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The k-factor Gegenbauer Asymmetric Power GARCH approach for modelling electricity spot price dynamics

Abdou Kâ Diongue,* Dominique Guégan†

Abstract

Electricity spot prices exhibit a number of typical features that are not found in most financial time series, such as complex seasonality patterns, persistence (hyperbolic decay of the autocorrelation function), mean reversion, spikes, asymmetric behavior and leptokurtosis. Efforts have been made worldwide to model the behaviour of the electricity's market price. In this paper, we propose a new approach dealing with the stationary k-factor Gegenbauer process with Asymmetric Power GARCH noise under conditional Student-t distribution, which can take into account the previous features. We derive the stationary and invertible conditions as well as the δth-order moment of this model that we called GG_k-APARCH model. Then we focus on the estimation parameters and provide the analytical form of the likelihood which permits to obtain consistent estimates. In order to characterize the properties of these estimates we perform a Monte Carlo experiment. Finally the previous approach is used to model electricity spot prices coming from the Leipzig Power Exchange (LPX) in Germany, Powernext in France, Operadora del Mercado Español de Electricidad (OMEL) in Spain and the Pennsylvania-New Jersey-Maryland (PJM) interconnection in United States. In terms of forecasting criteria we obtain very good results comparing with models using hederoscedastic asymmetric errors.

Keywords: Asymmetric distribution function - Electricity spot prices - Leptokurtosis - Persistence - Seasonality - GARMA - A-PARCH.

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1 Introduction

The appropriate modelling of electricity price processes is of interest for several reasons. First of all, the forecasting of electricity prices is of interest by itself in the management and trading in electricity markets. Second, the operation of electricity markets can be considered as similar as the operation of financial markets with electricity power derivatives being priced and traded in highly competitive markets. Thus, dynamic modelling of means and variances appears essential for this kind of data sets. A lot of propositions have already been done in the literature to model them. They are based on the fact that most of empirical works on electricity prices tend to focus on several features: namely seasonality, mean-reversion, spikes, high volatility, asymmetry and long-memory persistence. We explain the reasons of these interests.

1. Electricity spot prices display a pronounced seasonality at intra-daily, weekly and monthly levels, Escribano *et al.* (2002), Koopman *et al.* (2007) or Knittel and Roberts (2005). Since electricity is a commodity that cannot be stored, the demand and supply of electricity are highly inelastic and very sensitive to weathers and business cycles.

2. Other simple consequence of the real nature of this commodity is that the prices are mean-reverting, Bosco *et al.* (2007).

3. The spot electricity prices exhibit also infrequent and large jumps cause by extreme load fluctuations (due to severe weather conditions, generation outages, transmission failures, etc.), Clewlow and Strickland (2000) and Weron *et al.* (2004). It is uncommon that prices from one day to the next one, or even within just a few hours, can rise by a factor of ten or more. The time periods of considerable prices, are normally short, and prices tend to fall back down to more "normal" levels after just a few hours. Such rapid price changes are of uttermost importance to take into consideration if one wants to understand and/or characterize the electricity spot price process.

4. A growing body of literature is emerging that examines the general characteristics and extreme price volatility in electricity markets. For example, Hadsell *et al.* (2004) estimate conditional volatility in five U.S markets and show that deregulated electricity markets exhibit levels of price volatility unparalleled in traditional commodity markets.

5. Further research asserts the importance of high kurtosis on electricity spot price, Diongue *et al.* (2004) or Higgs and Worthington (2005).

6. While the GARCH specification of Bollerslev (1986) captures the conditional variance in the spot returns, periods of high volatility followed

by extended periods of relative calm suggest an asymmetric response in electricity prices. This so-called leverage effect, firstly introduced by Black (1976), is usually stated as the existence of a negative correlation between past returns and future volatility, but not the other way around. To our knowledge, this feature is poorly reported in the modelling of electricity spot prices.

7. The significant volatility effects in the conditional standard deviation and also significant asymmetric responses of volatility, have been modelled by Hadsell *et al.* (2004) with a TARCH model while Higgs and Worthington (2005) investigated the intraday price volatility process for four Australian wholesale electricity markets using five models; namely GARCH, RiskMetrics (Gaussian Integrated GARCH), Gaussian APARCH, Student APARCH and skewed Student APARCH.

8. Some recent studies have shown that electricity prices can be described by long-memory processes. Leon and Rubia (2001), using Hylleberg, Engle, Granger and Yoo (1990) tests, cannot reject the hypothesis that electricity prices from OMEL market (Spain) have unit roots at the long-run frequency as well as at seasonal frequencies. Fractional differencing and non-stationarity are also detected by Koopman *et al.* (2007) in a periodic time series framework while Soares and Souza (2006) used the generalized long memory process of Gray *et al.* (1989) to forecast electricity demand from a Brazilian data set.

In this paper we propose a new approach which permit to take into account mainly all these features. Most of the studies that we list before use models which permit the modelling of one or two features but not more. Specifically they do not model the long memory behavior inside the seasonalities. Recall that the long memory models introduced by Granger and Joyeux (1980) and Hosking (1981) permit to model an infinite cycle which is too restrictive for the electricity prices. One of the main characteristic of the high frequencies data sets is the presence of volatility clustering and leptokurtosis, as soon as persistence and cyclical patterns in the conditional mean of the series combined with conditional heteroscedasticity. All these characteristics are presented inside electricity spot prices. The model introduced by Guégan (2000) permit to take these features into account and it has been applied to describe the German electricity spot prices with symmetric distribution function by Diongue *et al.* (2004). One limit of this last model concerns the non-existence of asymmetry. In another hand, to take into account the so-called leverage effects, we can use different nonlinear extensions of the GARCH model. We can cite the exponential GARCH (EGARCH) model of Nelson (1991), the threshold ARCH (TARCH) model of Zakoian (1994), the asymmetric power ARCH (APARCH) model of Ding, Granger and Engle (1993) or the GJR-GARCH(1,1) model introduced by Glosten, Jagannathan and Runkel (1993). These models allow past negative (resp. positive) shocks to have a deeper impact on current conditional volatility than past positive (resp. negative) shocks, but these models do not integrate in their stucture seasonalities or long memory behavior.

In the present paper, we examine the issue of long memory process with asymmetry power GARCH innovations that belong to the family of conditionally heteroscedastic processes with conditional Student-t distribution function in order to apply it on electricity prices. This new class of processes, that we call the GG_k-APARCH *(k-factor Gegenbauer with Asymmetry Power GARCH)* process, permits the modelling of long persistence, pseudo seasonnalities, volatility clustering, leverage effect, asymmetry and leptokurticity. We focus on two objectives. The first one consists in providing the probabilistic properties of the model (existence of a stationary invertible solution and expression of the higher order moments). The second one deals with the estimation theory. In order to provide consistent estimates, we use the maximum likelihood approach based on the analytical expression of the likelihood. Thus, we derive the exact expression of the likelihood, the score functions and of the Hessian matrix. This approach is new for such a model and we keep it all along the paper, even for the applications when we use competitive asymmetric GARCH innovations to compare the accuracy of the model developed in this paper. A Monte carlo experiment permit also to examine the properties of the empirical estimates for several GG_k-APARCH.

The paper is organized as follows. In section 2 we specify some notations, introduce the new model and give the conditions for the existence of a stationary solution. We also study the existence of the higher order moments. In Section three we deal with the estimation theory and provide the exact analytical expression of the maximum likelihood function when the innovations follow a Student-t distribution function. In Section four we exhibit numerical simulations in order to examine the properties of the parameters estimates. The Section five is devoted to the study of four electricity markets: we compare the results obtained from different versions of the GG_k-APARCH model with other models using classical asymmetric noises. Section six concludes.

2 The stationary GG_k-APARCH

The k-factor GIGARCH process $(X_t)_{t\in\mathbb{Z}}$, introduced by Guégan (2000, 2003), permitting to model existence of k pseudo seasonalities associated with persitence in the observations and existence of volatility on the conditional variance has the following expression: for all t,

$$\phi(B)\prod_{i=1}^{k}\left(I-2\nu_i B+B^2\right)^{d_i}(X_t-\mu)=\theta(B)\,\varepsilon_t, \tag{2.1}$$

where μ is the mean of the process $(X_t)_{t\in\mathbb{Z}}$, $d_i, i = 1, \cdots, k$ are the long memory parameters such that $0 < d_i < \frac{1}{2}$ if $|\nu_i| < 1$ or $0 < d_i < \frac{1}{4}$ if $|\nu_i| = 1$ for $i = 1, \cdots, k$ with k a nonzero integer. The polynomials $\phi(B)$ and $\theta(B)$ denote the well known ARMA operators and B the backshift operator. In the k-factor GIGARCH model, Guégan (2000) assumes that the process $(\varepsilon_t)_t$ follows a GARCH process with a symmetric distribution function, Bollerslev (1986). In the following we extend this model in a general setting and investigate its probabilistic properties.

2.1 The GG_k-APARCH process : Definition

The new GG_k-APARCH process $(X_t)_{t\in\mathbb{Z}}$ that we consider in this paper is such that the observations X_t are explained through the expression (2.1) and the noise $(\varepsilon_t)_{t\in\mathbb{Z}}$ follows an extension of the APARCH model introduced by Ding, Granger and Engle (1993) that we introduced now:

$$\varepsilon_t = h_t \eta_t, \tag{2.2}$$

and

$$h_t^\delta = \alpha_0 + \sum_{i=1}^{r} \alpha_i \left(|\varepsilon_{t-i}| - \gamma\varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{s} \beta_j h_{t-j}^\delta, \tag{2.3}$$

where $(\eta_t)_{t\in\mathbb{Z}}$ is a sequence of independent identically distributed random variables with zero mean and finite mδ-th unconditional absolute moments, m being a positive integer. The parameter γ ($|\gamma| < 1$) reflects the so-called leverage effect. A positive (resp. negative) value of the asymmetric volatility response γ means that the negative (resp. positive) shocks have a deeper impact on current conditional volatility than past positive shocks. The parameter δ ($\delta > 0$) plays the role of a Box-Cox transformation of the conditional standard deviation h_t and, the parameters $\alpha_0 > 0$, $\alpha_i \geq 0$ ($i = 1, \cdots, r$) are real numbers with at least one $\alpha_i > 0$ or $\beta_i \geq 0$ ($i = 1, \cdots, s$).

This model (2.1)- (2.3) couples the flexibility of a varying exponent with the asymmetry coefficient and the possiblity of long persistence on seasonalities as well. It permits to answer to the modelling of the different features of electricity prices as described in the previous section. It includes the k-factor GIGARCH process $(X_t)_{t\in\mathbb{Z}}$ of Guégan (2000, 2003) and many other models among them, the ARCH models of Engle (1982) ($\phi(B) = \theta(B) = 1$, $d_i = 0$ for $i = 1, \cdots, k$, $\delta = 2$, $\gamma = 0$ and $\beta_i = 0$, $i = 1, \cdots, s$), the GARCH model of Bollerslev (1986) ($\phi(B) = \theta(B) = 1$, $d_i = 0$ for $i = 1, \cdots, k$, $\delta = 2$ and $\gamma = 0$), the Taylor (1986) and Schwert (1990) GARCH models ($\phi(B) = \theta(B) = 1$, $\delta = 1$, $\gamma = 0$ and $d_i = 0$ for $i = 1, \cdots, k$), the GJR model of Glosten, Jagannathan and Runkle (1993) ($\phi(B) = \theta(B) = 1$, $d_i = 0$ for $i = 1, \cdots, k$ and $\delta = 2$), the TARCH model of Zakoian (1994) ($\phi(B) = \theta(B) = 1$, $d_i = 0$ for $i = 1, \cdots, k$ and $\delta = 1$), the ARMA-ARCH model of Weiss (1984) ($d_i = 0$ for $i = 1, \cdots, k$, $\delta = 2$, $\gamma = 0$ and $\beta_i = 0$, $i = 1, \cdots, s$), the FARIMA model of Granger and Joyeux (1980) and Hosking (1981) ($k = 1$, $\nu = 0$, $\delta = 0$ and $\gamma = 0$), the FARIMA-GARCH model of Ling and Li (1997) ($k = 1$, $\nu = 0$, $\delta = 2$ and $\gamma = 0$) and the k-factor GARMA model of Chung (1994) or Woodward *et al.* (1998) ($\delta = 0$).

In order to give the properties of the process (2.1)- (2.3), we recall the definition of the Gegenbauer polynomials $C_j(d, u)$, Magnus, Oberhettinger and Soni (1966) or Rainville (1960). They are defined by:

$$\left(1 - 2uz + z^2\right)^d = \sum_{j \geq 0} C_j(d, u) z^j, \tag{2.4}$$

where $|z| \leq 1$ and $|u| \leq 1$ and they can be easily computed by the following recursion formula

$$\begin{cases} C_0(d, u) = 1 \\ C_1(d, u) = 2du \\ \forall j > 1, C_j(d, u) = 2u(\frac{d-1}{j} + 1)C_{j-1}(d, u) - (2\frac{d-1}{j} + 1)C_{j-2}(d, u). \end{cases} \tag{2.5}$$

2.2 Existence of a stationary solution for the GG$_k$-APARCH process

In this part, we prove the existence of a stationary solution for the process $(X_t)_{t \in \mathbb{Z}}$ defined by (2.1)-(2.3). First of all we define the notion of δ-order stationary process.

