Intergenerational transfers and the stability of public debt with short-lived governments

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Abstract. Time consistent policies and reforms of intergenerational transfers are analyzed in an overlapping generation model. Governments have preferences, which give much weight to the living generations and they cannot commit themselves to future taxes and transfers, which will be decided by future governments with different objectives. The economy follow one of two equilibrium paths with perfect foresight. On one path, governments finance the costs of their transfers to the living by increasing public debt recklessly. Consumers pay more and more taxes to finance the cost of this debt, and the successive generations will enter a process of immiserisation. On the other path, in spite of their preference bias, governments borrow less and put the economy on a path of egalitarian consumption flows for the successive generations, with a constant ratio of public debt to national income. The mechanisms, which put an economy on one or the other equilibrium paths, are unconnected to the fundamentals of the model.

Keywords: Intergenerational transfers, Markov perfect equilibrium, overlapping generation model, time consistent policies.

JEL classification: E62, F43, H55.

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1. **Introduction**

In his excellent survey of retirement Cairncross (2004) writes: “A larger generation of old folk than ever before will need support for longer than ever before from a population of working age that is shrinking continuously in absolute size for the first time since the Black Death. Moreover, if things look bad in America and worse in continental Europe, they will one day look calamitous in some parts of the developing world”. There are two causes of this demographic transition: a widespread fall in fertility and an increase in life expectancy. This demographic transformation creates a financing problem for pay-as-you-go systems. To maintain the level of pensions unchanged, contributions to pension funds must be repeatedly increased. Moreover, the gap between the market interest rate and the implicit rate of return of the system becomes wider, which causes a decline in the economic condition of the young. The most natural reform to reduce the burden on the young is to decrease the level of pensions, which will lower the increase in contributions and free some income of the young generation. This saving can be invested in the financial markets at better rates. This policy will increase the welfare of the young and unborn, but it will decrease the income of the pensioners and the expected income of the people who are planning to retire early.

We can think of adding an ingredient to this reform. The loss of income of pensioners can be compensated by a transfer from the government, financed by public borrowing. This increase in public debt will be financed by the income of the young, freed by the decrease or the slower increase of their contribution. However, the consequence of this policy will be to increase the taxation of future generations. It will transfer a sacrifice from the elderly, currently alive, to people who are not yet born. This proposal is often canvassed in current political discussions. For instance, we can refer to a very good article in The Economist (“From slogan to legacy”, 13 November 2004), on the state of the debate inside the American administration and the Republican party or to the Economic Focus published by this magazine on 11 December 2004. Feldstein (2005) also gives a stimulating discussion of these issues.

A quotation by Miles and Cerny (2001) help understand why a government could be forced to borrow as an element of its intergenerational transfer policy: “The result that a large proportion of those alive now would be worse off if the unfunded state scheme is phased out – even though every future generation is better off – illustrates the nature of the transition problem rather clearly. Democratically elected governments facing voters who focus on the direct implications to them (and not to all future generations) of changes to state pension systems would find it hard to get support for this kind of transition plan. Table 2 suggests that once a transition from an unfunded to a funded scheme is complete welfare for all subsequent generations will be higher, but without relying on deficit financing the transition will cause certain generations to be worse off, and those generations could form a majority of voters permanently blocking any change."

We investigate these questions in an overlapping generation model with the hypothesis that successive governments organize any intergenerational transfer between living and still unborn people, under the only constraint that they always face their obligations to their creditors. A government gives more weight to the living than to the unborn. Moreover, if it can set transfers and taxes for the current period, it cannot commit itself to future taxes and transfers, which will be decided by the next governments with different objectives. However, a government can set the level of public debt, which will be transmitted to its successor. A government can give generous transfers to the living and finance the cost of this policy by borrowing. Its successors can follow the same policy and this will initiate a process of increasing public debt, which will lead to the immiserisation of future generations. We suggested that this policy can result from the difficulties currently met by the public pension system and by the political resistance to reform it. More generally, it can result from the increasing cost of the Welfare state, which governments can allow themselves to be tempted to finance by public borrowing. Heller (2003) gives a fascinating account of the dangers for a society, which puts off costs that should be paid in the current time and which increases the burden of future generations.

The results, which will be proven, partly confirm these concerns. We find that the economy can follow any of two equilibrium paths. In one of them, governments borrow more and more to improve the conditions of the living. Then, the increasing cost of public debt will lead taxes to always higher levels, and the successive generations will become poorer and poorer. However, there exists another equilibrium. Governments, which cannot commit to future decisions and are especially sensitive to the welfare of the living, will however put the economy on a path of egalitarian consumption flows of the successive generations. The explanation of this result is that if a government borrowed too much, the following government would punish the generations, which were already alive under the first government, by increasing their taxes. The less the first government cared for these generations, the harsher this punishment will be. Thus, the expectation of this punishment will discipline the first government and discourage it of borrowing too much. We will also show that, for the last equilibrium, in times of demographic transition, the sacrifice, which will have to be made, will be shared in an egalitarian way across generations.

A difference between the first and the second equilibrium is the extent of the punishment by a government of the generation, which was already alive under the previous government, the one which could borrow too much. This punishment is higher for the second than for the first equilibrium. However, in both situations, expectations are perfect. The mechanisms, which put an economy on one or the other equilibrium paths, are unconnected to the fundamentals of the model. We are left with the open question of which sunspots will decide if the future generations will enter or not an immiserisation process. The fact that these sunspots cannot be directly connected to demographic transition, does not prevent the possibility that we have entered a period when some Western economies have switched from a path of stable public debt to a path of increasing indebtedness. Public and foreign deficits and debts have followed worrisome trends in some industrialized countries, in the current decade.

The analysis of intergenerational transfers is usually made by using an overlapping generation model (Azariadis, 1993; de la Croix and Michel, 2002). We assume that our economy is small and open on the rest of the world. This assumption, or more precisely the possibility of international borrowing and deficits of the balance of trade, is necessary to make the transfers from future to current generations possible. We will see that the problem is not so much the increase in the public debt as the share of this increase, which leads foreign debt up. Successive governments and generations of consumers will participate in a dynamic game where governments will adopt Markov strategies, and where the private sector will adapt its decentralized decisions to the current and expected decisions of governments.

In Grossman and Helpman (1998), short-lived governments have an objective function similar to ours and the similar concept of perfect Markov equilibrium is used, but the economy is closed, governments must always balance their budgets and the production and utility functions are linear. The conclusions differ and, instead of two equilibria, there are an infinity of them.

The model is presented in the first section. We will describe the dynamic game played between successive governments and generations of people in the second section. We will also explain how governments determine their public transfers and debts. Several lemmas characterizing the equilibrium paths of the model will be given in the third section. We will investigate the first equilibrium where the consumption and the welfare of successive generations remain constant over time in the fourth section. The second equilibrium where successive generations enter an immesirising process will be analyzed in the fifth section. A discussion of an alternative definition of equilibrium will be given in the sixth section.

