Real adjustment of current account imbalances with firms' heterogeneity

Francesco Pappadà

Documents de Travail du
Centre d'Economie de la Sorbonne

CES Working Papers

Real adjustment of current account imbalances with firms' heterogeneity

Francesco PAPPADA

2008.96

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://ces.univ-paris1.fr/cesdp/CES-docs.htm
ISSN : 1955-611X

Real Adjustment Of Current Account Imbalances With Firms' Heterogeneity

Francesco Pappadà*

December 8, 2008

Abstract

This paper investigates the impact of a real current account adjustment on terms of trade, aggregate productivity and welfare-based exchange rate in a two-country general equilibrium model. As in Melitz (2003), firms are heterogeneous in productivity, and endogenously enter and exit their domestic and export markets. The real adjustment of the current account leads to the increase of Home exports through the intensive and the extensive margins of trade: incumbent firms export more and new exporters endogenously enter the market. In the benchmark case, the extensive margin of trade accounts for about 19% of the overall adjustment and the depreciation of the national currency is lower with respect to models where this margin is not considered. In the literature, the change in the terms of trade is lower when goods are more substituable. This common finding is overturned by the endogenous entry of new exporters. For a given dispersion of productivity across firms, a higher elasticity of substitution reduces the role played by the extensive margin on the adjustment and yields a higher depreciation of the exchange rate.

Classification code: F4

Key words: global imbalances, real adjustment, depreciation, extensive margin, firms'heterogeneity.

*Paris School of Economics, University of Paris 1 Pantheon-Sorbonne, 106-112 boulevard de l'Hopital, 75013 Paris. E-mail: francesco.pappada@univ-paris1.fr. I thank Philippe Martin for his helpful comments.

1 Introduction

The current account deficit of the United States of America has steadily grown since 2001, reaching a peak at 7 % of the US GDP in 2006 and then slightly recovering to 5.3 % of US GDP in 2007.

As Obstfeld and Rogoff (2000, 2004, 2005) point out in their series of papers on this topic, the global current account imbalances do not represent a benign worldwide equilibrium among current account positions, and a sudden reversal of the global imbalances could be very costly for the US economy and the rest of the world. In particular, they show that the real adjustment of the US current account deficit could lead to a large depreciation of the US dollar (about 30 %) and imply a sharp reduction in US consumption and welfare.

In this paper, I build a two-country general equilibrium model to reproduce the global current account imbalances, and investigate the consequences of an exogenous real adjustment of the global imbalances on the exchange rate and the aggregate productivity in a long-run steady-state dynamic. Firms are heterogeneous in productivity, and endogenously enter and exit their domestic and foreign markets. The assumption of firms' heterogeneity is the main innovation with respect to models with representative firms, and is fundamental to assess the role played by the entry of new exporter firms on the current account adjustment. The endogenous entry of new exporters, that is the extensive margin of trade, dampens the required depreciation of the exchange rate because the increase in the aggregate of exports occurs for a smaller change in the terms of trade. Furthermore, I find that the magnitude of the extensive and intensive margin of trade over the current account adjustment strictly depends on the interaction between the elasticity of substitution and the dispersion of productivity across firms. For given elasticity of substitution, when the dispersion of productivity across firms is low, the extensive margin of trade plays a greater role on the adjustment and the depreciation of the exchange rate is low.

In the literature on the effects of transfers, and more recently on the effects of global rebalancing, the attention is focused on the elasticity of substitution between home and foreign-produced goods. The elasticity of substitution is the key parameter in the calibration of these Macroeconomic models, and the magnitude of the results on the terms of trade relies directly on the choice of this parameter. A common finding in the literature is that a higher level of elasticity of substitution among goods leads to a lower change in the terms of trade.

In this paper, the endogenous entry of new exporter firms overturns this common result. There are two opposite effects of a higher elasticity of substitution. On the one hand, as in standard models, a higher elasticity of substitution means that less of a price change is required to increase net exports on the intensive margin. However, when the elasticity of substitution is higher, the sales of firms that enter the export market in response to the current account adjustment are more sensitive to productivity.

As new exporter firms are less productive and smaller than incumbent exporters, a higher elasticity of substitution requires more of the adjustment through the intensive margin and therefore a higher price change. For a given dispersion of productivity across firms, I show that the second mechanism is the predominant one. This result is related to the one of Chaney (2008) on the effects of trade liberalization in a similar model.

The model is built in a long-run perspective and does not focus on the financial channel[1] of adjustment of the global current account imbalances. Likewise, the financial channel plays a minor role in the long term, and the changes in net US exports are the main source of the adjustment. In Obstfeld and Rogoff (2004), the real adjustment of the global imbalances occurs through a transfer of resources coming from the deficit country towards the rest of the world. The transfer affects positively the income of households in the rest of the world, and increases their demand for US exports. On the contrary, US households suffer a reduction in their income, they demand less goods, and US imports shrink. Notwithstanding, the well-known debate between Keynes and Ohlin teaches us that the transfer of resources has effects both on income and terms of trade. This means that the adjustment requires a depreciation of the exchange rate, which is around 30 % according to Obstfeld and Rogoff (2004). In their two-sector model with representative firms producing either one tradable or one non-tradable good, they claim that the rise in the productivity of the foreign non-tradable producers could make the adjustment of the US current account easier. On the contrary, if also the foreign exporters are more productive, their products are cheaper for US households and the current account position could eventually be worsened.

I focus on a long-term perspective of current account adjsutment and therefore I put the emphasis on how the structure of trade patterns may be affected by the adjustment[2]. Several empirical papers have shown the importance of the extensive margin in international trade, that is the exports of new firms or new products into a foreign market. Galstyan and Lane (2008) show that the extensive margin of trade plays a substantial role in the recent trade dynamics of major surplus and deficit countries. The seminal paper by Melitz (2003) gives an elegant theoretical explanation of such stylized fact as, in this paper, firms endogenously decide to enter or exit the foreign market according to their level of productivity. Chaney (2008) extends the work of Melitz in a multi-country model and shows that, in a model where the extensive margin of trade is at work, the impact of trade barriers is dampened and not magnified by the elasticity of substitution, as expected in Krugman (1980).

Corsetti, Martin and Pesenti (2008) introduce the extensive margin of trade in their model of current

[1]Further discussions on this subject can be found in Lane and Milesi-Ferretti (2006), Gourinchas and Rey (2007a, 2007b), and Curcuru, Dvorak and Warnock (2007).

[2]Dekle, Eaton and Kortum (2008) look at the implications of the reversal of the US current account deficit on relative wages, relative GDPs and real wages.

account adjustment. Once the transfer is done, some firms producing for the non-traded sector enter in the set of the exporter firms, and so contribute to the adjustment. They find that the extensive margin of trade dampens the required depreciation of the exchange rate as new firms become exporters and the adjustment occurs for a lower change in terms of trade. In Corsetti, Martin and Pesenti (2008), firms do not differ in productivity and the impact of the extensive margin over the adjustment depends on their hypotheses on the convexity of the cost function and the elasticity of the labor supply.

This paper studies the impact of a real adjustment of global imbalances in a model with firms' heterogeneity and endogenous self-selection mechanism amongst firms, as in Melitz (2003) and Chaney (2008). The interaction between the elasticity of substitution and the dispersion of productivity across firms is fundamental to assess the role played by the extensive and the intensive margin of trade on the overall adjustment. The dispersion of productivity across firms becomes a key parameter to look at, and opens the way to unexplored findings in the International Macroeconomics' literature.

The remainder of the paper is organised as follows. Section 2 introduces a two-country model with firms' heterogeneity and derives the analitical results of a real current account adjustment at the general equilibrium. Section 3 analyses the implications of such adjustment on all the endogenous variables in the model. In section 4, I report the results of numerical simulations, which show the impact of different levels of dispersion of productivity across firms over the exchange rate depreciation. Section 5 concludes.

2 The adjustment of current account imbalances in a model with firms' heterogeneity

2.1 Model framework

In a simple model framework with two countries, Home and Foreign, and one tradable sector, each firm produces one variety and pays a fixed and a variable cost, which depends negatively on the firm's productivity. In this model with only one tradable sector, all firms are potentially exporters.

Free entry conditions on the profits made by Home firms on domestic and foreign market determine two levels of productivity thresholds. These two cut-offs on productivity endogenously define the number of firms producing for the domestic and the foreign market. Actually, the exporter firms are those with a level of productivity above the threshold required to be active on the foreign market. Other firms, with a lower level of productivity, are active just on the domestic market and finally, least productive firms are forced to be inactive. This means that this simple model can account for the endogenous non-tradedness of less productive firms and so match the stylized facts on productivity and exports.

Suppose that the Home country is running a current account deficit. I define the adjustment of the current account of the Home country as a real adjustment implying an increase in the exports and a decrease in the imports. This means that the demand addressed to Home exports has to increase while the demand of Home households for imports has to decrease.

Without any kind of shock on preferences or technology, the only way to obtain this adjustment is an increase in Foreign income and a decrease in Home income. The simplest way to modelize this change in the income of the Home and Foreign country is to introduce a transfer of resources, T, going from the Home country (in deficit) towards the Foreign country. In reality, this amount of resources is residual to the adjustment, but here I take it as the exogenous driving force which makes the adjustment being possible. As a result, the exogenous transfer T determines the level of the exchange rate ϵ, which is endogenous in the model.