Definition 2.1 *We call δ-order stationary process, a process $(\varepsilon_t)_{t \in \mathbb{Z}}$ such that, for all t and all $\delta \in \mathbb{N}$, $E\left(\left|\varepsilon_t^{\delta}\right|\right) < \infty$.*

Now, we give an existence theorem.

Theorem 2.1 *Let be $(X_t)_t$ the process defined by (2.1)-(2.3). We assume that $\alpha_0 > 0$, α_i, for $i = 1, \cdots, r$, $\beta_i \geq 0$, for $i = 1, \cdots, s$, $\delta \geq 0$, $|\gamma| < 1$ and $|d_i| < \frac{1}{2}$ if $|\nu_i| < 1$ or $|d_i| < \frac{1}{4}$ if $|\nu_i| = 1$, for $i = 1, \cdots, k$. We assume also that $\phi(B)$ and $\theta(B)$ have no common factors and their roots lie outside the unit circle. Moreover, let be $Z_t = (|\eta_t| - \gamma\eta_t)^{\delta}$ with $E(Z_t) \sum_{i=1}^{r} \alpha_i + \sum_{j=1}^{s} \beta_j < 1$. Thus,*
(i) there exists a unique δ-order stationary solution $(X_t, \varepsilon_t)_{t \in \mathbb{Z}}$ for the model (2.1)-(2.3).
(ii) The solution has a causal representation given by

$$X_t - \mu = \sum_{k=0}^{\infty} \beta_k(d, \nu, \phi, \theta) \varepsilon_{t-k} \quad a.s, \tag{2.6}$$

where the process $(\varepsilon_t)_{t\in\mathbb{Z}}$ *is given by the relationship*

$$\varepsilon_t = \eta_t \left[\alpha_0 + \sum_{j=1}^{\infty} c^{'} \left(\prod_{i=1}^{j} A_{\delta t-i} \right) \xi_{\delta t-j} \right]^{\frac{1}{\delta}} \quad a.s, \tag{2.7}$$

where $\xi_{\delta t} = (\alpha_0 Z_t, 0, \cdots, 0, \alpha_0, 0, \cdots, 0)^{'}_{(r+s)\times 1}$. *The first component of this vector is* $\alpha_0 Z_t$ *and the* $(r+1)th$ *component is* α_0. *The vector* c *in (2.7) is defined by* $c = (\alpha_1, \cdots, \alpha_r, \beta_1, \cdots, \beta_s)^{'}$ *and the matrix* $A_{\delta t}$ *is equal to:*

$$A_{\delta t} = \left(\begin{array}{ccc|ccc} \alpha_1 Z_t & \cdots & \alpha_r Z_t & \beta_1 Z_t & \cdots & \beta_s Z_t \\ & \mathbf{I}_{(r-1)\times(r-1)} & O_{(r-1)\times 1} & & O_{(r-1)\times s} & \\ \hline \alpha_1 & \cdots & \alpha_r & \beta_1 & \cdots & \beta_s \\ & O_{(s-1)\times r} & & \mathbf{I}_{(s-1)\times(s-1)} & O_{(s-1)\times 1} & \end{array} \right), \tag{2.8}$$

with $\mathbf{I}_{r\times r}$ *the* $r \times r$ *identity matrix. The coefficients* $\beta_j(d, \nu, \phi, \theta)$ *are such that* $\beta_0(d, \nu, \phi, \theta) = 1$ *and for all* $j \geq 1$ *we have*

$$\beta_j(d, \nu, \phi, \theta) = \psi_j(d, \nu) + \sum_{i=1}^{\min(j,p)} \phi_i \beta_{j-i}(d, \nu, \phi, \theta) - \sum_{i=1}^{\min(j,q)} \theta_i \psi_{j-i}(d, \nu), \tag{2.9}$$

where

$$\psi_j(d, \nu) = \pi_j(-d, \nu), \tag{2.10}$$

with

$$\pi_j(d, \nu) = \sum_{\substack{0\leq l_1,\cdots,l_k\leq j,\\ l_1+\cdots+l_k=j}} C_{l_1}(-d_1, \nu_1) \ldots C_{l_k}(-d_k, \nu_k). \tag{2.11}$$

The coefficients C_{l_k} *corresponds to the Gegenbauer polynomials given by (2.5).*
(iii)Moreover, the process $(\varepsilon_t, X_t)_{t\in\mathbb{Z}}$ *is strictly stationary and ergodic.*

Proof 2.1 *(i) Multiplying (2.3) by* Z_t, *we obtain,*

$$(|\varepsilon_t| - \gamma\varepsilon_t)^{\delta} = \alpha_0 Z_t + \sum_{i=1}^{r} \alpha_i (|\varepsilon_{t-i}| - \gamma\varepsilon_{t-i})^{\delta} Z_t + \sum_{i=1}^{s} \beta_i h^{\delta}_{t-i} Z_t. \tag{2.12}$$

Now, we rewrite the expression (2.12) in a vector form:

$$\tilde{\varepsilon}_t = A_{\delta t}\tilde{\varepsilon}_{t-1} + \xi_{\delta t}, \tag{2.13}$$

where $\tilde{\varepsilon}_t = \left[(|\varepsilon_t| - \gamma\varepsilon_t)^{\delta}, \cdots, (|\varepsilon_{t-r+1}| - \gamma\varepsilon_{t-r+1})^{\delta}, h^{\delta}_t, \cdots, h^{\delta}_{t-s+1} \right]^{'}$, $\xi_{\delta t}$ *is given in the expression (2.7) and the matrix* $A_{\delta t}$ *is defined by (2.8). Now, let be for* $n = 1, 2, \cdots$

$$S_{n,\delta t} = \xi_{\delta t} + \sum_{j=1}^{n} \left(\prod_{i=0}^{j-1} A_{\delta t-i} \right) \xi_{\delta t-j}, \tag{2.14}$$

and denote $(s_{n,\delta t})_k$ *the kth element of* $\left(\prod_{i=0}^{j-1} A_{\delta t-i}\right)\xi_{\delta t-j}$. *Then,*

$$\begin{aligned} E\left[(s_{n,\delta t})_k\right] &= \eta_k^{'} E\left[\left(\prod_{i=0}^{j-1} A_{\delta t-i}\right)\xi_{\delta t-j}\right] \\ &= \eta_k^{'}\left[\prod_{i=0}^{j-1} E\left(A_{\delta t-i}\right)\right] E\left(\xi_{\delta t-j}\right) \\ &= \eta_k^{'} A^j c_1, \end{aligned} \tag{2.15}$$

because $(Z_t)_{t\in\mathbb{Z}}$ *is a sequence of iid variables and each element of* $A_{\delta t}$ *and* $\zeta_{\delta t}$ *is nonnegative. Here* $\eta_k = (0,\cdots,0,1,0,\cdots,0)^{'}$ *with 1 appearing at the kth position,* $c_1 = E(\xi_{\delta t})$ *is a constant vector and*

$$A = \left(\begin{array}{ccc|ccc} \alpha_1 E(Z_t) & \cdots & \alpha_r E(Z_t) & \beta_1 E(Z_t) & \cdots & \beta_s E(Z_t) \\ & \mathbf{I}_{(r-1)\times(r-1)} & O_{(r-1)\times 1} & & O_{(r-1)\times s} & \\ \hline \alpha_1 & \cdots & \alpha_r & \beta_1 & \cdots & \beta_s \\ & O_{(s-1)\times r} & & \mathbf{I}_{(s-1)\times(s-1)} & O_{(s-1)\times 1} & \end{array}\right). \tag{2.16}$$

It is straightforward to verify that the characteristic polynomial, $f(\lambda)$ *of* A *is equal to:*

$$f(\lambda) = \det(\lambda I - A) = \lambda^{r+s} - \lambda^s E(Z_t)\sum_{i=1}^{r}\alpha_i\lambda^{r-i} - \lambda^r\sum_{i=1}^{s}\beta_i\lambda^{s-i}.$$

Then, under the condition $E(Z_t)\sum_{i=1}^{r}\alpha_i + \sum_{j=1}^{s}\beta_j < 1$, *the eigenvalues of the matrix* A *lie outside the unit circle. This means that the spectral radius* ρ *of* A *is less than 1. Thus, the rigth side of the expression (2.15) is less than* $c\rho^j$ *for some constant* c. *Then the existence of the* δ*-order stationary solution* $(X_t,\varepsilon_t)_t$ *derives from the Theorem 2.1 in Ling and Li (1997). Hence the assertion (i) is proved.*
(ii) Under the previous conditions given for the parameters of the model (2.1)-(2.3), the stationary solution of the $(\varepsilon_t,X_t)_{t\in\mathbb{Z}}$ *verifies the relationships (2.6) and (2.7). Details are given in the Proposition 1 of Guegan (2003). Thus the stationary solution is causal under the appropriate conditions for the parameters.*
(iii) The process $(\varepsilon_t,X_t)_{t\in\mathbb{Z}}$ *appears as measurable function of the independent identically distributed random variables* Z_t, *then it is strictly stationary and ergodic.*

2.3 Existence for the higher moments

The knowledge of the expression and existence of higher order moments for a model defined by the relationships (2.1)-(2.3) is important particularly for the applications. Ling and McAleer (2002) provide the necessary and suffcient condition for the existence of the asymmetric power GARCH(r,s) model of Ding, Granger and Engle (1993). Here we extend their result for the GG$_k$-APARCH process introduced in (2.1)-(2.3).

Theorem 2.2 *Let be the process $(X_t)_{t\in\mathbb{Z}}$ defined by (2.1)-(2.3). We assume that it is centered and that the hypotheses of theorem 2.1 are verified. Now, if $\rho\left[E\left(A_{\delta t}^{\otimes m}\right)\right] < 1$, then $E\left(|X_t|^{m\delta}\right) < \infty$, with $A_{\delta t}$ given by the equation (2.8).*

Proof 2.2 *The δmth-order moments of the process $(|\varepsilon_t|)_{t\in\mathbb{Z}}$ are finite, as soon as $\rho\left[E\left(A_{\delta t}^{\otimes m}\right)\right] < 1$, (Theorem 3.2 in Ling and McAleer (2002)). Under the assumptions of the previous Theorem 2.1, we obtain:*

$$\begin{aligned} E\left(\left|X_t^{m\delta}\right|\right) &\leq \sum_{k_1,\cdots,k_m=0}^{\infty} \beta_{k_1}^{\delta}(d,\nu,\phi,\theta)\cdots\beta_{k_m}^{\delta}(d,\nu,\phi,\theta)\, E\left(\left|\varepsilon_{t-k_1}^{\delta}\right|\cdots\left|\varepsilon_{t-k_m}^{\delta}\right|\right) \\ &= \left(\sum_{k=0}^{\infty}\beta_k^{\delta}(d,\nu,\phi,\theta)\right)^m E\left(\left|\varepsilon_t^{m\delta}\right|\right), \end{aligned} \tag{2.17}$$

where the coefficients β_k are given by (2.9). Further, because $\sum_{k=0}^{\infty}\beta_j^{\delta}(d,\nu,\phi,\theta) < \infty$, then the right hand side of (2.17) is finite.

3 Estimation theory

In this paragraph, we investigate a two steps maximum likelihood method to estimate all parameters,

$$\varpi = (\phi_1,\cdots,\phi_p,\theta_1,\cdots,\theta_q,d_1,\cdots,d_k,\alpha_0,\alpha_1,\cdots,\alpha_r,\beta_1,\cdots,\beta_s,\gamma,\delta)',$$

of the GG$_k$-APARCH process defined by equations (2.1)-(2.3). The first step consists to estimate the long-memory parameters $(d,\nu) = ((d_1,\nu_1),\cdots,(d_k,\nu_k))$ and the ARMA(p,q) parameters $\psi = (\phi_1,\cdots,\phi_p,\theta_1,\cdots,\theta_q)$ using the Whittle's approach, Ferrara and Guégan (2001). In the second step, the APARCH(r,s) parameters $\omega = (\varphi,\delta,\gamma)$ is a vector of $(r+s+3)$ unknown parameters where $\varphi = (\alpha_0,\alpha_1,\cdots,\alpha_r,\beta_1,\cdots,\beta_s)$. To estimate the parameterse, we can use the maximum likelihood method applied to the residuals of the long-memory process. We detail now the two-steps procedure.

