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Nonlinear Stock Prices Adjustment in the G7 Countries

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Abstract

This paper aims to modeling stock prices adjustment dynamics toward their fundamentals. We used the class of Switching Transition Error Correction Models (STECM) and we showed that stock prices deviations toward fundamentals could be characterized by nonlinear adjustment process with mean reversion. First, according to Anderson (1997), De Grauwe and Grimaldi (2005) and Boswijk *et al.*(2006), we justify these nonlinearities by the presence of heterogeneous transaction costs, behavioural heterogeneity and the interaction between shareholders expectations. After, we present STECM specification. We apply this model to describe the G7 indexes adjustment dynamics toward their fundamentals. We showed that the G7 stock indexes adjustment is smooth and nonlinearly mean-reverting and that the convergence speeds vary according to the disequilibrium extent. Finally, using two indicators proposed by Peel and Taylor (2000), we determine phases of under- and overvaluation of stock prices and measure intensity of stock prices adjustment strengths.

Keywords : Stock Prices, Heterogeneous Transaction Costs, Nonlinear Adjustment and STECM.

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I. Introduction

Lately, stock markets have experienced a development without precedent. At the end of 1987, Dow Jones displayed a rise of 250% in relation to its low level of 1982. It was situated around 3600 points in 1994, it has more that tripled in five years to clear the rod of 11000 points in 1999 and it passed the rod of 11700 in the beginning of 2000. German, British, Spanish, French and Italian stock markets have also at least doubled between 1994 and 1999. Nevertheless, this stock market efflorescence has sometimes been accompanied by a strong period of falls. Thus, history is marked by many episodes of collapse of stock prices. For example, Dow Jones lost 22,6% in one night. S&P500 has more lately recorded a fall of 40% in January 2003, while being located around 910 points.

According to rational explanation, the rapid variations of stock prices reflect changes occurring in fundamentals. But, Campbell and Shiller (2001) suggested that changes in fundamental factors are not enough to explain changes in stock indexes. Summers (1986) showed that "irrational" fads could create some persistent deviations between courses and their fundamentals. Empirically, many authors such as Poterba and Summers (1988), Fama and French (1988), Cecchetti *et al.* (1990) and Manzan (2003) showed that stock prices are mean-reverting. However, it was not often clear whether this finding is due to the fact that price diverges really from its intrinsic value, or rather because of a misspecified fundamental process that does not account for the switching dynamics of dividends. For example, Cecchetti *et al.*(1990) suggested that dividends are characterised by two regimes : high growth rate of dividends and negative growth of dividends that drive stock prices between an "expansionary" state and "contractionary" one.

In order to understand the logic that generated stock price periods of prosperity and the explanatory factors of the possible "dysfunctions" of stock markets, we raised several questions : Why do prices deviate so much from their fundamental ? How can we explain a strong undervaluation of stock markets ? Is there any fundamental factors that can justify this rise of stock prices ? Is this rise the result of fundamental features or rather the reflection of an excessive enthusiasm ?

Certainly, these questions are not new, but they are reconsidered because of the progress recorded concerning stock prices modeling. Shiller (1981) and LeRoy and Porter (1981) suggested that stock markets exhibit excess volatility. Otherwise, Hirshleifer (2001) and Barberis and Thaler (2003) proposed a behavioral explanation while describing shareholders behavior at short and long-term. Authors suggested that, at short-term, investors underreact news about economic fundamentals and they may slowly adjust their valuations to incorporate these news. However, as in Boswijk *et al.* (2006), investors could drive prices too far from what is warranted by fundamental news.

Such empirical evidence was explained differently by several models. Barberis *et al.*(1998) developped a model with two regimes : trend regime and mean-reverting regime. In the first regime, investors are too conservative while they overreact to a stream of positive fundamental news in the second one assuming that it is a sign of a new regime of higher growth. Daniel et *al.*(1998) explained aggregate markets dynamics by the fact that investors are sometimes overconfident and overestimate then the precision of private information. The confirmation or not of private signal by public information could considerably affect shareholders reactions and stock prices adjsutment dynamics. Boswijk *et al.*(2006) considered also a model with behavioural heterogeneity. Authors showed that behavioural heterogeneity affect significantly prices dynamic, while presenting two regimes : A “mean-reversion regime” and a “trend following regime” and respectively two types of investors : fundamentalists and trend followers. The coexistence of these investors explains markets fluctuations over time. Indeed, the first regime is dominated by fundamentalists and prices should move toward their fundamentals. However, in the second regime, shareholders would expect positive stock returns and market is then dominated by trend followers. Empirically, studying S&P500 adjustment dynamic, authors showed that before the 90s, the trend regime is activated only occasionaly and it does not persist for many years while after the 90s this regime persisted for long time.

This paper investigates stock prices misalignment and explains stock markets fluctuations, while studying stock courses adjustment toward fundamentals. In particular, the following questions are raised : Does a gap exist between financial asset price and its intrinsic value that is not a white noise? In the affirmative, does the price fit continually or not to its equilibrium value ? Is price adjustment symmetrical or asymmetric ? Linear or nonlinear ?

These questions and the exploration of stock courses deviations dynamics are not yet very developed probably because of the difficulties associated with fundamental value estimation and the complexity of stock prices deviations modeling. In this paper, it is proposed, on the one hand, an estimate of stock prices fundamental value using Dividend Discount Model (DDM) for which expected variables are replaced by the deterministic part of Smooth Transition Autoregressive Models (STAR). On the other hand, stock prices adjustment dynamics toward these estimated fundamental values are being investigated and stock prices deviations dynamics are being modelled on a nonlinear framework using STECM.

Thus, the originality of this paper is triple. First, the hypothesis of stock prices adjustment is studied not only for the American market as in the most previous studies, but also for an original field of application : The group of G7 countries. Secondly, besides the evaluation of fundamental value using nonlinear techniques, we studied the stock prices adjustment toward fundamentals, we measured this adjustment speed while evaluating transition functions and we determined periods of under and over-valuation and the adjustment strengths using two indicators proposed by Peel and Taylor (2000), but never applied on stock markets. Third, we kept a hypothesis often forgotten in the literature relative to stock prices adjustment : The effect of interdependence and contagion between stock markets on prices adjustment.

Otherwise, to justify the nonlinearity characterizing stock prices adjustment dynamics, we retained, according to Anderson (1997), De Grauwe and Grimaldi (2005) and Boswijk *et al.* (2006), hypotheses of heterogeneous transaction costs, behavioural heterogeneity and interaction between shareholders expectation. This implied implicitly rejection of instantaneous prices adjustment and efficiency hypotheses for which information whole is instantaneously and completely integrated in price and new information is strictly unforeseeable, completely uncertain and absolutely not correlated with the old information (i.e. Fama (1965)). Indeed, prices adjustment can not, in practice, be immediate since some delay is sometimes necessary to integrate correctly the new information to the course (i.e. Obstfeld and Taylor (1997)).

Moreover, transaction costs appear as a limit to arbitrage and efficiency and they can have many considerable repercussions, for example, when investors expected that potential gain is lower than assumed costs. Thus, transaction costs imply discontinuous prices adjustment and persistent deviations of stock prices from fundamentals. Deviations from equilibrium last for a very long time, which suggests that they may be governed by nonlinear adjustment process that is mean-reverting with an adjustment speed that increases directly with the extend of the deviations from equilibrium (i.e. Manzan (2005) and Boswijk *et al.*(2006), Jawadi (2006)). STECM is then appropriate to describe stock prices dynamics in presence of heterogeneous transaction costs.

The rest of this article is organized as follows. Nonlinearities characterizing stock prices adjustment are economically justified in section II. Section III presents the methodology and describes the empirical results. Conclusions are summarised in section IV.

II. Stock Prices Adjustment Dynamics within Market Frictions

2-1 Stock Prices Adjustment and Transaction Costs

Stock prices independence hypothesis has been tested against its linear or nonlinear dependence alternative and it was often rejected (i.e. Gallagher and Taylor (2001), Schaller and Van Norden (2002), Psaradakis et *al.*(2004), Boswijk et *al.*(2005)). This result is assimilated to a rejection of efficiency and instantaneous prices adjustment hypotheses. Moreover, authors suggested that fundamentals do not show the required persistence that could explain stock prices evolution. Thus, prices dynamic is characterized by two regimes : persistent regime and quick mean reversion regime. In the first regime, stock prices deviations are persistent and contribute to drive prices away from their fundamental values. Instead, in the second regime, prices are strongly mean-reverting. Boswijk *et al.*(2006) define also two regimes : “fundamentalists regime” for which agents believe in mean reversion of stock prices toward the benchmark fundamental value and a “chartist or trend following regime” for which investors expect deviations from fundamental to this trend.

Authors justified these findings by presence of market frictions such as transaction costs, mimetic behaviour and noise traders. Market frictions induce some delays, slowness and intertias effects in stock prices adjustment dynamics, rejecting then linear and instantaneous adjustment hypotheses of financial asset prices (i.e. Anderson (1997), Michael et *al.*(1997) and Manzan (2005)). Indeed, linear modeling forces stock prices adjustment to be linear and symmetrical. However, in presence of transaction costs and heterogeneous expectations (De Grauwe and Grimaldi (2005)), prices adjustment can not be neither linear, nor continuous.

Dumas (1992) showed that, in presence of proportional transaction costs, exhange and arbitrage are shown to be persistent and that prices adjustment is nonlinearly mean-reverting with a convergence speed that depends on desequilibrium size. This persistence is due the fact that transaction costs create two zones : A first region of no trade called also "transaction band" and a region of exchange. Stock prices adjustment dynamic is different owing to the fact that it is within transaction band or not. Indeed, within no-trade zone, arbitrage and adjustment are not active because expected returns are lower than transaction costs and prices could deviate from their fundamental values. Deviations would diverge, would be left uncorrected as long as they are small relative to transaction costs and they would be near-unit root in this zone. Consequently, within this band, no arbitrage takes place and adjustment process is divergent so that prices spend most of the time away from fundamental and deviations could last a very long time.

Disequilibrium is corrected only when prices deviations and arbitrage opportunities are large enough to compensate for transaction costs, notably in the second zone of exchange, arbitrage and adjustment become possible since adjustment opportunities are enough to pay transaction costs. Thus, prices deviations are white noise in this region as prices could join their fundamentals with a convergence speed that varies directely with the size of deviations from equilibrium. Thus, this implies that, in presence of transaction costs, adjustment prices dynamics toward fundamentals are rather nonlinear. It indicates also that prices deviations are persistent and that are shown not to follow a random walk but a nonlinear process that is mean-reverting with an adjustment speed that is increasing according to the desequilibrium size. So, as in Dumas's analysis, stock prices adjustment process is nonlinear in which the larger the prices deviations from fundamental, the stronger the tendency to return to equilibrium.

Anderson (1997) showed also that this adjustment process can be parsimoniously reproduced using nonlinear error correction models (NECM)[1]. In particular, Anderson (1997) proved that nonlinear models that capture adjustment in presence of market frictions are STAR-ECM or STECM, originally proposed by Anderson and Teräsvirta (1992) and developed more recently by Van Dijk *et al.* (2002). Anderson (1997) explained nonlinear adjustment arising because of presence of a portfolio adjustment that is an "on-off" process which occurs or not in function of disequilibrium size. In addition, he suggested the smoothness of adjustment since the presence of heterogenous transaction costs that define the strength of error correction mechanism according to the costs distribution and the desequilibrium extend. Thus, this modeling could represent an aggregate adjustment process that is strong for an important desequilibrium size and weak as the market approaches equilibrium and price nears its fundamental value.

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[1] Anderson (1997) focused on the study of yield movements in the US Tresaury Bill Market.

Formally, Anderson (1997) defined three types of adjustment dynamics. In a first step, market is assumed to be frictionless and transaction costs are null. Thus, adjustment dynamic is rather linear, continuous, symmetrical and then described by the following linear model :

$$\Delta rt = -\rho \left(S_{i,t-1} - \eta_{i,t-1}\right) + \Phi(L)\Delta r_{t-1} + \varepsilon_t \qquad (1)$$

Where : $r_{i,t} = \sum_{t=1}^{T}\left(P_{i,t} - F_{i,t}\right), \quad \forall \; t = 1, 2, ..., T.$

$P_{i,t}$ is price of asset (i) and $F_{i,t}$ is its fundamental value. $\Phi(L)$ is an operator of delays and ε_t is a white noise. $S_{i,t}$ is observed deviation between asset price and its fundamental value, while $\eta_{i,t}$ is the minimal prices differential expected by investors. For this representation, adjustment speed is limited to be constant and it is measured by the adjustment term (ρ).

Thus, within absence of transaction costs, investors have to profit from all prices deviations. They still compare $S_{i,t}$ to $\eta_{i,t}$. So, if $S_{i,t} = \eta_{i,t}$, arbitrage opportunities are not important. But, if $S_{i,t} > \eta_{i,t}$ (resp. $S_{i,t} < \eta_{i,t}$), asset (i) is over-valuated (resp. under-valuated), then some investors could continue to detain this asset, while others would profit immediately. Therefore, arbitrage would be active and adjustment process bringing course toward equilibrium is linear, continuous and with a constant speed of adjustment (equation (1)).

However, markets are not frictionless in practice. Transaction costs exist and can reduce arbitrage opportunities. Indeed, if one unit of an asset (i) is exchanged, only $\frac{1}{(1+\tau)}$ units are actually transferred[2]. Thus, arbitrage is not always active and prices adjustment may be neither continuous nor with a constant speed. Some rigidities could then be induced in stock prices adjustment dynamics notably when transaction costs are higher than expected returns. Therefore, investors reaction will be rather dependant to the importance of deviation between assumed transaction costs and net yields $[(S_{i,t} - \mu_{i,t}) - \tau]$.

Indeed, if $S_{i,t} - \mu_{i,t} > \tau$, investors would arise their detention of the asset (i), while for $S_{i,t} - \mu_{i,t} < -\tau$, they have to reduce their horizon of investment. Investors would be undecided for $-\tau < S_{i,t} - \mu_{i,t} < \tau$. Absence of arbitrage in this zone could then generate a no-trade that is centered around equilibrium $[(S_{i,t} - \mu_{i,t}) = 0]$ and its size is determined by transaction costs level. Consequentely, presence of transaction costs induce a discontinuous stock prices adjustment dynamics for which arbitrage is not active in all zones and adjustment speed is not any more constant.

