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R&D Delegation in a Duopoly with Spillovers

Abstract

There is evidence that competing firms delegate R&D to the same independent profit-maximizing laboratory. We draw on this stylized fact to construct a model where two firms in the same industry offer transfer payments in exchange of user-specific R&D services from a common laboratory. Inter-firm and within-laboratory externalities affect the intensity of competition among delegating firms on the intermediate market for technology. Whether competition is relatively soft or tight is reflected by each firm's transfer payment offers to the laboratory. This in turn determines the laboratory's capacity to earn profits, R&D outcomes, delegating firms' profits, and social welfare. We compare the delegated R&D game to two other ones where firms (i) cooperatively conduct in-house R&D, and (ii) non-cooperatively choose in-house R&D. The delegated R&D game Pareto dominates the other two games, and the laboratory earns positive profits, only if within-laboratory R&D services are sufficiently complementary but inter-firm spillovers are sufficiently low. We find no room for policy intervention, because the privately profitable decision to delegate R&D, when the laboratory participates, always benefits consumers.

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1 Introduction

Examples of firms delegating (i.e. outsourcing) R&D to for-profit laboratories abound. The National Science Foundation (2006) indicates that "[t]he average [real] annual growth rate of contracted-out R&D from 1993 to 2003 (9.4%) was about double the growth rate of in-house company-funded R&D (4.9%). For manufacturing companies, contracted-out R&D grew almost three times as fast as R&D performed internally, after adjusting for inflation."[1] Chemical companies are at the forefront of this phenomenon. They report $2.8 billon in contracted-out R&D in 2003, of which $2.7 billion were for pharmaceuticals and drugs. More surprising is the fact that rival firms often delegate their R&D to a common independent laboratory. For example, Bayer and ICI (two European firms in the chemical industry which compete on world markets) signed multi-year contracts in 1999 and 2000 respectively with Symyx, a U.S.-based private laboratory. Symyx receives payments by providing access to a proprietary technology for the production of high-value specialty polymers. Similar arrangements are observed in other sectors. In the steel industry, for example, ThyssenKrupp and Arcelor (two major European suppliers), contracted in 1995 with VAI, a laboratory which specializes in the design of new steel production methods. The R&D services received from VAI aim at producing wide thin strips of stainless and carbon steel directly from the molten metal, omitting the stages of slab casting and rolling.

There is evidence that firms which delegate their R&D to an independent laboratory place restrictions on the uses of the research. R&D contracts typically state the targeted outcome in exchange of a payment scheme which specifies either discriminatory clauses where rival firms are listed or exclusivity conditions. Such conditions include the right of first refusal, whereby the contracting firm can purchase the rights to specific R&D outcomes before anyone else, or a veto power over the ability of the laboratory to offer a license to any other party. For example, restrictions of this kind appear in a 1997 contract between Millennium (a U.S.-based private laboratory in the biotechnology sector) and

Monsanto (a US provider of agricultural products) for gene-sequencing R&D services. In this contract, Millennium agrees not to make any other significant agricultural enterprise benefit from the outcomes collaboration, in the near future (i.e., for an agreed-upon period of time), unless Monsanto has given its written consent. Rival firms may also place clauses that refer explicitly to each other in R&D contracts, in order to restrict who might get access to a common laboratory's output. For example, a 1998 R&D contract between Millennium and a pharmaceutical division of Bayer stipulates that the firm may not benefit from the output of existing bilateral collaborative research agreements signed in the past by the laboratory and a list of competitors, including Hoffmann-La Roche, Eli Lilly, and Pfizer.[2]

In most cases, independent laboratories do provide information on the content of the contracts they sign, and on the identity of the firms they serve, in order to signal the relevance of their technological skills to potential clients. This information is very often supplied through press releases or on the web sites of the laboratories.

We draw from these industry practices to construct a game where firms may delegate R&D to an independent laboratory. We compare the outcomes of this game with two other ones where R&D is conducted in-house either cooperatively or non-cooperatively. We allow firm-specific R&D services to be either complements, substitutes, or independent inside the laboratory's R&D cost function. We (i) ask when the laboratory earns positive profits, (ii) compare R&D outcomes, firms' profits, and social welfare in a delegated game with those in cooperative and non-cooperative in-house R&D games, and (iii) derive conditions for R&D delegation to Pareto-dominate cooperative and non-cooperative R&D.

While we know of no theoretical model of R&D delegation to a common laboratory, even though such contracts are common, there is an extensive literature on the supply of technology licenses and in-house R&D. The literature on licenses typically considers a monopolistic laboratory which sells a patented process innovation to vertically-related firms by making take-it-or-leave-it offers to downstream firms. Most analyses build on Katz and Shapiro's (1986) complete information model where the laboratory incurs no cost (i.e., R&D costs have been paid in a previous period), and each downstream firm is a potential user of one unit of the innovative input.[3] These analyses base an inventor's ability to earn benefits on the strategic interaction among potential licensees.

The in-house R&D literature pays particular attention to how technological spillovers affect R&D outcomes, firms' profits, and social welfare, when firms may choose R&D cooperatively or non-cooperatively. In their seminal analysis, d'Aspremont and Jacquemin (1988) consider duopolistic firms which invest in deterministic cost-reducing R&D. They show that cooperation is R&D augmenting and welfare improving when between-firm technological spillovers are sufficiently high. The numerous extensions to their model assume in-house R&D, either in each firm's separate laboratory or in a jointly owned one, with firms sharing the operating costs.[4] Amir, Evstigneev and Wooders (2003) unify and generalize the results of this literature without relying on specific functional forms. They confirm two central results of this research stream: (i) R&D cooperation increases firms' profits; and (ii) the profitability of R&D cooperation increases with the level of R&D spillovers.[5]

As in d'Aspremont and Jacquemin (1988), we set up a model where two symmetric firms behave *à la* Cournot on a final market and benefit from cost-reducing R&D outputs. We build on this benchmark framework by giving firms the option to delegate R&D non-cooperatively to an independent laboratory. As in Katz and Shapiro (1986), the laboratory is a profit-maximizer, and may serve none, one, or two firms. However, we abandon their assumption that the laboratory sells at no cost, and as a price maker, the fruit of past R&D efforts. Rather, we assume the laboratory responds to payment schemes by providing firm-specific R&D services at some costs.[6] This assumption captures situations where a laboratory derives income from tailor-made R&D which it provides to firms.

We consider that R&D generates two externalities: (i) the usual between-firm cost-reducing technological spillovers, and (ii) positive or negative within-laboratory externalities depending on whether R&D services are complements or substitutes. We refer to the first externalities as *direct* externalities and to the second as *indirect* ones. We allow firm-specific R&D services to be either complements or substitutes in the laboratory, i.e. indirect externalities can be positive or negative respectively.[7] We use the natural ability of the common agency framework to capture the antagonistic action of two forces: (i) the congruent objectives of the two users to share the resources of the same laboratory so as to benefit from economies of scope, and (ii) the competing attempts by each firm to pull the production of R&D services towards its individual needs. In this model, we assume the laboratory (an agent) may accept the contracts offered by the two firms (principals), or only one firm, or none. To capture the large contracting possibilities observed in real-world contracts, we also specify that each firm may observe and verify the R&D services delivered to its rival.

Using the terminology introduced by Bernheim and Whinston (1986a), we construct a *delegated* common agency model. This means that the agent may accept only a subset of contracts.[8] Our common agency model is also of the *public* kind, a definition which appears in Martimort (2005). This says that each principal may observe and verify the level of output which is delivered to the other principal, and thus condition its payment scheme on it.[9]

We establish a number of interesting and novel results. This is done by investigating the effect of harmonized or conflicting requirements by R&D users, as a function of direct and indirect externalities, on the ability of firms and the laboratory to earn more benefits than in alternative options. This leads to identify the situations in which the laboratory finds it profitable to deliver R&D services not only to one firm, but to both of them. In the latter case, while one could expect the laboratory to always earn positive benefits when R&D services demanded by the firms are complements, we prove this is not the case. We find that the laboratory earns positive benefits only if the firm-specific R&D services it produces are substitutable, or not too complementary, and inter-firm spillovers are sufficiently low.

Intuitively, the ability of the laboratory to earn positive benefits depends on the degree of rivalry between the two firms for its services. This rivalry is a function of the degree of the complementarity or substitutability of the research projects inside the laboratory (indirect externalities) and of inter-firm spillovers (direct externalities). On the one hand, if the R&D services are rival, in the sense that the marginal cost of supplying one firm increases with the level of R&D supplied to the other firm, then the two firms will compete for the laboratory's services. This effect is reinforced by a sufficiently low level of spillovers, implying that the R&D output received by a firm yields benefits almost exclusively to itself. On the other hand, if the R&D services which the two firms purchase from the laboratory are complements, then they benefit from each other's projects and are thus not inclined to ask for exclusive services from the laboratory. A sufficiently high degree of inter-firm spillovers has a similar effect because in this case each firm benefits substantially from the R&D services received by the other player. Either one of the two effects may dominate. When the combination of direct and indirect externalities is positive each firm is interested in seeing the other one contract also. In this case, competition for the control of laboratory's choices is soft. This is unfavorable to the laboratory. When combined direct and indirect externalities are negative, competition for the control of R&D services supplied by the laboratory is tight, a situation which is favorable to the laboratory. Eventually, whether competition for the laboratory's services is soft or tight is reflected by each firm's payment offers to the laboratory, and hence drives the ability of the latter to earn excess benefits. Finally, if research projects are extremely substitutable, the laboratory finds it profitable to serve one firm only.

In more technical terms, we are able to partition the space of indirect and direct externalities into two subspaces. In the first one the laboratory exactly breaks even, whereas in the second one the laboratory earns positive benefits. This result is robust to changes in the specific functional forms we use in the model. Although the algebraic expression of the frontier separating the two subspaces depends on the linear demand and constant marginal cost which we borrow from the standard R&D literature, the partition of the externalities space can be obtained with other cost and demand functions. This is demonstrated by Billette de Villemeur and Versaevel (2004), who propose mild sufficient conditions on the algebraic form of the laboratory's cost function, and of firms' gross profit functions, for either negative or positive externalities in R&D dimensions to dominate, and therefore for the laboratory to earn or not benefits.[10]

Beyond this preliminary result, the comparison of the delegated R&D game with the two benchmark games depends on the specific form of the cost and demand functions. This does not mean that qualitatively similar findings may not be obtained with alternative specifications (for example, by assuming that R&D inputs are chosen in lieu of outputs, or that firms compete in prices instead of quantities on the final market). To our knowledge, this still remains to be checked on a case by case basis for a detailed comparison of R&D levels, firms' profits, and welfare outcomes in the three games.

Equilibrium R&D in the non-cooperative game is known to be lower (higher) than in the cooperative setting for high (low) direct spillovers (d'Aspremont and Jacquemin 1988). We show that R&D is greater (smaller) in the delegated game than in the cooperative one for positive (negative) indirect spillovers. This occurs because the laboratory internalizes the strategic interaction of the two firms on the intermediate market for technology and on the final market for products via the payment schemes it receives. As a result the laboratory's choice of R&D is equivalent to it maximizing the sum of its benefits and the two firms' profits. In particular, zero within-laboratory externalities mean the equilibrium outcomes of the delegated R&D and cooperative R&D games are identical. When firms delegate R&D, the complementarity of their research projects means that the laboratory can produce R&D more efficiently than the firms. Hence, R&D in the delegated game can exceed the non-cooperative solution even when direct externalities are negative – in which case R&D in the non-cooperative game exceeds that in the cooperative one – provided indirect externalities are sufficiently positive. For similar reasons delegated R&D may exceed the non-cooperative solution when direct externalities are positive provided indirect externalities are not too negative.

When firms delegate R&D to the laboratory, they earn higher profits as indirect externalities increase. This arises because (i) it is relatively cheaper for the laboratory to perform R&D, than for firms to conduct it in house, as indirect externalities increase, while (ii) simultaneously the increased complementarity between the firms' R&D services means they can reduce their transfer payments to the laboratory. We show that firms' profits are higher when they delegate R&D to the laboratory, than in the other two organizational forms, for sufficiently high indirect externalities.

However, for reasons given above, there is no guarantee that the laboratory will choose to operate at such high levels of indirect externalities. If the laboratory must earn positive benefits to participate, the firms choose to delegate R&D only when direct externalities are low. This result differs sharply from the well established claim that R&D cooperation (as opposed to delegation) becomes more profitable with increasingly high direct externalities.

The welfare analysis proceeds by observing that higher R&D implies lower prices, more consumption, and consequently higher consumer surplus. We find that R&D delegation Pareto-dominates cooperation and non-cooperation, and the laboratory earns positive benefits, if and only if R&D services are sufficiently complementary inside the laboratory and inter-firm spillovers are sufficiently low. This occurs because (i) the laboratory operates only for sufficiently low indirect externalities, whereas (ii) firms earn higher profits and consumers obtain more surplus with delegated R&D than in the other two settings only for sufficiently high indirect externalities. This opposition prevents at least one of the parties (the laboratory, the firms, or consumers) to gain strictly more in the delegated R&D game than in the other two games when spillovers are too high. From a policy perspective, we prove that a firm's choice to delegate R&D to an independent profit-making laboratory never harms consumers. Hence, there is no room for policy intervention when R&D delegation takes place along the lines described in this paper.

The present analysis complements those on the industrial organization of R&D, in the spirit of Aghion and Tirole (1994) and Ambec and Poitevin (2001), which examine the impact of non-deterministic R&D on the relative efficiency of a separate governance structure (where a single user buys an innovation from an independent unit) and an integrated structure (in which the user sources R&D internally). Both Aghion and Tirole (1994) and Ambec and Poitevin (2001) assume a unique R&D user. We, on the other hand, are interested in the strategic interaction of several firms which not only contract with a common laboratory but also compete on the product market.[11] Another more recent strand of the R&D literature analyzes cooperative R&D in vertically-related industries (Banerjee and Lin 2001, Banerjee and Lin 2003, Atallah 2002, Brocas 2003, Ishii 2004, for examples). In these papers, firms may benefit from imperfectly appropriable process R&D produced not only by a direct competitor, but also by upstream or downstream firms. What is transacted by firms between successive stages of production is an homogeneous input to be transformed in some final good, not R&D services. Although this framework is perfectly valid for some settings, the examples provided here concern the delegation of R&D services.

The remainder of the paper is as follows. Section 2 presents the three R&D games, defines and discusses the equilibrium concepts. Section 3 establishes that the laboratory maximizes aggregate benefits and derives conditions under which it earns zero benefits. Section 4 ranks the outcomes of the three R&D games as a function of firm-level technological spillovers and within-laboratory (anti-)complementarities, and illustrate the results graphically in the direct and indirect externalities plane. Next, section 5 investigates whether one of the three games can Pareto-dominate the other two and discusses policy implications. Finally, section 6 concludes. All proofs and figures are in the Appendix.

