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► **To cite this version:**

Elsa Martin, Hubert Stahn. ON THE COORDINATION OF THE EUROPEAN AGRI-ENVIRONMENTAL AND WATER INTERNALIZING POLICIES. 2009. halshs-00360993

**HAL Id: halshs-00360993**

**<https://shs.hal.science/halshs-00360993>**

Preprint submitted on 12 Feb 2009

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**Document de Travail  
n°2009-**

## **ON THE COORDINATION OF THE EUROPEAN AGRI-ENVIRONMENTAL AND WATER INTERNALIZING POLICIES**

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**January 2009**

**DT-GREQAM**

# On the coordination of the European agri-environmental and water internalizing policies

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January 6, 2009

## Abstract

The point of departure of this work is the lack of coordination of european environmental internalizing policies. At the national level, while the water authority generally has to internalize the negative externalities of water extraction, the agricultural one aims at encouraging environmentally friendly one. More locally, considering an aquifer as being the only vector of environmental effects, we show that the externalities occurring can compensate themselves in such a way that the open-loop Nash game played by the two distinct authorities in charge of these policies is inefficient. In this special case, we propose to implement a coordinated policy based on a double fiscal scheme also showed budget balanced.

*Keywords:* water policy, agricultural policy, externalities.

*J.E.L. classification numbers:* H23, Q18, Q28.

## 1. Introduction

The negative externalities of natural resources exploitation are a well documented problem in the economic literature and many internalizing instruments had been theoretically studied. Nowadays, these well-known instruments are also effectively used in the real world. For instance, in the European Union (EU), the concepts of "*full cost recovery*" had been introduced in the Water Framework Directive (WFD) of 2000<sup>1</sup>: this directive asks the member states to make sure that all externalities of water resources extraction are internalized. For this purpose, the recommendation to the national water authorities is to incorporate them into the price of the resource.

On the other hand, in 1998, the OECD Agriculture Ministers adopted the concept of multifunctionality as a policy principle; this concept "*recognizes that beyond its primary function of supplying food and fibre, agricultural activity can also shape the landscape, provide environmental benefits such as land conservation, the sustainable management of renewable natural resources and*

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<sup>1</sup>See European parliament (2000).

*the preservation of biodiversity, and contribute to the socioeconomic viability of many rural areas*" (OECD, 2001). It is why, among other things, in 2003, the CAP reform intensified environmental concerns in agricultural policies through proposing to internalize the environmental amenities linked with the agricultural activities.

At the member states level, these two sets of policies are implemented by two different authorities: one is in charge of the water sector and the other of the agricultural one. The lack of coordination between both when they internalize environmental externalities can induce inefficiencies. Dworak, Kranz, Karaczun and Herbke (2006) recommend to coordinate these authorities intervention. We are going to propose a model illustrating this view.

From the best of our knowledge, the seminal conceptual framework concerned with interactions between agricultural and environmental policies was proposed by Just and Antle (1990). They demonstrated that the agricultural policies are structured so that undesirable environmental effects can be mitigated if policies are appropriately designed and administrated. Johnson, Wolcott and Aradhyula (1990) supported this view by emphasizing that the programs structure and the institutional setting are important conditioning factors. Finally, during the same debate, Hrubovcak, Leblanc and Miranowski (1990) illustrated some of the difficulties of coordinating agricultural and environmental policies when altering agricultural chemical use.

But, as underlined by Just, Bust and Donoso (1991), these studies simply serve to *"better understand the interface of agricultural and resource policy"*. Following this point of view, Weinberg and Kling (1996) proposed to go further: they studied the opportunities to better coordinate the two types of policies. They concentrated on irrigation water policies in the West and policies for the control of drainage from irrigated agriculture. They specified a social objective in order to examine, both conceptually and empirically, the welfare losses associated with uncoordinated policy making. More recently, Gomez-Limon, Arriaza and Berbel (2002), within the framework of the European WFD and CAP policies, examine the relationships between these policies. Their empirical work, applied to a case study in central Spain, shows that the two policies must be coordinated in order to meet socioeconomic goals (farmers' income and labour demand) and environmental protection (water-use efficiency). Horan, Shortle and Abler (2004) studied the coordination and design of point-nonpoint trading programs and agri-environmental policies. They looked at the condition under which coordination of these policies can induce efficiency gains.

All these papers are primarily concerned with agricultural production externalities, these latter one possibly being either positive or negative. In our work, we want to concentrate on a less studied case: the one in which environmental amenities are jointly produced by the agricultural sector, the negative environmental externalities coming from another economic activity. Concentrating on an organic agricultural production, we will then look at the strategic interactions between a water authority and an agricultural one, both of them having environmental concerns in mind. The water authority will be in charge of the regulation of the resource extraction and the agricultural one of the amenities supplied by the organic production of food. Furthermore, taking the strong links between groundwater and agriculture as a point of departure, we will assume that all the environmental

effects pass through an aquifer. This will add some dynamic into the problem. Following Provencher and Burt's (1993) typology, we will consider two types of quantity externalities: the stock one will be related to the existence value of the aquifer reserve and the pumping cost one will capture the congestion effects of the withdrawals on the stock height. Since agricultural production is often blamed for its negative impact on water quality, we will also take into account a quality externality. In order to do so, we are going to partly follow Roseta-Palma (2002) who introduced for the first time both quantity and quality concerns into a groundwater management setting. She proposed to distinguish between a dilution effect depending on the groundwater stock size and a filter one linked with the farmers' irrigations. Since the aquifer considered here will only be replenished thanks to surface irrigations, we will assume that above an aquifer height threshold, when the stock increases, the irrigation water that is percolating is less filtered and the quality deteriorated.

More precisely, we are going to consider, on the one hand, a group of farmers producing food in a traditional organic way (using surface water conveyed thanks to gravity and with very few chemicals added) and selling it on a competitive market to consumers. This production will be assumed to strongly participate to an aquifer replenishment, thanks to percolations allowed by the irrigations<sup>2</sup>; for clarity sake, the relation between food production and percolations will be postulated linear. On the other hand, the groundwater hence generated will be assumed to be purchased on another competitive market by the consumers to a water distribution agency. We will then assume that an equilibrium with respect to the choice of the resource had already been reached: surface water is used for agricultural production and groundwater for domestic consumption. Such a scheme can induce several quantity and quality externalities. But in the real world, some aquifers are more concerned with quality problems and some other with quantity one. We will thus concentrate on the second type, focusing on a setting in which the externalities previously presented finally compensate themselves in such a way that only a discounting effect is remaining.

In such a case, the basic optimal intertemporal groundwater policy of a water authority consists in taxing the resource extractions in order to slow down the extractions of the resource otherwise subject to overexploitation: this corresponds to the European Union recommendations laying in the WFD. But because of some inefficiencies remaining, an agricultural policy can also be at work at the same time in order to internalize them, in such a way that it is as if both the authorities were playing an open-loop Nash game. Since this is the case within the framework of the CAP reform of 2003, we will refer to this situation as an European Uncoordinated Water and Agricultural Internalizing Policies (UWAIP) setting. According to our parameters values, we will see that in such a setting, the optimal intertemporal agricultural policy can either consists in taxing or subsidizing the food traditional organic production. But we will show that the implementation of such an UWAIP rarely coincides with the efficient solution. We will then propose to implement a coordinated policy consisting in taxing the groundwater consumption and always (whatever the set of parameters is) subsidizing the organic farming production in order to reach the socially-optimal level of surface

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<sup>2</sup>Note that this particular situation is not a pure fiction since it is frequent in most areas characterized by a Mediterranean climate. It is why Brown and Deacon (1972) proposed to take into account percolations when defining a pigovian tax on groundwater extractions.

irrigation water percolations to the aquifer located below. We will finally conclude by showing that in such a situation, it is always possible to finance the agri-environmental fiscal transfer with the money collected thanks to the groundwater taxation.

Our argument will be organized in the following way. We introduce in section 2 our dynamic model. Section 3 will be devoted to the characterization of the environmental externalities of food and groundwater consumption. In section 4, we will explain why the lack of coordination between water and agricultural internalizing policies can induce inefficiencies. Section 5 will be devoted to the coordinated policy being able to decentralize the efficient solution thanks to a double fiscal scheme. We will finally conclude and propose some potential extensions.

## 2. The model

Since we assume that all environmental external effects pass through an aquifer, its law of motion will represent the ecological system considered. The consumer's environmental preferences will thus be fully determined by this height.

Futhermore, we do not want to enter into the debate related to the conjunctive use of water resources. We thus assume that an equilibrium with respect to the choice of the water resource had already been reached: groundwater will be consumed for domestic uses and the agricultural production will be made possible only thanks to surface water irrigations.

### 2.1. The simplified ecological system

We consider a small closed economy composed of consumers and producers who are located above an aquifer. At time  $t$ , the first one consume an amount  $g_t$  of groundwater for domestic use (this water is assumed to be clean: it is drinkable without the need of any treatment) and an amount  $f_t$  of agricultural food. The latter one is assumed to be sold on a pure and perfect competitive market by homogeneous atomistics farmers and the first one by a distribution agency. Also considering homogeneous consumers, we will reason on a representative consumer and a representative producer.

The production of food generates a by-product: percolations of an amount  $bf_t$  of surface water to the aquifer. For clarity sake, a fixed linear relation between the amount of irrigated food produced and the aquifer height is assumed. Crops are supposed to be grown up in a traditional organic way.

The water table is described as a function of time which is obtained, as in most of the papers of the literature on the subject, by equating "rate in" minus "rate out" with the impact on the water table height,  $\dot{h}(t) := \dot{h}_t$  where  $h_t \in ]0, 1[ \forall t \neq 0$  is a percentage of height. The aquifer height motion is thus described by the following differential equation:

$$\dot{h}_t = bf_t - g_t \tag{2.1}$$

where  $b > 0$ .

