$$\bar{y}_{0v}^* = \bar{y}_{01}^* = \frac{r - \alpha}{\pi^F} I \frac{\beta}{\beta - 1}, \tag{13}$$

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[13]In a similar continuous-time investment model, Smit and Trigeorgis (2004, chapter 9) suppress the Stackelberg equilibrium. They emphasize that, if quantities are chosen sequentially, the first mover's choice is not a best reply to the quantity chosen by the second mover. When the market subgame is repeated, the leader has an incentive to reduce its output. More generally, here the Stackelberg scenario may refer to a large class of circumstances in which the first player benefits from any advantage reflected by (12). The many possible sources of the advantage include brand loyalty effects, switching costs, and other lock-in phenomena. See Cottrell (2002) for several examples.

[14]In the present paper, the possibility that the leader earns higher instantaneous profits when the follower has invested than in monopoly, that is $\pi_{10} < \pi^L$, is ruled out. This case is precisely investigated by Mason and Weeds (2006, Proposition 5), where unusual comparative statics results characterize situations in which the follower's investment benefits the leader so much as to outweigh the effect of increased competition. However we may have $\pi^F = \pi_{11} = \pi^L$, so that the Stackelberg market substage equilibrium coincides with the Cournot equilibrium (this occurs if $x^c = 1 < x^m$).

all $v \geq 1$. By comparing (13) with (6), one observes that $\bar{y}_{01}^* > (=)y_{01}^*$ if and only if $\pi^F < (=)\pi_{11}$. Then substitute $\bar{y}_{01}^*$ for y_{01}^* and also π^L for π_{11} in (7) and (8) to obtain $\bar{L}(y)$ and $\bar{L}_{y_{00}}(y)$, respectively. The maximum of $\bar{L}_{y_{00}}(y)$ with respect to y_{00} is obtained at the same value as the maximum of $L_{y_{00}}(y)$, that is y_{00}^L as displayed in (9). This leads to a first claim:

Proposition 1 *If the first entrant may commit at $t = 0$ relative to the level of Y_t that triggers entry, it chooses to enter when $Y_t = y_{00}^L$, independently of the nature of competition on the product market in the post-entry period, while $\pi_{11} < \pi^L$ implies $\bar{L}^*(y_{00}^L) > L^*(y_{00}^L)$ and $\bar{F}^*(y_{00}^L) < F^*(y_{00}^L)$.*

On the contrary, in the absence of commitment, the nature of competition impacts the timing of entry decisions. The possibility of playing first in the market subgame makes a difference. To see that, compare the slopes of $\bar{F}^*(y)$ and $\bar{L}(y)$ with the slopes of $F^*(y)$ and $L(y)$, respectively. Define $\mathcal{F} \equiv \bar{F}^* - F^*$ and $\mathcal{L} \equiv \bar{L} - L$, then recalling that $\beta > 1$, to find

$$\frac{\mathrm{d}\mathcal{F}(y)}{\mathrm{d}y} = \frac{I}{y}\frac{\beta}{\beta - 1}\left[\left(\frac{y}{\bar{y}_{01}^*}\right)^\beta - \left(\frac{y}{y_{01}^*}\right)^\beta\right] < 0 \tag{14}$$

if $\pi^F < \pi_{11}$, and

$$\frac{\mathrm{d}\mathcal{L}(y)}{\mathrm{d}y} = \frac{\beta}{r - \alpha}\left[\left(\frac{y}{y_{01}^*}\right)^{\beta - 1}(\pi_{10} - \pi_{11}) - \left(\frac{y}{\bar{y}_{01}^*}\right)^{\beta - 1}(\pi_{10} - \pi^L)\right] > 0 \tag{15}$$

if $\pi_{11} < \pi^L$, all $y > 0$. In words, for all positive values of y, in the Stackelberg scenario the slope of the second entrant's value function is strictly lower than in the Cournot scenario, while the slope of the first entrant's value function is higher than in the Cournot scenario. Together with $\bar{F}^*(0) = F^*(0) = 0$, and $\bar{L}(0) = L(0) = -I$, this leads to conclude that $\bar{F}^*$ and $\bar{L}$ intersect at a lower value of y, we denote by $\bar{y}_{00}^p$, than F^* and L. In addition, it is sufficient to check that $\bar{F}^*(\bar{y}_{01}^*) < \lim_{y \uparrow \bar{y}_{01}^*} \bar{L}(y)$ when $\pi^F < \pi^L$ to conclude that $\bar{F}^*$ and $\bar{L}$ cannot intersect for any $y > \bar{y}_{00}^p$ in the Stackelberg scenario.

Proposition 2 *There exists $\bar{y}_{00}^p < y_{00}^p$ such that $\bar{F}^*(\bar{y}_{00}^p) = \bar{L}(\bar{y}_{00}^p)$, and $\bar{F}^*(y) < \bar{L}(y)$ for all $y \in (\bar{y}_{00}^p, \bar{y}_{01}^*)$.*

On the time scale, this means that the two values $\bar{F}^*$ and $\bar{L}$ equalize earlier in the Stackelberg scenario than in the Cournot setting, that is $\bar{\tau}_{00}^p = \inf\{t \geq 0 : Y_t \geq \bar{y}_{00}^p\} < \tau_{00}^p$.

It remains to examine when firms choose to be either the first or the second entrant, or to enter simultaneously. Let $Y_0 \leq \bar{y}_{00}^p$ hereafter, for simplicity. There are three possible cases.

Suppose first that $y < \bar{y}_{00}^p$, which implies that $\bar{L}(y) < \bar{F}^*(y)$. In that case each firm waits for the other to enter first, since leadership is less profitable. Suppose now that $y \geq \bar{y}_{01}^*$ and that no firm has entered yet. Then both firms find it profitable to enter immediately. If one of the two firms leads by entering at $\bar{y}_{01}^*$ immediately before its rival, its value is $\frac{\pi^L}{r-\alpha}y - I$, and the follower's value is $\frac{\pi^F}{r-\alpha}y - I$. If the two firms enter simultaneously, their value is $\frac{\pi_{11}}{r-\alpha}y - I$ (see the dotted lines in Figure 1). Eventually, when $\bar{y}_{00}^p \leq y < \bar{y}_{01}^*$, it is valuable to take the lead since $\bar{F}^*(y) < \bar{L}(y)$. Therefore each firm has an incentive to "undercut" its rival on the segment $[\bar{y}_{00}^p, y_{00}^L]$ to preempt the lead position.

The dynamics of competition for the lead position in the investment stage when firms play *à la* Stackelberg in the short-run market subgame is the same as the one examined by BLM when firms play *à la* Cournot, other things remaining equal. All properties of the preemption equilibrium, including rent-dissipation and equalization, are robust to the change in specifications from (3) to (12). The only differences are that 1) the leader enters earlier, 2) the follower enters later, and 3) each firm's value is lower in equilibrium.[15] The following claim summarizes the main features of the comparison.