2 R&D Games

We consider a duopoly which faces a linear inverse demand function:

$$p_i(\mathbf{q}) = a - b(q_i + \theta q_j), \tag{1}$$

for $i, j = 1, 2,\ i \neq j$, where $\mathbf{q} \equiv (q_i, q_j) \in \mathrm{R}^2_+$ describes output quantities, p_i is firm i's unit price, a and b are positive parameters, and $\theta \in [0, 1]$ captures the degree of substitutability between the two products. Each firm incurs a constant unit cost of production which it can reduce through process innovations. We also assume, as in d'Aspremont and Jacquemin (1988), a unit cost of production:

$$c_i(\mathbf{x}) = c - x_i - \beta x_j, \tag{2}$$

for $i, j = 1, 2,\ i \neq j$, where $\mathbf{x} \equiv (x_i, x_j) \in R^2_+$ is the vector of R&D outputs obtained by firms, the marginal cost parameter $c \in (0, a)$, and $\beta \in [0, 1]$ denotes technological spillovers. It follows that firm i's gross profit function is:

$$\pi_i(\mathbf{q}, \mathbf{x}) = [p_i(\mathbf{q}) - c_i(\mathbf{x})]q_i. \tag{3}$$

The next section formalizes three cost-reducing R&D games in extensive forms.

2.1 Cooperative R&D

In a first stage, the duopoly cooperatively chooses in-house R&D outcomes in the two proprietary laboratories by maximizing joint profits. The cost of R&D is given by:

$$r(x_i) = \frac{\gamma}{2} x_i^2, \tag{4}$$

for $i = 1, 2$, and where γ is a positive parameter. In a second stage, given the chosen R&D outcomes each firm non-cooperatively maximizes individual profits by choosing its output. We solve this game by backward induction. We start with stage two by looking for the Nash equilibrium in quantities on the final market.

Definition 1 (NE) *The symmetric final market outcome* $\mathbf{q}^*(\mathbf{x})$ *is a Nash equilibrium if:*

$$\pi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x}) \geq \pi_i(q_i, q_j^*(\mathbf{x}), \mathbf{x}), \tag{5}$$

all $\mathbf{x}$ *and all* q_i, $i, j = 1, 2$, $i \neq j$.

Substituting $\mathbf{q}^*(\mathbf{x})$ into the gross profit equation (3), we define firm i's concentrated profits as $\pi_i(\mathbf{x}) \equiv \pi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x})$. This last expression describes gross profits where a firm behaves *à la* Cournot on the final market for all levels of R&D. Using this concentrated profits function, we can express firm i's net profits in the first stage of the game, as a function of $\mathbf{x}$, by:

$$\pi_i^{\mathrm{c}}(\mathbf{x}) \equiv \pi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x}) - r(x_i). \tag{6}$$

We then maximize the sum of the two firms' profits to obtain the symmetric equilibrium level of R&D, denoted by x^c.[12] Substituting $\mathbf{x}^c \equiv (x^c, x^c)$ in (6) we can calculate a firm's symmetric net equilibrium profits as $\pi^{\mathrm{c}} = \pi_i^{\mathrm{c}}(\mathbf{x}^c)$. The vector of output quantities in the cooperative game is obtained by evaluating $\mathbf{q}^*$ at $\mathbf{x}^c$, denoted by $\mathbf{q}^c$.

Instead of cooperatively choosing their R&D, firms may decide to do so non-cooperatively, as explained below.

2.2 Non-Cooperative R&D

In a first stage, firms non-cooperatively conduct R&D in-house by maximizing their individual profits in their own R&D, with each firm's R&D costs given by (4). The second stage is as in the cooperative R&D game. In this game, we denote firm i's net profit as a function of $\mathbf{x}$ by:

$$\pi_i^{\mathrm{n}}(\mathbf{x}) \equiv \pi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x}) - r(x_i). \tag{7}$$

We solve the game by backward induction.

Definition 2 (SPNE) *The symmetric equilibrium quantities and in-house* R&D *outcomes* $(\mathbf{q}^{\mathrm{n}}, \mathbf{x}^{\mathrm{n}})$ *are a subgame-perfect Nash equilibrium if:*

i) $\mathbf{q}^{\mathrm{n}} \equiv \mathbf{q}^*(\mathbf{x}^{\mathrm{n}})$, *where* $\mathbf{q}^*(\mathbf{x})$ *is a* NE *as in Definition 1;*

ii) $\mathbf{x}^{\mathrm{n}}$ *is a* NE, *that is* $\pi_i^{\mathrm{n}}(\mathbf{x}^{\mathrm{n}}) \geq \pi_i^{\mathrm{n}}(x_i, x_j^{\mathrm{n}})$, *for all* x_i, $i, j = 1, 2$, $i \neq j$.

Substituting $\mathbf{x}^{\mathrm{n}}$ into (7) we obtain each firm's symmetric net equilibrium profits, denoted by π^{n}.

This game is identical to another one where, in lieu of in-house R&D production, there are two independent laboratories. In that alternative game, each firm writes a contract with a dedicated laboratory to obtain specific R&D services in exchange of transfer payments. This gives two symmetric principal-agent relationships. It follows that a firm's problem is as in (7), but $r(x_i)$ is now firm i's payment for x_i, in lieu of an R&D cost. In a complete information set-up, if each laboratory must at least break even to participate, each firm's payment can be chosen so that each laboratory earns exactly zero benefits. [13] The problem is however different if there is a unique, common, and independent laboratory, from which the two firms buy R&D services. We tackle this next in a new framework.

2.3 Delegated R&D

We assume there is one independent laboratory which produces R&D at a cost which increases with the quantity supplied to each firm. However, within-laboratory externalities can arise when the laboratory produces R&D for both firms. The sign of externalities depends on the nature of the R&D services which the firms request. The production of R&D services is said to be complementary (substitutable) when the marginal cost of supplying one firm decreases (increases) with the quantity supplied to the other firm.

For the sake of tractability, and comparison with the numerous cooperative and non-cooperative R&D games explored in the literature using the framework of d'Aspremont and Jacquemin (1988), we specify the following laboratory's cost function:

$$s(\mathbf{x}) = \frac{\gamma}{2}(x_1^2 + x_2^2) - \delta x_1 x_2, \tag{8}$$

for $i = 1, 2$ and $i \neq j$, and $\delta \in [-\gamma, \gamma)$. The parameter δ captures complementary (substitutable) R&D services in the laboratory if it is positive (negative). If δ equals zero, the laboratory is as efficient as each firm's proprietary laboratory and (8) collapses to the sum of (4) over the two firms. Note that the term $\delta x_1 x_2$ is the simplest way to capture complementarity or substitutability between two variables. A nice aspect of (8) is that complementarity or substitutability is reflected by the sign of a single parameter as suggested by Milgrom and Roberts (1990, p. 517) in an illustrative example. The same algebraic specification appears in the complete information version of the cost function of a common agent in Martimort and Stole (2003a), and in the utility function of a common agent in Martimort and Stole (2003b).

In the delegated R&D game, we let each firm's transfer to the laboratory depend on both its R&D, and that of its competitor, purchased from the laboratory. Formally, we denote for each firm the set of transfer payments by:

$$T \equiv \{t | t(\mathbf{x}) \geq 0 \quad \text{for all} \quad \mathbf{x}\}. \tag{9}$$

This specification differs from much of the common agency literature which usually focuses on the consumer goods market where each principal supplies a common agent which is a retailer or a consumer (Gal-Or 1991, Stole 1991, Martimort 1996, Bernheim and Whinston 1998, Calzolari and Scarpa 2004, Martimort and Stole 2004, for examples). In these models, the transfers to the common agent depends only on the amount each principal transacts with the agent. This assumption has two justifications. The first one relates to the informational set-up. If the contractual clauses that organize product flows are either not observable by the agent's clients or not verifiable in court, then each principal's transfer payments should depend exclusively on its own quantity. The second justification has its roots in the institutional context. Indeed, product market transactions are subject to no-discrimination rules which are enforced by antitrust authorities.

Our paper builds on other well-established specifications of a distinct literature on the industrial organization of R&D as discussed in the introduction (d'Aspremont and Jacquemin 1988, Kamien, Muller and Zang 1992a, Amir 2000). That literature assumes that information is complete and that observable R&D efforts are verifiable. Consequently, firms may (horizontally) contract on their respective choice variables when they choose to cooperate in the R&D stage. Moreover, all papers which compare R&D cooperation *versus* non-cooperation are based on the fact that antitrust rules on R&D are less stringent than those on product markets.[14]

Many cooperative agreements typically involve rivals that collaborate horizontally at the pre-competitive stage by coordinating the use of proprietary R&D facilities. These types of agreements are well analyzed by the means of existing models of R&D cooperation. However, the stylised facts that are descriptive of some other agreements are better captured by the specifications of our delegated R&D model. For example, Katz *et al.* (1990, pp. 186-190) describe the case of MCC (Microelectronics and Computer Technology Corporation), which became "an open-market supplier of contract R&D" to supply application-oriented "deliverables" to large firms (*e.g.*, Hewlett-Packard, Motorola, Honeywell, Westinghouse Electric). More recently, Majewski (2004), who uses detailed contract-level data on R&D projects that filed for protection under the US National Cooperation Research Act, documents many cases where horizontal competitors in product markets actually outsource their R&D project to an independent for-profit R&D entity.[15]

In addition, many R&D contracts have complex contractual clauses that authorize a fine tuning between the payments of a given firm and the R&D services the laboratory may supply to other firms (see http://contracts.onecle.com for a large sample of contracts). For example, in the late 1990s Tularik – a Californian independent laboratory that specializes in the research and development of therapeutic pharmaceutical products based on a proprietary technology – signed a series of bilateral multi-annual contracts for the delivery of firm-specific and nevertheless technologically related R&D services to a set of American, Japanese, and European firms. The latter include Merck, Sumitomo Pharmaceuticals, and Roche Bioscience, which can be rivals on final markets for pharmaceutical products. In these contracts, one reads clauses that explicitly acknowledge the existing contractual links which involve the common laboratory, with a nominative list of third parties. Some other clauses also stipulate that the laboratory may not transfer any R&D output resulting from the contractual agreement without the prior written consent of the particular firm that originated it. In some cases, the firm has an option to purchase the rights to some identified R&D outcomes of the laboratory, before such opportunity is offered to any third party (the so-called "right of first refusal"), for a certain period of time from the date of the result. That period, together with the technological breadth of the rights, and the financial terms, are all negotiable. This actually forms a link between a given firm's transfer payments and the R&D outputs received by all other laboratory's clients.

In view of our objective to compare the performance of the delegated R&D game with standard cooperative and non-cooperative models, we substitute the specifications of the literature of reference on R&D for the assumptions that apply in the case of contractual arrangements between wholesalers and a common retailer. The assumed verifiability of choice variables, the specificity of antitrust rules toward R&D agreements, and the observed clauses in sample contracts, justify the assumption that firms condition their payments to an independent laboratory on the level of R&D services their competitors may receive.[16]

Several explanations for the fact that buyers of new knowledge write contracts exist in the literature. In a cross-sectorial empirical analysis, Veugelers (1997) remarks that when in-house facilities are available, as we assume here, the capacity to go for it alone increases a firm's bargaining power in negotiating with an external laboratory. On the intermediate market for biotechnology, where R&D buyers are large pharmaceutical, agribusiness, or chemical firms, Lerner and Merges (1998) evoke the financial constraints faced by specialized laboratories, and Argyres and Liebeskind (2002) refer to a high rate of entry on the supply side. In the words of Hagedoorn and Roijakkers (2006), we focus on cases in which "one partner, typically a large firm, contracts another, usually a small, partner to develop a specific technology" (p. 434).

The timing of the delegated R&D common agency game is as follows. In a first stage, the two firms (principals) simultaneously and non-cooperatively offers contingent transfer payments $t_i(\mathbf{x})$ to the independent laboratory (an agent). In a second stage, given $\mathbf{t} \equiv (t_1, t_2)$, the laboratory accepts either both contracts, or only one, or none. Then it chooses the verifiable amounts of firm-specific R&D services, at a cost $s(\mathbf{x})$, to maximize its own benefits given by:

$$\mathcal{L}(\mathbf{x}) = t_1(\mathbf{x}) + t_2(\mathbf{x}) - s(\mathbf{x}). \tag{10}$$

The third stage is as the final stage in the other two games.

Note that if the laboratory refuses all contracts it produces no R&D services and earns zero benefits. This outside option leads to the following participation constraint:

$$\mathcal{L} \geq 0. \tag{11}$$

When discussing policy implications later, we shall consider situations where (11) holds with strict inequality. This would be the case if the laboratory incurs positive (arbitrarily small) installation costs, or faces a profitable outside option. Finally, we denote the set of R&D services which, given strategies $\mathbf{t}$, maximize the laboratory's benefits by:

$$X(\mathbf{t}) \equiv \arg\max_{\mathbf{x}} \mathcal{L}(\mathbf{x}(\mathbf{t})). \tag{12}$$

In this model, identical firms address symmetric contingent transfer proposals to the laboratory. This does not imply that equilibrium R&D services, and the related equilibrium payments, are symmetric across firms. While in the cooperative R&D game the ex-ante equal treatment of firms (in the sense of Leahy and Neary (2005)) applies when side-payments between firms are ruled out, in the delegated R&D game there is no reason to constrain *a priori* the laboratory to supply symmetric R&D output. In fact, Section 3 establishes that for some parameter values the laboratory may serve only one firm in equilibrium.

To compare the delegation of R&D with the cooperative and non-cooperative reference cases, we maintain the assumption that information is complete among firms. However, this does not extend to the laboratory which needs not know downstream cost and demand functions. An outcome of the delegated R&D game is a three-tuple $(\mathbf{x}^{\mathrm{d}}, \mathbf{t}^{\mathrm{d}}, \mathbf{q}^{\mathrm{d}})$, where $\mathbf{x}^{\mathrm{d}}$ denotes the laboratory's equilibrium choice, $\mathbf{t}^{\mathrm{d}}$ firms' equilibrium payments, and $\mathbf{q}^{\mathrm{d}}$ equilibrium quantities on the final market. In this game firm i's net profit, as a function of $\mathbf{x}$, equals:

$$\pi_i^{\mathrm{d}}(\mathbf{x}) \equiv \pi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x}) - t_i^{\mathrm{d}}(\mathbf{x}). \tag{13}$$

The laboratory bears all R&D costs, while the functional form of firms' net profits in the delegated R&D game is similar to Crémer and Riordan (1987) who model multilateral transactions with bilateral contracts, but with transfer payments that are here contingent on the laboratory's choice of R&D outputs.[17] For this game, by equilibria we mean truthful subgame-perfect Nash equilibria, which are defined as follows.

