Welfare improvement properties of an allowance market in a production economy

Antoine Mandel

Documents de Travail du
Centre d'Economie de la Sorbonne

CES Working Papers

Welfare improvement properties of an allowance market in a production economy

Antoine MANDEL

2007.29

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://ces.univ-paris1.fr/cesdp/CES-docs.htm
ISSN : 1955-611X

Welfare improvement properties of an allowance market in a production economy

Antoine Mandel[1] [2]

Centre d'Economie de la Sorbonne, UMR 8174, CNRS-Université Paris 1.

Abstract

This paper studies the welfare improvement properties of a market of allowances in an economy with a single type of externality. We show that thanks to the opening of such a market the Pareto optima can be decentralized as marginal pricing equilibria. However, the set of equilibria is much larger than this of Pareto optima. In order to discriminate the efficient equilibria we introduce a demand revealing mechanism tailored for this framework.

Key Words: General Equilibrium Theory, Pareto optimality, Externalities, Markets of allowances.

[1] The author is grateful to Professor Jean-Marc Bonnisseau for his guidance and many useful comments. All remaining errors are mine.
[2] antoine.mandle@malix.univ-paris1.fr

1 Introduction

It has long been recognized that the interactions between the economic activities and the environment have effects which are not properly reflected by the market prices. Those effects have thus been referred to as externalities. An abundant theoretic literature has focused on the means to overcome this failure of the market, that is to design economic models encompassing externalities whose equilibria have suitable pareto optimality properties. A first branch of the literature pioneered by Arrow [2], proposed the creation of artificial commodities (hereafter called Arrovian commodities) which associate to every couple of agents and every commodity in the economy, the influence caused by the use of this commodity by the first agent on the second. This work and its extensions by Laffont [9], Bonnisseau [3] and others can be seen as a formalization of the Coase theorem in a general equilibrium framework. On another hand, in [4], Boyd and Conley argue that externalities can be defined intrinsically and treated as public bads (or public goods in case of positive externalities). They then introduce markets of allowances for externalities and show that the associate Lindhal equilibria are Pareto optimal.

Contemplating the growing interest in "pollution permits" markets such as the European Union Emission Trading Scheme, one may assume that this literature has influenced the governmental policies dealing with environmental economic issues. Indeed the creation of those markets seem at first sight a direct application of this theoretic work. However, a closer look, brings to light that those markets actually do not feet in any of the models cited above. Indeed they are markets of allowances for public bads in the sense of Boyd and Conley, but in general the allowance is used only as a private good by the polluters. This is underlined by the fact that the justification for the creation of those markets is that they allow for reduction of the pollution at the least possible cost, not that they lead to Pareto optimality. Even when consumers have access to those markets, the free rider problem make it doubtful that a Lindhal equilibrium may be implemented. On the opposite, our approach is to underline the fact that when it creates an allowance market the government also creates a public good consisting in the difference between the situation that prevails under *laissez-faire* and the level of allowances it supplies to the economy. Our aim is to determine wether this particular type of public good provision may lead to Pareto optimal outcomes.

We focus on a general equilibrium model with a finite number of goods and agents. The production of each firm causes a similar external effect on each consumer. The government forces by legal means the producers to use as input the quantity of allowances corresponding to the externality they cause and initially endow the agents with a certain quantity of allowances. Agents may then trade these on a market. We consider various possible structure for the allowance market: the consumers may or may not have access to the market as buyers (i.e use the allowance as a public good), the consumption of allowances by consumers may be subsidized by the government. Building upon a result of Bonnisseau [3], we show that Pareto optima can be decentralized as marginal pricing equilibria in such an economy independently of the market structure. However the analogous of the first welfare theorem does not hold, that is competitive equilibria are not necessarily Pareto optimal. In particular when the consumers have access to the allowance market as buyers, a necessary condition for an equilibrium to be Pareto optimal is paradoxically that no allowance is purchased by the consumers (as in Smith and al. (10)). Hence, the only way to obtain a Pareto optimal outcome is that the government chooses a proper initial allocation. In this respect, we provide a simple mechanism to implement optimality: the government diminishes its supply of allowances proportionally to the consumers purchase.