We also assume that the initial groundwater height is equal to its natural hydrologic equilibrium:  $h_0 = 1$ . Rubio and Casino (2001) define this level as the one "*corresponding to the maximum water*

table elevation at which the water reserves coincide with the storage capacity of the aquifer". They added that with such an assumption, "the human activity, justified by economic parameters, consists of mining the aquifer until an economic hydrologic equilibrium has been reached". So, with such an assumption, the aquifer considered must face a quantity management problem and will never overflow.

## 2.2. The producers

Food and groundwater are the two market goods considered in our economy.

### 2.2.1. The representative groundwater distribution supplier

Drinkable water distribution is generally associated with strong and expensive networks of canals and pipes. The presence and the importance of the costs associated with such networks can be the origin of natural monopoly situations. It is why the water distribution is generally delegated by local or central governments to a private firm through an auction mechanism: the networks building investments are made by this authority (thanks to lump-sum taxes imposed to consumers for instance) and the private firm is in charge of operating and maintaining costs. The government then chooses the firm proposing the cheapest price.

Considering atomistics and homogeneous firms makes them compete on a pure and perfect market in order to win the water distribution task. But we do not want to model this mechanism here. We will rather focus on its outcome which is a groundwater unit price,  $\pi_t$ , equal to its marginal cost:

$$\pi_t(h_t) = \kappa(h_t)$$

where the marginal cost  $\kappa(h_t)$  is a decreasing function since a low height is associated with high energy needs and thus high pumping costs.

In order to fully characterize and compare the solutions of the optimal control problems that we are going to study, we need to specify all our functions. The total cost of groundwater distribution at time  $t$ ,  $\kappa(h_t)g_t$ , will be assumed to depend on both the amount of water extracted and on the pumping lift rate  $(1 - h_t)$ , in a linear way. The marginal cost of pumping groundwater will thus take the following form:

$$\kappa(h_t) := \kappa - h_t$$

It is thus assumed to be composed of:

- a fixed part  $\kappa > 0$  due to the hydrologic cone,
- another part negatively proportional to the groundwater height due to the energy needed in order to pump water.

Notice that  $\kappa$  corresponds to the aquifer height from which extraction costs are null.

### 2.2.2. The representative organic farmer

The organic agricultural production is assumed to be sold on a pure and perfect competitive market. The production costs,  $c(f_t)$ , are assumed proportional to the amount of food produced:

$$c(f_t) := cf_t$$

where  $c > 0$ . At the equilibrium of the market, the unit price of the food is equal to its marginal cost such that:

$$p_t = c$$

Note that since the marginal costs of this producer are equal to their average one, considering a pure and perfect competitive markets means that, at the equilibrium, his profits are null.

### 2.3. The representative consumer

The consumer is assumed to consume an amount  $f_t$  of food and  $g_t$  of groundwater at time  $t$ . For clarity sake, the utility generated from this consumption is assumed fully separable in both goods:  $u(f_t)$ , resp.  $v(g_t)$ , denotes the willingness to pay for food, resp. for groundwater consumption. Futhermore, the complete utility function will be written as:  $U(f_t, g_t, h_t) = u(f_t) + v(g_t) + e(h_t) + m(f_t, g_t, h_t)$  where:

- $m(f_t, g_t, h_t)$  is the amount of money remaining to the consumer after having bought groundwater and food;
- and  $e(h_t)$  is an indicator of this consumer's environmental preferences. These environmental concerns are assumed to be fully determined by the aquifer height,  $h_t$ .

Recalling that the market prices are set at their competitive level (given by the marginal costs of the goods), the surplus from food,  $s(f_t)$ , and groundwater,  $\sigma(g_t)$ , consumption are given by the following expressions:

$$\begin{aligned} s(f_t) &= u(f_t) - cf_t \\ \sigma(g_t, h_t) &= v(g_t) - \kappa(h_t)g_t \end{aligned}$$

More specifically, the utility from the consumption of both goods will be assumed quadratic (and thus concave):  $v(g_t) := \alpha g_t - \frac{1}{2}g_t^2$  and  $u(f_t) := af_t - \frac{1}{2}f_t^2$  where  $\alpha, a > 0$  and quasi-linear in money.<sup>3</sup> Hence, if the consumer is endowed with a constant budget  $B^4$  through time, the amount of money that is remaining for the other goods when food and water have been consumed is:

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<sup>3</sup>Note that since these functions are increasing with the first units and decreasing with the last one the position of the maximum is of importance. It is why it would have been more correct to rather reason on the functions  $v(g_t) := \text{Max} \{ \alpha g_t - \frac{1}{2}g_t^2, \frac{1}{2}\alpha^2 \}$  and  $u(f_t) := \text{Max} \{ af_t - \frac{1}{2}f_t^2, \frac{1}{2}a^2 \}$ . But since parameters  $a$  and  $\alpha$  can always be chosen sufficiently high in order to check the "good position" of the maximum, we will work on the simpler functions proposed in the text.

<sup>4</sup>We can consider that the building costs needed by the government to construct the groundwater distribution networks are deducted from this amount, through a lump-sum tax for instance.



$m(f_t, g_t, h_t) := B - cf_t - \kappa(h_t)g_t$ . For clarity sake and because of not modifying our results, the budget variable  $B$  will be omitted. The consumption of food generating the maximum amount of surplus,  $s(f_t)$ , is assumed strictly positive:  $d := a - c > 0$ . Furthermore, when the aquifer tends to be empty ( $h_t \rightarrow 0$ ), the consumer must be able to use another type of water resource, the demand for groundwater hence going to zero. Thus, even if this possibility is outside the scope of this work, it is reasonable to assume that  $\alpha - \kappa := 0$ .

We also announced that our consumer worries about the environment and that these concerns are fully represented by the aquifer height. On the one hand, we assume that the groundwater quality can get deteriorated when the stock height is increasing: a higher stock means that the surface water which is infiltrating to the ground is less filtered. Note that this is not true for every heights of the stock. So, we are going to assume that this phenomena is occurring when the groundwater stock level is higher than a threshold such that  $0 < \bar{h} < 1$ . But when it is lower than this height, the groundwater can be considered as becoming physically scarce, also inducing a disutility to the consumer (which can be linked with an existence value for instance). So it is as if there was a physical threshold of the groundwater stock height,  $\bar{h}$ , inducing no disutility to the consumer. These environmental concerns are captured by the function  $e(h_t)$  which properties can be summed up as:

- $\frac{de(h_t)}{dh_t} > 0$  for  $h_t < \bar{h}$ ,
- $\frac{de(h_t)}{dh_t} < 0$  for  $h_t > \bar{h}$ .

It is why we decided to specify this function as:  $e(h_t) := -\frac{1}{2}(h_t - \bar{h})^2$ . Note that this function captures non-marketable environmental effects.

Finally, the expressions used in order to fully characterize and compare our dynamic solutions will be the following one:

$$\begin{aligned} U(f_t, g_t, h_t) &= -\frac{1}{2}g_t^2 + h_tg_t + df_t - \frac{1}{2}f_t^2 - \frac{1}{2}(h_t - \bar{h})^2 \\ \sigma(g_t, h_t) &= -\frac{1}{2}g_t^2 + h_tg_t \\ s(f_t) &= df_t - \frac{1}{2}f_t^2 \end{aligned}$$

### 3. The environmental dynamic externalities of food and groundwater consumption

In the myopic competitive case, the consumer does not take into account the impact of his consumption choices on the hydrologic system. This leads to inefficiencies with respect to a central planner program. We are going to check this after having defined the myopic competitive and the efficient solutions.

**Definition 3.1.** *The myopic competitive equilibrium paths of groundwater and food consumption  $\{g_t^m(h_t), f_t^m\}$  are given by the maximization of the consumer's utility at each time  $t$ :*

$$\{g_t^m(h_t), f_t^m\} \in \arg \max_{f_t, g_t} U(f_t, g_t, h_t)$$

The necessary FOC (First Order Conditions) of this problem give the short-run demand functions for groundwater and food:

$$\begin{aligned} \frac{dv(g_t)}{dg_t} &= \kappa(h_t) \\ \frac{du(f_t)}{df_t} &= c \end{aligned}$$

These equations illustrate a classical result according to which, at each time  $t$ , the myopic competitive allocation of groundwater and food are such that the private marginal utility of each one consumption equals its private marginal cost.

**Proposition 3.2.** *Within the framework of our specific functional forms<sup>5</sup>, the myopic competitive intertemporal paths can be written as:*

$$\begin{aligned} h_t^m &= g_t^m = h_e^m + (1 - h_e^m) e^{-t} \quad \forall t \\ f_t^m &= f_e^m = d \quad \forall t \end{aligned}$$

where the aquifer height at the steady state is denoted  $h_e^m = bd$ .

At the steady state of this myopic competitive setting, which is denoted by  $e$  and for which  $t \rightarrow \infty$ , we observe that the amount of groundwater consumed is equal to the by-product maximizing the food utility function. Recalling that the initial groundwater height is always higher than the steady state one since it is equal to unity, we have that:

- the aquifer stock is decreasing along time and so is the amount of groundwater consumed;
- the myopic competitive demand of food is the same one at each time and so are the percolations to the aquifer induced by this food production.

We now move to the characterization of an efficient path of consumption. In such a setting, it is as if a central planner was choosing the optimal paths of food and groundwater to consume, knowing perfectly the impacts of these consumptions on the aquifer height.

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<sup>5</sup>Since we focus on the interior solutions, we implicitly restrict our parameters in such a way that  $bd < 1$ . Furthermore, if parameters  $b$  and  $d$  are too small, this can lead to a negative utility because of the externalities hence becoming too severe.