Definition 3 (TSPNE) *The equilibrium delegated* R&D *outcomes, transfer payments, and equilibrium quantities* $(\mathbf{x}^{\mathrm{d}}, \mathbf{t}^{\mathrm{d}}, \mathbf{q}^{\mathrm{d}})$ *are a truthful subgame-perfect Nash equilibrium if:*

i) $\mathbf{q}^{\mathrm{d}} \equiv \mathbf{q}^*(\mathbf{x}^{\mathrm{d}})$, *where* $\mathbf{q}^*(\mathbf{x})$ *is a* NE *as in Definition 1;*

ii) $(\mathbf{x}^{\mathrm{d}}, \mathbf{t}^{\mathrm{d}})$ *is a* NE*, that is* $\mathbf{x}^{\mathrm{d}} \in X(\mathbf{t}^{\mathrm{d}})$ *and there is no* $i = 1, 2$*,* $t_i \in T$*, and no* $\mathbf{x} \in X(t_i, t_j^{\mathrm{d}})$ *such that* $\pi_i^{\mathrm{d}}(\mathbf{x}) > \pi_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$*;*

iii) t_i^{d} *is truthful relative to* $\mathbf{x}^{\mathrm{d}}$*, that is for all* $\mathbf{x}$ *either* $\pi_i^{\mathrm{d}}(\mathbf{x}) = \pi_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$*, or* $\pi_i^{\mathrm{d}}(\mathbf{x}) < \pi_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$ *and* $t_i^{\mathrm{d}}(\mathbf{x}) = 0$*,* $i, j = 1, 2$*,* $i \neq j$*.*

Substituting $\mathbf{x}^{\mathrm{d}}$ into (13) gives a firm's net equilibrium profits which, when symmetric, are denoted by π^{d}.

Intuitively, in any truthful equilibrium, a firm offers a transfer $t_i^{\mathrm{d}}(\mathbf{x})$ that reflects exactly its individual valuation of the laboratory's choice of $\mathbf{x}$ with respect to $\mathbf{x}^{\mathrm{d}}$, all $\mathbf{x}$. Definition 3-iii) refers to two possible cases. Either gross profits $\pi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x})$ exceed net equilibrium profits $\pi_i(\mathbf{q}^{\mathrm{d}}, \mathbf{x}^{\mathrm{d}}) - t_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$, and the difference between transfer offers $t_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$ and $t_i^{\mathrm{d}}(\mathbf{x})$ is set equal to the difference between gross profits $\pi_i(\mathbf{q}^{\mathrm{d}}, \mathbf{x}^{\mathrm{d}})$ and $\pi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x})$. Or principal i's gross profits with $\mathbf{x}$ are strictly less than net equilibrium profits obtained with $\mathbf{x}^{\mathrm{d}}$, in which case the transfer $t_i^{\mathrm{d}}(\mathbf{x})$ is set to zero.

As formally stated in Definition 3, the truthful equilibrium concept refers to a menu of payments which, when non zero, are equal to the difference between each principal's gross payoff for any possible output of the agent and a constant. In this game, the constant is the net equilibrium profit. Only payments for the output that is actually supplied are received by the agent in equilibrium. Bernheim and Whinston (1986b), who first introduced the truthfulness refinement, also derive several properties to characterize truthful equilibria in a class of delegated common agency games that includes our set-up. A first property, when expressed in the terms of the present context, says that for any set of transfer offers by any one of the two firms, there exists a truthful strategy in the other firm's best-response correspondence. This implies that a firm can restrict itself to truthful strategies. A second property is that all truthful Nash equilibria are coalition-proof. In our case, this means that joint net profits, as obtained in a truthful subgame-perfect equilibrium by the two firms, are higher than in any other subgame-perfect Nash equilibria.[18] These two properties hold for all given choices of $\mathbf{q}$ in the final stage, including $\mathbf{q}^*$.

We acknowledge that non-truthful strategies may also be obtained as a Nash equilibrium in delegated common agency games. This property is made clear by Kirchsteiger and Prat (2001) who introduce the definition of very simple strategies, which they call "natural". A given principal's transfer payment is natural when it is strictly positive on only one agent's output, and zero otherwise. This describes an example of a forcing contract. Interestingly, while the first property (existence of truthful strategies in a principal's best-response correspondence) still applies to Nash equilibria in natural strategies, the second property (coalition-proof) does not. Moreover, when there are only two principals, as in our model, a coalition-proof equilibrium is Pareto-efficient among principals.[19] This offers a strong justification for choosing truthful (subgame-perfect Nash) equilibrium as a solution concept.

3 Profits Maximization and Distribution

In this section, we examine how the laboratory's choice compares with joint profits maximization, and establish that the two firms purchase R&D services in equilibrium. We then derive a condition under which the laboratory earns positive benefits. This condition partitions the (β, γ) space, which we refer to as the externalities plane in the remainder of the paper.

We start be defining two externalities which will be helpful in discussing the results. Concentrated profits $\pi_i(\mathbf{x})$ vary with the level of technological spillovers as measured by β. These spillovers are a *direct* externality because firm i's gross profits not only depend on x_i, but also on x_j for all $\beta > 0$. These externalities are negative (positive) if an increase in x_j has a negative (positive) impact on firm i's concentrated profits.

Property 1 (Direct Externalities) *For* $i, j = 1, 2,$ *and* $i \neq j$

$$\frac{\mathrm{d}\pi_i(\mathbf{x})}{\mathrm{d}x_j} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \textit{if and only if} \quad \beta \begin{matrix} > \\ = \\ < \end{matrix} \theta/2.$$

In what follows, we identify positive (negative) direct externalities with $\beta > (<)\theta/2$.

Within-laboratory technological conditions give rise to *indirect* externalities which appear only in the delegated R&D game. We shall say that indirect externalities are negative (positive) if serving higher quantities to a firm makes it more (less) costly for the laboratory to serve the other one, i.e. if the production of R&D services are substitutable (complementary). More formally, for the cost function (8):

Property 2 (Indirect Externalities) *For* $i, j = 1, 2,$ *and* $i \neq j$

$$\frac{\mathrm{d}^2 s(\mathbf{x})}{\mathrm{d}x_i \mathrm{d}x_j} \begin{matrix} < \\ = \\ > \end{matrix} 0 \quad \textit{if and only if} \quad \delta \begin{matrix} > \\ = \\ < \end{matrix} 0.$$

Typically, R&D services are complements (i.e., $\delta > 0$) when the laboratory can serve the two firms by using the same resources. They are substitutes (i.e., $\delta < 0$) when there are bottlenecks in the laboratory's capacity to simultaneously supply the two firm-specific services.

Let the aggregate benefits function for the two firms and the laboratory be:

$$\Lambda(\mathbf{x}) = \pi_1(q^*(\mathbf{x}), \mathbf{x}) + \pi_2(q^*(\mathbf{x}), \mathbf{x}) - s(\mathbf{x}). \tag{14}$$

Proposition 1 (Joint Profits Maximization and Common Laboratory) *In all TSPNE in which* $\mathcal{L} \geq 0$*,*

i) The laboratory's choice of R&D *services to maximize its benefits (10) is equivalent to maximizing aggregate benefits (14), and*

ii) There exists a continuous strictly decreasing frontier in the externalities plane (β, δ)*, denoted by* $\bar{\delta}$*, which takes values in the interval* $[-\gamma, \gamma)$ *and lies below* $(\theta/2, 0)$*, such that:*

(a) if $\delta \leq \bar{\delta}$ *the laboratory serves one firm only;*

(b) if $\delta > \bar{\delta}$ *the laboratory serves the two firms.*

Proposition 1 contains two related claims. Claim i) is a restatement of Bernheim and Whinston (1986b) adapted to our context. It states that the non-cooperative attempt by firms to maximize individual profits, by delegating R&D, leads the laboratory to deliver a pair of R&D services which maximizes the aggregate benefits of all parties, that is the two firms and the laboratory. This "efficiency" (from the viewpoint of firms and the laboratory only) result is rooted in the truthfulness refinement concept, as discussed above. It specifies that the laboratory is offered two transfer schedules which exactly reflect the respective shape of each firm's gross profit function (that is, $\pi_i(\mathbf{q}^{\mathrm{d}}(\mathbf{x}), \mathbf{x})$, all $\mathbf{x}$, $i = 1, 2$). The laboratory thus internalizes both direct and indirect externalities, and in equilibrium maximizes the sum of the two firms' profits, net of R&D costs. This result is silent on consumers' welfare. We will be able to address this issue once we compute the quantities of R&D services produced by the laboratory and compare them to the two other games. This is the subject of section 4.1.

Claim ii) of Proposition 1 establishes that, depending on the level of δ, in equilibrium the laboratory delivers R&D services either to one firm only (either firm 1 or firm 2, indifferently), or to both of them. The latter situation occurs when technological conditions do not penalize the production of a two-dimensional R&D output, and transfer payments incentivize the laboratory to serve the two firms. In this case, there exists an interior solution to the joint-profit maximization problem, where (*14*) is the maximand.

For a specific example, let $a = 1$, $b = 1$, $c = 3/4$, $\gamma = 2$, and $\theta = 1$. Then consider two points in $[0, 1] \times [-2, 2)$ above the frontier $\bar{\delta} = 2(1-\beta)^2 - 2$ (see Figure 1), say $(1/5, -1/5)$ and $(4/5, -1/5)$. The latter point lies in the South-East quadrant of the externalities plane, above the frontier $\delta_{\mathcal{L}=0}$, implying that firms absorb all R&D benefits. Each of them receives $x^{\mathrm{d}} \approx 0.068$ in exchange of a payment $t^{\mathrm{d}} \approx 0.050$ and earns $\pi^{\mathrm{d}} \approx 0.010$, while $\mathcal{L} = 0$. By contrast, the former point is located below $\delta_{\mathcal{L}=0}$ in the South-West quadrant, therefore the laboratory earns positive benefits. At that point, each firm receives $x^{\mathrm{d}} \approx 0.035$, pays $t^{\mathrm{d}} \approx 0.004$, and earns $\pi^{\mathrm{d}} \approx 0.005$. The laboratory earns $\mathcal{L} \approx 0.005$.[20]

In the next section, we shall focus on symmetric interior solutions, denoted by $x_1^{\mathrm{d}} = x_2^{\mathrm{d}} = x^{\mathrm{d}}$, to compare the equilibrium outcomes of the delegated R&D game with the ones obtained in the benchmark cooperative and non-cooperative R&D games.

When δ is close to the lower bound $-\gamma$, a problem of congestion arises, in which case firms' payment schemes incentivize the laboratory to specialize in the production of only one line of R&D services. Since the frontier $\bar{\delta}$ decreases when β increases, the congestion is more frequent when direct externalities are low. Then the joint-profit maximization problem calls for a corner solution.[21] In the latter situation, recall that the asymmetry in the supplied R&D levels is captured by firms' respective transfer payments to the laboratory. From the truthfulness property, we know that each firm's offered payments exactly reflect its individual valuation of $\mathbf{x}$. This implies that, in equilibrium, the two identical firms earn the same net positive profits, we denote by $\bar{\pi}^{\mathrm{d}}$, although R&D levels are asymmetric, for $\bar{x}_1^{\mathrm{d}} = 0 < \bar{x}_2^{\mathrm{d}}$ (when the laboratory does not serve firm 1, say). In that case, the laboratory receives asymmetric payments $\bar{t}_1^{\mathrm{d}} = \pi_1\left(0, \bar{x}_2^{\mathrm{d}}\right) - \bar{\pi}^{\mathrm{d}}$ and $\bar{t}_2^{\mathrm{d}} = \pi_2\left(0, \bar{x}_2^{\mathrm{d}}\right) - \bar{\pi}^{\mathrm{d}}$. It is straightforward to check that $0 \leq \bar{t}_1^{\mathrm{d}} < (=) \bar{t}_2^{\mathrm{d}}$ for all $\beta < (=) 1$. Whenever $\bar{t}_1^{\mathrm{d}}$ is positive, firm 1 participates in the financing of the R&D output, although it does not receive anything directly from the laboratory. It benefits from firm 2's purchased R&D through spillovers.

Denote by Λ the maximum aggregate benefits obtained by maximizing (14) with respect to $\mathbf{x}$. The following proposition partitions the externalities plane.

Proposition 2 (Joint Profits Distribution) *There exists a continuous strictly decreasing frontier in the externalities plane* (β, δ)*, denoted by* $\delta_{\mathcal{L}=0}$*, and which includes the point* $(\theta/2, 0)$*, such that in all TSPNE the laboratory earns positive benefits if* $\delta < \delta_{\mathcal{L}=0}$*, and exactly breaks even otherwise.*

Proposition 2 says that the magnitude of indirect externalities (δ), for a given value of direct externalities (β), determines the laboratory's ability to appropriate a share of innovation benefits, and thus laboratory's participation constraint (11) to be slack or binding. This is because indirect externalities, in combination with inter-firm technological spillovers, impact the nature of competition between the two firms on the intermediate market for R&D. This competition is reflected by their offers of transfer payments $(t_1^{\mathrm{d}}(\mathbf{x}), t_2^{\mathrm{d}}(\mathbf{x}))$. On the one hand, if both externalities are negative, a firm's concentrated profits decrease with the other firm's R&D (Property 1), and serving one firm increases the laboratory's cost of serving the other (Property 2). This is a case of tough competition between the two firms for the laboratory's services, which is a source of positive profits for it. On the other hand, if both externalities are positive, a firm's concentrated profits are increasing in the other firm's R&D, and serving one firm decreases the laboratory's cost of serving the other. Thus, competition for the laboratory's resources is relatively soft and the laboratory earns no benefits. When the externalities are of opposite signs, the laboratory's ability to appropriate benefits depends on their magnitudes. This opposition gives rise to $\delta_{\mathcal{L}=0}$, which can thus be viewed as a weighted sum of direct and indirect externalities.

Propositions 1 and 2 are useful for the comparison of the outcomes of the three R&D games at the pivotal no-externalities point $(\beta, \delta) = (\theta/2, 0)$.

Proposition 3 (The No-Externalities Case) *The outcomes of the three games are the same at the pivotal no-externalities point.*

At the pivotal point, there are no direct and indirect externalities. This implies that solutions in $\mathbf{x}$ are the same in the three R&D games. In the delegated game the laboratory earns zero benefits, as if firms were relying on in-house R&D capabilities, because $(\beta, \delta) = (\theta/2, 0)$ is on $\delta_{\mathcal{L}=0}$. We now solve the three R&D games by backward induction and rank the performance of the three games in the externalities plane. The explicit solutions of the games are in Appendix A.

4 Comparing the Three Games

We partition the externalities plane by deriving frontiers on which R&D, profits, or welfare are equal in the delegated R&D game and in one of the two alternative games. By welfare, we mean the sum of consumer surplus, firms' profits, and the laboratory's benefits. For the sake of completeness, we also include the comparison of the outcomes of the cooperative and non-comparative games as established by d'Aspremont and Jacquemin (1988). In the delegated R&D game, we focus on interior solutions to the joint-profit maximization problem, as solved by the laboratory by reacting to payment offers to maximize its own benefits. Note from the onset that, as a result of Proposition 3 all such frontiers include the pivotal no-externalities point.