2. An Overlapping Generations Model

The model is constructed for a small open economy. Domestic and foreign goods are perfect substitute and the international capital market is perfect. Time is assumed to be discrete, 0 denotes the current period, and agents hold perfect expectations. All economic variables are expressed in real terms. $N(t)$ consumers were born at the beginning of period t. They will die at the end of period $t+1$. They work for the first period of their life, then retire. Each worker receives wages $w(t)$. The total number of births increases over time at rate $1+n>0$. We have $N(t)=N(1+n)^{t+1}$, for $t\geq -1$, where $N=N(-1)$ denotes the total number of people entering the second period of their life at the beginning of period 0. We norm the population by setting $N=1$. The domestic interest rate is equal to the interest rate in the rest of the world, $i^*>0$, which is exogenous and constant over time. We need the following assumption:

Assumption 1. The interest rate in the rest of the world is higher than the growth rate of population: $i^*>n$.

Domestic output in period t, $Y(t)$, is proportional to the employment in the period.

$$Y(t) = PN(t) = P(1+n)^t \tag{1}$$

To simplify the model, we assume that the productivity term P stays constant over time. The natural growth rate of the economy is equal to the growth rate n of the active population. The wage rate is equal to: $w(t) = w = P$.

1.1. The Public Sector

$s_1(t)$ and $s_2(t)$ denote net public transfers made by the government, respectively to each young and to each old person, in period t. To simplify the model we assume that the government does not consume goods. Public debt at the end of period t, deflated by the active population living in the period, is denoted as $B(t)$. The budgetary equilibrium of the government is

$$(1+n)B(t) = (1+n)s_1(t) + s_2(t) + (1+i^*)B(t-1) \tag{2}$$

Eq. (2) determines the dynamics of total public debt $(1+n)^{t+1}B(t)$ (the initial level of total public debt $B(-1)$ is given). We assume that the sequence of net public transfers is consistent with a bounded public debt per worker $B(t)$. Under Assumption 1, this condition implies the inter-temporal solvency of the government: $B(t)\left[(1+n)/(1+i^*)\right]^t \underset{t\to\infty}{\to} 0$.

1.2. Plan of a Consumer Born at the Beginning of Period $t \geq 0$

We assume that consumers receive no endowment at their birth and leave no bequest when they die. The non capital incomes of a consumer born at the beginning of period t, in the two periods of its life, respectively are: $w + s_1(t)$, and: $s_2(t+1)$. The wealth at birth of this consumer is

$$W(t) \equiv w + s_1(t) + s_2(t+1)/(1+i^*) \tag{3}$$

This agent consumes $C_1(t)$ and $C_2(t+1)$ in the two periods of his life. These consumption flows must satisfy the budgetary constraint

$$C_1(t) + C_2(t+1)/(1+i^*) = W(t) \tag{4}$$

The wealth of this consumer at the end of period t, which is identical to national private wealth per worker, is

$$A(t) = w + s_1(t) - C_1(t) \tag{5}$$

We assume that consumers take their lifetime decisions at the time of their birth. They perfectly forecast their future incomes. Their discount rate is $\beta > 0$ and their preference for consumption is logarithmic. The utility at birth of a consumer born at the beginning of period t is

$$U(t) = \ln[C_1(t)] + \ln[C_2(t+1))]/(1+\beta) \tag{6}$$

Nothing substantial would change in the analysis if we assumed an incompressible level of consumption per period: $\overline{C} > 0$, or it the utility function belonged to the more general class of CRRA functions: $C^{1-\sigma}/(1-\sigma)$, with $\sigma > 0$.

The maximization of the utility function in Eq. (6) under the budgetary constraint (4) gives the consumption of this consumer in the two periods of his life

$$C_1(t) = W(t)(1+\beta)/(2+\beta) \tag{7}$$

$$C_2(t+1) = W(t)(1+i^*)/(2+\beta) \tag{8}$$

1.3. Plan of a Consumer Born at the Beginning of Period -1

This consumer enters the second part of his life at the beginning of period 0. His wealth inherited from the past is $A(-1)$. Moreover, this consumer receives public transfers $s_2(0)$. His consumption is equal to the sum of his wealth, the interest income earned on this wealth and his non-wealth income

$$C_2(0) = (1+i^*)A(-1) + s_2(0) \tag{9}$$

The utility of this consumer at the beginning of period 0 is

$$U(-1) = \ln[C_2(0)] \tag{10}$$

The total wealth at birth of each consumer is assumed positive:.

Assumption 2. The government's transfer policy must satisfy the constraints: $s_2(0) + (1+i^*)A(-1) > 0$, and: $w + s_1(t) + s_2(t+1)/(1+i^*) > 0$, for $t \geq 0$.

Finally, if $F(t)$ with $t \geq -1$ represents the net foreign assets owned by the economy, at the end of period t and deflated by the active population, we have

$$F(t) = A(t) - B(t) \tag{11}$$

2. How Does the Government Determine Public Transfers and Debt?

We can easily prove that the sum of the discounted wealth at birth of all generations, computed in period 0, is equal to the sum of the discounted consumption of these generations and is independent of intergenerational transfers:

$$\sum_{t=0}^{\infty}\left[C_1(t) + C_2(t+1)/(1+i^*)\right]\left[(1+n)/(1+i^*)\right]^{t+1} + C_2(0)/(1+i^*) = \\ (1+n)w/(i^*-n) + A(-1) - B(-1) \tag{12}$$

We assume that these sums are positive.

Assumption 3. The initial conditions of the economy satisfy: $(1+n)w + (i^*-n)\left[A(-1) - B(-1)\right] > 0$.

Assumption 2 implies the validity of Assumption 3. Eq. (12) shows that a reform of the public transfer policy to the successive generations improve the welfare of some generations and lead to a decline in the utility of other generations. This efficiency result is well known (Azariadis, 1993; Feldstein and Liebman, 2002). Its consequence is that the system of intergenerational transfers, which prevails in an economy, results from government's arbitrage between the various generations.

We introduce an objective function of the government at time 0:

$$\Omega(0) = U(-1) + A(1+n)U(0)\,,\ A > 0 \tag{13}$$

This function gives different weights to the utilities $U(-1)$ of the elderly alive in period 0, and $(1+n)U(0)$ of the cohort of the youth who are also currently alive. The government does not care for generations, which will be born in later periods. Grossman and Helpman (1998) give political foundations for this function. More general objective functions could be considered. However, this specification is simple and will be sufficient to establish our results.