3.1 Estimation of the long memory parameters and the ARMA parameters

The first step consists of estimating the long-memory $(d_1,\cdots,d_k)$ parzmeters and the ARMA (ψ) parameters using the well-known Whittle maximum likelihood method following Ferrara (2000), Ferrara and Guégan (2001) or

Giraitis and Leipus (1995) works. In these papers, the authors estimate simultaneously the long and short memory parameters of model (2.1) using Whittle's method assuming that the noise is stationary. The Whittle estimator works in the spectral domain and the Whittle likelihood of the (mean-corrected) sample $X_1, \cdots, X_n$ is defined, for any frequency λ in $(-\pi, \pi]$, as

$$L_W(X, \psi, d, \nu) = \sum_{j=1}^{\left[\frac{n}{2}\right]} \frac{I_n(\lambda_j)}{f(\lambda_j, \varpi)}, \tag{3.1}$$

where $f(\lambda, \varpi)$ is the spectral density function of the process $(X_t)_{t\in\mathbb{Z}}$ introduced in (2.1), defined by

$$f(\lambda, \varpi) = h(\lambda, \psi) \prod_{i=1}^{k} |2(\cos(\lambda) - \nu_i)|^{-2d_i} g_\varepsilon(\lambda, \omega), \tag{3.2}$$

where

$$h(\lambda, \psi) = \left| \frac{\theta(e^{-i\lambda})}{\phi(e^{-i\lambda})} \right|^2. \tag{3.3}$$

The function g_ε is the spectral density function associated to the process $(\varepsilon_t)_{t\in\mathbb{Z}}$ and $I_n(\lambda)$ is the periodogram :

$$I_n(\lambda) = \frac{1}{n} \left| \sum_{t=1}^{n} \left(X_t - \overline{X}\right) e^{-i\lambda t} \right|^2, \tag{3.4}$$

where $\bar{X}$ denotes the sample mean. Assuming that we observe the previous data set $X_1, \cdots, X_n$, then the periodogram is evaluated at the Fourier frequencies

$$\lambda_j = \frac{2\pi j}{n},\ j = -\left[\frac{n-1}{2}\right], \cdots, \left[\frac{n}{2}\right].$$

Now, under the stationary conditions on the parameters of the model (2.1)-(2.3) given in theorem (2.1), we get the following result:

Theorem 3.1 *Let $\{X_t\}_{t=1}^n$ be the process defined by the equation (2.1). Under the stationary conditions given in theorem (2.1), then*

1. $\sqrt{n}\left(\hat{\psi}_n - \psi_0\right) \xrightarrow{D} N\left(0, 4\pi V(\psi)^{-1}\right)$, *as $n \to \infty$, where,*

$$V(\psi)_{ij} = \int_{-\pi}^{\pi} h^2(\lambda, \psi) \frac{\partial h^{-1}(\lambda, \psi)}{\partial \psi_i} \frac{\partial h^{-1}(\lambda, \psi)}{\partial \psi_j} d\lambda. \tag{3.5}$$

Here $h(\lambda, \psi)$ denotes the spectral density of the ARMA part of the process $(X_t)_{t\in\mathbb{Z}}$, given in (3.3).

2. *Moreover* $\sqrt{n}\left(\hat{d}_n - d\right) \xrightarrow{D} N\left(0, 4\pi V\left(d\right)^{-1}\right)$, *with*

$$V\left(d\right)_{ij} = \int_{-\pi}^{\pi} \log\left|4\sin\left[\frac{(\lambda-\lambda_i)}{2}\right]\sin\left[\frac{(\lambda+\lambda_i)}{2}\right]\right| \log\left|4\sin\left[\frac{(\lambda-\lambda_j)}{2}\right]\sin\left[\frac{(\lambda+\lambda_j)}{2}\right]\right| d\lambda. \tag{3.6}$$

Proof 3.1 *For the linear part of the proof of this theorem we refer to Whittle (1952) and Hosoya and Taniguchi (1982). The second part of the theorem has been proven by Hosoya (1997), Theroem 2.3., assuming that the innovations are ergodic. This last condition is verified under stationary conditions given in Theorem (2.1).*

3.2 Estimation of the APARCH parameters under Conditional Student - t distribution

In order to estimate the APARCH parameters, we need to filter the process $(X_t)_t$ to get the residuals $(\varepsilon_t)_t$ using the previous estimating parameters. The calculation of (ε_t) from (X_t) involves a finite approximation of the infinite sum which appears in the definition of the Gegenbauer differencing operator (2.4). The fractional differencing operator associated to the process $(X_t)_t$, with mean μ , if $\theta\left(B\right) \neq 0$, is equal to:

$$\frac{\phi\left(B\right)}{\theta\left(B\right)} \prod_{i=1}^{k} \left(I - 2\nu_i B + B^2\right)^{d_i} \left(X_t - \mu\right) = \mu + \sum_{j=0}^{\infty} \alpha_j\left(d,\nu,\phi,\theta\right) X_{t-k}, \tag{3.7}$$

where the weights $\alpha_j\left(d,\nu,\phi,\theta\right)$ are given by the following recursive expression:

$$\alpha_j\left(d,\nu,\phi,\theta\right) = \pi_j\left(d,\nu\right) - \sum_{i=1}^{\min(j,p)} \phi_i \pi_{j-i}\left(d,\nu\right) + \sum_{i=1}^{\min(j,q)} \theta_i \alpha_{j-i}\left(d,\nu,\phi,\theta\right), \tag{3.8}$$

where $\pi_j\left(d,\nu\right)$ are given in equation (2.10).

This last expression involves the unobserved quantities $X_0, X_{-1}, X_{-2}, \cdots$. If these quantities are replaced by their mean μ, the previous operator $\frac{\phi(B)}{\theta(B)} \prod_{i=1}^{k} \left(I - 2\nu_i B + B^2\right)^{d_i}$ can be calculated from the data. But the substitution of X_t, $t \leq 0$ by μ is unrealistic for a long-range dependent series. So a compromise method, as suggested in Beran (1994), is to replace X_t by μ only for $t \leq M$, where M is chosen large enough. Thus, the expression (3.7) becomes:

$$\varepsilon_t = \mu + \sum_{j=0}^{t+M-1} \alpha_j\left(d,\nu,\phi,\theta\right) X_{t-k}. \tag{3.9}$$

In practice, a high-order AR(p) model for (X_t) is built as auxiliary model which is used to estimate the presample values using the "backtesting" method developed in Box and Jenkins (1976).

Now the residuals $(\varepsilon_t)_t$ provided by (3.9) permit to estimate the parameters of the APARCH(r, s) model (2.3). As we are going to use a maximum likelihood approach to solve this problem, we need now to specify the probability distribution function of the noise $(\eta_t)_t$ which appears in expression (2.2). In the following, we will work with the assumption H_0 :

Assumption H_0 : The innovations $(\eta_t)_{t\in\mathbb{Z}}$ have a conditional Student-t distribution with l degrees of freedom.

The assumption H_0 permits to introduce some asymmetric in the modelling of our data sets. This assumption is more flexible than the Gaussian one. Recall that for the APARCH model Laurent (2004) derives the expression of the likelihood in that latter case. We now derive the log-likelihood function $L_n(\omega)$, the scores function and the Hessian matrix for the APARCH model (2.2)-(2.3). It is equal to:

$$L_n(\omega) = n\left[\log\Gamma\left(\frac{l+1}{2}\right) - \log\Gamma\left(\frac{l}{2}\right) - \frac{1}{2}\log\pi(l-2)\right] - \frac{1}{2}\sum_{t=1}^{n}\ell_t, \quad (3.10)$$

with

$$\ell_t = \log(h_t^2) + (l+1)\left[\log(1+\frac{\varepsilon_t^2}{h_t^2(l-2)})\right],$$

and $\Gamma(.)$ represents the Gamma function. The lower limit for l is zero. For $l < 3$, the unconditional variance does not exist and the lower l is the fatter the tails are. To estimate ω, we need to know the first-order derivatives and to solve the equation $\frac{\partial L_n(\omega)}{\partial\omega} = 0$. Differentiating ℓ_t under the assumption H_0 with respect to the full set of parameter ω yields

$$\frac{\partial\ell_t(\omega)}{\partial\omega} = \left[\frac{1}{h_t^2} - \frac{l+1}{(l-2)}\frac{\varepsilon_t^2}{h_t^2}\left(1+\frac{\varepsilon_t^2}{(l-2)h_t^2}\right)^{-1}\right]\frac{\partial h_t^2}{\partial\omega}. \quad (3.11)$$

To obtain the Hessian matrix, we need to differentiate $\frac{\partial\ell_t(\omega)}{\partial\omega'}$ with respect to ω. Thus, we obtain

$$\begin{aligned}\frac{\partial^2\ell_t(\omega)}{\partial\omega\partial\omega'} &= -\frac{1}{h_t^2}\frac{\partial h_t^2}{\partial\omega}\frac{\partial h_t^2}{\partial\omega'} \\ &\quad -\frac{(l+1)}{(l-2)^2}\frac{\varepsilon_t^4}{h_t^4}\left(1+\frac{\varepsilon_t^2}{(l-2)h_t^2}\right)^{-2}\frac{\partial h_t^2}{\partial\omega}\frac{\partial h_t^2}{\partial\omega'} \\ &\quad +2\frac{(l+1)}{(l-2)}\frac{\varepsilon_t^2}{h_t^3}\left(1+\frac{\varepsilon_t^2}{(l-2)h_t^2}\right)^{-1}\frac{\partial h_t^2}{\partial\omega}\frac{\partial h_t^2}{\partial\omega'}. \quad (3.12)\end{aligned}$$

Equations (3.11) and (3.12) require the computation of $\frac{\partial h_t^2}{\partial \omega}$. However, we can remark that in the APARCH specification, a power transformation of the conditional variance is modelled by h_t^δ. Hence, we rewrite the conditional variance h_t^2 as $\left(h_t^\delta\right)^{\frac{2}{\delta}}$ which leads to:

$$\frac{\partial h_t^2}{\partial(\gamma,\varphi)} = \frac{2}{\delta}\frac{h_t^2}{h_t^\delta}\frac{\partial h_t^\delta}{\partial(\gamma,\varphi)}, \tag{3.13}$$

and

$$\frac{\partial h_t^2}{\partial \delta} = \frac{2}{\delta}\frac{h_t^2}{h_t^\delta}\left(\frac{\partial h_t^\delta}{\partial \delta} - \frac{h_t^\delta \log h_t^\delta}{\delta}\right). \tag{3.14}$$

Now, the derivatives of h_t^2 with respect to φ, δ and γ require the computation of $\frac{\partial h_t^\delta}{\partial \varphi}$, $\frac{\partial h_t^\delta}{\partial \gamma}$ and $\frac{\partial h_t^\delta}{\partial \delta}$. We provide them :

$$\frac{\partial h_t^\delta}{\partial \varphi} = d_t + \sum_{j=1}^{s} \beta_j \frac{\partial h_{t-j}^\delta}{\partial \varphi}, \tag{3.15}$$

where $d_t = \left(k\left(\varepsilon_{t-1}\right)^\delta, \cdots, k\left(\varepsilon_{t-r}\right)^\delta, h_{t-1}^\delta, \cdots, h_{t-s}^\delta\right)$ and

$$\frac{\partial h_t^\delta}{\partial \varphi} = 0, \text{ for } t \leq 0.$$

Moreover,

$$\frac{\partial h_t^\delta}{\partial \gamma} = d_t^* + \sum_{j=1}^{s} \beta_j \frac{\partial h_{t-j}^\delta}{\partial \gamma}, \tag{3.16}$$

where d_t^* is a $(1 \times r)$ vector whose i^{th} component is $\alpha_i \frac{\partial k(\varepsilon_{t-i})^\delta}{\partial \gamma}$ with

$$\frac{\partial k\left(\varepsilon_{t-i}\right)^\delta}{\partial \gamma} = \begin{cases} -\delta k\left(\varepsilon_{t-i}\right)^{\delta-1} \varepsilon_{t-i} \text{ if } t > 0 \\ -\frac{\delta}{n}\sum_{j=1}^{n}\left(\left|\varepsilon_j - \gamma\varepsilon_j\right|\right)^{\delta-1}\varepsilon_j \text{ if } t \leq 0, \end{cases} \tag{3.17}$$

and

$$\frac{\partial h_t^\delta}{\partial \delta} = 0, \text{ for } t \leq 0.$$

Let I_t the indicatrice function defined by

$$I_t = \begin{cases} 1 \text{ if } t > 0 \\ 0 \text{ if } t \leq 0, \end{cases}$$

then, differentiating with respect to δ gives

$$\begin{aligned} \frac{\partial h_t^\delta}{\partial \delta} &= \sum_{i=1}^{r} \alpha_i \left[k\left(\varepsilon_{t-i}\right)^\delta \log k\left(\varepsilon_{t-i}\right)\right]^{I_{(t>i)}} \left[\frac{1}{n}\sum_{s=1}^{n}\left(\left|\varepsilon_s\right| - \gamma\varepsilon_s\right)^\delta \log\left(\left|\varepsilon_s\right| - \gamma\varepsilon_s\right)\right]^{1-I_{(t>i)}} \\ &\quad + \sum_{j=1}^{s} \beta_j \left(\frac{\partial h_{t-j}^\delta}{\partial \delta}\right)^{I_{(t>j)}} \left[0.5\left(\frac{1}{n}\sum_{s=1}^{n}\varepsilon_s^2\right)^{\frac{\delta}{2}} \log\left(\frac{1}{n}\sum_{s=1}^{n}\varepsilon_s^2\right)\right]^{1-I_{(t>i)}}. \end{aligned} \tag{3.18}$$

In the following we use the previous expressions (3.11) - (3.18) to estimate the parameters of the APARCH part of the model (2.1)-(2.3).