Proposition 3 *Suppose $\pi^F < (=)\pi_{11} < (=)\pi^L$: (1) There exists only one MPE outcome of the investment game, in which firm f (with probability 1/2) invests immediately before Y_t reaches $\bar{y}_{00}^p < (=)y_{00}^p$ for the first time, at the stochastic time $\bar{\tau}_{00}^p < (=)\tau_{00}^p$, while firm $-f$ (with probability 1/2) enters at $\bar{y}_{01}^* > (=)y_{01}^*$, that is waits until the stochastic time $\bar{\tau}_{01}^* > (=)\tau_{01}^*$ to invest; (2) Firms' equilibrium value is $\bar{F}^*(\bar{y}_{00}^p) = \bar{L}(\bar{y}_{00}^p) < (=)F^*(y_{00}^p) = L(y_{00}^p)$.*

[Insert Figure 1]

In more intuitive terms, the preemption MPE is characterized by more competition when the two firms are interested in leading not only the order of entry in the investment stage, but also the sequence of moves in the short-run market stage. The possibility given by entering

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[15]The proof of the result is simple since it relies exclusively on the comparison of $\bar{F}^*$ and $\bar{L}$ with F^* and L, respectively. Therefore there is no need to demonstrate again the characterization of a preemption equilibrium offered in the original setup, including the claim that only one firm invests at y_{00}^p (or $\bar{y}_{00}^p$ in the present paper). In BLM, this result is grounded on a continuous time representation of limits of discrete time mixed strategy equilibria, adopted by Fudenberg and Tirole (1985), with firm f's strategy defined as a function $s_{\nu f}^f(k^f, k^{-f}, y) \in [0, 1]$ representing the "intensity" with which f invests in ν^f capacity units, given the existing stocks (k^f, k^{-f}) and the shock $Y_t = y$. A simpler alternative, as in see Grenadier (1996), is to assume that, when firms choose the same point in time to enter, an exogenous random mechanism assigns the lead position to one of them with a given propability.

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first, and then to anticipate the quantity choice of the second entrant to earn π^L (in lieu of π_{11}) in the product market stage, is a bonus for the leader, a penalty for the follower. It makes the prize sweeter, and the defeat bitter. However, in equilibrium of the investment game this results in the leader entering earlier, and the follower entering later, that is in a longer installation period. (Here "installation" is the counterpart to "diffusion" in the terminology introduced by Fudenberg and Tirole (1985, 1987) for the analysis of preemption in technology adoption). No rent results from the extended incumbency period, since the rent equalization property holds. Therefore no "mobility barrier" (Gilbert (1989)) originates from the first-mover advantage. Actually there is more dissipation, in the sense that firms' equal equilibrium values $\bar{L}\left(\bar{y}_{00}^p\right)$ and $\bar{F}^*\left(\bar{y}_{00}^p\right)$ are lower than $L\left(y_{00}^p\right)$ and $F^*\left(y_{00}^p\right)$, although the difference between $\bar{L}$ and $\bar{F}^*$ is higher than between L and F^* for all values of y. The first-mover bonus at the market level turns into more competition at the investment level, and *in fine* in a lower value for both parties.[16]

Note that firms may not escape that situation by cooperating, in order to enter later, at some stochastic date $\tau > \bar{\tau}_{00}^p$. This would not be a self-enforcing deal (here it is assumed that an agreement of this kind is not contractible for legal reasons). Supposing it takes place, the agreement would lead firms to obtain the same value $\bar{S}(y) = \left(\frac{y}{y_{00}}\right)^\beta \left(\frac{\pi_{11}}{r-\alpha} y_{00} - I\right)$, where y_{00} triggers simultaneous entry for both firms (their instantaneous gross profit is as in the Cournot scenario because there is no first-mover). A maximum value $\bar{S}^*(y)$ obtains for $y_{00} = \bar{y}_{00}^s$. When Y_t hits this threshold, each firm's value is $\bar{S}^*(\bar{y}_{00}^s)$. If a firm, say f, chooses to deviate while $-f$ sticks to the collusive strategy, it may enter immediately before Y_t hits

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[16]The extension of the assumption $\pi^F < \pi_{11} < \pi^L$, and consequently of Proposition 3, to more than two firms, is not obvious. In a static hierarchical Stackelberg model where n firms choose outputs sequentially, Anderson and Engers (1992) provide a necessary and sufficient condition on a demand parameter and the number of firms for the first mover to earn *lower* profits in Stackelberg than in Cournot if $n > 2$. This is done with a specific form for the demand function that allows for the linear specification as a particular case, and marginal costs equal to zero. With the same demand and variable cost specifications, Anderson and Engers (1994) construct a discrete time sequential investment model, in which $n \geq 2$ firms choose the different times at which they enter. This is done by paying a fixed cost and choosing an output level, one after another, before reaching time 0, that is the opening date of the market where outputs are sold once and for all. (In BLM the two firms may enter anytime while the market develops, so that simultaneous entry is not excluded *a priori*, and firms compete in the market subgame at each point in time afterwards.) When fixed entry costs are not too high, it is found that all n firms enter and there is equalization of actualized profits, as in the present Stackelberg scenario (see Proposition 2, p. 839). Further research could thus consist in using the ranking of profits revealed by Anderson and Engers (1992) in the hierarchical Stackelberg model with more than two firms to generalize the results obtained here with a duopoly.

$\bar{y}_{00}^s$, that is at $\bar{y}_{00}^s - \varepsilon$, with ε arbitrarily small, for a value $\bar{L}(\bar{y}_{00}^s)$. Accordingly, in the case of simultaneous entry each colluding firm earns the instantaneous gross profit π_{11} from $\bar{\tau}_{00}^s$ on, for ever, while in case of deviation f earns π_{01} from $\bar{\tau}_{00}^s - \varepsilon$ to $\bar{\tau}_{01}^*$, and then π^L from $\bar{\tau}_{01}^*$ on, again for ever (in the two cases firms pay the entry cost I at – almost – the same time). Since $\pi^L \geq \pi_{11}$, the collusive entry agreement cannot be implemented for all $\pi_{01} > \pi_{11}$.[17]

Figure 1 illustrates Proposition 3 in the particular case of a linear demand function. The graphs of L and F^* converge to the same point as y approaches y_{01}^* from below. The distance between $\bar{L}$ and $\bar{F}^*$, when y approaches $\bar{y}_{01}^*$, can be measured by observing that $\lim_{y\uparrow\bar{y}_{01}^*} \bar{L}(y) - \bar{F}^*(\bar{y}_{01}^*) = I\frac{\beta}{\beta-1}\frac{\pi^L-\pi^F}{\pi^F}$, which is proportional to the difference between the profits earned by the leader and the follower in the short-run market subgame.[18] One also obtains a longer time period during which the first entrant is the unique supplier. Although it does not lead to a blockaded entry, playing first in the short-run market subgame postpones the other firm's investment for a while. This comes at some cost, since the leader enters earlier, hence at more risk, and the two firms' values are lower, than in the benchmark situation.[19]

The comparison of MPE outcomes across the two versions can be made more precise. First, it depends on the difference between π^F and π^L. One way to formalize this is to define the non-negative differences γ and δ, with $\pi^F = \pi_{11} - \gamma$ and $\pi^L = \pi_{11} + \delta$, before substituting $\pi_{11} - \gamma$ for π^F and $\pi_{11} + \delta$ for π^L in (14) and (15). Then one obtains $\frac{\mathrm{d}^2\mathcal{F}(y)}{\mathrm{d}\gamma\mathrm{d}y} = -\frac{\mathrm{d}^2\mathcal{L}(y)}{\mathrm{d}\delta\mathrm{d}y} = -\frac{\beta}{r-\alpha}\left(\frac{y}{\bar{y}_{01}^*}\right)^{\beta-1} < 0$, while $\frac{\mathrm{d}^2\mathcal{L}(y)}{\mathrm{d}\gamma\mathrm{d}y} = \beta\frac{M-L}{F}\left(\frac{y}{\bar{y}_{01}^*}\right)^{\beta-1}\frac{\beta-1}{r-\alpha} > 0$. In words, given π_{11}, when γ increases the graph of $\bar{F}^*$ becomes flatter, while the graph of $\bar{L}$ becomes steeper (also when δ increases). Therefore:

Remark 2 $\bar{y}_{00}^p$ and $\bar{F}^*\left(\bar{y}_{00}^p\right) = \bar{L}\left(\bar{y}_{00}^p\right)$ are monotone decreasing when profit differences γ and/or δ increase.