At time 0, the government sets only its decisions for this period and cannot commit itself to these decisions, which will be implemented by future governments. When this government makes its choice it must anticipate the reactions of the governments of the future, the objective functions of which are: $\Omega(t) = (1+n)^t \left[U(t-1) + A(1+n)U(t)\right],\ t \geq 1$. In this expression, $U(t-1)$ and $U(t)$ represent the utility of each old person and of each young person, alive in period t, computed for the whole rest of their life. The relative weights of the various cohorts in the government's objective function change over time. Therefore, the solution to the problem exhibits dynamic inconsistency: the actions for the future periods that the government would choose in period 0, if it were able to pre-commit, would differ from those it finds optimal when in periods 1, 2, etc. The standard approach to analyze sequential decisions problems with time-inconsistent preferences is to view the decision-maker in each period t as a distinct player, in the sense of non-cooperative game theory (Vieille and Weibull, 2005). One obtains a sequential game with infinitely many players, each one acting only once, but caring not only about the material payoff in his own period but also about the payoffs in subsequent periods.

The Markov perfect equilibrium is a concept of solution, which is well adapted to our problem. According to Maskin and Tirole (2001), "consider a dynamic game in which, in every period t, player i's payoff π_t^i depends only on the vector of players' actions, a_t, that period, and on the current (payoff-relevant) `state of the system' $\theta_t \in \Theta_t$. That is, $\pi_t^i = g_t^i(a_t, \theta_t)$. Suppose, furthermore, that player i's possible actions A_t^i depend only on θ_t: $A_t^i = A_t^i(\theta_t)$ and that θ_t is determined by the previous actions a_{t-1} and state θ_{t-1}. Finally, assume that each player maximizes a discounted sum of per period payoffs: $\sum_t \delta^{t-1}\pi_t^i$. In period t, the history of the game, h_t, is the sequence of previous actions and states $h_t = \left((a_1, \theta_2), \ldots, (a_{t-1}, \theta_t)\right)$. But the only aspect of history that directly affects player i's payoffs and action sets starting in period t is the state θ_t. Hence, a Markov strategy in this model should make player i's period t action dependent only on the state θ_t rather than on the whole history h_t… We shall define a Markov Perfect Equilibrium to be a subgame perfect equilibrium in which all players use Markov strategies". These authors add: "Many economic models entail games that are stationary in the sense that "they look the same" starting in any period", i.e., they do not depend on calendar time. For these games it is natural to make Markov strategies independent of calendar time as well".

In our dynamic game, the initial state in period $t \geq 0$ is defined by the amounts of public debt $B(t-1)$ and of national private wealth $A(t-1)$, deflated by the number of living old people. The government of this period sets the transfers to the living youth and elderly, $s_1(t)$ and $s_2(t)$. Then, these people determine their consumption. The behavior of the elderly is passive: each of them consumes his total wealth and income: $C_2(t) = (1+i^*)A(t-1) + s_2(t)$. Each young person determines his consumption of the period and his expected consumption of next period, $C_1(t)$ and $C_2(t+1)$, according to Eq. (7) and (8). He needs to forecast the transfer he will receive from the government of period $t+1$, $s_2(t+1)$. Although he knows that this transfer depends on the aggregated choices of all the young consumers, he is also aware that his actions have no significant effect on these aggregated choices, and considers his expectation of $s_2(t+1)$ as a value and not as a function.

The government of period t perfectly forecasts the reaction to its actions by the consumers of the same period. Moreover, it knows that the transfers decided by the next government are a function of the state of the economy at the beginning of period $t+1$: $s_2(t+1) = f(B(t), A(t))$. The government of period t makes the decision maximizing its objective function under the constraints given in Assumption 2.

The definition of Markov strategies does not put any constraint on the shape of the reaction function f. This function is affine is taken affine, an assumption discussed in Lemma 4:

$$s_2(t+1) = s_2' - a_1 B(t) - a_2 A(t) \text{, for } t \geq 0 \tag{14}$$

From Eq. (3), (5) and (7)

$$(2+\beta)A(t) - w = s_1(t) - \frac{1+\beta}{1+i^*} s_2(t+1)$$

Under the condition: $a_2 \neq (1+i^*)\dfrac{2+\beta}{1+\beta}$, the reaction of the government of period $t+1$ expected by the government of period t is

$$s_2(t+1) = s_2 - \lambda B(t) - \mu s_1(t), \qquad \text{with:} \qquad s_2 = \frac{(2+\beta)s_2^{'} - a_2 w}{2+\beta - a_2(1+\beta)/(1+i^*)},$$

$$\mu = \frac{a_2}{2+\beta - a_2(1+\beta)/(1+i^*)}, \ \lambda = \frac{a_1(2+\beta)}{2+\beta - a_2(1+\beta)/(1+i^*)} \tag{16}$$

In the rest of the text we use again the constant parameters s_2, λ and μ, instead of the original parameters $s_2^{'}$, a_1 and a_2.

3. Characterization of Equilibria

The current period is $t=0$. We compute the decisions executed in this period by the current government and the decisions, which are expected to be executed in future periods by future governments. We will see that these decisions are consistent with the assumed reaction function. The government of period t only cares for the living youth and elderly and its transfers can only increase the satisfaction of these agents. This government can increase these transfers by borrowing more. However, the governments of period $t+1$ will react by punishing the living youth of period t, and this punishment will limit the profligacy of the government of this period.

From Eq. (3) and (16), we deduce that the wealth at birth of somebody young in period t is

$$W(t) = w + \frac{s_2}{1+i^*} - \frac{\lambda}{1+n}B(t-1) + \left(1 - \frac{\lambda+\mu}{1+i^*}\right)s_1(t) - \lambda\frac{s_2(t)}{(1+n)(1+i^*)} \tag{17}$$

If the government of period t increases its transfers to each old person of this period by 1, then the government of period $t+1$ will reduce $W(t)$ by $\lambda/(1+n)(1+i^*)$. If the first government increases the transfers to each young people by 1, then its successor will reduce $W(t)$ by $(\lambda+\mu)/(1+i^*)$. Then, the total effect on $W(t)$ will be $1-(\lambda+\mu)/(1+i^*)$. So, λ and $\lambda+\mu$ measure the extent of the punishment of the profligacy of a government by its successor.

In the second case, if the punishment is low that is if $\lambda + \mu < 1 + i^*$, then the government of period t will borrow as much as it can and set the transfers to its youth at the maximum level. If the punishment is severe: $\lambda + \mu > 1 + i^*$, the government of period t taxes its youth at a very high level so that its successor more than compensates this rigor by giving generous benefits to this cohort in the next period. The successor finances the cost of this policy by borrowing. In both cases, the rise in $W(t)$ is only constrained by the borrowing capacity of governments, which is reached when the whole income of the following generations is used to stabilize the public and foreign debts. Assumption 4 will exclude these corner solutions of the model by constraining the reaction function of the governments.

Assumption 4. The reaction function of governments must satisfy the constraint: $\mu + \lambda = 1 + i^*$.