The previous relationships permit to obtain the maximum likelihood estimates of the parameters ω solving these analytical expressions. They permi to avoid the use of numerical techniques and approximations. Then, we get the following proposition:

Proposition 3.1 *Let $\{\varepsilon_t\}_{t=1}^n$ be the process defined by the equation (2.2) - (2.3) . Under the stationary conditions given in theorem (2.1) and the assumption H_0:*

1. *There exists a MLE $\hat{\omega}_n$ such that it satisfies the equation $\frac{\partial \ell_t(\omega)}{\partial \omega} = 0$ and $\hat{\omega}_n \to \omega_0$ in Probability as soon as $n \to \infty$.*

2. *For such a sequence, $\sqrt{n}\left(\hat{\omega}_n - \omega_0\right) \xrightarrow{D} N\left(0, \Omega_0^{-1}\right)$, as $n \to \infty$, where,*

$$\Omega_0 = -\frac{1}{2} E\left(\frac{\partial^2 \ell_t(\omega)}{\partial \omega \partial \omega'}\right), \tag{3.19}$$

3. *Moreover, a consistent estimator of the matrix $E\left(\frac{\partial^2 \ell_t(\omega)}{\partial \omega \partial \omega'}\right)$ is expressed as follows:*

$$\begin{aligned}
\hat{\Omega}_0 \quad = \quad & -\frac{1}{n}\sum_{t=1}^{n}\frac{1}{h_t^2}\frac{\partial h_t^2}{\partial \omega}\frac{\partial h_t^2}{\partial \omega'} \\
& -\frac{1}{n}\frac{(l+1)}{(l-2)^{-2}}\sum_{t=1}^{n}\frac{\varepsilon_t^4}{h_t^4}\left(1+\frac{\varepsilon_t^2}{(l-2)\,h_t^2}\right)^{-2}\frac{\partial h_t^2}{\partial \omega}\frac{\partial h_t^2}{\partial \omega'} \\
& +2\frac{1}{n}\frac{(l+1)}{(l-2)^{-1}}\sum_{t=1}^{n}\frac{\varepsilon_t^2}{h_t^3}\left(1+\frac{\varepsilon_t^2}{(l-2)\,h_t^2}\right)^{-1}\frac{\partial h_t^2}{\partial \omega}\frac{\partial h_t^2}{\partial \omega'}.
\end{aligned} \tag{3.20}$$

Proof 3.2 *The same main lines of the proof of theorem 3.2 in Diongue and Guégan (2004) can be used to obtain this proposition.*

3.3 Estimation of the APARCH parameters under Conditional GED distribution

In the applications we will use also a GED distribution for the $(\eta_t)_t$ innovations in order to compare our results with classical fat-tailed distribution. We provide now the expression of the likelihood that we will use in the simulations.

If the innovations are assumed to follow a GED distribution with tail index υ, the density is equal to

$$f(x) = \frac{\upsilon 2^{-\left(1+\frac{1}{\upsilon}\right)}}{\lambda_\upsilon \Gamma\left(\frac{1}{\upsilon}\right)} e^{-\frac{1}{2}\left|\frac{x}{\lambda_\upsilon}\right|^\upsilon}, \ -\infty < x < \infty, \tag{3.21}$$

with $\lambda_\upsilon = \sqrt{\frac{\Gamma(\frac{1}{\upsilon})2^{-\frac{2}{\upsilon}}}{\Gamma(\frac{3}{\upsilon})}}$ and $0 < \upsilon < \infty$ is the tail-thickness parameter. The GED includes the Gaussian distribution ($\upsilon = 2$) as a special case, along with many other distributions: some more fat-tailed than the Gaussian one (e.g. the double exponential distribution corresponding to $\upsilon = 1$) and some more thin-tailed (e.g the Uniform distribution on the interval $\left[-\sqrt{3}, \sqrt{3}\right]$ when $\upsilon \to \infty$). Then the log-likelihood function is given by:

$$\begin{aligned} L_n(\omega) &= n\left[\log\left(\frac{\upsilon}{\lambda_\upsilon}\right) - \left(1+\upsilon^{-1}\right)\log(2) - \log\Gamma\left(\frac{1}{\upsilon}\right)\right] \\ &\quad -\frac{1}{2}\sum_{t=1}^{n}\left[\log\left(h_t^2\right) + \frac{1}{h_t}\left|\frac{\varepsilon_t}{\lambda_\upsilon}\right|^\upsilon\right], \end{aligned} \tag{3.22}$$

where $0 < \upsilon < \infty$ and $\lambda_\upsilon = \sqrt{\frac{2^{-\frac{2}{\upsilon}}\Gamma\frac{1}{\upsilon}}{\Gamma\frac{3}{\upsilon}}}$. The analytic expression of the gradient vector and the Hessian matrix for the GED log-likelihood could be obtained very easily following the previous method developed for the Student-t distribution. Details can be obtained under request.

We provide in the next section the properties of the estimates obtained using this method by Monte Carlo simulations.

4 Numerical simulations

In this section, several numerical simulations are presented to characterize the empirical properties of the estimated parameters, using the previous framework. We exhibit the mean, the mean absolute error (MAE) and root mean square error (RMSE) statistics defined by

$$MAE = E\left(|\hat{a} - a_0|\right), \quad RMSE = \sqrt{E\left((\hat{a} - a_0)^2\right)}, \tag{4.1}$$

where $\hat{a}$ is an estimator of the true parameter a_0.

In order to be accurate for a high frequency financial time series, we simulated several GG$_k$-APARCH processes with length $n = 1000$ and $n = 2500$. Indeed, we are interested to apply our method on high frequency data sets whose size is important. We use several kinds of noises: conditionally Gaussian, conditionally Student-t with 3 degrees of freedom and conditionally GED distributions noises with tail index equal to 5. We specify now the GG_k-APARCH models.

1. For the conditional variance we consider an APARCH$(1,1)$ model:

$$h_t^\delta = \alpha_0 + \alpha_1 \left(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1}\right)^\delta + \beta_1 h_{t-1}^\delta. \tag{4.2}$$

   For all experiments, we set $\alpha_0 = 0.1$, $\alpha_1 = 0.3$, $\beta_1 = 0.4$. The parameter vector (γ, δ) takes three different set of values:

$$(\gamma, \delta) = (0, 2)$$
$$(\gamma, \delta) = (-0.1, 2)$$
$$(\gamma, \delta) = (-0.1, 1.2).$$

2. For the conditional mean model, we consider two different processes: a GG_1 and GG_2 processes, without short memory terms, defined by:

$$\left(1 - 1.72B + B^2\right)^{0.4} X_t = \varepsilon_t,$$
$$\left(1 - 1.72B + B^2\right)^{0.4} \left(1 - 1.41B + B^2\right)^{0.3} X_t = \varepsilon_t.$$

The main purpose of these experiments is to assess the finite sample performance of parameter estimator in the GG_k-APARCH model using several elliptical distributions, namely the Normal, the Student-t and the GED distributions. The choice of these distributions is motivated by the empirical features observed inside the electricity spot price time series. The simulations have been computed using 100 replications for each data generating process. The results [1] are reported in Tables 1 - 6.

Results from the Gaussian GG_k-APARCH model are given in Tables 1 and 4. The estimation method performs well as soon as the MAE and RMSE are small. it is the case for all models and mainly when the sample size increases. It appears that the presence of the APARCH part does not effect the properties of the long memory parameters' estimators.

In order to study the robustness of the estimator when the errors come from a non-normal distribution, we report in Tables 2 and 5 the estimation obtained using a Student-t GG_k-APARCH model, while Tables 3 and 6 summarize the results for a GED GG_k-APARCH model. The results are quite good whatever the number of factors $k = 1, 2$. The MAE and the RMSE

[1] Estimates are obtained using MATLAB codes that are available to the authors upon request. The estimation respects the nonnegative and stationary conditions.

values are always very small. Thus, we can conclude that the performance of the estimated parameters is not affected by the conditional distribution. Moreover, the Monte Carlo experiments show the impact of the sample size n using this method: when n increases, the results improve.

To summarize:

1. The parametric Whittle estimator performs as expected in the sense that the RMSE and the MAE values for the long memory parameter are generally small.

2. The estimation method seems to be unaffected by the presence of the APARCH errors.

3. The maximum likelihood method stays robust even if the fourth moment of the errors is not finite, in case, for instance, of the Student conditional distribution with 3 degrees of freedom.

4. In all examples the innovation distribution turns out to have a little affect on the RMSE and MAE values.

5. Finally, and not surprisingly, the performance of the estimators improve substancially as the sample size n increases from 1000 to 2500.

5 Application to electricity spot prices

We now model the hourly electricity spot prices[2] from Leipzig Power eXchange (LPX) in Germany, Powernext in France, Operadora del Mercado Español de Electricidad (OMEL) in Spain and Pennsylvania-New Jersey-Maryland (PJM) interconnection in United States, using the k-factor Gegenbauer APARCH model. With respect to the previous discussion, we consider that this class of models can take into account different features of these data sets. The period begins the 5 november 2004 and finishes the 5 november 2007 for all these electricity markets.

The figures (1) - (4) show the evolution of the hourly electricity spot prices for the four markets. We observe price spikes for the LPX, PJM and Powernext markets and we notice that the Spanish electricity prices present very often small values. On these figures we can detect calm periods followed by turmoil periods with very sharp jumps.

The figure 5 exhibits the average price for each hour over the whole sample for the four Power Exchanges. The prices begin to increase around 5:00 a.m. and continues to increase until 12:00 a.m. when the first and biggest peak of

[2]Source: Bloomberg

the day appears. Then the prices decreases until 5:00 p.m. and, after reaching its locally lowest point, they increase again between 7:00 p.m. and 8:00 p.m., when the second peak of the day is reached. However, the electricity prices in the spanish market is very stable compared with the other markets. We osberve a miximum price around 10 euros per MWh for this last market.

In the following, we work with the electricity spot prices in their logarithmic scale, replacing all negative price values by 0.01. Table 7 summarizes the descriptive statistics of the whole sample of hourly logarithm prices for the four electricity markets. Sample means, medians, maxima, minima, standard deviations, skewness, kurtosis and Jarque-Bera test (JB) statistics are reported. The highest average logarithm prices are given by the PJM and the LPX markets. The standard deviation of the logarithm prices ranges from 0.151 for OMEL market to 0.593 for PJM market.

The empirical distribution of the logarithm prices series is non Gaussian. Indeed, all markets are significantly negatively skewed ranging from -6.148 (PJM) to -0.478 (OMEL). They also exhibit an excess kurtosis, ranging from 8.118 for OMEL to 68.005 for PJM, indicating leptokurtic or heavy-tailed distributions. The Jarque-Bera statistics and corresponding p-value in Table 7 is used to test the null hypothesis that the intraday distribution of the logarithm prices is normally distributed. All p-values are smaller than the 0.01 level of significance indicating that the null hypothesis is rejected.

Plots of the sample autocorrelation functions are provided in Figures (6)-(9). The autocorrelation functions decay with an hyperbolic rate towards zero at the seasonal lags. In Figures (10)-(13), we plot the classical periodogram for the logarithm hourly electricity prices of LPX, PJM, Powernext and OMEL markets. The figures (10), (12) and (13) exhibit three distinct peaks, corresponding to the three seasonal frequencies $\omega_1 = 0.0375$, $\omega_2 = 0.2618$ and $\omega_3 = 0.5236$ while the figure (11) exhibits three different peaks corresponding to the frequencies $\omega_1 = 0$, $\omega_2 = 0.2618$ and $\omega_3 = 0.5236$.

From this first graphical and statistical analysis, it seems reasonable to model these series using a generalised long memory model introduced in the equation (2.1) with $k = 3$, whose representation is:

$$\prod_{i=1}^{3} \left(I - 2\nu_i B + B^2\right)^{d_i} \left(Y_t - \bar{Y}\right) = \varepsilon_t,$$

where $\nu_1 = \cos(\omega_1)$, $\nu_2 = \cos(\omega_2)$ and $\nu_3 = \cos(\omega_3)$, Y_t representing the log-electricity spot prices and $\bar{Y}$ the empirical mean of the series. The following models are estimated for the four electricity power markets.

1. For the LPX market, we get:

$$\phi(B)\,\phi_1(B)\,\phi_2(B)\,\phi_3(B)\,(Y_t - 3.699) = \theta(B)\,\varepsilon_t,$$

where

$$\phi(B) = \left(1 - 0.6372B + 0.3768B^2\right)\left(1 - 0.9088B^{168}\right),$$

$$\theta(B) = \left(1 - 0.6483B + 0.3393B^2\right)\left(1 + 0.07461B^{24} - 0.6803B^{168} - 0.0675B^{336}\right),$$

$$\phi_1(B) = \left(1 - 1.9886B + B^2\right)^{0.128},$$

$$\phi_2(B) = \left(1 - 1.9318B + B^2\right)^{0.091},$$

and

$$\phi_3(B) = \left(1 - 1.7320B + B^2\right)^{0.036}.$$

2. For the PJM market, we have

$$\left(1 - 0.9852B^{24}\right)\phi_1(B)\,\phi_2(B)\,\phi_3(B)\,(Y_t - 3.861) = \left(1 - 0.9474B^{24} - 0.02075B^{48}\right)\varepsilon_t,$$

where

$$\phi_1(B) = (1 - B)^{0.230},$$

$$\phi_2(B) = \left(1 - 1.9318B + B^2\right)^{0.101},$$

and

$$\phi_3(B) = \left(1 - 1.7320B + B^2\right)^{0.038}.$$

3. For the Powernext market, we obtain:

$$\phi(B)\,\phi_1(B)\,\phi_2(B)\,\phi_3(B)\,(Y_t - 3.691) = \theta(B)\,\varepsilon_t,$$

where

$$\phi(B) = (1 - 0.1105B)\left(1 - 0.8462B^{168}\right),$$

$$\theta(B) = \left(1 - 0.2644B + 0.0566B^2 - 0.0371B^3\right)\left(1 + 0.134B^{24} + 0.0645B^{48} - 0.6378B^{168}\right),$$

$$\phi_1(B) = \left(1 - 1.9886B + B^2\right)^{0.209},$$

$$\phi_2(B) = \left(1 - 1.9318B + B^2\right)^{0.129},$$

and

$$\phi_3(B) = \left(1 - 1.7320B + B^2\right)^{0.083}.$$

4. For the OMEL market, we get:

$$\phi(B)\,\phi_1(B)\,\phi_2(B)\,\phi_3(B)\,(Y_t - 1.55) = \theta(B)\,\varepsilon_t,$$

where

$$\phi(B) = (1 - 0.1614B)\left(1 - 0.9317B^{168}\right),$$

$$\theta(B) = \theta_1(B)\,\theta_2(B),$$

with

$$\theta_1(B) = \left(1 - 0.2766B + 0.0809B^2\right),$$

$$\theta_2(B) = \left(1 + 0.0899B^{24} + 0.0492B^{48} + 0.0756B^{72} + 0.0586B^{96} - 0.6376B^{168} - 0.0486B^{336}\right),$$

$$\phi_1(B) = \left(1 - 1.9886B + B^2\right)^{0.191},$$

$$\phi_2(B) = \left(1 - 1.9318B + B^2\right)^{0.106},$$

and

$$\phi_3(B) = \left(1 - 1.7320B + B^2\right)^{0.094}.$$

The parameter estimation is done using the Whittle estimation method developped previously. The estimation of the long memory parameter d is highly significant accross all time series indicating that the log electricity spot prices have persistent behaviors, with orders of integration inside $\left(0, \frac{1}{2}\right)$, implying stationarity, but also long memory behavior.

