How Can Gender Discrimination Explain Fertility Behaviors and Family-friends Policies ?

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How Can Gender Discrimination Explain Fertility Behaviors and Family-friendly Policies ?

by Magali Recoules[*]

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Abstract

This paper focuses on the interaction between gender discrimination and household decisions. It develops a general equilibrium model with endogenous fertility, endogenous labor supply and endogenous size of government spending. Family policies are assumed to decrease the time that parents spend on their children. The model shows that gender discrimination may explain differences in household decisions between countries. The solution shows a U-shaped relationship between fertility and gender discrimination. An increase in the discrimination level implies a related decrease in fertility, women's participation in the labor force and in family-friendly policies.

JEL Classification: D13, H31, J13, J71

Keywords: Gender discrimination, Fertility, Labor supply, Public policies.

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1 Introduction

Since the middle of the 1980s empirical studies have shown an inversion of the cross-country correlation between the female labor supply and the fertility rates in OECD countries (N.Ahn and P.Mira 2002 [3]). The correlation has become positive, throwing back into question the traditional idea of substitution between childbearing and women's labor force participation choices. Now, the countries exhibiting the lowest levels of female employment are also those that have low fertility rates. On the other hand, the countries that are characterized by the highest levels of female employment are also those that have high fertility rates. Some authors propose to explain this situation by changes in institutional context that have helped to reconcile child-rearing with the participation of women in the labor market (A.Adesrà 2004 [1]; K.L.Brewster and R.R.Rindfuss 2000 [9]).[1] The institutional context, such as labor market arrangements and family-friendly policies, also differs considerably from country to country both in the type and the extent of these policies.

How can such differences in household decisions as family-friendly policies, fertility and the female labor supply, be explained? My paper proposes an explanation based on gender discrimination in the labor market. More precisely, this paper studies, through a general equilibrium model, the way in which gender discrimination affects the related decisions on fertility, the female labor supply and family policies. Labor market discrimination, by reducing the wage of women, influences household decisions through three direct effects. An increase in gender discrimination leads to both an increase in the specialization of women in household activities and to a decrease in the child-rearing opportunity cost in terms of pay, as well as a decrease in the total household income. The two former effects play in favour of fertility, while the latter tends to reduce it. Moreover, the joint decrease in household income and the opportunity cost of children tends to decrease the willingness to pay for family-friendly policies. The model provides a U-shaped relationship between fertility and gender discrimination. More precisely, it shows that an increase in gender discrimination may lead to a related decrease in fertility, the female labor supply and family policies.

This paper is based on two crucial assumptions. First, that there is gender discrimination in the labour market that leads to a gender wage gap. Thus for the same skills and working time, women receive a lower wage than men because of gender discrimination. In the literature, gender discrimination partly accounts for the gender wage gap (G.S.Becker 1957 [5]; Aigner and Cain 1977 [5]; S.Coate and G.C.Loury 1993 [12]) and thus for the differences in child-rearing opportunity cost between spouses. Secondly, family-friendly policies may exist that decrease the

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1. Jaumotte (2003)[18] studies the factors determining the female labor supply in OECD countries and finds that public spending in child care stimulates female employment. Cristina d'Addio and Mira d'Ercole (2005)[13] study the determinants of fertility and find that "total fertility rates are higher in OECD countries with wider childcare availability and lower direct costs of children".

time cost of rearing children. These policies are endogenous and result from a vote of agents. By decreasing the child-rearing time of parents and especially the child-rearing time of women, these public policies influence both fertility and female employment decisions (K.L.Brewster and R.R.Rindfuss 1996 [8]; P.Apps and R.Rees 2004 [4]).[2]

The economy is composed of men and women organized as couples. Each man and woman having the same preferences, all households are identical. Household decisions are determined through a two-stage decision process as in Cavalcanti and Tavares 2007 [14]. The first stage refers to fertility, labor supply and individual consumption choices. These decisions are the result of the maximization of a weighted sum of individual utilities under household budget constraint, the weightings being the bargaining power of each partner. The second stage refers to the size of public spending, more precisely the taxation level. The extent of family-friendly policies is determined by a vote of households. Each member of the couple chooses the taxation level which maximizes his or her indirect utility. If spouses have different preferences there is no consensus concerning the expected tax rate within the household, so the theory of probabilistic voting is used in the second stage (A.Lindbeck and J.W.Weibull 1987 [20]; T.Persson and G.Tabellini 2000 [23]).

The model shows that different intensities of gender discrimination may explain the differences in household decisions across countries. Gender discrimination, by acting on the female wage, modifies the allocation of tasks within the household and implies a specialization of gender roles. The solution of the model shows a U-shaped relationship between fertility and gender discrimination. If the discrimination is not too great, its increase raises the cost of having children and puts off the childbearing decision. Beyond a discrimination threshold there is an inversion of this relationship and households choose to have more children. Moreover, an increase in gender discrimination discourages the participation of women in the labor market as it reduces female wage. By remaining at home for longer, women's demand for public services decreases and agents vote for a lower tax rate. Beyond a discrimination threshold, spouses choose a tax rate equal to zero as the gains given by the public policies are not enough to offset their costs. Hence, an increase in the discrimination level may imply a related decrease in fertility, women's employment and family policies.

To complement these results, the model is extended by introducing a child-rearing function with imperfect substitutability of parents' time and a collective decision-making process within household. The first extension allows the analysis of the gender discrimination effects on men's decisions and shows the negative effect of the gender discrimination on the fathers' childrearing time. The second is divided into two parts. First, it is assumed that spouses have different preferences, which allows to take into account the effect of bargaining power on overall

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2. Apps and Rees (2004) find that ''countries which have individual rather than joint taxation, and which support families through child care facilities rather than child payments, are likely to have both higher female employment and higher fertility''.

household decisions. Secondly, it is assumed that the wife's bargaining power depends negatively on the gender discrimination. Bargaining power is affected by relative wages of spouses, which are also influenced by changes in discrimination on the labor market. This extension allows us to study the way in which a collective approach to the decision-making process within the household modifies the results of the benchmark model (Bourguignon and Chiappori 1992 [7]; Chiappori 1997 [11]).

This work complements the literature relating fertility, the female labor supply and public spending decisions in which usually only women take care of children and a unitary decision-making process is used. The model presented in this paper is based on Cavalcanti and Tavares 2007 [14] and Galor and Weil 1996 [15] in which the gender wage gap is due to differences in physical strength and reduces as the economy grows. By taking into account the individual utilities of spouses, the current paper analyzes the specific behavior of each partner in the household decision-making process. It discusses the voting system when spouses have different preferences, as well as the relative weight of agents in the household decision-making process and the effect of changes in bargaining power on household decisions. It also analyzes the way in which gender discrimination may affect related decisions of fertility, the female labor supply and family policies, while Cavalcanti and Tavares are mainly interested in the link between the female labor supply and the level of public spending.

The paper is structured as follows. Section 2 provides an overview of some empirical evidence regarding fertility rates, labor supply and family policies in OECD countries. In Section 3, a general equilibrium model with gender discrimination is developed. Section 4 presents the main results. Section 5 proposes some extensions of the benchmark model in which are successively introduced a child-rearing function with imperfect substitutability, heterogeneity within household and a collective household decision-making process (Bourguignon and Chiappori 1992 [7]; Chiappori 1997 [11]). A discussion about the main results follows. Section 6 concludes.