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[17]Of course this holds also in the Cournot alternative, and also if $\bar{y}_{00}^s > \bar{y}_{01}^*$, in which case $\pi^L \geq \pi_{11}$ is sufficient to conclude.

[18]Beyond $\bar{y}_{01}^*$, each firm's value is a linear function of y. This is because, for $y \geq \bar{y}_{01}^*$, and if no entry occurred, both firms will enter immediately. With probability $1/2$, firm f enters first and its value is $\frac{\pi^L}{r-\alpha}y - I$; firm $-f$ follows and its value is $\frac{\pi^F}{r-\alpha}y - I$ (see Figure 1).

[19]In real-world circumstances, a first-mover advantage is likely to erode over time. It will not be sustainable for ever. Brand loyalty, or consumption habits, are certainly not immutable, so that the difference between profit levels π^F, π_{11}, and π^L is likely to shrink progressively (see Cottrell and Sick (2005) for historical evidence). Therefore firms' equilibrium values, as derived in the present setting, should be seen as benchmarks vis-à-vis more realistic situations in which the first investor's superiority is only temporary.

Moreover, (9) and (13) imply that the difference between y_{00}^L and $\bar{y}_{01}^*$ depends on π^F and π_{10}, but not on π^L, while (14) and (15) imply that $\bar{y}_{00}^p = y_{00}^p$ and $y_{01}^* = \bar{y}_{01}^*$ if and only if π^F and π^L are equal also (i.e., in the Cournot setting). More precisely:

Remark 3 $\bar{y}_{00}^p = y_{00}^p = y_{00}^L = y_{01}^* = \bar{y}_{01}^*$ *if and only if* $\pi^F = \pi_{11} = \pi^L = \pi_{10}$.

There are only two particular situations in which we do not obtain either only equality signs (as in the latter remark) or only strict inequality signs throughout the comparison of all threshold values of y: (i) if $\pi^F = \pi_{11} = \pi^L < \pi_{10}$, we have $\bar{y}_{00}^p = y_{00}^p < y_{00}^L < y_{01}^* = \bar{y}_{01}^*$, which simply means that the Stackelberg situation "degenerates" to the Cournot case; (ii) if $\pi^F < \pi_{11} = \pi^L = \pi_{10}$, we have $\bar{y}_{00}^p < y_{00}^p = y_{00}^L = y_{01}^* < \bar{y}_{01}^*$, which compares a particular Stackelberg outcome with the benchmark situation in which the leader may commit to enter when Y_t reaches y_{00}^L (the leader's profits can be made equal to both monopoly profits and one-shot duopoly profits by relaxing the strict monotonicity assumption of the demand function). We may thus connect the ranking of investment triggers, across the two versions of the game, to all possible rankings of instantaneous profit levels a firm may earn in the short-run market subgame.

4 The Case of Limited Mobility

From Proposition 3, we know that in equilibrium each firm may lead the investment schedule, or follow, with probability $1/2$, in the Cournot and Stackelberg scenarios alike. The indeterminacy of the identity of the leader/follower is a consequence of the assumption that firms are *a priori* symmetric. To avoid relying on a random selection mechanism that distributes roles without any economic rationale, one may introduce an investment cost asymmetry as in Pawlina and Kort (2001), a marginal cost asymmetry, or a quality differential, as suggested by Smit and Trigeorgis (2004). The firm with a lower fixed or marginal cost, or with a higher demand, would be the one that preempts its less profitable competitor.

An alternative approach, adopted here, is to relax the assumption that firms are equally able to mobilize resources they need *with no delay* at any point in time, from $t = 0$ onward. Clearly this perfect mobility specification does not fit many real-world circumstances. The fact that a new demand takes off is certainly not obvious at early stages. Once detected, the installation of production and distribution resources of all kinds, as required to start supplying, cannot be immediate. As Prescott and Visscher (1977, p. 379) put it, "*some*

entrants become aware of a profitable market before others or require longer periods of time in which to "tool up"." To illustrate, consider again the market for legal music downloads mentioned in introduction. In a press conference concerning the first anniversary of the iTunes Music Store, Steve Jobs emphasized the fact that Apple was prompter than rivals to assess the sales potential of that market when it was only embryonic: *"Zero to 70 million in one year, you know, if a year ago anyone had predicted that iTunes would sell 70 million songs during its first year, they would have been laughed out of town."* Following Apple's early entry, after a couple of years the firm's market value rose significantly more than the ones of subsequent entrants.[20] Can this fact be considered as *prima facie* evidence that rents may originate from the superior ability to assess the potential of an emerging market?

To answer this question, a simple – and most realistic – departure from the frictionless world described so far consists in assuming that, at least in the early development stages of a new market, firms may not be equally able to explore emerging opportunities, or to set up the assets they need to exploit them. In other words, firms may be imperfectly and asymmetrically able either to identify the potential of a burgeoning market, or to move fast and invest as soon as they find it profitable to do so. The simplest possible way to capture this idea is to introduce mobility constraints $Y_t \geq y^f$ and $Y_t \geq y^{-f}$ for the two firms f and $-f$, respectively, with $0 \leq y^f \leq y^{-f}$.[21] The constraints formalize the assumption that the management is unable to seize market opportunities, or to initiate an investment process, if the market has not grown up to a certain level, which can be firm-specific. The levels y^f and y^{-f} cannot be adjusted by firms in the short-run; they are technological (or organizational) parameters.[22] I assume they are known by all parties, and are unchanged across the Cournot and Stackelberg versions of the model. Since the analysis focuses on an infant industry, and the level y_{00}^L is not only lower than y_{01}^* and $\bar{y}_{01}^*$, but also a reference point common to

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[20]See the transcript of the press conference from Steve Jobs concerning the first anniversary of the iTunes Music Store, dated April 29th, 2004, available at: http://www.macobserver.com.

[21]Mobility thresholds can be given a chronological interpretation by defining the corresponding stochastic dates τ^f and τ^{-f}. Since the time to "tool up" is measured from date $t = 0$, that is from the opening date of the market onwards, τ^f and τ^{-f} can be understood as durations. The more mobile firm f is able to enter immediately after date τ^f. Firm $-f$ is slower, as it takes more time τ^{-f} to satisfy the technological conditions that allow it to enter. If $y^f = 0$, then $\tau^f = 0 \leq \tau^{-f}$ (i.e., the more mobile firm is perfectly mobile as it is able to invest immediately at all market development levels).

[22]The mobility constraints clearly differ from the decision lags in Gilbert and Harris (1984, Section 3) where a firm is assumed to be able to invest strictly before its rival. Hence the first investment can occur at any t, from the very beginning. The decisions lags, although arbitrarily close to zero, are always positive. To compare, in the present model firms can be mobile only for $t > \tau_{00}^L$. Moreover we may have $\tau^f = \tau^{-f}$.

the Cournot and Stackelberg scenarios (see Proposition 1), it is convenient to specify that $y^{-f} \leq y_{00}^L$, implying that $\tau^f \leq \tau^{-f} \leq \tau_{00}^L$ (i.e., both firms are sufficiently mobile to enter as soon as Y_t hits y_{00}^L).[23]

How are entry decisions, and firm values, impacted by the limited mobility constraints? If the current industry-wide shock is strictly less that y^f, obviously no entry may occur, for all y^f, y^{-f}. Otherwise the exact timing of entry depends on the comparison of mobility parameters with y_{00}^p in the original setup (or $\bar{y}_{00}^p$ with a first-mover advantage). When both firms are *almost* perfectly mobile, in the sense that $y^{-f} \leq y_{00}^p$ (or $y^{-f} \leq \bar{y}_{00}^p$), we already know that a free MPE outcome obtains in which only one firm enters at y_{00}^p (resp. $\bar{y}_{00}^p$) while the other firm waits until τ_{01}^* (resp. $\bar{\tau}_{01}^*$).[24] Any firm can be either the leader or the follower in the investment subgame, with equiprobability. In that case, the specification that mobility parameters y^f and y^{-f} may differ across firms plays no role.