2 The Model

We consider an economy with a finite number L of commodities labelled by $\ell = 1 \cdots L$, lying within an environment which is characterized by a real parameter τ.

There are n firms in the economy indexed by $j = 1 \cdots n$ whose production possibilities are described by a closed set Y_j. As they produce, firms influence the environment. We measure according to the differentiable function $f_j : \mathbb{R}^L \to \mathbb{R}_-$ the damage caused to the environment by firm j. The actual state of the environment when the firms choose a production scheme $(y_j) \in \prod_{j=1}^n Y_j$ is $\sum_{j=1}^n f_j(y_j)$.

There are m consumers in the economy indexed by $i = 1 \cdots m$. They gain utility from the consumption of strictly positive quantities of commodities 1 to L and are sensitive to the state of the environment. Their preferences are represented by a quasi-concave and differentiable utility function u_i defined on $\mathbb{R}^L_{++} \times \mathbb{R}$ which associates to an environmental parameter $\tau \in \mathbb{R}_-$ and to a bundle of commodities $x_i \in \mathbb{R}^L_{++}$ an utility level $u_i(x_i, \tau)$. We shall denote by $P_i(x_i, \tau)$ the set of elements $\{(x'_i, \tau') \in \mathbb{R}^L_{++} \times \mathbb{R}_- \mid u_i(x'_i, \tau') \geq u_i(x_i, \tau)\}$ corresponding to the pair of state of the environment and commodity bundle preferred to (x_i, τ) by agent i. We shall assume in the remaining of the paper that those utility functions are monotone, locally non-satiated in commodities, increasing with the environment, and that one of them is strictly monotone [3]. This will ensure in paticular equilibrium prices are positive.

The initial resources of the economy are set equal to $\omega \in R^L_{++}$.

The remaining of this paper is concerned with the Pareto optimal outcomes

[3] So that one can apply theorem 3 of Bonnisseau (3).

of this economy defined as:

Definition 1 *An element* $((x_i),(y_j) \in (\mathbb{R}^L_{++})^m \times \prod_{j=1}^n Y_j$ *is an attainable allocation if* $\sum_{i=1}^m x_i = \sum_{j=1}^n y_j + \omega$.
An element $((x_i),(y_j)) \in (\mathbb{R}^L_{++})^m \times \prod_{j=1}^n Y_j$ *is a Pareto optimum if it is an attainable allocation and if there exist no attainable allocation* $((x'_i),(y'_j))$ *such that for all* i, $(x'_i, \sum_{j=1}^n f_j(y'_j)) \in P_i(x_i, \sum_{j=1}^n f_j(y_j))$, *and for at least an* i_0, $(x'_{i_0}, \sum_{j=1}^n f_j(y'_j)) \in int P_{i_0}(x_{i_0}, \sum_{j=1}^n f_j(y_j))$

A result of Bonnisseau (3) entails a general characterization of these Pareto optima:

Theorem 1 (theorem 3 in Bonnisseau (3)) *If* $((\overline{x}_i),(\overline{y}_j))$ *is a pareto optimum of* $\mathcal{E}$, *then there exist* $(\overline{p},\overline{q}) \in \mathbb{R}^{L+1}_+$ *such that*

(1) For all i, *there exist* $\overline{q}_i$ *such that* $(\overline{p},\overline{q}_i) \in N_{P_i(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))}(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))$ *and* $\sum_{i=1}^m \overline{q}_i = \overline{q}$
(2) For all j, $\overline{p} + \overline{q}\nabla f_j(\overline{y}_j) \in N_{Y_j}(\overline{y}_j)$

Building on this result, which extends the optimality properties of marginal pricing (see Guesnerie (6)) to a framework encompassing externalities, we focus on the possibility to decentralize Pareto optima as marginal pricing equilibria for appropriate market structures. Let us recall that marginal pricing equilibria coincide with the standard competitive equilibria under additional convexity assumptions but also allows to deal with increasing returns to scale in the production sector.