Lemma 1. The policy, which is implemented by the government of period $t \geq 0$ is such that

$$\frac{1}{C_2(t)} = \frac{2+\beta}{1+i^*}\frac{\lambda A}{1+\beta}\frac{1}{W(t)} \tag{18}$$

Proof. In Appendix.

Lemma 2. The sequence of transfer policies is determined by the reaction function

$$s_2(t+1) = s_2 - \lambda B(t) - (1+i^* - \lambda)s_1(t) \tag{19}$$

by Eq. (2), (15) and (20):

$$C_2(t+1) = \left[s_2(t+1) + (1+i^*)A(t)\right] = \left[s_2(t) + (1+i^*)A(t-1)\right]\lambda A/(1+\beta) = C_2(t)\lambda A/(1+\beta) \tag{20}$$

for $t \geq 0$, with $A(-1)$ and $B(-1)$ given.

Proof. Eq. (20) results from Eqs (8) and (18).

Lemma 3 establishes a condition, which must be satisfied by the reaction function of governments:

Lemma 3. The existence of an equilibrium requires that either: $\lambda A = 1+\beta$, or: $\lambda = \frac{1+i^*}{1+n}\frac{1}{A/(1+\beta)+1/(1+n)}$.

Proof. In Appendix.

We have assumed that the reaction function of governments is affine. Lemma 4 shows that this assumption is weak.

Lemma 4. The reaction function of the government of period $t+1$: $s_2(t+1) = f[B(t), s_1(t)]$ cannot simultaneously satisfy the two following conditions

a) The function f is continuously differentiable.

b) The function

$$g[s_2(t)+(1+i^*)B(t-1)] = \arg\underset{s_1(t)}{Max}\{s_1(t) + f\{s_1(t) + [s_2(t)+(1+i^*)B(t-1)]/(1+n), s_1(t)\}/(1+i^*)\}$$

is point-to-point and continuously differentiable.

Proof. In Appendix.

This lemma excludes many reaction functions, but not the affine one. However, it is insufficient to prove that the reaction function must be affine. There is still the possibility that there exist other reaction functions, which would for instance be discontinuous. Even if we limit our analysis to the affine case, Lemma 3 suggests that the model has two equilibrium paths that we investigate now.

4. An Egalitarian Equilibrium. Case $\lambda = (1+\beta)/A$

Proposition 1 gives all the features of this equilibrium.

Proposition 1. The first equilibrium path of the economy is described by:

a) The transfers to the elderly in period 0 are given by

$$s_2(0) = \frac{(1+i^*)(1+n)}{(1+n)(2+\beta)+(i^*-n)}\{w-(2+\beta)[A(-1)-B(-1)]\} - (1+i^*)B(-1) \tag{21}$$

b) The constant part of the transfers to the elderly in the following periods is given by

$$s_2 = -\frac{(i^* - n - \lambda)(1+i^*)}{(1+n)(2+\beta)+(i^*-n)}\left\{w-(2+\beta)\left[A(-1)-B(-1)\right]\right\} \tag{22}$$

c) The consumption flows of each old and young persons are constant over time and given by

$$C_2(t) = \frac{1+i^*}{(1+n)(2+\beta)+(i^*-n)}\left\{(1+n)w+(i^*-n)\left[A(-1)-B(-1)\right]\right\}, \tag{23}$$

$$C_1(t) = \frac{1+\beta}{(1+n)(2+\beta)+(i^*-n)}\left\{(1+n)w+(i^*-n)\left[A(-1)-B(-1)\right]\right\} \tag{24}$$

d) The transfers to the youth in period $t \geq 0$, $s_1(t)$, can be set to arbitrary levels. Public indebtedness per worker, is given by

$$B(t) = s_1(t) + \frac{1+i^*}{(1+n)(2+\beta)+(i^*-n)}\left\{w-(2+\beta)\left[A(-1)-B(-1)\right]\right\}, \text{ for } t \geq 0 \tag{25}$$

e) Assumption 3 implies the validity of Assumption 2.

f) Net foreign assets per worker are constant over time and given by

$$F(t) = A(t) - B(t) = A(-1) - B(-1), \text{ for } t \geq 0 \tag{26}$$

g) The transfers to the elderly after period 0 are given by

$$s_2(t+1) = -(1+i^*)s_1(t) - \frac{(i^*-n)(1+i^*)}{(1+n)(2+\beta)+(i^*-n)}\left\{w-(2+\beta)\left[A(-1)-B(-1)\right]\right\}, \ t \geq 0 \tag{27}$$

Proof. In Appendix.

To understand the economic meaning of Proposition 1, we introduce the concept of admissible paths of the consumption flows of the successive generations. First, such paths must satisfy the inter-temporal budget constraint (12). Moreover, they must be consistent with the ratio set by each consumer between its consumption in the two parts of its life, given by Eq. (7) and (8)

$$C_2(t+1)/C_1(t) = (1+i^*)/(1+\beta), \text{ for } t \geq 0 \tag{28}$$

If we substitute this expression in Eq. (12), the admissible paths of the consumption of the elderly are given by

$$\frac{2+\beta}{1+i^*}\sum_{t=0}^{\infty} C_2(t+1)\left[(1+n)/(1+i^*)\right]^{t+1} + C_2(0)/(1+i^*) = (1+n)w/(i^*-n) + A(-1) - B(-1) \tag{29}$$

The steady state solution of Eq. (29), $C_2(t) = C_2$ for $t \geq 0$, is identical to Eq. (23).

The utility of the old generation living in period 0 is given by Eq. (10): $U(-1) = \ln[C_2(0)]$. The utility of the generation born in period $t \geq 0$ can easily be deduced from Eq. (6)

$$(1+n)^{t+1}U(t) = (1+n)^{t+1}\frac{2+\beta}{1+\beta}\ln[C_2(t+1)] \tag{30}$$

If for a moment the government of period 0 could commit itself to implement all its decisions, and if it cares for all generations and uses the social welfare function

$$U(-1) + D\sum_{t=0}^{\infty}[E(1+n)]^{t+1}U(t)\text{, with } D > 0 \text{ and } 0 < E < 1/(1+n) \tag{31}$$

then this government would select the optimal admissible path of the consumption of the elderly by maximizing expression (31) under constraint (29). The first order conditions of this program is

$$C_2(t+1) = [D/(1+\beta)][(1+i^*)E]^{t+1}C_2(0) \tag{32}$$

We will get the same result as Eq. (23) if and only if

$$D = 1+\beta\text{, and: } E = 1/(1+i^*) \tag{33}$$

The social welfare function of the government becomes $U(-1) + \frac{1+\beta}{1+i^*}\sum_{t=0}^{\infty}\frac{(1+n)^{t+1}U(t)}{(1+i^*)^t}$. The optimal policy is implemented by the same system of transfers as in Proposition 1. We sum up the above considerations:

Proposition 2. The first equilibrium path of the model is the only admissible steady state path consistent with the initial conditions. It would be selected by a government who could commit itself to the execution of its decisions in the future if its social welfare function satisfied the constraints (33).