Using the finite approximation given in the expression (3.9), we can calculate the residuals $\varepsilon_1, \cdots, \varepsilon_n$ from $Y_1, \cdots, Y_n$ using the previous filters. The figures (14) - (17) display the sample autocorrelation of the conditional mean's residuals for all the markets. We remark that apart for a very small numbers of autocorrelation lags, all the lags are inside the confidence interval. This permits to conclude that those residuals correspond to white noise processes.

In Table 8, we summarize the descriptive statistics for the residuals and some diagnostic tests. We notice that the mean and the standard deviation are small for all markets. The skewness numbers are significantly negative for all the markets indicating that the conditional distribution of the long memory residuals is left skewed. The kustosis values are higly significant far from 3. This suggest to model the residuals using a conditional distribution that allows greater kurtosis than the Normal distribution. The Jarque-Bera normality test whose values are given in Table 8, is far beyong his critical value at 5% (5.99), indicating that the Normal distribution for the residuals of all these spot price residuals series should clearly be rejected.

Moreover, the Lagrange Multiplier (LM) test of Engle (1982) can be used to test the presence of conditional heteroskedasticity and evidence of ARCH effects. The LM test of order one and five (in the last row of Table 8) indicates that the log electricity spot prices residuals exhibit ARCH effects with probability equal to one. To model such a behavior, in the following, we use GARCH, EGARCH and APARCH models in order to describe the conditional volatility of the residuals, along with a fat-tailed error distribution.

Despite the theoritical interest of the (r, s) GARCH models, the GARCH$(1, 1)$ model is, in general, satisfactory when modelling financial assets returns volatility, Bollerslev *et al.* (1992) and Hansen and Lunde (2005). Thus, in this current paper, for all models we use $r = 1$ and $s = 1$. Moreover, following the previous remarks on the conditional distribution of the residuals, we will use several density functions: the Normal, Student-t and GED distribution functions. We give now the results of these various adjustments.

- General comments.
  The estimated values for the parameters of the various volatility conditional models are reported in Tables 9 - 12 with their standard deviations in brackets for the four markets. The ARCH coefficient $\hat{\alpha}_1$ in all four markets are significantly different to zero, indicating the presence of significant ARCH effects, while the lagged volatility $\hat{\beta}_1$ are also significant, except for the OMEl market. The sum of the ARCH and GARCH effects is equal to one for all these four markets, suggesting that the shocks are persistent. The tail coefficients in the Student-t and the GED-GARCH specification are significant for the four markets. This indicates that the Student-t and GED distributions have taken into account the fat-tailed characteristic of the series.

  The Box Pierce Q-statistics, $Q^2(10)$ and the p-values are computed on the squared standardized residuals to test the null hypothesis that there is no remaining heteroskedasticity: some residuals heteroskedasticity remains for the Powernext and the OMEL markets. Furthermore, the goodness-of-fit statistics are provided in the tables 9 - 12: They correspond to the log-likelihood (Log-L), the Akaike Information Criterion (AIC), and the Bayesian Information Criterion (BIC). They indicate that the Student-t GARCH model better describes the behavior of residuals for the LPX, PJM and Powernext markets, while for the OMEL market, the GED-GARCH model is favoured.

- Comments fior each specific conditional volatility models.

  1. EGARCH specification. The EGARCH estimation results are summarized in the columns 5 - 7 inside the Tables 9 - 12. For all markets, the innovation and the volatility spillovers are significantly different from zero (GARCH coefficients and parameter $\hat{\alpha}_1$

negative). The leverage effect parameter $\hat{\gamma}$ is positive and signifiant at a level of 5% for the four markets. This means that there exists negative correlation between volatility and log electricity spot prices. The tail parameter of the Student-t EGARCH model indicates thinner tails for the PJM market than for the other electricity markets. The Box Pierce statistics, $Q^2(10)$, is non significant except for the Powernext and the PJM markets. Regarding the goodness-of-fit statistics, the GED-EGARCH is favoured for all these markets, except for the PJM market.

2. APARCH specification. In the three last columns of Tables 9 - 12, we provide the results for the APARCH model. The parameter $\hat{\alpha}_1$ is significant for all markets, indicating presence of ARCH effects. The lagged volatility parameter $\hat{\beta}_1$ is also significantly different from zero. The estimated asymmetric volatility response coefficients to market news given by $\hat{\gamma}$ is significant and positive for all markets. The power coefficient $\hat{\delta}$ varies between 1 and 2 and can be very high for the OMEI market. This result motivates our intuition that it appears highly appropiate to use a non-linear representation for the conditional variance in order to take into account the volatility effects which appear in the residuals. The tail coefficients in the Student-t and in the GED distributions are also significant for the four markets. The Box Pierce Q-statistics for the LPX and the PJM markets indicate that the Student-t APARCH model has overcome the problem of heteroskedasticity. For OMEL and Powernext markets, the Box Pierce statistics suggest to use a Normal APARCH model. Further, the goodness-of-fit statistics point out that the Student-t APARCH model performs better for the LPX, PJm and Powernext markets while for the OMEL market the GED-APARCH seems a better adjustment.

3. Skewed Student-APARCH specification. The goodness-of-fit statistics as well as the residuals diagnostics indicate that the Student-t APARCH perform well in describing the conditional power standard deviation for all markets. However, to deal with the left skewed observed on log electricity market, we compare in the following the Skewed Student-t APARCH model with the symmetric Student-t APARCH model. The table 13 presents the results for the Skewed Student-t APARCH model. The ARCH effect parameter is significantly different from zero as well as the volatility parameter. This implies that the last period's volatility shocks in log electricity prices have a small effect. The estimated asymmetric volatility response coefficients to market news $\hat{\gamma}$, except for the OMEL marke are significantly positive. The skew pa-

rameters in the Skewed Student-t distribution are negative for all markets. The negative asymmetric coefficients appear consistent with the behavior of the data. When we compare the Student-t APARCH and the Skewed Student-t APARCH model, the latter parforms better according to the goodness-of-fit and Box Pierce statistics. However, the Box Pierce Q-statistics indicates that these models are not able to take into account the serial dependencies in the condiational variance of the log electricity spot price for Powernext and OMEL markets. We guess that the FI-APARCH model of Tse (1998) with fat-tailed distribution would be appropriate to deal with these serial dependencies in the condiational variance of these specific markets. This will be studied in a companion paper.

In summary, it appears that the APARCH model is appropriate to modelling the conditional vartiances of the residuals of the log prices. The choice of the conditional distribution can mainly improve this adjustment. Thus this work shows the interest to use competitive models in order to detect the best models when we have complex and several behaviors characterizing a data set. Indeed, in that latter case, tests are not sufficient to specify the best model. The next step will be to compare the various specifications that we have obtained, using a forecasting strategy. This will be done in a companon paper.

6 Conclusion

In this paper, we present a new model, called GG_k-APARCH model in order to study the long memory dynamics and fat tailed distribution as well as asymmetry in the conditional variance of specific data sets. We apply this new modelling to hourly electricity spot prices for LPX, Powernext, PJM and OMEL markets. The estimated adjustments of GG_k-GARCH, GG_k-EGARCH and GG_k-APARCH models under Normal, Student's t and GED conditional distributions are compared in terms of in sample fit. We find significant long memory component in the conditional mean of hourly log electricity spot prices and model the high volatility using appropriate conditional variance models.

A general conclusion is that the APARCH model with Student-t errors are found to be useful in modelling conditional volatility of hourly log electricity spot prices. When we compare the former approach with the asymmetric Student-t APARCH model, which accommodates both left-skewed and fat-tailed features encountered in electricity spot price data, the results indicate that intraday price volatility in all four electricity markets, except for the OMEL market where the GED APARCH model, appears better. In any case, it seems that the Skewed Student-t APARCH can perform the description of the serial dependencies in the conditional variance.

However, Figure 5 shows that the electricity spot prices depend on the time-of-day effects. It will be interesting to take into account also these features. Moreover, we recommand to investigate the impact of other effects such as the week-of-month and month-of-year and the corresponding volume of demand per hour. In addidtion, other competitive models like the FI-APARCH model can be considered. For future work, it will be interesting also to study the causality between European markets.

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Table 1: Gaussian GG_1-APARCH

	APARCH model with $\gamma = 0$ and $\delta = 2$											
	$n = 1000$						$n = 2500$					
	$\hat{d}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{d}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$
Estimates	0.4152	0.1648	0.2912	0.3304	0.0027	1.9738	0.4080	0.1184	0.2947	0.3861	-0.0015	1.9831
MAE	0.0201	0.1248	0.0468	0.1381	0.0669	0.5680	0.0115	0.0704	0.0270	0.0790	0.0382	0.3069
RMSE	0.0258	0.1605	0.0604	0.1692	0.0839	0.7364	0.0153	0.0888	0.0338	0.0993	0.0478	0.4051
	APARCH with $\gamma = -0.1$ and $\delta = 2$											
Estimates	0.4169	0.1521	0.2727	0.3682	-0.1079	1.9328	0.4082	0.1179	0.2999	0.3875	-0.0999	1.9590
MAE	0.0223	0.1166	0.0519	0.1259	0.0702	0.5556	0.0141	0.0828	0.0263	0.0864	0.0398	0.3261
RMSE	0.0274	0.1499	0.0647	0.1541	0.0861	0.7138	0.0169	0.1018	0.0319	0.1066	0.0494	0.4032
	APARCH with $\gamma = -0.1$ and $\delta = 1.2$											
Estimates	0.4709	0.0963	0.3311	0.3695	-0.0993	1.2567	0.4373	0.1022	0.3049	0.3808	-0.0978	1.2511
MAE	0.0709	0.0393	0.0713	0.0916	0.11697	0.3892	0.0374	0.0257	0.0281	0.0524	0.0428	0.2251
RMSE	0.0744	0.0541	0.1554	0.1312	0.2456	0.5562	0.0397	0.0332	0.0347	0.0685	0.0557	0.2958

This table summarizes the estimates coefficients, MAE - Mean Absolute Errors, RMSE - Roots Mean Squared Errors for the Gaussian GG_1-APARCH$(1, 1)$. Monte simulations are computed with 100 replications. Each replication gives a sample size $n = 1000$ and 2500 of observations. The setting parameters are $d_1 = 0.4$, $\alpha_0 = 0.1$, $\alpha_1 = 0.3$, $\beta_1 = 0.4$, $(\gamma = 0, \delta = 2)$, or $(\gamma = -0.1, \delta = 2)$ and $(\gamma = -0.1, \delta = 1.2)$.

Table 2: Student GG_1-APARCH

	APARCH with $\gamma = 0$ and $\delta = 2$													
	$n = 1000$							$n = 2500$						
	$\hat{d}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{v}$	$\hat{d}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{v}$
Estimates	0.4168	0.1728	0.3085	0.3326	-0.0001	1.8722	3.1783	0.4102	0.1294	0.2929	0.3632	0.0254	2.009	3.1765
MAE	0.0310	0.1346	0.0823	0.1493	0.1198	0.5166	0.3283	0.0250	0.0838	0.0532	0.1072	0.0923	0.3468	0.2945
MSE	0.0384	0.1687	0.1046	0.1818	0.1588	0.6182	0.5190	0.0323	0.1033	0.0722	0.1364	0.1777	0.4093	0.9917
	APARCH with $\gamma = -0.1$ and $\delta = 2$													
Estimates	0.4205	0.1686	0.3016	0.3312	-0.1272	2.0046	3.1219	0.4082	0.1283	0.2965	0.3759	-0.1035	1.9413	3.0586
MAE	0.0381	0.1284	0.0926	0.1592	0.1122	0.6324	0.2912	0.0221	0.0903	0.0413	0.1006	0.0793	0.3376	0.1489
MSE	0.0527	0.1589	0.1219	0.2014	0.1551	1.1251	0.3728	0.0314	0.1083	0.0539	0.1296	0.1266	0.4021	0.1871
	APARCH with $\gamma = -0.1$ and $\delta = 1.2$													
Estimates	0.4608	0.1194	0.2977	0.3704	-0.1021	1.2782	3.1042	0.4339	0.1076	0.3091	0.3726	-0.1072	1.2641	3.0517
MAE	0.0649	0.1003	0.0619	0.1132	0.1235	0.3885	0.2900	0.0364	0.0618	0.0446	0.0684	0.0786	0.2737	0.1559
MSE	0.0725	0.1298	0.0834	0.1484	0.1551	0.4839	0.3916	0.0419	0.0759	0.0579	0.0916	0.0979	0.3745	0.1983

This table summarizes the estimates coefficients, MAE - Mean Absolute Errors, RMSE - Roots Mean Squared Errors for the Student-t with $l = 3$ degrees of freedom GG_1-APARCH$(1,1)$. Monte simulations are computed with 100 replications. Each replication gives a sample size $n = 1000$ and 2500 of observations. The setting parameters are $d_1 = 0.4$, $\alpha_0 = 0.1$, $\alpha_1 = 0.3$, $\beta_1 = 0.4$, $(\gamma = 0, \delta = 2)$, or $(\gamma = -0.1, \delta = 2)$ and $(\gamma = -0.1, \delta = 1.2)$.