Of more interest are all situations in which at least one constraint bites. When limited mobility constraints are symmetric, and sufficiently strong so that no free preemption equilibrium obtains, the "floor" $y^f = y^{-f}$ prevents rent competition to fully dissipate and equalize monopoly rents. In the constrained preemption equilibrium, one of the two firms, say f, enters immediately before Y_t reaches the constraint, and $-f$ again enters at y_{01}^* (or at $\bar{y}_{01}^*$). The limited mobility constraints may also differ across firms. If $y^f < y^{-f}$ firm f is given the possibility *not* to invest immediately when Y_t hits y^f. When $Y_t < y^{-f}$, firm f faces no threat of entry by the rival $-f$, and thus may choose not to invest until Y_t reaches any higher level y_{00} in the interval $[y^f, y^{-f}]$. In the time dimension, it may postpone entry to any point τ_{00} in $[\tau^f, \tau^{-f}]$, during which it is protected from preemption. When positive, the difference between mobility parameters makes it impossible for firm $-f$ to contest the lead position for a while. Firm f may thus exploit the lack of market awareness or of technological agility of its rival to postpone entry, and nevertheless to enter first, at less risk, and at some profit. Since opting for leadership in the entry process is a dominant strategy, and the value function is monotone increasing with y, firm f's optimal choice is to enter at $y^{-f} - \varepsilon$, with ε arbitrarily small (henceforth consider ε is negligible to simplify the notation).[25] In other words, the more mobile firm finds it optimal to postpone entry as much as possible, to enter an instant before the rival is able to contest the lead position. To summarize:

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[23]With no upper bound on mobility parameters, *immobile* firms would be defined for $\tau^f = \tau^{-f} = \infty$.

[24]Here $\bar{y}_{00}^p \leq y_{00}^p$ implies that, in the Cournot scenario, if $y^{-f} \leq \bar{y}_{00}^p$ the leader enters at y_{00}^p, not at $\bar{y}_{00}^p$, in a free premption equilibrium.

[25]The reasoning is similar to the choice of price in a Bertrand duopoly, with asymmetric marginal costs, when the low cost supplier maximizes profits by charging a price almost equal to the rival's marginal cost.

Proposition 4 *Suppose $y_{00}^p \leq y^f$ (or $\bar{y}_{00}^p \leq y^f$) so that the MPE is constrained: (1) If constraints are symmetric, firm f (with probability $1/2$) invests immediately before Y_t reaches $y^f = y^{-f}$ for the first time, and firm $-f$ (with probability $1/2$) enters at y_{01}^* (resp. at $\bar{y}_{01}^*$); (2) If constraints are asymmetric, firm f (with probability 1) invests immediately before Y_t reaches $y^{-f} > y^f$ for the first time, and firm $-f$ enters at y_{01}^* (resp. at $\bar{y}_{01}^*$).*

[**Insert Figure 2**]

Obviously, when the constraint $Y_t \geq y^{-f}$ is sufficiently stringent, the identity of the first entrant is no more indeterminate. Leadership is more cumulative than in the case of perfect mobility. The firm that is aware in advance of the market potential, or requires a shorter period of time to trigger the investment procedure, is also the one that leads the entry process. However, for a given y^{-f} the leading firm does not necessarily enter at the same point in time in the Cournot and Stackelberg contexts. Therefore the exact comparison of firm values, in equilibrium, across the two scenarios of product market competition, depends on the level of mobility parameters. To see that, first define $\hat{y}$ in $[\bar{y}_{00}^p, y_{00}^p]$ such that $L(y_{00}^p) < (=)\bar{L}(y)$ if and only if $y > (=)\hat{y}$, and also $\check{y}$ in $[y_{00}^p, \bar{y}_{01}^*]$ such that $F^*(y_{00}^p) < (=)\bar{F}^*(y)$ if and only if $y > (=)\check{y}$.[26] Then refer to Figure 2, which represents all the properties of the value functions we need, together with the comparison of equilibria across the Cournot and Stackelberg scenarios, to visualize the rankings of firm values for all levels of y^{-f}, as made formal in the following claim.

Proposition 5 *Suppose $\pi^F < \pi_{11} < \pi^L$: if $y^{-f} \leq y_{00}^p$ in the Cournot setup, or if $y^{-f} \leq \bar{y}_{00}^p$ in the Stackelberg scenario, limited mobility has no impact on the free MPE of the investment game. Otherwise:*

(i) if $\bar{y}_{00}^p < y^{-f} \leq \hat{y}$

$$\bar{F}^*(\bar{y}_{00}^p) = \bar{L}(\bar{y}_{00}^p) < \bar{F}^*(y^{-f}) < \bar{L}(y^{-f}) < F^*(y_{00}^p) = L(y_{00}^p); \qquad (16)$$

(ii) if $\hat{y} < y^{-f} \leq y_{00}^p$

$$\bar{F}^*(\bar{y}_{00}^p) = \bar{L}(\bar{y}_{00}^p) < \bar{F}^*(y^{-f}) < F^*(y_{00}^p) = L(y_{00}^p) < \bar{L}(y^{-f}); \qquad (17)$$

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[26]The existence of $\hat{y}$ and $\check{y}$ follows from the continuity of value functions, together with $\bar{L}(\bar{y}_{00}^p) < L(y_{00}^p) < \bar{L}(y_{00}^p)$ for the first threshold, and $\bar{F}^*(y_{00}^p) < F^*(y_{00}^p) < \bar{F}^*(\bar{y}_{01}^*)$ for the second one. The monotonicity of value functions guarantees uniqueness.

(iii) $\;$ if $y_{00}^p < y^{-f} \leq \check{y}$

$$\bar{F}^*(\bar{y}_{00}^p) = \bar{L}(\bar{y}_{00}^p) < \bar{F}^*(y^{-f}) < F^*(y_{00}^p) = L(y_{00}^p) < F^*(y^{-f}) < L(y^{-f}) < \bar{L}(y^{-f});$$
$$(18)$$

(iv) $\;$ if $\check{y} < y^{-f} \leq y_{00}^L$

$$\bar{F}^*(\bar{y}_{00}^p) = \bar{L}(\bar{y}_{00}^p) < F^*(y_{00}^p) = L(y_{00}^p) < \bar{F}^*(y^{-f}) < F^*(y^{-f}) < L(y^{-f}) < \bar{L}(y^{-f}).$$
$$(19)$$

In less formal terms, in cases (i) and (ii), that is when firms are still relatively mobile (i.e., $y^{-f} \leq y_{00}^p$), firm f enters at y_{00}^p in the Cournot case, and $-f$ at y_{01}^* as in the absence of constraints.[27] However, in the Stackelberg scenario, the more mobile firm f finds it profitable to enter at y^{-f}, whereas $-f$ enters at $\bar{y}_{01}^*$, and both entities benefit from it. More precisely, in case (i) there is rent equalization and dissipation in the Cournot scenario, as in the benchmark model. However, f's value is higher than $-f$'s value in the Stackelberg scenario. In the latter case the two firms' respective values, although higher than with no limited mobility constraints, remain lower than in Cournot. In that sense, the first-mover advantage in the product market stage does not pay. In (ii), the follower's value in the Stackelberg scenario is higher than with perfect mobility, although it remains lower than any of the two firms' values with Cournot. An important outcome is that the Stackelberg leader's value is highest. That is, cumulative leadership is profitable for the leader.