Formally, Anderson (1997) showed that, in presence of homogeneous transaction costs, representation (1) is not appropriated anymore to reproduce prices adjustment dynamics as this model can not replicate discontinuous arbitrage and adjustment that its speed is variable. He extended then

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[2] τ is fraction of transaction costs.

this model to the nonlinear framework. He got the following nonlinear specification that reproduces adjustment dynamic not only in no-trade zone but also when arbitrage is strong :

$$\begin{aligned}&\Delta r_t = -\rho\,[F\left|S_{i,t-1}-\mu_{i,t-1}\right| \times (S_{i,t-1}-\mu_{i,t-1})] + \Theta(L)\Delta r_{t-1} + \varepsilon_t \\ &\text{Where : } F[\left|S_{i,t-1}-\mu_{i,t-1}\right|] = 1 \text{ si } \left|S_{i,t-1}-\mu_{i,t-1}\right| > \tau \\ &\qquad\qquad\qquad\quad = 0 \text{ si } \left|S_{i,t-1}-\mu_{i,t-1}\right| \le \tau.\end{aligned} \tag{2}$$

This representation is adequate to reproduce stock prices adjustment in presence of homogeneous transaction costs. Nevertheless, in practice, transaction costs are not usually homogeneous but also heterogeneous, as investors do not necessarly have the same expenses and stock markets do not often apply the same costs. However, presence of individual transaction costs could generate different thresholds that their aggregation on only one threshold could not reproduce accurately stock prices adjustment dynamic. Then, specification (2) is not anymore adequate to study stock prices adjustment toward fundamental. Therefore, Anderson (1997) extended representation (2) while introducing individual thresholds.

This extension is theoretically founded as in practice investors have specific thresholds because they have different individual transaction costs associated to their portofolios. These thresholds can be “smeared” when they are aggregated and could not reproduce accurately the behaviour of all regimes (i.e. Jawadi (2006) and Jawadi and Chaouachi (2006)). Therefore, an adjustment process integrating heterogeneous transaction costs and allowing adjustment to be smooth and gradual rather than brutal would be an appropriate adjustment model to reproduce stock prices adjustment.

Let $\tau_{i,j}$ be transaction costs threshold associated to purchase of asset (i) by investor j. Rational Investor reacts after a price deviation only if $(S_{i,t} - \mu_{i,t}) > \tau_{i,j}$ or $(S_{i,t} - \mu_{i,t}) < -\tau_{i,j}$. Following Anderson (1997), this means that, while noting τ the sum of transaction costs associated to purchase of asset (i), the response of investor j to a price deviation is proportional to the intensity of the attraction that exercises $\tau_{i,j}$ (specific transaction costs) on τ (total of transaction costs).

Therefore, if we note $H(\tau)$ the cumulative density function of these expenses, the function $H(|S_{i,t} - \mu_{i,t}|)$ measured the proportion of assets for which investors find beneficial to answer to prices deviations. Operators answer is then measured by $H(|S_{i,t} - \mu_{i,t}|)$ and prices adjustment is closely bound to investors reaction. Formally, integrating heterogeneous transaction costs spreading to equation (2) implies the following adjustment process that could reproduce this adjustment dynamic :

$$\begin{aligned}&\Delta r_t = -\rho\,[H(\left|S_{i,t-1}-\mu_{i,t-1}\right|) \times (S_{i,t-1}-\mu_{i,t-1})] + \Theta(L)\Delta r_{t-1} + \varepsilon_t \\ &\text{Where : } H(.) \text{ is cumulative density function of} \\ &\qquad\quad\ \text{threshold transaction costs.}\end{aligned} \tag{3}$$

According to Anderson (1997), the transition function is defined as follows :

$$H_s(\tau) = 1 - \exp\left[-\beta(\tau)^2\right],\ \beta > 0 \text{ et } \forall\, 0 \leq \tau < \infty$$
$$\text{Where : } \beta \text{ measures transition speed or steepness of investors impulse function.} \quad (4)$$

$H_s(\tau)$ is ranging between 0 and 1. $H_s(\tau)$ corresponds to exponential function but it constitues an explicit part of a theoritical model. Furthermore, the above nonlinear representation (equation (3)) can be assimilated and interpretated as a structural form from which would be drifted the nonlinear error correction processes and STECM.

Otherwise, it is important to note that, besides transaction costs, nonlinearity and slowness characterising stock prices adjustments can be explained by the coexistence and interaction between distinct shareholders (i.e. chartists, fundamentalists and noise traders), mimetic behaviour[3] and heterogeneous expectations[4] and information asymmetry. For example, if one group observes a private signal while the second one has to learn information only from public information, asymmetry information could imply heterogeneous expectations and induce slowness in prices adjustment as a public signal could be interpreted in different ways by investors[5].

Thus, in presence of investors with different sentiments, fundamental value is common knowledge but investors have heterogeneous beliefs about speed of stock prices mean reversion. Thus, if markets are overvalued, pessimistic investors (i.e. fundamentalists) would believe that this situation will soon be corrected, while optimistic agents (i.e. trend followers) believe that in the short-term the price trend will continue[6]. Furthermore, as in Shiller (2000), the sentiment of investors can vary significantly over time as investors can become more optimistic (resp. pessimistic) in response to significant stock prices increases (resp. decreases) and switch between different beliefs to change their investment strategies.

Consequently, we understand that behavioural heterogeneity, transaction costs and heterogeneous beliefs might play an important role in asset pricing and could explain stock markets dysfunctions and persistent deviations of stock prices from fundamental valuations.

Formally, previous studies focused on stock prices adjustment such as Manzan (2003), Boswijk, Hommes and Manzan (2005) showed that NLECM is appropriate to study stock prices adjustment dynamics within market frictions. Indeed, these processes allow adjustment to be asymmetric and define different regimes depending on whether course is far of its fundamental value or not. The following paragraph describes briefly the most results of these previous studies.

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[3] See Jawadi (2006) and Jawadi and Chaouachi (2006).

[4] For more details, see De Grauwe and Grimaldi (2005)) and Boswijk *et al.*(2006).

[5] See also Harris and Raviv (1993) and Hong and Stein (1999).

[6] Vissing and Jorensen (2003) suggested that at the beginning of 2000, 50% of investors supposed that stock market is overvalued, 25% believed that market was fairly valued and less that 10% thought that it was undervalued, implying that individual investors are often heterogeneous and that they have different bout stock market prospect .

2-2 Empirical Literature Review

Stock prices adjustment is not yet very developed and there is not any unanimous conclusion on prices adjustment nature. However, because of the important stock prices rising relative to fundamentals, several studies have recently focused on this subject. On the one hand, LeRoy and Porter (1981) and Shiller (1981) used DDM to estimate fundamental value and showed its smooth character. Shiller (1981) concluded also on "*volatility puzzle*" while showing that $\sigma(P_t) < \sigma(P^*_t)$ for S&P500 and Dow Jones[7]. On the other hand, Campbell and Shiller (1987)) used linear cointegration techniques to study relationships between prices and dividends, but these modeling procedures limit adjustment to be symmetrical and linear. Indeed, within market frictions, prices deviate often from fundamentals and their adjustment is rather asymmetrical, slow and discontinuous. Allen and Yang (2001) studied British stock price deviations over the period 1986-2000 and showed that 35% of prices deviations are not explained by fundamentals.

These differences are probably due to difficulties associated with fundamental value estimation. Indeed, a possible explanation is due to the fact that it is not usually easy to identify prices deviations that are not explained by fundamental. Fundamental value estimation is often restricted by some assumptions (i.e. discount rate, cash flows and expectations process), and no fundamental value modeling is chosen with unanimity. Thus, literature review is confronted with three main questions : Which discount rate is appropriate ? Which expectation process is it necessary to consider ? How can we measure expected future cash flows ? In practice, answers to these questions are always conditioned by some assumptions.

For example, cash flows are often measured by expected future dividends[8]. Dividends were estimated differently but often under rational expectations hypothesis. Gordon (1962) used linear combination of "normal" dividend and dividend growth. However, Gordon methodology has been criticized because using Mobile Average process to measure "normal" dividend can not reproduce dividends notably in periods of growth. Campbell and Shiller (1987) and West (1988) used an unaltered process to estimate dividends while showing stock prices volatility excess. Nevertheless, Froot and Obstfeld (1991) showed more recently that this hypothesis is restraining and they retained a random walk process with derive to estimate S&P500 dividends. Authors retained the following relation[9] : $\Delta d_t = \mu + \varepsilon_t$.

Authors retained bubble hypothesis to explain the S&P deviations, but they conclude while considering the alternative of threshold processes to reproduce dividends evolution to explain stock prices deviations[10]. Cecchetti, Lam and Mark (1990, 1993) and Bonomo and Garcia (1994) showed also that process generating dividends is nonlinear. They used a markov model to reproduce dividends

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[7] P^*_t is Shiller rational expost price.

[8] For more details, see Shiller (1981, 1989, 2000), Manzan (2005), Boswijk *et al.*(2006)).

[9] $d_t = \log(D_t)$.

[10] Culter, Poterba and Summers (1988) showed that the S&P500 *Dividend Yield* ratio (D_t/P_t) was around 4,8% since 1871, 4% since 1950 and 1,17% in 1999, indicating a change in dividends growth.

dynamic. Timermann (1994) justified this nonlinearity by the feed back exercised by stock prices on dividends as prices reflect information asymmetry that can affect dividends policies.

In a such context, Driffill et Sola (1998) rejected the bubbles model of Froot et Obstfeld (1991) and showed that a markov model with two regimes is appropriate to reproduce dividends evolution. Authors showed that dividends dynamics are characterized by the presence of several regimes reflecting stylised facts of 50, 60 and 70 years[11]. In the same context, Gutièrrez and Vazquez (2000) suggested the presence of a *feed back* between American stock prices and their dividends and authors showed that the following threshold ARMA process is adequate to reproduce dividends adjustment dynamics[12] :

$$d_t = \beta_0 + \beta_1 \pi_0 + \left(\beta_1 \alpha_i + \beta_2\right) d_{t-1} + \frac{\beta_1}{1 - \delta \beta_1 \left(1 + \alpha_i\right)} u_t + \varepsilon_t \qquad (5)$$

Where : $0 < \delta < 1$ *et* $\forall\, i = 1, 2.$

Nielsen and Olesen (2000) used also Hamilton process to estimate *Dividend Yield* ratio of Denmark stock indexes on the period 1927-1996 and showed the presence of two regimes (persistent and dynamic regime). Authors justify this persistence by the presence of cyclic component in dividends. Schaller and Van Norden (2000) used also the markov model to estimate dividends and showed presence of two regimes : slow and high dividends growth. More recently, Berdin et Hyde (2005) proposed to capture nonlinearity and cyclic behaviours characterising the relation between courses and dividends while using STAR models for eight countries : (Belgium, Canada, France, Germany, Ireland, Japan, United Kingdom and United States). Authors showed that relation between fundamental and returns is nonlinear and that returns reaction vis-à-vis fundamental depends on the regime or state of economy : phase of growth or recession.

These results showed overall that using threshold models is appropriate to reproduce dividends dynamics. Thus, according to Driffill and Sola (1998) and Berdin and Hyde (2005), we used STAR models to generate expected dividends and estimate fundamental value in order to study stock prices adjustment toward these fundamentals in a nonlinear framework[13].

In the literature, a few studies focused on adjustment stock prices in a nonlinear framework (i.e. Manzan (2005) and Boswijk et al.(2006)). Authors justified nonlinearity by transaction costs and behavioural heterogeneity and showed that S&P adjustment is rather asymmetrical and nonlinear and that STAR model is appropriate to reproduce its mean reversion. However, these studies concerned only the American stock market (S&P500). In addition, they are based on strong restricted hypotheses (i.e. constant risk-free and constant dividend growth). They assumed also that investors know perfectly fundamental value but they are unaware of

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[11] See Jawadi (2007), for more details on this model.

[12] δ désigne le facteur d'actualisation.

[13] STAR models are particularly adequate to reproduce nonlinearity and persistence characterising dividends dynamics.

prices deviations adjustment process. In what follows, we propose an alternative empirical study, using STECM. Furthermore, our empirical study concerns an original application field : the G7 countries, while reproducing interdependence between these stock markets.

Formally, stock prices adjustment hypothesis would be studied under the double angle of STAR processes and STECM. Using these nonlinear modeling techniques could help not only to reproduce dynamics of stock prices deviations in presence of market frictions, but also to provide an illustration of stock prices phases of under- and overvaluation over the recent years and to determine the speed of prices mean-reversion toward fundamentals. In particular, STAR models are useful to propose a new nonlinear fundamental value estimation[14]. While this value would then define a long-run relationship, STECM would be used characterize stock prices deviations toward equilibrium.

The next section presents the methodology and describes the empirical results.

III. Stock Prices Adjustment Modeling

We study, first, the fundamental value modeling and its estimation results, then we focused on stock prices adjustment results toward fundamentals.

3-1 Fundamental Value Modeling

In a perfect foresight world with absence of vote rights and under transversality condition, the Dividend Discount Model (DDM) of future cash flows[15] leads to the following fundamental value :

$$\tilde{F}_t = \frac{D_{t+1}}{(1+i_t)} + \frac{D_{t+2}}{(1+i_t)(1+i_{t+1})} + \frac{D_{t+3}}{(1+i_t)\,(1+i_{t+1})(1+i_{t+2})} + \ldots \tag{6}$$

Writing this empirical expression of fundamental value and subtracting it from (6), we got a recurrent equation which corresponds to Shiller ex post price :

$$\tilde{F}_{t+1} = \tilde{F}_t \left(1 + i_{0t}\right) - D_{t+1} \tag{7}$$

Where : i_{t+1} is required return at (t+1) and D_{t+1} is dividend distributed during the period [t, t+1][16].

Thus, we retained this formulation to estimate fundamental value under hypothesis of rational expectations, notably for estimating expected future dividends. The discount rate was defined as the sum of a constant risk premium and a time-varying risk-free interest rate of one month. Furthermore, an additional assumptions were introduced to estimate the risk premium. The discount rate was defined as the sum of risk-free rate and constant risk premium. This risk premium is estimated by the

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[14] The most previous studies were limited to linear fundamental value estimations (i.e. Manzan (2005) and Boswijk et al.(2006)).

[15] Several authors such as Shiller (1981,2000), Campbell and Shiller (1987), Manzan (2005) and Boswijk *et al.* (2006) assimilated cash flows to dividends.