Table 3: GED GG_1-APARCH

	APARCH with $\gamma = 0$ and $\delta = 2$													
	$n = 1000$							$n = 2500$						
	$\hat{d}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{v}$	$\hat{d}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{v}$
Estimates	0.4159	0.1318	0.2939	0.3521	-0.0058	2.0803	5.1417	0.4114	0.1059	0.2953	0.3916	0.0027	2.0404	5.0776
MAE	0.0196	0.0963	0.0269	0.0906	0.0336	0.4657	0.4582	0.0130	0.0586	0.0193	0.0594	0.0236	0.2951	0.2829
MSE	0.0246	0.1168	0.0335	0.1211	0.0421	0.5763	0.5723	0.0164	0.0715	0.0335	0.0767	0.0287	0.3750	0.5039
	APARCH with $\gamma = -0.1$ and $\delta = 2$													
Estimates	0.4089	0.1293	0.2898	0.3853	-0.1009	1.9048	4.9942	0.4182	0.1198	0.2995	0.3756	-0.1166	2.0263	5.1033
MAE	0.0132	0.0644	0.0225	0.0533	0.0276	0.2877	0.2593	0.0218	0.0893	0.0308	0.0838	0.0415	0.4189	0.4392
MSE	0.0160	0.0787	0.0291	0.0686	0.0351	0.3594	0.3302	0.0268	0.1093	0.0369	0.1051	0.0536	0.5595	0.5293
	APARCH with $\gamma = -0.1$ and $\delta = 1.2$													
Estimates	0.4478	0.1042	0.2942	0.3927	-0.1041	1.2506	4.8473	0.4219	0.1103	0.2996	0.3850	-0.1011	1.2080	4.9914
MAE	0.0481	0.0577	0.0312	0.0571	0.0411	0.2568	0.4427	0.0229	0.0415	0.0186	0.0309	0.0248	0.1739	0.2512
MSE	0.0524	0.0749	0.0400	0.0724	0.0515	0.3117	0.5688	0.0260	0.0535	0.0234	0.0413	0.0307	0.2294	0.3148

This table summarizes the estimates coefficients, MAE - Mean Absolute Errors, RMSE - Roots Mean Squared Errors for the GED (with shape equal to 5) GG_1-APARCH$(1,1)$. Monte simulations are computed with 100 replications. Each replication gives a sample size $n = 1000$ and 2500 of observations. The setting parameters are $d_1 = 0.4$, $\alpha_0 = 0.1$, $\alpha_1 = 0.3$, $\beta_1 = 0.4$, $(\gamma = 0, \delta = 2)$, or $(\gamma = -0.1, \delta = 2)$ and $(\gamma = -0.1, \delta = 1.2)$.

Table 4: Gaussian GG_2-APARCH

	$n=1000$							$n=2500$						
	$\hat{d}_1$	$\hat{d}_2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{d}_1$	$\hat{d}_2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$
APARCH model with $\gamma = 0$ and $\delta = 2$														
Estimates	0.4194	0.2954	0.1536	0.2814	0.3446	-0.0034	2.0127	0.4139	0.2964	0.1186	0.2939	0.3841	0.0016	1.9908
MAE	0.0299	0.0271	0.1113	0.0465	0.1273	0.0687	0.5201	0.0192	0.0154	0.0818	0.0274	0.0728	0.0413	0.3352
RMSE	0.0366	0.0334	0.1390	0.0592	0.1547	0.0888	0.6699	0.0257	0.0189	0.0947	0.0344	0.0919	0.0512	0.4047
APARCH with $\gamma = -0.1$ and $\delta = 2$														
Estimates	0.4189	0.2944	0.1372	0.2862	0.3752	-0.1064	1.9657	0.4130	0.2928	0.1066	0.2946	0.4019	-0.1027	1.9902
MAE	0.0281	0.0274	0.1170	0.0449	0.1295	0.0706	0.5251	0.0182	0.0170	0.0734	0.0251	0.0723	0.0392	0.3272
RMSE	0.0355	0.0337	0.1488	0.0568	0.1580	0.1004	0.6716	0.0224	0.0219	0.0893	0.0320	0.0948	0.0486	0.4185
APARCH with $\gamma = -0.1$ and $\delta = 1.2$														
Estimates	0.4597	0.2879	0.1232	0.2780	0.3847	-0.0988	1.2318	0.4354	0.2849	0.1120	0.2943	0.3861	-0.0996	1.2109
MAE	0.0611	0.0312	0.0924	0.0406	0.0745	0.0726	0.3239	0.0359	0.0202	0.0503	0.0273	0.0571	0.0408	0.2002
RMSE	0.0693	0.0421	0.1239	0.0517	0.0968	0.0907	0.4128	0.0399	0.0263	0.0673	0.0335	0.0729	0.0519	0.2675

This table summarizes the estimates coefficients, MAE - Mean Absolute Errors, RMSE - Roots Mean Squared Errors for the Gaussian GG_2-APARCH$(1,1)$. Monte simulations are computed with 100 replications. Each replication gives a sample size $n = 1000$ and 2500 of observations. The setting parameters are $d_1 = 0.4$, $d_2 = 0.3$, $\alpha_0 = 0.1$, $\alpha_1 = 0.3$, $\beta_1 = 0.4$, $(\gamma = 0, \delta = 2)$, or $(\gamma = -0.1, \delta = 2)$ and $(\gamma = -0.1, \delta = 1.2)$.

Table 5: Student GG_2-APARCH

	$n = 1000$								$n = 2500$							
	$\hat{d}_1$	$\hat{d}_2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{v}$	$\hat{d}_1$	$\hat{d}_2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{v}$
APARCH model with $\gamma = 0$ and $\delta = 2$																
Estimates	0.4003	0.2965	0.1543	0.2906	0.3489	0.0138	1.9366	3.1200	0.4124	0.2957	0.1298	0.2857	0.3578	-0.0041	2.0859	3.0501
MAE	0.0242	0.02221	0.119	0.0537	0.1204	0.0779	0.3939	0.2415	0.0278	0.0228	0.0971	0.0574	0.1236	0.0821	0.4693	0.1787
RMSE	0.0319	0.0281	0.1601	0.0713	0.1521	0.1611	0.4754	0.5306	0.0365	0.0305	0.1245	0.0783	0.1552	0.1357	0.7194	0.2701
APARCH with $\gamma = -0.1$ and $\delta = 2$																
Estimates	0.4269	0.2915	0.1869	0.3029	0.3211	-0.0584	1.9981	3.0378	0.4117	0.2953	0.1198	0.2854	0.3906	-0.0890	1.9802	3.2761
MAE	0.0376	0.0336	0.1451	0.0829	0.1846	0.1095	0.6029	0.2410	0.0249	0.0221	0.0906	0.0552	0.1040	0.0941	0.4069	0.3829
RMSE	0.0479	0.0435	0.1956	0.1058	0.2184	0.1669	0.9565	0.2979	0.0319	0.0289	0.1122	0.0749	0.1318	0.1803	0.6019	0.2359
APARCH with $\gamma = -0.1$ and $\delta = 1.2$																
Estimates	0.4825	0.2822	0.1013	0.3068	0.3476	-0.1151	1.5228	3.1486	0.4513	0.2729	0.0956	0.2994	0.3808	-0.0963	1.2897	3.1060
MAE	0.0843	0.0377	0.0822	0.0641	0.1053	0.1369	0.5166	0.3022	0.0536	0.0338	0.0590	0.0447	0.0601	0.0701	0.2629	0.1983
RMSE	0.0981	0.0509	0.1105	0.0826	0.1433	0.1872	1.6362	0.4140	0.0598	0.0424	0.0734	0.0587	0.0832	0.0903	0.3329	0.2520

This table summarizes the estimates coefficients, MAE - Mean Absolute Errors, RMSE - Roots Mean Squared Errors for the Student-t GG_2-APARCH$(1,1)$. Monte simulations are computed with 100 replications. Each replication gives a sample size $n = 1000$ and 2500 of observations. The setting parameters are $d_1 = 0.4$, $d_2 = 0.3$, $\alpha_0 = 0.1$, $\alpha_1 = 0.3$, $\beta_1 = 0.4$, $(\gamma = 0, \delta = 2)$, or $(\gamma = -0.1, \delta = 2)$ and $(\gamma = -0.1, \delta = 1.2)$.

Table 6: GED GG_2-APARCH

	$n=1000$								$n=2500$							
	$\hat{d}_1$	$\hat{d}_2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{v}$	$\hat{d}_1$	$\hat{d}_2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{v}$
APARCH model with $\gamma=0$ and $\delta=2$																
Estimates	0.4227	0.2892	0.1446	0.2901	0.3631	-0.0002	1.9457	5.0712	0.4114	0.2926	0.1246	0.2926	0.3864	-0.0028	1.9406	5.0670
MAE	0.0278	0.0252	0.1033	0.0313	0.0990	0.0409	0.4305	0.4574	0.0146	0.0153	0.0653	0.0214	0.0603	0.0264	0.2912	0.2686
RMSE	0.0344	0.0318	0.1304	0.0388	0.1232	0.0504	0.5226	0.5818	0.0186	0.0194	0.0805	0.0275	0.0771	0.0332	0.3774	0.3430
APARCH with $\gamma=-0.1$ and $\delta=2$																
Estimates	0.4203	0.2922	0.1370	0.2862	0.3755	-0.1021	1.9487	5.0785	0.4098	0.2943	0.1025	0.3007	0.3939	-0.1019	2.0294	5.0309
MAE	0.0297	0.0262	0.1019	0.0301	0.1150	0.0395	0.3546	0.4203	0.0165	0.0155	0.0647	0.0188	0.0559	0.0278	0.2552	0.2511
RMSE	0.0369	0.0327	0.1245	0.0388	0.1426	0.0498	0.4781	0.5696	0.0198	0.0192	0.0762	0.0231	0.0711	0.0349	0.3223	0.3137
APARCH with $\gamma=-0.1$ and $\delta=1.2$																
Estimates	0.4508	0.2904	0.1145	0.2928	0.3841	-0.1051	1.2242	4.4940	0.4352	0.2830	0.1089	0.2957	0.3961	-0.0995	1.1978	4.9251
MAE	0.0519	0.0285	0.0594	0.0311	0.0597	0.0399	0.2479	0.4741	0.0357	0.0217	0.0428	0.0208	0.0339	0.0222	0.1736	0.2792
RMSE	0.0602	0.0353	0.0715	0.0397	0.0743	0.0505	0.3264	0.5971	0.0448	0.0324	0.0571	0.0258	0.0431	0.0279	0.2254	0.3634

This table summarizes the estimates coefficients, MAE - Mean Absolute Errors, RMSE - Roots Mean Squared Errors for the GED GG_2-APARCH$(1,1)$. Monte simulations are computed with 100 replications. Each replication gives a sample size $n=1000$ and 2500 of observations. The setting parameters are $d_1=0.4$, $d_2=0.3$, $\alpha_0=0.1$, $\alpha_1=0.3$, $\beta_1=0.4$, $(\gamma=0,\delta=2)$, or $(\gamma=-0.1,\delta=2)$ and $(\gamma=-0.1,\delta=1.2)$.

