Wage-led Regime, Profit-led Regime and Cycles: a Model(French title for the publication: Régime wage-led, régime profit-led et cycles : un modèle)

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HAL Id: halshs-00149942

https://shs.hal.science/halshs-00149942

Submitted on 13 Jun 2007

WAGE-LED REGIME, PROFIT-LED REGIME AND CYCLES: A MODEL

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We propose a dynamic model which deals with the impact of income distribution variations on growth. In that goal, we use two models : the classical Goodwin model (1967) and the Bhaduri-Marglin model (1990), which also focuses on the links between income distribution and growth, but in a Keynesian frame. We introduce Keynesian demand constraints within the Goodwin model and modify its investment function, which becomes non-linear. With these new hypotheses, we show that Goodwin cycles may either be maintained or disappear. If most trajectories oscillate around a classical equilibrium, the economy may also fall during a cycle into a Keynesian unemployment state. In that case, cycle dynamic is broken because wages are squeezed whereas the economy is in a wage-led regime. This model allows to capture some specific characteristics of the French economic situation that took place in the 1980s-1990s.

The impact of income distribution on growth has been studied by two different strands of economic literature, belonging respectively to the classical and post-Keynesian traditions. On the classical side, Goodwin (1967) proposes a formalization of endogenous cycles where investment is directly related to saving. Because the propensity to save of entrepreneurs is higher than that of workers, an increase (respectively a decrease) in the profit share always boosts (respectively hinders) accumulation and growth. The latter assumption has however been questioned by some Keynesian economists. According to Bhaduri and Marglin (1990), the effect on growth of a rise in profits remains uncertain: it may either be positive (*profit-led* regime) or negative (*wage-led* regime). Nevertheless, the Bhaduri-Marglin model remains incomplete as it does not tackle dynamic issues.

A few contributions have already introduced Keynesian hypotheses in a classical framework. Skott (1989) integrates a class struggle dynamics in a Kaldorian trade cycle model. Duménil and Levy (1999) build a model combining a short term Keynesian dynamics with a long term classical one, with adjusting prices. In this article, we try to incorporate both configurations of Bhaduri-Marglin's model within the Goodwin framework. We so underline that the effects of income distribution evolutions on growth fundamentally depend on the growth regime existing in the economy. Besides, we assume that the two regimes emphasized by Bhaduri and Marglin (1990) are endogenous and are themselves related to the *level* of the wage share. Thus we propose a dynamic model where growth affects income distribution, which itself may modify in return the nature of the growth regime and lead to a new income distribution. One main conclusion is that "Goodwin cycles" can either be maintained or disappear. In this last case, the dynamics jumps from one equilibrium to another that includes long lasting unemployment. This conclusion allows to capture some specific characteristics of the French economic situation during the 1980s-90s when both profit share and unemployment rate durably increased together. Such a fact cannot entirely be explained by the Goodwin model and needs therefore to take into account Bhaduri-Marglin's assumptions.

The first section is briefly devoted to Goodwin's and Bhaduri-Marglin's models, and the way they can be articulated. This combination is then formalized in the second section. The third section emphasizes our main conclusions that are illustrated by simulations. In the fourth section our results are confronted to the French economic dynamics of the 1980s-90s. Lastly a short conclusion is given.

I. – THE GOODWIN (1967) AND BHADURI-MARGLIN (1990) MODELS

The Goodwin model is a well-known mathematical representation of endogenous cycles rooted in the Marxian tradition. According to Goodwin, cycles are the consequence of the class struggle upon the sharing of the value added between workers and capitalists (wages against profits). To give a better approach of the mechanisms of this model, let us introduce a situation characterized by a high unemployment. Real wages increase less than labour productivity according to the iron law. Consequently profit share increases, investment being proportionally raised. Accumulation and growth then push the demand for labour that leads to a decline of unemployment. This phenomenon restores the wage bargaining power of trade-unions, which

progressively brings about a profit-squeeze. This weaker profitability (and thus the weaker level of saving) leads to a drop of investment that provokes accumulation slowdown as well as an increase in unemployment. Thus, a new cycle begins.

