Production Externalities and Expectations Application to the Economics of Climate Change

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Abstract

In this paper, we extend the problem of decentralization of Pareto optima in an economy with production externalities to the case where the production capacities upon which Pareto optimality is defined may differ from the aggregate of the firms expectations about their production possibilities. This issue is raised in order to deal with the seemingly diiferent exectations of firms and governments about the economic consequences of climate change. We show the government can create a "production allowance" market in order to force the firms to produce in a way it considers as optimal. The results are then applied to the analysis of the economic and welfare consequences of climate change.

Key Words: General Equilibrium Theory, Pareto optimality, Externalities.

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1 Introduction

This paper focuses on the following decentralization problem: given initial resources ω and a set of production technologies Z, how can the Pareto optima with regards to Z and ω be decentralized as competitive equilibria in an economy with general externalities where the individual production technologies are given by correspondences Y_j, sensitive to the other firms production choices? The standard decentralization problem à la Arrow-Laffont is encompassed in this setting when $Z = \{z \in \mathbb{R}^L \mid \exists(y_j) \in \prod_{j=1}^n Y_j(y_{-j}) \ s.t \ \sum_{j=1}^n y_j = z\}$, but it also allows us to deal with a more general problem when Z is a strict subset of $\{z \in \mathbb{R}^L \mid \exists(y_j) \in \prod_{j=1}^n Y_j(y_{-j}) \ s.t \ \sum_{j=1}^n y_j = z\}$. In the latter case, Z can be interpreted as the information the government has gained on the aggregate production possibilities in the economy through statistics and economic studies. It may be less optimistic than the aggregate of the firms expectations Y_j on the production possibilities, which may be erroneous because firms are imperfectly informed of the long term production possibilities or do not compute accurately the external effects they face.

The general equilibrium literature on decentralization with externalities was pioneered by Arrow (2) which builds upon the idea of the Coase theorem (8) and considers the decentralization of externalities as a problem of missing market. Arrow defines external effect as a relative notion: the influence of the use of good x by agent a on agent b, and therefore proposes the opening of one market of external effect per commodity and per couple of agents as a mean to restore Pareto optimality. Laffont (11) and Bonnisseau (4) extended this analysis to encompass consumption externalities and non-convexities. Another approach is this of Boyd and Conley (7) which consider externalities as a well defined physical entity: smoke, sulfur dioxide or the flowers of an orchard. They propose the use of allowances for externalities as public goods by the pollutees in order to implement Pareto optimality at a Lindhal like equilibrium. Now all those authors only study the case where $Z = \{z \in \mathbb{R}^L \mid \exists(y_j) \in \prod_{j=1}^n Y_j(y_{-j}) \ s.t \ \sum_{j=1}^n y_j = z\}$ and propose solution concepts which require the opening of a large number of markets and are subject to market failures, due to the exiguity of the market and the presence of non-convexities in the case of Arrow and followers and to the free-riding problem in the case of Boyd and Conley.

The main contribution of this paper is to introduce the possibility of differences between the set of efficient aggregate production techniques Z and the aggregate of the firms expectations about their production possibilities $\{z \in \mathbb{R}^L \mid \exists(y_j) \in \prod_{j=1}^n Y_j(y_{-j}) \ s.t \ \sum_{j=1}^n y_j = z\}$. Also the decentralization mechanism we propose is based on the opening of a single allowance market.

The motivation for introducing a distinction between the set of efficient production techniques Z and the aggregate of of the firms expectations Y_j comes from the following remark on the economics of climate change: many of the po-

tential consequences of climate change, such as changes in agricultural yields and in localization of crops, disruption of ecosystems or increased vulnerability of physical capital (see the IPCC report (1) for an extensive list) are likely to affect the production possibilities of economies. On the other hand, the production sector is partly responsible for climate change because of its greenhouse gases emissions. We therefore have a typical production externality. Moreover, the polluters, energy intensive industries, are well identified. However the potentially damaged firms have never claimed for a compensation and have even less advocated the opening of markets of allowances thanks to which they could influence the state of the environment. On the contrary, markets of allowances for greenhouse emission gases have been launched by governments after they had limited the firms greenhouse gases emission allowances. The central idea of this paper is that this divergence between market and public concern comes from the fact that both have different expectations on the influence of climate change on future production possibilities. That is the lack of a spontaneous creation of an emission market can be interpreted as an aggregate expectation of the production sector that losses due to climate change are not considerably higher than the transaction costs associated with the operation of an emission market. Public action is then unnecessary if the government shares this opinion of the production sector on the influence of climate change. On the contrary, we argue that the government judges it is necessary for him to intervene because it doesn't share the aggregate beliefs of the producers on their inter-temporal production possibilities. It is indeed less optimistic.

This conclusion is the rational to build a model which encompasses differences between the efficient aggregate productions, which represent the government expectations, and the aggregate of the firms expectations about production possibilities. We identify thanks to the standard first and second welfare theorems the Pareto optima of the economy with the competitive equilibria with regards to the "government production set" Z and focus on the decentralization of those Pareto optima in the economy with production externalities described by production correspondences Y_j. A possibly puzzling feature of the model is that, until the application to climate change of the last section, we do not introduce time explicitly. Even though the problematic is clearly intertemporal, the density of the general equilibrium model allows us to encompass time implicitly by considering that the goods are dated and that there exist a complete set of markets. The only rational that would remain to introduce explicitly time is to account for incompleteness of financial markets, but it seems to us this would unnecessarily complicate the analysis.

The solution concept we propose is the opening of a single market of "production allowances" which represent the right to lead the aggregate production away from the efficiency frontier. Indeed the distance to the efficiency frontier can be seen as a summary of the quantity of "bad" in the economy. This solution concept is on two grounds inspired by the work of Luenberger. First,

the definition of the production allowance is related to the shortage function introduced in the literature by Luenberger (12) and Bonnisseau-Cornet (6). Second the production allowance "alters the individual [profit] functions so that they correspond to the appropriate social [profit] functions" and "Once individual [profit] functions are corrected, individual actions, designed to maximize these functions, will lead to Pareto efficiency." (see Luenberger ((13)). Indeed we show that the opening of a "production allowance" market allows for decentralization of Pareto optima when the firms are more "optimistic" than the government as well as in the standard setting of economies with externalities.

In the last section, we further specify the model and consider explicitly an economy undergoing climate change. In this framework the production allowance market is needed to transfer the government expectations about climate change to the firms but an emission allowance market or markets for external effects à la Arrow can then be used in order to allocate efficiently the cost of reducing externalities. A tentative interpretation of our results in this framework is to state that a precise view of the actions needed to adapt to climate change is necessary for the firms to address efficiently the mitigation issue.

2   The model

We consider a general equilibrium economy [2] with a finite number of goods indexed by $\ell = 1 \cdots L$, a finite number of producers indexed by $j = 1 \cdots n$, a finite number of consumers indexed by $i = 1 \cdots m$ and a government.

We allow for general externalities between producers and therefore represent, following Arrow (2) and Laffont (11), firm j production capacities by a correspondence $Y_j : (\mathbb{R}^L)^{(n-1)} \to \mathbb{R}^L$. It associates to an environment [3] $y_{-j} \in (\mathbb{R}^L)^{(n-1)}$ corresponding to the other firms production choices, the set $Y_j(y_{-j}) \subset \mathbb{R}^L$ of production plans firm j then considers as feasible. Such a representation allows to encompass every possible relation between the production process and the environment. We shall assume those characteristics satisfy the standard assumptions needed to define competitive behavior in presence of externalities (11):

Assumption (P) *For all j, Y_j is lower semi-continuous, has a closed graph*

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[2] Notations: $\mathbb{R}^L_{++}$ will denote the positive orthant of $\mathbb{R}^L$ and $\mathbb{R}^L_+$ its closure. Given an index set A and a family of elements indexed by A $(x_a)_{a \in A}$, x_{-a} denotes the family of elements indexed by $A - \{a\}$, $(x_b)_{b \in A - \{a\}}$. Given a convex set X and $x \in X$, $N_X(x)$ denotes the normal cone to X at x and $T_X(x)$ the tangent cone to X at x.