[16] This equation is compatible with that resulted from Model of Lucas (1978). Indeed, while solving Euler equation that defines the equilibrium (see Prat (2007)) under perfect expectations, we got the following relations : $P_{t+1} = P_t \lambda_t \left(1 + i\right) - D_{t+1}$ and $P_{t+1} = P_t \lambda_t \left(1 + i_t\right) - D_{t+1}$. Thus, under absence of bubble assumption, this equation corresponds to our empirical fundamental value formulation (equation (8)). where : $\lambda_t = \frac{U'(C_t)}{U'(C_{t+1})}$ *and* $i_t = \lambda_t (1+i) - 1$, C_t is agent consumption at t, U'(C_t) is marginal utility of C_t, P_t is stock price, D_t is dividend paid in the period [t-1, t] and i_t is discount rate.

method of sweep. We fixed, first, an initial value (F_0) for the fundamental value (F_t). Then, the fundamental value is given by the forward resolution of the following relation :

$$F_{t+1} = F_t \left(1 + i_{0t} + \rho_0\right) - E_t\left(D_{t+1}\right) \tag{8}$$

Where : $E_t(.)$ is the conditional expectations upon available information at t, i_{0t} is risk-free rate and ρ_0 is constant risk premium.

Expected future dividends $E_t\left(D_{t+1}\right)$ is then replaced by the determinist part of estimated STAR model, while several values are given for F_0 and ρ_0. Optimal values for $\hat{F}_0$ and $\hat{\rho}_0$ are those that minimize the statistic Q measuring the squared sum of logarithmic deviations between asset price and its fundamental value (equation (9)).

$$Q = \sum_{t=1}^{n=T} \left(p_t - f_t\right)^2 \tag{9}$$

Where : p_t and f_t are respectively stock price and fundamental value in logarithm[17].

3-2 Fundamental Value Estimation Results

In practice, we estimated fundamental value (equation (8) for the G7 countries (Canada, France, Germany, Italy, Japan, United Kingdom (UK) and United States of America (USA)) while using monthly data over the period 1969-2005[18]. In order to get dividend series, we used Gross Indexes and Price Indexes that we obtained from Morgan Stanley Capital International[19]. In addition, monthly free-risk discount rates, defined by Monetary Market Rate (MMR), and CSA-industrial production series (Corrected of the seasonal variations) were obtained from the International Monetary Fund's International Financial Statistics. All data are expressed in local currencies. Otherwise, using monthly data provides us with a reasonably large sample to apply linearity tests.

In a first step, we checked the stationarity hypothesis before applying linearity tests. Thus, the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were applied to logarithmic stock prices series. Both ADF and PP tests rejected the null hypothesis and showed that the G7 stock prices are I(1). In addition, the G7 stock returns seem to have asymmetric distributions, to be characterized by a *leptokurtic* effect and are not normal[20]. This could be assimilated to a signs of nonlinearity characterizing prices dynamics.

We estimated, in a second step, the fundamental value based on equation (8), under rational expectations hypothesis, while replacing expected future dividends by STAR estimation[21] and determining optimal values for risk premium and initial fundamental value so that the square sum of

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[17] In practice, we swept F_0 around P_0 in the interval [P_0 –50% P_0, P_0 –50% P_0] and ρ_0 in the interval [1% 7%]

[18] Canada : 1969:12-2005:02, France : 1970:01-2004:10, Germany : 1969:12-2005:02, Italy : 1971:01-2005:02, Japan : 1969:12-2005:02, UK : 1969:12-2005:01 and USA : 1969:12-2005:02.

[19] Gross index is a course measure that takes into account dividends investment while price index excludes it. All courses are closing prices.

[20] Returns are defined as the prices logarithmic difference : $R_t = \log(P_t) - \log(P_{t-1})$.

[21] In the same context, Driffill and Sola (1998) used Markov Models to reproduce the S&P500 dividends evolution.

prices deviations toward fundamentals is minimised (equation (9)). Thus, first, dividends (for Germany, Italy and Japan) or dividends growth (for Canada, USA, France and UK) –depending on unit root tests results applied on dividends series - were estimated using STAR process. Secondly, after replacing expected dividend by deterministic part of estimated STAR (respectively equations (10) and (11)), an initial fundamental value was fixed to generate a fundamental value set, using the recurrent expression (equation (8)). We fixed, in particular, $F_0 = P_0$, then, we swept F_0 and ρ_0 respectively in the intervals $I_1 = [P_0\text{-}50\%, P_0\text{+}50\%]$ and $I_2 = [1\%, 7\%]$. Optimal values of F_0 and ρ_0 have to minimize Q.

$$D_t = \left(\alpha_0 + \alpha_1 D_{t-1} + \cdots + \alpha_p D_{t-p}\right) + \left(\beta_0 + \beta_1 D_{t-1} + \cdots + \beta_p D_{t-p}\right) \times F\left(D_{t-d}, \gamma, c\right) + \varepsilon_t \tag{10}$$

$$\Delta D_t = \left(\alpha_0 + \alpha_1 \Delta D_{t-1} + \cdots + \alpha_p \Delta D_{t-p}\right) + \left(\beta_0 + \beta_1 \Delta D_{t-1} + \cdots + \beta_p \Delta D_{t-p}\right) \times F\left(\Delta D_{t-d}, \gamma, c\right) + \nu_t \tag{11}$$

Thus, this specification for dividends or dividends growth[22] allows for two regimes associated with the extreme value of the transition function (F(.) = 0 and F(.) = 1) but also it allows for a "continuum" of regimes that are associated with a different values of F.

Empirically, we estimated the selected STAR models on dividends for Germany, Italy and Japan and on dividends growth for Canada, USA, France and UK, by the Nonlinear Squares (NLS) Method (equations (10) and (11))[23]. Thus, STAR modeling for dividend series showed that dividend adjustment dynamics are nonlinear in all considered countries[24]. Specification tests showed that dividends dynamics can be reproduced while using LSTAR processes for Germany and USA and ESTAR models for Canada, France, UK, Italy and Japan. STAR estimation results are presented briefly in the first appendix, whereas a more detailed presentation of these results is in Jawadi (2007).

Overall, analysis of these results showed superiority of nonlinear model in relation to linear process and indicated the presence of two meaningful regimes in dividends dynamics. Transition between these regimes is rather slow as the estimated transition speed ($\hat{\gamma}$) is relatively weak for the most cases. In addition, misspecification tests that we used to evaluate the adequacy of the estimated STAR models[25] showed that residuals have the good statistical properties. Indeed, residuals sets seem

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[22] Both equations (10) and (11) correspond to STAR models that are developed by Anderson and Teräsvirta (1992) and Teräsvirta (1994). (α_0, α_1, …, α_p) and (β_0, β_1, …, β_p) are respectively Autoregressive coefficients in the first and the second regime and d is the delay parameter defining transition variable that determines transition between dividends regimes ($d \geq 1$). F(.) is the transition function, it is a continuous function that is bounded between 0 and 1. F(.) is either logistic ($F\left(D_{t-d}, \gamma, c\right) = \left(1 + \exp\left\{-\gamma \left(D_{t-d} - c\right)\right\}\right)^{-1}, \gamma > 0$) or exponential ($F\left(D_{t-d}, \gamma, c\right) = 1 - \exp\left\{-\gamma \left(D_{t-d} - c\right)^2\right\}, \gamma > 0$) and we have respectively Logistic STAR (LSTAR) model or Exponential STAR (ESTAR) model. γ measures the transition speed between regimes or the smoothness of the transition from one regime to the other and c is the threshold parameter.

[23] In practice, STAR modeling is done in many steps (i.e. Specification tests, linearity tests and estimation). For more details on this modeling, see Van Dijk *et al.*(2002) and Jawadi and Koubbaa (2007).

[24] We applied five Lagrange Multiplier (LM) tests that are presented rigorously in Van Dijk *et al.*(2002).

[25] See Eitrheim and Teräsvirta (1996), for more details on these tests.

to be near a white noise and reproducing expected dividends series by deterministic estimation of STAR models is therefore coherent and compatible with the rational expectations hypothesis that we retained to estimate the fundamental value.

Otherwise, estimating constant risk premium and initial fundamental value, we got the following results :

Table 1: Initial Fundamental Value and Constant Risk Premium Estimation Results

Series	**Germany**	**Canada**	**USA**	**France**	**UK**	**Italy**	**Japan**
$\hat{F}_0$	73.11	80.32	85.12	72.57	86.13	57.25	129.15
P_0	100	100	100	103.67	100	80.51	100
$\hat{\rho}_0$	3.8%	4.8%	5.4%	3.95%	4.29%	6.01%	6.58%

Note : $\hat{F}_0$ are respectively initial observed price and optimal initial fundamental value. $\hat{\rho}_0$ is optimal risk premium value.

This implies that excepting Japan, all indexes were over-valuated. Otherwise, risk premium values estimations are plausible as the average estimation for risk premium for the G7 is equal to 4,97%. Our estimations are also compatible with those of Mehra and Prescott (1985), Siegel (1992), Cochrane (1997), Clauss and Thomas (1999), Pastor and Stambaugh (2000) and Fama and French (2001). Indeed, Mehra and Prescott (1985) showed presence of an *Equity Premium Puzzle* as risk premium was too elevated in the past (≅ 6%). Siegel (1992) suggested that the estimated mean per annual risk premium over the period 1802-1990 is about 4,62%. Pastor and Stambaugh estimations showed that risk premium fluctuates since 1834 between 4% and 6% but that it would be reduced in the last decade. Otherwise, the more elevated risk premium value for USA can be explained, as in Reitz (1988), by a survivorship bias characterizing American investors.

In a third step, fundamental value was estimated while retaining the following empirical formulation :

$$\hat{F}_{t+1} = \hat{F}_t \left(1 + i_{0t} + \hat{\rho}_0\right) - \hat{D}^d_{t+1}$$

$$\text{Where : } \hat{D}^d_{t+1} \text{ is deterministic part of estimated STAR of future dividends (estimation of equation (10) or (11)).} \qquad (12)$$

The originality of this method in relation to those of Black *et al.*(2003), Manzan (2005) and Boswijk *et al.*(2006) is double. On the one hand, this methodology is based on less restraining hypotheses such as rational expectations hypothesis rather than perfect expectations assumption. On the other hand, we allow the introduction of the nonlinearity not only in adjustment process but also in estimating the fundamental value.

Fundamental value estimation results, that are reported in appendix 2, showed that our results are compatible with those of the previous studies (i.e. Manzan (2005) and Boswijk et *al.*(2006)). Estimated fundamental value is more slower and persistent than observed stock prices for all studied countries. In particular, prices fluctuate around fundamentals inducing periods of under-valuations since 1970 followed of overvaluation phases. As in Black *et al.*(2003) and Manzan (2005), courses deviations are meaningfully very persistent and prices last for long time away from their fundamentals. However, at the end of the period, courses seem to be over-valuated for Germany, Canada and USA while a mean reversion in stock prices is observed for UK, Japan, France and Italy.

Furthermore, analysis of these graphics implied two principal common features. First, there is a common period of under-valuation that began in 1970 and ended in 1980 (in 1995 for USA). This period of depression is due to shocks of 1973 and 1979, debts crisis (1982) and the introduction of a new International Monetary System (IMS). Secondly, there is a long period of overvaluation in German, American and Japanese stocks markets in 1990 due probably to bondholder crash (1994), Asian crisis (1997), but also to the telecommunication development, the increase of transaction volumes and the reduction of transaction costs.

Thus, there is an alternation of under- and overvaluation phases as stock prices deviate sometimes from their fundamentals but they are then mean-reverting. However, this can escape the linear modeling and a these dynamics can not be reproduced by linear modeling techniques because this modeling limits adjustment to be symmetrical and continuous. For that, we focused on nonlinear cointegration processes (STECM) to reproduce prices adjustment. STECM are appropriate to study prices adjustment within market frictions.

3-3 STECM for Stock Prices Deviations

STECM was introduced by Swanson (1996) and Koop *et al.*(1996), whereas their statistical properties were developed by Van Dijk *et al.*(2002). This modeling takes into account essentially two properties : non stationnarity and nonlinearity. Indeed, STECM defines an adjustment dynamic that depends on the sign (Logistic STECM (LSTECM)) or the size (Exponential STECM (ESTECM)) of prices deviations. We focused, on what follows, on modeling stock prices deviations while using STECM. These deviations are defined as follows :

$$p_t = f_t + z_t \tag{13}$$

Where : p_t and f_t are respectively asset stock price and its fundamental value in logarithm and z_t is stock price deviations.

STECM is appropriate to reproduce adjustment dynamic of non stationary series (i.e. p_t and f_t) for which long-run relation (equation (13)) is linear, but the process generating adjustment of p_t toward f_t is nonlinear. Furthermore, as in Anderson (1997), ESTECM could be more appropriate than Linear ECM to study stock prices adjustment in presence of heterogeneous transaction costs and discontinuous arbitrage. Indeed, LECM implies a linear process for z_t and a continuous and symmetrical adjustment process for p_t with constant speed of adjustment. However, transaction costs imply discontinuities in arbitrage and prices and induce nonlinear mean reversion in prices and an adjustment with a time varying speed that varies with the extend of prices deviations. In addition, as transaction costs are often heterogenous, adjustment is smooth rather than discrete and STECM more adequate to characterise prices deviations.

Formally, under absence of transaction costs hypothesis, stock prices adjustment is symmetrical, continuous and with constant speed of adjustment. Deviations dynamic can than be described by the following linear representation :

$$\Delta z_t = k + \rho \, z_{t-1} + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \varepsilon_t \tag{14}$$

Where : ρ is the adjustment term characterizing prices mean reversion and ε_t is white noise.

However, this representation limits adjustment to be invariant, symmetrical and linear and it is conditioned by absence of transaction costs. As in Anderson (1997) and Jawadi and Chaouachi (2006), market are not frictionless, transaction costs not only exist but also are heterogeneous. Therefore, LECM (equation (14)) is not appropriate. Its extension while introducing heterogeneous transaction costs and discontinuities and asymmetry in arbitrage, implies Nonlinear ECM that can reproduce stock prices "misalignements" : STECM.

Formally, STECM is defined as follows :

$$\Delta z_t = k' + \rho_1 z_{t-1} \times \left[1 - F\left(\gamma, z_{t-d}, c\right)\right] + \rho_2 \, z_{t-1} \times F\left(\gamma, z_{t-d}, c\right) + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \mu_t \tag{15}$$

Where : ρ_1 and ρ_2 are respectively adjustment terms in the first and second regime, z_{t-1} is lagged error correction term, z_{t-d} is transition variable, ϕ_i are AR parameters and $\mu_t \rightarrow N(0, \sigma_\mu^2)$.