The hypotheses of the model are:

- The production factors are labour L and capital K
- Labour productivity a and labour force n grow at a constant rate α and β:

$$
\begin{aligned}
a &= a_0 e^{\alpha t} \qquad \alpha > 0 \\
n &= n_0 e^{\beta t} \qquad \beta > 0
\end{aligned}
$$

- Wages are entirely consumed, profits entirely saved and invested, then

$$
\dot{K} = Y - wL = \left(1 - \frac{w}{a}\right) Y = (1 - \omega)Y \tag{1}
$$

where w is the wage rate and ω the wage share in the value added.

- The capital-output ratio σ is constant. Thus

$$
\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = \frac{(1 - \omega)}{\sigma} \tag{2}
$$

- The real wage growth rate $\frac{\dot{w}}{w}$ is related to the employment level v

$$
\frac{\dot{w}}{w} = -\gamma + \rho v \qquad \gamma, \ \rho > 0 \tag{3}
$$

By solving this model, we get a two differential equations system

$$
\begin{cases}
\dot{v} = \left[\left(\frac{1}{\sigma} - (\alpha + \beta) \right) - \frac{\omega}{\sigma} \right] v \\
\dot{\omega} = [\rho v - (\alpha + \gamma)] \omega
\end{cases} \tag{4}
$$

The equilibrium is given by $\omega^* = 1 - (\alpha + \beta)\sigma$ and $v^* = \frac{\alpha + \gamma}{\rho}$. At the equilibrium, the Jacobian has a trace equalling zero and a positive determinant ($= \frac{\rho}{\sigma} \omega^* v^*$). The eigenvalues are complex and conjugate, their real part equalling zero. The dynamics of the system is a cycle oscillating around the center (vortex E on figure 1) that corresponds to the system equilibrium.

Fig. I. The Goodwin cycle.

Say's law (according to which growth is never demand constrained) is implicit in the Goodwin model and implies that the level of investment always equals – and is determined by– profits (profits generate investment and not the contrary

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like in the Keynesian analysis of Kalecki). This hypothesis has been criticized by Keynesian economists, highlighting that the amount of investment does not depend directly on saving level. Following this argument, a specification of investment is needed for building a new model. However this should not prevent from taking into account profitability as a central determinant of both investment decision and saving behaviour, as Robinson (1962) emphasized it. In that sense, the Bhaduri-Marglin model analyzes the effects of income distribution on growth through the impact of profit-share evolutions upon investment on the one hand and saving on the other hand. The authors show that output growth can be supported either by wages increase (*wage-led* regime) or profits increase (*profit-led* regime).

In their model, consumption is given by:

$$C = (1 - s_w)W + (1 - s_r)R \tag{5}$$

where W is the payroll, R are the total profits ($Y = R + W$), s_w and s_r are saving propensities of workers and capitalists respectively, with $s_r > s_w$. It follows that:

$$\frac{S}{K} = \frac{1}{\sigma}\left[(s_r - s_w)\pi z + s_w z\right] \tag{6}$$

with K, the level of physical capital of the economy, S the saving, $\pi = \frac{R}{Y}$, $z = \frac{Y}{\overline{Y}}$ (where $\overline{Y}$ is the output level when the capital stock is fully used), and $\sigma = \frac{K}{\overline{Y}}$ (constant)[†].