2   Some empirical evidence

Since the inversion of the cross-country correlation between fertility and the female labor supply in the middle of the 1980s, OECD countries with the lowest levels of female employment are also those that have low fertility rates (Bettio and Villa 1998 [6]). And the countries with the highest levels of female employment are also those that have high fertility rates (see Figure 6 in Appendix 1).
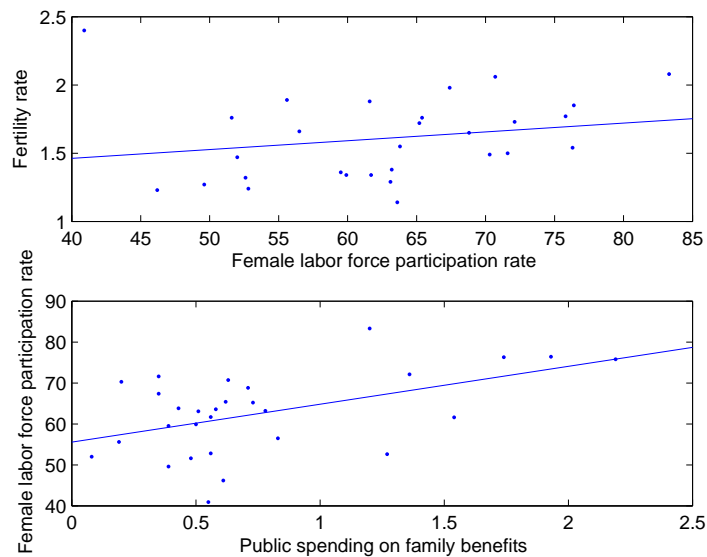
Figure 1. Family Decisions in OECD countries in 2000

Source: Total fertility rates correspond to the number of children aged 15 to 49 years old per woman. Female labor force participation rates are those for persons aged 15-64 years. Public spending on family benefits is family spending on services percentage of GDP.

The data come from the OECD database. All OECD countries are taken into account except Turkey.

Figure 1 points out a positive relationship between fertility and the female labor supply, and between the female labor supply and family-friendly policies.

The differences of behavior regarding fertility and women's employment choices may be explained by institutional changes in family policies and labor market institutions (A.Adesrà 2004 [1]; T.Kögel 2004 [19]). Family policies, by influencing the cost of having children, modify family behavior in terms of female employment and fertility (A.C. d'Addio and M.Mira d'Ercole 2005 [13]).

It has been observed that European countries which have the highest levels of fertility rates and female labor supply are also those that have high state intervention concerning the family. Countries can be organized into different clusters according to their respective behaviors in terms of the fertility rate, women's labor force participation rate and social policy (Chesnais[10] and Handrais [16]).[3]

Gender discrimination acting on wages may be an explanation of these various household behaviors. Family decisions in European countries have been employed to illustrate this assumption. Two indicators of gender discrimination have been selected: the percentage of the gender wage gap which is unexplained by differences in characteristics between men and women[4], and the female economic

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3. Some authors have organized countries in clusters according to their respective behavior concerning family policies. For example, according to the classification proposed by Gauthier (2002) [2], there are three groups of countries. The first includes Denmark, while the second includes France, the United Kingdom, Austria, Germany and Ireland. And the last would comprise the Southern European countries. We can see that this classification matches that of the discrimination index in Table 1.

4. This index is calculated for the year 2000 and is taken from Meurs and Ponthieux 2005. They

activity rate as a percentage of the male rate. A similar classification of countries selected is obtained by using both these indexes.[5]

Countries	Part of the Gender Wage Gap Unexplained by Gender Differences in Characteristics(%)	Female Economic Activity Rate (aged 15 and above) as % of Male Rate
Denmark	26,24	84
France	27,72	76
United Kingdom	39,01	74
Austria	40,98	65
Germany	45,53	69
Ireland	50,78	52
Italy	58,25	58
Spain	62,02	56
Greece	88,84	58
Portugal	117,44	71

Table 1. Gender Discrimination Index in 2000

Sources : The index of the gender wage gap is calculated and taken from Meurs and Ponthieux 2005 [22].The female economic activity rate as a percentage of the male rate comes from the Human Development Report of 2002, published by the United Nations Development Program (UNDP) [24].

As this paper focuses on the effects of gender discrimination on household decisions the former indicator, "the part of the gender wage gap unexplained by gender differences in characteristics", has been employed to study the relationships between household decisions and gender discrimination in the following figures.

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analyze the composition of the gender wage gap by dividing it into the gap due to characteristics and that due to returns for these ten European countries. The part of the gender wage gap which is unexplained by differences in characteristics is used in this paper as a discrimination index. The sample studied, in the current paper, is limited to that of Meurs and Ponthieux, as it is difficult to procure the first discrimination index for many countries.