Now, when there exist markets for standard commodities only, a price equilibrium with marginal tarification of the economy may be defined as:

Definition 2 *An allocation* $(\overline{x}_i),(\overline{y}_j))$ *in* $(\mathbb{R}^L_{++})^m \times \prod_{j=1}^n Y_j$ *is a marginal pricing equilibrium with transfers if there exist a price* $p \in \mathbb{R}^L_+$ *and a wealth allocation* $(w_1,\cdots,w_m) \in \mathbb{R}^m$ *with* $\sum_{i=1}^m w_i = p\cdot(\omega + \sum_{j=1}^n y_j)$ *such that:*

(1) For every i, $\overline{x}_i$ *maximizes* $u_i(\cdot, \sum_{j=1}^n f_j(\overline{y}_j))$ *among the feasible consumption plans* $\{x_i \in \mathbb{R}^L_{++} \mid p\cdot x_i \le w_i\}$;
(2) For all j, $\overline{p} \in N_{Y_j}(\overline{y}_j)$;
(3) $\sum_{i=1}^m \overline{x}_i = \sum_{j=1}^n \overline{y}_j + \sum_{i=1}^m \omega_i$;

and it is well-known [4] that, when there are non-trivial environmental effects none of these equilibria is Pareto optimal. Indeed, at such an equilibrium the negative external effects of production on the environment are not reflected by the market prices.

[4] what one can also check by comparing the first order necessary conditions for equilibria with the characterization of Pareto optima given in lemma 1.

3 Markets of allowances

A solution to this failure of standard markets is given by the Coase theorem. It advocates the opening of allowance markets on which consumers sells to producers the right to deteriorate the environment against some financial compensation. However the first temptative implementations of the Coase theorem in a general equilibrium framework by Arrow (2) and Laffont (9) requested the opening of one allowance market per commodity and per couple of agents. On those markets agent i and j were supposed to trade the influence on agent j of the use of the good l by agent i. As those authors themselves acknowledge, such a setting is far from realistic. In practice, in the US SO2 market or in the European Union Emission Trading Scheme, a single market of allowances has been opened whose functioning can be summarized as follows.

Firms are forced by legal means to use as input in their production process a quantity of allowances corresponding to their actual influence on the environment. Namely, in order to produce y_j, firm j should use as input a quantity $f_j(y_j)$ of allowances. Meanwhile the government supplies allowances to the economy by initially allocating a quantity A among consumers and producers. This leads to the opening of an allowance market on which firms may purchase from the other agents the quantity of allowances they need to set in motion their production plans.

This type of market, where the allowance bares on an externality whose essence is well defined, has been studied by Boyd and al. in (4) and Conley and al. in (5). These authors treat allowances symmetrically to public good and use Lindhal-like personalized prices in order to obtain decentralization results. Focusing on the symmetry with public goods, it seems to us these authors do not take in consideration an important particularity of the allowance market: by fixing the endowment in allowances the government freely supplies a public good to consumers: the difference between this endowment and the situation that prevails under *laissez-faire.* This particular way of providing public goods partly relax the free-riding problems. Hence, one may obtain decentralization for simple market mechanisms, and it is not necessary to introduce personalized prices.

4 Private Equilibria

Indeed, let us consider the simplest situation where the allowance is exchanged only as a private good among producers. The associated equilibrium concept is:

Definition 3 *An allocation* $((\overline{x}_i), (\overline{y}_j, f_j(\overline{y}_j)))$ *in* $(\mathbb{R}^L_{++})^m \times \prod_{j=1}^n (Y_j \times \mathbb{R}_-)$ *is a* *marginal pricing equilibrium with transfers and private use of the allowance* [5] *if there exist a price* $(\overline{p}, \overline{q}) \in \mathbb{R}_+^{L+1}$ *and a wealth allocation* $(w_1, \cdots, w_m) \in \mathbb{R}^m$ *with* $\sum_{i=1}^m w_i = p \cdot (\omega + \sum_{j=1}^n \overline{y}_j)$ *such that:*

(1) For every i, $\overline{x}_i$ *maximizes* $u_i(\cdot, \sum_{j=1}^n f_j(\overline{y}_j))$ *in the budget set* $\{x_i \in \mathbb{R}_{++}^L \mid p \cdot x_i \leq w_i\}$;
(2) For all j, $\overline{p} + \overline{q}\nabla f_j(\overline{y}_j) \in N_{Y_j}(\overline{y}_j)$;
(3) $\sum_{i=1}^m \overline{x}_i = \sum_{j=1}^n \overline{y}_j + \omega$;
(4) $\sum_{j=1}^n f_j(\overline{y}_j) + A = 0$.