**Definition 3.3.** The optimal paths of groundwater and food consumption  $\{g_t^*(h_t), f_t^*(h_t)\}$  are given by the maximization of the present value of the consumer's utility stream:

$$\begin{aligned} & \max_{f_t, g_t} \int_0^{\infty} U(f_t, g_t, h_t) e^{-rt} dt \\ \text{s.t.} \quad & \dot{h}_t = bf_t - g_t \text{ with } h_0 = 1 \end{aligned}$$

where  $0 < r < 1$  is the discount factor.

In order to solve this problem, we are going to use the maximum principle. Let  $H$  denotes the current value Hamiltonian of this problem:

$$H(h_t, g_t, f_t, \mu_t) = v(g_t) - \kappa(h_t)g_t + u(f_t) - cf_t + e(h_t) + \mu_t(bf_t - g_t)$$

The efficient solution thus satisfies the conditions:

$$\begin{aligned} \frac{dv(g_t)}{dg_t} = \kappa(h_t) + \mu_t & \quad \dot{h}_t = bf_t - g_t \\ \frac{du(f_t)}{df_t} + b\mu_t = c & \quad \frac{\dot{\mu}_t}{\mu_t} = r + \frac{d\kappa(h_t)}{dh_t} \frac{g_t}{\mu_t} - \frac{de(h_t)}{dh_t} \frac{1}{\mu_t} \end{aligned}$$

which, along with the following transversality condition, are necessary:

$$\lim_{t \rightarrow \infty} e^{-rt} \mu_t = 0$$

The shadow price of the groundwater stock,  $\mu_t$ , reflects the opportunity costs (resp. benefits) associated with the unavailability (resp. availability) in the future of any unit of water consumed (resp. which is percolating) in the present. Its rate of variation along time,  $\frac{\dot{\mu}_t}{\mu_t}$ , reflects the externalities that a myopic competitive solution fails to internalize with respect to a central planner solution:

- the pumping cost externality lies in this rate decrease with the consumption of groundwater:  $\frac{d\kappa(h_t)}{dh_t} \frac{g_t}{\mu_t}$ ,
- the quality externality occurs when the aquifer height is higher than  $\bar{h}$  and lies in the rate increase with  $-\frac{de(h_t)}{dh_t} \frac{1}{\mu_t}$ ,
- the stock externality occurs when the aquifer height is lower than  $\bar{h}$  (when it becomes physically scarce) and lies in the rate decrease with  $-\frac{de(h_t)}{dh_t} \frac{1}{\mu_t}$ .

Note that there is a market for the pumping cost externality, through  $\kappa(h_t)$ , but not for the quality and stock one since  $e(h_t)$  represents an environmental preference.

**Proposition 3.4.** Using our specific functional forms<sup>6</sup>, the efficient intertemporal paths can be written as:

$$\begin{aligned} \mu_t^* &= \frac{\bar{h}}{r+1}, \quad f_t^* = d + \frac{b\bar{h}}{r+1} \quad \forall t \\ h_t^* &= h_e^* + (1 - h_e^*) e^{-t} \\ g_t^* &= g_e^* + (1 - h_e^*) e^{-t} \end{aligned}$$

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<sup>6</sup>We need to impose the following restriction on our parameters in order to avoid that the aquifer overflows:  $d < \frac{r+1-\bar{h}(b^2+1)}{b(r+1)}$ . Notice that this condition is stronger than the one needed in the myopic competitive case,  $bd < 1$ .

where  $g_e^* = bd + \frac{\bar{h}b^2}{r+1}$  and  $h_e^* = bd + \frac{\bar{h}(b^2+1)}{r+1}$ .

We can then directly deduce from the previous elements that:

- the shadow price of the groundwater stock is positive and constant along time which means that our specifications make the three previous externalities compensate themselves in such a way that only a discounting effect proportional to the physical aquifer height inducing no disutility to the consumer, i.e.  $\bar{h}$ , is remaining;
- the efficient amount of food consumed is the same one at each period of time and it is increasing with the shadow price of the groundwater stock, in a way proportional to the percolation coefficient,
- the efficient paths of groundwater consumption and aquifer height are decreasing along time and the efficient paths of groundwater consumed is related to the shadow price of the aquifer in a negative way: the resource extraction is slowed down with respect to the myopic competitive case.

It is because of the form of the shadow price,  $\mu_t^*$ , that we can say that the aquifer considered here is more facing quantity problems than quality one. This is consistent with the two previous assumptions according to which: (i) the aquifer is only replenished thanks to traditional organic agriculture (no rainfall) and (ii) the human activity, justified by economic parameters, consists of mining the aquifer until an economic hydrologic equilibrium has been reached ( $h_0 = 1$ ).

In order to check the inefficiency of the myopic competitive solution, we are now going to compare it with the efficient one.

**Proposition 3.5.** *The comparisons of the efficient and myopic competitive intertemporal solutions tell us that:*

- (i) *the efficient aquifer height is always higher,*
- (ii) *the efficient volume of groundwater consumed can also be higher after a transition phase during which it is lower,*
- (iii) *the efficient amount of food consumed is always higher,*
- (iv) *all the differences are increasing along time except the one in food consumption which is temporally constant.*

The phenomena lying in (ii) is due to the fact that the amount of food consumed (and hence the percolations to the aquifer) is also controlled in the efficient case, making it possible to consume higher amounts of groundwater without reducing the aquifer height. Notice also that the date of the end of this transition phase,  $t = -\ln \frac{b^2}{1+b^2}$ , is increasing with the percolation coefficient.

All these results strongly depend on the fact that only a discounting externality is remaining in our setting.

#### 4. The lack of coordination between water and agricultural internalizing policies

National policies aiming at internalizing food and groundwater consumption externalities as the one identified in the previous section are generally delegated to different administrations without any coordination between both. In our setting, we can think of a water authority and an agricultural one playing an open-loop Nash game: they commit themselves at the moment of starting to an entire temporal path of groundwater consumption for the first one and of food consumption for the second one, each one maximizing the present value of the consumer's surplus stream with respect to each good, the consumption path chosen by the other being given and incorporated into the groundwater height motion.

These authorities will also take into account the environmental concerns of the consumers. But according to the weight put by each one of these administrations on these preferences, the inefficiencies due to the lack of coordination between both will differ. We are going to focus on these inefficiencies.

##### 4.1. The general case of Uncoordinated Water and Agricultural Internalizing Policies (UWAIP)

In the more general case, the agricultural authority puts a weight  $0 \leq w \leq 1$  on environmental concerns and the water one another one denoted  $0 \leq \omega \leq 1$ . Why would they put different weights on these concerns? Precisely because they do not coordinate on this goal.

**Definition 4.1.** *The equilibrium paths of groundwater and food consumption  $\{g^{ol}(h_t), f^{ol}(h_t)\}$  in an UWAIP setting are given by the simultaneous maximizations of the present value of the consumer's surplus with respect to (i) food and to (ii) groundwater consumption, both programs taking into account the consumer's preferences for environmental concerns in a weighted way:*

(i)

$$\max_{f_t} \int_0^{\infty} [s(f_t) + we(h_t)] e^{-rt} dt$$

$$s.t. \quad : \quad \dot{h}_t = bf_t - \bar{g}_t \text{ with } h_0 = 1$$

where  $\bar{g}_t$  is the optimal groundwater consumption path chosen by the water authority;

(ii)

$$\max_{g_t} \int_0^{\infty} [\sigma(g_t, h_t) + \omega e(h_t)] e^{-rt} dt$$

$$s.t. \quad : \quad \dot{h}_t = b\bar{f}_t - g_t \text{ with } h_0 = 1$$

where  $\bar{f}_t$  is the optimal food consumption path chosen by the agricultural authority.

This double problem can be solved using the maximum principle; it admits two current value Hamiltonians:

$$\begin{aligned} H_f(h_t, f_t, p_t) &= s(f_t) + we(h_t) + p_t(bf_t - \bar{g}_t) \\ H_g(h_t, g_t, \lambda_t) &= \sigma(g_t, h_t) + \omega e(h_t) + \lambda_t(b\bar{f}_t - g_t) \end{aligned}$$

where  $\lambda_t$  is the shadow price of the aquifer taking into account by the water authority and  $p_t$  the one by the agricultural authority. The necessary conditions are the following one:

$$\begin{aligned} \frac{dv(g_t)}{dg_t} &= \kappa(h_t) + \lambda_t & \frac{du(f_t)}{df_t} + bp_t &= c \\ \dot{h}_t &= bf_t - g_t \\ \frac{\dot{\lambda}_t}{\lambda_t} &= r + \frac{d\kappa(h_t)}{dh_t} \frac{g_t}{\lambda_t} - \frac{de(h_t)}{dh_t} \frac{\omega}{\lambda_t} & \frac{\dot{p}_t}{p_t} &= r - \frac{de(h_t)}{dh_t} \frac{w}{p_t} \\ \lim_{t \rightarrow \infty} e^{-rt} \lambda_t &= 0 & \lim_{t \rightarrow \infty} e^{-rt} p_t &= 0 \end{aligned}$$

The main difference between this UWAIP case and the efficient one lies in the presence of two shadow prices. We can furthermore notice that the rate of variation along the time of the shadow price taking into account by the water administration is the same one as in the efficient case. But it is not true for the one considered by the agricultural authority since it only concentrates on non-market effects, through  $e(h_t)$ , and not on the market one linked with the pumping costs.