5. Except for Portugal. In Table 1, as in Portugal the unexplained part is larger than the total gap, the first discrimination index is higher than 100%. "This suggests that the productive characteristics of employed women are on average higher than those of men".
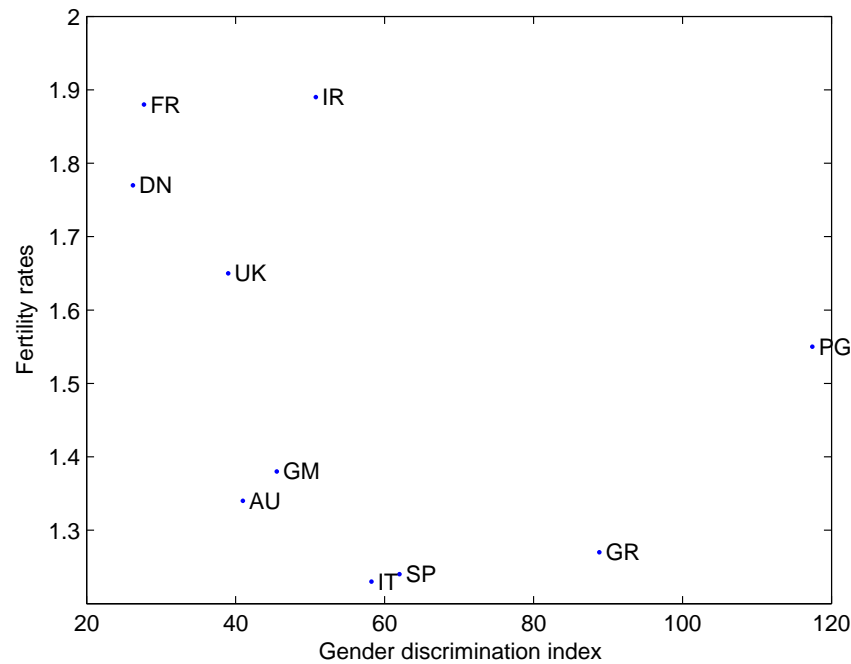
Figure 2. Fertility Rate and Gender Discrimination Index
Source : Fertility rates for 2000 come from the OECD database. The indicator of gender discrimination represents the percentage of the gender wage gap which cannot be explained by differences in characteristics between men and women. Calculated for the year 2000, from Meurs and Ponthieux 2005 [22].
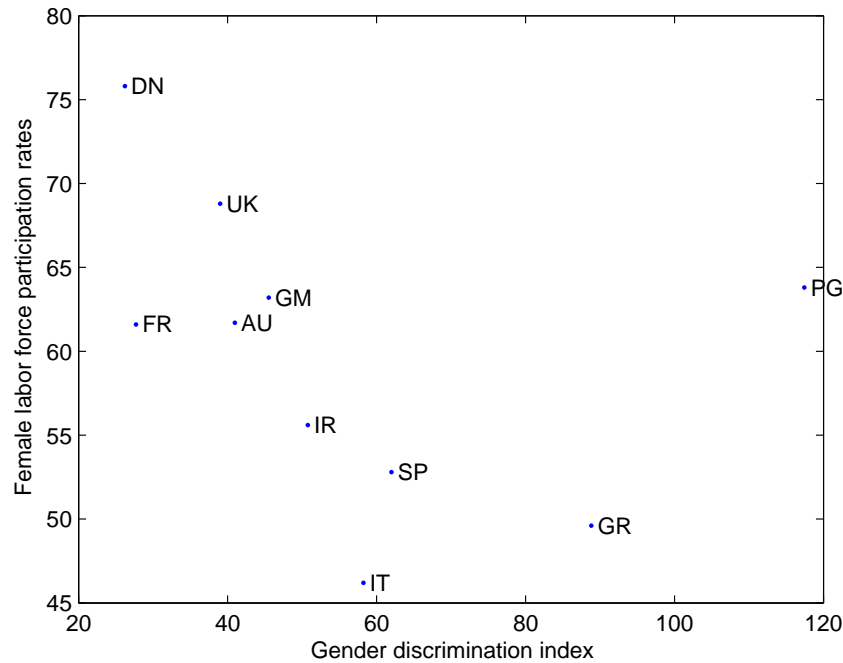
Figure 3. Female Labor Force Participation Rate and Gender Discrimination Index
Source : Women's labor force participation rates in 2000 from the OECD database. The indicator of gender discrimination represents the percentage of the gender wage gap which cannot be explained by differences in characteristics between men and women. Calculated for the year 2000, from Meurs and Ponthieux 2005 [22].

Figure 4. Family-friendly Policies and Gender Discrimination Index
Source : Public spending on family benefits is family spending on services as a percentage of GDP in 2000 from the OECD database. The indicator of gender discrimination represents the percentage of the gender wage gap which cannot be explained by differences in characteristics between men and women. Calculated for the year 2000, from Meurs and Ponthieux 2005 [22].
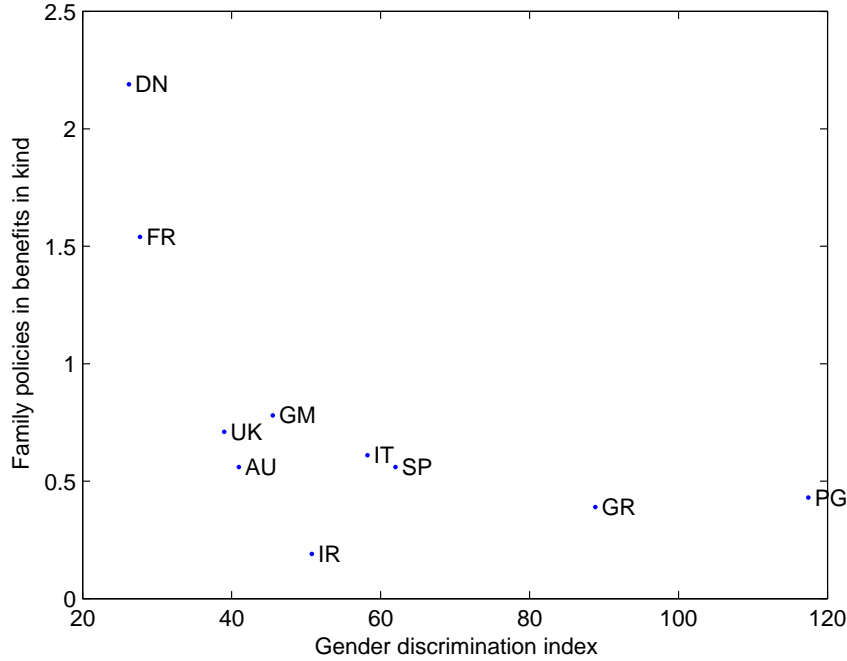
These figures allow us to give an idea of the sort of relationship we can expect to find in the theoretical model. They show, overall, a negative relationship between household decisions and the gender discrimination index. Moreover, in the light of these data a U-shaped relationship could be suspected between the fertility rate and the gender discrimination index.

The next section presents a general equilibrium model that analyzes the way in which gender discrimination can affect family decisions.

3 The model

The relationship between household decisions and gender discrimination is studied through a general equilibrium model with endogenous fertility, endogenous labor supply and endogenous size of government spending. The framework of the model is based on the article, written by Galor and Weil in 1996 [15] and that by Cavalcanti and Tavares in 2007 [14] which introduces public spending to the model. The economy is composed of men and women organized as couples and the level of family-friendly policies is endogenously determined by a vote of agents. Family policies are assumed to decrease the time that parents spend on their children.

3.1 Firm

The production technology uses one production factor, labor. There are two kinds of worker, female workers, L^f, and male workers, L^m, which are perfect substitutes. The marginal productivity of men and women is the same.

The production function is,

$$f(L^f + L^m) = A(L^f + L^m),$$

where $A > 0$ is the total productivity of factors.

Given the technology and the input prices, the representative firm chooses inputs in order to maximize its profits.

$$\max_{L^f, L^m} \Pi(L^f, L^m) = f(L^f, L^m) - w^m L^m - (w^f + d) L^f$$

Here the parameter d captures the problem of discrimination and can be interpreted as the taste for discrimination of employers as in the discrimination theories based on discriminative preferences pioneered by Gary Becker 1957 [5].[6]

The first order conditions associated with the representative firm's problem are:

$$w^f = A - d \text{ and } w^m = A, \quad \text{with } d \in [0, A[$$

As this model is taking place in an economy in which men and women have the same level of human capital, d represents the wage gap between men and women per hour worked. Thus it determines the level of gender discrimination in the labor market.

For the same skills and working time, women receive a lower wage than men because of the gender discrimination, $w^f < w^m$.

3.2 Household

All households are identical in this society. Each agent has one unit of time which is divided between child care and paid work, and has the same level of human capital. Thus, the wage-difference between spouses comes from gender discrimination in the labor market.

The preferences of spouses are assumed to be the same and are represented by the following utility function:

$$U^i = \beta \ln(c^i) + \gamma \ln(n) \quad s.t \quad i = f, m$$

where n is the number of children per couple and c^i the individual consumptions.

An additional assumption on parameters is made, $\gamma + \beta = 1$.[7]

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6. The choice of taste-based discrimination can present some problems concerning the persistence of discrimination in the long run. However, it has been selected for its clarity in exposing the gender discrimination effects on household decisions which is the aim of this paper. For a survey of gender discrimination theories see Nathalie Havet (2004) [17].