[3] y_{-j} denotes the vector $(y_1, \cdots, y_{j-1}, y_{j+1}, \cdots y_n) \in (\mathbb{R}^L)^{(n-1)}$.

and convex values. One has the possibility of inaction: $0 \in Y_j(0)$ *and free-production is impossible asymptotically:* [4] *for all* $(\zeta_j) \in (\mathbb{R}^L)^n$, $\mathcal{A}(\prod_{j=1}^n Y_j(\zeta_{-j})) \cap \{(\zeta_j) \in (\mathbb{R}^L)^n \mid \sum_{j=1}^n \zeta_j \geq 0\} = \{0\}$.

The consumers are standard utility maximizers. Following Arrow (2) we do not take in consideration externalities in the consumption sector as our main concern is the efficiency of the production process. Agent i consumption set is $\mathbb{R}_+^L$ and its preferences are represented by an utility function $u_i : \mathbb{R}_+^L \to \mathbb{R}$. We assume:

Assumption (C) *For all i, u_i is continuous, quasi-concave and locally non-satiated. At least one of the u_i is strictly monotone.*

The initial resources of the economy are set equal to $\omega \in \mathbb{R}_{++}^L$.

On the other hand, we introduce the set, $Z \subset \mathbb{R}^L$, of production plans the government considers as feasible in the aggregate. We assume it fits into a framework *à la Arrow-Debreu* (9):

Assumption (G) *Z is closed, convex, satisfies free-disposability, production irreversibility and possibility of inaction.*

We shall also assume the government only anticipates production plans that are technically feasible from the producers point of view:

Assumption (Decentralizability) *For all $z \in Z$, there exist $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n y_j = z$.*

That is, the government can not be more "optimistic" than the firms.

2.1 The government point of view

With regards to the government production set, Z, an allocation $(\overline{x}_i) \in (\mathbb{R}_{++}^L)^m$ is Pareto optimal if $\sum_{i=1}^m \overline{x}_i - \omega \in Z$ and if there does not exist an allocation $x_i' \in \mathbb{R}_+^L$ with $\sum_{i=1}^m x_i' - \omega \in Z$ such that $u_i(x_i') \geq u_i(\overline{x}_i)$ for all i with a strict inequality for at least an i_0. Note that we focus on the Pareto optima lying in the interior of the consumption sets. So that, according to the seminal first and second welfare theorems (9), the set of those Pareto optima coincide with the competitive equilibria of an economy whose production set is Z. That is:

Proposition 1 *An allocation $(\overline{x}_i) \in (\mathbb{R}_{++}^L)^m$ is Pareto optimal if and only if there exist an aggregate production plan $\overline{z} \in Z$, a price $\overline{p} \in \mathbb{R}_{++}^L$ and an assignment of wealth levels $(w_1, \ldots, w_m)$ with $\sum_{i=1}^m w_i = \overline{p} \cdot (\overline{z} + \omega)$ such that:*

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[4] $\mathcal{A}Z$ denotes the asymptotic cone to Z.

(1) $\overline{z}$ maximizes profit at price $\overline{p}$ in Z
(2) $\overline{x}_i$ maximizes u_i in the budget set $\{x_i \in \mathbb{R}^L_+ \mid p \cdot x_i \le w_i)\}$
(3) $\sum_{i=1}^m \overline{x}_i = \overline{z} + \omega$

An allocation $((\overline{x}_i), \overline{z}, \overline{p})$ satisfying conditions (1) to (3) is a competitive equilibrium "from the government point of view": it is the type of outcome which should emerge if the government expectations about the production possibilities are accurate and if the economy follows an efficient productive and exchange process. Note that the existence of such an equilibrium, and hence of a Pareto Optimum, is a direct consequence of the standard existence proof *à la* Arrow-Debreu under assumptions [C] and [G]. In the following, taking the government point of view, we investigate which policies the government can implement in order to promote the decentralization of these Pareto optima.

2.2 Production Allowance Market

In our framework, competitive behavior of the firms may lead to two types of failures. The first is a seminal problem in presence of externalities: the improper aggregation by the commodities prices of the cost of external effects leads to improper internalization of those effects by the firms (see Laffont (11)). It may then be that the decentralized choices of the firms lead to an aggregate production below the efficiency frontier ∂Z.

A second type of failure may occur when the firms are *over-optimistic* in the sense that Z is a strict subset of $\{z \in \mathbb{R}^L \mid \exists (y_j) \in \prod_{j=1}^n Y_j(y_{-j}) \ s.t \ \sum_{j=1}^n y_j = z\}$. It can then be that firms choices correspond to an aggregate production outside Z which firms might, from the government point of view, finally fail to produce. This may well disorganize the whole economy and thus lead to heavy welfare losses. The consideration of such a failure is consubstantial to the distinction we make between the government and the firms production sets (which may in particular correspond to different beliefs on the extent of external effects).

Hence, the government objective is to maintain the aggregate production on the thin line drawn by the boundary of Z in between inefficiency and unrealizability. Due to the welfare losses they cause, inefficiency and unrealizability may be considered as public bads. It then is very tempting, thinking of the Coase theorem and of the previous general equilibrium literature on decentralization with externalities (2), (7), (9), to consider the use of a market of allowances as a mean to overcome these failures. Moreover, given the duality between inefficiency and unrealizability, a single market might well be sufficient to overcome both failures.

In all generality, we can describe the creation of an allowance market as follows. The government defines throw an "allowance function" $h_j : (\mathbb{R}^L)^n \to \mathbb{R}$, the

quantity of allowances $h_j(y_j, \overline{y}_{-j})$ firm j should use as input in order to produce y_j within an environment $(\overline{y}_{-j})$. That is firm j production correspondence is turned to $G_j : (\mathbb{R}^L)^{n-1} \to \mathbb{R}^{L+1}$ defined by

$$G_j(\overline{y}_{-j}) = \{(y_j, \alpha_j) \in \mathbb{R}^{L+1} \mid y_j \in Y_j(y_{-j}) , \ \alpha_j \leq -h_j(y_j, \overline{y}_{-j})\}.$$

Competitive behavior of the producers is well defined in this setting provided the allowance function satisfies the following assumption:

Assumption (Allowance) *For all j, for all $(\overline{y}_j) \in (\mathbb{R}^L)^n$ the mapping $h_j(\cdot, \overline{y}_{-j})$ is continuous, convex and satisfies $h_j(0,0) = 0$.*

On the other hand, the government supplies the economy with a quantity $A \in \mathbb{R}$ of allowances by initially allocating the agents (If $A < 0$ one should consider the government imposes "initial obligations"). Trades occur on the market so that each agent might fulfill its requirements. Given a governmental supply of allowances $A \in \mathbb{R}$, we can define a price equilibrium of the economy with production allowances as:

Definition 1 (Price Equilibrium with Allowances) *A collection of production plans $(\overline{y}_j, \overline{\alpha}_j) \in \prod_{j=1}^n G_j(y_{-j})$ together with a collection of consumption plans $(\overline{x}_i) \in (\mathbb{R}^L_{++})^m$ is a Price Equilibrium with Allowances if there exist a price $(\overline{p}, \overline{q}) \in \mathbb{R}^{L+1}_{++}$ and an assignment of wealth levels $(w_1, \ldots, w_m)$ with $\sum_{i=1}^m w_i = (\overline{p}, \overline{q}) \cdot (\sum_{j=1}^n \overline{y}_j + \omega, \sum_{j=1}^n \overline{\alpha}_j + A)$ such that:*

(1) For all j, $(\overline{y}_j, \overline{\alpha}_j)$ maximizes profit, $(\overline{p}, \overline{q}) \cdot (y_j, \alpha_j)$, in $G_j(\overline{y}_{-j})$;
(2) For all i, $\overline{x}_i$ maximizes $u_i(x_i)$ in the budget set $\{x_i \in \mathbb{R}^L_+ \mid p \cdot x_i \leq w_i\}$;
(3) $\sum_{i=1}^m \overline{x}_i = \sum_{j=1}^n \overline{y}_j + \omega$;
(4) $\sum_{j=1}^n \overline{\alpha}_j + A = 0$.