Using the characterization of Pareto optima in lemma 1, it appears clearly that:

Proposition 1 *Every pareto Optimum can be supported as a private marginal pricing equilibrium with transfers, provided the initial allocation of allowances is well chosen.*

Proof: *Given the quasi-concavity of the utility function, a sufficient condition for an* $\overline{x}_i$ *satisfying the budget constraint to solve the consumer problem at a private marginal pricing equilibrium is that there exist* $q_i \in \mathbb{R}$ *such that* $(q_i, \overline{p}) \in N_{P_i(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))}(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))$. *This condition as well as* (2) *and* (3) *are clearly satisfied at a Pareto optimum according to theorem* 1. *Hence given a Pareto optimum, it suffices to choose a wealth allocation letting each consumption plan satisfying the budget constraint and an initial allocation of allowances equal to the firms demand, in order to decentralize it as a marginal pricing equilibrium with transfers and private use of the allowance.* □

Hence whenever the environment acquires (through the allowance market) a value, Pareto optima may be decentralized as marginal pricing equilibria. Nevertheless there remains a huge indeterminacy on the allocations of allowances which entail Pareto optimality. *A priori* the probability to reach a Pareto optima is rather small, so to say, negligeable: when the set of equilibria is a non-empty differentiable manifold, the optimality condition $\sum_{i=1}^m q_i = q$ implies the set of Pareto optimal equilibria is a submanifold of codimension 1. To overcome this indeterminacy, we shall study refined notions of equilibria.

5 Public Equilibria

First, let us determine to which extent the opening of the allowance market to public use by the consumers can diminish the number of non-optimal equilibria. When the allowance market is opened to consumers they may purchase it as a public good in order to improve the state of the environment and their

[5] which we will refer to as private marginal pricing equilibrium for short.

program is turned to maximize, given the other agents purchase of allowances $(\overline{s}_k)_{k\neq i}$, the utility $u_i(x_i, -A+\Sigma_{k\neq i}\overline{s}_k+s_i)$ they get from the consumption bundle $(x_i, s_i) \in \mathbb{R}^L_{++} \times \mathbb{R}_+$ of allowance and regular commodities. Accordingly the equilibrium concept is turned to:

Definition 4 *An allocation $(\overline{x}_i, \overline{s}_i), (\overline{y}_j, f_j(\overline{y}_j)))$ in $(\mathbb{R}^L_{++} \times \mathbb{R}_+)^m \times \prod_{j=1}^n (Y_j \times \mathbb{R}_-)$ is a public marginal pricing equilibrium with transfers [6] if there exist a price $(\overline{p}, \overline{q}) \in \mathbb{R}^{L+1}_+$ and a wealth allocation $(w_1, \cdots, w_m) \in \mathbb{R}^m$ with $\sum_{i=1}^m w_i = (\overline{p}, \overline{q}) \cdot (\omega + \sum_{j=1}^n \overline{y}_j, A + \sum_{j=1}^n f_j(\overline{y}_j))$ such that:*

(1) For every i, $(\overline{x}_i, \overline{s}_i)$ maximizes $u_i(x_i, -A+\sum_{k\neq i}\overline{s}_k+s_i)$ among the feasible consumption plans $\{(x_i, s_i) \in \mathbb{R}^L_{++} \times \mathbb{R}_+ \mid \overline{p} \cdot x_i + \overline{q}s_i \leq w_i\}$;
(2) For all j, $\overline{p} + \overline{q}\nabla f_j(\overline{y}_j) \in N_{Y_j}(\overline{y}_j)$;
(3) $\sum_{i=1}^m \overline{x}_i = \sum_{j=1}^n \overline{y}_j + \omega$;
(4) $\sum_{i=1}^m \overline{s}_i = \sum_{j=1}^n f_j(\overline{y}_j) + A$.