7. It is assumed that $\gamma + \beta = 1$ only to simplify equations. This assumption does not change the results.

The budget constraint of the household is

$$[w^f(1-h^f) + w^m(1-h^m)](1-\tau) = c^f + c^m \qquad (1)$$

where h^i $s.t$ $i = f, m$ is parents' time spent on parental care and child-rearing, and τ is the tax rate. Notice that prices of consumption goods are normalized at one.

In this model, government policies have an influence on household decisions. Public revenues are collected by the government through a proportional tax τ on household income. The government budget is equilibrated and taxes are employed to finance the per-couple government spending, g, intended to decrease the per-child cost of rearing children.

The time allocated by parents to their children is

$$H = n\,h(g) = h^f + h^m,$$

where $h(g)$ represents the total time devoted by parents to each child.

Notice that the time spent by a woman or a man on children are perfect substitutes.

Household Decisions

Couples determine the number of children, the size of government and individual consumptions subject to a budget constraint that reflects the allocation of time between labor supply and child-rearing.

Household decisions are fixed through a two-step decision process. The first stage refers to fertility, labor supply and individual consumption choices. These decisions are the result of the maximization of a weighted sum of individual utilities under the household budget constraint. In the maximization, the weightings are the bargaining power of each spouse. In the rest of the paper, these decisions will be noted as intra-family decisions.

The second stage refers to the size of public spending, more precisely the taxation level. Each member of the couple chooses the taxation level which maximizes his or her indirect utility. Public spending is exclusively devoted to decreasing the time that parents spend on their children.

Intra-family Decisions and Specialization :

The couple's program for intra-family decisions :

$$\underset{c^f, c^m, n, h^f, h^m}{\text{Max}} \theta[(1-\gamma)\ln(c^f) + \gamma\ln(n)] + (1-\theta)[(1-\gamma)\ln(c^m) + \gamma\ln(n)]$$

$$s.t.\,[w^f(1-h^f) + w^m(1-h^m)](1-\tau) = c^f + c^m,$$

$$H = n\,h(g) = h^f + h^m,$$

where θ is the bargaining power of the wife.

Notice that the opportunity cost of child-rearing is stronger for men than for women, because of $w^f < w^m$. Due to the gender discrimination, there is a specialization of gender roles within the couple based on comparative advantage and budget constraint (1). In the household only the woman takes care of the children and the man spends all his time on the labor market.

$$h^m = 0 \;\; \text{and} \;\; n\,h(g) = h^f$$

Based on gender specialization, the new couple's program for intra-family decisions is :

$$\max_{c^f, c^m, n, t^f} \theta[(1-\gamma)\ln(c^f) + \gamma\ln(n)] + (1-\theta)[(1-\gamma)\ln(c^m) + \gamma\ln(n)]$$

$$s.t. \, [w^f t^f + w^m](1-\tau) = c^f + c^m,$$

$$1 = t^f + n\, h(g),$$

where t^f is the time spent on the labor market by the woman.

The intra-family decisions are expressed in relation to government spending. So each partner votes for the optimal level of public spending taking into account its effects on the woman's trade-off between the labor market and child-rearing.

The level of fertility choice is

$$n = \frac{\gamma(w^f + w^m)}{w^f h(g)},$$

The number of children is limited by the time constraint of women and depends on household income. It is also a decreasing function of women's child-rearing opportunity cost, $w^f h(g)$.

The individual consumption decisions are

$$c^f = \theta(1-\gamma)[w^f + w^m](1-\tau) \quad \text{and} \quad c^m = (1-\theta)(1-\gamma)[w^f + w^m](1-\tau)$$

The individual consumptions depend exclusively on total disposable income.

Tax rate determination

Having fixed their intra-family choices, each spouse chooses the level of tax rate that maximizes his or her indirect utility.

The budget of the government is balanced throughout. Therefore,

$$g = \tau(1-\gamma)(w^f + w^m),$$

where τw^m is tax on the husband's paid work and $\tau w^f (1 - h(g)n)$ is tax on the wife's paid work. Thus,

$$\tau = \frac{g}{(1-\gamma)(w^f + w^m)},$$

As in the article by Cavalcanti and Tavares (2007) [14], the tax rate τ is endogenously determined by a vote of the adult population.[8]

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8. However, contrary to Cavalcanti and Tavares 2007 [14], the tax rate decision is made at individual level and not at household level, as in my paper the determination of tax rate is non-cooperative. This assumption introduces the discussion about voting systems when men and women vote for different tax rates in Section 5.2.

The indirect utility of each spouse is

$$V^i(w^i; g) = (1-\gamma)\ln(c^i(g)) + \gamma\ln(n(g)) \ \ s.t \ \ i = m, f.$$

Each partner fixes the tax rate which maximizes his/her indirect utility,

$$\bar{g} = \underset{g \geqslant 0}{\mathrm{argmax}} \ V(w^i, g)$$

The time allocated by women to each child is assumed to be a decreasing function of public spending,

$$h(g) = \phi[1 + g]^{-\varepsilon},$$

where $\varepsilon > 0$ and ϕ is the minimal time that parents have to devote to each child. More precisely, ϕ represents the time cost of children for parents when there is no public spending. The parameter ε captures the efficiency of family policies.

Each partner chooses his/her prefered level of taxation in solving the following maximization problem :

$$\underset{g^i}{\mathrm{Max}} \, V^i = (1-\gamma)\ln[c^i(g^i)] + \gamma\ln[n(h(g^i))], i = f, m$$

The preferred tax rate of agents is given by the following expression,[9]

$$\tau^i = \frac{\gamma\varepsilon}{\gamma\varepsilon + (1-\gamma)} - \frac{1}{(w^f + w^m)(\gamma\varepsilon + (1-\gamma))}, i = f, m$$

Spouses choose the same level of tax rate. So there is a consensus within the couple concerning the expected tax rate in the society.

The tax rate is positively linked with the household income. The tax rate has two effects on the labor supply decisions. A high tax rate discourages the labor supply of the household. But it also reduces the opportunity cost of child-rearing for women and increases women's labor force participation. So the final effect of the tax rate on labor supply is ambiguous.

Before examining the equilibrium, some intermediate results can be quoted :

$$w^m = A, \ w^f = A - d, \ L^f = t^f \text{ and } L^m = t^m,$$

where w^m, w^f, L^f, t^f, L^m and t^m are respectively male wage, female wage, women's labor demand, women's labor supply, men's labor demand and men's labor supply.

4 Implications of gender discrimination

Proposition 1. *At the equilibrium, two solutions could be identified by different gender discrimination levels. An interior solution which is characterized by a positive tax rate, $\tau > 0$. And a corner solution which is specified by a tax rate equal to zero, $\tau = 0$ if $A > d > 2A - \frac{1}{\gamma\varepsilon}$.*

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9. For intermediate results see Appendix 2.

