The demand for health insurance in a multirisk context

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The demand for health insurance in a multirisk context

La demande d'assurance santé dans un contexte multirisque

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Abstract

Using a model of bivariate decision under risk, we analyse the health insurance demand when there are two sources of risk: a health risk and an uninsurable one. We examine how the uninsurable risk affects the coverage of the health risk. We show that the determinants of the demand for health insurance are not only the correlation between the health and uninsurable risks as shown by Doherty and Schlesinger (1983a) and the variation of the marginal utility of wealth with respect to the health status (Rey, 2003) but also the way in which the occurrence of the uninsurable risk affects the marginal utility of wealth.

Résumé

En utilisant un modèle *bivarié* de décision dans le risque, nous analysons la demande d'assurance santé dans un contexte où un individu fait face à deux risques de nature différente : un risque de santé et un risque non assurable. Nous examinons l'impact d'un risque non assurable sur la couverture du risque de santé. Nous montrons que les principaux déterminants de la demande d'assurance santé sont non seulement la corrélation entre le risque de santé et le risque non assurable comme l'ont montré Doherty et Schlesinger (1983a) et l'utilité marginale de la richesse en fonction de l'état de santé (Rey, 2003) mais aussi la manière dont la réalisation du risque non assurable affecte l'utilité marginale de la richesse.

Mots clés : risques corrélés, assurance santé, utilité contingente
Keywords: Correlated risks, Health insurance, State-dependent utility

JEL CLASSIFICATION: D81, I11, I18

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1 Introduction

The analysis of the optimal health insurance was strongly influenced by three propositions.
The first proposition is the Bernoulli principle. It states that risk-averse agents will choose full coverage when the premium is actuarially fair. The second proposition derives from by Mossin-Smith (1968). In their pioneering paper, Mossin-Smith (1968) showed that if the premium for insurance is loaded, the individual chooses less than full insurance. The third proposition is proposed by Arrow (1963). It states that a risk-averse agent will prefer a franchise contract to a coinsurance contract.
Doherty and Schlesinger (1983a) show, in the presence of an uninsurable financial risk, that sufficient conditions for the validity of these propositions depend on the correlation between insurable and uninsurable risks. However, Doherty and Schlesinger (1983a) use a one-argument utility function. Rey (2003) takes into account this limit by using a bivariate utility function. She shows that the determinants of the demand for health insurance are not only the correlation between the health and uninsurable risks but also the variation of the marginal utility of wealth with respect to the health status. However, Rey (2003) imposes restrictive assumptions on the health risk. She assumes that illness is characterized only by a decrease in health status. However, since there exist curative cares, which are costly for individuals, the health risk induces both a loss in the health status and a financial loss. This paper extents the result obtained by Rey (2003) to this framework. We also consider an uninsurable risk (For example an accident risk). The aim of this paper is to examine how the uninsurable risk affects the coverage of the health risk.
We show that the optimal health insurance depends crucially on the way in which the occurrence of the uninsurable risk affects the marginal utility of wealth.
The organization of this article is as follows. The next section introduces the model. The section that follows examines the optimal coinsurance contracts. Section 4 analyses the optimal insurance policy. The last section concludes.

2 The model

We consider an individual who derives utility from wealth W and from its health stock H.
We use a Von Neumann Morgenstern two-arguments utility function U(W,H). We assume for

U standard concavity assumptions:

$U_1 > 0, U_2 > 0, U_{11} < 0, U_{22} < 0$ and $U_{11} U_{22} - (U_{12})^2 > 0$.

We don't impose any restriction on the sign of U_{12}, the cross second derivative of U.

The agent has an initial health stock H_0, an initial wealth stock W_0 and he becomes ill with a probability p_1. The disease implies a health loss D. In case of disease, there exist exogenous curative cares L at cost $c(L)$ for the individual.

With a probability p_2, the agent faces an uninsurable loss. We denote V(W,H) the utility function when the uninsurable loss occurs, with V(W,H) < U(W,H). Indeed, the realisation of the uninsurable loss always decreases the utility of wealth and health for risk-averse preferences (Cook and Graham (1977)).

We assume for V standard assumptions: $V_1 > 0, V_2 > 0, V_{11} < 0, V_{22} < 0$ and

$V_{11} V_{22} - (V_{12})^2 > 0$.