STECM is a combination of two LECM for which adjustment is gradually rather than abruptly and adjustment speed is time varying with the deviations size or sign. STECM leads to a LECM for F = 0 or F = 1. The crucial parameters for STECM are ρ_1 and ρ_2. Indeed, in presence of transaction costs, the larger the deviation from equilibrium, the stronger the tendency to move back to equilibrium. This implies that even if $\rho_1 \geq 0$, ρ_2 must be strictly negative, the linear adjustment parameter ρ has to lie between ρ_1 and $(\rho_1 + \rho_2)$ and $(\rho_1 + \rho_2) < 0$ in order that prices would be nonlinearly mean-reverting and to have a nonlinear process that is globally stable (i.e. Michael *et al.*(1997)).

This means that, in the first regime (central regime), when deviations are small, z_t nears an unit root process ($z_t \rightarrow I(1)$ and approaches a random walk process) or may have an explosive behaviour ($\rho_1 \geq 1$), deviations are persistent, last for a very long time and are left uncorrected so that stock prices spend most of the time away from fundamentals. Instead, in the outer regimes, when deviations are sufficiently enough to pay transaction costs, the process would be mean-reverting with a convergence speed that varies directly with the size of deviations and z_t would approach to a white noise. However, for the continuum states, adjustment is described by of combination of the two adjustment terms ρ_1 and ρ_2 that are pondered by F(.). The important is stock prices deviations toward fundamentals the stronger is affected weight of ρ_2 relative to ρ_1. Therefore, such a behaviour could escape to conventional linear cointegration framework.

In practice, ESTECM was often used to study asset prices adjustment dynamics. It has been used to study the relationship between spot and futures prices of the FTSE100 index by Taylor *et al.*(2000), the exchange rates adjustment toward PPP and fundamentals by Michael *et al.*(1997), and Peel and Taylor (2000), the interest rates dynamics by Anderson (1997), Van Dijk and Franses (2000) and Liu (2001), the relationship between output and money by Swanson (1999) and Rothman *et al.*(2001) and more recently stock prices adjustment toward fundamentals by Manzan (2005) and Jawadi (2006). LSTECM and threshold ECM were rather applied by Dwyer *et al.*(1996), Martens *et al.*(1998) and Tsya (1998) notably to study the relationship between spot and futures prices of the S&P500 index.

In what follows, ESTECM was specified and estimated according to Van Dijk and Fransess (2000) modeling procedures. Indeed, after estimating fundamental value and defining long-run relationship, linear cointegration hypothesis was first tested while testing the null hypothesis : $H_0 : \rho = 0$ (no linear cointegration) against its alternative $H_1 : \rho < 0$ on stock prices deviations (z_t). However, since transaction costs effects may have implications on the conventional cointegration tests, H_0 may not be rejected even though prices are nonlinearly mean-reverting ($(\rho_1 + \rho_2) < 0$), probably since conventional cointegration is based on a linear model and because of the low power of tradional unit root tests (i.e. Taylor and Sarno (2001)). Thus, we tested, secondly, linear adjustment hypothesis against its alternative of nonlinearity while using LM tests[26].

After testing linearity, selected STECM is then estimated by NLS method. However, in order to distinguish the random walk behaviour of stock prices deviations from that of white noise process and reproduce mean reversion in stock prices, in presence of market frictions, not only in the central regime but also in the outer regimes, we developed, according to Michael, Nobay and Peel (1997)[27], an extension of the basic STECM (equation (15)).

This extension is useful to determine, on the one hand, stock prices periods of under- and overvaluation. On the other hand, it allows to define a new measure of stock prices adjustment strengths and the intensity of the mean reversion in stock prices. Indeed, with this extension, we estimated two indicators, proposed by Peel and Taylor (2000), never applied on stock markets, to estimate stock prices under and over-valuation periods and mean reversion characterizing their dynamics.

Formally, the following hypotheses were maintained :

$$H_0^a : k' = c = 0 , \qquad (16)$$

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[26] See Van Dijk *et al.*(2002) for more details.

[27] Michael *et al.*(1997) study focused on the exchange rate adjustment toward PPP.

$H_0^b : \rho_1 + \rho_2 = -1 \ / \ H_0^a$,

$H_0^c : \rho_1 = 0 \ / \ H_0^a$ and H_0^b .

Under these hypotheses, the basic STECM (equation (15)) implies :

$$\Delta z_t = -z_{t-1} \times F(\gamma, z_{t-d}, c) + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \mu_t \tag{17}$$

This restricted STECM is useful to reproduce stock prices adjustment and describe their deviations dynamic while distinguishing clearly two different dynamics for zt : zt approaches a random walk in the central regime while it nears a white noise in the outer regimes. Indeed, H^a_0 suggests that c = 0 to identify periods of under- and overvaluation, H^b_0 tests the presence of white noise process for z_t in the outside regimes, whereas H^c_0 indicates that z_t is near unit root in the central regime. H^b_0 and H^c_0 should also indicate the presence of different speeds of convergence that are varying with the desequilibrium size. Otherwise, these hypotheses allow the possibility of estimating two indicators proposed by Peel and Taylor (2000), but never applied on stock markets, to determine stock prices periods of under and over-valuation and the speed of mean reversion in their dynamics.

In practice, no-restricted and restricted STECM (equations (15) and (17)) were, in a first time, estimated independently. In a second one, hypotheses (H^a_0, H^b_0 and H^c_0) were tested while using the following likelihood ratio test :

$$LR = 2\left[L(\theta_1) - L(\theta_0)\right] \tag{18}$$

Where : $L(\theta_0)$ is log-likelihood of restricted STECM, $L(\theta_1)$ is log-likelihood of no-restricted STECM. LR → $\chi^2(q)$ d q is the number of tested constraints.

Empirically, we focused now on STECM estimation modeling. Thus, we describe, on the one hand, results of preliminary tests and STECM specification tests. On the other hand, we centred on STECM estimation results[28].

3-3-1 Preliminary Tests

First, we applied linear cointregation tests while checking the stationarity of (z_t) in order to check whether (13) is a long-run relationship. Secondly, nonlinear adjustment hypothesis was tested while testing linearity of z_t. Rejecting linearity hypothesis after establishing long-run relationship between p_t and f_t implies that prices are nonlinearly cointegrated and that adjustment could be reproduced by STECM.

In practice, ADF tests showed that both p_t and f_t are I(1) for all series. This implies that, under the stationarity hypothesis of (z_t), p_t and f_t can be linearly cointegrated and prices adjustment is necessarily linear, continuous and symmetric and could be reproduced by LECM. However, as in Michael *et al.*(1997), cointegration tests could be affected by the nonlinearity as these tests are based

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[28] We describe briefly STECM methodology, see Van Dijk *et al.*(2002) and Jawadi (2006) for more details.

on linear specifications. Therefore, the failure to reject the null hypothesis of no-cointegration does not mean necessarily the non stationarity of (z_t). Indeed, adjustment could be governed by asymmetric and nonlinear process. For example, in presence of transaction, prices deviations are persistent and courses are mean-reverting only if disequilibrium size is largely enough to compensate these costs.

ADF tests showed that no-cointegration hypothesis is rejected at 5% for Canada, UK and Italy. z_t is not stationary at 10% only for USA implying then a linear mean reversion in six indexes. However, this instability in results can be due to nonlinearity effect. Linear cointegration tests could lose power and be misspecified if adjustment is governed by nonlinear process, and reject then cointegration hypothesis (Taylor *et al.* (2001)).

Otherwise, these tests verify simultaneously two properties : stationarity in mean and stationarity in variance. Thus, rejection of stationarity of z_t can be due to presence of an ARCH effect. Thus, we analysed, first, the statistical properties of z_t and we noticed asymmetry and volatility excess as skewness coefficient is significantly negative and statically meaningful and standard deviations are elevated notably for USA and Japan. In addition, normality hypothesis is rejected.

Secondly, we applied ARCH tests z_t and we concluded on ARCH effect in the most prices deviations series. Thus, these results confirm our doubts relative to linear cointegration tests. Indeed, the failure to reject the null hypothesis of non-cointegration for USA can be explained by the presence of an ARCH effect. In what follows, we studied stock prices adjustment while using STECM. But, besides usual nonlinear tests, we applied nonlinear adjustment tests, proposed by Wooldridge (1990, 1991) and Van Dijk *et al.*(2002), that are robust to heteroscedasticity in order to take into account ARCH tests results

3-3-2 STECM Specification Tests

STECM specification is defined in three stages : i) Specification of linear model, ii) Linearity tests and iii) Selection of transition function. First, while specifying linear model, we have considered the interdependence and contagion between stock markets and notably the dependence of these markets to the American one[29]. Thus, we introduced current and previous American prices deviations as an explanatory variable for the adjustment models. Germany (resp. French) deviations were introduced for France (resp. Germany) adjustment model in order to reproduce interdependence between French and Germany markets, while Japanese deviations were introduced in American prices adjustment model. Secondly, we introduced the free-risk interest rates as an explanatory variable in stock prices adjustment models to reproduce the sensitivity of stock markets to that of risk-free assets. Thirdly, we introduced also industrial production in prices adjustment dynamic in order to replicate the impact of economic state on stock market adjustment (i.e. Prat(1982)).

Formally, we rewrite equation (14) as follows :

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29 See Ammer and Mei (1996), Erb, Harvey and Viskanta (1996) and Aglietta and Berrebi (2005)) for more details.

$$\Delta z_t = k + \rho z_{t-1} + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \sum_{j=0}^{p'} \alpha_j \Delta z_{t-j}^{USA} + \sum_{j=0}^{p'} \theta_j \Delta i_{0,\, t-j} + \sum_{j=0}^{p'} \theta'_j \Delta q_{t-j} + \varepsilon_t \qquad (19)$$

Where : z_t^{USA} is American stock prices deviations, i_0 is risk-free interest rate and q_t is industrial production in logarithm.

In practice, many specifications have been tested in order to determine the number of lags, using AIC, BIC, Ljung-Box Statistics and Autocorrelation functions. Thus, we retained p = 1, for Germany, USA, France, Italy and Japan; p = 2 for UK and p = 3 for Canada. Linear models are estimated by OLS[30]. Results showed that the most AR parameters are statistically meaningful at 5% and 10% implying that previous tendencies are useful. Adjustment term ($\hat{\rho}$) is negative and significative implying a mean reversion in stock prices for all countries except for Italy.

Otherwise, a strong evidence of contagion effect is seen at 5%, as American market affects strongly the other MSCI indexes. A mutual contagion effect is also shown respectively between Germany and French Markets and American and Japanese markets. Furthermore, industrial production seems affect positively and meaningfully prices adjustment for Canada, USA, UK and Japan with some delays, due probably to time taken to integrate new information. However, interest rates variations affect negatively and meaningfully courses adjustment. This can be associated with the degree of the competition between these two markets.

Overall, these results are good but residuals are neither normal nor homoscedastics and this can be seen as a sign of nonlinearity. Furthermore, linear specification limits arbitrage to be active all the time, markets to be frictionless and stock prices adjustment to be continuous, linear, symmetrical and to have a constant adjustment speed ($\hat{\rho}$). However, within market frictions, arbitrage is naturally active by regime and adjustment is rather discontinuous.

In the second stage, we check nonlinear stock prices adjustment hypothesis while using LM tests developed by Van Dijk *et al.*(2002). Data are monthly, thus, we applied linearity tests for $1 \le d \le 12$ and we considered d $\in$ [1,2,…,12] as a plausible values for the delay parameter (d). Transition variable is delayed deviations (z_{t-d}). Besides usual linearity tests, we applied also tests that are robust to heteroscedasticity, in order to take into account ARCH effect characterizing data. Rejecting linearity using these tests means that this nonlinearity is in mean and not in variance. Results are reported in table 2.

Table 2 : Linearity Tests

Delay	**Germany**	**Canada**	**USA**	**France**	**UK**	**Italy**	**Japan**
p	1	3	1	1	2	1	1
$\hat{d}$	2	2	9	1	2	8	1
$\hat{d}_r$	10	2	6	2	1	6	10

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[30] Results are presented in appendix 3.

p is lags number in linear model. $\hat{d}$ and $\hat{d}_r$ indicate optimal values of d according respectively to standard and robust linearity tests.

Thus, for both tests, linearity was strongly rejected at the 5%. This is in line with Manzan (2005) and Boswijk *et al.*(2006)[31]. However, as financial data are often volatile, we retained according to Van Dijk *et al.*(2002), results of robust linearity tests. Thus, linearity is also rejected, but more strongly for Germany, Canada and Italy respectively for d = 10, d = 2 and d = 6. Values of d varies across countries with three couples of countries : (Germany and Japan), (Canada and France) and (USA and Italy) and are respectively d = 10, d = 2 and d = 6, while for UK, d = 1. Overall, linearity was strongly rejected for all MSCI indexes. This result is very important as it confirmed the theoretical due to heterogeneous transaction costs. So, stock prices adjustment could be studied in a nonlinear framework using STECM.

The last step of STECM specification is transition function choice. Although transaction costs suggested exponential function and several previous studies (i.e. Michael *et al.*(1997), Manzan (2005) and Boswijk *et al.*(2006)) retained exponential function, ESTECM or LSTECM was selected on the basis of a sequence of tests developed by Teräsvirta (1994) and Escribano and Jordă (1999)[32]. Results of these tests are presented in table 3.

Table 3 : Selecting Transition Function Results

Series	**Delay**		**P-values of Teräsvirta Tests**			**P-values of Escribano and Jordă Test**		
	$\hat{d}$	$\hat{d}_r$	H_{03}	H_{02}	H_{01}	H_{0L}	H_{0E}	***Model***
Germany	1	10	0.09	0.01	0.00	0.00	0.001	ESTECM
Canada	3	2	0.01	0.00	0.00	0.008	0.00	ESTECM
USA	1	6	0.0009	0.00	0.001	0.003	0.00	ESTECM
France	1	2	0.15	0.008	0.04	0.002	0.00	ESTECM
UK	2	1	0.00	0.00	0.01	0.00	0.00	ESTECM or LSTECM
Italy	1	6	0.21	0.002	0.54	0.007	0.00	ESTECM
Japan	1	10	0.24	0.004	0.001	0.00	0.00	ESTECM or LSTECM

From table 3, ESTECM was often retained to reproduce prices adjustment for the most studied indexes. This resultant is important as it confirmed heterogeneous transaction costs suggestion. It is on line with previous studies (i.e. Manzan (2005) and Boswijk *et al.*(2006)). For UK and Japan, both models are good, but while estimating these two models, information criteria conclude in favor of ESTECM. ESTECM is then estimated for all indexes.