4.1 Interior Solution :

The tax rate is given by,

$$\tau = \frac{\gamma\varepsilon}{\gamma\varepsilon + (1-\gamma)} - \frac{1}{(2A-d)(\gamma\varepsilon + (1-\gamma))}$$

Proposition 2. *If $d < 2A - \frac{1}{\gamma\varepsilon}$, the marginal gain given by public spending compensates for the marginal cost of the latter and adults vote for a strictly positive tax rate, $\tau > 0$.*

The tax rate is a decreasing function of the gender discrimination, $\frac{d\tau}{dd} < 0$ and so the higher the gender discrimination, the smaller the tax rate.

Moreover, the condition under which the tax rate τ is positive could also be analyzed as an efficiency constraint concerning family policies, in other words a constraint on ε. Indeed, if the welfare services offered by the state are too low, the voters choose a low tax rate.

The number of children chosen by the couple is given by,

$$n = \frac{\gamma(\gamma\varepsilon)^\varepsilon(2A-d)[1+(1-\gamma)(2A-d)]^\varepsilon}{\phi(\gamma\varepsilon + (1-\gamma))^\varepsilon(A-d)}$$

Proposition 3. *There is a U-shaped relationship between fertility and gender discrimination if $\varepsilon > \left(1 + \frac{1}{2A(1-\gamma)}\right)$.[10] If this condition is not satisfied, the fertility decision is an increasing function of gender discrimination.*

The interpretation of gender discrimination effects on fertility decisions can be made in two parts.

If the level of gender discrimination is not too high, fertility is negatively linked to gender discrimination, $\frac{dn}{dd} < 0$ and an increase in gender discrimination discourages fertility. To explain this, three effects can be pointed out : two price effects and an income effect.

The price effect is shown in studying the woman's opportunity cost of child-rearing, $h(g)\,w^f$, and can be divided into two effects. The direct price effect implies that a decrease of female wage, w^f, due to a higher discrimination level, reduces the child-rearing opportunity cost in terms of pay. Thus, as women are paid less, they are discouraged from participating in the labor market and they might decide to have more children. This effect has a positive impact on child-bearing choice. The indirect price effect implies that a higher discrimination level leads to less public spending, because in this case the taxation level is smaller. So a smaller tax rate implies a rise in women's opportunity cost in terms of time, $h(g)$, and has a negative impact on fertility choice.

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10. For intermediate results see Appendix 3.

The income effect can be set out as follows. A higher discrimination level reduces the female wage[11] and therefore the household income. Having children is costly, so a reduction of the household income discourages the couple from having more children. This has a negative impact on fertility choice.

In conclusion, the negative effects dominate and a higher gender discrimination level discourages fertility choice.

However, when the discrimination is beyond a certain threshold there is an inversion of the relationship between fertility and gender discrimination. The childbearing decision becomes an increasing function of the gender discrimination.

The woman's labor supply is given by,

$$t^f = 1 - \frac{\gamma(2A - d)}{(A - d)}$$

The woman's labor supply is negatively correlated with the gender discrimination level, $\frac{\mathrm{d}t^f}{\mathrm{d}d} < 0$.

The higher the gender discrimination level, the smaller the women's labor force participation. A higher level of discrimination against women decreases women's wage and discourages women's labor force participation. Furthermore, as a consequence of the specialization of gender roles, the gender gap in employment widens when the number of children increases, all other things being equal.

Moreover, as an increase of the gender discrimination level reduces the extent of family-friendly policies, mothers' family responsibilities and the limited availability of adequate child-care services may also reduce women's labor force participation.

Individual consumptions are also negatively correlated with the discrimination level. A higher gender discrimination level reduces household income for a fixed working time and decreases individual consumptions,

$$c^f = \theta(1 - \gamma)\frac{[(1 - \gamma)(2A - d) + 1]}{(\gamma\varepsilon + (1 - \gamma))} \text{ and } c^m = (1 - \theta)(1 - \gamma)\frac{[(1 - \gamma)(2A - d) + 1]}{(\gamma\varepsilon + (1 - \gamma))}.$$

4.2 Corner solution :

Proposition 4. *The corner solution exists if* $A > d > 2A - \frac{1}{\gamma\varepsilon}$ *and corresponds to the situation in which there is no public spending,* $\tau = 0$.

As the gender discrimination level is very high, $d > 2A - \frac{1}{\gamma\varepsilon}$, the marginal gain given by public spending does not compensate for the marginal cost of the latter. Adults vote for a tax rate equal to zero.

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11. The establishment of gender discrimination only takes into account the disadvantage of women on the labor market and not the possible advantage of men.

The number of children chosen by the household is given by,

$$n = \frac{\gamma(2A - d)}{(A - d)\phi}$$

Fertility is positively associated with gender discrimination, $\frac{\mathrm{d}n}{\mathrm{d}d} > 0$. A higher level of gender discrimination encourages fertility. To explain this result, two effects can be pointed out: a price effect and an income effect.

The price effect is illustrated by studying women's opportunity cost of child-rearing, $w^f h(g)$. As there is no public spending, there is no indirect effect. The price effect implies that a decrease in women's wage, w^f, due to a higher discrimination level, reduces the child-rearing opportunity cost in terms of pay. Thus as women are paid less, they are discouraged from participating in the labor market which leaves them more time free to take care of children. And they can decide to have more children. This has a positive effect on fertility choice.

The income effect can be analyzed in the following way. A higher gender discrimination level reduces women's wage and household income. Having children is costly, so a reduction of household income discourages couples from having a lot of children. This has a negative impact on fertility choice.

In short, the price effect dominates the income effect and a higher gender discrimination level stimulates fertility choice. This result coincides with the literature which specifies that childbearing decisions are negatively linked with female wages.

If there is a U-shaped relationship between fertility and discrimination, this case corresponds to the increasing part of the U-shaped function.

The woman's labor supply is given by,

$$t^f = 1 - \frac{\gamma(2A - d)}{(A - d)},$$

The gender discrimination level also has a negative impact on the female labor supply, $\frac{\mathrm{d}t^f}{\mathrm{d}d} < 0$.

Individual consumptions are still negatively correlated with the discrimination level. A higher discrimination level reduces household income for a fixed working time and decreases individual consumptions,

$$c^f = \theta(1 - \gamma)(2A - d) \text{ and } c^m = (1 - \theta)(1 - \gamma)(2A - d)$$

To sum up, gender discrimination, by acting on wages, modifies the allocation of tasks within the household. If discrimination is not too great, its increase raises the cost of having children and puts off the childbearing decision. Beyond a discrimination threshold there is an inversion of this relationship and households have more children. As gender discrimination reduces female wage, its increase discourages the entry of women into the labor market. By remaining at home for longer, the female demand for public services decreases and spouses vote for a lower tax rate.

It seems that the gender wage gap affects household decisions and, more precisely, the allocation of time between paid and unpaid work.[12] Some extensions of the benchmark model are now proposed in order to analyze the effects of gender discrimination on men's decisions, and on household decisions when spouses differ in their preferences.

5 Further Issues

The purpose of the following extension is to study the gender discrimination effects on men's behavior. Indeed, as men do not take care of children in the benchmark model, gender discrimination does not affect the male labor supply. But, by introducing a childrearing function with an imperfect substitutability of the parents' time into the benchmark model, men's decisions are no longer independent of gender discrimination.