We don't impose any restriction on the sign of the variation of the marginal utility of wealth with respect to the uninsurable loss. Three cases are possible.

- The occurrence of the uninsurable loss may leave the individual's marginal utility of wealth unchanged: U_1(W,H) = V_1(W,H). In this case, the uninsurable loss does not effect the wealth uncertainty.

- The occurrence of the uninsurable loss may increase the individual's marginal utility of wealth: U_1(W,H) < V_1(W,H). In this case, the uninsurable loss plays as a hedge against wealth uncertainty.

- The occurrence of the uninsurable loss may decrease the individual's marginal utility of wealth: U_1(W,H) > V_1(W,H). In this case, the uninsurable loss plays as an amplifier of wealth uncertainty.

3 Optimal Coinsurance Contracts

To examine the optimal coinsurance contracts[4], we consider a coinsurance health contract in which the insurance reimburses $\alpha\ c(L)$ of the expense in health care. The premium for insurance level α is $P(\alpha) = (1+m)\ p_1\ \alpha\ c(L)$, where m is the loading factor, $m \geq 0$.

Four states of nature can appear. Utility levels and probabilities of occurrence are characterized as follows.

State 1: U(W_0 - $P(\alpha)$, H_0) no loss occurs

State 2: U(W_0 - $P(\alpha)$- $(1-\alpha)\ c(L)$, H_0 - D + L) only the insurable loss occurs

State 3: V (W_0 - $P(\alpha)$, H_0) only the uninsurable loss occurs

State 4: V(W_0 - $P(\alpha)$- $(1-\alpha)\ c(L)$, H_0 - D + L) the two loss occurs.

We note Π_i the probability of occurrence of state i (i = 1 ….4).

They are defined as follows:

$\Pi_1 = 1 - p_1 - p_2 + p_1\, p_{2/1}$

$\Pi_2 = p_1\ (1 - p_{2/1})$

$\Pi_3 = p_2 - p_1\, p_{2/1}$

$\Pi_4 = p_1\, p_{2/1}$

Where p_1 (resp. p_2) denotes the probability of occurrence of the insurable (resp. uninsurable) loss and $p_{2/1}$ the conditional probability of the uninsurable loss given the insurable loss.

We don't impose restriction on the correlation between the losses. The tree cases are possible: the insurable loss and the uninsurable loss are independent ($p_2 = p_{2/1}$), the insurable loss and the uninsurable loss are positively correlated ($p_2 < p_{2/1}$) and the insurable loss and the uninsurable loss are negatively correlated ($p_2 > p_{2/1}$).

The optimal level of insurance is solution of the following program:

$$\max_{\alpha} E(\alpha) = \Pi_1\ U(W_0 - P(\alpha), H_0) + \Pi_2\ U(W_0 - P(\alpha) - (1-\alpha)\ c(L), H_0 - D + L)$$
$$+ \Pi_3\ V(W_0 - P(\alpha), H_0) + \Pi_4\ V(W_0 - P(\alpha) - (1-\alpha)\ c(L), H_0 - D + L) \qquad (1)$$

[4] The optimal coinsurance contracts is the individual's optimal insurance demand for given health expenditures.

First order condition writes[5]:

$$E'(\alpha) = p_1\ c(L)\ [Z_1(W_2, H_0 - D + L)(1 - (1+m)\ p_1) - (1 - p_1)(1+m)\ Z_1(W_1, H_0)]$$
$$+\ p_1(p_{2/1} - p_2)\ c(L)\{(1+m)\ p_1\ [F_1(W_2, H_0 - D + L) - F_1(W_1, H_0)]$$
$$- F_1(W_2, H_0 - D + L)\} \qquad (6)$$

Where:

$W_1 = W_0 - P(\alpha)$ et $W_2 = W_0 - P(\alpha) - (1-\alpha)\ c(L)$

$Z(W, H) = (1 - p_2)\ U(W, H) + p_2\ V(W, H)$

$F(W,H) = U(W, H) - V(W, H)$.

To examine the optimal coinsurance contracts, we consider in turn two situations: the situation where the premium is actuarially fair $(m = 0)$ and the situation where the premium is loaded $(m > 0)$.