2.3 Example of Allowance Functions

The model can represent the actual markets of allowances for greenhouse gases emissions in Europe or SO2 emissions in the united states, but the types of allowance we shall consider to obtain positive decentralization results are more elaborate and more abstract. Their construction is based on the idea that the level of "bad" in the economy can always be measured by the distance between the actual production and the production efficiency frontier ∂Z.

In order to construct such allowance functions, the shortage function as defined in Luenberger [10] and in Bonnisseau-Cornet [4] proves to be very useful. Given, a reference bundle of commodities $\gamma \in \mathbb{R}^L_{++}$, the shortage function for Z is defined on $\mathbb{R}^L$ by

$$g(z) = min\{s \in \mathbb{R} \mid z - s\gamma \in Z\}.$$

It provides an intrinsic measure of the distance between the actual production and the frontier of Z and can be interpreted wether as how many reference commodity bundles will fail to be produced when the producers are over-optimistic wether as how many more reference commodity bundles could be produced if the external effects were properly internalized. It moreover characterize Z in the sense of the following lemma whose proof is straightforward:

Lemma 1 *Under assumption* (G), *g is a convex and continuous function such that:*

(1) $z \in Z$ if and only if $g(z) \leq 0$;
(2) For every $z \in \partial Z$, $N_Z(z) = < \partial g(z) >$.

Based on this shortage function, one can define firm j allowance function by:

- A share in the aggregate level of "bad"

$$h_j^1(y_j, \overline{y}_{-j}) = \frac{g(y_j + \sum_{k \neq j} \overline{y}_k)}{n}$$

- The difference between the aggregate level of "bad" when it produces and this when it does not produce

$$h_j^2(y_j, \overline{y}_{-j}) = g(y_j + \sum_{k \neq j} \overline{y}_k) - g(\sum_{k \neq j} \overline{y}_k).$$

- A convex and increasing transformation of the preceding:

$$h_j^3(y_j, \overline{y}_{-j}) = \phi(g(y_j + \sum_{k \neq j} \overline{y}_k)) - \psi(g(\sum_{k \neq j} \overline{y}_k)).$$

Thanks to the properties of the shortage function, those functions satisfy the assumption (Allowance) and the assumption (Exact Compensation) introduced below.

2.4 Decentralization of Pareto optima

In order to allow for decentralization of Pareto optima, the allowance must compensate the differences between the aggregate (relative to Z) marginal rates of substitution and the individual ones (relative to Y_j):

Assumption (Compensation) *For every production plan z associated to a Pareto Optimum, there exist $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ with $\sum_{j=1}^n y_j = z$ and $\lambda > 0$ such that for all j,*

$$N_Z(\sum_{j=1}^n \overline{y}_j) \quad \subset \quad N_{Y_j(y_{-j})}(y_j) + \lambda \partial h_j(\cdot, y_{-j})(y_j).$$

Note that the allowance functions h^1, h^2, and h^3 introduced in the preceding satisfy this condition as they are constructed upon the transformation function for Z and satisfy $<\partial h_j(\cdot, \overline{y}_{-j})(\overline{y}_j) >= N_Z(z)$, while for all $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$, one has $0 \in N_{Y_j(\overline{y}_{-j})}(\overline{y}_j)$.

This condition is in fact sufficient to obtain a decentralization result. One has:

Theorem 1 *Assume assumptions (P), (C), (G), (Decentralizability), (Allowance) and (Compensation) hold. Any Pareto Optimum can be decentralized as an equilibrium with allowances.*

Proof: *Let $(\overline{z}, (\overline{x}_i))$ be a Pareto optimal allocation and $\overline{p}$ the associate equilibrium price given by proposition 1. One clearly has $\overline{p} \in N_Z(\overline{z})$. Under assumption (Decentralizability) and (Compensation) there exist $(\overline{y}_j) \in \prod_{j=1}^n Y_j(\overline{y}_{-j})$ and $\overline{q} > 0$ such that $\sum_{j=1}^n \overline{y}_j = \overline{z}$, and $\overline{p} \in N_{Y_j}(\overline{y}_{-j}) + \overline{q}\partial h_j(\cdot, \overline{y}_{-j})(\overline{y}_j)$. Due to the convexity of $G_j(\overline{y}_{-j})$, this is a sufficient condition for $(\overline{y}_j, -h_j(\overline{y}_j, \overline{y}_{-j}))$ to maximize profit at price $(\overline{p}, \overline{q})$ in $G_j(\overline{y}_{-j})$. Choosing A such that $A = \sum_{j=1}^n h_j(\overline{y}_j, \overline{y}_{-j})$ the allowance market is cleared and one can implement the wealth distribution $(\overline{p} \cdot \overline{x}_1, \ldots, \overline{p} \cdot \overline{x}_n)$ in order to implement the equilibrium consumption $\overline{x}_i$ as solutions to the consumers problems.*

Hence, the opening of an allowance market based on one of the functions h_1, h_2 or h_3 allows the decentralization of the Pareto optima.

Moreover, in the differentiable case, (Compensation) is necessary to obtain a complete decentralization result:

Theorem 2 *Assume Z has a smooth boundary [5] and one of the utility functions is smooth[6] and strictly concave. If (Compensation) does not hold, there exist at least a Pareto Optimum which can not be decentralized as an equilibrium with allowances.*

Proof: *Assume (Compensation) does not hold, that is there exist a Pareto optimal allocation $(\overline{z}, (\overline{x}_i))$, and an associated price $\overline{p}$ such that for every $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ with $\sum_{j=1}^n y_j = \overline{z}$ and for every $\lambda > 0$ there exist j such that $N_Z(\sum_{j=1}^n y_j) \not\subset N_{Y_j(y_{-j})}(y_j) + \lambda \partial h_j(\cdot, y_{-j})(y_j) >$. As Z is smooth, $N_Z(\sum_{j=1}^n y_j)$ is a half-line and hence one in fact has that whatever (y_j) and λ may be, for some j one has: $N_Z(\sum_{j=1}^n y_j) \cap N_{Y_j(y_{-j})}(y_j) + \lambda \partial h_j(\cdot, y_{-j})(y_j) = \emptyset$.*

Now, assume $(\overline{z}, (\overline{x}_i))$, can be decentralized as an equilibrium with allowances, $((y_j, \alpha_j), (\overline{x}_i))$. The strict concavity and the smoothness of one of the utility function imply the equilibrium price must be colinear to the price $\overline{p}$ given by proposition 1. Hence the equilibrium price must be of the form $(\overline{p}, q)$ for some $q > 0$. This equilibrium price must satisfy for every j, the first order condition for profit maximization at $(y_j, \alpha_j) : \overline{p} \in N_{Y_j(y_{-j})}(y_j) + q\partial h_j(\cdot, y_{-j})(y_j)$. This

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[5] That is ∂Z is a C^2-submanifold of $\mathbb{R}^L$ of codimension 1.
[6] C^2.

contradicts the preceding and hence ends the proof.