5.1 Imperfect Substitutability of Parents' Childrearing Time

Even if the data show that men spend less time with children than women, all the same they are allocating a small part of their available time to child care. So a childrearing function with imperfect substitutability of parents' time is introduced into the benchmark model.

The childrearing function is,

$$h(g)n = (h^f)^\eta \left(h^m\right)^{(1-\eta)}$$

where η means the efficiency of female childrearing time and $1 - \eta$ means the efficiency of male childrearing time.[13]

Men, like women, have to trade off between childrearing time and working time. The higher η is, the more time women devote to children and vice versa for men.

A U-shaped relationship between gender discrimination and fertility can also be observed if the condition $\varepsilon > \frac{(2\eta - 1)}{2A(1 - \gamma)} + (2\eta - 1)$ is satisfied (see Appendix to Section 4). Gender discrimination still has a negative impact on women's labor supply and on the tax rate level.

However, gender discrimination has a positive effect on men's labor supply. As higher gender discrimination discourages his wife from participating in the labor force, the husband has to work longer in order to compensate for the loss of income due to the decrease in his wife's working time.

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12. These decisions also depend on cultural attitudes. See Fernandez, Fogli and Olivetti (2004) [25].

13. If $\eta = 1$, there is a total specialization of gender roles within the household, and the results of this model correspond to those of the benchmark model.

The woman's childrearing time is positively linked to the discrimination level, $\frac{dh^f}{dd} > 0$:

$$h^f = \frac{\eta\gamma(2A - d)}{(A - d)}$$

The man's childrearing time is negatively related with the discrimination level, $\frac{dh^m}{dd} < 0$:

$$h^m = \frac{(1 - \eta)\gamma(2A - d)}{A}$$

Women are devoting more time to children than men because of gender discrimination, whatever η. Thus, as women are discriminated against in the labor market, men have to offset the loss of income implied by the gender discrimination by working longer and decreasing their childrearing time. So discrimination in the labor market may be one of the explanatory factors for the low investment of men in domestic activities that all empirical studies reveal (C.Sofer and S.S.Rizavi 2008 [26]; M.Burda, D.S.Hamermesh and P.Weil 2007 [21]).[14]

To sum up, the introduction of a childrearing function with imperfect substitutability of parents' time allows us to take into account the effects of gender discrimination on individual male decisions. Because of gender role specialization in the benchmark model, gender discrimination has no influence on men's employment. However, in this extension, due to the new trade-off for men between children and market work, a positive relationship is observed between the level of gender discrimination and the level of men's labor force participation. This result lets us presume that male employment is negatively related to the extent of family policies, as the latter is still negatively related to gender discrimination.[15]

Up to now, it has been assumed that spouses have the same preferences. However, it is feasible to think that men and women can differ in terms of preferences.

5.2 Heterogeneity within household and a collective approach to the decision-making process:

5.2.1 Heterogeneity of preferences within the household:

It is now assumed that spouses have different preferences and individuals are similar within a gender group.

The program of a representative couple after the specialization of gender roles is given as follows,

$$\max_{c^f, c^m, n} \theta[\beta^f \ln(c^f) + \gamma^f \ln(n)] + (1 - \theta)[\beta^m \ln(c^m) + \gamma^m \ln(n)]$$

$$s.t. \ [w^f(1 - h(g)n) + w^m](1 - \tau) = c^f + c^m$$

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14. In the Southern European countries such as Italy where gender discrimination is high, the participation of men in the domestic sphere is low. In the Northern European countries such as Denmark, where gender discrimination is lower, male participation in domestic activities is much greater.

15. as in Cavalcanti and Tavares (2007) [14]

It is assumed that $\gamma^f + \beta^f = 1$ and $\gamma^m + \beta^m = 1$.[16]

In a non-cooperative decision-making process of tax rate, spouses choose different levels of taxation:

$$\tau^i = \frac{\gamma^i \varepsilon}{((1 - \gamma^i) + \gamma^i \varepsilon)} - \frac{(1 - \gamma^i)}{(2A - d)(\theta(1 - \gamma^f) + (1 - \theta)(1 - \gamma^m))(1 - \gamma^i + \gamma^i \varepsilon)}, i = f, m$$

Proposition 5. *If women have higher preferences for children than men, $\gamma^f > \gamma^m$, women would vote for a higher tax rate than men, $\tau^f > \tau^m$.*

As spouses have different preferences and each member of the household does not vote for the same tax rate, a discussion about the voting system could follow. If women have no right to vote or are constrained in their voting, the tax rate applied is the men's one. However, if women have voting rights and are free in their voting decisions, there is no consensus concerning the tax rate applied in the society.

The economy is composed only of two kinds of individual, men who vote for τ^m and women who vote for τ^f. As there is no majority in the society because the proportions of men and women are the same, every tax rate between that chosen by men and that chosen by women could be a solution. So the theory of probabilistic voting is used to fix the tax rate applied (A.Lindbeck and J.W.Weibull 1987 [20]; T.Persson and G.Tabellini 2000 [23]).

The level of taxation is fixed by solving the following maximization problem :

$$\underset{g}{\text{Max}} \left[\frac{1}{2} V^f(c^f, n) + \frac{1}{2} V^m(c^m, n) \right]$$

where V^f and V^m are respectively the indirect utility of the woman and of the man.[17]

The tax rate is given by the following expression:

$$\tau(d, \theta) = \frac{\varepsilon(\gamma^f + \gamma^m)}{B} - \frac{(1 - \gamma^f) + (1 - \gamma^m)}{(2A - d)B[(1 - \gamma^f)\theta + (1 - \gamma^m)(1 - \theta)]}$$

$$\text{with } B = [(1 - \gamma^f) + (1 - \gamma^m) + \varepsilon(\gamma^f + \gamma^m)]$$

The woman's labor supply is given by,

$$t^f(d, \theta) = 1 - \frac{(2A - d)(\theta \gamma^f + (1 - \theta)\gamma^m)}{(A - d)}$$

Proposition 6. *If women value children more than men, $\gamma^f > \gamma^m$, an increase of the wife's bargaining power, θ, will both reduce the tax rate and the time spent by women on the labor market $\left(\frac{d\tau(d, \theta)}{d\theta} < 0 \text{ and } \frac{dt^f(d, \theta)}{d\theta} < 0 \right)$.*

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16. This assumption allows us to change individual preferences without modifying the bargaining powers within the household. Moreover, this assumption does not limit the set of possibilities concerning preferences.