3.1 The premium is actuarially fair

In this section, we examine optimal coinsurance contracts where the premium is actuarially fair $(m = 0)$. The actuarially fair insurance premium is such premium when the expected loss of the insurance company equals exactly the revenue from insurance premium.

Two situations are possible: the situation where the individual treats himself perfectly ($L = D$) and the situation where he does not ($L < D$). Firstly, let us consider the case where the individual treats himself perfectly ($L = D$).

If the premium for insurance is actuarially fair then equation (6) may be written as:

$$E'(\alpha) = p_1\ c(L)(1 - p_1)\ [Z_1(W_2, H_0 - D + L) - Z_1(W_1, H_0)]$$
$$+\ p_1(p_{2/1} - p_2)\ c(L)\{\ p_1\ [F_1(W_2, H_0 - D + L) - F_1(W_1, H_0)]$$
$$- F_1(W_2, H_0 - D + L)\} \qquad (8)$$

From equation (8), we obtain in the case where the individual treats himself perfectly ($L = D$):

[5] see proof developped in Appendix 1.

Proposition 1 (Complete curing) If the premium for insurance is actuarially fair $(m = 0)$, the individual chooses full insurance ($\alpha^* = 1$) if one of the following conditions is true:

- The insurable loss and the uninsurable loss are positively correlated ($p_{2/1} > p_2$) and the occurrence of the uninsurable loss increases the individual's marginal utility of wealth ($U_1 < V_1$).
- The insurable loss and the uninsurable loss are negatively correlated ($p_{2/1} < p_2$) and the occurrence of the uninsurable loss decreases the individual's marginal utility of wealth ($U_1 > V_1$).

Proof. See appendix 2.1.

Let us now turn to the case where the individual doesn't treat himself perfectly ($L < D$).
We obtain:

Proposition 2 (Partial curing) If the premium for insurance is actuarially fair $(m = 0)$, the individual chooses full insurance ($\alpha^* = 1$) if one of the following conditions is true:

- The insurable loss and the uninsurable loss are independent ($p_{2/1} > p_2$) and the cross derivative of U is negative or null ($U_{12} \leq 0$).
- The insurable loss and the uninsurable loss are positively correlated ($p_{2/1} > p_2$), the cross derivative of U is negative or null ($U_{12} \leq 0$) and the occurrence of the uninsurable loss increases the individual's marginal utility of wealth ($U_1 < V_1$).
- The insurable loss and the uninsurable loss are negatively correlated ($p_{2/1} < p_2$), the cross derivative of U is negative or null ($U_{12} \leq 0$) and the occurrence of the uninsurable loss decreases the individual's marginal utility of wealth ($U_1 > V_1$).

Proof. See appendix 2.2.

The positive sign of the cross derivative of U ($U_{12} > 0$) means that the marginal utility of the composite material good (W) increases with health status (H) and reciprocally.
Rey (2003) obtains that when the insurable and the uninsurable risks are negatively correlated and $U_{12} < 0$, full coverage is optimal if the premium is actuarially fair. We show that this result is true if the occurrence of the uninsurable loss increases the individual's marginal utility of

wealth. The sign of the variation of the marginal utility of wealth with respect to the uninsurable loss is crucial when studying individual's optimal insurance demand.

3.2 The premium is loaded

In this section we examine the individual's optimal insurance demand when the premium for insurance is loaded $(m > 0)$.

Firstly, let us consider the case where the individual treats himself perfectly ($L = D$).

We obtain:

Proposition 3 (Complete curing) If the premium for insurance is loaded $(m > 0)$, the individual chooses less than full insurance ($\alpha^* < 1$) if one of the following conditions is true:

- $p_{2/1} = p_2$
- The insurable loss and the uninsurable loss are positively correlated ($p_{2/1} > p_2$) and the occurrence of the uninsurable loss decreases the individual's marginal utility of wealth ($U_1 > V_1$).
- The insurable loss and the uninsurable loss are negatively correlated ($p_{2/1} < p_2$) and the occurrence of the uninsurable loss increases the individual's marginal utility of wealth ($U_1 < V_1$).

Proof. See appendix 2.3.

Let us now turn to the case where the individual doesn't treats himself perfectly ($L < D$).