The latter claim holds in the other two cases, where firms are less mobile (i.e., $y_{00}^p < y^{-f}$). In (iii) and (iv) the more mobile firm, f, enters at y^{-f} (i.e., immediately before the shock reaches that level), which is higher than y_{00}^p, in the Cournot and Stackelberg cases alike ($-f$ enters at y_{01}^* and at $\bar{y}_{01}^*$, respectively). An important result is that the leader benefits from the first-mover advantage (i.e., $\bar{L}(y^{-f}) > L(y^{-f})$, for all y^{-f} in $(\bar{y}_{00}^p, y_{00}^L]$), while the follower is worse off (i.e., $\bar{F}^*(y^{-f}) < F^*(y^{-f})$). By reducing competition for the lead position in the investment game, asymmetric limited mobility constraints turn the first-mover advantage – a property that characterizes the short-run market subgame – into more value for firm f at equilibrium in the long-run investment game, at some cost for the rival $-f$. Formally,

$$\frac{\mathrm{d}^2 \bar{L}(y^{-f})}{\mathrm{d}\delta \mathrm{d}y^{-f}} = -\frac{\mathrm{d}^2 \bar{F}^*(y^{-f})}{\mathrm{d}\gamma \mathrm{d}y^{-f}} = \left(\frac{y^{-f}}{\bar{y}_{01}^*}\right)^{\beta-1} \frac{\beta}{r-\alpha} > 0.$$

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[27]The values $F^*(y^{-f})$ and $L(y^{-f})$ do not appear in (16-17) because in the Cournot scenario no firm will enter at $y^{-f} < y_{00}^p$.

The larger the distance δ between Cournot instantaneous profits π_{11} and the leader's profits π^L, the higher the marginal value, for f, as a function of firm $-f$'s mobility parameter y^{-f}.

Note that in (iii) the two Cournot players' equilibrium value in a free preemption equilibrium (that is $F^*(y_{00}^p) = L(y_{00}^p)$) is lower than their respective values in a constrained MPE. This means that, when no first-mover advantage characterizes the product market, and $-f$'s limited mobility constraint bites, there is less rent dissipation than when y^{-f} is lower than y_{00}^p. Moreover firm values are not equal as f benefits more than $-f$ from the delayed entry process ($F^*\left(y^{-f}\right) < L\left(y^{-f}\right)$). The only difference in (iv) is that the follower's value, in the Stackelberg scenario, is now larger than the benchmark, that is Cournot firms' equal equilibrium value with perfect mobility.[28] This is because the leader enters late, at a relatively high y^{-f}, above $\check{y}$. This compensates for the reduction in the instantaneous profits firm $-f$ earns, on the product market, as a second mover.

5 Growth and Volatility: Comparative Statics

This section discusses the effect of a change in the parameters that appear in the geometric Brownian motion, namely α (growth) and σ (volatility), on the level of Y_t at which entry takes place, and on the associated value of firms, in the Cournot and Stackelberg scenarios, with or without mobility limitations.

Define the difference $\mathcal{D}(y) \equiv \bar{L}(y) - \bar{F}^*(y)$, for any y in the interval $(0, y_{00}^L]$, which includes y_{00}^p and $\bar{y}_{00}^p$. In what follows, for conciseness I use the latter Stackelberg notation only, since it encompasses formally the Cournot case (it suffices to specify $\gamma = \delta = 0$). Recall that $\bar{y}_{00}^p$ is implicitly defined by $\mathcal{D}(\bar{y}_{00}^p) = 0$, and that $\mathcal{D}(y^{-f}) > 0$ for all y^{-f} in $(\bar{y}_{00}^p, y_{00}^L]$.

■ *Growth* For all $y \leq y_{00}^L$, if $0 < \pi^F \leq \pi^L \leq \pi_{10}$, one finds

$$\frac{\mathrm{d}\mathcal{D}(y)}{\mathrm{d}\alpha} = \underbrace{\frac{\partial \mathcal{D}(y)}{\partial \beta}}_{>0} \underbrace{\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}}_{<0} + \underbrace{\frac{\partial \mathcal{D}(y)}{\partial \alpha}}_{\geq 0}, \tag{20}$$

hence the effect of the growth parameter α on $\bar{y}_{00}^p$, and on firms' value, for all y^f, y^{-f}, is ambiguous.[29] A higher drift may either decrease or increase the difference between $\bar{L}(y)$ and

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[28] For some parameter values we may have $\check{y} \geq y_{00}^L$, implying that (iv) is irrelevant.

[29] All derivations appear in the appendix.

$\bar{F}^*(y)$, at any level of $y \leq y_{00}^L$, implying in particular that $\bar{y}_{00}^p$ either increases or decreases, respectively.

Example 1 Let $\alpha = \frac{1}{2}\left(r - \sigma^2\right)$, implying that $\beta = 2$. Then pick $r = 3/4$, $\sigma = 1/2$, together with $\pi^F = 1/4$, $\pi^L = 1/2$, and solve $\mathcal{D}(y) = 0$. This leads to $\bar{y}_{00}^p$, which can be written as a function of π_{10} and I. Then $\mathrm{d}\mathcal{D}(y)/\mathrm{d}\alpha$, when evaluated at $\bar{y}_{00}^p$, reduces to a simple expression which is positive (zero) if and only if π_{10} is less than (equal to) $7/4$.

It follows that the impact of a change in α on firms' (constrained) preemption equilibrium value can also be either negative or positive, depending on parameter values.

■ *Volatility* For all $y \leq y_{00}^L$, we have

$$\frac{\mathrm{d}\mathcal{D}(y)}{\mathrm{d}\sigma} = \underbrace{\frac{\partial\mathcal{D}(y)}{\partial\beta}}_{>0}\underbrace{\frac{\mathrm{d}\beta}{\mathrm{d}\sigma}}_{<0}, \tag{21}$$

which is therefore negative. The effect of σ on the difference $\mathcal{D}$ is thus univocal. Since (21) holds on the interval $(0, y_{00}^L]$, the impact of a change in σ on the level of Y_t at which entry takes place can be inferred from the negative sign of $\mathrm{d}\mathcal{D}(y)/\mathrm{d}\sigma$, as follows:

(1) In a free preemption MPE, when σ increases, the leader enters later, at a higher level $\bar{y}_{00}^p$ (a consequence of $\bar{L}(y) \gtreqless \bar{F}^*(y)$ if and only if $y \gtreqless \bar{y}_{00}^p$).[30] The follower enters latter also, at a higher level $\bar{y}_{01}^*$ (this is because $\mathrm{d}\bar{y}_{01}^*/\mathrm{d}\sigma = \left(\partial\bar{y}_{01}^*/\partial\beta\right)\left(\mathrm{d}\beta/\mathrm{d}\sigma\right) > 0$).[31] Here the sign of the change of the difference between $\bar{y}_{00}^p$ and $\bar{y}_{01}^*$, and in the two firms' equal value $\bar{F}^*(\bar{y}_{00}^p) = \bar{L}(\bar{y}_{00}^p)$, at this level of generality, is indeterminate.

(2) In a constrained preemption MPE, when σ increases, the leader's entry date remains the same (y^{-f} does not depend on σ), and again the follower enters later, so that the "installation" gap between the two investment triggers increases also. Since a higher σ results in a narrower distance $\mathcal{D}(y^{-f})$, the value differential between the leader and the

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[30] An increase in σ will not change the level of y at which the leader enters until $\bar{y}_{00}^p$ (which does depend on σ) reaches y^{-f} from below, in which case one gets back to a free preemption equilibrium. The conclusion that a higher σ delays investments is intuitive and standard. To compare, Mason and Weeds (2008) find that greater uncertainty can *lower* the leader's trigger point when the follower's investment benefits the leader, so that $\pi_{10} < \pi^L$ is possible, a case we do not consider here.