**Proposition 4.2.** *Using our specifications<sup>7</sup>, the uncoordinated intertemporal paths can be written as:*

$$\begin{aligned} f_t^{ol} &= f_e^{ol} + \frac{bw(1-h_e^{ol})}{\rho-r} e^{\rho t} & g_t^{ol} &= g_e^{ol} + \frac{(1-h_e^{ol})(r-\rho+\omega)}{r+1-\rho} e^{\rho t} \\ h_t^{ol} &= h_e^{ol} + (1-h_e^{ol}) e^{\rho t} \\ p_t^{ol} &= p_e^{ol} + \frac{w(1-h_e^{ol})}{\rho-r} e^{\rho t} & \lambda_t^{ol} &= \lambda_e^{ol} + \frac{(1-\omega)(1-h_e^{ol})}{r+1-\rho} e^{\rho t} \end{aligned}$$

where  $g_e^{ol} = \frac{br[d(r+\omega)+b\bar{h}w]}{wb^2(r+1)+r(r+\omega)}$ ,  $f_e^{ol} = \frac{r[d(r+\omega)+b\bar{h}w]}{wb^2(r+1)+r(r+\omega)}$ ,  $h_e^{ol} = \frac{b(r+1)(dr+\bar{h}wb)+r\bar{h}\omega}{wb^2(r+1)+r(r+\omega)}$ ,  $p_e^{ol} = \frac{w[-bd(r+1)+r\bar{h}]}{wb^2(r+1)+r(r+\omega)}$ ,  $\lambda_e^{ol} = \frac{rbd(1-\omega)+\bar{h}(wb^2+\omega r)}{wb^2(r+1)+r(r+\omega)}$  and  $\rho < 0$ .

It is first interesting to note that the shadow price of the groundwater stock taken into account by the water authority is always positive in this general UWAIP setting. This means that she always (whatever the weights are) gives incentives to reduce the amount of groundwater consumed, with respect to a myopic competitive solution.

Furthermore, it is easy to check that the amount of groundwater consumed and the aquifer height are decreasing along time. On the contrary, the amount of food consumed is increasing along time; the shadow price taken into by the agricultural policy can be either positive or negative, hence possibly inducing either a decrease or an increase of food consumption with respect of the myopic competitive case.

All our results are summed up in the synoptic table of results presented in figure E.1 (see the appendix).

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<sup>7</sup>When  $p_t^{ol} < 0$ , since the agricultural authority has no incentive to slow down the decrease of food consumption, we have to restrict our parameters such that  $f_e^{ol}(r-\rho) > bw(1-h_e^{ol})$  in order to check that this consumption is positive all along the path. Finally, we also need to restrict our parameters values in order to insure that the aquifer never overflow by imposing  $h_e^{ol} < 1$ .

## 4.2. The European case after the Water Framework Directive (WFD) of 2000

In practice, the over-extraction of water resources is a widely known problem. It is why, in 2000, the European Union proposed a common framework to its members states concerning their water resource management: the Water Framework Directive. Among others, this directive is asking the states to make sure that water is sold at its real value, including the non-market one. Following this law, a national authority in charge of the water taxation policy concentrates on groundwater consumption surplus and environmental preferences.

The level of consumption of food is thus considered as being fixed at the myopic competitive one. But this authority is perfectly able to know the amount of percolations to the aquifer induced by this consumption. So the problem to solve is a particular case of the one stated in definition 4.1, with  $(w, \omega) = (0, 1)$ .

Proposition 4.2 directly gives us the stable intertemporal WFD paths leading to the stationary equilibrium  $(g_e^\omega, h_e^\omega, \lambda_e^\omega)$  as:

$$\begin{aligned}\lambda_t^\omega &= \frac{\bar{h}}{r+1} \quad \forall t \\ h_t^\omega &= h_e^\omega + (1 - h_e^\omega) e^{-t} \\ g_t^\omega &= g_e^\omega + (1 - h_e^\omega) e^{-t}\end{aligned}$$

where  $g_e^\omega = bd$  and  $h_e^\omega = bd + \frac{\bar{h}}{r+1}$ .

We observe that in a WFD setting, the shadow price of the aquifer is constant along time and equal to the efficient one since the same discounting effect is remaining.

**Proposition 4.3.** *The comparison of these WFD paths and steady states,*

- (i) *with the myopic competitive one, tells us that they are always higher or equal,*
- (ii) *and with the efficient one, that they are always lower.*

So, even if the WFD can induce a welfare increase with respect to a myopic competitive situation, it is always worse than an efficient solution, as defined previously. Furthermore, the proof of proposition 4.3 tells us that, at the WFD and myopic competitive steady states, the consumptions are the same one although the aquifer height are different. This comes from the fact that, at each time before the steady state, the amount of groundwater consumed in the myopic competitive setting is strictly higher than the one consumed in the WFD setting.

## 4.3. A new framework: the second pillar of the CAP

In practice, the agricultural sector is often blamed for the inefficiencies remaining. It is why, among other reasons, in 2003, the European Common Agricultural Policy has moved to recommendations focusing more on environmental aspects of the agricultural production.

So the European context is quite different from the previous isolated WFD one which constitutes a very particular case of our general UWAIP setting since no game is really played because only

the water authority is intervening. Here, we are going to add a national agricultural authority intervention to the water one and a game is now going to be played between both. Each one is going to take into account the full environmental preferences of the consumers:  $(w, \omega) = (1, 1)$ . We will refer to such a case as the European UWAIP one.

We then directly deduce the paths leading to the new stationary equilibrium  $(g_e^{\omega w}, h_e^{\omega w}, \lambda_e^{\omega w}, p_e^{\omega w})$  characterizing the European uncoordinated setting as:

$$\begin{aligned} f_t^{\omega w} &= f_e^{\omega w} + \frac{b(1-h_e^{\omega w})}{\phi-r} e^{\phi t} & g_t^{\omega w} &= g_e^{\omega w} + (1-h_e^{\omega w}) e^{\phi t} \\ h_t^{\omega w} &= h_e^{\omega w} + (1-h_e^{\omega w}) e^{\phi t} \\ p_t^{\omega w} &= p_e^{\omega w} + \frac{1-h_e^{\omega w}}{\phi-r} e^{\phi t} & \lambda_t^{\omega w} &= \frac{\bar{h}}{r+1} \quad \forall t \end{aligned}$$

where  $g_e^{\omega w} = \frac{br[d(r+1)+b\bar{h}]}{(r+1)(r+b^2)}$ ,  $f_e^{\omega w} = \frac{r[d(r+1)+b\bar{h}]}{(r+1)(r+b^2)}$ ,  $h_e^{\omega w} = \frac{b(r+1)(dr+b\bar{h})+r\bar{h}}{(r+1)(r+b^2)}$ ,  $p_e^{\omega w} = \frac{-bd(r+1)+r\bar{h}}{(r+1)(r+b^2)}$  and  $\phi = -\frac{1}{2} + \frac{1}{2}r - \frac{1}{2}\sqrt{1+2r+r^2+4b^2}$ .

In such an European UWAIP setting, the shadow prices of the aquifer taken into account by the water and by the agricultural administrations differ in such a way that:

- the one of the water authority is constant along time and equal to the efficient one (and consequently to the one corresponding to the WFD setting) since only the negative discounting effect of myopic competitive groundwater withdrawals is internalized by this authority,
- the one of the agricultural authority evolves along time according to the aquifer height, in a way strongly dependant on the value of  $\bar{h}$  since the agricultural authority internalizes the positive or negative (according to the set of parameters) quality and stock externalities but not the pumping cost one.

**Proposition 4.4.** (i) *In an European UWAIP setting, the shadow price of the aquifer taken into account by the agricultural authority is always lower than the one of the efficient case. It can furthermore either be positive or negative since, depending on the parameters values, the quality externality can be considered as being more important than the stock one, or the opposite.*  
(ii) *The amount of food consumed in an European UWAIP setting can either be lower or higher than the one obtained in a WFD setting, depending on the sign of  $p_e^{\omega w}$ . But it is always lower than the efficient one.*

Because of the exponential functions, comparisons of the groundwater consumption and aquifer height equilibrium paths between the European UWAIP settings and the one previously obtained are less obvious. We thus propose to further work on numerical examples. Considering the more frequent situation in which  $p_t^{ol} < 0 \quad \forall t$ , the simulations example stated in appendix H tells us that there are some parameters sets for which the European UWAIP setting is worse than the WFD one, all along the paths. Furthermore, appendix I shows that the UWAIP setting equilibria are lower than the one of the efficient setting at each time if the shadow price of the agricultural authority is negative and that the two solutions becomes very similar if the set of parameters induces a possible



positivity of this shadow price. But if we recall that the amount of food consumed in an UWAIP setting is always lower than the efficient one, we can conclude that the UWAIP setting is always inefficient. This conclusion can be checked by computing the Net Present Value (NPV) of future utility stream in the two case:  $NPV^* - NPV^{ol} = 5,01$  for the set of parameters for which  $p_t^{ol} < 0 \forall t$  and  $NPV^* - NPV^{ol} = 0,001$  for the other set.<sup>8</sup>

#### 4.4. Where do the inefficiencies come from?

We shew that a European UWAIP setting is inefficient. The main question here is to know why. In order to answer, we are going to check if other special cases of UWAIP are also inefficient. The two other likely settings are:

(i) the one where only the agricultural authority is internalizing the quality and the stock externalities:  $(w, \omega) = (1, 0)$ , denoted  $w$ ;

(ii) and the one where both authorities internalize them in equal share:  $(w, \omega) = (\frac{1}{2}, \frac{1}{2})$ , denoted  $es$ .

Since the comparisons with the myopic and efficient cases are as difficult as in the European UWAIP setting (because of the exponential function), we will directly work on simulations. For this purpose, we will run the NPV and compare them in the efficient, the myopic and the various UWAIP cases for different sets of parameters<sup>9</sup>. Values are summed up in appendix J.

The main result is that any of the UWAIP cases studied is efficient although UWAIP settings are welfare improving with respect to the myopic competitive case. The exception pointed out by our simulation is the case (i): the NPV can then be even worse than the myopic competitive one, specially for a low value of the aquifer height threshold,  $\bar{h}$ . The sensibility of this case with respect to this threshold is special since we also observe that all the differences increase with  $\bar{h}$  except the one related to this case: they are decreasing for the set of parameters such that  $p_t^{ol} < 0 \forall t$  and there is no clear trend for the other set.