We obtain:

Proposition 4 (Partial curing) If the premium for insurance is loaded $(m > 0)$, the individual chooses less than full insurance ($\alpha^* < 1$) if one of the following conditions is true:

- $p_{2/1} = p_2$ and $U_{12} \geq 0$
- $p_{2/1} > p_2$, $U_{12} \geq 0$, $U_1 > V_1$ and $U_{12} > V_{12}$
- $p_{2/1} < p_2$, $U_{12} < 0$, $U_1 < V_1$ and $U_{12} < V_{12}$.

Proof. See appendix 2.4.

Contrary to Rey (2003), the determinants of the demand for health insurance depend not only on the correlation between the insurable and the uninsurable losses risks and the variation of the marginal utility of wealth with respect to the health status, but also on the way in which the occurrence of the uninsurable risk affects the marginal utility of wealth.

4 Optimal Insurance Policy

The Arrow theorem states that a risk-averse agent will prefer a franchise contract to a coinsurance contract.

To examine the validity of this theorem to our framework, we extent our previous model to the case where the individual faces three losses:

- a small health loss L_S
- a large health loss L_L
- an uninsurable loss

We define six states of nature. Utility levels and probabilities of occurrence are characterized as follows:

State 1: U(W_0 , H_0). No loss occurs.

State 2: U(W_0 - $c(L_S)$, H_0 - D_S + L_S). Only the insurable small health loss occurs.

State 3: U(W_0 - $c(L_L)$, H_0 - D_L + L_L). Only the insurable large health loss occurs

State 4: V(W_0 , H_0). Only the uninsurable loss occurs.

State 5: V(W_0 - $c(L_S)$, H_0 - D_S + L_S). The uninsurable and the insurable small health losses occurs.

State 6: V(W_0 - $c(L_L)$, H_0 - D_L + L_L). The uninsurable and the insurable large health losses occurs.

We note Π_i the probability of occurrence of state i (i = 1 ….6).

They are defined as follows:

$\Pi_1 = 1 - p_S - p_L - p_2 + p_S\, p_{2/S} + p_L\, p_{2/L}$

$\Pi_2 = p_S - p_S\, p_{2/S}$

$\Pi_3 = p_L - p_L\, p_{2/L}$

$\Pi_4 = p_2 - p_S\, p_{2/S} - p_L\, p_{2/L}$

$\Pi_5 = p_S\, p_{2/S}$

$\Pi_6 = p_L\, p_{2/L}$

Where p_S and p_L denotes the probabilities of L_S and L_L, respectively.

We consider two insurance policies defined as follows:

- Franchise contract

$$F = \begin{cases} c(L_L)\text{-}c(L_S) & \text{in states 3 and 6} \\ 0 & \text{otherwise} \end{cases}$$

F is a deductible policy with deductible level $c(L_S)$.

- Coinsurance contract

$$C = \begin{cases} \alpha\ c(L_S) & \text{in states 2 and 5} \\ \alpha\, c(L_S)\ +\ (1 - f(\alpha))\,[c(L_L) - c(L_S)] & \text{in states 3 and 6} \\ 0 & \text{in states 1 and 4} \end{cases}$$

C is a proportional basis policy.

However, coinsurance and franchise contracts are available at the same premium. We obtain:

$$P\ = \alpha\ c(L_S)\ (\,p_S\ +\ p_L\,) + (1\text{-}\,f(\alpha))\ [\,c(L_L)\text{-}\ c(L_S)]\,p_L\ =\ [\,c(L_L)\text{-}\ c(L_S)]\,p_L\ .$$

with $f(\alpha) = \left(\dfrac{\alpha\ c(L_S)}{c(L_L)\ -\ c(L_L)}\right)\left[1\ +\ \dfrac{p_L}{p_S}\right]$[6].