3-3-3 ESTECM Estimation Results

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[31] Previous studies applied only standard tests.

[32] Eitrheim and Teräsvirta (1996) argued that strict application of Teräsvirta tests may lead to wrong conclusions.

ESTECM was estimated by the NLS Method. In practice, first, the no-restricted ESTECM (equation (15)) was estimated. Then, H^a_0, H^b_0 and H^c_0 were tested while applying a likelihood ratio test (equation (16)). Finally, after accepting these hypotheses, we estimated the restricted ESTECM (equation (17)).

Thus, a no-restricted ESTECM was estimated for Germany, Canada, USA, France and Italy while both no restricted ESTECM and LSTECM were estimated for UK and Japan. Estimation results indicated that ESTECM is adequate to reproduce stock prices deviations dynamics for all MSCI indexes. Indeed, threshold parameter (c) and transition speed (γ) are statically significant at 5%. Adjustment term in the first regime (ρ_1) is not significant for Germany, Canada, Italy and Japan indicating that prices are not mean-reverting in this regime. ρ_1 is positive and significant only at 10% for France and USA implying an explosive behavior near equilibrium for these prices

However, adjustment term in the second regime (ρ_2) is negative and statistically significant at 5% for Germany, France and UK and at 10% for USA, Italy and Japan[33]. In addition, ($\rho_2 < 0$) and ($\rho_1 + \rho_2 < 0$) indicating that prices are nonlinearly mean-reverting and confirming a nonlinear mean reversion in stock prices. In other words, z_t is near a random walk and has an unit root ($\rho_1 = 1$) or an explosive beahavior ($\rho_1 > 1$) in the central regime for small deviations, whereas for larges deviations, prices are strongly mean-reverting and z_t is near a white noise.

Otherwise, interest rates effects are negative and significant while industrial production is statistically significant only for Canada, USA and Japan. There is also a significant contagion effect between Germany and French markets and a strong dependence of MSCI indexes to the American stock market.

In order to reproduce more explicitly stock prices adjustment toward fundamentals near the equilibrium but also when prices are away from equilibrium, we followed Michael, Nobay and Peel (1997) and we estimated a restricted ESTECM. This could reproduce stock prices adjustment for small and large deviations, locate phases of under- and overvaluation and estimate adjustment strengths that lead prices toward fundamentals. It is also useful to test the presence of random walk near the equilibrium in the central regime and the white noise behavioral in the outside regimes. Thus, the restrictions H^a_0, H^b_0 and H^c_0 were tested while applying likelihood ratio test and we got the following results :

Table 4 : Likelihood Ratio Test Results

Serie	**Germany**	***Canada***	***USA***	***France***	***UK***	***Italy***	***Japan***
LR^a	0.8*	0.79	0.85	0.58	0.12	0.79	0.28
LR^b	0.89	0.93	0.98	0.82	0.09	0.77	0.11
LR^c	0.93	0.74	0.97	0.90	0.08	0.67	0.80

(*) are the p-values of LR Test.

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[33] For Canada, ρ_2 is significant only at 13%.

Results showed that H^a_0, H^b_0 and H^c_0 are accepted for all MSCI indexes and more accepted for France and USA. This implies that z_t has a random walk behavior near equilibrium and a white noise process away from equilibrium. However, these restrictions are not strongly accepted for Japan and are accepted for USA only at 10% for USA. Finally, we estimated ESTECM under H^a_0, H^b_0 and H^c_0 and we reported results in table 5.

Table 5 : Nonlinear Estimation Results[34]

	Germany	**Canada**	**USA**	**France**	**UK**	**Italy**	**Japan**
p	1	3	1	1	2	1	1
$\hat{d}_r$	10	2	6	2	1	6	10
$\hat{\gamma}$	0.62 $(3.8)^*$	0.1 $(4.4)^*$	0.57 $(3.6)^*$	8.53 $(3.29)^*$	0.64 $(1.63)^{**}$	9.94 $(2.7)^*$	7.65 $(2.18)^*$
$\hat{\phi}_1$	-0.06 $(-1.75)^{**}$	-0.08 $(-1.63)^{**}$	-0.03 $(-1.69)^{**}$	0.06 $(2.1)^*$	-0.02 (-0.44)	0.14 $(2.9)^*$	-0.02 $(-1.63)^{**}$
$\hat{\phi}_2$	-	-0.02 (-1.1)	-	-	-0.46 $(-9.7)^*$	-	-
$\hat{\phi}_3$	-	0.17 $(5.4)^*$	-	-	-	-	-
$\hat{\alpha}_0$	0.16 $(3.07)^*$	0.68 $(16.1)^*$	-	0.44 $(7.9)^*$	1.08 $(21.7)^*$	0.98 $(13.1)^*$	0.06 (1.2)
$\hat{\alpha}_1$	0.12 $(2.4)^*$	0.16 $(2.9)^*$	-	-	-0.05 (-0.9)	0.38 $(5.08)^*$	0.35 $(6.07)^*$
$\hat{\alpha}_2$	-	-	-	-	0.37 $(6.2)^*$	0.42 $(5.8)^*$	-
$\hat{\alpha}'_0$	0.19 $(3.6)^*$	-	-	-	-	-	-
$\hat{\alpha}''_0$	-	-	-	0.9 $(20.4)^*$	-	-	-
$\hat{\beta}_0$	-	-	0.18 $(3.9)^*$	-	-	-	-
$\hat{\theta}_0$	-0.007 $(-1.73)^{**}$	-0.01 $(-4.2)^*$	-0.03 (-6.06)	-0.02 $(-5.8)^*$	-0.005 $(-1.8)^{**}$	-0.06 $(-10.3)^*$	-0.01 $(-2.3)^*$
$\hat{\theta}'_0$	-	-	-	-	-	-	0.34 $(1.98)^*$
$\hat{\theta}'_1$	-	0.11 (0.8)	0.41 $(1.8)^{**}$	-	-	-	-
$\hat{\gamma} \times \sigma_z^2$	0.07	0.006	0.08	1.2	0.04	1.3	1.1
V	0.88	0.9	0.94	0.93	0.97	0.92	0.95
ADF (p)	-13.9^* (p = 0)	-14.3^* (p = 0)	-14.8^* (p = 0)	-14.6^* (p = 0)	-20.3^* (p = 0)	-14.6^* (p = 0)	-14.07^* (p = 0)
DW	1.97	2.04	2.02	2.03	2.01	2.0	2.02
Q(4)	0.12	0.6	2.07	1.5	0.95	4.6	2.2
Q(12)	5.31	29.2	9.34	13.07	14.2	15.5	6.7
ARCH (q)	5.06^* (q = 1)	10.8^* (q =1)	14.3^* (q =1)	0.55^* (q =1)	17.7^* (q =1)	7.9^* (q =1)	18.8^* (q =2)
N	18	47	30	45	27	25	28

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[34] Note : Values under regression coefficients are the t-ratios of estimators. Q(4) and Q(12) are Ljung-Box statistics. (*) and (**) designate respectively the significativity at 5% and 10%. ADF and ARCH are respectively the statistics of ADF and ARCH tests. V is ratio of residual variances of linear and nonlinear models and N is iterations number.

Results showed that ESTECM has good statistical properties. Indeed, AR parameters are often statistically significant at 5%. There is a strong evidence of contagion and interdependence between the MSCI stock indexes. In particular, current and previous American deviations affect significantly the other stock prices adjustment dynamics. There is also significant mutual effect between respectively French and Germany markets and American and Japanese markets. Furthermore, interest rates affect negatively and significantly stock markets adjustment, while industrial production has a significant effect only for Japan at 5% and for USA at 10%.

Otherwise, γ is often statistically significant at 5%, it is significant at 10% for UK. Its estimated values are relatively weak confirming hypotheses of nonlinear adjustment and smooth transition. This implies prices deviations adjustment dynamic is nonlinear with Mean-Reversion. Indeed, prices are nonlinearly mean-reverting with an adjustment speed that varies with their deviations size from equilibrium. For small deviations, prices deviate from fundamentals and last for long-time away from their fundamentals, but for a large deviations- notably when they exceed the assumed transaction costs-, arbitrage and adjustment would be active and prices reverted back to fundamental quickly with an adjustment speed that grows with the desequilibrium extend.

Otherwise, computing the ratio of residual variances of linear and nonlinear models, we showed the superiority of ESTECM compared to LECM in reproducing stock prices adjustment toward fundamentals. This ratio showed a reduction of 12% in the residual variance for USA compared to the linear model. Such results are on line with those of Black, Fraser and Groenewold (2003) and Bohl (2003) that showed also the inability of linear model to reproduce stock prices adjustment.

Finally, we estimated transition functions and plotted them (on the vertical axis) against lagged values of stock prices deviations (appendix 4) in order to show more explicitly the slowness characterizing prices adjustment. Results suggested many stylised facts. First, transition between regimes is slowly confirming by the weaker values of γ. Transition is more quick for French, Italian and Japanese cases.

Secondly, observations are distributed symmetrically around equilibrium and a considerable number of observations is around It, confirming the choice of exponential function. Thirdly, these functions showed clearly presence of differentiated adjustment speeds that varie with the desequilibrium size. These function are elevated for large deviations notably for France, Italy and Japan, but relatively weak for small deviations. For example, transition function did not exceed respectively 0,45 for Germany; 0,03 for Canada; 0,40 for USA, whereas it reached the unity in French, Italian and Japanese Indexes. This implies that AR coefficient measuring adjustment strength (second indicator of Peel and Taylor (2000))[35] is not equal to 0 and that adjustment is often active to conduct prices toward fundamentals. Overall, a nonlinear mean reversion in MSCI indexes was shown that is

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[35] See next paragraph.

quicker in French, Italian and Japanese cases, notably for larges deviations so that deviations in this regime nearest a white noise process.

In conclusion, linear adjustment hypothesis was rejected and a strong evidence of nonlinear mean reversion in MSCI stock indexes was shown, for which adjustment speed is rising with prices desequilibrium size. Indeed, stock prices deviations are near unit roots around the equilibrium, while they approach a white noise process in the extreme regimes. Thus, restricted ESTECM is appropriate to reproduce courses adjustment in presence of distortions and slowness induced by transaction costs.

In order to check the validity of these results, three tests of misspecification were applied : Tests of residual autocorrelation, tests of omitted linearity and tests of parameters constancy. Results of these tests are given in appendix 5. Results are globally positive retaining ESTECM and confirming our empirical analysis and conclusions. Indeed, residues are not correlated for all MSCI indexes. The hypothesis of constance parameters is rejected at 5% only for UK. In addition, applying standard and robust linearity tests to ESTECM residues for several value of d, ($1 < d < 12$), indicated that nonlinearity has been captured for most studied countries by ESTECM. The null hypothesis is rejected only for UK.

In the next step, first, stock prices movements were gauged and located. Then, we proposed a new measure of adjustment strengths characterizing mean reversion in stock prices.

3-4 Gauging Under-valuation and Overvaluation Phases

ESTECM estimation is then used to gauge the degree of stock prices under- and overvaluation toward fundamentals while using estimated transition function. Indeed, according to two indicators developed by Peel and Taylor (2000), it is possible to use transition function to determine the degree of mean reversion in stock prices. Thus, we showed a lower mean reversion degree for small deviations and a higher mean reversion one for large deviations.

In practice, we used these two indicators to gauge the degree of under- and overvaluation and measure stock prices adjustment forces toward equilibrium. The first indicator is defined as follows :

$$\Pi(z_t) = 100 \times F(z_t) \times \text{sign}(z_t) \tag{20}$$

Where : $sign(z_t) \equiv \frac{z_t}{|z_t|}$ and $-100 \leq \Pi(z_t) \leq 100$

$\Pi(z_t)$ is defined in term of transition function and corrected by deviations sign because in reason of the symmetric nature of exponential transition function, F(.) measures the importance of prices deviations from equilibrium regardless of sign. F(.) takes the same value for the same deviations but that have different signs. $\Pi(z_t)$ defines a measure of the degree of overvaluation if it is positive or under-valuation when $\Pi(z_t) < 0$. $\Pi(z_t) \rightarrow 0$ implies that stock price approaches its fundamental value.

The second indicator is given by the following representation :

$$\Psi(z_t) = 1 - F(z_{t-d}) \tag{21}$$

Where : $0 \leq \Psi(z_t) \leq 1$

This indicator is a function of the AR parameter of restricted ESTECM. It implies a measure of the degree of mean reversion in stock prices. $\Psi(z_t)$ is bounded between 0 and 1. More $\Psi(z_t) \rightarrow 1$, much more adjustment is lower and z_t leads to random walk. However, when $\Psi(z_t) \rightarrow 0$, stock prices adjustment toward fundamental becomes active so that z_t leads to a white noise.

Overall, these two indexes are appropriate to evaluate prices deviations and their correcting mechanisms. But, the comparison of these two indicators showed the presence of a temporal shift between these two indicators. Indeed, $\Pi(z_t)$ is function of current stock prices deviations while $\Psi(z_t)$ depends on lagged deviations. Thus, estimating $\Pi(z_t)$ and $\Psi(z_t)$ implies a measure of deviations when they occurred and an indication on their correction with an adjustment time delay (d).

Otherwise, a new empirical contribution is proposed while estimating $\Pi(z_t)$ and $\Psi(z_t)$ in order to gauge periods of stock prices over or under-valuation and to measure adjustment strengths of stock prices toward equilibrium.

In practice, first, $\Pi(z_t)$ is estimated and gotten graphics are reported in appendix 6. We showed a strong periods of under and over-valuation of MSCI stock indexes over the studied period except for UK index for which only a strong under-valuation of about –50% was located in 1973. In addition, an important episodes notably for Italian, American, Japanese and French indexes was showed. In particular, stock prices are characterized by an under-valuation phase in the beginning of 1973 due probably to the first oil shock. For Germany, this under-valuation is about 16% in March 1973 while it is more strongly for France[36]. This phase last 21 months in USA but it was strong for Japan and fast in the UK case. Markets were also under-valuated in the beginning of 1980 in reason of the second oil shock in 1979 and the debts crisis in 1982. However, G7 markets were globally characterised by a significant correction of stock prices deviations after signing of the *Plazza* Accord in 1985. But in reason of the effects of the against-oil shock, the decrease of the price of oil barrel implied a phase of fall of stock prices (i.e. German index lost 30%).