We have assumed that a government cannot commit itself to its decisions, which will be implemented by future governments, and that it only cares for the living generations. However, in spite of this selfishness, the decisions taken by the successive governments result in a consumption flow per head, which does not change over time. This egalitarian allocation of consumption between generations implies that the government of a period does not transfer the cost of its generosity with the living to the unborn, and does not deteriorate the fate of the later generation. The first equilibrium path of the model is independent of the weight A which the government gives to the youth in comparison to the elderly.

The first solution of the model has the property of hysteresis: the net foreign assets per worker in each period and the consumption of each person are constant over time and permanently depend on the initial values of the net foreign assets (Laffargue, 2004). The transfers to the youth $s_1(t)$ are undetermined. This indetermination is transmitted to the transfers to the elderly $s_2(t)$ and to public and private debts, $B(t)$ and $A(t)$. However, the value at birth of the transfers to an individual over its life cycle, $s_1(t)+s_2(t+1)/(1+i^*)$, is uniquely determined and constant over time. It is set in a way that keeps constant the amount of foreign assets per worker: $F(t)=A(t)-B(t)$.

The government of period t sets $s_2(t)$ and the welfare of the old generation in period t to the level it wants because no future government can correct this decision. However, the government of period $t+1$ can punish the government of period t by increasing its taxes on the generation, which was young in period t. The parameter λ measures the extent of this punishment, which is inversely proportional to the weight of the youth, A in the objective function of the government of period t. The more this government cares for this generation, the milder the punishment necessary to forbid this government of borrowing too much.

AT last, we investigate how the first equilibrium reacts to the demographic transition. This will be defined as a decrease in the population growth rate n. We assume that n is constant and relatively large until period $T-1>0$, then it is set to a lower value at and after period T. Eq. (26) shows that the net national wealth per worker, $A(t)-B(t)$, does not change at and after T. From Eq. (23) and (24), at and after T, the consumptions per head $C_1(t)$ and $C_2(t)$ decrease (increase) by the same proportion if $w-(2+\beta)[A(-1)-B(-1)]$ is positive (negative), that is if the value at birth of the transfers received over the lifetime is positive (negative).

5. An Equilibrium immesirising Future Generations. Case: $\lambda = \frac{1+i^*}{1+n}\frac{1}{A/(1+\beta)+1/(1+n)}$

Lemma 2 showed that the expansion rate of consumption per head is equal to $\lambda A/(1+\beta)$. Assumption 5 below implies that this rate is less than 1, that: $i^* - n < \lambda < (1+\beta)/A$. The successive generations will consume less and less.

Assumption 5. The weight given to the young in the objective function of governments satisfies the constraint: $A < (1+\beta)/(i^* - n)$.

In section 4, we suggested that $A \sim DE = (1+\beta)/(1+i^*)$. If: the discount rate of consumers is close to the international interest rate, $\beta \sim i^*$, we have: $A \sim 1$. This implies that the constraint of Assumption 5 is approximately equivalent to $n > -1$, which is a constraint that we set at the beginning of section 1. Under this assumption, parameter λ, which measures the severity with which a government reduces its transfers in reaction to the profligacy of its predecessor, is lower for the second equilibrium than for the first. The first equilibrium is based on the perfect foresight that the next government will adopt a rigorous budgetary rule. This expectation disciplines the present government, which adopts the same rule. The net foreign assets per worker remains unchanged over time and there is no transfer of the cost of the current government policy to the following generations. In the second equilibrium, the current government anticipates that its successor will follow a laxer budgetary rule. It lacks incentive to follow itself a rigorous fiscal policy and leaves the net foreign assets of the economy to decrease. Successive generations will become poorer and poorer.

Proposition 3. The second equilibrium path of the economy is described by:

a) The constant part of the transfers to the elderly in periods after 0 is:

$$s_2 = w[\lambda/(i^* - n) - 1](1+i^*) \tag{34}$$

b) The transfers to the elderly in period 0 are:

$$s_2(0) = \frac{1+i^*}{(2+\beta)\lambda A/(1+\beta)+\lambda/(1+n)}\left\{w\lambda/(i^*-n)-(2+\beta)A(-1)\lambda A/(1+\beta)-\lambda B(-1)/(1+n)\right\} \tag{35}$$

c) The consumption flows of each old and young person in period $t \geq 0$ are:

$$C_2(t) = \frac{\lambda}{i^*-n}\frac{1+i^*}{(1+n)(2+\beta)\lambda A/(1+\beta)+\lambda}\{(1+n)w+(i^*-n)[A(-1)-B(-1)]\}\left(\frac{\lambda A}{1+\beta}\right)^t \tag{36}$$

$$C_1(t) = \frac{\lambda}{i^*-n}\frac{1+\beta}{(1+n)(2+\beta)\lambda A/(1+\beta)+\lambda}\{(1+n)w+(i^*-n)[A(-1)-B(-1)]\}\left(\frac{\lambda A}{1+\beta}\right)^{t+1} \tag{37}$$

d) The transfers to the youth in period $t \geq 0$, $s_1(t)$, can be set to arbitrary levels. Public indebtedness per worker is:

$$B(t) = s_1(t) + w(1+i^*)/(i^*-n) - (2+\beta)C_2(t+1)/\lambda \text{, for } t \geq 0 \tag{38}$$

e) Assumption 3 implies the validity of Assumption 2.

f) Net foreign assets per worker are:

$$F(t) = -w(1+n)/(i^*-n) + [1 + A(1+n)(2+\beta)/(1+\beta)]C_2(t+1)/(1+i^*) \text{, for } t \geq 0 \tag{39}$$

g) The transfers to the elderly after period 0 are given by

$$s_2(t+1) = -(1+i^*)s_1(t) - w(1+i^*) + (2+\beta)C_2(t+1) \text{, for } t \geq 0 \tag{40}$$

Proof. In Appendix.

This second equilibrium path of the economy is still admissible and efficient. We can easily show that the consumption in period 0 of each old person is higher than for the first equilibrium. However, the consumption of a person in each stage of his life decreases from generation to generation and converges to zero. We get again the same indetermination of the transfers to the youth as for the first equilibrium. This indetermination is transmitted again to the transfers to the elderly and to public and private debts. The value at birth of the transfers to an individual over his life cycle, $s_1(t) + s_2(t+1)/(1+i^*) = -w + (2+\beta)C_2(t+1)$, is uniquely determined, it decreases over time and converges to $-w$. Net foreign assets per worker, $A(t) - B(t)$, decrease over time and converge to the negative amount $-w(1+n)/(i^*-n)$. In the long run, each person pays the total amount of his wealth at birth to the government in order to finance the stabilization of the foreign debt, which results from the profligacy of previous governments.