The investment function has two explicative variables: the profit rate $\frac{R}{K}$ and the rate of capacity utilization z, which captures the demand effect:

$$\frac{I}{K} = i_0 + i_r\frac{R}{K} + i_z\frac{Y}{\overline{Y}} = i_0 + z(i_r\frac{\pi}{\sigma} + i_z) \qquad i_r, i_z > 0 \tag{7}$$

As $\frac{I}{K} = \frac{S}{K}$, we get a relation between z and π:

$$z = \frac{\sigma i_0}{(s_r - s_w - i_r)\pi + (s_w - \sigma i_z)} \tag{8}$$

An economy is *profit-led* if $\frac{\partial z}{\partial \pi} > 0$ and *wage-led* otherwise. The nature of the economic regime rests only on the sensitivity of both saving and investment to variations of π : the economy is *wage-led* if $i_r > s_r - s_w$ (we suppose $i_0 > 0$). Therefore the market equilibrium condition implies either an increasing or a decreasing relation between wage share and output. The effect of profit on growth is then mitigated.

[†]Note that in Goodwin model, Y always equals $\overline{Y}$.

In the Goodwin model, recession periods generate their own recovery thanks to investment takeoff (boosted by higher profits). Here, this recovery might not happen because:

- either investment grows less than savings when profit share increases, so that global effect on growth is negative. This arises if the economy is *wage-led*.

- or recession has a direct negative impact on investment due to aggregate demand decrease. Investment evolutions depend on two antagonistic effects: the positive effect of profit recovery (Goodwin), and the negative impact of economic activity slowdown (negative Keynesian accelerator).

In his review of the Bhaduri-Marglin model, Taylor (1991) suggests that the nature of the economic regime could directly depend on the "initial" level of wages: the greater it is, the more likely is the economy to be *profit-led*. This rather intuitive interpretation can be illustrated by an inverted U-shaped curve, which can be used in a fruitful manner in a dynamic model[‡].

II. – THE PROPOSED MODEL

On Figure 2, we combine the "Taylor curve" with a wage-setting (WS) curve (i.e. a wage equation where the level of real wage depends positively on output, see Jackman, Layard, Nickell (1991)). In this framework, output is determined by aggregate demand which itself depends on the wage share. There is a wage share level ω^* which separates *wage-led* and *profit-led* areas from each other. As long as wage share is below ω^*, the positive effect of wage share increase on consumption outweighs the (possible) negative effect on investment (note that investment itself may even be supported by an accelerator effect in this case). On the contrary, beyond ω^*, profits are threatened to be squeezed so that every wage increase will depress investment more than it boosts consumption. Only a decrease of the wage share could recover investment and growth in this area. It is important to note that the dynamical dimension of our model comes from the endogeneity of the wage share. We see that this dynamics generates cycles around one first equilibrium but may bifurcate to another one, where unemployment and profit share are both very high. Indeed, a too large wage share reduction during the recession period of the cycle may prevent investment to recover (in spite of the increase of profit share), that

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[‡]Note that this representation does not correspond at all to the analytical model of Bhaduri-Marglin, since the sign of $\frac{\delta z}{\delta \pi}$ from (8) does not depend on π. In the next section, we will try to give a mathematical formalization of this "Taylor curve".

actually breaks the cycle. This new equilibrium is characterized by unemployment and weakness of global demand. This is the reason why we call it a "Keynesian equilibrium" (see figure 2 below).

Fig. II. Combining the "Taylor curve" with a wage-setting (WS) curve.

In the rest of this article, we propose a macroeconomic model which captures these dynamics. For that purpose, we keep the framework of the Goodwin model, so the wage equation is a Phillips curve (not a WS curve). Nevertheless, as we shall see, we modify the investment function, which allows us to establish an inverted U-shaped relation between wage share and output growth rate.

Our model is built in discrete time. All coefficients are supposed positive. The firms production function is given by:

$$Y_t = a_t L_t \tag{9}$$

The rates of growth of labour productivity a_t and labour force n_t are α and β respectively. We call w the real wage rate, ω the wage share and π the profit share.

II.1. Goods market

We suppose that production depends on aggregate demand and not on the current stock of capital as in Goodwin model. At each period t, consumption and investment determine production, which is then shared between wages and profits. Consumption and investment of the following period are thus directly related to the income distribution of the current period.