[31] It is easy to check that $\partial\bar{y}_{01}^*/\partial\beta < 0$, while $\mathrm{d}\beta/\mathrm{d}\sigma < 0$ is proved in the appendix.

follower decreases. This occurs always at some benefit for firm $-f$, since one checks that

$$\frac{\mathrm{d}\bar{F}^*(y)}{\mathrm{d}\sigma} = \underbrace{\frac{\partial \bar{F}^*(y)}{\partial \beta}}_{<(=)0} \underbrace{\frac{\mathrm{d}\beta}{\mathrm{d}\sigma}}_{<0} > (=)0, \text{ all } y < (=)y_{00}^L. \tag{22}$$

Moreover, without adding more structure to the model, $\pi_{01} > \pi^L$ implies

$$\frac{\mathrm{d}\bar{L}(y)}{\mathrm{d}\sigma} = \underbrace{\frac{\partial \bar{L}(y)}{\partial \beta}}_{>(=)0} \underbrace{\frac{\mathrm{d}\beta}{\mathrm{d}\sigma}}_{<0} < (=)0 \text{ iff } y < (=)\bar{y}_{01}^* \exp(-1/\beta). \tag{23}$$

When the derivative is negative (positive), more volatility penalizes (benefits) the leading firm f. Since $\bar{y}_{01}^* \exp(-1/\beta) < y_{00}^L$ if and only if $\pi^F/\pi_{01} > \exp(-1/\beta)$, firm f's value will increase with σ only when the profit ratio π^F/π_{01} is sufficiently high for the entry threshold y^{-f} to be possibly in the interval $(\bar{y}_{01}^* \exp(-1/\beta), y_{00}^L]$, in which case more volatility benefits firm f also.

Example 2 *Let $\alpha = \frac{1}{2}\left(r - \sigma^2\right)$, implying that $\beta = 2$. Then pick $r = 3/4$, $\sigma = 1/2$, together with $\pi^F = 1/2$, $\pi_{01} = 3/4$, and $I = 1/2$. With these values $\pi^F/\pi_{01} > \exp(-1/\beta)$, so that $\mathrm{d}\bar{L}(y^{-f})/\mathrm{d}\sigma$ is negative (zero) if $y^{-f} < (=)\bar{y}_{01}^* \exp(-1/\beta)$, and positive otherwise. In particular $\mathrm{d}\bar{L}(y^{-f})/\mathrm{d}\sigma\big|_{y^{-f}=\bar{y}_{00}^p} < 0$, while $\mathrm{d}\bar{L}(y^{-f})/\mathrm{d}\sigma\big|_{y^{-f}=y_{00}^L} > 0$.*

To summarize:

Proposition 6 *In a mobility constrained preemption equilibrium, the effect of the growth parameter is ambiguous, while more uncertainty reduces the positive rent differential $\mathcal{D}(y^{-f})$, down to zero when $\bar{y}_{00}^p$ reaches y^{-f} from below, at the advantage of the follower, $-f$. It also increases the gap between the two investment triggers y^{-f} (a parameter) and $\bar{y}_{01}^*$ (which increases with σ).*

6   Final Remarks

If firms are perfectly mobile, by assuming that, when the other firm enters, the leading investor benefits from a first-mover advantage in the duopoly market subgame (to capture, say, a brand loyalty, or consumption habits), a preemption equilibrium obtains in which the leader invests earlier, and the follower later, than in the benchmark scenario. However,

the longer incumbency period does not result in additional rents to the leader. The two firms' equilibrium values are equal. They are also lower than in the absence of first-mover advantage. The rent-equalization outcome is preserved, with more dissipation. More profits for one player at each point in time results in less expected value for both in a dynamic setup. This result thus reinforces the usual conclusion that, by competing for the lead position on a growing market with development uncertainty, firms actually *take* in value. It also clearly rationalizes the case of real-world firms that enter very early on a new market, seeking the benefit of a pioneering move, and nevertheless demonstrate no superior financial performance in the long run.

However, another conclusion of the paper is that the latter result is rooted in the crucial – albeit usually implicit in the literature – assumption that firms may enter at any point in time, with no delay, at the early stages of a new market development process. This perfect mobility assumption does not reflect all observed real-world circumstances. Indeed a very different picture is obtained once firm-specific limited mobility constraints are introduced. They capture situations in which investors are not equally able to explore new market opportunities, or to install promptly the assets they need to exploit them. The constraints serve resolving the indeterminacy of the identity of the leader/follower in a preemption equilibrium, in a similar way as a cost asymmetry or a quality differential. They also lead to unfamiliar outcomes.

In the absence of first-mover advantage, the dampening-of-competition effect of mobility constraints results in higher and asymmetric firm values. The first investor, in equilibrium, is the firm that may detect in advance the potential of a growing market, and/or is capable to trigger the entry process at a lower level of demand. It invests immediately before the less mobile rival may enter. With a first-mover advantage in the short-run market stage, what was a Pyrrhic victory for the leader in the case of perfect mobility turns into a profitable success story when mobility is imperfect. The market advantage amplifies the impact of mobility constraints on firms' performance. The leader's value is higher, while the follower's one is lower, than in the benchmark Cournot case, other things (including mobility constraints) remaining equal. While a change in growth has an ambiguous effect, more volatility is shown to reduce the positive difference in firms' value, to the benefit of the follower, and to increase the distance between the two firms' respective investment triggers.

Arguably, these findings contradict common wisdom, which sees only virtue in the ability to move fast. In the model, the leading firm finds it most profitable to enter immediately *before* its rival, though possibly much *later* than with perfect mobility. The point is certainly

not to enter early, but only earlier than a competitor. In this framework, a long duration between the entry dates of two firms, as observed on a growing market, is no indication of a proportionally large gap in the managers' ability to seize new opportunities. To illustrate, on the market for legal music downloads, where Apple clearly benefits from a first-mover advantage, there is no reason to interpret the late introduction of Zune by Microsoft as an evidence of extreme bureaucratic inertia. It is consistent with a value maximizing behavior, with only a limited difference in the two firms' swiftness or their managers' "*vista*", and with a highly uncertain demand. This also resulted in a large value differential, as predicted by the model with a first-mover advantage and limited mobility.

The source of rent this analysis points to is actually a technological or managerial limitation of at least one firm's ability to detect a burgeoning demand, or to mobilize resources. When firms compete for the lead position, the limitation is a necessary condition for an entrepreneurial competitor to benefit from the advantage it seeks by entering first. This theoretical output can be tested empirically, as one may check whether limited mobility constraints play a role in real-world circumstances. On a given emerging market, if the value of an early investor – which can be approximated by the capital market value – can be observed to benefit from a first-mover advantage, one may conclude that the industry was not prompt at mobilizing the resources needed to enter, to the advantage of the first investor. If early entry does not result in intra-industry profit differentials, it reveals that unleashed rent-seeking behavior has taken place. There is no reason to conjecture, on purely analytical grounds, that the latter case is more frequent than the former.