6. An Alternative Definition of the Equilibrium

Each government has two control variables, which are independent of one another, the transfers to its youth and the transfers to its elderly. To prevent corner solutions we constrained the reaction function of governments by Assumption 4. Under this condition we proved the existence of two equilibria. For each of them the paths of consumption of the young and the old and of the net foreign assets of the economy are uniquely determined. However, there exists a large indetermination of the paths followed by public transfers and debt. As in a Ricardian world, increasing the transfers to the youth and financing the cost of this policy by issuing public bonds, which are subscribed by the young who need this saving to face the reduction in the transfers to be received in the next period, has no economic consequence (Eq. (25) and (27)). However, we dislike this indetermination, and look for an alternative to Assumption 4. Instead of constraining the reaction function of governments, we constrain their decisions, for example, by forbidding a government to increase (decrease) the transfer to the elderly without increasing (decreasing) the transfers to the youth, by the same amount:

Assumption 6. The transfers to the two generations in a same period must satisfy: $s_2(t)/(1+n) = s_1(t) + \psi$, with $\psi > 0$, $t \geq 0$.

Proposition 4. Under Assumption 6, the economy takes one of two equilibrium paths, each including an infinity of paths indexed by the parameter μ of the governments' reaction function.

a) In the first path, consumption and transfers are constant over time, but their levels depend on parameter μ.

b) In the second path, consumption and transfers grow at the expansion rate $\frac{1}{1+n}\left(\frac{(1+i^*)\mu}{1+(1+\beta)/A/(1+n)}\right)^{1/2}$. Their initial values depend on parameter μ.

Proof. In Appendix.

What its new is that, for each of the two equilibria, the difference in punishment of the profligacy in favor of the young relatively to the old, μ, can be set at an arbitrary value. Expressions given in the proof determine the extent λ of the punishment of the profligacy on the old. We have an infinity of each of the two kinds of equilibria, indexed by the parameter μ of the reaction function of governments. However, for each value of this parameter and for each kind of equilibrium, we obtain a unique path of the transfers and public debt: the indetermination with the previous definition of equilibrium has disappeared.

For the second kind of equilibria, the expansion rate of the consumption of the elderly can be less or greater than 1. It increases with the values of parameters μ and A. Beside immesirisation, we also have the opposite case where the accumulation of public and foreign assets and transfers to the future generations improves over time their welfare.

The new definition of equilibrium rests on an *ad hoc* constraint on the choices of a government. Other constraints would give different results. Moreover, the indeterminacy in the equilibriums obtained in sections 4 and 5 is rather innocuous and meaningful. Now, we get a much larger set of very different equilibria.

Conclusion

We have used an overlapping generations model of an open economy to analyze the setting of time consistent intergenerational transfers policies. Governments make their decisions without putting weight on the welfare of future generations. They cannot commit to decisions, which will be executed by their successors. However, they can constrain the choices of the governments of the future by setting the level of public debt (governments always face their obligations to their creditors). A government can punish a predecessor, which borrowed too much, by increasing taxes on people who were already alive under this previous government (which cared for the welfare of these consumers).

The model has two perfect foresight equilibrium paths. In the first one where the expected rate of punishment is high, governments follow rigorous policies and consumption and foreign debt per head remain constant over time. Future generations will not be sacrificed for the benefit of the current ones. The demographic transition does not change this result. It only makes all the consumers uniformly poorer. In the second equilibrium where the expected rate of punishment is lower, governments adopt laxer policies. The initial consumption of the elderly is higher than for the first equilibrium. However, consumption decreases over time and converges to zero. The foreign debt per head increases over time and converges to a high level.

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APPENDIX. Proofs

Proof of Lemma 1

The program, which determines the choices of the government of time $t \geq 0$ is

$$\underset{s_1(t), s_2(t)}{Max} \left[U(t-1) + A(1+n)U(t) \right] \quad (41)$$

$$U(t-1) = \ln\left[(1+i^*)A(t-1) + s_2(t)\right] \quad (42)$$

$$U(t) = \frac{2+\beta}{1+\beta} \ln\left[w + s_1(t) + s_2(t+1)/(1+i^*)\right] \quad (43)$$

$$s_2(t+1) = s_2 - \lambda B(t) - \mu s_1(t) \quad (44)$$

$$B(t) = s_1(t) + s_2(t)/(1+n) + \left[(1+i^*)/(1+n)\right] B(t-1) \quad (45)$$

with $A(t-1)$, $B(t-1)$, s_2, λ given and $\mu = 1 + i^* - \lambda$.

This program is equivalent to the program

$$\underset{s_1(t), s_2(t)}{Max} \left[U(t-1) + A(1+n)U(t)) \right] \quad (46)$$

$$U(t-1) = \ln\left[(1+i^*)A(t-1) + s_2(t)\right] \quad (47)$$

$$U(t) = \frac{2+\beta}{1+\beta} \ln\left[w + s_2/(1+i^*) - \lambda s_2(t)/(1+n)/(1+i^*) - \lambda B(t-1)/(1+n)\right] \quad (48)$$

with $A(t-1)$, $B(t-1)$, s_2, λ given.

The value taken by the objective function of this program is independent of the transfers $s_1(t)$ to the youth. Its maximization relatively to the transfers to the elderly $s_2(t)$ gives Eq. (18). The second order condition is satisfied.

Proof of Lemma 3

Eqs (15) and (2) can be written

$$(2+\beta)C_2(t+1) - w(1+i^*) = s_2 + \lambda[s_1(t) - B(t)] \quad (49)$$

$$(1+n)[s_1(t) - B(t)] - (1+i^*-\lambda)[s_1(t-1) - B(t-1)] + s_2 = 0 \text{, for } t \geq 1 \quad (50)$$

We use the first of these equations to eliminate $s_1(t) - B(t)$ and $s_1(t-1) - B(t-1)$ from the second equation. We get, for $t \geq 1$

$$(i^*-n)s_2 = -(2+\beta)(1+n)\left[C_2(t+1) - \frac{1+i^*-\lambda}{1+n}C_2(t)\right] - w(1+i^*)(i^*-n-\lambda) =$$
$$-(2+\beta)(1+n)\left[\frac{\lambda A}{1+\beta} - \frac{1+i^*-\lambda}{1+n}\right]\left[(1+i^*)A(-1) + s_2(0)\right]\left(\frac{\lambda A}{1+\beta}\right)^t - w(i^*-n-\lambda)(1+i^*)$$

(51)

As $i^* > n$, s_2 can be constant over time only if: $\lambda A = 1+\beta$ or if: $\lambda = \frac{1+i^*}{1+n}\frac{1}{A/(1+\beta)+1/(1+n)}$.