We suppose that saving propensities of workers and capitalists are respectively equal to 0 and 1. Consumption in $(t + 1)$ equals the payroll of period t:

$$C_{t+1} = w_t L_t \tag{10}$$

Investment level in $(t + 1)$ depends on profitability and on a variable capturing the demand faced by firms. Profitability is measured by the profit share of the preceding period t. Consumption of the current period replaces the capacity utilization rate to capture the demand effect. Nevertheless, demand affects investment as soon as profitability is high enough to allow firms to invest (for example, imagine that banks demand a minimum level of profitability to lend). That's why we propose a multiplicative form between profitability and demand variables in the function:

$$I_{t+1} = \eta C_{t+1} \pi_t = \eta C_{t+1} (1 - \omega_t) \qquad \eta > 0 \tag{11}$$

In the following, we will suppose $\eta > 1$. From (10) and (11), the goods market equilibrium $Y = C + I$ is given by:

$$Y_{t+1} = Y_t \left[1 - (1 - \omega_t)(1 - \eta \omega_t) \right] \tag{12}$$

This is an inverted U-shaped curve between the output growth rate and the wage share. Both *profit-led* and *wage-led* configurations don't depend only on model parameters anymore (like in the Bhaduri-Marglin model) but on an endogenous variable, i.e. the wage-share. Taking (9) into account, we easily obtain from (12) a relation between v_{t+1} and v_t (v is the employment rate).

II.2. Labour market

We keep the Goodwin wage equation (in discrete time):

$$\frac{w_{t+1} - w_t}{w_t} = \rho v_{t+1} - \gamma \qquad \rho, \gamma > 0 \tag{13}$$

From (13), it follows:

$$\omega_{t+1} = \left(\frac{\rho}{1 + \alpha} v_{t+1} + \frac{1 - \gamma}{1 + \alpha} \right) \omega_t \tag{14}$$

Note immediately that $\frac{1-\gamma}{1+\alpha} < 1$. In the following, we suppose that $-1 < \frac{1-\gamma}{1+\alpha} < 1$.
We finally obtain a two equations system:

$$
\begin{cases}
v_{t+1} = \frac{[1-(1-\omega_t)(1-\eta\omega_t)]}{(1+\alpha)(1+\beta)} v_t = f(v_t, \omega_t) \\[2ex]
\omega_{t+1} = \frac{1}{1+\alpha} \left[\frac{[1-(1-\omega_t)(1-\eta\omega_t)]}{(1+\alpha)(1+\beta)} \rho v_t + (1-\gamma) \right] \omega_t = g(v_t, \omega_t)
\end{cases}
\tag{15}
$$

In the following, we note $\mu = (1 + \alpha)(1 + \beta) > 1$.

II.3. Equilibria determination

Solving the system for fixed points, i.e., $v_{t+1} = v_t$ and $\omega_{t+1} = \omega_t$ (demonstrations of this section are in Appendix 1), we then can draw the corresponding phase diagram (figure 3). On figure 3, we have replaced employment rate v by unemployment rate $u = 1 - v$.

Fig. III. Phase Diagram

Note that if $u = 1$ ($v = 0$), then $\dot{u} = 0$; if $\omega = 0$, then $\dot{\omega} = 0$. Therefore $(u^* = 1, \omega^* = 0)$ is one equilibrium of the system (E_1). The other equilibria are

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a saddle-point (E_2) and a center (locally), E_3. Thus the system has a center - in accordance with the Goodwin model - but exhibits two other equilibria, especially a locally stable one, E_1. This latter result contradicts the Goodwin model.

III. – SIMULATIONS

In this section we have calibrated our model so as to exhibit various dynamics that we can obtain from our model. All these simulations are presented in the plan (wage share, unemployment rate).