The balanced current account for the Home country can be therefore written as follows:

$$EXP - IMP - T = 0$$

The adjustment of the current account deficit passes through several effects. First, the Ohlin's income effect: the Home country exports more than before because the Foreign country is richer and demands more of the Home produced goods. Second, the Keynes's terms-of-trade effect: the transfer of resources produces a change in relative prices and leads to the depreciation of the Home currency. These two effects are analysed in a framework where firms are heterogeneous in productivity and endogenously decide to serve only their local market or to be also exporters.

Production

Firms in the economy produce the final tradable good for the domestic and the export market by using the labor as the only input. Wages are taken as the numeraire and equalized to 1. The output is equal to:

$$Y_i = \alpha(x) \cdot L_i$$

where $i = H, F$. I define $\alpha(x) \equiv \tilde{x}_i$ as the aggregate productivity of labor in the country i with $\frac{\partial \alpha}{\partial x} > 0$. The labor employed in the production is free to move across firms in the same country but cannot be employed by firms in the other country (there is not migration), and the total amount of labor workforce in this economy is given simply by $L = L^H + L^F$.

Firms have access to the same technology but the productivity of labor (x) that each of them uses as input differs across firms, and so generates firms' heterogeneity. The productivity of labor (x) is drawn from a Pareto distribution with the shape parameter γ, so that the cumulative distribution function of prductivity can be written as follows:

$$F(x) = 1 - \left(\frac{b}{x}\right)^{\gamma}, \quad \text{with} \quad x \geq b > 0$$

where b is a scale parameter that binds the support $[b, +\infty)$ from below. Log x is then distributed exponentially with a standard deviation equal to $1/\gamma$. Any truncation from below x is also distributed Pareto with the same shape parameter γ and, by definition, x has a finite variance if and only if $\gamma > 2$.

A long-run adjustment

The real adjustment of the current account is analysed in a steady state long-run dynamic. At a given moment in the time, the transfer shocks the economy, and I focus on the implications of this shock on the productivity thresholds of the domestic-oriented and the exporter firms at the general equilibrium. In this framework, I consider a given mass M_i of firms which productivity is distributed following the Pareto law of distribution as previously shown. With respect to Melitz (2003), there is no pure entry of firms[3] into the distribution but just a reallocation of the existing M_i firms, after the adjustment of the current account.

As in Chaney (2008), the number of potential entrepreneurs drawing a productivity shock from the distribution is given and proportional to the size of the country i. Therefore $M_i = \beta_i \cdot L_i$ where β_i is the ratio of entrepreneurial activity in country i. For simplicity I take $\beta_i = 1 \; \forall i$ in the model and the potential number of firms in the economy is equal to L_i: some of them sell only in the domestic market, some sell also in the export market, others shut down and stay inactive.

Finally, the only degree of heterogeneity across firms concerns their productivity of labor (x), therefore each firm has the same size and employs one unit of labor. This results in more simple expressions for the aggregate of the labor demand and the total output.

Function Cost

Firms producing for the domestic and foreign market face the same function cost. It consists in two parts: the variable cost which depends negatively on the productivity of labor and increases in the quantity of goods produced ($\frac{q}{x}$), and the fixed cost (F_j) which is a given amount of worked hours that firms have to pay in order to enter the local (D) or the export (X) market:

$$\overline{C}(x) = w\left(\frac{q}{x} + F_j\right) \quad \text{with} \quad F_j = \begin{cases} F_D & \text{if} \;\; j = D \\ F_X & \text{if} \;\; j = X \end{cases}$$

The function cost is the same across the two countries because the fixed cost of entering the local market (F_D), in terms of worked hours, is assumed to be the same for country H and F as well as the fixed cost of entering the export market (F_X).

Also, for both the Home and the Foreign country, the export fixed cost is

$$F_X = (1 + \delta)F_D \quad \text{with} \quad \delta > 0$$

[3]Actually, in this model, firms know their level of productivity and do not have to pay an initial sunk cost to draw one productivity shock and enter the market.

Parameter δ reflects the difference between the fixed export cost and the fixed local cost. Actually, when firms export, they face transaction costs greater than those payed when they sell on their local market. As a consequence, the exporter firms need more worked hours to pay these additional transaction costs. The presence of these fixed costs generates a process of self-selection amongst the firms serving the local or the export market. According to this process, the less productive firms cannot pay the fixed costs, and are therefore forced to be inactive. The remaining firms choose to sell their varieties only in their local market and can even choose to export. In the end, even if there's only one tradable sector in this economy, the model can account for the endogenous nontradedness by the less productive firms and the exit by the least productive ones.

Households

$$U = \log C$$

In the Home country the utility of the representative household is a positive function of consumption, C. Consumption for Home households is based on a panel of domestic-produced good and imports (foreign export), composed in the CES function shown below:

$$U = \log \left[\int_{\overline{x}_D}^{\infty} q_D(x)^{1-\frac{1}{\sigma}} dF(x) + \int_{\overline{x}_X^*}^{\infty} q_X^*(x^*)^{1-\frac{1}{\sigma}} dF(x^*) \right]^{\frac{\sigma}{\sigma-1}}$$

$\overline{x}_D$ is the threshold on the productivity of domestic firms which are active on the domestic market, and $\overline{x}_X^*$ is the threshold on the productivity of foreign firms who export in the Home country. σ denotes the elasticity of substitution across varieties, as well as the elasticity of substitution between import-competing goods $q_D(x)$ and imports $q_X^*(x^*)$.

The budget constraint of the representative Home household is

$$\int_{\overline{x}_D}^{\infty} p_D(x) c_D(x) dF(x) + \int_{\overline{x}_X^*}^{\infty} \epsilon p_X^*(x^*) c_X^*(x^*) dF(x^*) + I \leq 1 + \frac{\Pi}{L} - \frac{T}{L^H}$$

Note that $c_D(x)$ is the consumption of domestic-produced goods, $c_X^*(x^*)$ is the consumption of foreign-produced goods and the price of imports, p_X^*, is expressed in the Foreign currency. I is the investment in a well-diversified international portfolio of claims on firms' profits worldwide.

On the right hand side of the budget constraint, the wage of the Home household is equal to 1, and $\frac{\Pi}{L}$ is the share of the world total profits which are equally redistributed to the households in the economy. Finally, the income of each Home household is reduced by an amount equal to $\frac{1}{L^H}$ of the transfer of resources going from the Home country to the Foreign country (the rest of the world).

Prices

In this model all prices are defined in their national currency, so the term ϵ is used to identify the exchange rate between the national and the foreign currency. The exchange rate ϵ is expressed by home units of labor in terms of foreign ones and is therefore equal to the wage ratio between Home and Foreign country $\left(\frac{w_H}{w_F}\right)$. The prices can be written as:

$$p_D(x) = \frac{\sigma}{\sigma-1}\frac{1}{x}w \qquad p_X(x) = \frac{\sigma}{\sigma-1}\frac{\tau}{x}w \qquad p_D^*(x^*) = \frac{\sigma}{\sigma-1}\frac{1}{x^*}w^* \qquad p_X^*(x^*) = \frac{\sigma}{\sigma-1}\frac{\tau}{x^*}w^*$$

From the CES utility function, the price index for the Home country is

$$P = \left[\int_{\overline{x}_D}^{\infty} p_D(x)^{1-\sigma} dF(x) + \int_{\overline{x}_X^*}^{\infty} [\epsilon p_X^*(x^*)]^{1-\sigma} dF(x^*)\right]^{\frac{1}{1-\sigma}}$$

and, by the same means, the price index for the Foreign country is

$$P^* = \left[\int_{\overline{x}_D^*}^{\infty} [\epsilon p_D^*(x^*)]^{1-\sigma} \, dF(x^*) + \int_{\overline{x}_X}^{\infty} p_X(x)^{1-\sigma} dF(x)\right]^{\frac{1}{1-\sigma}}$$

Note that the two price indexes are both defined in terms of Home currency.

Exports

Now I turn to the analysis of the international trade and look at the profits made by firms who decide to export. The value of exports for one firm in the Home country is simply $h(x) = p_X(x) \cdot q_X(x)$, and the same is true for the exports of one firm in the Foreign country, $l(x^*)$. Therefore, the value of exports for one firm in the Home and the Foreign country, both expressed in Home currency, can be written as:

$$h(x) = \left[1 + \frac{\Pi}{L} + \frac{T}{L^F}\right] L^F \left[\frac{p_X(x)}{P^*}\right]^{1-\sigma} \quad \text{and} \quad l(x^*) = \left[1 + \frac{\Pi}{L} - \frac{T}{L^H}\right] L^H \left[\frac{\epsilon p_X^*(x^*)}{P}\right]^{1-\sigma}$$

The only variables which affect the sales on the domestic and foreign market are the exogenous transfer of resources T, and the endogenous exchange rate ϵ, while the key parameters are the elasticity of substitution σ and the Pareto shape parameter on productivity γ. With respect to these parameters, I assume that $\gamma > \sigma - 1$, which ensures that both the distribution of productivity draws and the distribution of firm's sales have finite variances.

2.2 Free entry conditions

This model provides endogenously the entry and exit of Home firms in the domestic and foreign market as they choose their local-oriented or export activity by comparing profits on the Home and Foreign market. Each firm is a monopolist for the variety it produces and faces free entry conditions, which imply that profits have to be equal to 0.