At such a public equilibrium, the first-order conditions for the consumers program are given by the following lemma:

Lemma 1 *Let $\overline{x}_i \in \mathbb{R}^L_{++}$:*

- *The bundle $(\overline{x}_i, \overline{s}_i)$ with $\overline{s}_i > 0$ solves the consumer problem if and only if $(\overline{p}, \overline{q}) \cdot (\overline{x}_i, \overline{s}_i) = w_i$ and $(\overline{p}, \overline{q}) \in N_{P_i(\overline{x}_i, -A+\sum_{i=1}^m \overline{s}_i)}(\overline{x}_i, -A + \sum_{i=1}^m \overline{s}_i)$.*
- *The bundle $(\overline{x}_i, 0)$ solves consumer i program if and only if $\overline{p} \cdot \overline{x}_i = w_i$ and there exist $\overline{q}_i \leq \overline{q}$ such that $(\overline{p}, \overline{q}_i) \in N_{P_i(\overline{x}_i, -A+\sum_{i=1}^m \overline{s}_i)}(\overline{x}_i, -A + \sum_{i=1}^m \overline{s}_i)$.*

Proof: *Given the quasi-concavity of the utility function, first-order conditions are necessay and sufficient. Moreover, non-satiation implies the budgetary constraint is necessarily binding.*

Now at a bundle $(\overline{x}_i, \overline{s}_i) \in \mathbb{R}^{L+1}_{++}$, no other constraint than the budgetary one may be binding so that the bundle is optimal if and only if $(\overline{p}, \overline{q}) \cdot (\overline{x}_i, \overline{s}_i) = w_i$ and $(\overline{p}, \overline{q}) \in N_{P_i(\overline{x}_i, -A+\sum_{i=1}^m \overline{s}_i)}(\overline{x}_i, -A + \sum_{i=1}^m \overline{s}_i)$.

At a bundle $(\overline{x}_i, 0)$ the constraint $\overline{s}_i \geq 0$ is binding so that it is sufficient and necessary for optimality that $\overline{p} \cdot \overline{x}_i = w_i$ and that there exist $\mu_i \geq 0$ such that $(\overline{p}, \overline{q}) \in N_{P_i(\overline{x}_i, -A+\sum_{i=1}^m \overline{s}_i)}(\overline{x}_i, -A + \sum_{i=1}^m \overline{s}_i) + \mu_i(0, 1)$.

The proof then proceeds easily.

This characterization yields that every Pareto optimum can be decentralized as a public equilibrium:

Proposition 2 *Every pareto Optimum can be supported as a public marginal*

[6] *in extenso:* marginal pricing equilibrium with transfers and public use of the allowances

pricing equilibrium with transfers, provided the initial allocation of allowances is well chosen.

Proof: *Let $(\overline{x}_i), (\overline{y}_j)$ be a Pareto optimal allocation. According to proposition* 1 *there exist a price $(\overline{p}, \overline{q})$ and a wealth distribution (w_i) such that it can be decentralized as a private equilibrium. In order to show, that it may also be decentralized as a public equilibrium, it suffices to show that $(\overline{x}_i, 0)$ solves the consumer program at a public equilibrium.*

Now, from the proof of proposition 1 *one knows that there exist $\overline{q}_i \in \mathbb{R}$ such that $(\overline{p}, \overline{q}_i) \in N_{P_i(\sum_{j=1}^n f_j(\overline{y}_j), \overline{x}_i)}(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))$. Using monotonicity of the utility functions, one in fact has $\overline{q}_i \geq 0$ for all i. Using Pareto optimality of the equilibrium, one has $\sum_{i=1}^m \overline{q}_i = \overline{q}$, and hence $\overline{q}_i \leq \overline{q}$. Hence there exist $\overline{q}_i \leq q$ such that $(\overline{p}, \overline{q}_i) \in N_{P_i(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))}(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))$. Using clearance of the allowance market, and the fact that $\overline{p} \cdot \overline{x}_i = w_i$, sufficient condition for $(\overline{x}_i, 0)$ to solve the consumers program are satisfied according to lemma* 1. □