As noted by Harvie (2000), the econometric estimates for the center in the Goodwin model are far from actual values, what is true for our own simulations too. Concerning ρ and γ in the wage equation, we have taken values between 0,5 and 1, like in Harvie (2000) estimates (for OECD countries where this relation is significant). Labour force and labour productivity growths are respectively of 2 % and 3 % ($\beta = 0,02$ and $\alpha = 0,03$) so that $\mu \simeq 1,05$. The η coefficient in investment function has been chosen such as ω_1^* and ω_2^* remain real ($\eta > 1,56$). η has been fixed at 1,6. We suppose that, at equilibrium E_3, v^* is equal to 0,95. To keep $v^* = \frac{\alpha+\gamma}{\rho}$ constant, we must modify γ when we modify ρ (i.e. the sensibility of real wage growth to employment). In our different simulations we take as initial conditions $v_0 = 0,98$ (unemployment rate of 2 %) and $\omega_0 = 0,83$ (ω_0 is between ω_1^* and ω_2^*). The unique aspect distinguishing our different simulations is the varying value for ρ (and consequently the value of γ too, to keep $v^* = 0,95$), which increases from 0,5 to 1.

The first simulation ($\rho = 0,5$, see figure 4) shows that our model allows us to reproduce Goodwin cycles.

Fig. IV. First simulation: Goodwin cycles.

Figure 5 (where $\rho = 0,7$) exhibits the attractive effect of E_2 on these cycles.

Fig. V. Second simulation: Goodwin cycles and attracting effect of E_2.

In the third simulation (figure 6), the dynamics "bifurcates" and is attracted by the equilibrium $E_1(0,0)$ and there is no cycle anymore.

Fig. VI. Third simulation: The Goodwin cycle disappears.

Here cycle dynamics is interrupted because of a too important wage squeeze during a recession phase. As a result, consumption brings down investment in its fall in spite of profit share recovery. As soon as investment increases when consumptions drops, cycles are maintained although the model is demand constrained. However, if consumption becomes too weak, that may offset the positive effect of profit recovery on investment. Consumption and investment may thus fall together what induces a cumulative slump. Cycles disappear because neither consumption nor investment can restore growth. Slowdown exacerbates wage-share fall, whereas the economy is in the *wage-led* area (i.e. wages should increase to boost growth). Moreover, the labour force growth exceeds the growth of labour demand as soon as wage-share is lower than ω_1^*. That is the reason why slump is cumulative and the economy collapses to $E_1(0,0)$ (which does not appear on the figure because of the retained scaling. See appendix 1). The dynamics leading to this equilibrium is thus characterized by a form of Harrodian instability (which reminds us Keynes's banana economy parable in *A Treatise on Money* (1930)): higher unemployment induces weaker wages, which strengthens recession and unemployment (in *wage-led* regime), and so on. Such an instability could be softened by supposing that high and persistent unemployment has no effect on wages anymore. Moreover, economic policy is a very efficient tool against Harrodian instability (Harrod (1948)), even if our model does not emphasize it.

Thanks to this model, we propose an interpretation of growing unemployment which characterized the French economy in the 1980s and 1990s.

IV. – THE CASE OF FRENCH UNEMPLOYMENT IN THE 1980s AND 1990s

Figure 7 sketches the French trajectory between 1960 and 2000 as it can be seen with the French Goodwin curve that we present.

Fig. VII. French "Goodwin curve", 1960-2000.

In the 1960s, unemployment remained below 3 %. This situation strengthened the bargaining power of trade unions in a context of labour productivity growth slowdown. That partly explained the takeoff of wage-share in the 1970s which squeezed profits and reduced investment (Bruno and Sachs, 1985). At the beginning of the

1980s, the competitive disinflation has been run with the purpose to diminish unemployment by promoting exportations through inflation fight and wage austerity (Blanchard and Muet, 1993). Unfortunately, the profit-share recovery in the 1980s came along with a very short decrease of unemployment. Therefore, since the middle of 1980s, we can observe a sharp breakdown in the French trajectory with the theoretical dynamics of Goodwin model. Our model proposes an explanation of the lasting and joint increase of unemployment and profit-share (which has reached historical high levels in the 1990s without sensitive effect on investment) from 1982 to 1997. Note however that our model can explain neither the profit-share increase in the 1960s nor the coming back of sustained growth from 1997 (not shown on the figure).