The following free entry condition applies to the Home firms producing for the domestic market:

$$\pi_D^H = \frac{1}{\sigma}\left[1+\frac{\Pi}{L}-\frac{T}{L^H}\right]L^H\left(\frac{p_D(x)}{P}\right)^{1-\sigma} - F_D = 0 \qquad \text{(FEC 1)}$$

As wages are the numeraire, F_D represents the value of worked hours paid by Home firms entering the Home market. This value is expressed in terms of Home currency, as well as all the profits in the free entry conditions. The solution to (FEC 1) provides the value of the Home local market threshold of productivity $\bar{x}_D$ obtained for given price index P. The free entry condition tells us that the Home firms with a level of productivity above $\bar{x}_D$ are active on the local market and produce import-competing goods, while the others are forced to exit.

The same free entry condition applies then on the firms in the Foreign country[4]:

$$\pi_D^F = \frac{1}{\sigma}\left[1+\frac{\Pi}{L}+\frac{T}{L^F}\right]L^F\left(\frac{\epsilon p_D^*(x^*)}{P^*}\right)^{1-\sigma} - \epsilon F_D = 0 \qquad \text{(FEC 2)}$$

The solution to (FEC 2) gives the Foreign local market threshold of productivity $\bar{x}_D^*$ for the given price index P*. When looking at the export market, the free entry condition for the Home exporter firm is

$$\pi_{exp}^H = \frac{1}{\sigma}\left[1+\frac{\Pi}{L}+\frac{T}{L^F}\right]L^F\left[\frac{p_X(x)}{P^*}\right]^{1-\sigma} - (1+\delta)F_D = 0 \qquad \text{(FEC 3)}$$

(FEC 3) determines the critical threshold of labor productivity for the Home exporter firm $\bar{x}_{exp}$. All Home firms with a level of productivity above $\bar{x}_{exp}$ are exporters on the Foreign market, whereas firms with a level of productivity $\bar{x}_D \leq x < \bar{x}_{exp}$ are active just on the domestic market, and finally, least productive firms are forced to be inactive.

By the same means, the value of profits for the Foreign exporter firm, expressed in the Home currency, can be written as:

$$\pi_{exp}^F = \frac{1}{\sigma}\left[1+\frac{\Pi}{L}-\frac{T}{L^H}\right]L^H\left[\frac{\epsilon p_X^*(x^*)}{P}\right]^{1-\sigma} - \epsilon(1+\delta)F_D = 0 \qquad \text{(FEC 4)}$$

Finally, this condition on the export market for Foreign firms gives the critical threshold of labor productivity $\bar{x}_{exp}^*$ for the firms entering the Home market.

2.3 Equilibrium price indexes and productivity thresholds

The equilibrium price indexes in the Home and Foreign market depend on the firms operating in that country and, as firms are heterogeneous in the model, their entry and exit on the Home and Foreign

[4]Once again note that also the profits of Foreign local-oriented firms are expressed in the Home currency and ϵF_D is the value, expressed in terms of Home currency, of the worked hours paid by the Foreign firms entering their local market.

market imply that the equilibrium price indexes depend on the productivity of active firms.

Plugging the productivity thresholds obtained from the equations (FEC 1) and (FEC 4) into the Home price index, the equilibrium price index for the Home country becomes

$$P = \left[\mu_3 \left(\frac{F_D}{L^H}\right)^{\frac{\sigma-\gamma-1}{\sigma-1}} \left(1 + A\tau^{-\gamma}\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}}\right)\right]^{-\frac{1}{\gamma}} \tag{1}$$

Where μ_3 depends positively on the transfer of resources T and on total profits Π which are constant and not affected by the transfer T. A is also a constant equal to $A = (1+\delta)^{\frac{\sigma-\gamma-1}{\sigma-1}}$.

In the Foreign country, the price index depends on the productivity thresholds of the Foreign local-oriented firms (FEC 2) and the Home exporter firms (FEC 3). The equilibrium price index for the Foreign country is

$$P^* = \left[\mu_4 \left(\frac{F_D}{L^F}\right)^{\frac{\sigma-\gamma-1}{\sigma-1}} \left(\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}} + A\tau^{-\gamma}\right)\right]^{-\frac{1}{\gamma}} \tag{2}$$

The equilibrium price indexes of the Home and Foreign country are both expressed in terms of Home currency and depend on the transport iceberg cost τ, on the exogenous fixed cost F_D, and on the transfer of resources T coming from Home toward the Foreign country. In the case of the Foreign price index, note that μ_4 depends negatively on the transfer of resources T[5].

From the point of view of the Home country, the outgoing transfer of resources reduces the national income and thus the demand of Home households is reduced as well. If the supply of goods is given, the Home price index is expected to decrease. On the other hand, the raise in the exchange rate, which is the depreciation of the Home currency, increases the Home price index as imports become more expensive. Once the transfer is done, the demand of goods increases in the Foreign country, and the Foreign price index tends to go up although a depreciation of the Home currency makes imports cheaper.

This is true when the supply is considered as given. In this model, the change in relative incomes between the Home and Foreign country, and the implications of the transfer on the exchange rate do affect the supply through the Free Entry Conditions. FECs on the Foreign and Home market give the critical thresholds on productivity for local-oriented and exporter firms operating on both countries. These thresholds are obtained for given price indexes, but the equilibrium value of the productivity thresholds on the Home and Foreign market can be found using the equilibrium price indexes of the Home and Foreign country previously determined. As a result, each productivity cut-off depends only on the exogenous transport cost τ, on the endogenous exchange rate ϵ and on the transfer which is the exogenous driving force in this model. Table 1 reports the equilibrium productivity thresholds[6] for

[5]See the Appendix for further details on the value of μ_3 and μ_4 and on the intermediate steps to find the equilibrium price indexes and thresholds.

[6]In the Home market, productivity thresholds $\bar{x}_D$ and $\bar{x}^*_{exp}$ depend positively on T (through λ_1) whereas in the Foreign market $\bar{x}^*_D$ and $\bar{x}_{exp}$ depend negatively on T (through λ_2).

local-oriented and exporter firms in the Home and Foreign market. These values are crucial to analyze the general equilibrium implications of the shock on the current account, the exchange rate and the change in productivity cut-offs.

[Table 1 about here.]

2.4 The balanced current account at the symmetrical equilibrium

The balanced current account at equilibrium is[7]

$$EXP - IMP - T = 0$$

The aggregate values of Home exports and imports, both expressed in terms of Home currency, are

$$EXP = L^H \int_{\bar{x}_{exp}}^{\infty} h(x) dF(x)$$

$$IMP = L^F \int_{\bar{x}^*_{exp}}^{\infty} l(x^*) dF(x^*)$$

For reasons of simplicity, as in Chaney (2008), I assume that the number of producers is proportional to the size of the country and there is no heterogeneity in the size of firms. In other words, each firm uses one unit of labor expressed in worked hours, so all firms have the same size but the unit of labor does not have the same productivity across firms (heterogeneity of x).

In order to analyze the impact of the transfer of resources over the balanced current account, I total differentiate it:

$$dEXP - dIMP - dT = 0 \qquad (3)$$

Using the Leibniz's integral rule to differentiate aggregate exports and imports, I can disentangle the intensive from the extensive margin of the adjustment of the current account through the transfer of resources and the exchange rate. The total differential of aggregate exports is

$$dEXP = \underbrace{\left[\int_{\bar{x}_{exp}}^{\infty} \frac{\partial h(x)}{\partial T} dF(x)\right]}_{\text{Intensive margin}} dT - \underbrace{\left[F'(\bar{x}_{exp}) \cdot h(\bar{x}_{exp}) \cdot \frac{\partial \bar{x}_{exp}}{\partial T}\right]}_{\text{Extensive margin}} dT +$$

$$+ \underbrace{\left[\int_{\bar{x}_{exp}}^{\infty} \frac{\partial h(x)}{\partial \epsilon} dF(x)\right]}_{\text{Intensive margin}} d\epsilon - \underbrace{\left[F'(\bar{x}_{exp}) \cdot h(\bar{x}_{exp}) \cdot \frac{\partial \bar{x}_{exp}}{\partial \epsilon}\right]}_{\text{Extensive margin}} d\epsilon$$

and, by the same means, the total differential of aggregate imports is

[7]The current account actually includes also the Net Foreign Asset position of the country, but here financial markets are perfectly diversified and therefore NFA is equal to 0.

$$dIMP = \underbrace{\left[\int_{\bar{x}^*_{exp}}^{\infty} \frac{\partial l(x^*)}{\partial T} dF(x^*)\right]}_{\text{Intensive margin}} dT - \underbrace{\left[F'(\bar{x}^*_{exp}) \cdot l(\bar{x}^*_{exp}) \cdot \frac{\partial \bar{x}^*_{exp}}{\partial T}\right]}_{\text{Extensive margin}} dT +$$

$$+ \underbrace{\left[\int_{\bar{x}^*_{exp}}^{\infty} \frac{\partial l(x^*)}{\partial \epsilon} dF(x^*)\right]}_{\text{Intensive margin}} d\epsilon - \underbrace{\left[F'(\bar{x}^*_{exp}) \cdot l(\bar{x}^*_{exp}) \cdot \frac{\partial \bar{x}^*_{exp}}{\partial \epsilon}\right]}_{\text{Extensive margin}} d\epsilon$$