Besides these common misalignment periods, German market knew a strong overvaluation after 1995 because of the increase of German exports and American deficits. Canadian index was characterised by three important phases of overvaluation (1970, 1980 and 2000) and two periods of under-valuation (1982 and 1998). It rejoined its equilibrium value between 1982 and 1987. New York stock market benefited from the rise of interest rate[37] and the *Plazza Accord* so that price reached fundamental in 1990. The American stock market climbed considerably until the elevated levels between 1994 and 2000 and it knew the most spectacular rise of its history, benefiting from the stock

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[36] The French index is characterised by three periods of decreasing on June 1973, February 1979 and October 1980. In this period, the French index lost more than 80%, but some correcting mechanism permitted it to be over-valuated lately. Π is about 96%.

[37] In the summer 1981, the Federal Reserve increased the interest rate of 20%.

crash of 1987, the changes crisis of the European System in 1990 and the Asian crisis in 1997. $\Pi(.)$ was around 50%.

British index adjustment is quiet and it is around its fundamental value at the most of the time. Nevertheless, for Italy, adjustment is more important and the degree of under or over-valuation is about 100%. Its transition speed was also relatively elevated ($\hat{\gamma} = 9{,}9$. This reflects probably the efforts provided by Italian government to assure the convergence of the Italian economy to respect *Maastricht* criterias. The size of this index adjustment is explained also by the importance of the government efforts notably after the exit of the Italian lira of the European Monetary System. Finally, for Japanese index, after an under-valuation period, the index seemed to be anchored to its fundamental value during the period 1976-1978 and until 1980. Then, it knew an important period of overvaluation of 100% that it finished on 1990 with the bursting of Japanese speculative bubble. It was also characterised by other overvaluation phases after 1995 in reason of the American Dollar devaluation, but it was under-valuated after 1998 because of the Asian crisis in 1997.

Therefore, $\Pi(z_t)$ allowed to identify the principal periods of under and overvaluation of the G7-MSCI indexes. Our results showed that the efficient market hypothesis is rejected and that there is an active adjustment process that describes stock prices deviations toward fundamental with a convergence speed that which varies with the desequilibrium size. Finally, $\Psi(z_t)$ is estimated and allowed for a new empirical measure of stock prices adjustment strengths.

3-5 Stock Prices Adjustment Strengths

$\Psi(z_t)$ defines a measure of stock indexes adjustment strengths. More $\Psi(z_t)$ is near 1, more price deviations nearest a random walk process, while more $\Psi(z_t)$ is far from 1, more stock prices adjustment process is active. As we explained, $\Psi(z_t)$ depends on z_{t-d} and not on z_t, meaning that there is a shift between the moment at which price is away from fundamental and the time for which the correcting adjustment process will be activated to correct the prices misalignments. In practice, our estimations showed the presence an average adjustment delay of about 5 months to correct the G7-MSCI indexes deviations.

From results reported in appendix 7, it was clearly showed the importance of adjustment strengths and the correcting mechanisms notably for Germany, Canada, USA, France, Italy and Japan. Indeed, stock prices adjustment toward equilibrium is discontinuous, asymmetrical and nonlinear. Prices are nonlinearly mean-reverting with an adjustment speeds that are variable but more volatile for Italian, French and Japanese cases. This convergence speed is volatile only at the end of the period for USA. It nearests the unity at the end of the period indicating that American deviations follow a random walk process. Otherwise, these results confirm that of Manzan (2005) who showed that American price (S&P500 index) is not mean-reverting after 1990.

Furthermore, results indicated also that estimated strengths last often long time away from 1 except Canadian index, indicating that adjustment is often active and that prices deviations do not follow necessary a random walk process. Thus, adjustment strengths are variable and are more important when prices deviations from fundamental are more significant. In addition, adjustment strengths are more elevated and adjustment is more strongly in periods of crises and crashes (i.e. 1973, 1979, 1987). Therefore, it was concluded that adjustment dynamics of the G7 stock prices are non-linear with a mean reversion in stock prices with a convergence speed that is more important in strong phases of under or over-valuation.

VI. Conclusion

An empirical study centred on stock prices adjustment toward fundamental of the G7 countries has been developed in a nonlinear framework, while using STECM. Nonlinearity has been justified by the presence of heterogeneous transaction costs and the coexistence of heterogeneous expectations. This nonlinearity has been introduced not only in evaluating stock prices adjustment but also in estimating fundamental values. Thus, the new fundamental value estimation that was proposed is in line in that of Lucas (1978) and Manzan (2005).

Concerning the study of nonlinear adjustment hypothesis, empirical results showed that prices are nonlinearly mean-reverting with a variable adjustment speeds that are varying with the size of the prices disequilibrium. Indeed, it is showed that near equilibrium (in the central regime,) prices deviations are near unit root, z_t follows a random walk process, while z_t nearests a white noise process for large prices deviations because of the mean reversion in stock prices in these regimes (i.e. outer regimes).

Finally, using two indicators of Peel and Taylor (2000), the principal periods of under and over-valuation of the G7-MSCI indexes were precisely located. In addition, while using estimated transition function, a new measures have been proposed to capture stock prices adjustment strengths and the intensity of correcting the G7-MSCI indexes misalignments.

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Appendix 1 : STAR Estimation Results

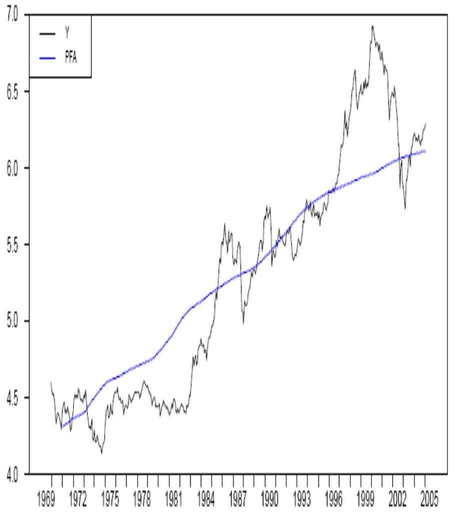
	Germany	**Canada**	**USA**	**France**	**UK**	**Italy**	**Japan**
α_0	0.06 (0.9)*	-0.26^{a} (-2.8)	0.28^{a} (2.2)	-0.006 (-0.1)	-0.008 (-0.27)	0.01 (0.1)	5.9a (12.7)
α_1	0.08 (1.1)	0.9^{a} (2.1)	-017^{a} (-10.8)	-0.73^{a} (-7.9)	-0.62^{a} (-4.6)	0.08^{a} (2.3)	-1.02^{a} (-5.7)
α_2	-0.002 (-1.04)	-0.27 (-0.4)	-3.5^{a} (-6.3)	-0.71^{a} (-7.5)	-0.78^{a} (-7.3)	-0.06^{a} (-2.3)	-2.3^{a} (-6.8)
α_3	0.2^{b} (1.9)	0.75 (1.1)	-2.0^{a} (-3.6)	-0.82^{a} (-9.7)	-0.49^{a} (-4.3)	0.03 (0.7)	-1.62^{a} (-13.9)
α_4	0.01^{a} (2.1)	-0.17 (-0.28)	-1.7^{a} (-3.2)	-0.64^{a} (-5.6)	-0.52^{a} (-4.7)	0.01 (0.5)	-1.6^{a} (-15.1)
α_5	-0.004 (-1.2)	1.5^{a} (2.2)	-0.33 (-0.7)	-0.68^{a} (-6.9)	-0.13 (-1.2)	-0.1^{a} (-3.3)	0.41 (1.3)
α_6	0.08^{a} (2.2)	3.1^{a} (4.0)	0.13 (0.2)	-0.95^{a} (-8.1)	0.57^{a} (3.8)	0.25^{a} (8.6)	-0.69^{a} (-5.6)
α_7	-0.04 (-0.7)	-3.09^{a} (-4.1)	0.39 (0.8)	-0.86^{a} (-7.3)	0.44^{a} (2.8)	-0.03 (-0.5)	-0.66^{a} (-3.7)
α_8	0.07 (1.6)	1.02^{a} (3.5)	1.07^{a} (2.2)	-0.84^{a} (-6.7)	0.34^{a} (2.1)	0.04^{b} (1.8)	1.7^{a} (6.0)
α_9	0.11^{b} (1.7)	0.15^{b} (1.7)	1.2^{a} (2.7)	1.02^{b} (1.6)	0.004 (0.03)	-0.11 (-1.0)	- -
α_{10}	0.09 (1.3)	- -	1.6^{a} (2.9)	-2.2^{a} (-10.7)	-0.09 (-0.7)	0.02 (0.4)	- -
α_{11}	0.06^{b} (1.9)	- -	0.59 (1.5)	-0.35^{a} (-2.1)	-0.32^{a} (-3.2)	0.06 (0.6)	- -
α_{12}	0.05^{a} (7.5)	- -	- -	0.27^{a} (2.8)	-0.16^{b} (-1.9)	-2.1^{a} (-2.0)	- -
β_0	1.9^{a} (5.5)	0.3^{a} (3.1)	-0.26^{a} (-2.0)	5.1^{a} (2.2)	2.36^{a} (5.5)	4.1^{a} (7.1)	-5.8^{a} (-12.6)
β_1	0.42^{a} (4.2)	-2.0^{a} (-4.8)	0.7^{a} (3.8)	3.3^{a} (2.5)	-1.06^{a} (-5.2)	0.01^{a} (0.1)	0.96^{a} (4.3)
β_2	-0.31^{a} (-3.7)	-0.99^{b} (-1.7)	2.5^{a} (-4.4)	-1.9^{a} (-0.9)	-1.4 (-0.6)	0.04 (0.2)	2.4^{a} (6.8)
β_3	0.1 (1.1)	-1.3^{b} (-1.9)	1.3^{a} (2.3)	4.6^{a} (2.4)	-1.7^{a} (-5.5)	-2.3^{a} (-5.3)	1.6^{a} (13.8)
β_4	-0.43^{a} (-5.2)	-0.3 (-0.5)	1.0^{b} (1.7)	-4.1^{a} (-1.6)	-1.6^{a} (-4.4)	-1.1^{a} (-5.8)	1.7^{a} (15.1)
β_5	-0.12 (-1.3)	-2.0^{a} (-2.7)	-0.34 (-0.7)	0.21 (0.22)	-2.03^{a} (-5.1)	0.06 (0.2)	-0.4 (-1.2)
β_6	-0.25^{a} (-2.4)	-3.7^{a} (-4.6)	-0.59 (-1.1)	1.5^{b} (1.8)	-2.4^{a} (-7.4)	0.1 (0.4)	1.4^{a} (10.2)
β_7	-0.09 (-1.0)	-3.4^{a} (-4.4)	-0.84 (-1.5)	1.8^{a} (2.1)	-2.5^{a} (-7.9)	-3.8^{a} (-8.7)	0.86^{a} (3.8)
β_8	-0.33^{a} (2.6)	-1.3^{a} (-4.2)	-1.7^{a} (-3.3)	1.1 (1.3)	-2.1^{a} (-8.1)	-0.03 (-0.1)	-1.6^{a} (-5.8)
β_9	0.46^{a} (4.9)	-0.08 (-0.3)	-1.8^{a} (-3.6)	-0.7 (-0.8)	-1.6^{a} (-6.6)	7.3^{a} (3.8)	- -
β_{10}	-0.26 (-0.9)	- -	-1.9^{a} (-3.3)	2.1^{a} (2.4)	-1.9^{a} (-7.1)	-1.2^{a} (-12.8)	- -
β_{11}	-0.09 (-0.3)	- -	-0.8^{a} (-2.0)	0.3 (0.4)	-1.1^{a} (-6.4)	-0.17^{a} (-1.0)	- -

β_{12}	0.29^a (2.0)	- -	- -	-0.5^b (-1.7)	-0.08^b (-1.8)	0.32^b (1.8)	- -
γ	5.3^a (2.8)	1.43^a (6.9)	0.24^a (2.5)	5.2^a (2.8)	0.17^a (4.9)	0.16^a (-3.8)	66.4^a (5.8)
c	0.78^a (14.8)	-0.34^a (-23.1)	-0.27^a (-6.9)	0.05^a (4.7)	-0.31^a (-2.0)	0.45^b (1.8)	0.04^a (22.1)
R^2	0.78	0.87	0.85	0.81	0.91	0.91	0.92
N	32	26	71	53	40	50	51

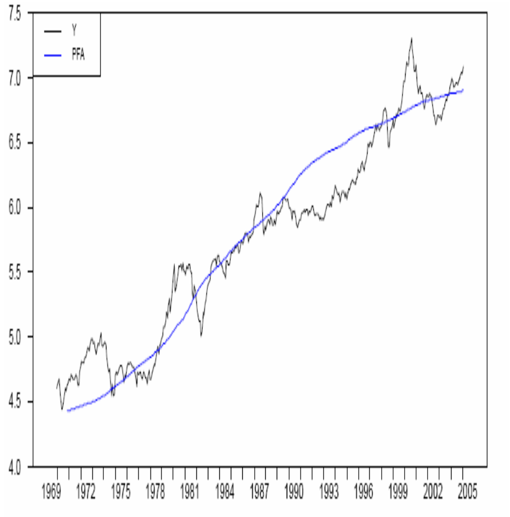
(*) : Values between brackets are the t-ratio of estimators. (a) and (b) designate respectively the significativity at 5% and 10%.