The sensibility of case (i) to the aquifer height threshold shows us that the inefficiencies do not only come from the lack of coordination of the authorities on stock and quality externalities. The pumping cost one also plays a crucial role. Indeed, in this case where only the agricultural authority is internalizing the non-marketable externalities, if, for instance, she has to give incentive to consume more food in order to increase aquifer replenishment and hence height, the water authority will refrain groundwater consumption fewer because of only internalizing pumping cost externalities. And the aquifer height will remain low. The two authorities will then make incentives in opposite ways because of pursuing conflicting goals.

In other words, the inefficiencies of UWAIP do not only come from the lack of coordination of agricultural and water administrations on environmental goals,  $e(h_t)$ , but also from the fact that

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<sup>8</sup>The reader can be surprised of these very low values but it is due to the scale chosen: the NPV are also very low.

<sup>9</sup>Note that the set of parameters inducing an interior solution will be different from the one previously chosen since the condition insuring that the aquifer never overflows becomes stronger in the  $w$  case than in the efficient one. Futhermore, with the new set of parameters chosen, the real eigenvalue  $\rho$  is going to possibly become complex because of the square root it contains. But the imaginary parts will be neglected because of being next to zero.

an authority in charge of the agricultural policy generally do not aim at internalizing marketable externalities (like the pumping cost one) which are under the jurisdiction of another authority: the water one. We explained that in the efficient case, the various externalities identified in our model compensate themselves in such a way that only what we called a discounting effect is remaining. But if various authorities aim at internalizing each of them without taking into account this compensation phenomena, the output can not be efficient.

## 5. A coordinated budget-balanced fiscal internalizing policy: groundwater taxation and food subsidy

The question that we want to answer now is: how to implement an efficient solution in such a setting? We are going to concentrate in this section on the main internalizing instruments used in Europe: fiscal one.

Theoretically, the implementation of the European UWAIP solution would consist in a double pigovian transfer:

- a first part implemented by the water authority: a tax constant along time,  $T^{\omega w} = \lambda_e^{\omega w}$ , imposed on every unit of groundwater consumed at each time,
- a second one by the agricultural authority: a transfer for every unit of food consumed,  $S_t^{\omega w} = bp_t^{\omega w}$ , evolving along time and possibly being either positive or negative depending on the shadow price of the resource, i.e. it can be either a tax or a subsidy. Note that this transfer is proportional to the percolation coefficient because it is the by-product of this food which needs to be internalized.

But in the real world, it would be very difficult to make such a theoretical fiscal scheme acceptable by the consumers since it is evolving at each period of time according to the aquifer height, possibly being either positive or negative. A stationary one would be more convenient and hence more realistic. So, more practically, we can think of the same tax implemented by the water authority and a stationary transfer,  $S^{\omega w} = bp_e^{\omega w}$ , on each every amount of agricultural food consumed.

**Proposition 5.1.** (i) *The implementation of a theoretical  $(T^{\omega w}, S_t^{\omega w})$  and of a practical  $(T^{\omega w}, S^{\omega w})$  fiscal scheme on food and groundwater consumptions both lead the myopic competitive steady state consumptions of these market goods to the stationary equilibrium characterizing an European UWAIP setting. The paths leading to this equilibrium within the framework of a theoretical scheme implementation follows the paths characterizing an European UWAIP. It is not the case for the paths induced by the practical scheme implementation.*

(ii) *But these latter one share the same properties as the one simulated in the European UWAIP: they can induce a better or a worse solution than in the WFD setting and even than in the myopic competitive one according to the set of parameters.*

(iii) *Whatever the set of parameters is, the uncoordinated implementation of a double stationary pigovian fiscal scheme is never efficient.*

More particularly, the proof of (ii), which is more general than the simulation examples previously taken for the European UWAIP, tells us that the paths of this implementation scheme are characterized by the following properties:

- if the set of parameters is such that the agricultural authority concentrates on the stock externality,  $p_e^{\omega w} > 0$ , a double fiscal scheme, even if implemented in an uncoordinated way, is always better than if a simple fiscal scheme on groundwater consumption had been implemented in an isolated WFD setting and thus than the myopic competitive solution;
- if the set of parameters is such that  $p_e^{\omega w} < 0$ , the lack of coordination between the policies is always worse than an isolated WFD setting and it can even also always<sup>10</sup> be the case than within a myopic competitive setting since the agricultural authority concentrates on a quality externality (which can only be internalized thanks to an aquifer height decrease) although the aquifer is facing quantity problems.

The question is now to study how to restore efficiency as characterized in definition 3.3. We propose to implement a distortionary fiscal scheme on food and groundwater consumptions. Such a policy would be very easy to implement in our setting since the scheme would be constant along time because of not being related to the groundwater stock: it is given by the aquifer shadow price:  $\mu_t^* = \mu_e^* \forall t$ . It should take the form of a fixed tax along time  $T^* = \mu_e^*$  imposed on each unit of groundwater consumed and a fixed subsidy along time  $S^* = b\mu_e^*$  on each unit of food consumed.

**Proposition 5.2.** *The introduction of a pigovian fiscal scheme  $(T^*, S^*)$  in the myopic competitive program leads to the optimal steady state, on the efficient path. Furthermore, a central government would always have enough money in order to implement this double fiscal scheme, at the steady state but also along the path. The lump-sum transfer that she has to distribute to the consumers at each time  $t$  during the transition phase to the steady state in order to be budget-balanced is:  $\mu_e^* (1 - h_e^{*i}) e^{-t}$ .*

## 6. Conclusion

When an aquifer is replenished thanks to an organic agricultural activity and that the groundwater is consumed in order to satisfy domestic uses, many externalities can occur: pumping cost, stock and quality one. Internalizing them can induce either an increase of the aquifer height or a decrease. Within the framework of our model, compensation phenomena occur between them in such a way that only a discounting effect remains. The absence of intervention can lead to the so-called groundwater over-exploitation.

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<sup>10</sup>It is true for the aquifer height when the parameters also check:  $b^2 p_e^{\omega w} + \frac{\bar{h}(b^2+r)}{(r+1)(r+b^2)} < 0$ .

The European Water Framework Directive precisely consists in avoiding such an output through taxing the groundwater consumptions to their real economic value. But we shew in proposition 4.3 that, in our setting, some inefficiencies are still remaining after the implementation of such a policy by a water authority. In practice, the agricultural authorities are also asked by the European Commission to take these inefficiencies into account when defining their policy guidelines. We then defined such a situation as an European UWAIP one because of the water and agricultural authorities playing an open-loop Nash game. Note that in such a setting, the groundwater consumption is still taxed and the food one can either be taxed or subsidized according to the parameters set.

In the special case illustrated by our model, we shew that neither the implementation of a European WFD setting nor the one of the European UWAIP one leads to an efficient solution. It is why we proposed to think of a coordinated policy consisting in taxing the groundwater consumption and always subsidizing the food one. Proposition 5.2 furthermore tells us that a central planner implementing such a double fiscal scheme would always have enough money to do it: the amount collected always compensates the one redistributed.

The sensibility of dynamic problems to the discount rate and to the initial conditions is widely known. It is why we decided to fix them: the initial aquifer height had been assumed to be at its maximum value as possible and the discount rate at the value proposed by the European Commission. More research is needed in order to study how our conclusions could differ according to these parameters values.

In order to implement the efficient solution, we proposed a distortionary fiscal scheme on food and groundwater consumption. Since we considered a closed economy, we could have proposed to implement the same fiscal scheme on the production of these goods. The results would have been the same. But it would also have been interesting to work in an open economy setting<sup>11</sup> and to look at the global implications of such a scheme. Furthermore, we know that water distribution is rarely made on a competitive market in the real world since our auction mechanism is often biased by some lobbying phenomena. It would also be interesting to incorporate such political economics issues in our model.

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<sup>11</sup>See Peterson, Boisvert and de Gorter (2002).

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## APPENDIX

## A. Proof of proposition 3.2

In order to fully characterize the solution, we solve the problem stated in definition 3.1 by using our functional forms. The necessary FOC, which are also sufficient, give us the short-run demand functions for groundwater and food as:

$$\begin{aligned} g_t^m &= h_t^m \\ f_t^m &= d \end{aligned}$$

We then need to study the long-run demand functions: the paths which are leading to the myopic competitive steady state  $(f_e^m, g_e^m, h_e^m)$ , characterized by  $\dot{h}_t^m = 0$ . It thus remains to solve, with respect to  $h_t$ , the following differential equation governing the change in the water table over time in the myopic competitive setting :

$$\dot{h}_t^m = bd - h_t^m$$

Thanks to a standard computation and recalling that we assumed that  $h_0 = 1$ , the myopic competitive intertemporal optimal path can then be written as:

$$\begin{aligned} h_t^m &= g_t^m = h_e^m + (1 - h_e^m) e^{-t} \quad \forall t \\ f_t^m &= f_e^m = d \quad \forall t \end{aligned}$$

where  $h_e^m = bd$ .

## B. Proof of proposition 3.4

In order to solve the problem stated in definition 3.3, we are going to use the maximum principle. Let  $H$  denotes the current value Hamiltonian of this problem:

$$H(h_t, g_t, f_t, \mu_t) = -\frac{1}{2}g_t^2 + h_t g_t + df_t - \frac{1}{2}f_t^2 - \frac{1}{2}(h_t - \bar{h})^2 + \mu_t (bf_t - g_t)$$

According to the maximum principle, the efficient solution  $(h_t^*, g_t^*, f_t^*, \mu_t^*)$  satisfies the following necessary conditions:

$$\begin{aligned} g_t^* &= h_t^* - \mu_t^* & \dot{h}_t^* &= bf_t^* - g_t^* \\ f_t^* &= d + b\mu_t^* & \dot{\mu}_t^* &= r\mu_t^* - (g_t^* - h_t^* + \bar{h}) \\ & & \lim_{t \rightarrow \infty} e^{-rt} \mu_t^* &= 0 \end{aligned}$$

**Remark 1.** Using the Arrow sufficiency theorem, it is obvious to check that these conditions are also sufficient since  $\partial_{hh}H^* = 0$ , where  $H^*$  denotes the maximized Hamiltonian corresponding to the efficient setting.