The value of $dEXP$ and $dIMP$ is obtained solving integrals and evaluating them at the symmetrical equilibrium. In the end, the equation (3) can be written as follows[8]:

$$\underbrace{2\left(\frac{\sigma-1}{\gamma}\right) B\, dT}_{\text{intensive}} + \underbrace{\left[1+\frac{\Pi}{L}\right] L^H \cdot B \left[\frac{\sigma-\gamma\sigma-1}{\gamma} \cdot \frac{A\tau^{-\gamma}-1}{1+A\tau^{-\gamma}} + \sigma - 1\right] d\epsilon}_{\text{intensive}} +$$

$$+ \underbrace{2\left(\frac{\gamma-\sigma+1}{\gamma}\right) B\, dT}_{\text{extensive}} + \underbrace{\left[1+\frac{\Pi}{L}\right] L^H \cdot B \left[\frac{\sigma-\gamma\sigma-1}{\gamma} \cdot \frac{A\tau^{-\gamma}-1}{1+A\tau^{-\gamma}} \cdot \frac{\gamma-\sigma+1}{\sigma-1} + \frac{\sigma}{\sigma-1} \cdot (\gamma-\sigma+1)\right] d\epsilon}_{\text{extensive}} +$$

$$- \underbrace{dT}_{\text{direct effect of T}} = 0 \tag{4}$$

The intensive and extensive margins of the adjustment of the current account are all positive, both with respect to a change in the exogenous driving force T and in the endogenous exchange rate ϵ. This means that when the current account adjusts, both the extensive and the intensive margin of trade increase and therefore move together, as several empirical contributions illustrate[9].

In particular, the first term in equation (4) represents the intensive margin of trade due to the transfer of resources. This term refers to the Ohlin's income effect, according to which the lower demand of the Home country with respect to the Foreign country reduces Home imports and increases the sales of the existing exporter firms without changes in relative prices. The second term refers to the intensive margin of trade dealing with the change in relative prices. This is the Keynes's terms-of-trade effect. The existing exporters increase their sales because the exchange rate depreciates after the transfer and their goods are cheaper for Foreign households.

The third and fourth terms in equation (4) replicate the Ohlin's and Keynes's effects but in a new extent: the extensive margin of trade. The Ohlin's income effect and the Keynes's terms-of-trade effect produce a change in exports and imports and contribute to the adjustment through the entry of new

[8]With B being a positive constant equal to $B = \frac{A\tau^{-\gamma}}{1+A\tau^{-\gamma}}$

[9]For an analysis of the size and the dynamics of the extensive and intensive margin of trade with respect to a trade liberalizaton, see Bernard, Redding and Schott (2006).

exporter firms in the Foreign market. This new element explains why, in this model with endogenous entry and exit of firms, the change in relative prices and so the required depreciation of the Home currency is lower than in a model with representative firms where the supply of exported goods does not change after the shock.

At this point, it is very important to compare the relative size of the extensive and the intensive margin, in order to understand which of the two margins is more important on the adjustment of the current account. Looking at the previous equation both with respect to T and ϵ, two cases arise about the relative size of the two margins, according to the level of γ and σ:

$$\begin{cases} \sigma - 1 < \gamma \leq (\sigma - 1)(2 - \frac{1}{2\sigma}) & \text{Intensive margin} \geq \text{Extensive margin} \\ \\ \gamma > (\sigma - 1)(2 - \frac{1}{2\sigma}) & \text{Extensive margin} > \text{Intensive margin} \end{cases}$$

For a given σ, the intensive margin is greater than the extensive margin for relatively low levels of γ, that is when the heterogeneity across firms is high. This means that the impact of new exporters over the adjustment of the current account is lower when there is more heterogeneity, as the more productive firms have a greater weight over the aggregate of exports. When γ is high, firms are less heterogeneous, therefore there is no great difference in term of productivity between exporters and domestic-oriented firms. As a result, the impact of the entry of new exporters over the aggregate of exports is relevant and so the extensive margin is greater than the intensive margin.

On the other hand, for a given level of γ[10], when the elasticity of substitution among goods σ is sufficiently low, the extensive margin is greater than the intensive margin. This happens because with a low level of σ, goods are highly differentiated and the market share of firms, even for the less productive, is high. Therefore, the entry of new exporters is relevant over the aggregate exports more than the change due to existing exporters (intensive margin). Conversely, when σ is high, goods are more substituable and the differences in productivity matter again because the more productive exporter firms charge smaller prices and sell more. As a result, the intensive margin is greater than the extensive margin.

In the end, the combination of a low level of σ and a high level of γ makes the extensive margin being very important with respect to the intensive margin. In this case, firms are quite homogeneous and not very productive, goods are highly differentiated, thus the endogenous entry of firms into the export market matters a lot for the adjustment.

On the other hand, when firms are very heterogeneous and goods are not very differentiated, the extensive margin still plays a role in the adjustment of the current account but its impact is very small

[10]As always, such that: $\gamma > \sigma - 1$ and $\gamma > 2$ in order to ensure a finite variance of the distribution of the productivity.

with respect to the intensive margin's impact.

The overall depreciation of the exchange rate

Now, I add the two margins of the adjustment with respect to dT and $d\epsilon$ in equation (4) in order to find the equilibrium value of $\frac{d\epsilon}{dT}$ which balances the current account after the introduction of a transfer towards the Foreign country. This results in:

$$\frac{d\epsilon}{dT} = \frac{\left(1+\frac{\Pi}{L}\right)^{-1} \frac{2}{L} \left(\frac{A\tau^{-\gamma}-1}{A\tau^{-\gamma}}\right)}{\frac{\sigma-\gamma\sigma-1}{\sigma-1}\left(\frac{2}{1+A\tau^{-\gamma}}\right)}$$

The value of total world profits made by firms in the Home and Foreign country is $\Pi = \frac{\sigma-1}{\gamma\sigma-\sigma+1}L$. Note that total world profits do not depend on the transfer T. This is due to the Pareto function of distribution of firms's productivity as the transfer affects the profits of some firms, but the extra-profits made by some are exactly equal to the losses of others. The previous expression is rearranged as

$$\frac{d\epsilon}{dT/L^H} = \frac{1}{2} \cdot \frac{\sigma-1}{\gamma\sigma}\left[\frac{1-\chi^2}{\chi}\right] > 0 \qquad (5)$$

where $0 < \chi < \tau^{1-\sigma}$ is an index of trade freeness and is equal to $A\tau^{-\gamma}$. This index reproduces the freeness of trade for the firms as it depends on the fixed cost to export δ and the iceberg transport cost τ. As it has been shown by Chaney (2008), these costs are sensitive to the level of heterogeneity γ, where high γ prevents firms to trade as the transport cost as well as the fixed export cost become much too expensive. Furthermore, the index of trade freeness can also be defined as $\chi = \frac{EXP}{DOM}$, as the fixed cost to export δ and the iceberg transport cost τ are the additive costs which differentiate the domestic sales from the export sales.
Given the definition of the index of trade freeness χ, I can write $\frac{1-\chi^2}{\chi} = \frac{1-\chi}{\text{openness rate}}$, where the openness rate is simply the ratio between exports and GDP. Equation (5) translates in

$$\frac{d\epsilon}{dT/L^H} = \frac{1}{2} \cdot \frac{\sigma-1}{\gamma\sigma} \cdot \frac{1-\chi}{\text{openness rate}} > 0 \qquad (6)$$

The depreciation of the exchange rate depends on the openness rate of the country. Focus on the third term in equation (6). A higher index of trade freeness χ generates a higher openness rate, and the third term in equation (6) therefore decreases. This means that the depreciation of the exchange rate is smaller when the economy is more open. Conversely, for a given openness rate, the depreciation of the exchange rate depends negatively on the dispersion of productivity γ. This is the extensive margin effect previously illustrated: for a given elasticity of substitution, when there is little dispersion in productivity (γ is high) the aggregate impact of the extensive margin of trade is higher and dampens the required depreciation[11].

Now look at the impact of a change in the elasticity of substitution on the required depreciation. Formally, this effect is captured by

$$\frac{\partial\left(\frac{d\epsilon}{dT/L^H}\right)}{\partial\sigma} = \underbrace{\frac{1}{2}\frac{\gamma\sigma-\sigma+1}{(\gamma\sigma)^2}\cdot\frac{1-\chi^2}{\chi}}_{(+)} + \underbrace{\frac{1}{2}\frac{1-\sigma}{\gamma\sigma}\frac{1+\chi^2}{\chi}\cdot D}_{(-)} \tag{7}$$

where $D \equiv ln(1+\delta)^{\frac{\gamma}{(\sigma-1)^2}} > 0$. The second term in equation (7) represents one common finding in the International Macroeconomics' literature. When the elasticity among goods is higher, the change in aggregate exports occurs for a smaller change in prices. In a standard model without entry of new exporter firms, this is the only impact of the elasticity of substitution on the required depreciation. In this model, the extensive margin of trade is at work, and the elasticity of substitution determines the aggregate impact of the entry of new exporter firms on the overall current account adjustment. The new exporter firms are less productive than the existing exporters, and this differential in productivity is strengthened by the higher elasticity of substitution. As a result, the aggregate impact of the entry of new exporter firms is smaller. A large part of the adjustment of the current account goes through the intensive margin of trade, and the depreciation of the exchange rate is greater (first term in equation (7)).