Hence the use of an allowance function satisfying (Compatibility) is a necessary and sufficient condition to obtain a complete decentralization result thanks to the opening of a single allowance market.

On the other hand, as in Laffont (11) decentralization results may also be obtained through the setting of an appropriate tax scheme on the firms. Indeed, consider that given an environment $\overline{y}_{-j}$, firm j is forced to pay a tax equal to $\lambda h_j(y_j, \overline{y}_{-j})$, the benefits of those taxes being allocated to consumers. One can then define a price equilibrium with production tax as :

Definition 2 (Price Equilibrium with tax) *A collection of production plans* $(\overline{y}_j) \in \prod_{j=1}^{n} Y_j(\overline{y}_{-j})$ *together with a collection of consumption plans* $(\overline{x}_i) \in (\mathbb{R}_{++}^{L})^{m}$ *is a price equilibrium with production tax if there exist a price* $\overline{p} \in \mathbb{R}_{+}^{L}$ *a level of tax* $\lambda > 0$ *and an assignment of wealth levels* $(w_1, \ldots, w_m)$ *with* $\sum_{i=1}^{m} w_i = \overline{p} \cdot \sum_{j=1}^{n} \overline{y}_j$ *such that:*

(1) For all j, $\overline{y}_j$ maximizes $\overline{p} \cdot y_j - \lambda h_j(y_j, \overline{y}_{-j})$ in $Y_j(\overline{y}_{-j})$;
(2) For all i, $\overline{x}_i$ maximizes $u_i(x_i)$ in the budget set $\{x_i \in \mathbb{R}_{+}^{L} \mid \overline{p} \cdot x_i \leq w_i\}$;
(3) $\sum_{i=1}^{m} \overline{x}_i = \sum_{j=1}^{n} \overline{y}_j + \omega$.

One then has

Theorem 3 *Assume assumptions (P), (C), (G), (Decentralizability), (Allowance) and (Compensation) hold. Any Pareto Optimum can be decentralized as a Price Equilibrium with production tax.*

Proof: *Let $(\overline{z}, (\overline{x}_i))$ be a Pareto optimal allocation and let us consider according to theorem 2 an equilibrium with allowances, $((\overline{p}, \overline{q}), (\overline{y}_j, \overline{\alpha}_j), (\overline{x}_i))$ which decentralize $(\overline{z}, (\overline{x}_i))$. Setting $\lambda = \overline{q}$ and implementing the revenue scheme $(p \cdot \overline{x}_i)$, such an equilibrium may be supported as an equilibrium with production tax as the consumers and producers programs are equivalent to those at the corresponding equilibrium with production allowances.*

In fact the tax scheme is chosen such that firm j has to pay the exact amount it was spending in production allowances at the competitive equilibrium decentralizing the Pareto Optimum under consideration. Now, the setting of efficient taxes is less convincing than the market decentralization as no mechanism can be used in order to determine the optimal tax rate.

3   First Welfare Like Theorems

The decentralization results *à la* Arrow-Laffont, (2) and (11), rely on the confrontation of supply and demand for external effects. Therefore the standard

first welfare theorem provide a strong intuition that a first welfare theorem will also hold in their framework. It is not the case here: the allowance market drives the price to a Pareto Optimum supporting direction but nothing guarantees that the equilibrium production always lies on the efficiency frontier ∂Z. In this section, we consider two means the government can use to strengthen its influence on the allowance market and on the equilibrium outcome. The first one, through quantities, is to choose adequately the initial allocation of allowances. The second one, through prices, is to provide additional allowances in exchange of commodity bundles and hence to influence the equilibrium relation between the allowance and the commodities prices. By either of these means, one can obtain a first welfare like theorem.

3.1   Choice of the initial allocation in allowances

By fixing the initial allocation of allowances at a suitable level, the government can control the efficiency of the production process, provided the level of allowances used characterize exactly the efficiency of the production process:

Assumption (Characterized Efficiency) *There exist $\overline{A} \in \mathbb{R}$ such that for all $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ one has:*

$$\sum_{j=1}^n h_j(y_j, y_{-j}) = \overline{A} \Leftrightarrow \sum_{j=1}^n y_j \in \partial Z$$

This assumption always holds for the allowance function h^1 defined above but not necessarily for h^2 or h^3.

On another hand, one must guarantee there will not exist equilibrium prices that differ from the aggregate marginal cost of production. Two conditions are necessary therefore. First, the influence of the allowance market must coincide with the aggregate marginal cost of production. That is one must have:

Assumption (Exact Compensation) *For all $y_j \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n y_j \in \partial Z$, $\partial h_j(\cdot, y_{-j})(y_j)$ is equal among all j and one has $N_Z(\sum_{j=1}^n \overline{y}_j) =< \partial h_j(\cdot, \overline{y}_{-j})(\overline{y}_j) > .$*

This condition is also satisfied by h^1 (but also by h^2 and h^3).

Second, the producers behavior shall be governed by the allowance market. Therefore we shall posit that:

Assumption (Over Optimism) *For all $y_j \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n y_j \in \partial Z$, one has $T_Z(\sum_{j=1}^n y_j) \subset \sum_{j=1}^n T_{Y_j(y_{-j})}(y_j)$ (or equivalently $\bigcap_{j=1\cdots n} N_{Y_j(y_{-j})}(y_j) \subset N_Z(\sum_{j=1}^n y_j)$)*

This assumption is satisfied in particular when one of the individual technical constraint is not binding. For example, in our framework, the over-optimism of the producers and/or the presence of externalities (see section(4)) are likely to lead to an interiority condition of the type for all $y_j \in \prod_{j=1}^{n} Y_j(y_{-j})$ such that $\sum_{j=1}^{n} y_j \in \partial Z$, one has for all j, $y_j \in int Y_j(y_{-j})$, which clearly implies (Over Optimism). Also note that this assumption is labeled (Over-Optimism) as it can be interpreted as stating the firms production sets encompass locally the government one.

With those three additional assumptions, one can guarantee that for a well chosen supply of allowances, the equilibria with allowances always are Pareto optimal:

Theorem 4 *Assume assumptions, (P), (C), (G), (Allowance), (Characterized Efficiency), (Over Optimism) and (Exact Compensation) hold. There exist an initial supply of allowances $\overline{A}$ such that $(y_j, \alpha_j), (x_i)$ is an equilibrium with allowances if and only if $(\sum_{j=1}^{n} y_j, (x_i))$ is a Pareto Optimum.*

Proof: *Let $\overline{A}$ be the level of allowances given by the assumption (Characterized Efficiency) and $(p, q, (y_j, \alpha_j), (x_i))$ be an equilibrium with production allowance for this level of allowances. As $q \neq 0$, for all j the constraint $\alpha_j \leq -h_j(y_j, y_{-j})$ is necessary binding and therefore using clearance of the allowance market, one gets $\sum_{j=1}^{n} h_j(y_j, y_{-j}) = \overline{A}$. Hence using (Characterized efficiency) one has $\sum_{j=1}^{n} y_j \in \partial Z$.*

Let us then prove that $\sum_{j=1}^{n} y_j$ maximizes profit in Z. As Z is convex, it suffices to show that $\sum_{j=1}^{n} y_j$ satisfies the first order condition for profit maximization, that is $p \in N_Z(\sum_{j=1}^{n} y_j)$. Now, one has under assumption (Over Optimism) that $\bigcap_{j=1\cdots n} N_{Y_j(y_{-j})}(y_j) \subset N_Z(\sum_{j=1}^{n} y_j)$ $(*)$.