Appendix 2 : Fundamental Value and Price Representation[38]

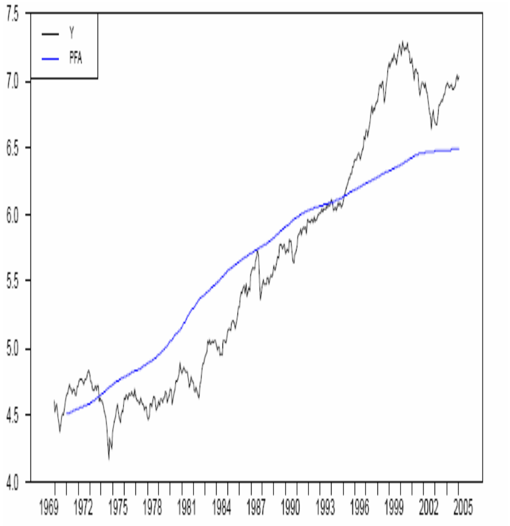
Germany



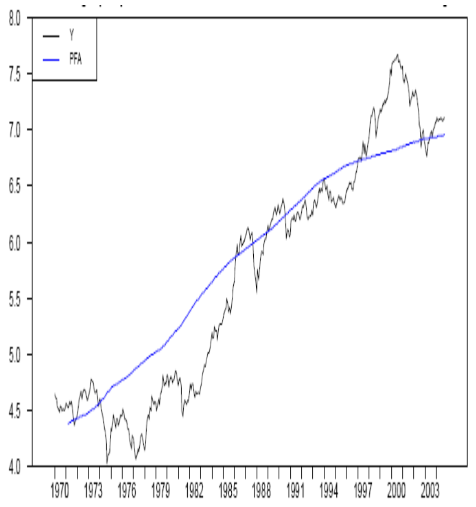
Canada



USA



France



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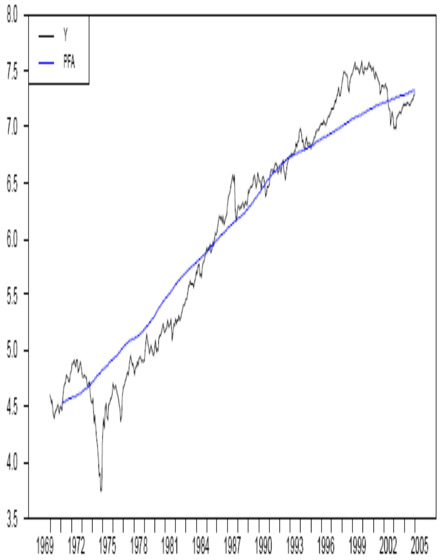
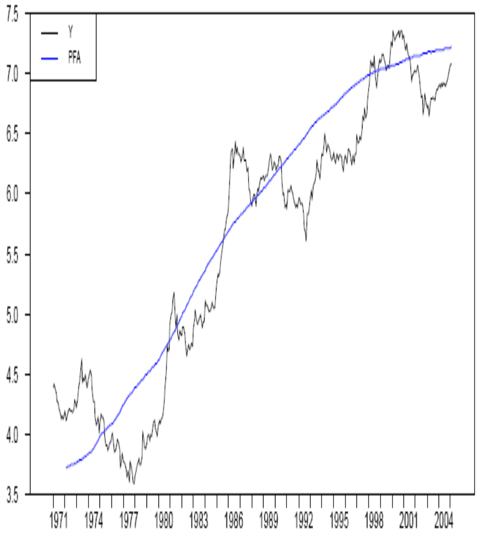
[38] **Note** : **Y** and **PFA** are respectively observed price and its estimated fundamental value in logarithm.

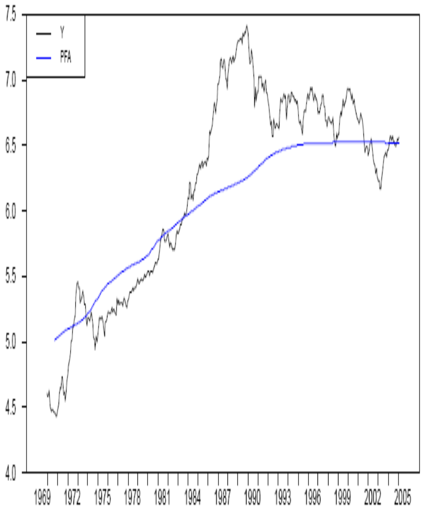
UK



Italy



Japan



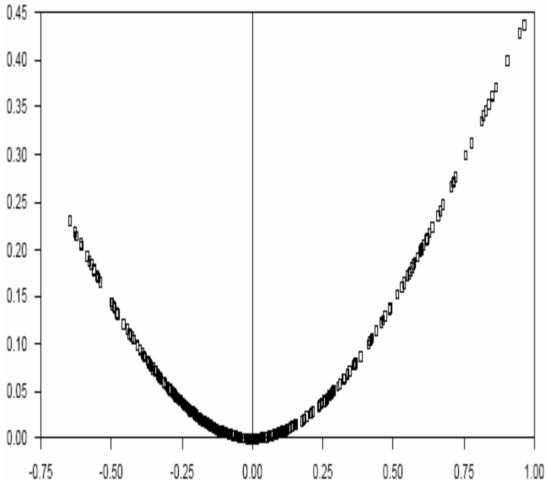
Appendix 3 :Linear Models Estimation Results

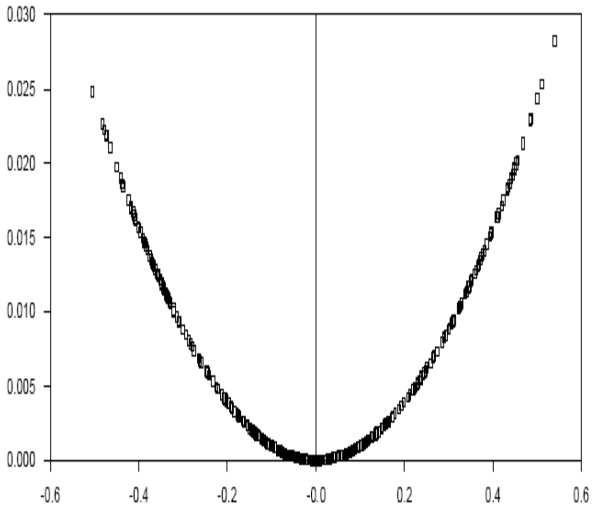
	Germany	**Canada**	**USA**	**France**	**UK**	**Italy**	**Japan**
p	1	3	1	1	2	1	1
$\hat{\rho}$	-0.015 (-2.31)*	-0.011 (-1.68)**	0.06 (0.9)*	-0.0001 (-1.74)**	-0.025 (-2.95)*	-0.005 (-0.63)	-0.012 (-2.05)*
$\hat{\phi}_1$	-0.013 (-1.63)**	0.017 (0.4)	-0.04 (-1.8)**	0.029 (1.71)**	-0.007 (-1.15)	-0.016 (-1.81)**	0.012 (1.83)**
$\hat{\phi}_2$	-	-0.019 (-1.74)**	-	-	-0.14 (-2.92)*	-	-
$\hat{\phi}_3$	-	0.102 (3.18)*	-	-	-	-	-
$\hat{\alpha}_0$	0.293 (5.14)*	0.83 (22.6)	-	0.4 (7.05)*	0.79 (16.2)*	0.52 (7.06)*	0.43 (7.91)*
$\hat{\alpha}_1$	0.131 (2.35)*	0.09 (1.65)**	-	-	0.09 (1.65)**	0.16 (2.0)	0.2 (3.37)*
$\hat{\alpha}_2$	-	-	-	-	0.13 (2.09)*	0.14 (1.96)*	-
$\hat{\alpha}_3$	-	-	-	-	0.15 (3.06)*	-	-
$\hat{\alpha}'_0$	0.49 (11.7)*	-	-	-	-	-	-
$\hat{\alpha}''_0$	-	-	-	0.51 (11.67)*	-	-	-
$\hat{\beta}_0$	-	-	0.15 (4.4)*	-	-	-	-
$\hat{\theta}_0$	-0.0007 (1.65)**	-0.011 (-3.8)*	-0.008 (2.57)	-0.011 (-2.4)*	-0.022 (-5.42)*	-0.011 (-2.16)*	-0.001 (-1.99)*
$\hat{\theta}'_0$	-	-	-	-	-	-	0.29 (1.64)**
$\hat{\theta}'_1$	-	0.22 (1.7)**	0.29 (1.69)**	-	-	-	-
$\hat{\theta}'_2$	-	-	-	-	0.25 (1.76)**	-	-
R^2	0.49	0.60	0.44	0.53	0.46	0.17	0.21
σ_L	0.04	0.03	0.03	0.04	0.04	0.06	0.04
Q(4)	0.09	0.46	2.37	1.77	1.25	3.18	1.84
Q(12)	3.56	31.01	10.06	13.1	14.9	17.56	5.8
J-B	31.95*	23.58*	7.66**	27.54*	372.2*	20.3*	24.55*

Note : Values under regression coefficients are the t-ratios of estimators. R^2 is coefficient of determination, J-B is statistic of Jarque-Berra test and σ_L is standard deviation of linear model. Q(4) and Q(12) are Ljung-Box statistics. (*) and (**) designate respectively the significativity at 5% and 10%.

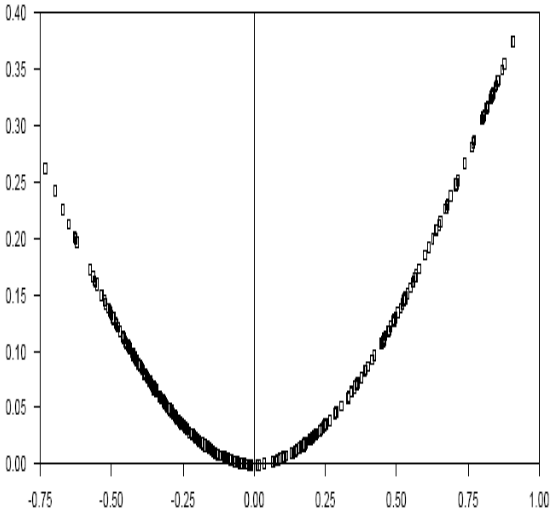
Appendix 4 : Estimated Transition Functions

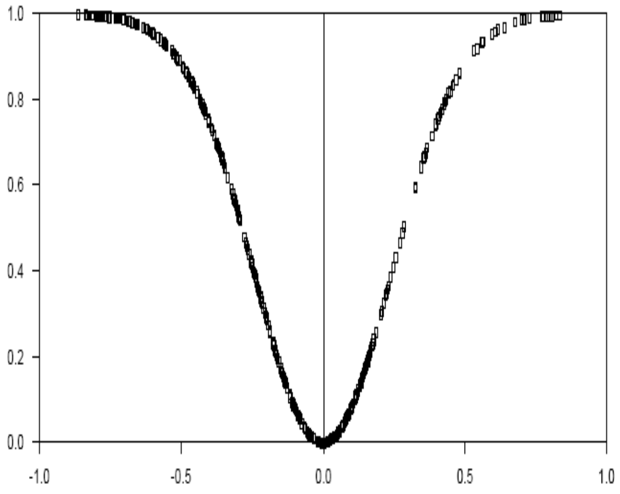
Germany **Canada**



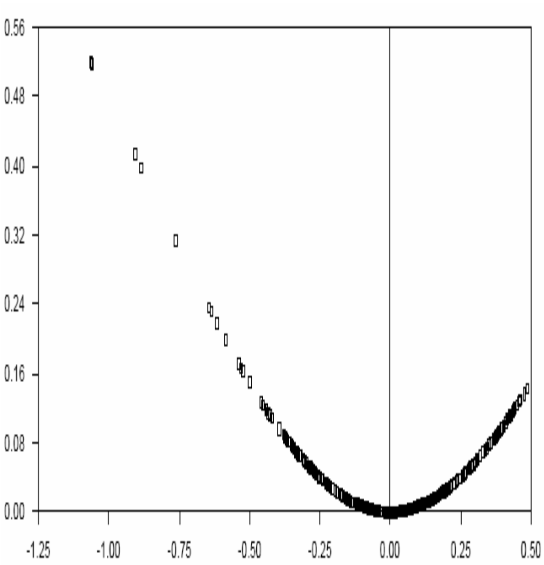


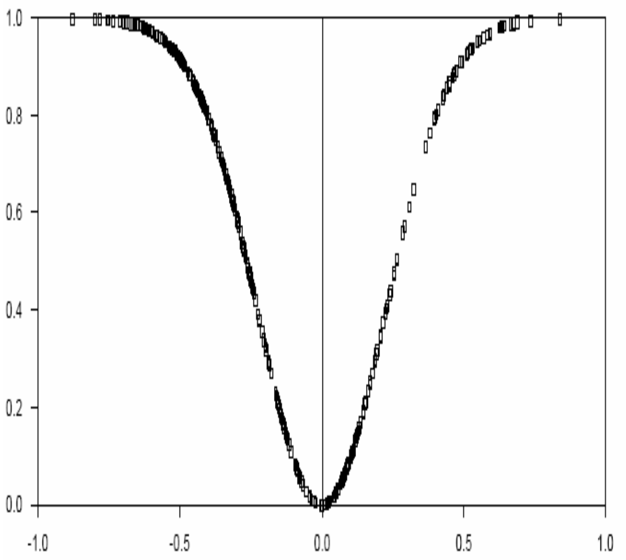
USA **France**



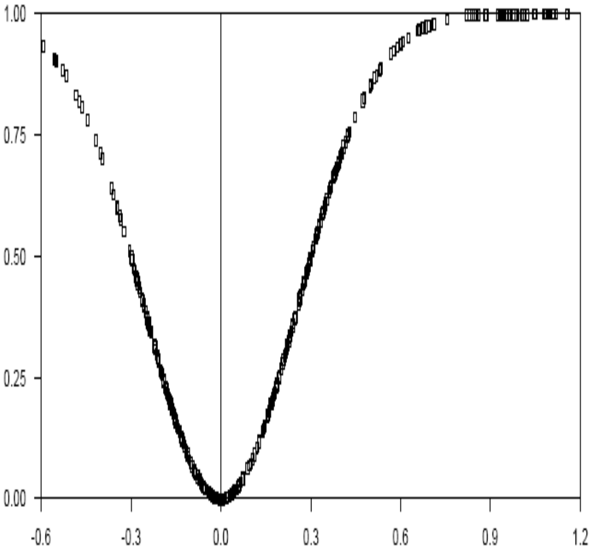


UK **Italy**





Japan



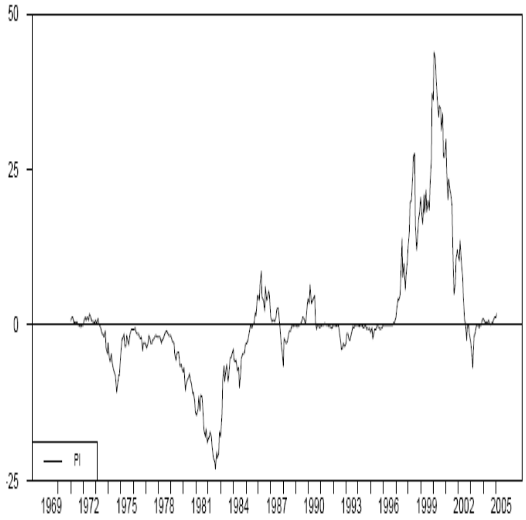
Appendix 5 : Misspecification Tests Results