For a given openness rate, the effect of a higher σ is always that the required depreciation is higher (see equation (6)). Furthermore, the positive effect of the elasticity of substitution over the depreciation of the exchange rate in equation (7) is predominant for reasonable values of the parameters γ and σ. The numerical simulations in section 4 show this effect. The depreciation of the exchange rate increases with the elasticity of substitution and the common finding in standard models is therefore overturned.

3 Implications of the current account adjustment

3.1 Productivity thresholds and exchange rates

This section shows the impact of the transfer on all the endogenous variables in the model. For a generic variable a, the impact of the transfer on a is captured by $\widehat{a} = \frac{da/a}{dT/L^H}$. The endogenous variables in the model are the following: the domestic and exporter thresholds for Home and Foreign country, the exchange rate ϵ, the welfare-based exchange rate RER (equal to $\frac{P^*}{P}$), the aggregate consumption C, and the welfare of the Home country. Table 2 reports these variables and shows the impact of the transfer on them at the symmetrical equilibrium.

[11] In section 4, I run numerical simulations to reproduce this dispersion of productivity's effect on the depreciation while controlling for the openness rate of the country.

[Table 2 about here.]

The total differentiation of the current account where $\epsilon = 1$, $T = 0$ and $L^H = L^F$ tells us that a marginal variation by the transfer reduces the required cut-off on the productivity of Home exporter firms. There is therefore entry of Home firms in the export activity while the cut-off on Home local-oriented firms goes up, so less productive Home firms that were active on the domestic market are forced to exit.

On the Foreign country, the ingoing transfer raises the level of productivity needed to export so only the more productive firms continue to export while other firms have to continue their activity on their local market. On the other hand, the transfer lowers the cut-off on the productivity of local-oriented Foreign firms; this means that there is entry of less productive Foreign firms on the Foreign market.

Why do these changes in cut-offs happen? The transfer of resources has a direct impact on the income of the Foreign country which is raised by T. For given exogenous supply of labor, the higher demand in Foreign country raises the prices of Home exports and allow less productive Home firms to pay the fixed export cost and enter the Foreign market. The productivity threshold to be exporter is reduced, and some new firms start to export and therefore increase the value of aggregate exports. Existing exporters sell more of their varieties, such that also the intensive margin contribute to the growth of exports. At the general equilibrium, the transfer of resources also leads to the raise of the exchange rate ϵ, but with respect to the case with only intensive margin, the depreciation of the exchange rate is dampened by the presence of the extensive margin. The increased demand of Foreign households allows also some Foreign firms to pay their entry cost on the Foreign market and there is therefore entry of local-oriented firms which were inactive before.

On the other hand, the transfer reduces the income of the Home country. Home households' consumption decreases and this translates directly in a reduction of Home households' welfare. It is more difficult for Home local-oriented firms to remain active on the Home market, as the cut-off on productivity increases and the less productive firms are forced to exit. The lower demand by Home households and the depreciation of the exchange rate affect the Foreign exporter firms too. The cut-off on the productivity of Foreign exporters goes up as only a few very productive firms can now pay the fixed cost, which is also augmented by the high ϵ, and enter the Home market.

3.2 GDP and Aggregate Productivity

As illustrated in the previous section, the shock on the Home economy induced by the transfer of resources T leads to a depreciation of the Home currency and to an endogenous exit of the less productive firms from the Home market. At the same time, some Home firms become exporters and enter the Foreign market. In this section, I investigate whether this switch among Home producers does imply a change on the total output of the country. The self-selection mechanism among Home firms pushes the worse firms in terms of productivity out of the market, and we should expect a change in the Home aggregate productivity as well.

In this model, GDP^H is equal to the GDP issued by the Home tradable sector[12], and can be defined as the remuneration of the factors of production. In this simple economy, this is equal to the sum of profits Π^H made by Home firms, both domestic-oriented and exporters, and the sum of wages payed to Home workers (remember that $w = 1$ as wage is the numeraire):

$$GDP^H = wL^H + \Pi^H$$

Total Home profits are net of the fixed costs of production. When adding the remuneration of the labor workforce to total profits, GDP^H is equal to the sales of the Home firms on the Home and the Foreign market. I define the value of domestic sales by one single Home firm as $g(x) = \left[1 + \frac{\Pi}{L} - \frac{T}{L^H}\right] L^H \left[\frac{p_D(x)}{P}\right]^{1-\sigma}$ while h(x) represents the single firm's export sales. GDP^H becomes

$$GDP^H = L^H \left[\int_{\bar{x}_D}^{\infty} g(x)dF(x) + \int_{\bar{x}_{exp}}^{\infty} h(x)dF(x)\right]$$

Both g(x) and h(x) depend on T and ϵ other than x, and as for the balanced current account, I use the Leibniz's rule to differentiate the Home GDP in order to disentangle the effects of the current account adjustment over the domestic sales (henceworth DOM) and the export sales (EXP):

$$dDOM = \underbrace{\left[\frac{1-\sigma}{\gamma}\frac{1}{1+\chi}\right]}_{\text{intensive}} dT - \underbrace{\left[\frac{\gamma-\sigma+1}{\gamma}\frac{1}{1+\chi}\right]}_{\text{extensive}} dT + \underbrace{\left[\frac{\gamma\sigma-\sigma+1}{\gamma}\frac{\chi}{(1+\chi)^2}\left(1+\frac{\Pi}{L}\right)L^H\right]}_{\text{intensive}} d\epsilon +$$
$$- \underbrace{\left[\frac{\sigma-\gamma\sigma-1}{\gamma}\frac{\gamma-\sigma+1}{\sigma-1}\frac{\chi}{(1+\chi)^2}\left(1+\frac{\Pi}{L}\right)L^H\right]}_{\text{extensive}} d\epsilon$$

$$dEXP = \underbrace{\left[\frac{\sigma-1}{\gamma}\frac{\chi}{1+\chi}\right]}_{\text{intensive}} dT + \underbrace{\left[\frac{\gamma-\sigma+1}{\gamma}\frac{\chi}{1+\chi}\right]}_{\text{extensive}} dT + \underbrace{\left[\frac{\gamma\sigma-\sigma+1}{\gamma}\frac{\chi}{(1+\chi)^2}\left(1+\frac{\Pi}{L}\right)L^H\right]}_{\text{intensive}} d\epsilon +$$
$$- \underbrace{\left[\frac{\sigma-\gamma\sigma-1}{\gamma}\frac{\gamma-\sigma+1}{\sigma-1}\frac{\chi}{(1+\chi)^2}\left(1+\frac{\Pi}{L}\right)L^H\right]}_{\text{extensive}} d\epsilon$$

The transfer of resources decreases DOM both at the extensive and the intensive margins of trade. This is due to the drop in the demand by Home households who transfer part of their revenue to the Foreign country. For the same reason, but with a positive sign, EXP are now increasing.

A depreciation of the Home Currency leads to an increase in export sales through the extensive and the

[12]The same holds for the Foreign country, with GDP^F equal to the the GDP of the Foreign tradable sector.

intensive margin as Home products are now cheaper for Foreign households. The upward movement in the exchange rate produces a positive effect also on DOM, as the price of imports in Home country raises due to depreciation. The higher price of imports allows Home firms to sell more on the Home market.

Nonetheless, this is a minor effect as the total impact of the adjustment over DOM is negative whereas it is positive for EXP. The substitution of the equilibrium value of $\frac{d\epsilon}{dT}$ into the previous equation yields

$$\frac{dDOM}{dT} = \underbrace{\frac{(\sigma-1)\left[(1-\chi)-2\right]}{2\gamma\cdot(1+\chi)}}_{\text{intensive (-)}} + \underbrace{\frac{(\gamma-\sigma+1)\left[(1-\chi)-2\right]}{2\gamma\cdot(1+\chi)}}_{\text{extensive (-)}} < 0$$

$$\frac{dEXP}{dT} = \underbrace{\frac{(\sigma-1)\left[(1-\chi)+2\chi\right]}{2\gamma\cdot(1+\chi)}}_{\text{intensive (+)}} + \underbrace{\frac{(\gamma-\sigma+1)\left[(1-\chi)+2\chi\right]}{2\gamma\cdot(1+\chi)}}_{\text{extensive (+)}} > 0$$

As expected, the aggregate exports increase both at the intensive and the extensive margin of trade. The Domestic sales decrease because the Demand is reduced by the transfer, and the existing local-oriented firms get lower sales (negative intensive margin). Additionally, the less productive firms are not able to pay the fixed cost, and are therefore forced to quit their activity (negative extensive margin). As a result, on the overall, $\frac{dGDP^H}{dT} = 0$ because the drop in domestic sales exactly offsets the increase in aggregate exports. This means that, at the symmetrical equilibrium, the current account adjustment has no impact over the total output of the Home country which stays unchanged.

Reallocation of labor and aggregate productivity

Although the current account adjustment does not produce any effect on the total output of the country, some firms are forced to exit the Home market while others start their new exporter life. The self-selection mechanism amongst firms and the reallocation of some of them toward the export market have an impact also on the reallocation of the labor force. More productive firms hire more workers, presumably those who were working for the least productive firms, so that in the end the labor market is cleared once again.