Moreover, for all j as $(y_j, -h_j(y_j, y_{-j}))$ is profit maximizing in $G_j(\overline{y}_{-j})$ at price (p, q), one has $p \in N_{Y_j}(y_j) + q\partial h_j(y_j, y_{-j})$. As $\partial h_j(y_j, y_{-j})$ is equal among j, this implies $p - q\partial h_j(y_j, y_{-j}) \in \bigcap_{j=1\cdots n} N_{Y_j(y_{-j})}(y_j)$ and hence $p - q\partial h_j(y_j, y_{-j}) \in N_Z(\sum_{j=1}^{n} y_j)$ because of (Over-Optimism). Using then (Exact Compensation) one has $\partial h_j(y_j, y_{-j}) \in N_Z(\sum_{j=1}^{n} y_j)$ and hence $p \in N_Z(\sum_{j=1}^{n} y_j)$.

Now, one can clearly implement the wealth distribution $(p \cdot \overline{x}_1, \ldots, p \cdot \overline{x}_n)$ in order to implement the consumptions $\overline{x}_i$ as solutions to the consumers problems. One can then apply proposition 1 and conclude.

Conversely, any Pareto Optimum can be decentralized as an equilibrium with allowances according to theorem 2. The (Characterized Efficiency) assumption implies that the corresponding allocation of allowances must be equal to $\overline{A}$.

Hence, one first obtains a "first welfare theorem" for allowance functions of type h_1.

The condition of constant endowment in allowances and the assumption of (Characterized Efficiency) can be dispensed with if (Over-Optimism) holds for all the production plans below the efficiency frontier:

Assumption (Strong-Over-Optimism) *For all $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n y_j \in Z$, one has $T_Z(\sum_{j=1}^n y_j) \subset \sum_{j=1}^n T_{Y_j(y_{-j})}(y_j)$.*

Note that this is satisfied in particular when for all $y_j \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n y_j \in Z$, there exist j such that $y_j \in int Y_j(y_{-j})$.

In this framework, efficiency can be achieved by linking allowance and commodities equilibrium prices through the government behavior. The mechanism applied is based on the idea that one of the aims of the government when it sets up the production allowance market is to prevent failures by the firms to deliver the production they had announced. If the government owns a stock of commodities corresponding to what it considers as the unrealizable part of the production, it may substitute for the firms if they fail to deliver and using its stock, supply the market at the announced level. The building of this stock may be related to the allowance market if one considers that firms may obtain from the government additional production allowances in exchange of commodities. Conversely if firms hold extra allowances they should be aloud to sell them to the government at the market price of some reference commodity bundle. The government hence clears the allowance market and imposes an equilibrium relation between the commodities and the allowance prices.

Expressly, let $\gamma \in \mathbb{R}_{++}^L$ be the reference commodity bundle used in the definition of the shortage function. The government is set to exchange allowances against commodity bundles γ. If the firm wishes to obtain additional allowances in exchange of commodities, the government simply create them thanks to its legal prerogatives. If the firm wishes to obtain commodities in exchange of allowances, the government purchases the corresponding amount of commodities on the market. Concerning the firms, the possibility to exchange allowances against commodities adds the technology $\{(t\gamma, -t) \in \mathbb{R}^{L+1} \mid t \in \mathbb{R}\}$ to their existing production capacities. The production correspondence of firm j is hence turned to

$$H_j(\overline{z}_{-j}) = \{(y_j, \alpha_j, \beta_j) \in \mathbb{R}^{L+2} \mid \exists z_j \in Y_j(\overline{z}_{-j}) \ y_j = z_j + \beta_j \gamma \ , \ \alpha_j + \beta_j \le -h_j(z_j, \overline{z}_{-j})\}$$

Note that the level of allowance exchanged against commodities, β_j, and the level of allowance obtained on the market,α_j, are treated as separate variables. This is a technical trick needed to keep track of the quantity of commodities actually produced by the firm as $z_j = (y_j - \beta_j \gamma)$, which one needs to know in order to compute the external effects and the allowance requirements. As those two allowances can be turn one into the other at no cost, they somehow

remain the same commodity, and the firms objective can be written as the maximization of the profit $(p, q) \cdot (y_j, \alpha_j + \beta_j)$ in $H_j(y_{-j} - \beta_{-j}\gamma)$.

When the government is assumed to systematically clear the market by exchanging the appropriate quantity $g \in \mathbb{R}$ of allowances against the value of the corresponding number of commodity bundles γ, an equilibrium for the initial allocation of allowances $\overline{A}$ is defined as:

Definition 3 [Equilibrium with allowance clearance]

A collection of production plans $(\overline{y}_j, \overline{\alpha}_j, \overline{\beta}_j) \in \prod_{j=1}^{n} H_j(y_{-j} - \beta_{-j}\gamma)$ together with a collection of consumption plans $(\overline{x}_i) \in (\mathbb{R}_{++}^L)^m$ form a price equilibrium with allowance clearance if there exist a price $(\overline{p}, \overline{q}) \in \mathbb{R}_{++}^{L+1}$, a government extra supply of allowances $g = -\sum_{j=1}^{n} \overline{\beta}_j$, and an assignment of wealth levels $(w_1, \ldots, w_m)$ with $\sum_{i=1}^{m} w_i = (\overline{p}, \overline{q}) \cdot (\sum_{j=1}^{n} \overline{y}_j + (g + \sum_{j=1}^{n} \overline{\beta}_j)\gamma + \omega, \sum_{j=1}^{n} \overline{\alpha}_j + \sum_{j=1}^{n} \overline{\beta}_j + \overline{A})$ such that:

(1) *For all $j, (\overline{y}_j, \overline{\alpha}_j, \overline{\beta}_j)$ maximizes profit, $(\overline{p}, \overline{q}) \cdot (y_j, \alpha_j + \beta_j)$, in $H_j(y_{-j} - \beta_{-j}\gamma)$;*
(2) *For all i $\overline{x}_i$ maximizes $u_i(x_i)$ in the budget set $\{x_i \in \mathbb{R}_+^L \mid p \cdot x_i \le w_i\}$;*
(3) $\sum_{i=1}^{m} \overline{x}_i = \sum_{j=1}^{n} \overline{y}_j + (g + \sum_{j=1}^{n} \overline{\beta}_j)\gamma + \omega$;
(4) $\sum_{j=1}^{n} \overline{\alpha}_j + \sum_{j=1}^{n} \overline{\beta}_j + \overline{A} = 0.$

One should remark that clearance of the allowance market by the government implies it must buy on the market the amount of commodities corresponding to the allowance it gets back or conversely that it supplies the commodities market with the bundles it obtains thanks to the extra allowances it supplies. Firms have a dual behavior. This is why the term $(\sum_{j=1}^{n} \beta_j + g)\gamma$ enters the equilibrium conditions on the commodities market, even though at equilibrium this quantity is null. Moreover, in order to balance its budget the government must wether set taxes on the consumers or subsidize them thanks to its surplus. Those operations are implicitly encompassed in the assignment of the wealth levels. An implicit assumption here is that the setting of those taxes (resp. subsidies) does not entail any form of strategic behavior of the consumers.