Moreover, lemma 1 also provides a testable necessary condition for optimality. This condition already underlined by Smith and al. in (10) is that at an optimum, consumers are priced out of the allowance market. In other words, a public equilibrium is Pareto optimal only if none of the consumers actually purchase allowances. Namely, one has

Proposition 3 *A public marginal pricing equilibrium $(\overline{x}_i, \overline{s}_i), (\overline{y}_j, f_j(\overline{y}_j)))$ is Pareto optimal only if for all i, $\overline{s}_i = 0$.*

Proof: *Let $(\overline{x}_i, \overline{s}_i), (\overline{y}_j, f_j(\overline{y}_j)))$ be a public marginal pricing equilibrium and $(\overline{p}, \overline{q})$ the corresponding equilibrium price. Let us assume that one of the s_i is positive. It implies according to lemma* 1 *that $(\overline{p}, \overline{q}) \in N_{P_i(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))}(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))$. Now, the regularity of the utility functions imply that $N_{P_i(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))}(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))$ is a half line whenever $(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j)) \in \mathbb{R}_{++}^L \times \mathbb{R}$. In other words, given $\overline{p}$, for all i there exist a single $\overline{q}_i$, such that $(\overline{p}, \overline{q}_i) \in N_{P_i(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))}(\overline{x}_i, \sum_{j=1}^n f_j(\overline{y}_j))$. The strict monotonicity of the utility functions with regards to the environment imply that all those q_i are positive. According to the preceding, one of those is equal to q. Therefore, one necessarily has $\sum_{i=1}^m q_i > q$ and the equilibrium can not be Pareto optimal.* □

Another advantage of the opening of the allowance market is that it guarantees the allowance price will be above the consumers marginal utility for the environment. Hence it strengthens the relation between equilibria and Pareto optima. Indeed while one can associate to every public equilibrium a private equilibrium by subtracting to every consumer endowment in allowances the amount it purchases at equilibrium, one can associate to a private equilibrium a public one only if at this private equilibrium every consumer marginal utility for the environment is greater than the allowance price. In this sense there are fewer public than private equilibria. Consequently the set of pareto

Optima is more closely surrounded by the set of public equilibria than by this of private equilibria. To sum up, the opening of the allowance market to the consumers withdraw part of the indeterminacy on the optimality properties of the equilibria and provides a testable necessary condition for optimality, that consumers are priced out of the allowance market.

6 Subsidized Equilibria

Pursuing in the direction of refinement through public use of the allowance, let us now consider situations where the government amplifies the consumers demand in allowances by diminishing proportionally to the consumers purchase the level of allowances it supplies to the economy. Namely one considers (k_i)-amplified equilibria at which the government announces a diminution of $(k_i - 1)s_i$ of its supply of allowances whenever consumer i purchases s_i allowances. Therefore each consumer considers that when it purchases s_i allowances, the state of the environment is improved of $k_i s_i$. If the consumers do not perceive the amplification of their purchase by the government may have consequences on their wealth (see below) through their endowment in allowances the equilibrium concept is turned to:

Definition 5 *An allocation* $(\overline{x}_i, \overline{s}_i), (\overline{y}_j, f_j(\overline{y}_j)))$ *in* $(\mathbb{R}^L_{++} \times \mathbb{R}_+)^m \times \prod_{j=1}^n (Y_j \times \mathbb{R}_-)$ *is a* (k_i)*-amplified public marginal pricing equilibrium with transfers*[7] *if there exist a price* $(\overline{p}, \overline{q}) \in \mathbb{R}^{L+1}_+$ *and a wealth allocation* $(w_1, \cdots, w_m) \in \mathbb{R}^m$ *with* $\sum_{i=1}^m w_i = (\overline{p}, \overline{q}) \cdot (\omega + \sum_{j=1}^n \overline{y}_j, A - \sum_{i=1}^m k_i \overline{s}_i + \sum_{j=1}^n f_j(\overline{y}_j))$ *such that:*