The standard measure of productivity used in national accounting is the ratio $\frac{output}{worker}$. In this model, the labor is the only factor of production and the aggregate productivity can be defined as

$$\text{aggregate productivity} \equiv \frac{GDP^H}{L^H} = 1 + \frac{\Pi^H}{L^H}$$

All workers supply the same amount of hours, there is no intensive margin of supplied labor and the number of worked hours is proportional to workers. Therefore, I can define the ratio $\frac{GDP^H}{L^H}$ indifferently as the output per worked hour or the output per worker.

As the total output of the country does not change in response to the transfer-driven adjustment of the current account, the aggregate productivity of the country is also constant at the level before the shock. Nonetheless, the labor market condition is cleared and the labor force is no more employed in low-productive firms as they are out of the market after the adjustment. Labor force flows toward the high-profits and more productive exporter firms, and is used to pay the additive fixed costs to be an exporter. More exporters do not only mean more profits but more fixed costs to be paid as well. At the symmetrical equilibrium, these costs (iceberg costs and higher fixed costs) exactly counterbalance the gain in productivity due to the exit of least productive firms. In the end, there is not any final effect over the aggregate productivity[13].

4 Numerical simulations

In this section, I run numerical simulations on the model in order to study the percentage variation of the exchange rate to a transfer of resources from the Home country to the Foreign country. Here the Home country represents the US whereas the Foreign country refers to the rest of the world.

The choice of parameters follows the benchmark set by Ghironi and Melitz (2005). The level of γ and σ is such that the standard deviation of log of US plant sales is equal to 1.67 as Bernard, Eaton, Jensen and Kortum [BEJK 2003] report in their article. Remember that the productivity of firms is Pareto-distributed and the standard deviation of log of sales is exactly equal to $\frac{1}{\gamma-\sigma+1}$. I take $\sigma = 3.8$ as reported by [BEJK 2003] on macro trade data and therefore set $\gamma = 3.4$ while the iceberg transport cost is equal to $\tau = 1.3$ such as in Obstfeld and Rogoff (2001). When looking at the fixed cost F_D, at general equilibrium the results don't depend on the level of F_D, therefore without loss of generalization, I can set $F_D = 0.3$ in order to ensure that cut-offs are greater than the support of the distribution b for every value of γ, while the additive export fixed cost δ is set at 0.2[14].

The Pareto distribution function of productivity is such that when the shape parameter γ is high, firms are less heterogeneous and less productive, that is more distributed at the bottom of the scale of productivity. In order to control for the decrease in productivity when γ is high, I set the support of the distribution $b = \frac{\gamma-1}{\gamma} \cdot Z$ where Z is the average of the distribution function which is exogenously

[13] The crucial hypothesis behind the self-selection mechanism among firms is that the reallocation of labor occurs without any kind of friction costs we could imagine (workers' training, labor market imperfections and so on). Further research should be addressed to this mechansim, in order to better specify what happens on the labor market when firms endogenously decide to enter or exit the Home and the Foreign market.

[14] This means that the fixed export cost is 20% higher than the fixed domestic cost. Results are robust to reasonable changes in the value of δ.

fixed at Z=1. Then, the average of the distribution and its standard deviation are

$$mean = Z \qquad \text{and} \qquad std.dev. = \frac{Z^2}{\gamma(\gamma-2)}$$

As a consequence, for the numerical simulations with a value of γ higher than in the benchmark, the standard deviation of the productivity distribution is lower, while the average of the distribution does not change and stays equal to Z. This is due to the automatic increase in the support b, depending itself on γ.

The present deficit of the current account of US is equal to 5.3% of US GDP, as reported by the Bureau of Economic Analysis for the year 2007. In the model, the transfer is set so to produce an increase in aggregate exports by 20%, the amount which is required to suddenly balance the CA. Notice that both aggregate exports and total output depend on the level of γ and σ chosen, therefore the amount of T varies according to the change of these parameters. When γ is greater than in the benchmark case, the economy becomes more closed and exports shrinks a lot as transport costs are very sensitive to a change in γ as shown by Chaney (2008). I therefore set the ratio of $\frac{T}{EXP} = 20\%$, before the adjustment, for any value of the dispersion of productivity γ.

[Table 3 about here.]

Table 3 presents the main numerical results of the simulations. The first and the second column report respectively the percentage change in the exchange rate and the real welfare-based exchange rate. The third and fourth column report the percentage impact of the intensive and the extensive margin of trade over the adjustment of the current account. Finally, the fifth column reports the ratio of the transfer over aggregate exports, exogenously fixed at 20%, and the sixth column refers to the ratio between the transfer and the total output of Home country.

In the benchmark case, the current account deficit is equal to 5.65% of total output and the transfer of resources increases aggregate exports by 20%. This shock implies a depreciation of the exchange rate of 4.22% and a depreciation of the real exchange rate of 1.20%. For the benchmark values of γ and σ, 80.87% of the current account adjustment passes through the channel of the intensive margin of trade whereas the remaining 19.13% of the adjustment passes through the extensive margin of trade. A model with representative firms misses about 19% of the adjustment and this leads to a higher depreciation of the exchange rate.

In table 3, I report the results of a sensitivity analysis for a change in the dispersion of productivity across firms. For higher values of γ, firms are more homogeneous and less productive, so just a few of them can pay the fixed costs and enter the Foreign market. In this case, the value of effective traded goods decreases and the Home economy becomes less open. Along with the lower openness of the economy, the transfer is also reduced and stays fixed at 20% of aggregate exports but it falls down to less than 1% of Home GDP when γ is higher than 10. At this point, the current account deficit is a minor trouble for the country. Firms are very homogeneous, the extensive margin of trade plays a major role for the adjustment, and the required depreciation is lower.

[Table 4 about here.]

In table 4, the sensitivity analysis is done for a change in the elasticity of substitution among goods. For a level of σ equal to 2.5, the extensive margin accounts for about 60% of the adjustment. Goods are very differentiated, and for given dispersion of productivity across firms γ equal to 3.4, even the low and middle productive firms have large market shares. As a result the impact of the entry of these new exporter firms is relevant over the adjustment, which occurs for a depreciation in the exchange rate of 2.60%. A higher level of elasticity of substitution progressively reduces the magnitude of the extensive margin. A larger part of the adjustment occurs through the intensive margin of trade, and the depreciation increases. These numerical simulations confirm the reversal of the common finding according to which a higher elasticity of substitution implies a lower change in terms of trade. In a model of global rebalancing with heterogeneous firms, the interaction between the productivity dispersion across firms and the elasticity of substitution determines the size of the extensive and intensive margin and the depreciation of the exchange rate.

5 Conclusion

The scope of this paper is to find new theoretical evidence on the impact that a real current account adjustment has in terms of currency depreciation, productivity thresholds, aggregate productivity and welfare-based exchange rate. The main innovation that this paper brings to the literature on the adjustment of the external account and the transfer debate is the introduction of firms' heterogeneity. The heterogeneity of firms has been of great interest since the paper of Melitz (2003), which was the cornerstone for following articles recently published on the New new theory of trade like Chaney (2008). Data actually confirms that only the most productive firms export and that the aggregate of exports is raised mainly by the export of new varieties that were not exported before.

A new important challenge for economic scholars is the application of the firms' heterogeneity on the field of International Macroeconomics. Ghironi and Melitz (2005) introduce the heterogeneity across firms in a model of international trade and macroeconomic dynamics. In this perspective, this paper analyzes the global rebalancing in a two-country model with firms' heterogeneity. The adjustment of the global imbalances has an impact on the level of productivity thresholds of exporters and local-oriented firms. The extensive margin of trade plays a key role in the adjustment and implies a lower depreciation of the real exchange rate (about 4%) with respect to models with representative firms.

In addition, this paper shows that the impact of the extensive margin of trade over the current account adjustment depends on the interaction between the level of dispersion of productivity across firms and the elasticity of substitution among goods. The new exporters are less productive than incumbent exporters, and therefore set higher prices. For given elasticity, the more the firms are productive and heterogeneous, the more the adjustment passes through the channel of the intensive margin of trade, and the depreciation is higher. Similarly, when goods are very substituable, the market shares of new exporters are very low. For this reason, when σ is high, the extensive margin of trade plays a minor role on the global adjustment and the depreciation is high.

Standard models focus uniquely on the good level of elasticity of substitution to adopt in simulations. In the macroeconomic literature, the values of σ vary in the range among 2 and 4, whereas in the trade literature, the elasticity of substitution is sensibly higher (around 10). In a model of external adjustment with firms' dynamics and heterogeneity in productivity, the relationship between σ and the depreciation of the exchange rate is more complex. In addition to the ambiguity around the value of σ, a new and not less important calibration issue arises on the level of dispersion of productivity across firms.

References

1. Bernard A., Eaton J., Jensen J., Kortum S., *Plants and Productivity in International Trade*, American Economic Review, v.93, pp.1268-1290, 2003.

2. Chaney T., *Distorted gravity: The Intensive and Extensive Margins of International Trade*, American Economic Review, forthcoming 2008.

3. Corsetti G., Martin P., Pesenti P., *Varieties and imbalances: the extensive margin of the current account adjustment*, CEPR Discussion Paper 6660, January 2008.

4. Curcuru S., Dvorak T., Warnock F., *Cross-Border Returns Differentials*, Federal Reserve Bank of Dallas working paper no.4, December 2007.