V. – CONCLUSION

In this article, Keynesian demand constraints have been introduced in the Goodwin model. We built a non-linear investment function including profitability and accelerator components. From the goods market equilibrium we thus get a mathematical relation between output growth rate and wage share. This relation is first increasing then decreasing, what corresponds to two distinct growth regimes studied by Bhaduri-Marglin: the *wage* and *profit-led* regimes. We keep the wage equation of Goodwin model to make our model dynamic. Therefore, the economy can be alternatively *wage* and *profit-led.* We then show that Goodwin cycles can be maintained in this new framework. However cycles are not the only possible trajectories of the system anymore: the economy may also be attracted by a cumulative slump path. This results from the drop of the too classical and mechanistic hypothesis of *ex ante* identity between profit and investment postulated by the Goodwin model. In our model a large wage decrease can reduce consumption so much that investment will not take off despite profit restoration. In that case investment and consumption decline together as in a typical Keynesian unemployment case. Nothing can then restore growth and the economy collapses toward a zero output level. We have not succeeded in stabilizing this "Keynesian" equilibrium before zero yet, which remains a weakness of our model. Thanks to it, we manage to sketch the French economic dynamics between 1970 and 1997 and the long lasting growth of both unemployment and profit share from the end of 1980s. Nevertheless, it cannot explain the economic recovery observed after 1997 in that country.

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APPENDIX A
MATHEMATICAL ANALYSIS OF THE DYNAMIC (SECTION II).

We suppose that:
$$\delta = \frac{\rho}{1+\alpha} \quad \text{and} \quad \theta = \frac{1-\gamma}{1+\alpha}.$$

From (15) it results that $v_{t+1} = v_t$ if $v^* = 0$ or if $(1-\omega^*)(1-\eta\omega^*) = 1-\mu$, where roots are called ω_1^* and ω_2^* ($\omega_1^* \leq \omega_2^*$) and belong to $[0;1]$.

The nature (real or complex) and values of ω_1^* and ω_2^* depend only on η and μ. We will now consider that both ω_1^* and ω_2^* are real, positive and inferior to 1, which implies restrictive conditions on η. For example, if $\mu = 1,05$, it is easy to show that η must be greater than $1,56$ (in the unlikely case where $\mu = 1,1$, we must have $\eta > 1,86$)[§]. In these conditions $\omega_1^*\omega_2^* = \frac{\mu}{\eta}$, therefore $\omega_1^* \leq \left(\frac{\mu}{\eta}\right)^{1/2}$ and $\omega_2^* \geq \left(\frac{\mu}{\eta}\right)^{1/2}$.

$\omega_{t+1} = \omega_t$ if $\omega^* = 0$ or if $v^* = \frac{(1-\theta)\mu}{\delta[1-(1-\omega^*)(1-\eta\omega^*)]}$, which is firstly decreasing then increasing in ω.

We finally obtain three equilibria :

$$E_1(v^* = 0, \omega^* = 0) \qquad E_2(v^* = \frac{(1-\theta)}{\delta}, \omega^* = \omega_1^*) \quad \text{and} \quad E_3(v^* = \frac{(1-\theta)}{\delta}, \omega^* = \omega_2^*)$$

The Jacobian is equal to:

$$J = \begin{pmatrix} \frac{1}{\mu}\left[1-(1-\omega^*)(1-\eta\omega^*)\right] & \frac{1}{\mu}v^*(\eta+1-2\eta\omega^*) \\ \frac{\delta}{\mu}(\omega^*)^2((\eta+1)-\eta\omega^*) & \frac{\delta}{\mu}v^*\omega^*(2(\eta+1)-3\eta\omega^*)+\theta \end{pmatrix}$$

In the neighborhood of E_1, both eigenvalues are real and lower than 1. E_1 is locally a stable equilibrium.