Proof of Lemma 4

We will assume in the proof that: $n = 0$, which will simplify the notations. The objective function of the government of period t is

$$\ln[(1+i^*)A(t-1) + s_2(t)] + A\frac{2+\beta}{1+\beta}\ln[w + s_1(t) + s_2(t+1)/(1+i^*)] \quad (52)$$

This government considers $A(t-1)$ as given and forecasts the reaction function of the government of period $t+1$: $s_2(t+1) = f[B(t), s_1(t)]$. We assume that this function is continuously differentiable. We recall that

$$B(t) = s_1(t) + s_2(t) + (1+i^*)B(t-1) \quad (53)$$

We assume that the function $s_1(t) + f[s_1(t) + s_2(t) + (1+i^*)B(t-1), s_1(t)]/(1+i^*)$ has a unique maximum in $s_1(t)$: $s_1(t) = g[s_2(t) + (1+i^*)B(t-1)]$, with function g continuously differentiable. We substitute this expression of $s_1(t)$ in the above function to obtain

$$s_1(t) + f[s_1(t) + s_2(t) + (1+i^*)B(t-1), s_1(t)]/(1+i^*) = h[s_2(t) + (1+i^*)B(t-1)] \quad (54)$$

The function h is continuously differentiable.

We maximize the objective function of the government relatively to $s_2(t)$ and obtain the necessary first-order condition

$$\frac{1}{(1+i^*)A(t-1)+s_2(t)}+A\frac{2+\beta}{1+\beta}\frac{h'[s_2(t)+(1+i^*)B(t-1)]}{w+h[s_2(t)+(1+i^*)B(t-1)]}=0 \quad (54)$$

We deduce from Eq. (15)

$$(1+i^*)A(t-1)=\frac{1+i^*}{2+\beta}[w+s_1(t-1)]-\frac{1+\beta}{2+\beta}s_2(t) \quad (55)$$

The first-order condition becomes

$$\frac{1}{(1+i^*)[w+s_1(t-1)]+s_2(t)}+\frac{A}{1+\beta}\frac{h'[s_2(t)+(1+i^*)B(t-1)]}{w+h[s_2(t)+(1+i^*)B(t-1)]}=0 \quad (56)$$

For $t\geq 1$ we have

$$\begin{aligned} s_2(t)&=h[s_2(t-1)+(1+i^*)B(t-2)]-g[s_2(t-1)+(1+i^*)B(t-2)]\\ &=h[B(t-1)-s_1(t-1)]-g[B(t-1)-s_1(t-1)] \end{aligned} \quad (57)$$

Let us set: $x=B(t-1)-s_1(t-1)$, and $y=s_1(t-1)$. We have the functional equation

$$\frac{1}{(1+i^*)(w+y)+h(x)-g(x)}=-\frac{A}{1+\beta}\frac{h'[h(x)-g(x)+(1+i^*)(x+y)]}{w+h[h(x)-g(x)+(1+i^*)(x+y)]} \quad (58)$$

or

$$\begin{aligned} &(1+i^*)(w-x)=F[h(x)-g(x)+(1+i^*)(x+y)]\equiv\\ &-\frac{1+\beta}{A}\frac{w+h[h(x)-g(x)+(1+i^*)(x+y)]}{h'[h(x)-g(x)+(1+i^*)(x+y)]}-\{h(x)-g(x)+(1+i^*)(x+y)\} \end{aligned} \quad (59)$$

As the right-hand side must be independent of y, the function F must be constant. However, in this case it cannot depend linearly on x.

Proof of Proposition 1

a) The last equation of the proof of Lemma 3 gives

$$(i^*-n)s_2=(i^*-n-\lambda)\{(2+\beta)[(1+i^*)A(-1)+s_2(0)]-w(1+i^*)\} \quad (60)$$

If we notice that: $C_2(0)=C_2(1)$, Eq. (15) written for $t=0$ gives

$$(2+\beta)\left[s_2(0)+(1+i^*)A(-1)\right]/\lambda - w(1+i^*)/\lambda = s_2/\lambda - \left[B(0)-s_1(0)\right] \quad (61)$$

Eq. (2) written for $t=0$ gives

$$B(0)-s_1(0) = s_2(0)/(1+n) + (1+i^*)B(-1)/(1+n) \quad (62)$$

We eliminate $B(0)-s_1(0)$ between the two last equations. We get

$$s_2 = \left[2+\beta+\lambda/(1+n)\right]s_2(0) + (2+\beta)(1+i^*)A(-1) - w(1+i^*) + \lambda(1+i^*)B(-1)/(1+n) \quad (63)$$

We eliminate s_2 between Eq. (63) and the Eq. (60). We get Eq. (21).

b) If we substitute the expression of $s_2(0)$ given by Eq. (21) in the last expression of s_2, we get Eq. (22).

c) If we substitute the expression of $s_2(0)$ given by Eq. (21) in Eq. (9), we get Eq. (23). We use Eqs (7) and (8) to get Eq. (24).

d) Eq. (15) written for $t \geq 0$ gives

$$B(t)-s_1(t) = -(2+\beta)C_2/\lambda + w(1+i^*)/\lambda + s_2/\lambda \quad (64)$$

If we substitute Eq. (64), the expressions of s_2 and C_2 given by Eq. (22) and (23), we get Eq. (25).

e) Assumption 2 is equivalent to $C_2(t) > 0$, which is equivalent to Assumption 3.

f) We subtract $B(t)$ from the expression of $A(t)$ given by Eq. (15)

$$(2+\beta)(1+i^*)\left[A(t)-B(t)\right] = w(1+i^*) - (1+\beta)s_2 + \left[(1+i^*)(2+\beta) - \lambda(1+\beta)\right]\left[s_1(t)-B(t)\right] \quad (65)$$

We substitute the expressions of s_2 and $B(t)$ given by Eq. (22) and (25). We get Eq. (26).

g) Eq. (27) is deduced from Eq. (19), (22) and (25).

Proof of Proposition 3

a) The last equation of the proof of Lemma 3 gives Eq. (34).

b) If we notice that: $C_2(0)\lambda A/(1+\beta) = C_2(1)$, Eq. (15) written for $t=0$ gives

$$(2+\beta)\left[s_2(0)+(1+i^*)A(-1)\right]\lambda A/(1+\beta) - w(1+i^*) = s_2 - \lambda\left[B(0)-s_1(0)\right] \quad (66)$$

Eq. (2) written for $t=0$ gives

$$B(0)-s_1(0) = s_2(0)/(1+n) + (1+i^*)B(-1)/(1+n) \quad (67)$$

We eliminate $B(0)-s_1(0)$ between Eq. (66) and Eq. (67). We get

$$s_2=[(2+\beta)\lambda A/(1+\beta)+\lambda/(1+n)]s_2(0)+(2+\beta)(1+i^*)A(-1)\lambda A/(1+\beta) \\ -w(1+i^*)+\lambda(1+i^*)B(-1)/(1+n) \tag{68}$$

We eliminate s_2 between Eq. (68) and Eq. (34) and get Eq. (35).

c) If we use Eqs (9) and (35) we get

$$C_2(0)=\frac{\lambda(1+i^*)}{(2+\beta)\lambda A/(1+\beta)+\lambda/(1+n)}\{w/(i^*-n)+[A(-1)-B(-1)]/(1+n)\} \tag{69}$$

We deduce Eq. (40) and (41) and Eq. (7) and (8).

d) Eq. (9) and (15) written for $t\geq 0$ give

$$\lambda[B(t)-s_1(t)]=s_2+w(1+i^*)-(2+\beta)C_2(t+1) \tag{70}$$

If we use the expressions of s_2 given by Eq. (34), we get Eq. (38).

e) Assumption 2 is equivalent to $C_2(t)>0$, which is equivalent to Assumption 3.

f) We deduce Eq. (39) from Eq. (9), (11) and (38).

g) We deduce Eq. (40) from Eq. (19), (34) and (38).