The optimal level of coinsurance is solution of the following program:

$$\alpha^* = \arg\max_{\alpha}\ E(\alpha) = \ \Pi_1\ \text{U}(\text{W}_0 - \text{P}\ ,\ \text{H}_0) + \Pi_2\ \text{U}(\text{W}_0 - \text{P}\ -\ (1-\alpha)\,c(L_S),\ \text{H}_0 - D_S\ +\ L_S)$$

$$+\ \Pi_3\ \ \text{U}(\text{W}_0 - \text{P}\ -\ \left(1 - \frac{p_S}{p_L}\alpha\right)c(L_L),\ \text{H}_0 - D_L\ +\ L_L)$$

$$+\ \Pi_4\ \ \text{V}(\text{W}_0 - \text{P}\ ,\ \text{H}_0)$$

$$+\ \Pi_5\ \ \text{V}(\text{W}_0 - \text{P}\ -\ (1-\alpha)\,c(L_S),\ \text{H}_0 - D_S\ +\ L_S)$$

$$+\ \Pi_6\ \ \text{V}(\text{W}_0 - \text{P}\ -\ \left(1 - \frac{p_S}{p_L}\alpha\right)c(L_S)\text{-},\ \text{H}_0 - D_S\ +\ L_S) \qquad (15)$$

First order condition writes:

$$E'(\alpha) = c(L_S)\,\Pi_2\ \ U_1(\text{W}_0 - \text{P}\ -\ (1-\alpha)\,c(L_S),\ \text{H}_0 - D_S\ +\ L_S)$$

$$-\ \left(\frac{p_S}{p_L}\right)c(L_S)\,\Pi_3\ \ U_1(\text{W}_0 - \text{P}\ -\ \left(1 - \frac{p_S}{p_L}\alpha\right)c(L_L),\ \text{H}_0 - D_L\ +\ L_L)$$

$$+\ \Pi_5\ \ c(L_S)\ V_1(\text{W}_0 - \text{P}\ -\ (1-\alpha)\,c(L_S),\ \text{H}_0 - D_S\ +\ L_S)$$

$$+\ \Pi_6\ \ \left(\frac{p_S}{p_L}\right)c(L_S)\ V_1\ (\text{W}_0 - \text{P}\ -\ \left(1 - \frac{p_S}{p_L}\alpha\right)c(L_S)\text{-},\ \text{H}_0 - D_S\ +\ L_S) \qquad (16)$$

[6] see proof developped in Appendix 3.

Proposition 5 The Arrow theorem only holds when one of the following conditions is true:

- $U_1 = V_1$
- $U_{12} = V_{12} = 0$
- $U_{12} < 0$ and $V_{12} > 0$

Proof. See appendix 4.

Proposition 5 shows the importance of the knowledge of the way in which the occurrence of the uninsurable risk affects the marginal utility of wealth to determine the optimal insurance policy.

5 CONCLUSION

In this article, we view health insurance as a combined hedge against the two consequences of falling ill: treatment expenditures and loss in health status. We use a bivariate utility function depending both on the wealth and health. We don't impose any restriction on the sign of the variation of the marginal utility of wealth with respect to the uninsurable loss. We have shown that it is difficult to obtain classical results of insurance theory in the health case. We have concluded that the determinants of the demand for health insurance are not only the correlation between the health and uninsurable risks and the variation of the marginal utility of wealth with respect to the health status but also the way in which the occurrence of the uninsurable risk affects the marginal utility of wealth.

Appendix

Appendix 1

The optimal level of insurance is solution of the following program:

$$\max_{\alpha} E(\alpha) = \Pi_1 \ U(W_0 - P(\alpha), H_0) + \Pi_2 \ U(W_0 - P(\alpha) - (1-\alpha)\ c(L), H_0 - D + L)$$
$$+ \Pi_3 \ V(W_0 - P(\alpha), H_0) + \Pi_4 \ V(W_0 - P(\alpha) - (1-\alpha)\ c(L), H_0 - D + L) \qquad (1)$$

Where: $P(\alpha) = (1 + m)\ p_1\ \alpha\ c(L)$ denote the premium for insurance.

Replacing the probabilities by their expressions, equation (1) writes as follows:

$$E(\alpha) = \{ p_1 [(1 - p_2)\, \mathrm{U}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)$$
$$+ \; p_2\; \mathrm{V}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)]$$
$$+ (1 - p_1)[(1 - p_2)\, \mathrm{U}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0) + p_2\; \mathrm{V}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0)]\}$$
$$+ \{ p_1 (p_{2/1} - p_2)\; [\mathrm{U}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0) - \mathrm{V}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0) - \mathrm{U}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\, c(L), \mathrm{H}_0 - D + L)$$
$$+ \mathrm{V}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)]\} \quad (2)$$