4.1 R&D outcomes

Lemma 1 (Cooperative, Non-Cooperative, and Delegated R&D)

i) There exists a continuous frontier $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$ *in the externalities plane such that in all TSPNE* $x^{\mathrm{d}} \gtreqless x^{\mathrm{c}}$ *if and only if* $\delta \gtreqless \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$*, with*

$$\delta_{\mathcal{L}=0} > \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 \quad \textit{for} \quad \beta < \theta/2;$$

$$\delta_{\mathcal{L}=0} = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 \quad \textit{for} \quad \beta = \theta/2;$$

$$\delta_{\mathcal{L}=0} < \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 \quad \textit{for} \quad \beta > \theta/2.$$

ii) There exists a continuous frontier $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ *in the externalities plane, such that in all TSPNE* $x^{\mathrm{d}} \gtreqless x^{\mathrm{n}}$ *if and only if* $\delta \gtreqless \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$*, with*

$$0 < \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\mathcal{L}=0} \quad \textit{for} \quad \beta < \theta/2;$$

$$\delta_{\mathcal{L}=0} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} = 0 \quad \textit{for} \quad \beta = \theta/2;$$

$$0 > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} > \delta_{\mathcal{L}=0} \quad \textit{for} \quad \beta > \theta/2.$$

Direct and indirect externalities combine to give Lemma 1. First consider Lemma 1-(i). The cooperative and delegated games yield the same R&D solution when there are no indirect externalities because of Proposition 1 (which says that the laboratory maximizes aggregate benefits in equilibrium), and of Property 2 (which implies that costs are the same in both games when $\delta = 0$). We know that the independent laboratory is more (less) efficient than in-house laboratories when indirect externalities are positive (negative), that is when $\delta > 0$ ($\delta < 0$). This completes the partitioning of the externalities plane for R&D output in the two games under scrutiny.

Second, consider Lemma 1-(ii). Recall that, from Property 1, optimal R&D is greater (smaller) in the cooperative than in the non-cooperative game for positive (negative) direct externalities. Let direct externalities be positive. If indirect externalities are also positive, the laboratory's higher efficiency means that delegated R&D exceeds the cooperative, and hence the non-cooperative, solutions. If indirect externalities are negative, the laboratory is at a disadvantage in the production of R&D over in-house laboratories. However, as it internalizes inter-firm direct externalities via the transfer payments it receives, it is only for sufficiently negative indirect externalities that non-cooperative R&D exceeds the delegated game solution. Consequently, $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ must cross in the South-East quadrant of the externalities plane.

Now let direct externalities be negative. If indirect externalities are also negative, the laboratory's lower efficiency than in-house laboratories means that the delegated solution is smaller than the cooperative and (by transitivity) the non-cooperative one. However, as the laboratory gains in efficiency as δ increases, there exist sufficiently high positive indirect externalities for the R&D outcome under the delegated game to exceed that under the non-cooperative game. Hence $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ must lie in the North-West quadrant of the externalities plane.

[Insert figure 1 about here]

The juxtaposition of $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$ and $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ in the externalities plane, as illustrated in Figure 1, allows us to rank optimal R&D across the three games. It is of interest that optimal R&D in the delegated game is greater than in either of the two games for sufficiently high indirect externalities, even when direct externalities are negative. This result stands in contrast with cooperative R&D always being less than non-cooperative one for negative direct externalities.

4.2 Firms' Profits

Lemma 2 (Cooperative, Non-Cooperative, and Delegated Profits)

i) There exists a continuous frontier $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ *in the externalities plane such that in all TSPNE* $\pi^{\mathrm{d}} \gtreqless \pi^{\mathrm{c}}$ *if and only if* $\delta \gtreqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$*, with*

$$
\begin{array}{lll}
0 < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} < \delta_{\mathcal{L}=0} & \textit{for} & \beta < \theta/2; \\
\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} = \delta_{\mathcal{L}=0} = 0 & \textit{for} & \beta = \theta/2; \\
0 = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} > \delta_{\mathcal{L}=0} & \textit{for} & \beta > \theta/2.
\end{array}
$$

ii) There exists a continuous frontier $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ *in the externalities plane such that in all TSPNE* $\pi^{\mathrm{d}} \gtreqless \pi^{\mathrm{n}}$ *if and only if* $\delta \gtreqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$*, with*

$$
\begin{array}{lll}
0 < \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} & \textit{for} & \beta < \theta/2; \\
\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} = 0 & \textit{for} & \beta = \theta/2; \\
0 = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} & \textit{for} & \beta > \theta/2.
\end{array}
$$

The intuition for $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ follows also from how the two externalities combine. For the same reasons as in Section 4.1, aggregate benefits are *ceteris paribus* increasing in indirect externalities. However, when part of the aggregate benefits accrue to the laboratory, which is the case for $\delta < \delta_{\mathcal{L}=0}$, then indirect externalities must be sufficiently positive to generate enough surplus to compensate for the laboratory's benefits. Hence, if direct externalities are negative, the locus which equalizes firms' profits in the delegated and cooperative games must lie in the North-West quadrant of the externalities plane. It cannot however lie above $\delta_{\mathcal{L}=0}$ where aggregate benefits in the delegated game exceed those in the cooperative game, but are divided equally between the two firms. If direct externalities are positive, the frontier is confounded with $\delta = 0$ because of Proposition 1, the cost structure being the same in both games and the laboratory earning zero benefits.

The intuition for the $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ locus is as follows. Recall that a firm's profits in the cooperative game always exceed those under the non-cooperative one because cooperation internalizes direct externalities and prevents R&D duplication. As a firm's profits in both the cooperative and delegated games are equal along $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$, by transitivity delegated profits exceed non-cooperative ones along that locus. Consider negative direct externalities. For $\delta = 0$, along that line cooperative profits are greater than those obtained in the delegated game. However, firms' profits in the delegated game are increasing in indirect externalities (Lemma D-2 in Appendix D). Hence, there exists a unique decreasing continuous locus in the North-West quadrant of Figure 2 such that $\pi^{\mathrm{d}} = \pi^{\mathrm{n}}$.

By the same token, there must exist a locus in the South-East quadrant of Figure 2 which equalizes profits in the delegated and non-cooperative games. That locus must lie below $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ for the following reason. Along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ optimal R&D expenditures are equal in both the delegated and non-cooperative games. However, the laboratory is less efficient than in-house R&D when there are negative indirect externalities. It follows that aggregate benefits in the non-cooperative game exceed those in the delegated game along that locus. As the laboratory does not earn negative profits, $\pi^{\mathrm{d}} < \pi^{\mathrm{n}}$ along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$. Therefore $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ lies above $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$.

[**Insert figure 2 about here**]

Figure 2 graphs $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ and $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ to compare firms' profits in the three games. As expected, firms' profits are highest in the delegated game when both externalities are positive. However, delegated R&D may yield the lowest profits even if direct externalities are weakly negative and indirect externalities are weakly positive (region below $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ in Figure 2). This occurs because in that region the laboratory earns positive benefits and indirect externalities do not have a high enough impact on aggregate benefits. Hence, positive indirect externalities are necessary but not sufficient for firms to prefer the delegated game to the other two. Note that the firms' profits results have a benchmark flavor, in the sense that the net benefits obtained by a laboratory endowed with some informational advantage, would be bounded from below by the equilibrium benefits obtained here.

4.3 Welfare

Lemma 3 (Cooperative, Non-Cooperative, and Delegated Welfare)

i) There exists a continuous frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$ in the externalities plane such that in all TSPNE $w^{\mathrm{d}} \gtreqless w^{\mathrm{c}}$ if and only if $\delta \gtreqless \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$, with

$$0 = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} < \delta_{\mathcal{L}=0} \quad \text{for} \quad \beta < \theta/2;$$

$$0 = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} = \delta_{\mathcal{L}=0} \quad \text{for} \quad \beta = \theta/2;$$

$$0 = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} > \delta_{\mathcal{L}=0} \quad \text{for} \quad \beta > \theta/2.$$

ii) There exists a continuous frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ in the externalities plane such that in all TSPNE $w^{\mathrm{d}} \gtreqless w^{\mathrm{n}}$ if and only if $\delta \gtreqless \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$, with

$$0 < \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} < \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} \quad \text{for} \quad \beta < \theta/2;$$

$$\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} = 0 \quad \text{for} \quad \beta = \theta/2;$$

$$0 > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} \quad \text{for} \quad \beta > \theta/2.$$

The frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$ is the direct consequence of Property 2, Proposition 1, and aggregate benefits being increasing in indirect externalities. To understand the intuition for $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$, let direct externalities be negative (i.e., $\beta < \theta/2$). If $\delta = 0$ in that region, both optimal R&D and firms' profits in the delegated game are smaller than in the non-cooperative game by Lemmas 1-(*ii*) and 2-(*ii*) respectively.

Therefore, when indirect externalities are negative, $w^{\mathrm{d}} < w^{\mathrm{n}}$ along $\delta = 0$. Second, aggregate benefits in the delegated game must be greater than in the non-cooperative game along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ because the laboratory is more efficient than in-house R&D facilities, and by definition the same amount of R&D is performed in both games. Moreover, w^{d} is increasing in δ (see Lemma D-3 in Appendix D). It follows that for each β in the region bounded by $\delta = 0$ and $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, there exists a value for δ such that welfare in the delegated and non-cooperative games are equal. The existence of $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ in the South-East quadrant of the externalities plane can be rationalized in the same way.

[**Insert figure 3 about here**]

It is worth noting that our analysis, and all the results, can be extended to a variant of the cooperative game. In this amended set-up, firms could still run their two independent laboratories but benefit from complementarities in the production of R&D outputs. In this case, the cost of doing R&D would be the same as in the delegated R&D game with a non-negative δ. Recall now from Proposition 1 that the laboratory acts as a "social planner" by choosing the quantities of R&D which maximize aggregate benefits, i.e. its own benefits and the sum of the two firms' profits. This implies that the objective function where both firms cooperate and there are complementarities across their R&D activities is identical to the objective function of the R&D delegated game.

It follows that, in this alternative cooperative R&D model, the optimal R&D level accruing to each firm, as well as the welfare level, would be identical to the ones of the delegated R&D game in equilibrium. Only firms' profits across the two games may differ for some parameter values. To see that, recall from Proposition 2 that the externalities plane (β, δ) can be partitioned into two subspaces: one where the laboratory earns no benefits, and another one where the laboratory earns positive benefits. In the alternative cooperative R&D game, any positive profits the laboratory earns in the delegated game would be equally divided between the two firms.

One could thus derive new frontiers in the externalities plane, on which firms' symmetric profits $\bar{\pi}^{\mathrm{c}}$ in the alternative cooperative R&D game are equal to those obtained in any of the other games. Section E in the Appendix proves that the profit frontiers for the comparison of $\bar{\pi}^{\mathrm{c}}$ with π^{c} and π^{n} coincide with the ones we derived in Lemma 2 for the comparison of π^{d} with π^{c} and π^{n}, whenever the laboratory earns no benefits, that is above the frontier $\delta_{\mathcal{L}=0}$ in Figure 2. This is because, in the latter subspace, firms earn the same equilibrium profits in the alternative cooperative R&D game as in the delegated R&D game. By contrast, strictly below $\delta_{\mathcal{L}=0}$, each firm earns higher equilibrium profits in the alternative cooperative R&D game than in the delegated R&D game where the common independent laboratory appropriates some benefits. One finds also that $\bar{\pi}^{\mathrm{c}} = \pi^{\mathrm{c}}$ if and only if $\delta = 0$, and that $\bar{\pi}^{\mathrm{c}} > \pi^{\mathrm{n}}$ for all $\delta \geq 0$. The intuition for this result is that, when complementarities are present, cooperative firms in the alternative model cannot be worse-off than in the benchmark cooperative R&D game where no cost complementarities are specified, and *a fortiori* in the non-cooperative R&D set-up.

From firms' viewpoint, this new cooperative game thus leads to the best of possible worlds. However, comparing this new model with the two benchmark ones and with our delegated R&D model is somewhat artificial. Indeed, the introduction of complementarities across firms as manna from heaven, on top of cooperation *stricto sensu*, is equivalent to assuming that firms jointly acquired the external laboratory without paying anything. Thus, the fact that firms would prefer the new cooperative game to anything else should not come as a surprise. In addition, the existence of cost complementarities in the production of R&D is more realistic in a R&D joint-venture – in which firms may also choose the level of spillovers – than in a simple R&D cartel. In a R&D joint-venture, firms will find it profitable to choose $\beta = 1$ (see Amir et al. (2003)), in which case equilibrium profits in the new model and in the delegated one are identical, as for any $\beta \geq \theta/2$. In what follows we keep focusing on the main three models introduced in Section 2.

5 Pareto Optimal R&D Organization and Policy Discussion

The juxtaposition of Proposition 2, Lemmas 1, 2 and 3 in the externalities plane allows us to investigate whether one of the three games can Pareto-dominate the other two.

Theorem 1 *The frontiers established in Proposition 2 and Lemmas 1, 2 and 3 are such that:*

$$\begin{aligned} &\delta_{\mathcal{L}=0} > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} > \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} > \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 \quad \textit{for } 0 \leq \beta < \theta/2; \\ &\delta_{\mathcal{L}=0} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} = \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 \quad \textit{for } \beta = \theta/2; \\ &0 = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} > \delta_{\mathcal{L}=0} \quad \textit{for } \theta/2 < \beta \leq 1. \end{aligned} \tag{15}$$

All frontiers are defined on $[-\gamma, \gamma)$, and the fact they intersect for $\beta = \theta/2$ stems from Proposition 3. For (β, δ) such that $0 \leq \beta < \theta/2$ and $\delta_{\mathcal{L}=0} < \delta < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$, the laboratory earns positive benefits (as opposed to zero profits otherwise). Moreover, in that region, consumer surplus (as inferred from R&D outcomes), and firms' equilibrium profits, are strictly higher in the delegated game than in the equilibria of cooperative and non-cooperative games. This does not hold elsewhere in the externalities plane, as can be checked from (15).

Corollary 1 (Delegation Dominance) *The delegated* R&D *game Pareto dominates the other two games, and the laboratory earns positive profits, for* $0 \leq \beta < \theta/2$ *and* $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} < \delta < \delta_{\mathcal{L}=0}$.

We have therefore established that for certain levels of externalities, consumers, firms, and the laboratory all benefit from the delegation of R&D. Therefore, delegated R&D is a Pareto optimal organizational form. For simple reasons, this cannot occur when direct and indirect externalities are positive. In that case, the delegated game yields the highest profits and consumer surplus, but the laboratory earns no benefits because firms' interests are congruent. For opposite reasons, welfare is minimized under the delegated game if both direct and indirect externalities are sufficiently negative, although in this case a laboratory would earn positive profits. What is crucial for the delegated R&D game to Pareto dominate the other two games, is that indirect externalities must not be too high, so that the firms must still compete for the laboratory's resources, which thus earns positive benefits and participates. But indirect externalities must be high enough to make welfare greater than in the other two games, and let firms obtain more of it than under the two other options.

[**Insert figure 4 about here**]

We can now use these results to examine when the interests of firms and consumers conflict or coincide. This is an important question because firms decide to delegate R&D only if it is profitable for them to do so, and if the laboratory participates. We find that, although no one asks for consumers' consent, firms' privately profitable decision to delegate R&D is always socially optimal. To see that, remark first that in all three games consumer surplus increases with R&D because lower costs lead to higher quantities and lower prices (see Appendix D-3). Second, in the externalities plane, for all values of direct spillovers, firms find it more profitable to delegate R&D than to do R&D in-house either cooperatively or non-cooperatively if and only if δ is above $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ (Theorem 1). Now observe that $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ is the "highest" profit frontier and that it is also always above the two frontiers $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$ and $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ which allow us to compare the consumer surpluses obtained in all three games equilibria (see Figures 1 and 2). Hence, firms never find it profitable to delegate R&D with consumers being worse off than in either of the two other games (as would be the case if we had, say, $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} < \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$ for some values of β).