In the neighborhood of E_2, the trace of J_{E_2} is strictly greater than 2 since $\omega_1^* \leq \left(\frac{\mu}{\eta}\right)^{1/2}$. Consequently, E_2 is a saddle-point.

In the neighborhood of E_3, the trace of J_{E_3} is lower than 2 since $\omega_2^* > \left(\frac{\mu}{\eta}\right)^{1/2}$. Roots are thus complex and conjugate. Their modulus is 1 so that E_3 is locally a center.

[§]This restriction on η is coherent with equation (11). In this model, the entire wage bill is spent on consumption (of the next period). Then new investment can only be financed by past profits and new bank credits. We then may expect that $\frac{I_{t+1}}{Y_t} > \pi_t$. To fit this consistency condition, our investment behaviour equation (11) must verify $\eta > \frac{1}{\omega}$ (note that η is a constant parameter and that is credit which is the adjusting variable here). Empirically η is very plausibly much bigger than $\frac{1}{\omega}$.

APPENDIX B
SECOND SIMULATION (SECTION III).

Here we reproduce the same figure as in the text (figure 6) but on a larger scale, so as to represent the complete trajectory towards $E_1(0,0)$:
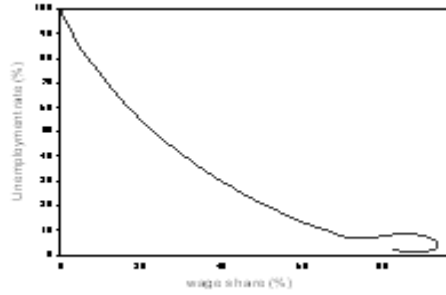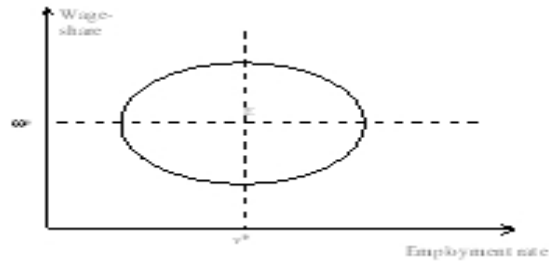
FIGURES
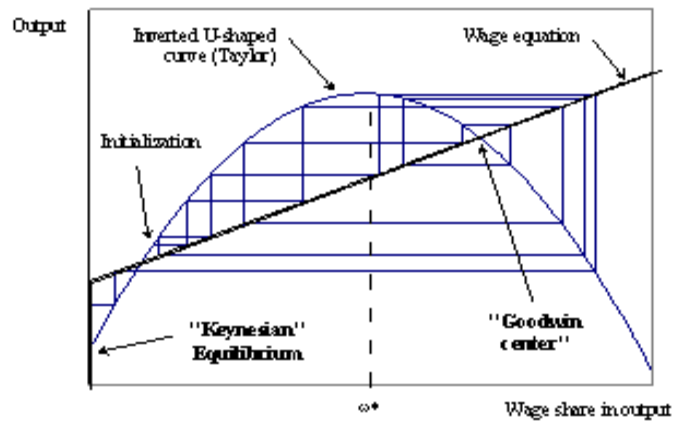