Or in a more compact form:

$$E(\alpha) = \{ p_1\; \mathrm{Z}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L) + (1 - p_1)\, \mathrm{Z}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0)\}$$
$$+\{ p_1 (p_{2/1} - p_2)[\mathrm{F}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0) - \mathrm{F}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)]\} \quad (3)$$

Where:

$$\mathrm{Z}(\mathrm{W}, \mathrm{H}) = (1 - p_2)\, \mathrm{U}(\mathrm{W}, \mathrm{H}) + p_2\; \mathrm{V}(\mathrm{W}, \mathrm{H})$$

$$\mathrm{A}(p_{2/1}, \mathrm{W}, \mathrm{H}) = p_1 (p_{2/1} - p_2)\; [\mathrm{U}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0) - \mathrm{V}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0)$$
$$- \mathrm{U}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)$$
$$+ \mathrm{V}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)]$$

$\mathrm{F}(\mathrm{W},\mathrm{H}) = \mathrm{U}(\mathrm{W}, \mathrm{H}) - \mathrm{V}(\mathrm{W}, \mathrm{H})$.

Consequently, the optimal level of insurance is solution of the following program:

$$\alpha^* = \arg \max_{\alpha}\; E(\alpha) = \{ p_1\; \mathrm{Z}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)$$
$$+ (1 - p_1)\, \mathrm{Z}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0)\}$$
$$+ \{ p_1 (p_{2/1} - p_2)[\mathrm{F}(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0) - \mathrm{F}(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)]\} \quad (4)$$

First order condition writes:

$$E'(\alpha) = p_1\; c(L)\; [Z_1(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)(1 - (1+m)\; p_1)$$
$$- (1 - p_1)(1+m)\; Z_1(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0)]$$
$$+ \; p_1 (p_{2/1} - p_2)\{(1+m)\; c(L)\; p_1\; [\mathrm{F}_1(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L) - \mathrm{F}_1(\mathrm{W}_0 - P(\alpha), \mathrm{H}_0)]$$
$$- c(L)\; \mathrm{F}_1(\mathrm{W}_0 - P(\alpha) - (1-\alpha)\; c(L), \mathrm{H}_0 - D + L)\} \quad (5)$$

Equation (5) can be written as follows:

$$E'(\alpha) = p_1 \; c(L) \; [Z_1(W_2, H_0 - D + L)(1 - (1+m)\, p_1) - (1 - p_1)(1+m) \; Z_1(W_1, H_0)]$$

$$+ \; p_1 (p_{2/1} - p_2) \{(1+m) \; c(L) \; p_1 \; [F_1(W_2, H_0 - D + L) - F_1(W_1, H_0)] - c(L) \; F_1(W_2, H_0 - D + L)\} \quad (6)$$

Where: $W_1 = W_0 - P(\alpha)$ et $W_2 = W_0 - P(\alpha) - (1-\alpha)\; c(L)$

Second order condition writes:

$$E''(\alpha) = \{\Pi_1 \,(1+m)^2 \,(p_1)^2 \,(c(L))^2 \; U_{11}(W_0 - P(\alpha), H_0)$$

$$+ \; \Pi_2 \,(c(L))^2 \,[1-(1+m)p_1]^2 \; U_{11}(W_0 - P(\alpha) - (1-\alpha)\; c(L), H_0 - D + L)$$

$$+ \; \Pi_3 \,(1+m)^2 \,(p_1)^2 \,(c(L))^2 \; V_{11}(W_0 - P(\alpha), H_0)$$

$$+ \; \Pi_4 \,(c(L))^2 \,[1-(1+m)p_1]^2 \; V_{11}(W_0 - P(\alpha) - (1-\alpha)\; c(L), H_0 - D + L)\} < 0 \quad (7)$$

The second order condition is satisfied because $E''(\alpha)$ is always negative.