Tests of No Error autocorrelation (p_value des tests LM_{SI})							
q / serie	Germany	Canada	USA	France	UK	Italy	Japan
q = 1	0.35	0.11	0.17	0.24	0.20	0.55	0.13
q = 2	0.62	0.13	0.24	0.44	0.22	0.46	0.28
q = 3	0.80	0.12	0.42	0.51	0.43	0.53	0.31
q = 4	0.90	0.23	0.53	0.63	0.57	0.33	0.29
q = 8	0.69	0.20	0.73	0.28	0.39	0.16	0.23
q = 12	0.89	0.35	0.75	0.27	0.10	0.17	0.40
Test of Parameter Constancy (p_value $LM_{c,i}$, $\forall$ i = 1, 2, 3.							
$LM_{c,1}$	0.48	0.22	0.18	0.17	0.02	0.34	0.23
$LM_{c,2}$	0.67	0.23	0.44	0.10	0.01	0.55	0.38
$LM_{c,3}$	0.88	0.55	0.68	0.30	0.03	0.75	0.63
Test of No Remaining Nonlinearity (p_value de LM_{AMR})							
$d' = 1$	0.84	0.63	0.97	0.19	0.11	0.11	0.11
$d' = 2$	0.92	0.49	0.94	0.27	0.01	0.59	0.07
$d' = 3$	0.94	0.57	0.87	0.46	0.13	0.11	0.06

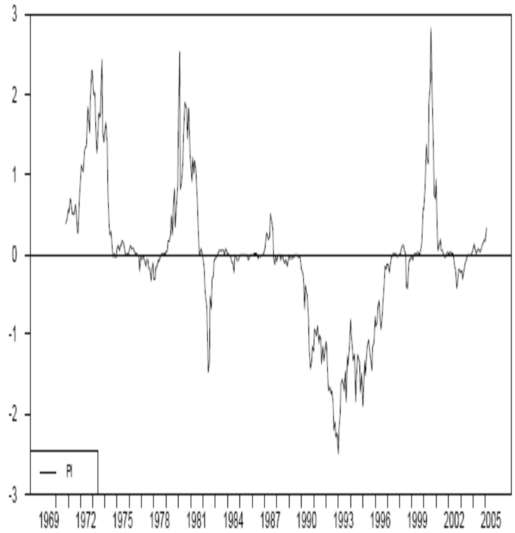
$d' = 4$	0.95	0.64	0.79	0.62	0.05	0.15	0.16
$d' = 5$	0.98	0.54	0.92	0.74	0.11	0.18	0.39
$d' = 6$	0.98	0.47	0.92	0.63	0.14	0.13	0.07
$d' = 7$	0.92	0.45	0.80	0.40	0.29	0.48	0.15
$d' = 8$	0.92	0.29	0.93	0.37	0.04	0.16	0.23
$d' = 9$	0.87	0.53	0.86	0.39	0.11	0.87	0.30
$d' = 10$	0.68	0.43	0.80	0.52	0.03	0.30	0.13
$d' = 11$	0.80	0.41	0.69	0.64	0.03	0.57	0.52
$d' = 12$	0.66	0.32	0.66	0.68	0.07	0.74	0.29

Appendix 6 : Phases of Under- and Overvaluation

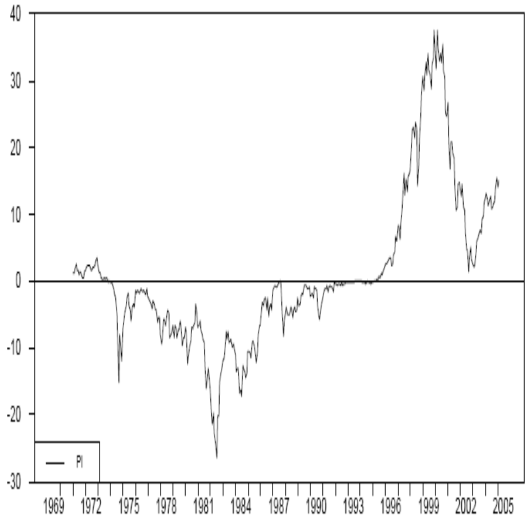
Germany



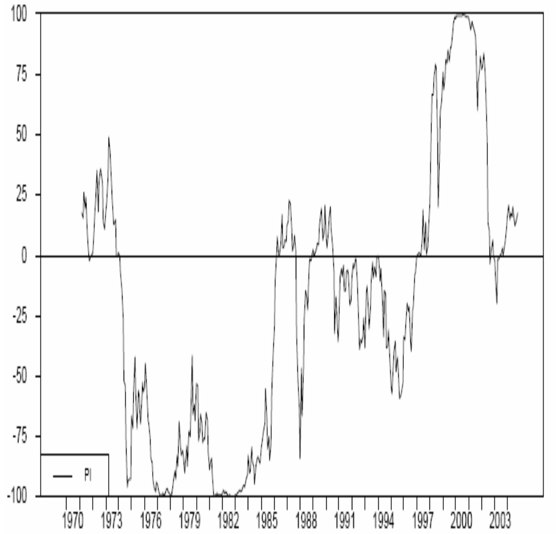
Canada



USA

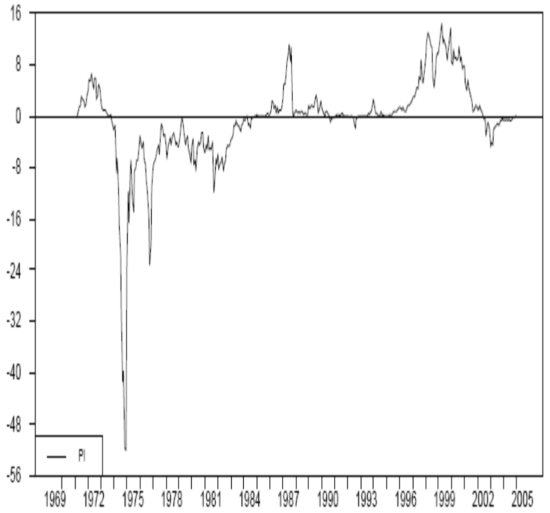


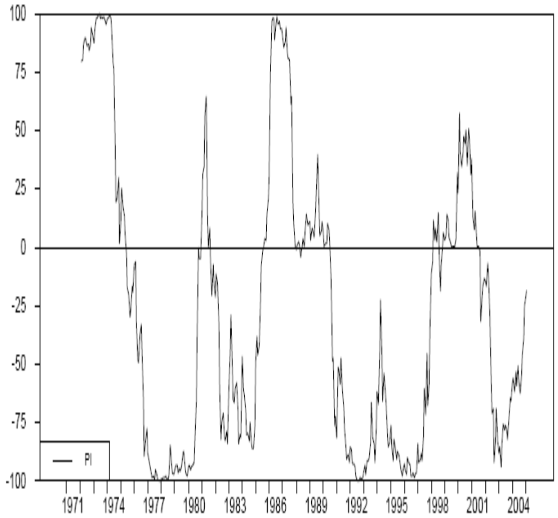
France



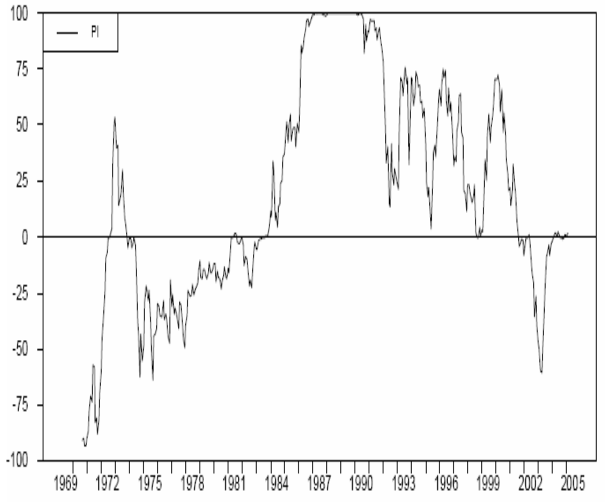
UK

Italy



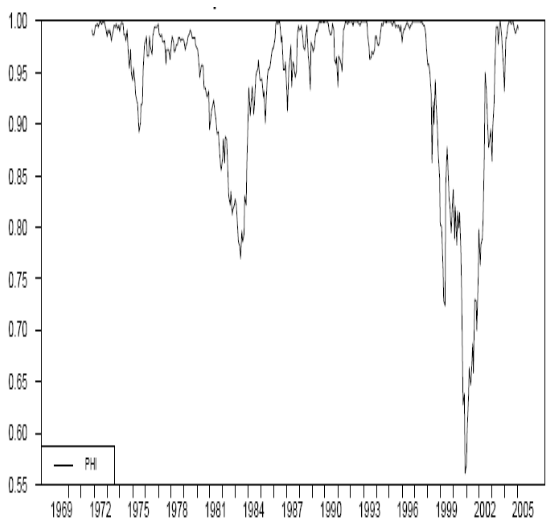


Japan

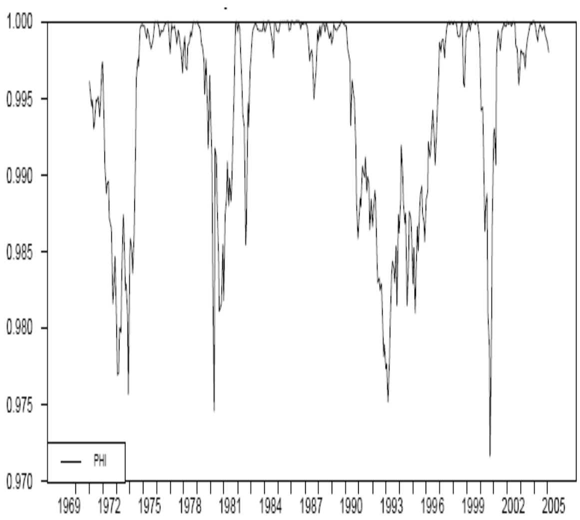


Appendix 7 : Stock Prices Adjustment Strengths

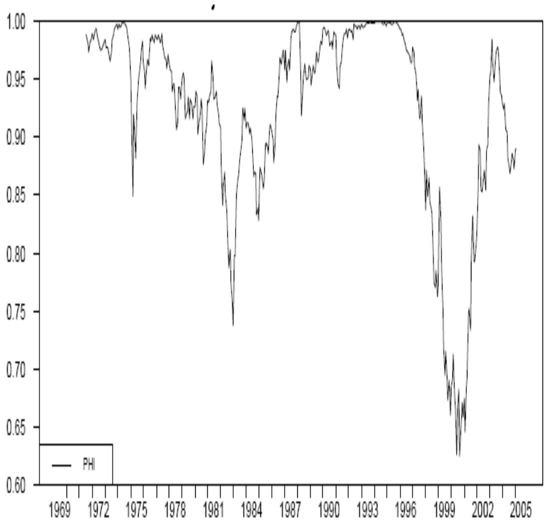
Germany **Canada**



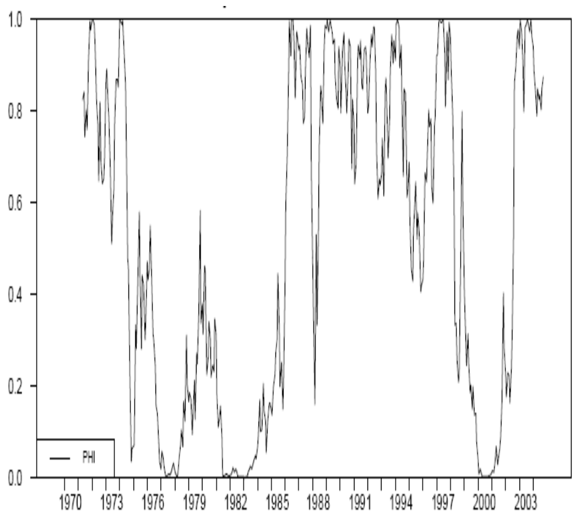
USA



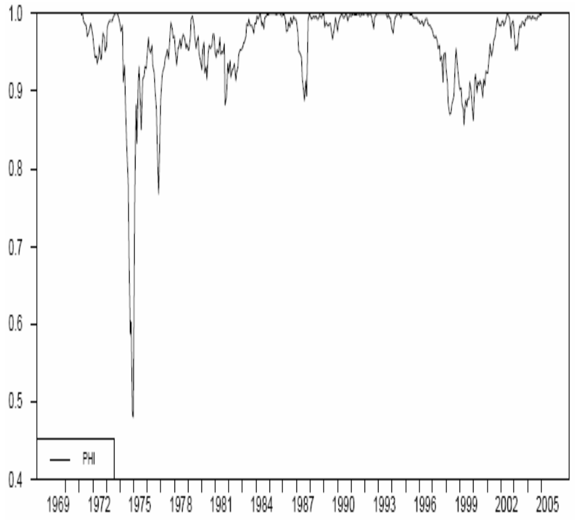
France



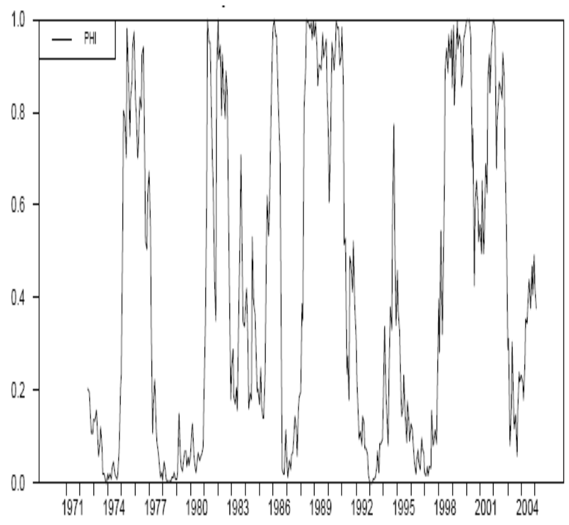
UK

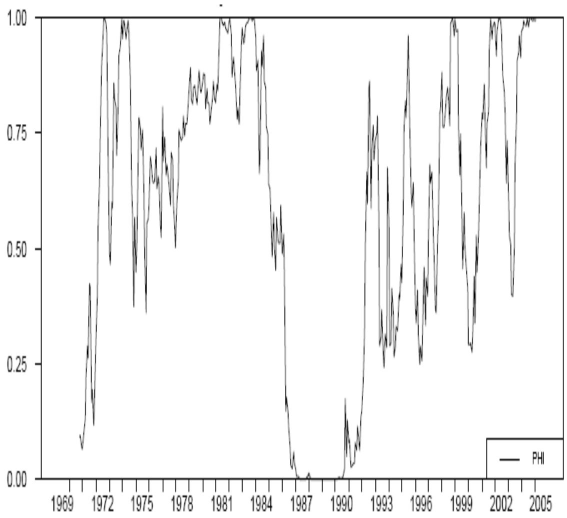


Italy



Japan

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