In order to fully characterize the efficient paths, we then solve the following dynamic system:

$$\begin{pmatrix} \dot{h}_t^* \\ \dot{\mu}_t^* \end{pmatrix} = \begin{pmatrix} bd \\ -\bar{h} \end{pmatrix} + \underbrace{\begin{pmatrix} -1 & b^2 + 1 \\ 0 & r + 1 \end{pmatrix}}_{A^*} \begin{pmatrix} h_t^* \\ \mu_t^* \end{pmatrix}$$

Since  $\det A^* = -r - 1 < 0$ , we deduce from the trace/determinant criteria that the path leading to the efficient steady state  $(g_e^*, h_e^*, \mu_e^*)$  characterized by  $\dot{h}_t^* = 0$  and  $\dot{\mu}_t^* = 0$ , is unstable except on the stable trajectories which directions are given by the eigenvector  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  associated to the eigenvalue  $-1$  which is satisfying the transversality condition

$\lim_{t \rightarrow \infty} e^{-rt} \mu_t^* = 0$ . The steady state is thus a saddle point.

After some simple computations, the stable intertemporal efficient paths are given by the following system:

$$\begin{aligned} \mu_t^* &= \frac{\bar{h}}{r+1}, \quad f_t^* = d + \frac{b\bar{h}}{r+1} \quad \forall t \\ h_t^* &= h_e^* + (1 - h_e^*) e^{-t} \\ g_t^* &= g_e^* + (1 - h_e^*) e^{-t} \end{aligned}$$

where  $g_e^* = bd + \frac{\bar{h}b^2}{r+1}$  and  $h_e^* = bd + \frac{\bar{h}(b^2+1)}{2r+1}$ .

### C. Proof of proposition 3.5

Some simple computations on the previous results tell us that:

$$\begin{aligned} f_t^* - f_t^m &= b\mu_e^* > 0 \quad \forall t \\ h_t^* - h_t^m &= (b^2 + 1)\mu_e^*(1 - e^{-t}) > 0 \quad \forall t \neq 0 \\ g_t^* - g_t^m &= b^2\mu_e^*(1 - e^{-t}) - \mu_e^*e^{-t} < 0 \quad \text{for } t < -\ln \frac{b^2}{1+b^2} \\ &> 0 \quad \text{for } t > -\ln \frac{b^2}{1+b^2} \end{aligned}$$

### D. Proof of proposition 4.2

The double problem stated in definition 4.1 can be solved using the maximum principle; it admits two current value Hamiltonians:

$$\begin{aligned} H_f(h_t, f_t, p_t) &= df_t - \frac{1}{2}f_t^2 - \frac{w}{2}(h_t - \bar{h})^2 + p_t(bf_t - \bar{g}_t) \\ H_g(h_t, g_t, \lambda_t) &= -\frac{1}{2}g_t^2 + h_tg_t - \frac{\omega}{2}(h_t - \bar{h})^2 + \lambda_t(b\bar{f}_t - g_t) \end{aligned}$$

According to the maximum principle, the solution  $(h_t^{ol}, g_t^{ol}, f_t^{ol}, \lambda_t^{ol}, p_t^{ol})$  satisfies the following necessary conditions:

$$\begin{aligned} g_t^{ol} &= h_t^{ol} - \lambda_t^{ol} & f_t^{ol} &= d + bp_t^{ol} \\ \dot{h}_t^{ol} &= bf_t^{ol} - g_t^{ol} \\ \dot{\lambda}_t^{ol} &= r\lambda_t^{ol} - [g_t^{ol} - \omega(h_t^{ol} - \bar{h})] & \dot{p}_t^{ol} &= rp_t^{ol} - [-w(h_t^{ol} - \bar{h})] \\ \lim_{t \rightarrow \infty} e^{-rt}\lambda_t^{ol} &= 0 & \lim_{t \rightarrow \infty} e^{-rt}p_t^{ol} &= 0 \end{aligned}$$

**Remark 2.** Using the Arrow sufficiency theorem, it is obvious to check that these conditions are also sufficient if and only if  $\omega = 1$ . Indeed, in such a setting, we can check that  $\partial_{hh}H_f^{ol} = -w$  and  $\partial_{hh}H_g^{ol} = 0$ , where  $H_f^{ol}$  and  $H_g^{ol}$  denote the maximized Hamiltonian in the open-loop setting corresponding respectively to the agricultural and water authorities' maximization problem.

We have then the following dynamic system to solve:

$$\begin{pmatrix} \dot{h}_t^{ol} \\ \dot{p}_t^{ol} \\ \dot{\lambda}_t^{ol} \end{pmatrix} = \begin{pmatrix} bd \\ -w\bar{h} \\ -\omega\bar{h} \end{pmatrix} + \underbrace{\begin{pmatrix} -1 & b^2 & 1 \\ w & r & 0 \\ -1 + \omega & 0 & r + 1 \end{pmatrix}}_{A^{ol}} \begin{pmatrix} h_t^{ol} \\ p_t^{ol} \\ \lambda_t^{ol} \end{pmatrix}$$

The characteristic polynomial of the matrix  $A^{ol}$  is a cubic-root one:  $-\rho^3 + 2r\rho^2 + \rho(r - r^2 + wb^2 + \omega) - r^2 - wrb^2 - wb^2 - \omega r$ .  $A^{ol}$  has thus three eigenvalues denoted  $\rho, \rho_1$  and  $\rho_2$ . We then directly deduce from Cardan's formula and  $r < 1$  that there are one real,  $\rho$ , and two complex eigenvalues, one being the conjugate of the other ( $\rho_2 := a + ib$  and  $\rho_3 := a - ib$ , for instance):

$$(-r^2 - wrb^2 - wb^2 - \omega r)^2 + \frac{4}{27}(r - r^2 + wb^2 + \omega)^3 > 0$$

We furthermore know that:

$$\det A^{ol} = \rho\rho_1\rho_2 = -r^2 - wrb^2 - wb^2 - \omega r < 0$$

Since  $\rho_2\rho_3 = a^2 + b^2 > 0$ , the real eigenvalue is always negative. Furthermore, we know from

$$\text{tr} A^{ol} = \rho + \rho_1 + \rho_2 = 2r > 0$$

that  $a > r$ . and, in case (iii), there exists a unique eigenvalue with a negative real part satisfying the transversality conditions  $\lim_{t \rightarrow \infty} e^{-rt}\lambda_t^{ol} = 0$  and  $\lim_{t \rightarrow \infty} e^{-rt}p_t^{ol} = 0$ : it is the real negative one  $\rho$ . But it is important to note that since this eigenvalue contains a square root, according to the values of our parameters, it can be either real or complex.

So, the path leading to the general UWAIP stationary equilibrium  $(g_e^{ol}, h_e^{ol}, \lambda_e^{ol}, p_e^{ol})$ , is unstable except on the stable trajectories which directions are given by the eigenvector associated to the negative eigenvalue:  $\begin{pmatrix} 1 \\ \frac{w}{\rho-r} \\ \frac{1-\omega}{r+1-\rho} \end{pmatrix}$ .

The stationary equilibrium is thus also a saddle point in this setting. After some simple computations, the stable intertemporal paths of the other variables can be deduced as:

$$\begin{aligned} f_t^{ol} &= f_e^{ol} + \frac{bw(1-h_e^{ol})}{\rho-r} e^{\rho t} & g_t^{ol} &= g_e^{ol} + \frac{(1-h_e^{ol})(r-\rho+\omega)}{r+1-\rho} e^{\rho t} \\ h_t^{ol} &= h_e^{ol} + (1-h_e^{ol}) e^{\rho t} \\ p_t^{ol} &= p_e^{ol} + \frac{w(1-h_e^{ol})}{\rho-r} e^{\rho t} & \lambda_t^{ol} &= \lambda_e^{ol} + \frac{(1-\omega)(1-h_e^{ol})}{r+1-\rho} e^{\rho t} \end{aligned}$$

where  $g_e^{ol} = \frac{br[d(r+\omega)+b\bar{h}w]}{wb^2(r+1)+r(r+\omega)}$ ,  $f_e^{ol} = \frac{r[d(r+\omega)+b\bar{h}w]}{wb^2(r+1)+r(r+\omega)}$ ,  $h_e^{ol} = \frac{b(r+1)(dr+\bar{h}wb)+r\bar{h}w}{wb^2(r+1)+r(r+\omega)}$ ,  $p_e^{ol} = \frac{w[-bd(r+1)+r\bar{h}]}{wb^2(r+1)+r(r+\omega)}$  and  $\lambda_e^{ol} = \frac{rbd(1-\omega)+\bar{h}(wb^2+\omega r)}{wb^2(r+1)+r(r+\omega)}$ .