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Appendix

A1. Derivative of $\mathcal{D}$ w.r. to α

For all $y \leq y_{00}^L$, we show that

$$\frac{\mathrm{d}\mathcal{D}(y)}{\mathrm{d}\alpha} = \underbrace{\frac{\partial\mathcal{D}(y)}{\partial\beta}}_{>0}\underbrace{\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}}_{<0} + \underbrace{\frac{\partial\mathcal{D}(y)}{\partial\alpha}}_{\geq 0}. \tag{24}$$

Consider the three terms in turn. First,

$$\frac{\partial\mathcal{D}(y)}{\partial\beta} = -\frac{I}{(\beta-1)\pi^F}\left(\ln\left(\frac{y}{\bar{y}_{01}^*}\right)\left(\pi^F + \left(\pi_{01} - \pi^L\right)\beta\right) + \pi_{01} - \pi^L\right)\left(\frac{y}{\bar{y}_{01}^*}\right)^{\beta},$$

which is positive (zero) if and only if $y < (=)\bar{y}_{01}^*/\tilde{y}$, where $\tilde{y} \equiv \exp\left(\frac{\pi_{01}-\pi^L}{\pi^F + \beta(\pi_{01}-\pi^L)}\right)$. Next y_{00}^L can be shown to be strictly lower than $\bar{y}_{01}^*/\tilde{y}$ for all parameter values. Indeed, recalling that $\bar{y}_{01}^* \equiv \frac{r-\alpha}{\pi^F}I\frac{\beta}{\beta-1}$ and $y_{00}^L \equiv \frac{r-\alpha}{\pi_{01}}I\frac{\beta}{\beta-1}$, one obtains $y_{00}^L < \bar{y}_{01}^*/\tilde{y}$ if and only if $\pi_{01} > \pi^F\tilde{y}$. Then observe that

$$\frac{\partial\left(\pi_{01} - \pi^F\tilde{y}\right)}{\partial\pi^F} = -\exp\left(\frac{\pi_{01}-\pi^L}{\pi^F + \left(\pi_{01}-\pi^L\right)\beta}\right)\frac{\left(\pi^F\right)^2 + \pi^F\left(2\beta-1\right)\left(\pi_{01}-\pi^L\right) + \beta^2\left(\pi_{01}-\pi^L\right)^2}{\left(\pi^F + \beta\left(\pi_{01}-\pi^L\right)\right)^2},$$

which is negative for all $\pi_{01} \geq \pi^L \geq \pi^F > 0$ and $\beta > 1$. The latter monotonicity implies that

$$\pi_{01} - \pi^F\tilde{y} \geq \pi_{01} - \pi^L\exp\left(\left(\pi_{01}-\pi^L\right)/\left(\pi^L + \beta\left(\pi_{01}-\pi^L\right)\right)\right).$$

As the expression on the right side of the inequality sign is monotone increasing with β, it is sufficient to impose $\beta = 1$ to find $\pi_{01} - \pi^F\tilde{y} > \pi_{01} - \pi^L\exp\left(\frac{\pi_{01}-\pi^L}{\pi_{01}}\right)$, which is zero if $\pi_{01} = \pi^L$, and positive otherwise. It follows that $\pi_{01} > \pi^F\tilde{y}$, therefore $y_{00}^L < \bar{y}_{01}^*/\tilde{y}$, and $\partial\mathcal{D}(y)/\partial\beta > 0$ for all $y \leq y_{00}^L$.

Second, for all $\beta > 1$ one finds

$$\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} = \frac{-\beta}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^2} < 0. \tag{25}$$

Third,

$$\frac{\partial\mathcal{D}(y)}{\partial\alpha} = \frac{(\beta-1)\pi_{01}\pi^F y - I\left(\frac{y}{\bar{y}_{01}^*}\right)^{\beta}\beta\left(r-\alpha\right)\left(\pi^F + \beta\left(\pi_{01}-\pi^L\right)\right)}{(\beta-1)\left(r-\alpha\right)^2\pi^F}.$$

Then observe that

$$\frac{\partial^2}{\partial y^2}\left(\frac{\partial\mathcal{D}(y)}{\partial\alpha}\right) = -I\beta^2\frac{\pi^F + \left(\pi_{01}-\pi^L\right)\beta}{\pi^F\left(r-\alpha\right)y^2}\left(\frac{y}{\bar{y}_{01}^*}\right)^{\beta} < 0,$$

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therefore $\partial \mathcal{D}(y)/\partial \alpha$ is concave in y, with two roots, namely

$$y' = 0 \quad \text{and} \quad y'' = y_{00}^L \left(\left(\frac{\pi_{01}}{\pi^F + (\pi_{01} - \pi^L)\beta} \right)^{\frac{\beta}{\beta-1}} \frac{\pi^F + (\pi_{01} - \pi^L)\beta}{\pi^F} \right) > 0,$$

for all $\pi_{01} \geq \pi^L \geq \pi^F > 0$. It can now be shown that the latter root cannot be strictly lower than y_{00}^L. We have $y'' = y_{00}^L$ if and only if $\beta = 0$, which is not admissible, or $\pi^L = \pi_{01} + \frac{\pi^F}{\beta} \left(1 - \left(\frac{\pi_{01}}{\pi^F} \right)^\beta \right)$. The latter solution is not admissible either. To see this, observe first that y'' is monotone increasing with π^L. Then recall that $\pi^F \leq \pi^L \leq \pi_{01}$, and remark that

$$\frac{\partial \left(\pi_{01} + \frac{\pi^F}{\beta} \left(1 - \left(\frac{\pi_{01}}{\pi^F} \right)^\beta \right) \right)}{\partial \pi^F} = \frac{1}{\beta} + \left(\frac{\pi_{01}}{\pi^F} \right)^\beta \left(1 - \frac{1}{\beta} \right) > 0,$$

and

$$\frac{\partial \left(\pi_{01} + \frac{\pi^F}{\beta} \left(1 - \left(\frac{\pi_{01}}{\pi^F} \right)^\beta \right) \right)}{\partial \pi_{01}} = 1 - \left(\frac{\pi_{01}}{\pi^F} \right)^{\beta-1} < (=)0,$$

if $\pi^F < (=)\pi_{01}$, so that $\pi_{01} + \frac{\pi^F}{\beta} \left(1 - \left(\frac{\pi_{01}}{\pi^F} \right)^\beta \right)$ reaches a maximum only if π^F is the highest possible given π_{01}, and π_{01} is the lowest possible given π^F, that is for $\pi^F = \pi^L = \pi_{01}$. In that case $\pi_{01} + \frac{\pi^F}{\beta} \left(1 - \left(\frac{\pi_{01}}{\pi^F} \right)^\beta \right) = \pi^L$, so that whenever $\pi^F < \pi^L \leq \pi_{01}$ or $\pi^F \leq \pi^L < \pi_{01}$ we obtain that π^L must be strictly higher than $\pi_{01} + \frac{\pi^F}{\beta} \left(1 - \left(\frac{\pi_{01}}{\pi^F} \right)^\beta \right)$, implying that $y'' > y_{00}^L$, all $\beta > 1$. It follows that the partial derivative of $\mathcal{D}(y)$ w.r. to α does not change sign on $(0, y_{00}^L]$. Then pick, say, $\pi^F = 1/3$, $\pi^L = 1/2$, $\pi_{01} = 1$ to conclude. $\square$

A2. Derivative of $\mathcal{D}$ w.r. to σ

For all $y \leq y_{00}^L$, we show that

$$\frac{d\mathcal{D}(y)}{d\sigma} = \underbrace{\frac{\partial \mathcal{D}(y)}{\partial \beta}}_{>0} \underbrace{\frac{d\beta}{d\sigma}}_{<0} < 0. \tag{26}$$

It has been established above that $\partial \mathcal{D}(y)/\partial \beta > 0$ for all $y \leq y_{00}^L$. Next, we have

$$\frac{d\beta}{d\sigma} = -2 \frac{2\sigma^2 (r - \alpha\beta)}{2 \left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \right) \sigma^5}.$$

From the expression of β in (4) we know the denominator is positive. The only root of the numerator is $\beta = r/\alpha$. Again from (4) the latter equality holds if and only if $r = \alpha$ or $r = 0$ (which is not admissible), otherwise $\beta < r/\alpha$. This leads to a negative sign. $\square$

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A3. Derivative of $\bar{F}^*$ and $\bar{L}$ w.r. to σ