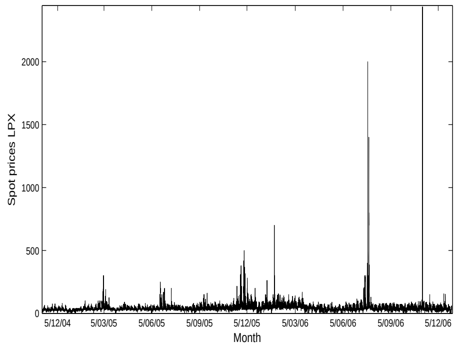

Figure 1: Time series of hourly electricity spot prices, euro/Mwh, at LPX in Germany

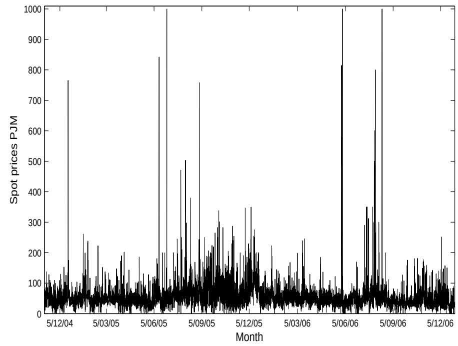

Figure 2: Time series of hourly electricity spot prices, dollar/Mwh, at PJM Power Exchange in New York

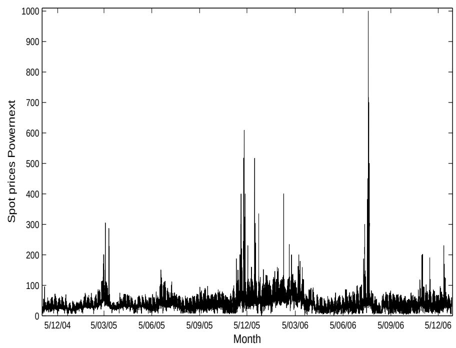

Figure 3: Time series of hourly electricity spot prices, euro/Mwh, at Powernext Power Exchange in France

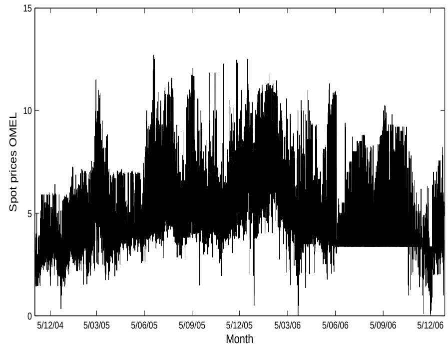

Figure 4: Time series of hourly electricity spot prices, euro/Mwh, at OMEL Power Exchange in Spain

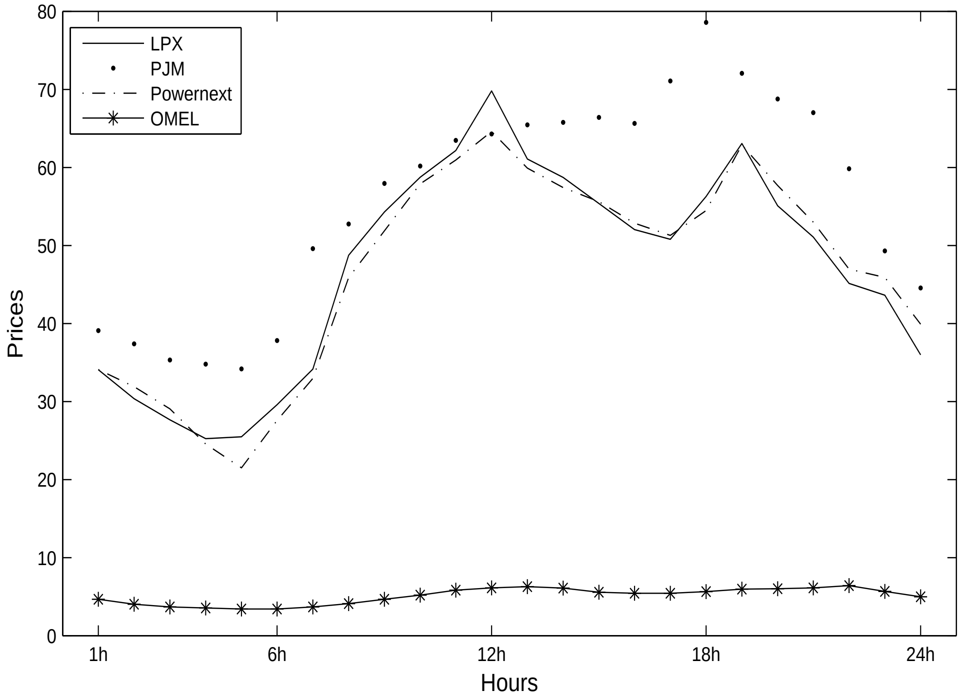

Figure 5: Average hourly electricity spot price accross the entire sample for the Power Exchange

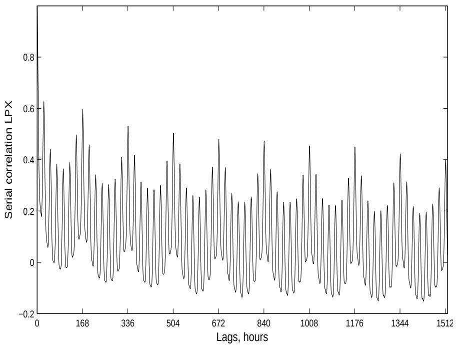

Figure 6: Serial correlogram of hourly electricity spot prices at LPX in Germany

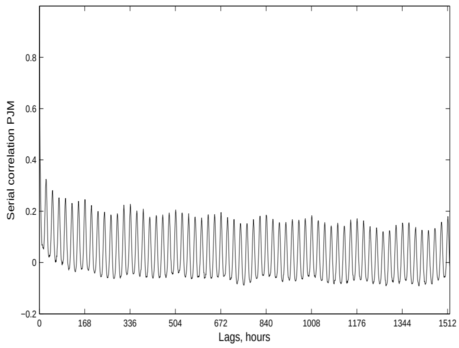

Figure 7: Serial correlogram of hourly electricity spot prices at PJM Power Exchange in New York

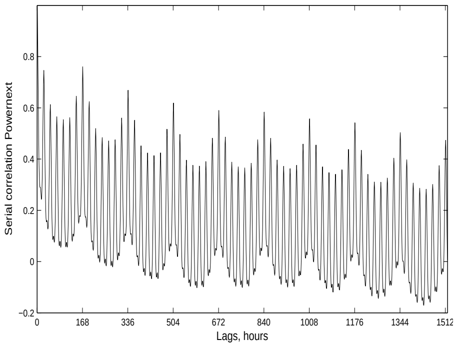

Figure 8: Serial correlogram of hourly electricity spot prices at Powernext Power Exchange in France

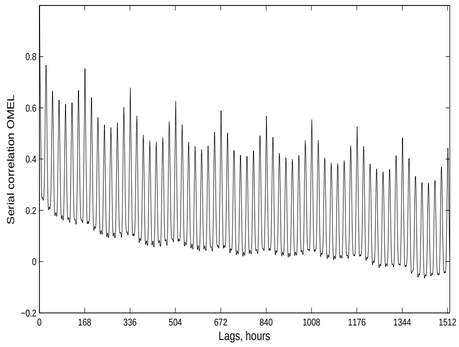

Figure 9: Serial correlogram of hourly electricity spot prices at OMEL Power Exchange in Spain

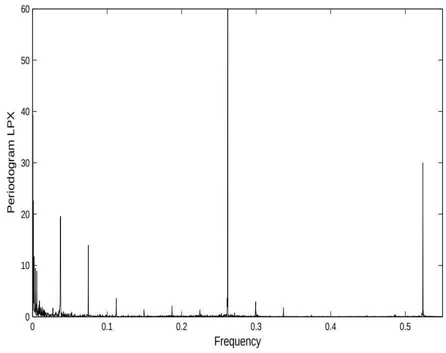

Figure 10: Classical periodogram applied to hourly electricity spot prices at LPX in Germany

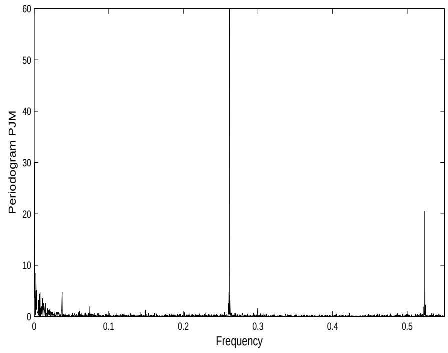

Figure 11: Classical periodogram applied to hourly electricity spot prices at PJM Power Exchange in New York

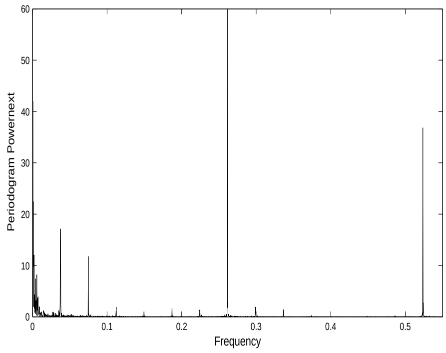

Figure 12: Classical periodogram applied to hourly electricity spot prices at Powernext Power Exchange in France

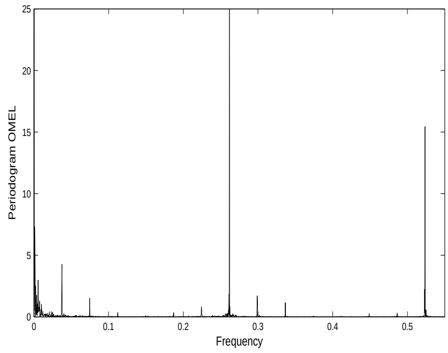

Figure 13: Classical periodogram applied to hourly electricity spot prices at OMEL Power Exchange in Spain

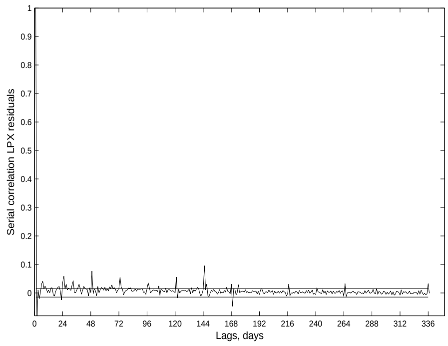

Figure 14: Serial correlogram of LPX SARIMA model residuals

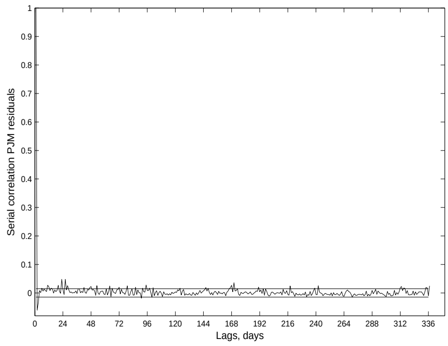

Figure 15: Serial correlogram of PJM SARIMA model residuals

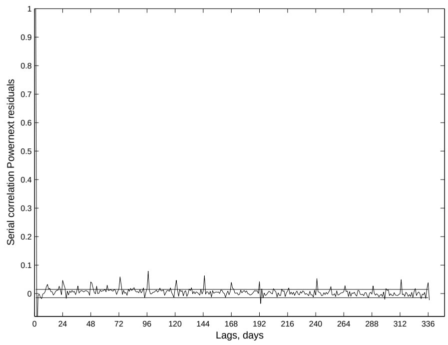

Figure 16: Serial correlogram of Powernext SARIMA model residuals

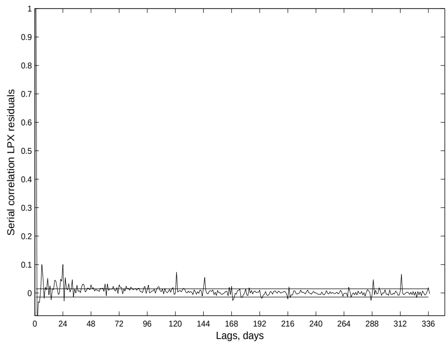

Figure 17: Serial correlogram of OMEL SARIMA model residuals

Table 7: Statistics of hourly electricity spot prices

	LPX	PJM	Powernext	OMEL
Observations	18912	18912	18912	18912
Mean	3.699	3.861	3.691	1.550
Meadian	3.708	3.917	3.713	1.5189
Maximum	7.798	6.907	6.907	2.541
Minimum	-4.605	-4.605	-4.605	-4.605
Standard deviation	0.376	0.593	0.318	0.151
Skewness	-3.616	-6.148	-0.833	-0.478
Kurtosis	46.877	68.005	11.912	8.118
Jarque-Bera statistic	$1.55E+06$	$3.448E+06$	$6.477E+06$	$2.136E+04$
JB p-value	0.0000	0.0000	0.0000	0.0000

This table provides measures of central tendency, dispersion and shape for the changes in the hourly spot prices for LPX - Leipzig Power Exchange, Germany - PJM - Pennsylvania-New Jersey-Maryland interconnection, United States - Powernext - France and OMEL - Operadora del Mercado Español de Electricidad, Spain from 5 november 2004 to 5 november 2007. JB - Jarque-Bera.

Table 8: Residual Diagnostic Tests. Diagnostic tests include probability in parentheses.

	LPX	PJM	Powernext	OMEL
mean	$5.3881\ 10^{-4}$	-0.0015	$5.2136\ 10^{-4}$	$5.7749\ 10^{-4}$
Standard deviation	0.2316	0.6110	0.1848	0.1476
Skewness	-5.541	-7.6153	-3.605	-3.635
Kurtosis	370.43	106.50	264.4	231.57
Jacque Bera	1.064810^{8}	8.6210^{6}	5.39010^{7}	4.12110^{7}
	(0.001)	(0.001)	(0.001)	(0.001)
ARCH(1)	2522	758.94	3386	2466
	0.000	0.000	0.000	0.000
ARCH(5)	2849	792.63	4137	2897
	0.000	0.000	0.000	0.000

This table presents the distributional properties of the GG_3 model with ARIMA component residuals for the four electricity markets - skewness, kurtosis and Jarque-Bera statistic. ARCH(k): the autoregressive conditional heteroskedasticity tests of order k and p-values in parentheses.