Fig. I. *The Goodwin cycle.*

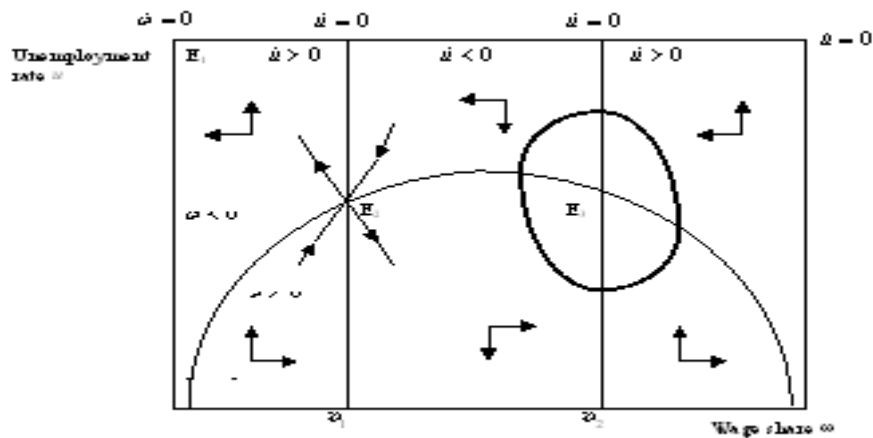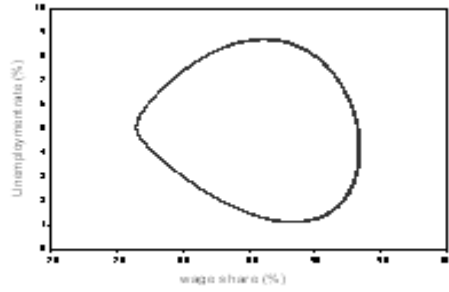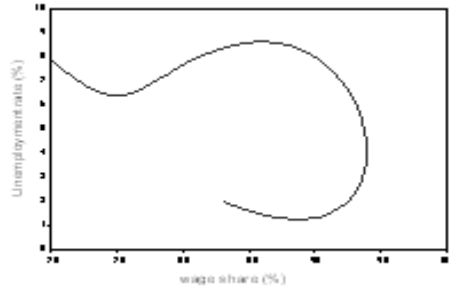**Fig. II.** *Combining the "Taylor curve" with a wage-setting (WS) curve.*

Fig. III. *Phase Diagram.*

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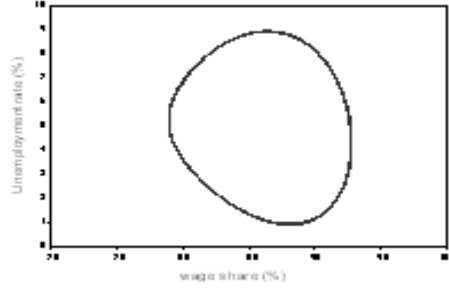
Fig. IV. *First simulation: Goodwin cycles.* $\rho = 0.50 \quad \gamma \simeq 0.45 \quad \eta = 1.60 \; \mu = 1.05$
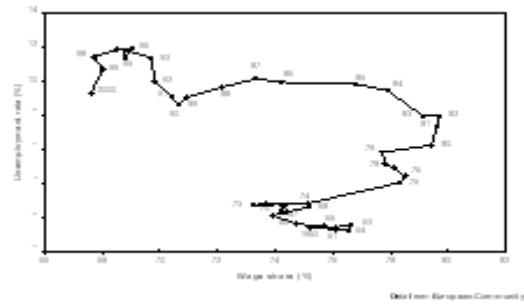$\omega_0 = 0.83 \quad v_0 = 0.98 \quad v^* \simeq 0.95 \quad \omega_1^* \simeq 0.75 \quad \omega_2^* \simeq 0.88$

Fig. V. *Second simulation: Goodwin cycles and attracting effect of* E_2. $\rho = 0.70$
$\gamma \simeq 0.45 \quad \eta = 1.60 \; \mu = 1.05 \quad \omega_0 = 0.83 \quad v_0 = 0.98 \quad v^* \simeq 0.95 \quad \omega_1^* \simeq 0.75$
$\omega_2^* \simeq 0.88$

Fig. VI. *Third simulation: The Goodwin cycle disappears.* $\rho = 0.90 \quad \gamma \simeq 0.83$
$\eta = 1.60 \; \mu = 1.05 \quad \omega_0 = 0.83 \quad v_0 = 0.98 \; v^* \simeq 0.95 \quad \omega_1^* \simeq 0.75 \quad \omega_2^* \simeq 0.88$

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Fig. VII. *French "Goodwin curve", 1960-2000.*

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