17. For intermediate results see Appendix 5.

The fertility is,[18]

$$n = \frac{(2A - d)(\varepsilon(\gamma^f + \gamma^m))^\varepsilon \tilde{\beta}[1 + (2A - d)(1 - \tilde{\beta})]^\varepsilon}{\phi(A - d)[(1 - \gamma^f) + (1 - \gamma^m) + (\gamma^f + \gamma^m)\varepsilon]^\varepsilon}$$

$$\text{With } \tilde{\beta} = (\theta\gamma^f + (1 - \theta)\gamma^m)$$

If it is assumed that $\gamma^f > \gamma^m$, an increase of θ will both increase the total time spent by women on children, h^f, and reduce public spending, g. The final effect on the time spent by women on each child, $h(g)$, is non-linear and depends on the public spending efficiency and on its level, respectively ε and g. Hence, as the cross derivative of $h(g)$ could be positive or negative in terms of ε and g ($\frac{\mathrm{d}^2 h(g)}{\mathrm{d}g\mathrm{d}\varepsilon} < 0 \,\text{or} > 0$), the effect of an increase of θ on fertility is non-monotonous. Three kinds of situation emerge and can be summarized by Figure 5.
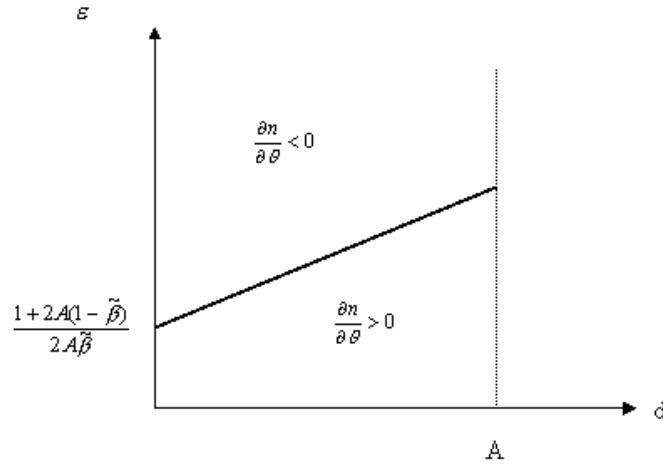

Figure 5. Effect of women's bargaining power on fertility

The effects of discrimination on household decisions are still the same. Indeed, a U-shaped relationship between gender discrimination and fertility can also be found, and gender discrimination still has a negative impact on women's labor supply and the tax rate level.

5.2.2 Collective approach to the decision-making process

The bargaining power of each member within the household is now assumed to depend on gender discrimination, d. The wife's bargaining power is assumed to be negatively linked to the gender discrimination level, $\frac{\mathrm{d}\theta(d)}{\mathrm{d}d} < 0$.

Female Labor Supply Decisions and Tax Rate Determination

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18. When spouses have different preferences, tax rates, female labor supply and fertility decisions are influenced by their bargaining power. Up to now, these decisions were only dependent on spouses' preferences and wages.

If men value children more than women, $\gamma^m > \gamma^f$, gender discrimination still has a negative effect on the female labor supply and the tax rate. However, if it is women who value children more, $\gamma^f > \gamma^m$, the discrimination effect on these decisions is ambiguous. Gender discrimination plays a part in the determination of these decisions in two different ways; a direct effect and an indirect effect through the bargaining power shifts (see Appendix 6). If the direct effect prevails, the gender discrimination still has a negative effect on both decisions, as has already been found. However, if the indirect effect prevails, an increase in the gender discrimination level has a positive impact on the female labor supply and on the tax rate. As gender discrimination reduces the female bargaining power within the household and men pay less attention to children, an increase in the discrimination plays in favour of consumption. Hence, women increase their labor supply and the chosen tax rate is higher. These results may be matched with Portuguese data. Indeed, while among Southern European countries Portugal has the highest level of gender discrimination, its family policies on benefits in kind are the most generous of the group and the female labor force participation is also the highest of the cluster (see Section one).

Fertility Decisions

The relationship between fertility and gender discrimination is less clear, as it depends on preferences and assumptions made concerning variables of the model such as the efficiency of family policies, ε, and the total productivity of factors, A (see Appendix 6).

6   Conclusion

In this paper, the relationships between gender discrimination and household decisions have been presented through a general equilibrium model. The model shows that different levels of gender discrimination may explain divergences in household decisions across countries. The solution shows a U-shaped relationship between fertility and gender discrimination. An increase in the discrimination level may lead to a related decrease in fertility, women's employment and family policies. Beyond a discrimination threshold, spouses vote for a tax rate equal to zero. These results match with the positive correlation between childbearing and women's labor supply which has been observed since the mid-1980s in OECD countries. Female labor force participation and the size of public spending vary in the same way as in Cavalcanti and Tavares 2007 [14].

Some extensions of this paper can be proposed. Discrimination can be analyzed as social norms which differ from country to country. Besides, beyond the problem of discrimination, there are also the cultural attitudes toward working mothers which play a role in female labor supply decisions and more generally in household decisions. This would be the subject of further research. An econometric analysis could also be done to test the relevance of this model.

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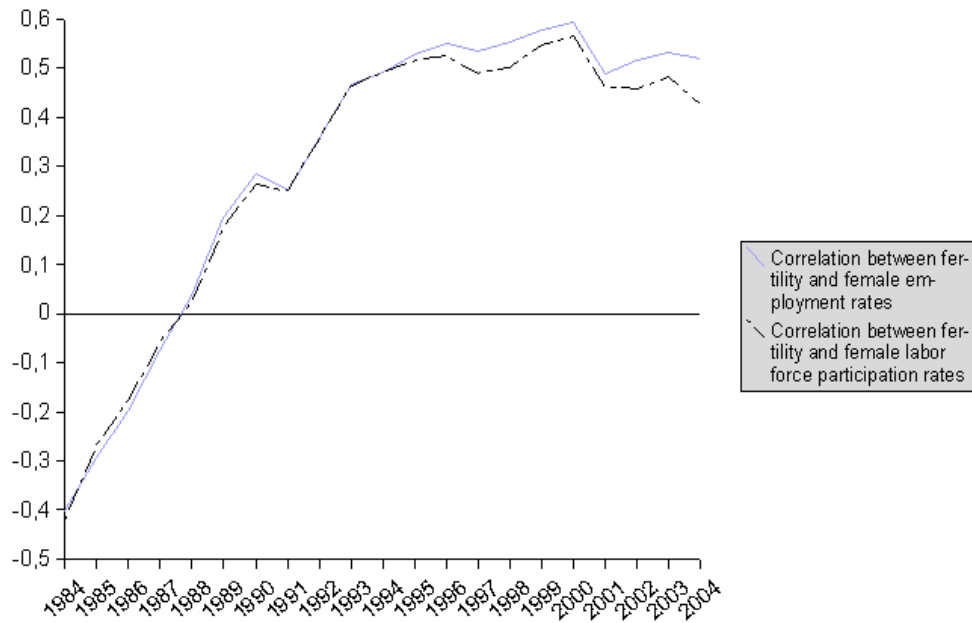
7   Appendix

Appendix 1

Figure 6. Correlation between fertility rates and female activity rates (line) and between fertility rates and female labor participation rates (dashed line)
Source : These correlations have been calculated for ten European countries: Austria, Denmark, France, Germany, Greece, Ireland, Italy, Portugal, Spain and the United Kindgom. Data come from the OECD database.