Appendix 2

Appendix 2.1

The strict concavity of $E(\alpha)$ ensures that full coverage is optimal ($\alpha^* = 1$) if and only if the first-order condition for maximizing $E(\alpha)$ evaluated at full coverage is nonnegative:

$\alpha^* = 1 \Leftrightarrow E'(1) \geq 0.$

$E'(1)$ writes:

$$E'(1) = -p_1 (p_{2/1} - p_2) \; c(L) \; [U_1(W_0 - p_1 \, c(L), H_0) - V_1(W_0 - p_1 \, c(L), H_0)] \quad (9)$$

If $p_{2/1} = p_2$ then $\alpha^* < 1$

If $p_{2/1} > p_2$ then $\alpha^* \begin{cases} = 1 \text{ if } U_1 < V_1 \\ < 1 \text{ if } U_1 > V_1 \end{cases}$

If $p_{2/1} < p_2$ then $\alpha^* \begin{cases} = 1 \text{ if } U_1 > V_1 \\ < 1 \text{ if } U_1 < V_1 \end{cases}$

Appendix 2.2

The strict concavity of $E(\alpha)$ ensures that full coverage is optimal ($\alpha^* = 1$) if and only if the first-order condition for maximizing $E(\alpha)$ evaluated at full coverage is nonnegative: $\alpha^* = 1 \Leftrightarrow E'(1) \geq 0$.

$E'(1)$ writes:

$$E'(1) = (1 - p_1)\, p_1\, c(L)\, [\, Z_1(W_0 - p_1\, c(L), H_0 - D + L) - Z_1(W_0 - p_1\, c(L), H_0)\,]$$
$$- p_1 (p_{2/1} - p_2)\, c(L)\, [(1 - p_1)\, F_1(W_0 - p_1\, c(L), H_0 - D + L) + p_1\, F_1(W_0 - p_1\, c(L), H_0)] \quad (10)$$

However, the sign of U_{12} is equal to the sign of Z_{12}.

From the equation (10), we obtain:

If $p_{2/1} = p_2$ then $\alpha^* \begin{cases} = 1 \text{ if } U_{12} \leq 0 \\ < 1 \text{ if } U_{12} > 0 \end{cases}$

If $p_{2/1} > p_2$ then $\alpha^* \begin{cases} = 1 \text{ if } U_{12} \leq 0 \text{ and } U_1 < V_1 \\ < 1 \text{ if } U_{12} > 0 \text{ and } U_1 > V_1 \end{cases}$

If $p_{2/1} < p_2$ then $\alpha^* \begin{cases} = 1 \text{ if } U_{12} \leq 0 \text{ and } U_1 > V_1 \\ < 1 \text{ if } U_{12} > 0 \text{ and } U_1 < V_1 \end{cases}$

Appendix 2.3

The strict concavity of $E(\alpha)$ ensures that full coverage is optimal ($\alpha^* = 1$) if and only if the first-order condition for maximizing $E(\alpha)$ evaluated at full coverage is nonnegative: $\alpha^* = 1 \Leftrightarrow E'(1) \geq 0$.

$E'(1)$ writes:

$$E'(1) = -[\, m\, Z_1(W_0 - p_1\, c(L), H_0) + p_1 (p_{2/1} - p_2)\, c(L)\, F_1(W_0 - p_1\, c(L), H_0)] \quad (11)$$

From the equation (10), we obtain:

If $p_{2/1} = p_2$ then $\alpha^* < 1$

If $p_{2/1} > p_2$ then $\alpha^* \begin{cases} = 1 \text{ is possible if } U_1 < V_1 \\ < 1 \text{ if } U_1 > V_1 \end{cases}$

If $p_{2/1} < p_2$ then $\alpha^* \begin{cases} = 1 \text{ is possible if } U_1 > V_1 \\ < 1 \text{ if } U_1 < V_1 \end{cases}$

Appendix 2.4

The strict concavity of $E(\alpha)$ ensures that full coverage is optimal ($\alpha^* = 1$) if and only if the first-order condition for maximizing $E(\alpha)$ evaluated at full coverage is nonnegative: $\alpha^* = 1 \Leftrightarrow E'(1) \geq 0$.

$E'(1)$ writes:

$$E'(1) = (1 - p_1)(1+m)\,[Z_1(W_2, H_0 - D + L) - Z_1(W_1, H_0)] - m\; Z_1(W_2, H_0 - D + L)$$
$$+\; p_1(p_{2/1} - p_2)\; c(L)\{(1+m)\; p_1\; [F_1(W_2, H_0 - D + L) - F_1(W_1, H_0)]$$
$$- F_1(W_2, H_0 - D + L)\} \qquad (12)$$

However, the sign of U_{12} is equal to the sign of Z_{12}.