Now, the fundamental issue is that at equilibrium the price of the allowance is necessarily equal to this of the commodity bundle γ. That is:

$$q = p \cdot \gamma \qquad\qquad (\star)$$

Otherwise the firms would buy on the market an infinite amount of allowances in order to exchange them against commodity bundles, or *vice-versa*. This condition characterizes the ratio between the allowance price and the other commodities prices. In order to control the equilibrium distance to ∂Z, it then suffices to let it depend on this ratio and hence to choose allowance functions such that the marginal rate of substitution between the production allowance and the commodities is itself sensitive to the distance to ∂Z. One can choose

for example, a particular case of h_3, an allowance function of the form

$$h_j(y_j, \overline{y}_{-j}) = \phi(g(y_j + \sum_{k \neq j} \overline{y}_k)) - \psi(g(\sum_{k \neq j} \overline{y}_k)).(\star\star)$$

where ϕ is a strictly convex function such that $\phi'(0) = 1$. Indeed, one can then check that:

(1) $\partial h_j(\cdot, y_{-j})(y_j) \cdot \gamma \leq 1$ if and only if $\sum_{j=1}^n y_j \in Z$
(2) $\partial h_j(\cdot, y_{-j})(y_j) \cdot \gamma = 1$ if and only if $\sum_{j=1}^n y_j \in \partial Z$.

The strict convexity of ϕ_j implies the marginal rate of substitution between production allowance and commodities increase with the distance to ∂Z ("with the level of bad"). The normalization of the derivative is not important *per se*. It must be understood in relation with the governmental exchange rate between commodities and allowances. Indeed, one will see below that an equilibrium price must satisfy $q\partial h_j(y_j, y_{-j}) = p$. Together with the price equilibrium condition $(\star)$ (determined by the governmental exchange rate), it implies that at equilibrium one must have $\partial h_j(y_j, y_{-j}) \cdot \gamma = 1$ which will guarantee according to the preceding that $\sum_{j=1}^n y_j \in \partial Z$. The same reasoning can be made whenever the derivative of ϕ in 0 equals the exchange rate between allowance and commodities. Finally, one has:

Theorem 5 *Assume assumptions (P), (C), (G), (Strong Over Optimism) hold and the allowance function is of the form $(\star\star)$. Any equilibrium with allowance clearance is Pareto optimal.*

Proof: *Let $(p, q, (y_j, \alpha_j, \beta_j), (x_i))$ be an equilibrium with allowance clearance. The first order conditions for profit maximization for firm j at (p, q) are $p \in N_{Y_j}(y_j) + q\partial h_j(y_j, y_{-j})$ and $p \cdot \gamma = q$. Taking the scalar product of this first equation by γ we get, $p \cdot \gamma = q \in N_{Y_j}(y_j) \cdot \gamma + q\partial h_j(y_j, y_{-j}) \cdot \gamma$. As $\gamma \in \mathbb{R}^L_{++}$ and $N_{Y_j(y_{-j})}(y_j) \in \mathbb{R}^L_+$, this implies $q \geq q\partial h_j(y_j, y_{-j}) \cdot \gamma$. and hence $\partial h_j(y_j, y_{-j}) \cdot \gamma \leq 1$. This implies according to the strict convexity assumption on the allowance function, that $\sum_{j=1}^n y_j \in Z$. Now, as $\partial h_j(y_j, y_{-j})$ is equal among j, one has $p - q\partial h_j(y_j, y_{-j}) \in \cap_j N_{Y_j(y_{-j})}(y_j)$. Under (Strong Over Optimism) this implies $p - q\partial h_j(y_j, y_{-j}) \in N_Z(\sum_{j=1}^n y_j)$. Now wether $p \neq q\partial h_j(y_j, y_{-j})$ and hence $N_Z(\sum_{j=1}^n y_j) \neq \{0\}$ which implies $\sum_{j=1}^n y_j \in \partial Z$, wether $p = q\partial h_j(y_j, y_{-j})$ which implies according to the preceding that $\partial h_j(y_j, y_{-j}) \cdot \gamma = 1$. The strict-convexity of the allowance function then imply that $\sum_{j=1}^n y_j \in \partial Z$ as underlined above. Anyhow, one has proved that $\sum_{j=1}^n y_j \in \partial Z$, and the remaining of the proof proceeds as in theorem 4.*

As the analogous of theorem 2 clearly holds for equilibria with allowance clearance, we have in fact proved that the equilibria with allowance clearance coincide with the Pareto optima. This solves in particular the problem of existence of an equilibrium with allowance clearance.

4 Applications

Until now, the government production set and hence the optimality criterium was exogenously given. Also, the relations between the firms and the government expectations was not explicit. We now introduce links between those in order to give a clearer interpretation of the preceding results.

4.1 Decentralization in an economy with production externalities

Let us first deal with the seminal problem of decentralization with externalities presented in Arrow (2) and Laffont (11). That is the decentralization of the Pareto optima with regards to the production capacities given by Y_j. If one then sets $Z = \{z \mid \exists (y_j) \in \prod_{j=1}^n Y_j(y_{-j}) \; s.t \; z = \sum_{j=1}^n y_j\}$ those Pareto optima coincide with the "government Pareto optima" studied in the preceding section.

Assumption (Decentralizability) clearly holds in this framework, so that if the allowance functions are well chosen (e.g h^1 to h^3 above), one obtains a second welfare theorem as a corollary of theorems 1 and 3 :

Corollary 1 *Assume assumptions (P), (C), (G), (Decentralizability), (Allowance) and (Compensation) hold. Any Pareto optimum with regards to Y_j can be decentralized as an equilibrium with production allowances or as an equilibrium with production tax.*

On the other hand, to implement first-welfare like theorems, one must check that one of the over-optimism assumption holds. The strong form is irrelevant here as if it holds one can check every competitive equilibrium (without any additional market) is Pareto optimal and there is no need to discuss the properties of the allowance market.

However, the weaker form is likely to be satisfied. Expressly, it states that when the aggregate production is efficient, the corresponding individual productions are inefficient from the firms point of view (firms hence are locally over optimistic). By contraposition, this is equivalent with saying that when the firms consider their productions are efficient, the aggregate outcome in fact is inefficient. That is the externalities always lead to inefficiency when the firms are competitive. This holds when there is a strong correlation between the production capacities and the environment, for example when the graphs of the correspondences Y_j are strictly convex (see the appendix for an explicit proof).

It then suffices to choose an allowance function satisfying (Exact Compensation) and (Characterized Efficiency) in order to apply theorem 4 and to obtain

a first-welfare like result. Namely:

Corollary 2 *Assume assumptions, (P), (C), (G), (Allowance), (Exact Compensation), (Over Optimism) and (Characterized efficiency) hold. There exist an allocation in allowances $\overline{A}$ such that $(y_j, \alpha_j), (x_i)$ is an equilibrium with allowance if and only if $((y_j), (x_i))$ is Pareto optimal with regards to the production capacities Y_j.*

4.2   Errors in the production sector

Let us now come closer to the problematic described in the introduction by explicitly considering the individual production correspondences are not accurate. To give a precise meaning to this sentence, we introduce explicitly "true" production possibilities at the individual level.

Indeed, we consider a situation where the "true " production possibilities are described by correpondences $Z_j : (\mathbb{R}^L)^n \to \mathbb{R}^L$, $Z_j((y_1, \cdots, y_n))$ being the production possibilities of firm j when the complete scheme of production plans in the economy is (y_j).[7] The government is informed of the aggregate production possibilities $Z = \{z \in \mathbb{R}^L \mid \exists (y_j) \in \prod_{j=1}^n Z_j(y_1, \cdots, y_n) \ s.t \ \sum_{j=1}^n y_j = z\}$ but not necessarily of the true individual production correspondences. On the other hand, the production possibilities perceived by the producers are given by correspondences $Y_j : (\mathbb{R}^L)^{(n-1)} \to \mathbb{R}^L$. We shall consider those are over-optimistic in the sense of one of the earlier assumptions and of course that they satisfy the (Decentralizability) requirement. One can for example think of the case where $Z_j \subset int Y_j$ or even that the producers are not aware they face an external effect and anticipate their production set are the $\cup_{(y_j) \in (\mathbb{R}^L)^n} Z_j((y_j))$.