Proof of Proposition 4

The wealth at birth of a person young in period t is

$$W(t)=w+\frac{s_2}{1+i^*}-\psi\left(1-\frac{\lambda+\mu}{1+i^*}\right)-\frac{\lambda}{1+n}B(t-1)-[2\lambda++\mu-(1+i^*)]\frac{s_2(t)}{(1+n)(1+i^*))} \tag{71}$$

Eq. (20) of Lemma 2 is substituted by

$$\frac{C_2(t+1)}{C_2(t)}=\frac{A}{1+\beta}[2\lambda+\mu-(1+i^*)] \tag{72}$$

We get a dynamic system with 4 variables, $s_1(t)$, $s_2(t)$, $B(t)$ and $C_2(t)$, and Eq. (16), (2), the relationship of Assumption 6 and

$(2+\beta)C_2(t+1)=w(1+i^*)+(1+i^*)s_1(t)+s_2(t+1)$, with $t\geq 0$, $B(-1)$ given

This system leaves the initial value $s_2(0)$ undetermined, but when this value is set, we compute the initial values of the two last variables $s_1(0)$ and $C_2(0)$ with Eq. (9) and the relationship of

Assumption 6. We will deduce the value of $s_2(0)$ and of parameters λ and s_2, from the fact that $C_2(t)$ must grow at the geometric rate. We obtain the dynamic system of two equations with two variables, $s_2(t)$ and $B(t)$.

$$s_2(t+1) = s_2 + (\lambda + \mu)\psi - (2\lambda + \mu)s_2(t)/(1+n) - \lambda[(1+i^*)/(1+n)]B(t-1) \tag{73}$$

$$B(t) = -\psi + 2s_2(t)/(1+n) + [(1+i^*)/(1+n)]B(t-1),\ t \geq 0,\ B(-1) \text{ given} \tag{74}$$

In this system, the steady state value $\hat{s}_2$ of $s_2(t)$ is given by

$$\left[(i^* - n) - 2\lambda + \frac{i^* - n}{1+n}\mu\right]\hat{s}_2 = (i^* - n)[s_2 + (\lambda + \mu)\psi] - \lambda(1+i^*)\psi$$

The eigenvalues of the dynamic system are the solutions ρ_1 and ρ_2 of the characteristic equation

$$\rho^2 - \frac{1+i^* - 2\lambda - \mu}{1+n}\rho - \frac{1+i^*}{(1+n)^2}\mu = 0 \tag{75}$$

To be brief, we consider the cases when $2\lambda \neq (i^* - n)[\mu/(1+n)+1]$ and $\mu > 0$. The system has a unique steady state and two different and real eigenvalues, which differ from 1.

$B(-1)$ is known. If we set $s_2(0)$, we can easily compute $B(0)$, then $s_2(1)$.

The dynamic path of the transfers to the elderly is

$$s_2(t) - \hat{s}_2 = b_1\rho_1^t + b_2\rho_2^t \tag{76}$$

with

$$b_1 = \{[s_2(1) - \hat{s}_2] - [s_2(0) - \hat{s}_2]\rho_2\}/(\rho_1 - \rho_2) \tag{77}$$

and $b_2 = -\{[s_2(1) - \hat{s}_2] - [s_2(0) - \hat{s}_2]\rho_1\}/(\rho_1 - \rho_2)$ (78)

Finally, the expression of the path followed by the consumption of the elderly is

$$(2+\beta)C_2(t+1) =$$
$$w(1+i^*) - (1+i^*)\psi + \left[\frac{1+i^*}{1+n} + 1\right]\hat{s}_2 b_1\left[\frac{1+i^*}{(1+n)\rho_1} + 1\right]\rho_1^{t+1} + b_2\left[\frac{1+i^*}{(1+n)\rho_2} + 1\right]\rho_2^{t+1} \tag{79}$$

We know that this consumption grows at the expansion rate $\frac{A}{1+\beta}[2\lambda + \mu - (1+i^*)]$. We have two different kinds of equilibrium paths for the model.

a) The consumption remains constant over time

$$2\lambda = (1+\beta)A + (1+i^*) - \mu \tag{80}$$

$$C_2(t+1) = C_2(0) = \left\{ w(1+i^*) - (1+i^*)\psi + \left[\frac{1+i^*}{1+n} + 1\right]\hat{s}_2 \right\} / (2+\beta) \tag{81}$$

$$b_1 = b_2 = 0 \tag{82}$$

We are in the case of an egalitarian equilibrium. The two last conditions, which are equivalent to $s_2(0) = s_2(1) = \hat{s}_2$ determine $s_2(0)$ and s_2.

b) The consumption grows at the rate of the eigenvalues of the dynamic system. This growth rate satisfies the characteristic equation

$$\left(\frac{A}{1+\beta}\right)^2 \left(1 + \frac{1+\beta}{A(1+n)}\right) [2\lambda + \mu - (1+i^*)]^2 = \frac{1+i^*}{(1+n)^2}\mu \tag{83}$$

As, consumption cannot be negative, we have

$$2\lambda = 1 + i^* - \mu + \frac{1+\beta}{A(1+n)} \left(\frac{(1+i^*)\mu}{1 + (1+\beta)/A/(1+n)} \right)^{1/2} \tag{84}$$

$$C_2(t+1) = \left[\frac{1}{1+n} \left(\frac{(1+i^*)\mu}{1 + (1+\beta)/A/(1+n)} \right)^{1/2} \right]^{t+1} C_2(0) \tag{85}$$

$$s_2(1) - \hat{s}_2 = \left[\frac{1}{1+n} \left(\frac{(1+i^*)\mu}{1 + (1+\beta)/A/(1+n)} \right)^{1/2} \right] (s_2(0) - \hat{s}_2) \tag{86}$$

and $w(1+i^*) - (1+i^*)\psi + \left[\frac{1+i^*}{1+n} + 1\right]\hat{s}_2 = 0$ (87)

The two last conditions determine $s_2(0)$ and s_2.