Appendix 2
First order condition of the maximization of spouses' indirect utilities:

$$\frac{\gamma\varepsilon}{(1+g)} - \frac{(1-\gamma)}{(1-\gamma)(w^f + w^m) - g} = 0$$

From this condition the expression of public spending can be found,

$$g = \frac{[\gamma\varepsilon(w^f + w^m) - 1](1-\gamma)}{(\varepsilon\gamma + 1 - \gamma)}$$

and also the time devoted by women to each child:

$$h(g) = \phi \left[\frac{\gamma \varepsilon [1 + (w^f + w^m)(1 - \gamma)]}{(\varepsilon \gamma + 1 - \gamma)} \right]^{-\varepsilon}$$

Appendix 3

The relationship between gender discrimination and fertility can be deduced from the derivative of fertility:

$$\frac{\mathrm{d} n(d)}{\mathrm{d} d} = Z \frac{[1 + (1 - \gamma)(2A - d)]^{\varepsilon - 1}}{(A - d)^2} \{ [1 + (1 - \gamma)(2A - d)]A - (2A - d)\varepsilon(1 - \gamma)(A - d) \}$$

$$\text{with } Z = \frac{\gamma (\gamma \varepsilon)^{\varepsilon}}{\phi (\varepsilon \gamma + 1 - \gamma)^{\varepsilon}}$$

Given $d \in [0, A[$, the derivative of fertility, $\frac{\mathrm{d} n(d)}{\mathrm{d} d}$, could be negative if the condition on public spending efficiency, $\varepsilon > (1 + \frac{1}{2A(1 - \gamma)})$, is satisfied.

Appendix 4

The couple's program for intra-family decisions :

$$\underset{c^f, c^m, n, h^f, h^m}{\text{Max}} \theta [\beta \ln(c^f) + \gamma \ln(n)] + (1 - \theta) [\beta \ln(c^m) + \gamma \ln(n)]$$

$$s.t. \, [w^f (1 - h^f) + w^m (1 - h^m)](1 - \tau) = c^f + c^m,$$

$$H = n \, h(g) = (h^f)^{\eta} (h^m)^{(1 - \eta)} \text{ and } \gamma + \beta = 1$$

The expression of the tax rate applied is the same as in the previous model. However, that of fertility differs from the benchmark model:

$$n = \frac{\gamma^{\varepsilon + 1}(1 - \eta)^{(1 - \eta)}\eta^{\eta} \varepsilon^{\varepsilon}(2A - d)[1 + (1 - \gamma)(2A - d)]^{\varepsilon}}{\phi(\gamma \varepsilon + (1 - \gamma))^{\varepsilon} A^{(1 - \eta)}(A - d)^{\eta}}$$

$$\frac{\mathrm{d} n(d)}{\mathrm{d} d} = s \{ -(1 - \gamma)\varepsilon(2A - d)(A - d) + [1 + (1 - \gamma)(2A - d)][A(2\eta - 1) + d(1 - \eta)] \}$$

$$\text{with } s = \frac{\gamma^{\varepsilon + 1}(1 - \eta)^{(1 - \eta)}\eta^{\eta}\varepsilon^{\varepsilon}}{\phi(\varepsilon \gamma + 1 - \gamma)^{\varepsilon} A^{(1 - \eta)}} \frac{[1 + (1 - \gamma)(2A - d)]^{\varepsilon - 1}}{(A - d)^{\eta + 1}}$$

Given $d \in [0, A[$, the derivative of fertility, $\frac{\mathrm{d} n(d)}{\mathrm{d} d}$, could be negative if the condition on public spending efficiency, $\varepsilon > \frac{(2\eta - 1)}{2A(1 - \gamma)} + (2\eta - 1)$, is satisfied.

Appendix 5

Results coming from the intra-family decisions program :

$$c^f(g) = \frac{\theta(1 - \gamma^f)(1 - \tau)}{\theta(1 - \gamma^f) + (1 - \theta)(1 - \gamma^m)} [(1 - \theta \gamma^f - (1 - \theta)\gamma^m)(2A - d)]$$

$$c^m(g) = \frac{(1 - \theta)(1 - \gamma^m)(1 - \tau)}{\theta(1 - \gamma^f) + (1 - \theta)(1 - \gamma^m)} [(1 - \theta \gamma^f - (1 - \theta)\gamma^m)(2A - d)]$$

$$n(g) = \frac{(2A - d)((1 - \theta)\gamma^m + \theta \gamma^f)}{\phi[1 + g]^{-\varepsilon}(A - d)}$$

Expressions of spouses' indirect utility are,

$$V^i = \gamma^i \ln(n(g)) + (1 - \gamma^i)\ln(c^i(g)) \, s.t. \, i = f, m$$

Following the probabilistic voting rule, the first order condition of the maximization program for fixing the tax rate is:

$$\frac{(\gamma^f + \gamma^m)\varepsilon}{(1 + g)} - \frac{(1 - \gamma^f) + (1 - \gamma^m)}{(2A - d)(1 - \theta \gamma^f - (1 - \theta)\gamma^m) - g} = 0$$

From this condition the expression of g can be found and after that of the tax rate in using the government budget constraint $(\tau = \frac{g}{(2A-d)(1-\theta\gamma^f-(1-\theta)\gamma^m)})$.

$$\frac{\mathrm{d}n(d,\theta)}{\mathrm{d}\theta} = \chi(\gamma^f - \gamma^m)\{1 + (2A-d)[1-(1+\varepsilon)((1-\theta)\gamma^m+\theta\gamma^f]\}$$

$$\text{with } \chi = \frac{(2A-d)[(\gamma^f+\gamma^m)\varepsilon]^\varepsilon[1+(2A-d)(1-\theta\gamma^f-(1-\theta)\gamma^m)]^{\varepsilon-1}}{(A-d)\phi[(\gamma^f+\gamma^m)\varepsilon+(1-\gamma^f)+(1-\gamma^m)]^\varepsilon}$$

Appendix 6

Effects of gender discrimination on household decisions:

$$\frac{\mathrm{d}t^f(d,\theta(d))}{\mathrm{d}d} = \frac{\mathrm{d}t^f(d,\theta(d))}{\mathrm{d}d} + \frac{\mathrm{d}t^f(d,\theta(d))}{\mathrm{d}\theta} \qquad \frac{\mathrm{d}\theta(d)}{\mathrm{d}d}$$

$$(-) \qquad\qquad (+)\,\text{if}\,\gamma^m > \gamma^f \qquad\qquad (-)$$

$$(-)\,\text{if}\,\gamma^f > \gamma^m$$

$$\frac{\mathrm{d}\tau(d,\theta(d))}{\mathrm{d}d} = \frac{\mathrm{d}\tau(d,\theta(d))}{\mathrm{d}d} + \frac{\mathrm{d}\tau(d,\theta(d))}{\mathrm{d}\theta} \qquad \frac{\mathrm{d}\theta(d)}{\mathrm{d}d}$$

$$(-) \qquad\qquad (+)\,\text{if}\,\gamma^m > \gamma^f \qquad\qquad (-)$$

$$(-)\,\text{if}\,\gamma^f > \gamma^m$$

$$\frac{\mathrm{d}n(d,\theta(d))}{\mathrm{d}d} = \frac{\mathrm{d}n(d,\theta(d))}{\mathrm{d}d} + \frac{\mathrm{d}n(d,\theta(d))}{\mathrm{d}\theta} \qquad \frac{\mathrm{d}\theta(d)}{\mathrm{d}d}$$

$$(-)\,\text{or}\,(+) \qquad\qquad (-)\,\text{or}\,(+) \qquad\qquad (-)$$