5. Dekle R., Eaton J., Kortum S., *Global Rebalancing with Gravity: Measuring the Burden of Adjustment*, IMF Staff Papers vol. 55 no.3, 2008.

6. Galstyan V., Lane P., *External Imbalances and the Extensive Margin of Trade*, mimeo, Trinity College of Dublin, July 2008.

7. Ghironi F. and Melitz M., *International trade and macroeconomics dynamics with heterogeneous firms*, Quarterly Journal of Economics CXX, August 2005: 865-915.

8. Gourinchas P., Rey H, 2007a, *From world banker to world venture capitalist: the U.S. external adjustment and the exorbitant privilege*, in R. Clarida (ed.) G7 Current Account Imbalances: Sustainability and Adjustment (Chicago, University of Chicago Press), 11-55.

9. Gourinchas P., Rey H, 2007b, *International financial adjustment*, Journal of Political Economy, forthcoming.

10. Lane P., Milesi-Ferretti G., *The External wealth of Nations Mark II: revised and extended estimates of foreign assets and liabilities*, IMF working paper, March 2006.

11. Melitz, M., *The Impact of Trade on Aggregate Industry Productivity and Intra-Industry Reallocations*, Econometrica, v. 71 no. 6, November 2003, pp. 1695-1725.

12. Obstfeld M., Rogoff K., *The Six Major Puzzles in International Macroeconomics: Is there a Common Cause?* in Ben S. Bernanke and Kenneth Rogoff (eds.), NBER Macroeconomics Annual 2000. Cambridge, MA: MIT Press, pp. 339-90.

13. Obstfeld M., Rogoff K., *The unsustainable US current account position revisited*, NBER, October 2004, revised version November 2005.

14. Obstfeld M., Rogoff K., *Global Current Account Imbalances and Exchange rates adjustments*, Working Paper, May 2005.

Appendix

A.1 Free entry conditions and productivity cut-offs

The following free entry condition applies to Home firms producing for the domestic market:

$$\pi_D^H = \frac{1}{\sigma}\left[1+\frac{\Pi}{L}-\frac{T}{L^H}\right]L^H\left(\frac{p_D(x)}{P}\right)^{1-\sigma} - F_D = 0$$

As wages are the numeraire, F_D represents the value of worked hours paid by Home firms entering the domestic market. This value is expressed in terms of Home currency, as well as all the profits in the free entry conditions. Then, substitute for the price of domestic-produced goods, and find

$$\bar{x}_D = \mu_2\left(\frac{F_D}{L^H}\right)^{\frac{1}{\sigma-1}}\frac{1}{P} \qquad (A1)$$

with

$$\mu_2 = \sigma^{\frac{1}{\sigma-1}}\frac{\sigma}{\sigma-1}\left[1+\frac{\Pi}{L}-\frac{T}{L^H}\right]^{-\frac{1}{\sigma-1}}$$

$\bar{x}_D$ is the Home local market cut-off of productivity, obtained for given price index P. Free entry condition tells us that Home firms with a level of productivity above $\bar{x}_D$ are active on the local market and produce import-competing goods, while the others are forced to exit.

The same free entry condition applies then on firms in the Foreign country[15]:

$$\pi_D^F = \frac{1}{\sigma}\left[1+\frac{\Pi}{L}+\frac{T}{L^F}\right]L^F\left(\frac{\epsilon p_D^*(x^*)}{P^*}\right)^{1-\sigma} - \epsilon F_D = 0$$

so that the threshold on the labor productivity of the Foreign firms producing for their local market is

$$\bar{x}_D^* = \mu_1\left(\frac{F_D}{L^F}\right)^{\frac{1}{\sigma-1}}\frac{1}{P^*}\epsilon^{\frac{\sigma}{\sigma-1}} \qquad (A2)$$

with

$$\mu_1 = \sigma^{\frac{1}{\sigma-1}}\frac{\sigma}{\sigma-1}\left[1+\frac{\Pi}{L}+\frac{T}{L^F}\right]^{-\frac{1}{\sigma-1}}$$

Now look at the exporter firms. I can easily define the profits made by the Home exporter firm as

$$\pi_{exp}^H = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[1+\frac{\Pi}{L}+\frac{T}{L^F}\right]L^F\left[\frac{x^{-1}\tau}{P^*}\right]^{1-\sigma} - (1+\delta)F_D$$

Once again the free entry condition tells us that $\pi_{exp}^H = 0$ so I find the critical threshold of labor productivity for the Home exporter firm:

$$\bar{x}_{exp} = \mu_1\left[\frac{(1+\delta)F_D}{L^F}\right]^{\frac{1}{\sigma-1}}\frac{\tau}{P^*} \qquad (A3)$$

All Home firms with a level of productivity above $\bar{x}_{exp}$ are exporters on the Foreign market, whereas firms with a level of productivity $\bar{x}_D \leq x < \bar{x}_{exp}$ are active just on the domestic market, and finally, least productive firms are forced to be inactive.

[15] Once again note that also the profits of Foreign local-oriented firms are expressed in the Home currency and ϵF_D is the value, expressed in terms of Home currency, of the worked hours paid by the Foreign firms entering their local market.

By the same means, I can write the value of profits for the Foreign exporter firm expressed in the home currency as

$$\pi^F_{exp} = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[1+\frac{\Pi}{L}-\frac{T}{L^H}\right]L^H\left[\frac{x^{*-1}\epsilon\tau}{P}\right]^{1-\sigma} - \epsilon(1+\delta)F_D$$

FEC on the export market for Foreign firms gives me the critical threshold of labor productivity for the firms entering the domestic country, so that $\bar{x}^*_{exp}$ is the productivity cut-off for Home imports:

$$\bar{x}^*_{exp} = \mu_2\left[\frac{(1+\delta)F_D}{L^H}\right]^{\frac{1}{\sigma-1}}\frac{\tau}{P}\epsilon^{\frac{\sigma}{\sigma-1}} \qquad (A4)$$

A.2 Equilibrium price indexes

Equations (A1),(A2),(A3) and (A4) represent the thresholds on productivity obtained for given prices. The equilibrium price indexes in the Home and Foreign markets depends on the firms operating in that country and, as firms are heterogeneous in the model, their entry and exit on Home and Foreign markets imply that the equilibrium price index of a country depends on the productivity of active firms.
Therefore, take the Home price index and substitute the prices of varieties and resolve integrals to obtain:

$$P = \left[\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\frac{\gamma b^\gamma}{\gamma-\sigma+1}\bar{x}_D^{\sigma-\gamma-1} + \left(\frac{\sigma}{\sigma-1}\epsilon\tau\right)^{1-\sigma}\frac{\gamma b^\gamma}{\gamma-\sigma+1}\bar{x}^{*\ \sigma-\gamma-1}_{exp}\right]^{\frac{1}{1-\sigma}}$$

Then plug the thresholds of the equations (A1) and (A4) into P, and after some algebraic manipulations the equilibrium price index for the Home country[16] is

$$P = \left[\mu_3\left(\frac{F_D}{L^H}\right)^{\frac{\sigma-\gamma-1}{\sigma-1}}\left(1+A\tau^{-\gamma}\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}}\right)\right]^{-\frac{1}{\gamma}} \qquad (A5)$$

where A is a constant equal to $A=(1+\delta)^{\frac{\sigma-\gamma-1}{\sigma-1}}$. In the Foreign country, the price index is

$$P^* = \left[\left(\frac{\sigma}{\sigma-1}\epsilon\right)^{1-\sigma}\frac{\gamma b^\gamma}{\gamma-\sigma+1}\bar{x}_D^{*\ \sigma-\gamma-1} + \left(\frac{\sigma}{\sigma-1}\tau\right)^{1-\sigma}\frac{\gamma b^\gamma}{\gamma-\sigma+1}\bar{x}_{exp}^{\sigma-\gamma-1}\right]^{\frac{1}{1-\sigma}}$$

Here, plug into P^* the productivity threshold of Foreign local-oriented firms (equation A2) as well as the cut-off of Home exporter firms (equation A3). These firms enter the Foreign market to sell varieties, and their prices have a weight in the Foreign price index. As a result[17]

$$P^* = \left[\mu_4\left(\frac{F_D}{L^F}\right)^{\frac{\sigma-\gamma-1}{\sigma-1}}\left(\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}}+A\tau^{-\gamma}\right)\right]^{-\frac{1}{\gamma}} \qquad (A6)$$

Equilibrium price indexes of Home and Foreign country both depend linearly on the transport iceberg cost τ as well as on the exogenous fixed cost F_D and on the transfer of resources T coming from Home toward the Foreign country.

[16] Where μ_3 is a constant equal to $\mu_3 = \left(\frac{\sigma}{\sigma-1}\right)^{-\gamma}\frac{\gamma b^\gamma}{\gamma-\sigma+1}\sigma^{\frac{\sigma-\gamma-1}{\sigma-1}}\left[1+\frac{\Pi}{L}-\frac{T}{L^H}\right]^{-\frac{\sigma-\gamma-1}{\sigma-1}}$. I am actually taking total profits Π as a constant too, and I prove later that Π isn't affected by the transfer T.