## E. Synopsis of results

	Efficient	Myopic competitive	UWAIP	
<b>Optimization problem</b>	$\max_{f_t, g_t} \int_0^{\infty} U(f_t, g_t, h_t) e^{-\rho t} dt$ $s.t.: \dot{h}_t = bf_t - g_t, h_0 = 1$	$\max_{f_t, g_t} U(f_t, g_t, h_t)$	$\max_{g_t} \int_0^{\infty} [\sigma(g_t, h_t) + \omega e(h_t)] e^{-\rho t} dt$ $s.t.: \dot{h}_t = bf_t - g_t, h_0 = 1$	$\max_{f_t} \int_0^{\infty} [s(f_t) + we(h_t)] e^{-\rho t} dt$ $s.t.: \dot{h}_t = bf_t - \bar{g}_t, h_0 = 1$
<b>Food consumption</b>				
Path	$d + \frac{b}{2(r+1)} \forall t$	$d \forall t$	$f_e^{ol} + \frac{bw(1-h_e^{ol})}{\rho-r} e^{\rho t}$	
Steady state			$\frac{r[d(r+\omega)+b\bar{h}w]}{wb^2(r+1)+r(r+\omega)}$	
<b>Groundwater consumption</b>				
Path	$g_e^* + (1-h_e^*) e^{-t}$	$h_e^m + (1-h_e^m) e^{-t}$	$g_e^{ol} + \frac{(1-h_e^{ol})(r-\rho+\omega)}{r+1-\rho} e^{\rho t}$	
Steady state	$bd + \frac{b^2}{2(r+1)}$	$bd$	$\frac{br[d(r+\omega)+b\bar{h}w]}{wb^2(r+1)+r(r+\omega)}$	
<b>Aquifer height</b>				
Path	$h_e^* + (1-h_e^*) e^{-t}$	$h_e^m + (1-h_e^m) e^{-t}$	$h_e^{ol} + (1-h_e^{ol}) e^{\rho t}$	
Steady state	$bd + \frac{b^2+1}{2(r+1)}$	$bd$	$\frac{b(r+1)(dr+\bar{h}wb)+r\bar{h}w}{wb^2(r+1)+r(r+\omega)}$	
<b>Shadow prices</b>				
Path	$\frac{1}{2(r+1)} \forall t$	$0 \forall t$	$\lambda_e^{ol} + \frac{(1-\omega)(1-h_e^{ol})}{r+1-\rho} e^{\rho t}$	$p_e^{ol} + \frac{w(1-h_e^{ol})}{\rho-r} e^{\rho t}$
Steady state			$\frac{rbd(1-\omega)+\bar{h}(wb^2+\omega r)}{wb^2(r+1)+r(r+\omega)}$	$\frac{w[-bd(r+1)+r\bar{h}]}{wb^2(r+1)+r(r+\omega)}$

Figure E.1: Synopsis of results

## F. Proof of proposition 4.3

Simple computations lead to the following results:

(i)

$$\begin{aligned} f_t^\omega - f_t^m &= 0 \forall t, g_e^\omega - g_e^m = 0 \\ h_t^\omega - h_t^m &= \lambda_e^\omega (1 - e^{-t}) > 0 \forall t \neq 0 \\ g_t^\omega - g_t^m &= -\lambda_e^\omega e^{-t} < 0 \forall t \rightarrow \infty \end{aligned}$$



(ii)

$$\begin{aligned} h_t^\omega - h_t^* &= -b^2 \mu_e^* (1 - e^{-t}) < 0 \quad \forall t \neq 0 \\ g_t^\omega - g_t^* &= -b^2 \mu_e^* (1 - e^{-t}) < 0 \quad \forall t \neq 0 \\ f_t^\omega - f_t^* &= -b \mu_e^* < 0 \quad \forall t \end{aligned}$$

## G. Proof of proposition 4.4

(i) According to our assumptions,  $\frac{1-h_e^{\omega w}}{\phi-r} < 0$  and

$$\mu_t^* - p_t^{\omega w} = \frac{bd(r+1) + \bar{h}b^2}{(r+1)(r+b^2)} - \frac{1-h_e^{\omega w}}{\phi-r} e^{\phi t} > 0 \quad \forall t$$

Furthermore, since  $p_t^{\omega w} = p_e^{\omega w} + \frac{1-h_e^{\omega w}}{\phi-r} e^{\phi t}$  and  $p_e^{\omega w} = \frac{-bd(r+1) + r\bar{h}}{(r+1)(r+b^2)}$ ,

- $bd(r+1) > r\bar{h} \Rightarrow p_t^{\omega w} < 0 \quad \forall t$ ,
- $bd(r+1) < r\bar{h} \Rightarrow p_t^{\omega w} > 0 \quad \forall t > \frac{1}{\phi} \ln \frac{p_e^{\omega w}(r-\phi)}{1-h_e^{\omega w}}$ .

(ii) The comparison between the amounts of food consumed in a European UWAIP setting and in a WFD one,

$$f_t^{\omega w} - f_t^\omega = bp_e^{\omega w} + \frac{b(1-h_e^{\omega w})}{\phi-r} e^{\phi t}$$

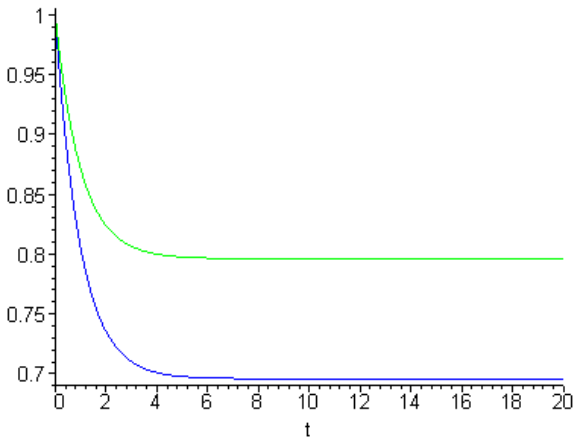
leads us to two different cases:

- $p_e^{\omega w} < 0 \Rightarrow f_t^{\omega w} - f_t^\omega < 0 \quad \forall t$ ;
- $p_e^{\omega w} > 0 \Rightarrow f_t^{\omega w} - f_t^\omega > 0 \quad \forall t > \frac{1}{\phi} \ln \frac{p_e^{\omega w}(r-\phi)}{1-h_e^{\omega w}}$ .

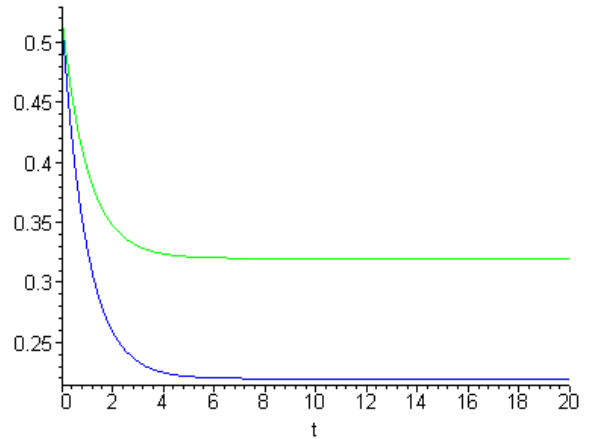
Finally, comparing the amount of food consumed in the European UWAIP setting with the efficient one leads to:

$$f_t^* - f_t^{\omega w} = \frac{b^2(dr + d + b\bar{h})}{(r+1)(r+b^2)} - \frac{b(1-h_e^{\omega w})}{\phi-r} e^{\phi t} > 0 \quad \forall t$$

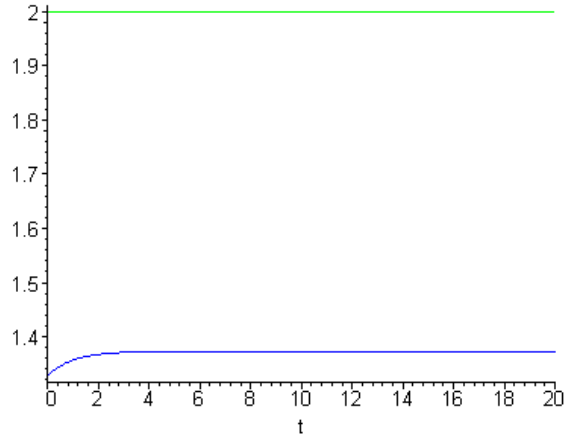
## H. Numerical example for which an UWAIP setting is worse than a WFD one at each period of time



Aquifer height paths



Groundwater consumption paths

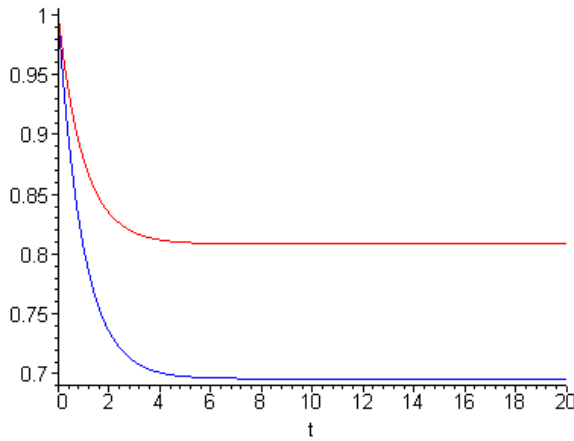


Food consumption paths

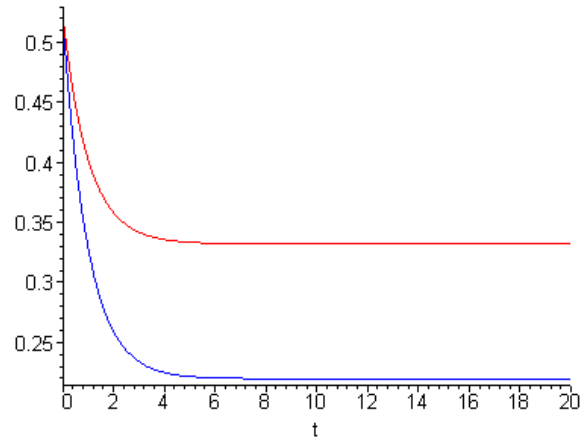
$b=0,16$   $d=2$  (i.e.  $p_t^{ol} < 0 \forall t$ )  $r=0,05$  and  $\bar{h}=1/2$

The green color is for the WFD setting and the blue one for an UWAIP one.