(1) For every i, $(\overline{x}_i, \overline{s}_i)$ *maximizes* $u_i(x_i, -A + \sum_{h \neq i} k_h s_h + k_i s_i)$ *among the feasible consumption plans* $\{(x_i, s_i) \in \mathbb{R}_+ \times \mathbb{R}^L_{++} \mid \overline{q} s_i + \overline{p} \cdot x_i \leq w_i\}$;
(2) For all j, $\overline{p} + \overline{q} \nabla f_j(\overline{y}_j) \in N_{Y_j}(\overline{y}_j)$;
(3) $\sum_{i=1}^m \overline{x}_i = \sum_{j=1}^n \overline{y}_j + \omega$;
(4) $\sum_{i=1}^m \overline{s}_i = \sum_{j=1}^n f_j(\overline{y}_j) + A$;

At a (k_i)-equilibrium, the first-order conditions for the consumers program are given by the following lemma:

Lemma 2 *Let* $\overline{x}_i \in \mathbb{R}^L_{++}$:

- *The bundle* $(\overline{x}_i, \overline{s}_i)$ *with* $\overline{s}_i > 0$ *solves the consumer problem if and only if* $(\overline{p}, \overline{q}) \cdot (\overline{x}_i, \overline{s}_i) = w_i$ *and* $(\overline{p}, \frac{\overline{q}}{k_i}) \in N_{P_i(\overline{x}_i, -A + \sum_{i=1}^m k_i \overline{s}_i)}(\overline{x}_i, -A + \sum_{i=1}^m k_i \overline{s}_i)$.
- *The bundle* $(\overline{x}_i, 0)$ *solves consumer* i *program if and only if* $\overline{p} \cdot \overline{x}_i = w_i$ *and there exist* $\overline{q}_i \leq \overline{q}$ *such that* $(\overline{p}, \frac{\overline{q}_i}{k_i}) \in N_{P_i(\overline{x}_i, -A + \sum_{i=1}^m k_i \overline{s}_i)}(\overline{x}_i, -A + \sum_{i=1}^m k_i \overline{s}_i)$.

[7] *in extenso:* a marginal pricing equilibrium with transfers and k times amplification of the public use of allowances

Proof: *The proof is similar to this of proposition* 1 *but for the* $\frac{1}{k_i}$ *coefficient whose presence is due to the amplification of the consumers purchases.*

This characterization yields that every Pareto optimum at which consumer i marginal utility for the environment [8] , $\overline{q}_i$, is lower than $\frac{\overline{q}}{k_i}$ can be decentralized as a public equilibrium:

Proposition 4 *Every Pareto optimum at which consumer i marginal disutility ,* $\overline{q}_i$*, is lower than* $\frac{\overline{q}}{k_i}$ *can be supported as a* (k_i)*-amplified equilibrium, provided the initial allocation of allowances is well chosen.*

Proof: *The proof is similar to this of proposition* 2 : *one considers the corresponding private equilibrium and show that the necessary conditions for the consumers program given by lemma* 2 *are satisfied when each consumer actually purchase* 0 *allowances.*

While $\sum_{i=1}^{m} \frac{1}{k_i}$ is greater than 1, one has a testable condition for optimality analogous to this given by proposition 3, an amplified equilibrium is optimal only if at least one consumer is priced out of the allowance market:

Proposition 5 *Let* (k_i) *such that* $\sum_{i=1}^{m} \frac{1}{k_i} > 1$*, a* (k_i)*-equilibrium* $(\overline{x}_i, \overline{s}_i), (\overline{y}_j, f_j(\overline{y}_j)))$ *is Pareto optimal only if there exist i such that* $s_i = 0$*.*

Proof: *The proof is similar to this of proposition* 3.