[17] Where μ_4 is a constant equal to $\mu_4 = \left(\frac{\sigma}{\sigma-1}\right)^{-\gamma}\frac{\gamma b^\gamma}{\gamma-\sigma+1}\sigma^{\frac{\sigma-\gamma-1}{\sigma-1}}\left[1+\frac{\Pi}{L}+\frac{T}{L^F}\right]^{-\frac{\sigma-\gamma-1}{\sigma-1}}$

A.3 Equilibrium thresholds and exchange rate

Free entry conditions on Foreign and Home market provide the critical thresholds on productivity for local-oriented and exporter firms operating on both countries. These thresholds are obtained for given price indexes but the equilibrium value of these thresholds can now be found just by plugging them into the equilibrium price indexes of Home and Foreign country. As a result, each productivity cut-off depends only on the endogenous exchange rate ϵ and on the exogenous transport cost τ and transfer T.

Firstly, substitute the Home price index in equations (A1) and (A4) to have

$$\bar{x}_D = \mu_2 \left(\frac{F_D}{L^H}\right)^{\frac{1}{\sigma-1}} \left[\mu_3 \left(\frac{F_D}{L^H}\right)^{\frac{\sigma-\gamma-1}{\sigma-1}} \left(1 + A\tau^{-\gamma}\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}}\right)\right]^{\frac{1}{\gamma}} \tag{A7}$$

$$\bar{x}^*_{exp} = \mu_2 \left[\frac{(1+\delta)F_D}{L^H}\right]^{\frac{1}{\sigma-1}} \tau\epsilon^{\frac{\sigma}{\sigma-1}} \left[\mu_3 \left(\frac{F_D}{L^H}\right)^{\frac{\sigma-\gamma-1}{\sigma-1}} \left(1 + A\tau^{-\gamma}\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}}\right)\right]^{\frac{1}{\gamma}} \tag{A8}$$

Then, plug the equilibrium Foreign price index into equations (A2) and (A3) and obtain

$$\bar{x}^*_D = \mu_1 \left(\frac{F_D}{L^F}\right)^{\frac{1}{\sigma-1}} \epsilon^{\frac{\sigma}{\sigma-1}} \left[\mu_4 \left(\frac{F_D}{L^F}\right)^{\frac{\sigma-\gamma-1}{\sigma-1}} \left(\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}} + A\tau^{-\gamma}\right)\right]^{\frac{1}{\gamma}} \tag{A9}$$

$$\bar{x}_{exp} = \mu_1 \left[\frac{(1+\delta)F_D}{L^F}\right]^{\frac{1}{\sigma-1}} \tau \left[\mu_4 \left(\frac{F_D}{L^F}\right)^{\frac{\sigma-\gamma-1}{\sigma-1}} \left(\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}} + A\tau^{-\gamma}\right)\right]^{\frac{1}{\gamma}} \tag{A10}$$

A.4 Implications of the current account adjustment

It is useful to total differentiate equations (A7)-(A10) in order to have more indications of the way the transfer affects the thresholds. It lowers the Home income and raises the Foreign income directly, and it has an indirect impact through the change in the exchange rate. Note that I total differentiate the productivity thresholds in the neighborhood of the symmetrical equilibrium with $\epsilon = 1$ and $T = 0$ so that I have:

$$d\bar{x}_D = \frac{1}{\sigma-1}\frac{1}{L^H}\left[1+\frac{\Pi}{L}\right]^{-1} \mu_2 \left(\frac{F_D}{L^H}\right)^{\frac{1}{\sigma-1}} P^{-1} dT + \mu_2 \left(\frac{F_D}{L^H}\right)^{\frac{1}{\sigma-1}} \frac{1}{\gamma}[...]^{-1} P^{-1} d[...]$$

I know that $\bar{x}_D = \mu_2 \left(\frac{F_D}{L^H}\right)^{\frac{1}{\sigma-1}} P^{-1}$ and $\frac{dP}{P} = \frac{1}{\gamma}[...]^{-1} d[...]$. Using the same substitution on the other thresholds, the total differential of each cut-off can be written as:

$$\frac{d\bar{x}_D}{\bar{x}_D} = \frac{1}{\sigma-1}\frac{1}{L^H}\left[1+\frac{\Pi}{L}\right]^{-1} dT - \frac{dP}{P}$$

$$\frac{d\bar{x}^*_{exp}}{\bar{x}^*_{exp}} = \frac{1}{\sigma-1}\frac{1}{L^H}\left[1+\frac{\Pi}{L}\right]^{-1} dT + \frac{\sigma}{\sigma-1}d\epsilon - \frac{dP}{P}$$

$$\frac{d\bar{x}^*_D}{\bar{x}^*_D} = -\frac{1}{\sigma-1}\frac{1}{L^F}\left[1+\frac{\Pi}{L}\right]^{-1} dT + \frac{\sigma}{\sigma-1}d\epsilon - \frac{dP^*}{P^*}$$

$$\frac{d\bar{x}_{exp}}{\bar{x}_{exp}} = -\frac{1}{\sigma-1}\frac{1}{L^F}\left[1+\frac{\Pi}{L}\right]^{-1} dT - \frac{dP^*}{P^*}$$

Finally $\frac{dP}{P}$, as well as $\frac{dP^*}{P^*}$, is function of $d\epsilon$ and dT. At the general equilibrium, the differentiation of the balanced current account gives the equilibrium relation $\frac{d\epsilon}{dT}$ which is fundamental to disentangle the direct effect of T on thresholds from its indirect effect through the exchange rate.

Home market
$\bar{x}_D = \lambda_1 \left(1 + A\tau^{-\gamma}\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}}\right)^{\frac{1}{\gamma}}$
$\bar{x}^*_{exp} = \left[(1+\delta)^{\frac{1}{\sigma-1}}\tau\right] \lambda_1 \left(1 + A\tau^{-\gamma}\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}}\right)^{\frac{1}{\gamma}} \epsilon^{\frac{\sigma}{\sigma-1}}$
Foreign market
$\bar{x}^*_D = \lambda_2 \left(\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}} + A\tau^{-\gamma}\right)^{\frac{1}{\gamma}} \epsilon^{\frac{\sigma}{\sigma-1}}$
$\bar{x}_{exp} = \left[(1+\delta)^{\frac{1}{\sigma-1}}\tau\right] \lambda_2 \left(\epsilon^{\frac{\sigma-\gamma\sigma-1}{\sigma-1}} + A\tau^{-\gamma}\right)^{\frac{1}{\gamma}}$

Table 1: Equilibrium productivity thresholds

$$\widehat{x}_D = \frac{1}{2\gamma}\left(1+\chi\right)\left[\frac{\gamma\sigma-\sigma+1}{\gamma\sigma}\right] > 0$$

$$\widehat{x}_{exp} = -\frac{1}{2\gamma}\left(\frac{1+\chi}{\chi}\right)\left[\frac{\gamma\sigma-\sigma+1}{\gamma\sigma}\right] < 0$$

$$\widehat{x}^*_D = -\frac{1}{2\gamma}\left(\frac{1+\chi}{\chi}\right)\left[\frac{\gamma\sigma-\sigma+1}{\gamma\sigma} + (1-\chi)\right] < 0$$

$$\widehat{x}^*_{exp} = \frac{1}{2\gamma}\left(1+\chi\right)\left[\frac{\gamma\sigma-\sigma+1}{\gamma\sigma} + \frac{1-\chi}{\chi}\right] > 0$$

$$\widehat{\epsilon} = \frac{1}{2}\cdot\frac{\sigma-1}{\gamma\sigma}\left[\frac{1-\chi^2}{\chi}\right] > 0$$

$$\widehat{RER} = \frac{1}{\gamma}\cdot\frac{\gamma\sigma-\sigma+1}{\gamma\sigma}\left[\frac{2(\gamma-\sigma+1)}{\sigma-1} + \frac{1}{2}\frac{(1-\chi)^2}{\chi}\right]$$

$$\widehat{C} = \frac{1}{\gamma}\cdot\frac{\gamma\sigma-\sigma+1}{\gamma\sigma}\left[\frac{\sigma-\gamma-1}{\sigma-1} + \frac{1}{2}(\chi-1)\right] - 1 < 0$$

$$\widehat{U} = \frac{\widehat{C}}{\log C} < 0$$

Table 2: Summary of main analytical results

	ϵ	RER	intensive	extensive	$\frac{T}{EXP}$	$\frac{T}{GDP}$
$\gamma = 3$	5.14	1.66	92.74	7.26	20	6.20
Benchmark	4.22	1.20	80.87	19.13	20	5.65
$\gamma = 5.23$	2.46	0.65	50.55	49.45	20	3.56
$\gamma = 7$	1.79	0.76	36.90	63.10	20	2.16
$\gamma = 10$	1.22	0.95	25.27	74.73	20	0.87

Table 3: Sensitivity analysis to the change in the productivity dispersion

	ϵ	RER	intensive	extensive	$\frac{T}{EXP}$	$\frac{T}{GDP}$
$\sigma = 2.5$	2.60	-1.85	40.08	59.92	20	4.91
$\sigma = 3$	3.24	-0.38	55.61	44.39	20	5.30
$\sigma = 3.5$	3.86	0.69	71.36	28.64	20	5.55
Benchmark	4.22	1.20	80.87	19.13	20	5.65
$\sigma = 4$	4.46	1.50	87.24	12.76	20	5.71

Table 4: Sensitivity analysis to the change in the elasticity of substitution