### I. The inefficiency of an UWAIP setting



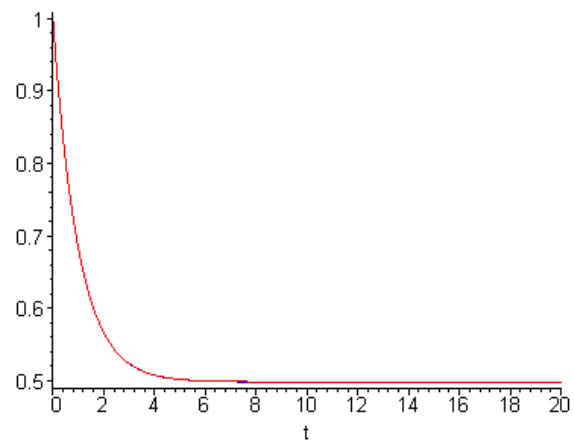
Aquifer height paths



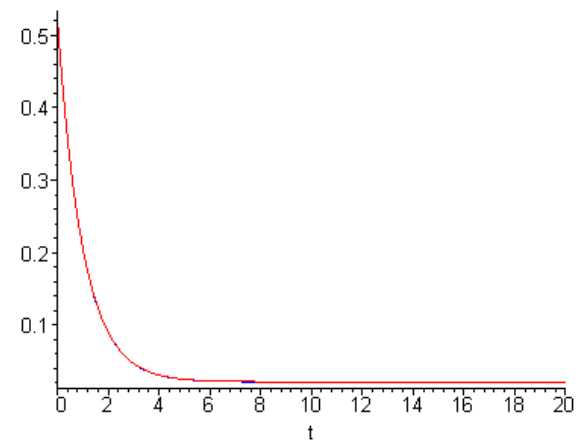
Groundwater consumption paths

$b=0,16$   $d=2$  (i.e.  $p_t^{ol} < 0 \forall t$ )  $r=0,05$  and  $\bar{h}=1/2$

The red color is for the efficient setting and the blue one for an UWAIP one.



Aquifer height paths



Groundwater consumption paths

b=0,018 d=1,2 (i.e.  $p_t^{ol} > 0 \forall t > 2,39$ ) r=0,05 and  $\bar{h}=1/2$

## J. NPV computations

	$\bar{h} = \frac{3}{8}$	$\bar{h} = \frac{1}{2}$	$\bar{h} = \frac{3}{4}$
$NPV^* - NPV^m$	1,31	2,33	5,23
$NPV^* - NPV^\omega$	0,03	0,06	0,13
$NPV^\omega - NPV^m$	1,27	2,27	5,10
$NPV^* - NPV^{\omega w}$	0,87	0,90	0,97
$NPV^{\omega w} - NPV^m$	0,44	1,43	4,26
$NPV^* - NPV^w$	5,59	5,18	4,45
$NPV^w - NPV^m$	-4,28	-2,86	0,78
$NPV^* - NPV^{es}$	0,88	0,90	0,96
$NPV^{es} - NPV^m$	0,43	1,42	4,27

	$\bar{h} = \frac{3}{8}$	$\bar{h} = \frac{1}{2}$	$\bar{h} = \frac{3}{4}$
$NPV^* - NPV^m$	1,27	2,27	5,10
$NPV^* - NPV^\omega$	0	0	0
$NPV^\omega - NPV^m$	1,27	2,27	5,10
$NPV^* - NPV^{\omega w}$	0	0	0
$NPV^{\omega w} - NPV^m$	1,27	2,27	5,10
$NPV^* - NPV^w$	0,39	0,15	0,45
$NPV^w - NPV^m$	0,88	2,11	4,65
$NPV^* - NPV^{es}$	0,01	0,01	0
$NPV^{es} - NPV^m$	1,26	2,26	5,10

r=0,05 b=0,16 d=0,8

r=0,05 b=0,018 d=1,2

## K. Proof of proposition 5.1

(i) In order to check that the implementation of the theoretical fiscal scheme  $(T^{\omega w}, S_t^{\omega w})$  leads to the European UWAIP policy as defined previously, we put it in the myopic competitive program. The necessary and sufficient FOC become the following one:

$$\begin{aligned} g_t^{\omega w i} &= h_t^{\omega w i} - \frac{\bar{h}}{r+1} \\ f_t^{\omega w i} &= d + b \left( p_e^{\omega w} + \frac{1 - h_e^{\omega w}}{\rho - r} e^{\rho t} \right) \end{aligned}$$

We can then directly check that the long-run demand functions coincide with the one defining an European UWAIP from the new differential equation governing the aquifer height variations along time,  $\dot{h}_t^{\omega w i} = bd + b^2 \left( p_e^{\omega w} + \frac{1 - h_e^{\omega w}}{\rho - r} e^{\rho t} \right) - h_t^{\omega w i} + \frac{\bar{h}}{r+1}$ , which admits the following solution:

$$h_t^{\omega w i} = h_e^{\omega w} + (1 - h_e^{\omega w}) e^{\rho t} = h_t^{\omega w}$$

since  $h_e^{\omega w i} = bd + b^2 p_e^{\omega w} + \frac{\bar{h}}{r+1} = h_e^{\omega w}$ .

In order to check that the implementation of the practical (because of being a stationary one) fiscal scheme  $(T^{\omega w}, S^{\omega w})$  leads to the uncoordinated policy as defined previously, we put it in the myopic competitive program. The necessary and sufficient FOC then become the following one:

$$\begin{aligned} g_t^{\omega w i r} &= h_t^{\omega w i r} - \frac{\bar{h}}{r+1} \\ f_t^{\omega w i r} &= d + \frac{-b^2 d(r+1) + \bar{h} b r}{(r+1)(r+b^2)} \end{aligned}$$

The long-run demand functions then directly come from the new differential equation governing the aquifer height variations along time,  $\dot{h}_t^{\omega w i r} = bd + \frac{-b^3 d(r+1) + \bar{h} b^2 r}{(r+1)(r+b^2)} - h_t^{\omega w i r} + \frac{\bar{h}}{r+1}$ :

$$\begin{aligned} h_t^{\omega w i r} &= h_e^{\omega w i r} + (1 - h_e^{\omega w i r}) e^{-t} \\ g_t^{\omega w i r} &= g_e^{\omega w i r} + (1 - h_e^{\omega w i r}) e^{-t} \\ f_t^{\omega w i r} &= f_e^{\omega w i r} \quad \forall t \end{aligned}$$

where  $h_e^{\omega w i r} = \frac{b(r+1)(dr + \bar{h}b) + r\bar{h}}{(r+1)(r+b^2)}$ ,  $g_e^{\omega w i r} = \frac{br[d(r+1) + \bar{h}b]}{(r+1)(r+b^2)}$  and  $f_e^{\omega w i r} = \frac{r[d(r+1) + \bar{h}b]}{(r+1)(r+b^2)}$ .

(ii) If we now compare these solutions with the one obtained previously, we have the following results:

$$\begin{aligned} h_t^{\omega wir} - h_t^\omega &= b^2 p_e^{\omega w} (1 - e^{-t}) \\ h_t^{\omega wir} - h_t^m &= \left[ b^2 p_e^{\omega w} + \frac{\bar{h}(b^2+r)}{(r+1)(r+b^2)} \right] (1 - e^{-t}) \\ g_t^{\omega wir} - g_t^\omega &= g_t^{\omega wir} - g_t^m = b^2 p_e^{\omega w} (1 - e^{-t}) \\ f_t^{\omega wir} - f_t^\omega &= f_t^{\omega wir} - f_t^m = b p_e^{\omega w} (1 - e^{-t}) \end{aligned}$$

which strongly depend on the aquifer shadow price taken into account by the agricultural authority sign.

(iii) We finally have that:

$$\begin{aligned} h_t^* - h_t^{\omega wir} &= \frac{b^3(dr+d+b\bar{h})}{(r+1)(r+b^2)} (1 - e^{-t}) > 0 \quad \forall t \neq 0 \\ g_t^* - g_t^{\omega wir} &= \frac{b^3(dr+d+b\bar{h})}{(r+1)(r+b^2)} (1 - e^{-t}) > 0 \quad \forall t \neq 0 \\ f_t^* - f_t^{\omega wir} &= \frac{b^2(dr+d+b\bar{h})}{(r+1)(r+b^2)} (1 - e^{-t}) > 0 \quad \forall t \neq 0 \end{aligned}$$

## L. Proof of proposition 5.2

(i) With such a fiscal scheme, the necessary FOC, which are also sufficient, are the following one:

$$\begin{aligned} g_t^{*i} &= h_t^{*i} - \frac{\bar{h}}{r+1} \\ f_t^{*i} &= d + \frac{b\bar{h}}{r+1} \quad \forall t \end{aligned}$$

The differential equation governing the change in the water table over time thus becomes:

$$\dot{h}_t^{*i} = bd - h_t^{*i} + \frac{\bar{h}(b^2+1)}{r+1}$$

And thanks to standard computations, the intertemporal path can then be written as:

$$\begin{aligned} h_t^{*i} &= h_e^{*i} + (1 - h_e^{*i}) e^{-t} \\ g_t^{*i} &= g_e^{*i} + (1 - h_e^{*i}) e^{-t} \end{aligned}$$

where  $h_e^{*i} = bd + \frac{\bar{h}(b^2+1)}{r+1}$  and  $g_e^{*i} = bd + \frac{\bar{h}b^2}{r+1}$ .

(ii) It obvious to check that:

$$T^* g_e^{*i} = \mu_e^* b \left( d + \frac{\bar{h}b}{r+1} \right) = S^* f_e^{*i}$$

and that:

$$T^* g_t^{*i} = bd\mu_e^* + \mu_e^* \frac{\bar{h}b^2}{r+1} + \mu_e^* (1 - h_e^{*i}) e^{-t} > bd\mu_e^* + \frac{\bar{h}b^2}{r+1} \mu_e^* = S^* f_t^{*i}$$