For all $y < (=)y_{00}^L$, we show that

$$\frac{\mathrm{d}\bar{F}^*(y)}{\mathrm{d}\sigma} = \underbrace{\frac{\partial \bar{F}^*(y)}{\partial \beta}}_{<(=)0} \underbrace{\frac{\mathrm{d}\beta}{\mathrm{d}\sigma}}_{<0} > (=)0. \tag{27}$$

This is because

$$\frac{\partial \bar{F}^*(y)}{\partial \beta} = \frac{I}{\beta - 1}\left(\frac{y}{\bar{y}_{01}^*}\right)^\beta \ln \frac{y}{y_{00}^L},$$

which is negative (or zero) for all $y < (=)y_{00}^L$, and it has been established above that $\mathrm{d}\beta/\mathrm{d}\sigma < 0$. Next, when $\pi^L < \pi_{01}$, we show that

$$\frac{\mathrm{d}\bar{L}(y)}{\mathrm{d}\sigma} = \underbrace{\frac{\partial \bar{L}(y)}{\partial \beta}}_{>(=)0} \underbrace{\frac{\mathrm{d}\beta}{\mathrm{d}\sigma}}_{<0} < (=)0, \tag{28}$$

if and only if $y < (=)\bar{y}_{01}^* \exp(-1/\beta)$. This is because

$$\frac{\partial \bar{L}(y)}{\partial \beta} = -\left(\beta \ln \frac{y}{\bar{y}_{01}^*} + 1\right)\left(\frac{y}{\bar{y}_{01}^*}\right)^\beta \frac{I}{\beta - 1}\frac{\pi_{01} - \pi^L}{\pi^F},$$

which is continuous in any positive y and admits a unique root $y = \bar{y}_{01}^* \exp(-1/\beta)$ for all $\pi_{01} > \pi^L$. Then $\exp(-1/\beta) < 1$ for all $\beta > 1$ implies that $\bar{y}_{01}^* \exp(-1/\beta) < \bar{y}_{01}^*$. Therefore $\partial \bar{L}(\bar{y}_{01}^*)/\partial \beta = -I\left(\pi_{01} - \pi^L\right)/\left(\beta - 1\right)\pi^F < 0$ implies that $\partial \bar{L}(y)/\partial \beta > 0$ if and only if $y < \bar{y}_{01}^* \exp(-1/\beta)$. Then recall from above that $\mathrm{d}\beta/\mathrm{d}\sigma < 0$ to conclude. It remains to compare y_{00}^L with $\bar{y}_{01}^* \exp(-1/\beta)$. It is easy to check that $y_{00}^L < (=)\bar{y}_{01}^* \exp(-1/\beta)$ if and only if $\beta > (=) - 1/\ln\left(\pi^F/\pi_{01}\right)$, or equivalently $\pi^F < (=)\exp(-1/\beta)\pi_{01}$. $\square$
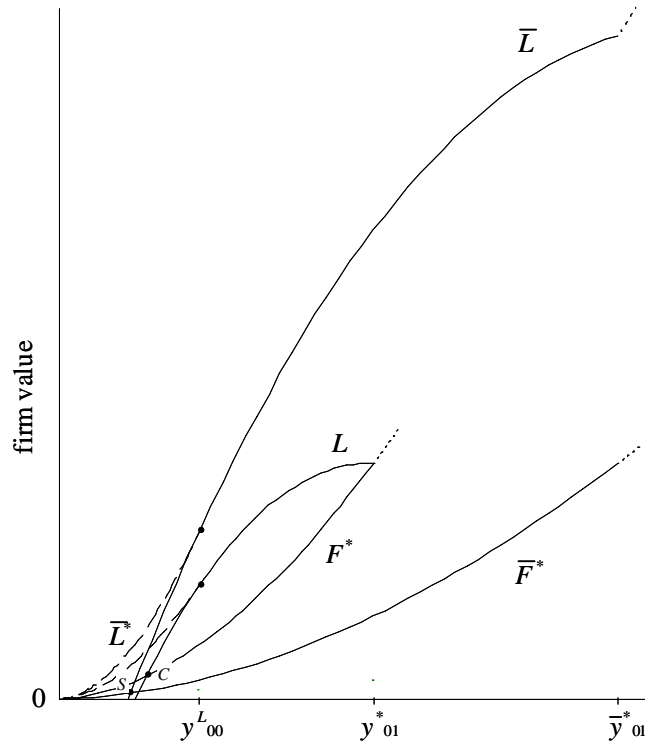
Figure 1: Firm values under alternative strategies with $P(X) = Y_t(1-X)$, $r = 1/2$, $\alpha = 1/4$, $\sigma = 1/4$, and $I = 1/10$. If entry can occur at any point in time with no delay, and a firm can commit to lead, it enters immediately before Y_t reaches y_{00}^L, in the Cournot and Stackelberg scenarios alike. With no commitment, each firm may lead with probability $1/2$. In the Cournot scenario (point C), in equilibrium the leader enters immediately before Y_t reaches y_{00}^p, the follower enters at y_{01}^*, firms' value is $L(y_{00}^p) = F^*(y_{00}^p)$. In the Stackelberg scenario (point S), in equilibrium the leader enters immediately before Y_t reaches $\bar{y}_{00}^p$, the follower enters at $\bar{y}_{01}^*$, firms' value is $\bar{L}(\bar{y}_{00}^p) = \bar{F}^*(\bar{y}_{00}^p)$.
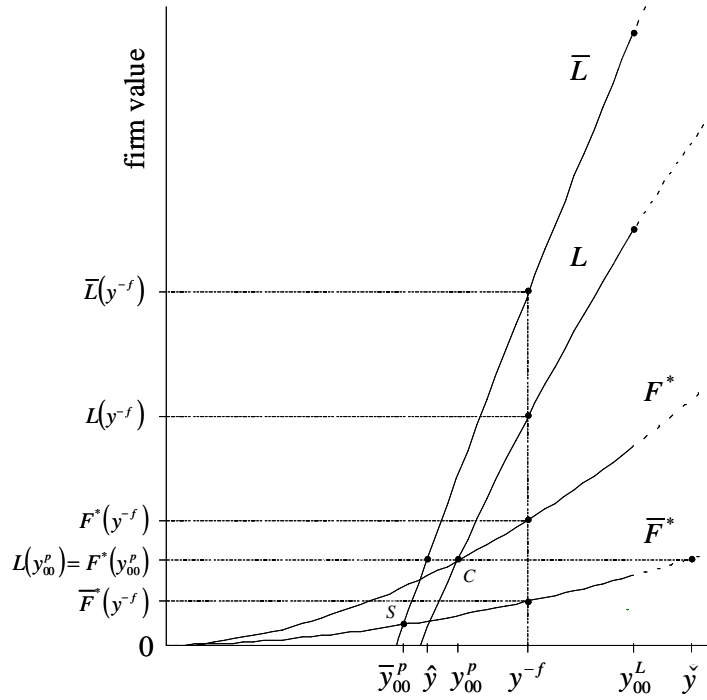
Figure 2: When $y^f < y^{-f}$, and $y_{00}^p < y^{-f}$, firm f leads by entering immediately before Y_t reaches y^{-f}. With no first-mover advantage on the market stage, the two firms' equal equilibrium value with perfect mobility (that is $F^*(y_{00}^p) = L(y_{00}^p)$) is lower than their respective values with limited mobility (i.e., $F^*(y^{-f})$ and $L(y^{-f})$) when $y_{00}^p < y^{-f} \leq \check{y}$. In the latter case, f benefits more than $-f$ from the delayed entry process ($F^*(y^{-f}) < L(y^{-f})$). In the Stackelberg scenario, f is better-off than in the Cournot case ($\bar{L}(y^{-f}) > L(y^{-f})$), while the follower is worse off ($\bar{F}^*(y^{-f}) < F^*(y^{-f})$).