Remark 1 *In fact, the preceding can be strengthen to: if* I_1 *is a subset of consumers such that* $\sum_{i \in I_1} \frac{1}{k_i} > 1$*, it can not be that every agent in* I_1 *purchases a positive level of allowances.*

According to lemma 2, as $\sum_{i=1}^{m} \frac{1}{k_i}$ decreases towards 1, the constraint baring on the consumers allowances choices become tighter and tighter (as every agent marginal utility must be at least k_i times smaller than the allowance price). There are fewer and fewer equilibria and consequently the set of equilibria surround more and more closely the set of Pareto Optima.

When $\sum_{i=1}^{m} \frac{1}{k_i}$ reaches 1, the (k_i)-amplified equilibria finally satisfy sufficient conditions for optimality in the sense of:

Proposition 6 *Let* (k_i) *such that* $\sum_{i=1}^{m} \frac{1}{k_i} = 1$*. At a* (k_i)*-amplified equilibrium* $(\overline{x}_i, \overline{s}_i), (\overline{y}_j, f_j(\overline{y}_j)))$ *such that for all i,* $\overline{s}_i > 0$*, the necessary conditions of theorem* 1 *hold. If moreover, the production sets and the environmental damages functions are convex, such an equilibrium is Pareto optimal.*

Proof: *Indeed according to lemma* 2*, at such an equilibrium one has* $(\overline{p}, \frac{\overline{q}}{k_i}) \in N_{P_i(\overline{x}_i, \sum_{j=1}^{n} f_j(\overline{y}_j))}(\overline{x}_i, \sum_{j=1}^{n} f_j(\overline{y}_j))$*. Together with the other equilibrium conditions, it implies the necessary conditions of theorem* 1 *are satisfied. As those*

[8] Here $\overline{q}_i$ and $\overline{q}$ are the elements given by theorem 1.

conditions are sufficient for optimality under the additional convexity assumptions on the production, the proof is complete.

Therefore the amplification of consumers purchase by a proportional diminution of the governmental supply of allowances seems a suitable mechanism to implement Pareto optimality. However we have not addressed yet the question of free-riding. Indeed consumers may anticipate that the amplification of their purchases by the government leads to a diminution of their own initial endowment in allowances and hence of their wealth. Taking this fact in consideration they may strategically reduce their purchase of allowances. Such a failure may be easily overcame. It suffices that the government announces it will amplify one agent purchase by diminishing only the other agents initial endowments. Such a mechanism can be related to the matching process described by Guttman in (8).

References

[1] IPCC (2001) IPCC Third Assessment Report, Climate Change 2001

[2] Arrow, K.J (1969) "The organization of economic activity : Issues pertinent to the Choice of Market versus Non-Market allocation", in Joint economic commitee, The analysis and evaluation of Public Expenditures ; The PPB System, Washington DC : Government Printing Office, pp 47-64.

[3] Bonnisseau, J-M. (1994) "Caractérisation des optima de pareto dans une économie avec effets externes." Annales d'économie et de statistique, No. 36

[4] Boyd, J. and Conley, John P. (1997) "Fundamental Nonconvexities in Arrovian Markets and a Coasian Solution to the Problem of Externalities" Journal of Economic Theory, Vol. 72, 1997, pp. 388-407.

[5] Conley John P and Stefani C. Smith, (2005). "Coasian equilibrium", Journal of Mathematical Economics, vol. 41(6), pp 687-704.

[6] Guesnerie, R. (2003) (sous la direction de) "Kyoto et l'économie de l'effet de serre ", Conseil d'analyse économique no. 39, Paris, La Documentation française.

[7] Guesnerie,R. (1975) "Pareto optimality in non-convex economies", Econometrica, 1975 vol 43, pp 1-29

[8] Guttman, J. (1978) "Understanding Collective Action: Matching Behavior," American Economic Review, vol 68, pp 251-55.

[9] Laffont, J, J. (1978) "Effets externes et théorie économique", Monographie du Séminaire d'économètrie. Centre National de la Recherche Scientifique, Paris.

[10] Smith, S. and Yates, A. (2003) "Should Consumers Be Priced Out of Pollution Permit Markets?" Journal of Economic Education Volume 34, No. 2 pp: 181-189