A more striking result is obtained when the laboratory must earn strictly positive profits to participate (i.e., (11) is replaced by $\mathcal{L} > 0$). This occurs if the laboratory has an outside option where it can earn some arbitrarily small positive net benefits. In that case, firms delegate R&D only if δ is between $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ and $\delta_{\mathcal{L}=0}$ when direct externalities are negative (i.e., $0 \leq \beta < \theta/2$). They cannot rely on the laboratory's R&D services when direct externalities are positive (i.e. $\theta/2 < \beta \leq 1$) because the frontier above which the laboratory earns zero benefits ($\delta_{\mathcal{L}=0}$) lies below all the other frontiers in that region (see Figure 2). Consequently, when the laboratory must make positive benefits to participate, firms will profitably delegate R&D services only when externalities fall in the Pareto dominating region defined in Corollary 1 and which corresponds to the shaded area in Figure 4. A straightforward policy implication is that, when firms behave as described here, there is no motivation for a regulator to constrain firms' choice to delegate R&D.

6 Conclusion

R&D outsourcing (delegation) is an increasingly important phenomenon, and many rivals delegate R&D to a common independent laboratory. While many theoretical industrial organization models compare the outcomes of firms doing R&D cooperatively or non-cooperatively, none has, to the best of our knowledge, investigated R&D delegation in a formal set-up and compared the three modes of R&D. This paper fills that gap by setting up a simple model where competitors can independently choose to delegate R&D to a profit-maximizing laboratory.

Our model of delegated R&D builds on the literature where a firm's R&D reduces not only its unit cost of production but also that of its competitor through technological spillovers (direct externalities). The R&D projects which firms delegate to the laboratory give rise to an additional externality because they can be complements (positive indirect externalities), substitutes (negative indirect externalities), or independent. Positive (negative) indirect externalities can be associated with economies of scope (a congestion) in the production of R&D. We characterize the impact of these two externalities on delegated R&D outcomes, firms' profits, social welfare, and the laboratory's benefits. We compare the outcomes of delegated R&D to an independent profit-maximizing laboratory to firms doing in-house R&D either non-cooperatively or cooperatively. One interesting aspect of our analysis is the ability to illustrate the results graphically by fully partitioning the plane of between-firm technological spillovers and within-laboratory R&D externalities, for all dimensions of the comparison.

We establish a number of novel results. First, only one firm receives R&D services if direct and indirect externalities are relatively low. Otherwise, assuming the laboratory participates whenever it earns non-negative benefits, sufficiently high direct and indirect externalities are necessary and sufficient for optimal R&D, firms' profits, and social welfare to be highest in the delegated R&D game. Second, assuming the common laboratory needs positive benefits to participate, positive indirect externalities are not sufficient for firms to delegate R&D. Firms will opt for R&D delegation to a for-profit common laboratory if indirect externalities are not too positive when direct externalities are negative. When indirect externalities are too high they counteract direct externalities and the laboratory only breaks even. Third, there always exists a region in the externalities plane where the laboratory earns positive benefits *and* the delegated R&D game Pareto-dominates the other two games if (i) direct externalities are sufficiently negative (for firms to compete for the laboratory's services); *and* (ii) indirect externalities are positive (so that firms benefit from economies of scope) but not too high (for the laboratory to earn positive benefits). Finally, our findings have a *laissez-faire* flavor: no regulatory intervention is required when firms decide to delegate R&D to a profit-seeking laboratory. This strong statement arises because, in this model, firms privately profitable choice to contract with a laboratory also benefits consumers.

This paper is a first step in the analysis of procurement markets for new technology with multiple buyers. Since we use standard yet specific algebraic specifications of the cost and demand functions, future work could also test the robustness of the results to changes in the formal specifications of the compared games. We proceed in that direction by using Amir (2000) to specify a modified cost function for the laboratory so as to investigate situations where R&D is conceived as expenditures (i.e., inputs), rather than outputs.[22] All our results remain valid. In another positive test of robustness, we show that our results are qualitatively similar when firms behave *à la* Bertrand in the product market stage of all three R&D games. These proofs are available from the authors upon request.

Another interesting extension would be to examine the relationship between R&D outsourcing and absorptive capacity. Instead of comparing R&D delegation with benchmark set-ups, as in the present paper, it is possible to construct a model in which each firm may simultaneously choose a level of internal R&D (as in the cooperative and non-cooperative R&D models) and tap technological knowledge from an external laboratory (as in the delegated R&D model). By introducing a formal representation of absorptive capacity adapted from Kamien and Zang (2000), one would specify that a firm may not benefit from the laboratory's output if it produces no R&D of its own.

Among the open questions it remains to investigate what happens when R&D efforts lead to product innovations. This could be done by assuming that R&D increases quality by using a framework in the spirit of Symeonidis (2003). It would be also natural to consider more than two firms. However, beyond usual tractability difficulties in computing the equilibrium, this would imply a qualitative leap in the characterization of the joint benefits function of the laboratory and any subset of firms.[23] Finally, our analysis, and the whole literature on which it is based, can be criticized by its assumption of deterministic R&D. To address that critique one may examine situations where the true cost of a R&D program is unknown before it starts.[24]

Appendix

A Explicit Solutions of the Three R&D Games

As standard we proceed backwards. Section A.1 solves for each firm's output on the final market, which is common to three games. Then section A.2 starts by defining some variables which to simplify the notation and solves for a firm's symmetric R&D in the cooperative (section A.2.1), non-cooperative (section A.2.2) and delegated games respectively (section A.2.3).

A.1 Final Market Stage

Each firm chooses output to maximize its gross profits (3). This yields two reaction functions, which we use to solve for each firm's subgame Cournot-Nash equilibrium output as a function of $\mathbf{x}$:

$$q_i(\mathbf{x}) = \frac{\alpha(2-\theta) + (2-\theta\beta)x_i - (\theta - 2\beta)x_j}{(2-\theta)(2+\theta)b}, \tag{16}$$

for $i, j = 1, 2$, $i \neq j$, and where $\alpha \equiv a - c$. Making use of (1) and (16) into (3) we obtain:

$$\pi_i(\mathbf{x}) = b[q_i(\mathbf{x})]^2, \tag{17}$$

for $i = 1, 2$. Now we turn to the R&D stage which is game-specific.

A.2 R&D Stage

Define the following two terms:

$$\begin{aligned} \Gamma_1 &\equiv b\gamma\,(2+\theta)^2 - 2\,(1+\beta)^2\,, \\ \Gamma_2 &\equiv b\gamma\,(2-\theta)^2\,(2+\theta)^2 - 2\,(2-\theta\beta)^2\,. \end{aligned}$$

In the remainder of the paper, we assume that Γ_1 and Γ_2 are positive. The assumption $\Gamma_1 > 0$ is a sufficient second-order condition for a symmetric optimum in the cooperative game when firms are treated equally ex-ante (in the sense of Leahy and Neary, 2005). The assumption $\Gamma_2 > 0$ ensures the objective functions in the non-cooperative game are concave. Note, however, that in the latter game, Henriques (1990) establishes that the reaction functions in the R&D space cross "correctly" when $|\partial x_i / \partial x_j|$ is less than 1. Using our notation this condition imposes a stronger restriction on Γ_2:

$$\Gamma_2 > |2\,(2 - \beta\theta)\,(2\beta - \theta)|. \tag{18}$$

Remark that (18) holds for all values of β when $b = \theta = 1$ and $\gamma = 2$, which are the values used to draw Figures 1 to 4.

Finally, we define:

$$\Gamma_3 \equiv b\gamma(2-\theta)(2+\theta)^2 - 2(1+\beta)(2-\theta\beta),$$

which we also assume to be positive for economic sense in what follows. We will use Γ_1, Γ_2, and Γ_3 to simplify the notation.

A.2.1 Cooperative R&D

The two firms' joint net profits are given by:

$$\pi_1(\mathbf{x}) + \pi_2(\mathbf{x}) - \frac{\gamma}{2}(x_1^2 + x_2^2), \tag{19}$$

where $\pi_1(\mathbf{x})$ and $\pi_2(\mathbf{x})$ are given by (17). We maximize (19) with respect to x_1 and x_2 to obtain the symmetric cooperative R&D outcome and the profits of each individual firm respectively as:

$$x^{\mathrm{c}} = 2\,(1+\beta)\,\alpha/\Gamma_1, \tag{20}$$

$$\pi^{\mathrm{c}} = \gamma\alpha^2/\Gamma_1. \tag{21}$$

A.2.2 Non-Cooperative R&D

Each firm chooses its R&D independently to maximize its net profits (7). This yields two reaction functions which we use to solve for a symmetric non-cooperative R&D outcome:

$$x^{\mathrm{n}} = 2\alpha(2-\theta\beta)/\Gamma_3. \tag{22}$$

We substitute (22) into (7), and use the cost function (4), to solve for the symmetric individual firm profits:

$$\pi^{\mathrm{n}} = \gamma\alpha^2\Gamma_2/\Gamma_3^2. \tag{23}$$

A.2.3 Delegated R&D

Proposition 1 states that the choice of R&D services by the laboratory is equivalent to maximizing aggregate benefits (14) with respect to x_1 and x_2. Assuming that the laboratory may treat firms differently (see the ex-post equal treatment case in Leahy and Neary, 2005), second-order conditions for a symmetric solution are:

$$b(\gamma-\delta)\left(2+\theta\right)^2 - 2\left(1+\beta\right)^2 > 0 \text{ and } b\left(\gamma+\delta\right)\left(2-\theta\right)^2 - 2\left(1-\beta\right)^2 > 0. \tag{24}$$

This can be rewritten as follows:

$$\frac{2\left(1-\beta\right)^2 - b\gamma\left(2-\theta\right)^2}{b\left(2-\theta\right)^2} < \delta < \frac{b\gamma\left(2+\theta\right)^2 - 2\left(1+\beta\right)^2}{b\left(2+\theta\right)^2}. \tag{25}$$

It is easy to check that the lower bound of the latter interval is greater than $-\gamma$, whereas the upper bound is strictly less than γ. This says that a symmetric solution is obtained if δ does not take extreme values inside $[-\gamma, \gamma)$.

Necessary first-order conditions are:

$$2b\left(q_i(\mathbf{x})\frac{\partial q_i(\mathbf{x})}{\partial x_i}+q_j(\mathbf{x})\frac{\partial q_j(\mathbf{x})}{\partial x_i}\right)-(\gamma-\delta)\,x_i=0, \tag{26}$$

$i,j=1,2,\ i\neq j$. This gives each firm's symmetric delegated R&D outcome:

$$x^{\mathrm{d}}=2\,(1+\beta)\,\alpha/\Gamma_4, \tag{27}$$

where $\Gamma_4\equiv b(\gamma-\delta)\,(2+\theta)^2-2\,(1+\beta)^2$, which is unambiguously positive from (24). Substituting (27) into (14), using the laboratory's explicit cost function (8), and simplifying gives aggregate benefits:

$$\Lambda=2\,(\gamma-\delta)\,\alpha^2/\Gamma_4. \tag{28}$$

B Proofs of Properties 1 and 2

To prove Property 1 we use (16) and (17) to obtain:

$$\frac{\mathrm{d}\pi_i(\mathbf{x})}{\mathrm{d}x_j}=\frac{2\,(2\beta-\theta)}{(2-\theta)\,(2+\theta)}q_i(\mathbf{x}), \tag{29}$$

for $i=1,2,\ i\neq j$. As $\theta\in[0,1]$, equation (29) is of the same sign as $(2\beta-\theta)$ for a positive output, or equals zero otherwise.

The proof of Property 2 follows directly from differentiating (8) with respect to x_i and x_j.

C Proof of Propositions

C.1 Proof of Proposition 1

The proof of Proposition 1-(i) is a simple adaptation in the notation from Bernheim and Whinston (1986b, first part of Theorem 2 on page 14, and proof on pages 24-25). It is available upon request from the authors. We capitalize on this first result to prove Proposition 1-(ii). Since the choice of

R&D services by the laboratory is equivalent to maximizing aggregate benefits (14) with respect to x_1 and x_2, we solve:

$$\max_{\mathbf{x}\geq 0} \Lambda\left(\mathbf{x}\right). \tag{30}$$

As a preliminary step, we first demonstrate that the stability condition (18) of the non-cooperative R&D game implies that:

$$\Gamma_2 - 2\left(2\beta - \theta\right)^2 > 0. \tag{31}$$

To see this, remark that (18) imposes:

$$\Gamma_2 + 2\left(2 - \beta\theta\right)\left(2\beta - \theta\right) > 0, \tag{32}$$

and also:

$$\Gamma_2 - 2\left(2 - \beta\theta\right)\left(2\beta - \theta\right) > 0. \tag{33}$$

This leads to two cases: (i) if $\beta < \theta/2$ then $-2\left(2\beta - \theta\right)^2 > 2\left(2 - \beta\theta\right)\left(2\beta - \theta\right)$, in which case (32) $\Rightarrow \Gamma_2 - 2\left(2\beta - \theta\right)^2 > 0$; (ii) if $\beta \geq \theta/2$ then $-2\left(2\beta - \theta\right)^2 \geq -2\left(2 - \beta\theta\right)\left(2\beta - \theta\right)$, in which case (33) $\Rightarrow \Gamma_2 - 2\left(2\beta - \theta\right)^2 > 0$. It follows that, for the comparison of the delegated R&D game with the non-cooperative R&D game to be meaningful, parameter values must be such that $\Gamma_2 - 2\left(2\beta - \theta\right)^2 > 0$.