In this framework, decentralization results are direct consequences of theorems 1 and 3:

Corollary 3 *Assume assumptions (P), (C), (G), (Decentralizability),(Allowance) and (Compensation) hold. Any Pareto optimum with regards to the production capacities Z_j can be decentralized in the economy with production capacities Y_j as an equilibrium with production allowance or as an equilibrium with production tax.*

Concerning first welfare like results, the introduction of the "true" production correspondences Z_j has add a new requirement on the individual choices of the producers: one must now guarantee that the production plans are "truly" feasible while theorems 4 and 5 only ensure the optimality and the feasibility

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[7] Such a definition for production correspondences is somehow unusual as it allows producers to have an external effect on themselves. The motivations for such a modelization are presented in the appendix.

at the aggregate level. They can be applied here if individual and aggregate feasibility coincide in the sense of:

Assumption (Feasibility Coincidence)

If $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ and $\sum_{j=1}^n y_j \in \partial Z$ then $(y_j) \in \prod_{j=1}^n Z_j((y_j))$.

Theorems 4 and 5 then respectively yield:

Corollary 4 *Assume assumptions, (P), (C), (G), (Convex Measures), (Exact Compensation), (Over Optimism), (Characterized Efficiency) and (Feasibility Coincidence) hold. There exist an initial allocation in allowances $\overline{A}$ such that $(y_j, \alpha_j), (x_i)$ is an equilibrium with allowances of the economy with production capacities Y_j if and only if $((y_j), (x_i))$ is Pareto optimal with regards to the production capacities Z_j.*

Corollary 5 *Assume assumptions, (P), (C), (G), (Strong Over Optimism) and (Feasibility Coincidence) hold and the allowance function is of the form $(\star\star)$. Any equilibrium with allowance clearance of the economy with production capacities Y_j is Pareto optimal with regards to the production capacities Z_j.*

Now, it seems clear that for *Feasibility Coincidence* to hold one must restrict the type of errors the firms may make and the type of external effects they may face. Roughly the assumption holds when errors and externalities are uniform among the firms. More precisely:

Assumption (Repartition Neutrality) *Consider an environment $(w_j) \in \mathbb{R}^n$ and a production scheme $(z_j) \in \prod_{j=1}^n Z_j((w_j))$ with $\sum_{j=1}^n z_j \in \partial Z$. For every $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n y_j = \sum_{j=1}^n z_j$ one has $y_j \in \prod_{j=1}^n Z_j((w_j))$.*

Assumption (Uniform Externalities) *For every $(z_j) \in \prod_{j=1}^n Y_j(z_j)$ and $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n z_j = \sum_{j=1}^n y_j \in \partial Z$, one has $Z_j((z_j)) = Z_j((y_j))$.*

Lemma 2 *Assumptions (Uniform Externalities) and (Repartition Neutrality) imply (Feasibility Coincidence).*

Proof: *Let $(y_j) \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n y_j \in \partial Z$. Hence there exist (z_j) such that $\sum_{j=1}^n z_j = \sum_{j=1}^n y_j$ and $z_j \in \prod_{j=1}^n Z_j(z_1, \cdots, z_n)$. Using assumption (Repartition Neutrality), one then has $y_j \in \prod_{j=1}^n Z_j(z_1, \cdots, z_n)$. On the other hand, assumption (Uniform Externalities) implies that for all j $Z_j(y_1, \cdots, y_n) = Z_j(z_1, \cdots, z_n)$. Hence one has $(y_j) \in \prod_{j=1}^n Z_j((y_j))$.*

(Repartition Neutrality) states that for a given environment, the "true" feasibility of a production scheme is independent of the repartition of the production among the firms. (Uniform Externalities) states that the externality faced by a producer depend only of the aggregate production level. It im-

plies in particular that the externalities are not directed (i.e the source of the externality does not matter) and that the set of goods is comprehensive enough to let the production process (including external effects) be unambiguously characterized by the input-output combination implemented. The relevance of those assumptions appears more clearly when the production correspondences are thought to represent industries rather than individual producers. Indeed, the state of the environment is then determined by the sum of outputs of all industries (Uniform Externalities) and because the industries are specialized there usually is a sole way to allocate among them the aggregate production (this implies (Repartition Neutrality)). Expressly, one can consider there are k types of industries in the economy. To those k types is associated a partition of the space of goods in k subsets such that a firm of type k uses as input and produces as output only goods in the kth subset. There exist an arbitrary number of firms of each type but the environmental constraint they face due to the other sectors of the economy is collective. That is to say if $Z_1, \cdots, Z_{n_k}$ and $Y_1, \cdots Y_{n_k}$ are respectively the "true " and " anticipated "production correspondences of the firms of type k, there exist a technico-environmental constraint function for the firms of type k, $E_k : \mathbb{R}^L \times (\mathbb{R}^L)^{n-n_k} \to \mathbb{R}$ such that given an environment set up by the other types firms production $(w_{n_k+1}, \cdots w_n) \in (\mathbb{R}^L)^{(n-n_k)}$ one has for $(y_1, \cdots y_{n_k}) \in \prod_{j=1}^{n_k} Y_j(y_1, \cdots y_{n_k}, w_{n_k+1}, \cdots w_n)$:

$$(y_j) \in \prod_{j=1}^n Z_j((y_1, \cdots y_{n_k}, w_{n_k+1}, \cdots w_n)) \Leftrightarrow E_k(\sum_{j=1}^n y_j, (w_{n_k+1}, \cdots w_n)) \leq 0.$$

Within such a framework, all the preceding assumptions hold.

5   Conclusion: an economy undergoing climate change

Let us now apply the preceding results to the model of an economy undergoing climate change. This will allow us to get further insight on the interpretation of the production allowance market and to compare its properties with those of emission allowance markets which are actually used in real economies.

We consider a very simple model: an economy with L goods and two periods of time. There is a single state of nature in the first period denoted by 0 and S states of nature in the second period, denoted by $s = 1 \cdots S$. Those different states may summarize the uncertainty about climate change. There is a complete set of contingent markets à la Debreu (9) and we assume all the transactions take place during the first period.

Climate change is due to greenhouse gases emissions in the first period and affect the production possibilities in the second period. We denote by $Y_j(E) =$

$(Y_j^0, Y_j^1(E), \cdots, Y_j^S(E))$ the production possibilities as expected by the producers given an aggregate emission level E in the first period. On the other hand, the true individual production possibilities are described by $Z_j(E) = (Y_j^0, Z_j^1(E), \cdots, Z_j^S(E))$ given an aggregate emission level E. Greenhouse gases emissions are measured according to a function f of the aggregate production in the first period. The government is well informed of the consequences of climate change and can compute accurately the emissions. Hence it considers the aggregate production possibilities are given by $Z = \{(z^0, z^1, \cdots, z^s) \mid \exists(z_j) \in \prod_{j=1}^n Z_j(f(\sum_{j=1}^n z_j^0)) , \sum_{j=1}^n z_j = z\}$.

In line with the arguments presented in the introduction, we shall assume (Decentralizability) and (Strong Over Optimism) hold [8] in order to translate the fact that the individual firms are less concerned than the government by the influence of climate change on future production possibilities. We can then embed this *Climate Change Economy* into the framework of section 4.2. In particular, one can construct as before a production allowance market and define equilibria with production allowances. Corollary 3 implies that every Pareto Optimum with regards to Z_j and ω, can be decentralized as a competitive equilibrium with production allowances. Corollaries 4 and 5 entail first-welfare like theorems. Hence the opening of a production allowance market seems a suitable solution to decentralize the Pareto optima of the climate change economy. According to theorem 2, it might even be the only one which requires the opening of only one market.