From the equation (8), we obtain:

If $p_{2/1} = p_2$ then $\alpha^* \begin{cases} = 1 \text{ is possible if } U_{12} < 0 \\ < 1 \text{ if } U_{12} \geq 0 \end{cases}$

If $p_{2/1} > p_2$ then $\alpha^* \begin{cases} = 1 \text{ is possible if } U_{12} < 0, U_1 < V_1 \text{ and } U_{12} < V_{12} \\ < 1 \text{ if } U_{12} \geq 0, U_1 > V_1 \text{ and } U_{12} > V_{12} \end{cases}$

If $p_{2/1} < p_2$ then $\alpha^* \begin{cases} = 1 \text{ is possible if } U_{12} \geq 0, U_1 > V_1 \text{ and } U_{12} > V_{12} \\ < 1 \text{ if } U_{12} < 0, U_1 < V_1 \text{ and } U_{12} < V_{12} \end{cases}$

Appendix 3

We consider two insurance policies defined as follows:

- Franchise contract

$$F = \begin{cases} c(L_L) - c(L_S) & \text{in states 3 and 6} \\ 0 & \text{otherwise} \end{cases}$$

F is a deductible policy with deductible level $c(L_S)$.

- Coinsurance contract

$$C = \begin{cases} \alpha \ c(L_S) & \text{in states 2 and 5} \\ \alpha \, c(L_S) \ + \ (1 - f(\alpha)) \, [c(L_L) - c(L_S)] & \text{in states 3 and 6} \\ 0 & \text{in states 1 and 4} \end{cases}$$

C is a proportional basis policy.

In the case of a coinsurance contract, the profit of the insurance company is defined as follows:

State 1 $\equiv P$ with Π_1

State 2 $\equiv P - \alpha \ c(L_S)$ with Π_2

State 3 $\equiv P - [\alpha \ c(L_S) + (1 - f(\alpha)) \, [c(L_L) - c(L_S)]]$ with Π_3

State 4 $\equiv P$ with Π_4

State 5 $\equiv P - \alpha \ c(L_S)$ with Π_5

State 6 $\equiv P - [\alpha \ c(L_S) + (1 - f(\alpha)) \, [c(L_L) - c(L_S)]]$ with Π_6

$\mathrm{E}(\pi) = P - [\alpha \ c(L_S) \, (p_S + p_L) + (1 - f(\alpha)) \, [c(L_L) - c(L_S)] \, p_L]$

However, $\mathrm{E}(\pi) = 0$

Thus $P = \alpha \ c(L_S) \, (p_S + p_L) + (1 - f(\alpha)) \, [c(L_L) - c(L_S)] \, p_L$ (13)

In the case of a franchise contract, the profit of the insurance company is defined as follows:

State 1 $\equiv P$ with Π_1

State 2 $\equiv P$ with Π_2

State 3 $\equiv P - [c(L_L) - c(L_S)]$ with Π_3

State 4 $\equiv P$ with Π_4

State 5 $\equiv P$ with Π_5

State 6 $\equiv P - [c(L_L) - c(L_S)]$ with Π_6

$\mathrm{E}(\pi) = P - [c(L_L) - c(L_S)] \, p_L$

However, $\mathrm{E}(\pi) = 0$ thus $P = [c(L_L) - c(L_S)] \, p_L$ (14)

However, coinsurance and franchise contracts are available at the same premium. We obtain:

$$P = \alpha\, c(L_S)\,(p_S + p_L) + (1 - f(\alpha))\,[c(L_L) - c(L_S)]\, p_L = [c(L_L) - c(L_S)]\, p_L .$$

with $f(\alpha) = \left(\dfrac{\alpha\, c(L_S)}{c(L_L) - c(L_L)}\right)\left[1 + \dfrac{p_L}{p_S}\right]$

Appendix 4

The Arrow theorem holds only if $V'(0) \leq 0$.

$$V'(0) = c(L_S)\, p_S\, [U_1(W_0 - P - c(L_S), H_0 - D_S + L_S) - U_1(W_0 - P - c(L_S), H_0 - D_L + L_L)]$$

$$- [V_1(W_0 - P - c(L_S), H_0 - D_S + L_S) - V_1(W_0 - P - c(L_S), H_0 - D_L + L_L)]\nu \qquad (17)$$

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