Table 9: Conditional volatility models parameter estimates - LPX electricity spot market

	GARCH			EGARCH			APARCH		
	Normal	Student-t	GED	Normal	Student-t	GED	Normal	Student-t	GED
α_0	0.0065029	0.0051966	0.0041	-1.3786	-0.1385	-1.88771	0.000002	0.000002	0.00002
	(0.000005497)	(0.0000001219)	(0.00001)	(0.0232)	(0.0247)	(0.0219)	(0.000003)	(0.000004)	(0.0000003)
α_1	0.6586	0.6823	0.6665	-0.2819	-0.2654	-0.0200	0.7302	0.5724	0.5062
	(0.0217)	(0.00129)	(0.0020)	(0.0102)	(0.0893)	(0.000065)	(0.0118)	(0.0009)	(0.0008)
β_1	0.3414	0.3177	0.335	0.9860	0.9648	0.6568	0.3859	0.3673	0.3966
	(0.0097)	(0.0004)	(0.0004)	(0.0214)	(0.0201)	(0.00078)	(0.0036)	(0.0009)	(0.0009)
γ	-	-	-	0.7876	0.7855	0.6589	0.3186	0.0882	0.1149
	(-)	(-)	(-)	(0.0009)	(0.0009)	(0.00109)	(0.0089)	(0.0001)	(0.0007)
δ	-	-	-	-	-	-	1.2929	1.3348	1.2839
	(-)	(-)	(-)	(-)	(-)	(-)	(0.0062)	(0.0018)	(0.0014)
shape	-	2.8856	0.8162	-	10000.4	30.8157	-	2.8117	0.8269
	(-)	(0.0037)	(0.0006)	(-)	(774.1)	(0.00054)	(-)	(0.0047)	(0.0005)
Log-L	8883.6	15936.1	15577	9759.5	9874	15509.89	102447.8	16069.9	15714.2
AIC	-17761.1	-31864.2	-31146	-1.9510.9	-19738.1	-31009.78	-20485.5	-32127.8	-31416.3
BIC	-17737.6	-31832.8	-31115	-19479.6	-19698.8	-30970.54	-20446.3	-32080.7	-1369.2
Q^2 (10)	0.1855	0.2332	0.3249	5.1076	4.6410	0.3036	4.3233	0.8482	1.1598
	(0.9999)	(0.9999)	(1.0000)	(0.8839)	(0.9138)	(0.9999)	(0.9316)	(0.9999)	(0.9997)

This table summarizes the estimates coefficients for the estimated models for LPX electricity spot market. These models are GARCH $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$, EGARCH $\log h_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{h_{t-1}} + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 \log h_{t-1}^2$, and APARCH $h_t^\delta = \alpha_0 + \alpha_1 \left(|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1}\right)^\delta + \beta_1 h_{t-1}^\delta$. Shape denotes the degrees of freedom parameter l for the Student's t distribution. Standard errors resulting from ML estimation are in parentheses. Log-L is the value of the maximized likelihodd. $Q^2\,(10)$ are the Box-Pierce statistic for remaining serial correlation in the squared standardized residuals with p-values in parentheses with critical value equal to 18.3070.

Table 10: Conditional volatility models parameter estimates - PJM electricity spot market

	GARCH			EGARCH			APARCH		
	Normal	Student-t	GED	Normal	Student-t	GED	Normal	Student-t	GED
α_0	0.1189	0.0645	0.0529	-0.7421	-1.1313	-1.3913	0.1304	0.000002	0.00002
	(0.0019)	(0.00002)	(0.00001)	(0.0234)	(0.0038)	(0.0302)	(0.0372)	(0.00005)	(0.00001)
α_1	0.7289	0.7943	0.7726	-0.2814	0.0646	0.1049	0.7050	0.6545	0.6342
	(0.0268)	(0.0038)	(0.0030)	(0.0109)	(0.0002)	(0.00404)	(0.0683)	(0.0024)	(0.0022)
β_1	0.2711	0.1182	0.1444	0.6489	0.5134	0.4382	0.4266	0.3179	0.3273
	(0.0269)	(0.0003)	(0.0007)	(0.0137)	(0.0008)	(0.00477)	(0.0288)	(0.0005)	(0.0008)
γ	-	-	-	0.6809	0.5968	-0.01539	0.4545	0.1059	0.1624
	(-)	(-)	(-)	(0.0076)	(0.0005)	(0.00000717)	(0.0196)	(0.0002)	(0.0005)
δ	-	-	-	-	-	-	1.0985	1.7605	1.6947
	(-)	(-)	(-)	(-)	(-)	(-)	(0.2087)	(0.0001)	(0.0054)
shape	-	2.7378	0.7665	-	2.6894	0.5571	-	2.7768	0.7759
	(-)	(0.0051)	(0.0004)	(-)	(0.0049)	(0.00179)	(-)	(0.0053)	(0.0004)
Log-L	-13809.9	-2217.9	-3223.9	-13622.9	-2389.07	-4456.27	-13051.9	-2131.1	-3134.8
AIC	27625.9	4443.8	6455.9	27253.8	4788.1	8922.54	26113.8	4274.2	6281.7
BIC	27649.4	4475.2	6487.3	27285.3	4827.4	8961.77	26152.9	4321.3	6328.8
$Q^2\,(10)$	13.3264	4.0372	3.3400	17.8630	8.4161	39.4480	25.5887	9.1798	10.5450
	(0.2059)	(0.9457)	(0.9723)	(0.0573)	(0.5882)	(0.0000)	(0.0043)	(0.5151)	(0.3940)

This table summarizes the estimates coefficients for the estimated models for PJM electricity spot market. These models are GARCH $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$, EGARCH $\log h_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{h_{t-1}} + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 \log h_{t-1}^2$, and APARCH $h_t^\delta = \alpha_0 + \alpha_1 \left(|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1}\right)^\delta + \beta_1 h_{t-1}^\delta$. Shape denotes the degrees of freedom parameter l for the Student's t distribution. Standard errors resulting from ML estimation are in parentheses. Log-L is the value of the maximized likelihodd. $Q^2\,(10)$ are the Box-Pierce statistic for remaining serial correlation in the squared standardized residuals with p-values in parentheses with critical value equal to 18.3070.

Table 11: Conditional volatility models parameter estimates - Powernext electricity spot market

	GARCH			EGARCH			APARCH		
	Normal	Student-t	GED	Normal	Student-t	GED	Normal	Student-t	GED
α_0	0.0039	0.0052	0.0043	-1.1014	-1.1109	-1.5074	0.0048	0.000002	0.00002
	(0.000001)	(0.0000001)	(0.0000001)	(0.0226)	(0.0223)	(0.00422)	(0.0004)	(0.00002)	(0.000006)
α_1	0.5308	0.6950	0.6656	-0.1007	-0.0991	-0.0479	0.4771	0.9507	0.5927
	(0.0026)	(0.0005)	(0.0008)	(0.0007)	(0.0006)	(0.00136)	(0.0026)	(0.0066)	(0.0008)
β_1	0.4692	0.3049	0.3344	0.6552	0.6563	0.7018	0.4983	0.2898	0.3022
	(0.0035)	(0.0004)	(0.0004)	(0.0014)	(0.0013)	(0.000598)	(0.0219)	(0.0015)	(0.0011)
γ	-	-	-	0.8246	0.8227	0.7473	0.2127	0.1270	0.1353
	(-)	(-)	(-)	(0.0011)	(0.0010)	(0.000179)	(0.0029)	(0.0003)	(0.0002)
δ	-	-	-	-	-	-	1.1722	1.4628	1.4013
	(-)	(-)	(-)	(-)	(-)	(-)	(0.0783)	(0.0128)	(0.00005)
shape	-	2.8451	0.7828	-	10000.87	0.7886	-	2.4406	0.7896
	(-)	(0.0013)	(0.0002)	(-)	(45.8463)	(0.000223)	(-)	(0.0025)	(0.0002)
Log-L	10447.4	14993.4	14866	10707.9	10725.2	14934.78	10856.7	15211.8	15014.1
AIC	-20888.9	-29978.9	-29724	-21407.8	-21440.5	-29859.57	-21703.5	-30411.7	-30016.2
BIC	-20865.3	-29947.5	-29693	-21376.5	-21401.2	-29820.03	-21664.2	-30364.6	-29969.1
$Q^2\,(10)$	31.1008	29.8671	30.6941	21.7496	21.9841	26.4565	15.9533	20.1787	19.3558
	(0.0005)	(0.0006)	(0.0007)	(0.016)	(0.015)	(0.0032)	(0.1009)	(0.0276)	(0.0359)

This table summarizes the estimates coefficients for the estimated models for Powernext electricity spot market. These models are GARCH $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$, EGARCH $\log h_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{h_{t-1}} + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 \log h_{t-1}^2$, and APARCH $h_t^\delta = \alpha_0 + \alpha_1 \left(|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1}\right)^\delta + \beta_1 h_{t-1}^\delta$. Shape denotes the degrees of freedom parameter l for the Student's t distribution. Standard errors resulting from ML estimation are in parentheses. Log-L is the value of the maximized likelihodd. $Q^2\,(10)$ are the Box-Pierce statistic for remaining serial correlation in the squared standardized residuals with p-values in parentheses with critical value equal to 18.3070.

Table 12: Conditional volatility models parameter estimates - OMEL electricity spot market

	GARCH			EGARCH			APARCH		
	Normal	Student-t	GED	Normal	Student-t	GED	Normal	Student-t	GED
α_0	0.00235	0.0052	0.0038	-0.2648	-0.2654	-2.1893	0.000002	0.000002	0.0596
	(0.000005)	(0.0000001)	(0.00000006)	(0.0172)	(0.0150)	(0.0223)	(0.0000001)	(0.0000003)	(0.0003)
α_1	0.2961	0.6599	0.5450	-0.0365	-0.0359	-0.000339	0.1369	0.0236	0.4144
	(0.0235)	(0.0009)	(0.0011)	(0.0004)	(0.0002)	(0.000000593)	(0.0019)	(0.0000008)	(0.0004)
β_1	0.6367	0.3401	0.3994	0.2466	0.2457	0.7959	0.8614	0.0000009	0.5405
	(0.0567)	(0.0005)	(0.0004)	(0.0062)	(0.0056)	(0.00059)	(0.0041)	(0.09 10^{-10})	(0.0008)
γ	-	-	-	0.9759	0.9756	0.6058	0.1851	-0.9999	0.0192
	(-)	(-)	(-)	(0.0004)	(0.0003)	(0.00116)	(0.0027)	(0.1838)	(0.00004)
δ	-	-	-	-	-	-	1.0282	8.2535	0.8393
	(-)	(-)	(-)	(-)	(-)	(-)	(0.0006)	(4.6080)	(0.0001)
shape	-	2.9173	0.8351	-	10000.27	0.8319	-	2.2439	0.8364
	(-)	(0.0024)	(0.0003)	(-)	(12.7295)	(0.000317)	(-)	(0.0557)	(0.0003)
Log-L	12413.7	15084.08	15201	12602.4	12613.5	15268.81	12586.6	14239.6	15307.4
AIC	-24821.4	-30160.2	-30393	-25196.8	-25217.1	-30527.62	-25163.2	-28467.3.7	-30602.8
BIC	-24797.8	-30128.8	-30362	-25165.5	-25177.8	-30488.38	-25123.9	-28420.2	-30555.8
Q^2 (10)	57.9522	61.6504	51.8912	727.68	671.96	39.7337	442.96	2466.08	29.3430
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0011)

This table summarizes the estimates coefficients for the estimated models for OMEL electricity spot market. These models are GARCH $h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$, EGARCH $\log h_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{h_{t-1}} + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 \log h_{t-1}^2$, and APARCH $h_t^\delta = \alpha_0 + \alpha_1 \left(|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1}\right)^\delta + \beta_1 h_{t-1}^\delta$. Shape denotes the degrees of freedom parameter l for the Student's t distribution. Standard errors resulting from ML estimation are in parentheses. Log-L is the value of the maximized likelihood. $Q^2\,(10)$ are the Box-Pierce statistic for remaining serial correlation in the squared standardized residuals with p-values in parentheses and critical value equal to 18.3070.

Table 13: Skewed Student-t APARCH model for all the markets

	LPX	PJM	Powernext	OMEL
α_0	0.000002	0.0070	0.000002	0.0789
	(0.000002)	(0.00002)	(0.00007)	(0.0075)
α_1	0.5745	0.7168	0.9524	0.5597
	(0.0032)	(0.0084)	(0.0167)	(0.0101)
β_1	0.3672	0.3267	0.2879	0.4599
	(0.0041)	(0.0021)	(0.0071)	(0.0025)
γ	0.0878	0.0703	0.1288	0.0042
	(0.0045)	(0.0023)	(0.0014)	(0.0058)
δ	1.3353	1.7235	1.4649	0.8176
	(0.0076)	(0.0111)	(0.0594)	(0.0848)
shape	2.8056	2.6562	2.4444	2.5889
	(0.0052)	(0.0047)	(0.0053)	(0.0113)
skew	-0.0105	-0.1332	-0.0318	-0.0078
	(0.000002)	(0.00006)	(0.00004)	(0.0005)
Log-L	16070.9	-1974.1	15223.9	15234.4
AIC	-32127.9	3962.2	-30433.8	-30454.8
BIC	-32073	4017.1	-30378.8	-30399.9
$Q^2\,(10)$	0.8191	7.4287	20.1446	28.2234
	(0.9999)	(0.6844)	(0.0279)	(0.0017)

This table summarizes the estimates coefficients for the estimated models for the Skewed Student-t APARCH model for all electricity spot markets. The model is APARCH $h_t^{\delta} = \alpha_0 + \alpha_1 \left(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1}\right)^{\delta} + \beta_1 h_{t-1}^{\delta}$. Shape denotes the degrees of freedom parameter l for the Student's t distribution.; Skew refers to the Skew Student-t skewness parameter. Standard errors resulting from ML estimation are in parentheses. Log-L is the value of the maximized likelihodd. $Q^2\,(10)$ are the Box-Pierce statistic for remaining serial correlation in the squared standardized residuals with p-values in parentheses. The Box-Pierce Q-statistic critical value is 18.3070.