However the interpretation of the production allowance is problematic as it has two types of effects on the firms behavior. On the one hand it prevents firms from emitting too much greenhouse gases in the first period and on the other hand it prevents them from setting up over optimistic production plans in the second period. Those two influences can be isolated and the production allowance market is necessary only for the second purpose. The first one can be dealt with an emission allowance market or with external effect markets *à la* Arrow.

Indeed, let us now consider that the government introduces an emission allowance market in the first period. Therefore we assume that the aggregate emission level $f(z^0)$ can be computed by adding individual emissions $f_j(z_j^0)$ and that after allocating a quantity E of emission allowances to the firms, the government forces them to detain the quantity of emission allowances corresponding to their actual emission level ; the firms being aloud to trade those emission allowances on a market. We moreover consider that the government knows only the aggregate production set of the second period as a correspondence depending of the aggregate level of emission and that this set can be represented by a convex shortage function $g(E, \cdot)$ with the total emission level as a parameter. Now by setting a production allowance market for the second period where firm j production allowance requirement is computed according

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[8] We also assume (P)(C)(G)

to the corresponding $h_j(E, \cdot)$, the government leads the economy to open two additional markets, one of emission allowance, one of production allowance. In this framework the production set of firm j becomes

$$C_j(E, y^s_{-j}) = \{(y_j, \alpha_j, \xi_j) \mid y_j \in Y_j(E) \ , \ \alpha_j \leq -h_j(E, y^s_j, y^s_{-j}) \ , \ \xi_j \leq -f_j(y^0_j)\}.$$

The behavior of the consumers remaining unchanged, we can then define a price equilibrium of the climate change economy with production and emission allowances as:

Definition 4 (Equilibrium of the climate change economy) *Given a supply A of production allowance and a supply E of emission allowances, a collection of production plans $(\overline{y}_j, \overline{\alpha}_j, \overline{\xi}_j), \in \prod_{j=1}^n C_j(E, \overline{y}_{-j})$ together with a collection of consumption plans $(\overline{x}_i) \in (\mathbb{R}^L_{++})^m$ is an equilibrium of the climate change economy if there exist a price $(\overline{p}, \overline{q}, \overline{r}) \in \mathbb{R}^{L+2}_+$ and an assignment of wealth levels $(w_1, \ldots, w_m)$ with $\sum_{i=1}^m w_i = (\overline{p}, \overline{q}, \overline{r}) \cdot (\sum_{j=1}^n \overline{y}_j + \omega, \sum_{j=1}^n \overline{\alpha}_j + A, \sum_{j=1}^n \overline{\xi}_j + E)$ such that:*

(1) For all j, $(\overline{y}_j, \overline{\alpha}_j, \overline{\xi}_j)$ maximizes profit, $(\overline{p}, \overline{q}, \overline{r}) \cdot (y_j, \alpha_j, \xi_j)$, in $C_j(E, \overline{y}_{-j})$
(2) For all i $\overline{x}_i$ maximizes $u_i(x_i)$ in the budget set $\{x_i \in \mathbb{R}^L_+ \mid \overline{p} \cdot x_i \leq w_i\}$
(3) $\sum_{i=1}^m \overline{x}_i = \sum_{j=1}^n \overline{y}_j + \omega$
(4) $\sum_{j=1}^n \overline{\alpha}_j + A = 0$
(5) $\sum_{j=1}^n \overline{\xi}_j + E = 0$

In a manner very similar to this of the proof of theorem 1, one can then prove that every Pareto Optimum can be decentralized as an equilibrium of the climate change economy: it suffices that the government chooses the optimal level of emissions for the first period and then lets the emission allowance and the production allowance markets operate. In the case of external effects *à la Arrow* or if firms use the emission allowance as a public good (See (7)) the optimal level of emissions might even be determined endogenously. Nevertheless, the production allowance market remains necessary to transfer to the firms information about the true production possibilities. Now, the main difference between this equilibrium concept and these of the preceding sections is that in the preceding, the production allowance market corrected indistinctly all the failures wether they were due to the external effects or to errors in expectations. Here, the production allowance market prevents firms from choosing unrealistic production plans for the second period while the emission allowance market controls the source of external effects.

As far as interpretation is concerned, the production allowance market can then be seen as a medium used by the government to transfer to the firms its expectations on the influence of climate change; that is a proxy for an adaptation policy. Emission allowance market or external effects markets *à la Arrow* are, on the other hand, means to allocate efficiently the costs of reducing greenhouse emission gases in the first period, tools for a mitigation policy.

Now emission allowance markets are not efficient unless production allowance markets also exist: recognizing the need for adaptation is a prerequisite for efficient mitigation through an endogenous determination of the optimal level of greenhouse gases emissions.

6   Appendix

6.1   Over-Optimism when the graphs of the production correspondences are strictly convex

Let us show that the (Over-Optimism) assumption holds when the graphs of the Y_j are strictly convex. The main remark needed therefore is that the strict convexity implies there is a unique way to write an element $z \in \partial Z$ as the sum of individual production plans $(y_j) \in \prod Y_j(y_{-j})$. Indeed assume there exist two distinct collections of production plans summing to $z \in \partial Z$. In other words there exist $(z_j), (y_j) \in \bigcap GraphY_j$ distinct and such that $\sum_{j=1}^n z_j = \sum_{j=1}^n y_j \in \partial Z$. Strict Convexity of the graphs then imply $\frac{1}{2}((z_j)+(y_j)) \in int(\bigcap GraphY_j)$. As $\sum_{j=1}^n \frac{1}{2}(z_j + y_j) = z$, this implies $z \in intZ$ which contradicts $z \in \partial Z$.

On the other hand it is easy to check that the Over-Optimism condition holds if for every $y_j \in \prod_{j=1}^n Y_j(y_{-j})$ such that $\sum_{j=1}^n y_j \in \partial Z$, one has:

$$v \in \bigcap N_{Y_j(y_{-j})}(y_j) \Rightarrow (v, \cdots, v) \in \bigcap_{\{(z_j) \in \cap GraphY_j | \sum_{j=1}^n z_j = z\}} \sum_{j=1}^n N_{GraphY_j}(z_j)$$

This is straightforward here, as according to the preceding $\{(z_j) \in \cap GraphY_j \mid \sum_{j=1}^n z_j = z\}$ is a singleton and for all j one has:
$v \in N_{Y_j(y_{-j})}(y_j) \Rightarrow (0, \cdots, 0, v, 0, \cdots, 0) \in N_{GraphY_j}(y_j)$.

6.2   Complement on the definition of the production correspondences Z_j

The justification for letting the producers have an external effect on themselves is that we do not want to distinguish two external effects of the same nature because they have a different source. It seems to us that this approach is more appropriate for the intertemporal externalities we want to deal with in the applications. Namely if there are m different types of externalities in the economy and that given an environment defined by a vector of externalities $(e_1, \cdots e_m)$, the production possibilities of firm j are $S_j(e_1, \cdots e_m)$, while the externalities are well determined as functions of the production $f_1(y_1, \cdots, y_n), \cdots, f_m(y_1, \cdots, y_n)$, the production correspondence we consider is $Z_j(y_1, \cdots y_n) = \{y_j \in \mathbb{R}^L \mid y_j \in S(f_1(y_1, \cdots, y_n) \cdots f_m(y_1, \cdots, y_n))\}$. Note

that one can easily turn to the usual framework by letting $Z'_j(y_{-j}) = \{y_j \in \mathbb{R}^L \mid y_j \in Z_j(y_{-j}, y_j)\}$.

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