Next, we turn to the Kuhn-Tucker necessary conditions of (30), that is:

$$\frac{\partial \Lambda\left(\mathbf{x}\right)}{\partial x_i} \leq 0, \qquad x_i \geq 0, \qquad x_i \frac{\partial \Lambda\left(\mathbf{x}\right)}{\partial x_i} = 0, \tag{34}$$

for $i = 1, 2$. We check that a candidate solution $\bar{\mathbf{x}}$, with $\bar{x}_1 = 0$ and $\bar{x}_2 \equiv \bar{x}^{\mathrm{d}} > 0$ (without loss of generality since the two firms are identical), satisfies these conditions. First, $x_2 \frac{\partial \Lambda(\mathbf{x})}{\partial x_2}\Big|_{\mathbf{x}=\bar{\mathbf{x}}} = 0$ if and only if $\frac{\partial \Lambda(\mathbf{x})}{\partial x_2}\Big|_{\mathbf{x}=\bar{\mathbf{x}}} = 0$, leading to:

$$\bar{x}^{\mathrm{d}} = \frac{2\alpha\left(1 + \beta\right)\left(2 - \theta\right)^2}{\Gamma_2 - 2\left(2\beta - \theta\right)^2}, \tag{35}$$

which is positive. Second, $\left.\frac{\partial\Lambda(\mathbf{x})}{\partial x_1}\right|_{x_1=0,x_2=\bar{x}_2} \leq 0$ if and only if:

$$\frac{2\alpha\left(1+\beta\right)}{b}\frac{b\left(\gamma+\delta\right)\left(2-\theta\right)^2-2\left(1-\beta\right)^2}{\Gamma_2-2\left(2\beta-\theta\right)^2} \leq 0. \tag{36}$$

Recalling that $\Gamma_2 - 2\left(2\beta-\theta\right)^2 > 0$, we obtain that (36) is verified if and only if $b\left(\gamma+\delta\right)\left(2-\theta\right)^2 - 2\left(1-\beta\right)^2 \leq 0$. Then define the frontier:

$$\bar{\delta} \equiv \frac{2\left(1-\beta\right)^2 - b\gamma\left(2-\theta\right)^2}{b\left(2-\theta\right)^2}. \tag{37}$$

This leads to two cases. Either:

$$\delta \in (\bar{\delta},\gamma) \Rightarrow \left.\frac{\partial\Lambda\left(\mathbf{x}\right)}{\partial x_1}\right|_{x_1=0,x_2=\bar{x}_2} > 0, \tag{38}$$

in which case (36) is violated and there is an interior solution, as computed in section A.2.3; or:

$$\delta \in [-\gamma,\bar{\delta}] \Rightarrow \left.\frac{\partial\Lambda\left(\mathbf{x}\right)}{\partial x_1}\right|_{x_1=0,x_2=\bar{x}_2} \leq 0, \tag{39}$$

in which case the corner solution $\bar{\mathbf{x}} = (0,\bar{x}_2)$ applies. For a sufficient second-order condition, we compute:

$$\frac{\partial^2\Lambda\left(0,x_2\right)}{\partial x_2^2} = -\frac{1}{b}\frac{\Gamma_2-2\left(2\beta-\theta\right)^2}{\left(2-\theta\right)^2\left(2+\theta\right)^2}, \tag{40}$$

which is always negative when the stability condition (18) of the non-cooperative game is satisfied (since this implies $\Gamma_2 - 2\left(2\beta-\theta\right)^2 > 0$, as shown above).

Eventually, to see that $\bar{\delta}$ exists in $[-\gamma,\gamma)$, first consider the slope and curvature of $\bar{\delta}$ in the plane (β,δ). This gives:

$$\frac{\partial\bar{\delta}}{\partial\beta} = -4\frac{1-\beta}{b\left(2-\theta\right)^2} < (=)0 \text{ iff } \beta < (=)1, \text{ and } \frac{\partial^2\bar{\delta}}{\partial\beta^2} = \frac{4}{b\left(2-\theta\right)^2} > 0. \tag{41}$$

Then compute $\bar{\delta}$ for two extreme values of β:

$$\bar{\delta}\big|_{\beta=0} = \frac{2}{b\,(2-\theta)^2} - \gamma, \text{ and } \bar{\delta}\big|_{\beta=1} = -\gamma. \tag{42}$$

Eventually, $\Gamma_2 - 2\,(2\beta - \theta)^2 > 0$ for $\beta = 0$ implies $\gamma > 8/b\,(2-\theta)^2\,(2+\theta)^2$. As the latter threshold is greater than $1/b\,(2-\theta)^2$, by transitivity one obtains $\gamma > 1/b\,(2-\theta)^2$, leading to $\bar{\delta}\big|_{\beta=0} < \gamma$. ■

C.2 Proof of Proposition 2

Let $N = \{1, 2\}$, and $2^N = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$. We build on Laussel and Le Breton (2001) – henceforth LLB – by associating to the common agency game, as defined in section 2.3, a transferable utility cooperative game with characteristic function $\Pi : 2^N \to \mathrm{R}$, such that:

$$\Pi(S) = \max_{\mathbf{x}} \sum_{i \in S} \pi_i(\mathbf{x}) - s(\mathbf{x}), \tag{43}$$

where $s(\mathbf{x})$ is given by (8) and $\pi_i(\mathbf{x})$ by (17). This function gives the highest joint-benefits of the laboratory and any subset S of firms in N, with $\Pi(\emptyset) = \pi_1(0, 0) + \pi_2(0, 0)$, that is the sum of concentrated profits with no R&D (a normalization). Then the proof consists in investigating additive properties of Π on 2^N in order to exploit a series of theorems that characterize the equilibrium outcomes of the delegated R&D game.

- From LLB's Theorem 3.1 (p. 102), if $\Pi(S)$ is strictly subadditive, that is:

$$\Pi(\{1, 2\}) < \Pi(\{1\}) + \Pi(\{2\}), \tag{44}$$

  then the laboratory earns positive benefits in all equilibria, that is $\mathcal{L} > 0$.

- From LLB's Theorem 3.3 (p. 104), if $\Pi(S)$ is strictly subadditive, then $\#N = 2$ implies that firms' symmetric profits in the delegated R&D game are:

$$\Pi^{\mathrm{d}} = \Lambda - \Pi(\{i\}), \tag{45}$$

$i = 1, 2$.

- From LLB's Theorem 3.2 (p. 103), if $\Pi(S)$ is superadditive, that is:

$$\Pi(\{1,2\}) \geq \Pi(\{1\}) + \Pi(\{2\}), \tag{46}$$

then $\mathcal{L} = 0$. In that case, the fact that the laboratory maximizes aggregate benefits $\Lambda(\mathbf{x})$ in $(\mathbf{x})$ (Proposition 1), together with the assumption that the symmetric firms are subject to "equal treatment", imply that a firm's equilibrium profits in the delegated R&D game are:

$$\pi^{\mathrm{d}} = \Lambda/2. \tag{47}$$

The remainder of the proof identifies values of δ which are such that $\Pi(S)$ is either strictly subadditive or superadditive. To that effect we solve for values of δ such that (46) holds with equality, to obtain a frontier which we denote by $\delta_{\mathcal{L}=0}$. However, the free maximization of $\Pi(S)$, for $S = \{i\}$, $i = 1, 2$, may yield negative maximands. Therefore, we consider in turn the free-maximum and constrained-maximum versions of (43), denoted by $\check{\Pi}(\{i\})$ and $\hat{\Pi}(\{i\})$, respectively. We thus obtain two frontiers $\check{\delta}_{\mathcal{L}=0}$ (free-maximum) and $\hat{\delta}_{\mathcal{L}=0}$ (constrained-maximum) each of which verify (46) with equality. We then calculate the values of δ for which the free maximand $\check{x}_j$ is equal to the constrained variable $\hat{x}_j \equiv 0$, and denote it by $\delta_{\check{x}_j=\hat{x}_j=0}$. Finally, we compare $\hat{\delta}_{\mathcal{L}=0}$ and $\check{\delta}_{\mathcal{L}=0}$ with $\delta_{\check{x}_j=\hat{x}_j=0}$ to verify for which parameter values (i) $\check{\delta}_{\mathcal{L}=0}$ verifies the positive maximand constraint, and (ii) $\hat{\delta}_{\mathcal{L}=0}$ verifies the non-positive maximand constraint. This allows us to derive the frontier $\delta_{\mathcal{L}=0}$ by "pasting" those two functions.

C.2.1 Free and Constrained Solutions

• Firstly, we solve the free-maximum version of (43), with $S = \{i\}$, $i = 1, 2$. Define the following:

$$\begin{aligned}
\Gamma_5 &\equiv b(\gamma^2 - \delta^2)\left(4 - \theta^2\right)^2 - 2((2\beta - \theta)^2 + (2 - \theta\beta)^2)\gamma + 4\,(2\beta - \theta)\,(2 - \theta\beta)\,\delta; \\
\Gamma_6 &\equiv b\gamma\left(4 - \theta^2\right)^2 - 2\,(\theta\beta - 2)^2\,; \\
\Gamma_7 &\equiv b\gamma\left(4 - \theta^2\right)^2 - 2\,(2 - \theta\beta)^2\,.
\end{aligned}$$

We assume Γ_5, Γ_6, and Γ_7 are positive for the following free-maximum problem to be concave:

$$\check{\Pi}(\{i\}) = \max_{\mathbf{x}}\left(\pi_i(\mathbf{x}) - s(\mathbf{x})\right), \tag{48}$$

where $s(\mathbf{x})$ is given by (8) and $\pi_i(\mathbf{x})$ by (17). Maximizing the right-hand side of (48) gives the following unconstrained R&D solutions:

$$\check{x}_i = 2\alpha\,(2 - \theta)\,[\delta\,(\theta - 2\beta) + \gamma\,(\theta\beta - 2)]/\Gamma_5; \tag{49}$$

$$\check{x}_j = 2\alpha\,(2 - \theta)\,[\delta\,(\theta\beta - 2) + \gamma\,(\theta - 2\beta)]/\Gamma_5. \tag{50}$$

Making use of (49) and (50) into (48) we obtain, for $i = 1, 2$:

$$\check{\Pi}(\{i\}) = \alpha^2\,(2 - \theta)^2\left(\delta^2 - \gamma^2\right)/\Gamma_5. \tag{51}$$

• Secondly, we solve the constrained-maximum versions of (43), with $S = \{i\}$, $i = 1, 2$. In this simpler problem, the only concavity condition imposes that $\Gamma_7 > 0$ to compute:

$$\hat{\Pi}(\{i\}) = \max_{x_i}\left(\pi_i(x_i, 0) - r(x_i)\right), \tag{52}$$

where $r(x_i)$ is given by (4), and $\pi_i(x_i, 0)$ is obtained by setting $x_j = \hat{x}_j \equiv 0$ in (17). Maximizing the right-hand side of (52) gives the constrained R&D:

$$\hat{x}_i \quad = \quad 2\,(a-c)\,(2-\theta\beta)\,(2-\theta)/\Gamma_7. \tag{53}$$

Substituting (53) into the right-hand side of (52) gives:

$$\hat{\Pi}(\{i\}) = \gamma\alpha^2(2-\theta)^2/\Gamma_7, \tag{54}$$

for $i = 1, 2$.

- We now derive the frontier $\delta_{\check{x}_j=\hat{x}_j=0}$, i.e. all values of δ which are such that $\check{x}_j = \hat{x}_j \equiv 0$. This yields:

$$\check{x}_j \quad \begin{matrix} > \\ = \\ < \end{matrix} \quad \hat{x}_j \equiv 0 \quad \text{if and only if} \quad \delta \quad \begin{matrix} > \\ = \\ < \end{matrix} \quad \frac{\gamma\,(2\beta-\theta)}{\theta\beta-2} \equiv \delta_{\check{x}_j=\hat{x}_j=0}. \tag{55}$$

C.2.2 The Laboratory's Zero-Benefits Free-maximum Frontier

We evaluate (46), assuming it holds with equality with free-maximum profits to obtain:

$$\Pi(\{1,2\}) - \check{\Pi}(\{1\}) - \check{\Pi}(\{2\}) = 0, \tag{56}$$

where $\check{\Pi}(\{1\})$ and $\check{\Pi}(\{2\})$ are given by (51). There are two roots to (56). The first is $\delta = \gamma$, which however violates (48). The second root is:

$$\delta = \frac{2\gamma\,(2-\theta\beta)\,(\theta-2\beta)}{(2-\theta\beta)^2 + (2\beta-\theta)^2} \equiv \check{\delta}_{\mathcal{L}=0}. \tag{57}$$

To check that (57) is compatible with free maximands, recall from (55) that (57) is defined only if $\check{\delta}_{\mathcal{L}=0} \geq \delta_{\check{x}_j=\hat{x}_j=0}$. We form the difference:

$$\delta_{\check{x}_j=\hat{x}_j=0} - \check{\delta}_{\mathcal{L}=0}, \tag{58}$$

and look for parameter values for which it is non-positive. Equating (58) to 0, we find that $\beta = \theta/2$ and $\beta = 1$ are the only admissible roots (the other roots are $\gamma = 0$, $\theta = \pm 2$, and $\beta = -1$.) Hence, (58) changes sign at most once in the domain of β. Evaluating (58) at some parameter values, we find that $\check{\delta}_{\mathcal{L}=0}$ is defined only for $0 \leq \beta \leq \theta/2$.

Finally, using (57), note that the frontier $\check{\delta}_{\mathcal{L}=0}$ includes $(\beta, \delta) = (\theta/2, 0)$.

C.2.3 The Laboratory's Zero-Benefits Constrained-maximum Frontier

As in C.2.2, we evaluate (46), assuming it holds with equality but using the constrained profits (54), and solve for its roots. The first root $a = c$, is not admissible by assumption. The second root is:

$$\delta = \frac{\gamma \left[2\left(2-\theta\beta\right)\left(\theta-2\beta\right)-\left(\theta-2\beta\right)^2\right]}{\left(2-\theta\beta\right)^2} \equiv \hat{\delta}_{\mathcal{L}=0}. \tag{59}$$

To check that the latter expression is compatible with a constrained maximand, recall from (55) that (59) is defined only if $\hat{\delta}_{\mathcal{L}=0} \leq \delta_{\check{x}_j=\hat{x}_j=0}$. Then form the difference:

$$\delta_{\check{x}_j=\hat{x}_j=0} - \hat{\delta}_{\mathcal{L}=0}, \tag{60}$$

and look for the parameter values for which it is non-negative.

Equating (60) to 0 gives $\beta = \theta/2$ as the only admissible root, and three non-admissible roots ($\gamma = 0$, $\theta = 2$, and $\beta = -1$). Hence, (60) changes sign once over the domain of β. Evaluating (59) at some parameter values leads to the conclusion that (60) is defined only for $\theta/2 \leq \beta \leq 1$.

Finally, using (59), note that, as for $\check{\delta}_{\mathcal{L}=0}$, the frontier $\hat{\delta}_{\mathcal{L}=0}$ includes $(\beta, \delta) = (\theta/2, 0)$.

C.2.4 The Laboratory's Zero-Benefits Frontier

By taking the conclusions of sections C.2.2 and C.2.3 together, we obtain that (46) holds with equality if and only if:

$$\delta = \delta_{\mathcal{L}=0} = \begin{cases} \check{\delta}_{\mathcal{L}=0} & \text{for} \quad 0 \leq \beta \leq \theta/2, \\ \hat{\delta}_{\mathcal{L}=0} & \text{for} \quad \theta/2 \leq \beta \leq 1, \end{cases} \tag{61}$$

where $\check{\delta}_{\mathcal{L}=0}$ and $\hat{\delta}_{\mathcal{L}=0}$ are explicitly given by (57) and (59). Note that $\beta = \theta/2$ implies $\delta_{\mathcal{L}=0} = \check{\delta}_{\mathcal{L}=0} = \hat{\delta}_{\mathcal{L}=0} = 0$. We now prove that $\delta_{\mathcal{L}=0}$ is decreasing in β, by considering $\check{\delta}_{\mathcal{L}=0}$ and $\hat{\delta}_{\mathcal{L}=0}$ in turn.

($\check{\delta}_{\mathcal{L}=0}$) Differentiating (57) with respect to β and equating to 0 yields 5 roots ($\gamma = 0$, $\theta = \pm 2$, or $\beta = \pm 1$), none of which is admissible. It follows that $\check{\delta}_{\mathcal{L}=0}$ is strictly monotone over the domain of β on which $\check{\delta}_{\mathcal{L}=0}$ is defined, that is $[0, \theta/2]$. To complete the proof, let for instance $\beta = \theta/2$, and check that $\mathrm{d}\check{\delta}_{\mathcal{L}=0}/\mathrm{d}\beta < 0$, as required.

($\hat{\delta}_{\mathcal{L}=0}$) Differentiating (59) with respect to β and equating to 0 yields 4 roots ($\gamma = 0$, $\theta = \pm 2$, or $\beta = -1$), none of which is admissible. It follows that $\hat{\delta}_{\mathcal{L}=0}$ is strictly monotone over the domain of β on which $\hat{\delta}_{\mathcal{L}=0}$ is defined, that is $[\theta/2, 1]$. To complete the proof, let, for instance $\beta = 1$, and check that $\mathrm{d}\hat{\delta}_{\mathcal{L}=0}/\mathrm{d}\beta < 0$, as required. ■

C.3 Proof of Proposition 3

Firstly, if $\beta = \theta/2$, for all δ, concentrated profits $\pi_i(\mathbf{x})$ depend only on each firm i's own R&D variable x_i (Property 1), in which case the cooperative and non-cooperative games coincide. Secondly, if $\delta = 0$, for all β, we have $r(x_1) + r(x_2) = s(\mathbf{x})$ (Property 2), and solving the cooperative game is equivalent to solving the delegated game (Proposition 1). Thirdly, if $\beta = \theta/2$ and $\delta = 0$, the laboratory earns no benefits (Proposition 2). By considering all three cases together, we conclude that the cooperative, non-cooperative, and delegated R&D games yield identical outcomes at the non-externalities point $(\theta/2, 0)$. ■

D Proof of Lemmas

We first establish how indirect externalities impact optimal outcomes in the delegated R&D game. This will be useful for the proofs of Lemmas $1-3$, which follow.

Lemma D-1 (R&D) $\mathrm{d}x^{\mathrm{d}}/\mathrm{d}\delta > 0$.

Differentiating (27) with respect to δ gives:

$$\frac{\mathrm{d}x^{\mathrm{d}}}{\mathrm{d}\delta} = 2\alpha b\,(1+\beta)\,(2+\theta)^2/\Gamma_4^2, \tag{62}$$

which is positive. ■

Lemma D-2 (Profits) $\mathrm{d}\pi^{\mathrm{d}}/\mathrm{d}\delta > 0$.

We consider $\delta > \delta_{\mathcal{L}=0}$ and $\delta \leq \delta_{\mathcal{L}=0}$ in turn.

($\delta \geq \delta_{\mathcal{L}=0}$) By Propositions 1 and 2:

$$\pi^{\mathrm{d}} = \Lambda/2. \tag{63}$$

Recalling that the laboratory's cost are given by (8), aggregate benefits by (14), and using the envelope theorem, we differentiate (63) with respect to δ to obtain:

$$\frac{\mathrm{d}\pi^{\mathrm{d}}}{\mathrm{d}\delta} = \frac{1}{2}\frac{\partial\Lambda}{\partial\delta} = \frac{1}{2}(x^{\mathrm{d}})^2, \tag{64}$$

which is unambiguously positive given that $x^{\mathrm{d}} > 0$.

($\delta \leq \delta_{\mathcal{L}=0}$) Assume that π^{d} is non increasing in δ, that is:

$$\pi^{\mathrm{d}}\Big|_{\delta<\delta_{\mathcal{L}=0}} \geq \pi^{\mathrm{d}}\Big|_{\delta=\delta_{\mathcal{L}=0}}, \tag{65}$$

and look for a contradiction. To do that, recall from Proposition 1 that the laboratory maximizes aggregate benefits, and from Proposition 2 that it exactly breaks even if $\delta = \delta_{\mathcal{L}=0}$. Consequently:

$$\pi^{\mathrm{d}}\Big|_{\delta=\delta_{\mathcal{L}=0}} = \frac{1}{2}\,\Lambda|_{\delta=\delta_{\mathcal{L}=0}}\,. \tag{66}$$

Moreover, recalling that the laboratory's cost are given by (8) and aggregate benefits by (14), by using the envelope theorem we obtain $\mathrm{d}\Lambda/\mathrm{d}\delta = (x^{\mathrm{d}})^2$, which is unambiguously positive given that $x^{\mathrm{d}} > 0$. It follows that:

$$\frac{1}{2}\,\Lambda|_{\delta=\delta_{\mathcal{L}=0}} > \frac{1}{2}\,\Lambda|_{\delta<\delta_{\mathcal{L}=0}}\,. \tag{67}$$

To conclude, taking (65), (66), and (67) together leads to:

$$\pi^{\mathrm{d}}\Big|_{\delta<\delta_{\mathcal{L}=0}} > \frac{1}{2}\,\Lambda|_{\delta<\delta_{\mathcal{L}=0}}\,, \tag{68}$$

by transitivity. Inequality (68) contradicts the result that $\pi^{\mathrm{d}} \le \Lambda/2$ for all $\delta \le \delta_{\mathcal{L}=0}$, as established by Propositions 1 and 2. Hence $\mathrm{d}\pi^{\mathrm{d}}/\mathrm{d}\delta > 0$. ■

Lemma D-3 (Welfare) $\mathrm{d}w^{\mathrm{d}}/\mathrm{d}\delta > 0$.

Welfare is defined as the sum of firms' profits and consumer surplus. It has been established above that profits π^{d} are increasing in δ. Here we turn to consumer surplus by investigating how $q_i(\mathbf{x}^{\mathrm{d}})$ and $p_i(\mathbf{x}^{\mathrm{d}})$ vary with δ.

($q_i(\mathbf{x}^{\mathrm{d}})$) Differentiating the Cournot-Nash symmetric equilibrium output (16) evaluated at $\mathbf{x}^{\mathrm{d}} = (x^{\mathrm{d}}, x^{\mathrm{d}})$, with respect to δ, we obtain:

$$\frac{\mathrm{d}q_i(\mathbf{x}^{\mathrm{d}})}{\mathrm{d}\delta} = \frac{(2+\theta)\,(1+\beta)\,x^{\mathrm{d}}}{\Gamma_4}. \tag{69}$$

As the denominator of (69), as well as the other parameters of the model and x^{d} are positive, it follows that (69) is also positive.

($p_i(\mathbf{x}^{\mathrm{d}})$) Differentiating the inverse demand function (1) evaluated at the symmetric equilibrium $\mathbf{x}^{\mathrm{d}} = (x^{\mathrm{d}}, x^{\mathrm{d}})$, with respect to δ, we obtain:

$$\frac{\mathrm{d}p_i(\mathbf{x}^{\mathrm{d}})}{\mathrm{d}\delta} = -\frac{b(1+\theta)\,(2+\theta)\,(1+\beta)\,x^{\mathrm{d}}}{\Gamma_4}, \tag{70}$$

which is negative since x^{d} and Γ_4 are positive.

Taking (69) and (70) together means that the consumer surplus is increasing in δ. The fact that firms' profits are also increasing in δ (Lemma D-2) completes the proof. ■

D.1 Proof of Lemma 1

D.1.1 Equal Delegated and Cooperative R&D Frontier $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$

As x^{d} is monotone increasing in δ (Lemma D-1) and x^{c} is invariant with δ, it follows that if there exists a value of δ for which $x^{\mathrm{d}} = x^{\mathrm{c}}$, it is unique. Moreover, for $\delta = 0$, (i) the costs of R&D are the same for the laboratory in the delegated R&D and for both firms in the cooperative game (see Property 2), and (ii) solving the delegated game for x^{d} is equivalent to solving the cooperative game for x^{c} (because of Proposition 1). Hence, $x^{\mathrm{d}} = x^{\mathrm{c}}$ for $\delta = 0$. Making use of that result, Lemma D-1, that x^{c} does not vary with δ and Proposition 2 gives Lemma 1-(i). ■

D.1.2 Equal Delegated and Non-Cooperative R&D Frontier $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$

Using (22) and (27) we define:

$$\Delta(\delta) \equiv x^{\mathrm{d}} - x^{\mathrm{n}}. \tag{71}$$

($\beta = \theta/2$)

Proposition 3 establishes that $\Delta(0) = 0$.

($\beta < \theta/2$)

Claim A: $\Delta(\delta) < 0$. Section D.1.1 establishes that $x^{\mathrm{d}} = x^{\mathrm{c}}$ for $\delta = 0$. Next, we know from d'Aspremont and Jacquemin (1988) that $x^{\mathrm{c}} < x^{\mathrm{n}}$ for $\beta \in [0, \theta/2)$ and all values of δ. Claim A follows by transitivity.

Claim B: $\Delta(\delta) > 0$ along $\delta_{\mathcal{L}=0}$. Recall from (61) that $\delta_{\mathcal{L}=0} = \check{\delta}_{\mathcal{L}=0} > 0$ for $\beta < \theta/2$. Then evaluating (71) at $\delta = \check{\delta}_{\mathcal{L}=0}$, and equating to 0 gives 7 roots ($\gamma = 0, \theta = \pm 2, \alpha = 0, \beta = \theta/2, \beta = 1, b = 0$), none of which is admissible. Therefore, $\Delta(\check{\delta}_{\mathcal{L}=0})$ does not change sign over this range of β. It is straightforward to check that claim B holds by computing $\Delta(\check{\delta}_{\mathcal{L}=0})$ at, say, $\beta = 0$ and any admissible values for the other parameters, and obtaining a positive value.

($\beta > \theta/2$)

Claim C: $\Delta(\delta) > 0$ for $\delta = 0$. Recall that $x^{\mathrm{d}} = x^{\mathrm{c}}$ for $\delta = 0$, as established in Section D.1.1. Then note that $x^{\mathrm{c}} > x^{\mathrm{n}}$ for $\beta \in (\theta/2, 1]$ from d'Aspremont and Jacquemin (1988), all δ. Claim C follows by transitivity.

Claim D: $\Delta(\delta) < 0$ along $\delta_{\mathcal{L}=0}$. Recall from (61) that $\delta_{\mathcal{L}=0} = \hat{\delta}_{\mathcal{L}=0} < 0$ for $\beta > \theta/2$, evaluate (71) at $\delta = \hat{\delta}_{\mathcal{L}=0}$, and equating to 0 gives 7 roots ($\gamma = 0, \theta = \pm 2, \alpha = 0, \beta = \theta/2, \beta = 2/\theta, b = 0.$), none of which is admissible. Therefore, $\Delta(\hat{\delta}_{\mathcal{L}=0})$ does not change sign over the relevant range of β. It is straightforward to check that $\Delta(\hat{\delta}_{\mathcal{L}=0})$ is negative by evaluating it at, say, $\beta = 1$ and any value for other parameters. Hence claim D is true.

Recall how $\delta_{\mathcal{L}=0}$ is constructed in (61). Using claims A to D, that x^{d} is continuous and monotone increasing in δ (Lemma D-1) while x^{n} does not vary with δ, means there exists a unique $\delta \equiv \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ such that $x^{\mathrm{d}} \gtreqless x^{\mathrm{n}}$ if and only if $\delta \gtreqless \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, with $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ as in Lemma 1-(*ii*). ■

D.2 Proof of Lemma 2

D.2.1 Equal Delegated and Cooperative Profits Frontier $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$

($\beta = \theta/2$)

We know from Proposition 3 that $\pi^{\mathrm{d}} = \pi^{\mathrm{c}}$ for $\delta = 0$ (the no-externalities case).

Now we consider $\beta < \theta/2$, for which $\delta_{\mathcal{L}=0} > 0$, and $\beta > \theta/2$, for which $\delta_{\mathcal{L}=0} < 0$.

($\beta < \theta/2$)

Claim A: $\pi^{\mathrm{d}} < \pi^{\mathrm{c}}$ for $\delta = 0$. On the one hand:

$$\pi^{\mathrm{d}} < \frac{1}{2}\, \Lambda|_{\delta=0}\,, \tag{72}$$

because the laboratory appropriates a share of maximized aggregate benefits as $\mathcal{L} > 0$ for $\delta = 0 < \delta_{\mathcal{L}=0}$, as can be inferred from Propositions 1 and 2. On the other hand:

$$\pi^{\mathrm{c}} = \frac{1}{2}\, \Lambda|_{\delta=0}\,, \tag{73}$$

from the specification of the cooperative R&D game. Putting (72) and (73) together gives $\pi^{\mathrm{d}} < \pi^{\mathrm{c}}$ for $\delta = 0$.

Claim B: $\pi^{\mathrm{d}} > \pi^{\mathrm{c}}$ for $\delta = \delta_{\mathcal{L}=0}$. On the one hand:

$$\pi^{\mathrm{d}} = \frac{1}{2}\, \Lambda|_{\delta=\delta_{\mathcal{L}=0}}\,, \tag{74}$$

from Propositions 1 and 2 because firms earn all maximized aggregate benefits as $\mathcal{L} = 0$ for $\delta = \delta_{\mathcal{L}=0}$. On the other hand:

$$\pi^{\mathrm{c}} < \frac{1}{2}\, \Lambda|_{\delta=\delta_{\mathcal{L}=0}}\,, \tag{75}$$

because of (73) and $\Lambda|_{\delta=0} < \Lambda|_{\delta=\delta_{\mathcal{L}=0}}$ as a result of (64). Putting (74) and (75) together means claim B holds.

($\beta > \theta/2$)

Claim C: $\pi^{\mathrm{d}} = \pi^{\mathrm{c}}$ for $\delta = 0$. On the one hand:

$$\pi^{\mathrm{d}} = \frac{1}{2}\, \Lambda|_{\delta=0}\,, \tag{76}$$

from Propositions 1 and 2 (as $\mathcal{L} = 0$ for $\delta = 0 > \delta_{\mathcal{L}=0}$). On the other hand:

$$\pi^{\rm c} = \frac{1}{2}\, \Lambda|_{\delta=0}\,, \tag{77}$$

from the specification of the cooperative R&D game. Claim C follows from (76) and (77).

Claim D: $\delta_{\mathcal{L}=0} < 0$ for $\beta > \theta/2$. That claim follows directly from Proposition 2.

Given claims A to D, that $\pi^{\rm d}$ is continuous and monotone increasing in δ (Lemma D-2) means there exists a unique $\delta \equiv \delta_{\pi^{\rm d}=\pi^{\rm c}}$ such that $\pi^{\rm d} \gtreqless \pi^{\rm c}$ if and only if $\delta \gtreqless \delta_{\pi^{\rm d}=\pi^{\rm c}}$, with $\delta_{\pi^{\rm d}=\pi^{\rm c}}$ as in Lemma 2-(i). ■

D.2.2 Equal Delegated and Non-Cooperative Profits Frontier $\delta_{\pi^{\rm d}=\pi^{\rm n}}$

($\beta = \theta/2$)

We know from Proposition 3 that $\pi^{\rm d} = \pi^{\rm n}$ for $\delta = 0$.

Now we consider $\beta < \theta/2$, for which $\delta_{\mathcal{L}=0} > 0$, and $\beta > \theta/2$, for which $\delta_{\mathcal{L}=0} < 0$. In both cases, we make use of $\delta_{x^{\rm d}=x^{\rm n}}$ as defined in Lemma 1. Note that $\delta_{x^{\rm d}=x^{\rm n}}$ is identical to $\delta_{\check{x}_j=\hat{x}_j=0}$, for all β. Indeed, the unique admissible root to $x^{\rm d} - x^{\rm n} = 0$ is:

$$\delta = \frac{\gamma(\theta - 2\beta)}{(2 - \theta\beta)},$$

which is the same as (55). The other non-admissible roots to $x^{\rm d} - x^{\rm n} = 0$ are $\alpha = 0$, $\theta = -2$, and $b = 0$.

($\beta < \theta/2$)

Claim A: $\pi^{\rm d} < \pi^{\rm n}$ for $\delta = \delta_{x^{\rm d}=x^{\rm n}}$. To prove claim A note that as $\delta = \delta_{\check{x}_j=\hat{x}_j=0} = \delta_{x^{\rm d}=x^{\rm n}}$, it follows that $\pi^{\rm d} = \Lambda - \Pi(\{i\})$ from Proposition 2,[25] with $\Pi(\{i\}) = \check{\Pi}(\{i\}) = \hat{\Pi}(\{i\})$ because of the definition of $\delta_{\check{x}_j=\hat{x}_j=0}$. Using the latter and (23) we define:

$$\tilde{\Delta}(\delta) = \pi^{\rm d} - \pi^{\rm n}. \tag{78}$$

As the roots to $\tilde{\Delta}(\delta_{x^{\rm d}=x^{\rm n}}) = 0$ $(\gamma = 0, \alpha = 0, \beta = \theta/2, \beta = 2/\theta)$ are not admissible, $\tilde{\Delta}(\delta_{x^{\rm d}=x^{\rm n}})$ does not change sign. It suffices to evaluate (78) at, say, $\beta = 0$, to check that claim A is valid.

Claim B: $\pi^{\rm d} > \pi^{\rm n}$ for $\delta = \delta_{\pi^{\rm d}=\pi^{\rm c}}$. We know that $\pi^{\rm d} = \pi^{\rm c}$ along $\delta_{\pi^{\rm d}=\pi^{\rm c}}$ by definition, while $\pi^{\rm c} > \pi^{\rm n}$ for $\beta < \theta/2$ and any δ from d'Aspremont and Jacquemin (1988). Therefore, by transitivity claim B is true.

$(\beta > \theta/2)$

Claim C: $\pi^{\rm d} < \pi^{\rm n}$ for $\delta = \delta_{x^{\rm d}=x^{\rm n}}$. Recall from (2) that the unit costs of production are equal under delegated and non-cooperative R&D along $\delta_{x^{\rm d}=x^{\rm n}}$. Thus, from (1) and (16), $q_i(\mathbf{x}^{\rm d}) = q_i(\mathbf{x}^{\rm n})$ and $p_i(\mathbf{x}^{\rm d}) = p_i(\mathbf{x}^{\rm n})$, for $i = 1, 2$, along $\delta_{x^{\rm d}=x^{\rm n}}$. It follows that gross concentrated profits (i.e., before R&D costs) are also equal, that is:

$$\pi_i(\mathbf{x}^{\rm d}) = \pi_i(\mathbf{x}^{\rm n}), \tag{79}$$

$i = 1, 2$. Moreover, we know from Lemma 1-(*ii*) that $\delta_{\mathcal{L}=0} < \delta_{x^{\rm d}=x^{\rm n}} < 0$. The first inequality sign means that the laboratory exactly breaks even along $\delta_{x^{\rm d}=x^{\rm n}}$ because of Proposition 2. This implies that firms' symmetric transfer payments exactly cover the laboratory's costs, that is $t_1^{\rm d}(\mathbf{x}^{\rm d}) + t_2^{\rm d}(\mathbf{x}^{\rm d}) = s(\mathbf{x}^{\rm d})$. The second inequality means indirect externalities are negative along $\delta_{x^{\rm d}=x^{\rm n}}$, because of Property 2. This implies that the laboratory's R&D costs are strictly greater than the firms' total R&D costs, that is $s(\mathbf{x}^{\rm d}) > r(x_1^{\rm n}) + r(x_2^{\rm n})$, with $x_1^{\rm n} = x_2^{\rm n} = x^{\rm n}$. It follows that $t_i^{\rm d}(\mathbf{x}^{\rm d}) > r(x_i^{\rm n})$ along $\delta_{x^{\rm d}=x^{\rm n}}$. It suffices to use (79) to obtain:

$$\pi_i(\mathbf{x}^{\rm d}) - t_i^{\rm d}(\mathbf{x}^{\rm d}) < \pi_i(\mathbf{x}^{\rm n}) - r(x_i^{\rm n}), \tag{80}$$

$i = 1, 2$. Inequality (80) says that claim C, which refers to net profits, is true.

Claim D: $\pi^{\rm d} > \pi^{\rm n}$ for $\delta = 0$. In the absence of indirect externalities, we have $s(\mathbf{x}) = r(x_1) + r(x_2)$. In that case, from Proposition 1, we know that solving the delegated game for $\mathbf{x}^{\rm d}$ is equivalent to solving the cooperative game for $\mathbf{x}^{\rm c}$. Then, from Proposition 2, because $\delta = 0 > \delta_{\mathcal{L}=0}$ implies that the laboratory exactly breaks even, we have $\pi^{\mathrm{d}} = \pi^{\mathrm{c}}$. As π^{c} is always greater than π^{n}, from d'Aspremont and Jacquemin (1988), claim D follows by transitivity.

Given claims A to D, that π^{d} is continuous and monotone increasing in δ from Lemma D-2, and π^{n} is invariant with δ, means there exists a unique $\delta \equiv \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ such that $\pi^{\mathrm{d}} \gtreqqless \pi^{\mathrm{n}}$ if and only if $\delta \gtreqqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$, with $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ as in Lemma 2-(ii). ■

D.3 Proof of Lemma 3

D.3.1 Equal Delegated and Cooperative Welfare Frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$

As w^{d} is monotone increasing in δ (Lemma D-3), and w^{c} does not vary with δ, it follows that if there exists a value of δ such that $w^{\mathrm{d}} = w^{\mathrm{c}}$, it is unique. Moreover, for $\delta = 0$, we know (i) the laboratory's costs in the delegated game are equal to both firms' total R&D costs in the cooperative game (Property 2), and (ii) solving the delegated game for x^{d} is equivalent to solving the delegated game for x^{c} (Proposition 1). Hence, the two games yield the same equilibrium quantities and prices, that is $q_i(\mathbf{x}^{\mathrm{d}}) = q_i(\mathbf{x}^{\mathrm{c}})$ and $p_i(\mathbf{x}^{\mathrm{d}}) = p_i(\mathbf{x}^{\mathrm{c}})$, $i = 1, 2$. Recalling that w^{d} is continuous and monotone increasing in δ, whereas w^{c} is invariant with δ, gives Lemma 3-(i). ■

D.3.2 Equal Delegated and Non-Cooperative Welfare Frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$

($\beta = \theta/2$)

We know from Proposition 3 that $w^{\mathrm{d}} = w^{\mathrm{n}}$ for $\delta = 0$.

Now we turn to the other values of β.

i) We compare consumer surpluses and gross concentrated profits (i.e., before R&D costs), in the delegated and non-cooperative games along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, and show they are the same. To see that, recall that $x^{\mathrm{d}} = x^{\mathrm{n}}$ along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, by definition. Therefore production costs, together with quantities and thus prices, are identical in the two games, that is $c_i(\mathbf{x}^{\mathrm{d}}) = c_i(\mathbf{x}^{\mathrm{n}})$, $q_i(\mathbf{x}^{\mathrm{d}}) = q_i(\mathbf{x}^{\mathrm{n}})$, and $p_i(\mathbf{x}^{\mathrm{d}}) = p_i(\mathbf{x}^{\mathrm{n}})$, $i = 1, 2$. It follows that firms' gross profits and consumer surpluses are the same in the delegated and non-cooperative R&D games along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$.

ii) We show that the sign of the difference between total R&D costs in the delegated and non-cooperative games along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ depends on the sign of direct externalities. To see that, observe that $-\delta x_1 x_2 \underset{>}{\overset{<}{=}} 0$ if and only if $\delta \underset{<}{\overset{>}{=}} 0$, for all $x_1, x_2 > 0$. Then recall from Lemma 1-(ii) that $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} \underset{<}{\overset{>}{=}} 0$ if and only if $\beta \underset{>}{\overset{<}{=}} \theta/2$. It follows that:

$$-\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} x_1 x_2 \underset{>}{\overset{<}{=}} 0 \quad \text{if and only if} \quad \beta \underset{>}{\overset{<}{=}} \theta/2, \tag{81}$$

for all $x_1, x_2 > 0$. Hence, (81) means that along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, the laboratory's costs are less than (equal to, greater than) firms' total in-house R&D costs if and only if β is less than (equal to, greater than) $\theta/2$.

As welfare is the sum of consumer surplus and firms' net profits, it follows from i) and ii) that:

$$w^{\mathrm{d}}\Big|_{\delta=\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}} \underset{<}{\overset{>}{=}} w^{\mathrm{n}} \quad \text{if and only if} \quad \beta \underset{>}{\overset{<}{=}} \theta/2. \tag{82}$$

We now use (82) to establish the existence of $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ for $\beta < \theta/2$, and $\beta > \theta/2$ respectively.

($\beta < \theta/2$)

Claim A: $w^{\mathrm{d}} > w^{\mathrm{n}}$ along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$. This claim follows from (82). Moreover $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ from Lemma 2-(ii), for $\beta < \theta/2$.

Claim B: $w^{\mathrm{d}} < w^{\mathrm{n}}$ for $\delta = 0$. Recall that $w^{\mathrm{c}} = w^{\mathrm{d}}$ for $\delta = 0$ by Proposition 1, and $w^{\mathrm{c}} < w^{\mathrm{n}}$ for $\beta < \theta/2$ from d'Aspremont and Jacquemin (1988). Claim B follows by transitivity.

($\beta > \theta/2$)

Claim C: $w^{\mathrm{d}} < w^{\mathrm{n}}$ along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, for $\beta > \theta/2$. The proof of claim C follows directly from (82).

Claim D: $w^{\mathrm{d}} > w^{\mathrm{n}}$ for $\delta = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$. On the firms' side, $\pi^{\mathrm{d}} = \pi^{\mathrm{n}}$ on $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ by definition. On the consumers' side, because $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ from Lemma 2-(*ii*), we obtain $x^{\mathrm{d}} > x^{\mathrm{n}}$, and consequently $c_i(\mathbf{x}^{\mathrm{d}}) < c_i(\mathbf{x}^{\mathrm{n}})$ leads to $q_i(\mathbf{x}^{\mathrm{d}}) > q_i(\mathbf{x}^{\mathrm{n}})$ and $p_i(\mathbf{x}^{\mathrm{d}}) < p_i(\mathbf{x}^{\mathrm{n}})$, $i = 1, 2$, on $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$. On the laboratory's side, we know that $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{\mathcal{L}=0}$ from Lemma 2-(*ii*) in the case of positive direct externalities, which implies that $\mathcal{L} = 0$ from Proposition 2. As firms and consumers are better-off in the delegated R&D game than in the non-cooperative one, while the laboratory earns zero benefits in either game, means that claim D is true.

Using claims A to D, that w^{d} is continuous and monotone increasing in δ (Lemma D-3), while w^{n} does not vary with δ, means there exists a unique $\delta \equiv \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ such that $w^{\mathrm{d}} \gtreqless w^{\mathrm{n}}$ if and only if $\delta \gtreqless \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$, with $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ as in Lemma 3-(*ii*). ■

E Comparison of $\bar{\pi}^{\mathrm{c}}$ with π^{c}, π^{n}, π^{d}

Recall that we denote by $\bar{\pi}^{\mathrm{c}}$ the profits each firm earns by cooperating in R&D with the rival firm when the R&D costs are the same as in the delegated R&D game, and $\delta \geq 0$.

Note first that $\bar{\pi}^{\mathrm{c}} = \Lambda/2 = \pi^{\mathrm{d}}$ if $\delta \geq \delta_{\mathcal{L}=0}$, and that $\bar{\pi}^{\mathrm{c}} = \Lambda/2 > \pi^{\mathrm{d}}$ if $\delta < \delta_{\mathcal{L}=0}$, by construction of the alternative cooperative R&D game. This implies that the profit frontiers for the comparison of $\bar{\pi}^{\mathrm{c}}$ with π^{c} and π^{n} coincide with the profit frontiers for the comparison of π^{d} with π^{c} and π^{n} at all points on $\delta_{\mathcal{L}=0}$ or above $\delta_{\mathcal{L}=0}$ in the plane of externalities (β, δ).

It thus remains to compare the new profit frontiers $\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{c}}}$ and $\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{n}}}$ with the frontiers defined in Lemmas 1, 2 and 3, that were demonstrated to lie strictly below $\delta_{\mathcal{L}=0}$. The only candidates are $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$, $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$, $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$, $\delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$, $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ for $\beta < \theta/2$. To do that, note first that $\bar{\pi}^{\mathrm{c}} = \Lambda/2$ by construction, and recall that $\mathrm{d}\Lambda/\mathrm{d}\delta > 0$, to conclude that $\mathrm{d}\bar{\pi}^{\mathrm{c}}/\mathrm{d}\delta > 0$. Then consider the frontiers $\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{c}}}$ and $\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{n}}}$ in turn, as follows.

($\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{c}}}$) Recall that $\bar{\pi}^{\mathrm{c}} = \pi^{\mathrm{c}}$ for $\delta = 0$, by construction. As $\bar{\pi}^{\mathrm{c}}$ is strictly monotone in δ, whereas π^{c} does not depend on δ, we obtain $\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{c}}} = 0$. It follows that $\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{c}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$, all $\beta < \theta/2$.

($\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{n}}}$) As $\pi^{\mathrm{n}} < \pi^{\mathrm{c}}$ and $\pi^{\mathrm{c}} = \bar{\pi}^{\mathrm{c}} = \Lambda/2$ for $\delta = 0$, the monotonicity of $\bar{\pi}^{\mathrm{c}}$ in δ implies that $\bar{\pi}^{\mathrm{c}}$ cannot be equal to π^{n} if δ is non-negative. It follows that $\delta_{\bar{\pi}^{\mathrm{c}}=\pi^{\mathrm{n}}}$ does not exist if $\delta \geq 0$, all $\beta < \theta/2$.

F. Figures

All four figures are drawn for $a = 1, b = 1, c = 3/4, \gamma = 2,\ \theta = 1$. Figures 1, 2, 3 include a reference to the following results by d'Aspremont and Jacquemin (1988): (*i*) $x^{\mathrm{c}} \gtreqless x^{\mathrm{n}}$ if and only if $\beta \gtreqless \theta/2$; (*ii*) $\pi^{\mathrm{c}} = \pi^{\mathrm{n}}$ at $\beta = \theta/2$, otherwise $\pi^{\mathrm{c}} > \pi^{\mathrm{n}}$; and (*iii*) $w^{\mathrm{c}} \gtreqless w^{\mathrm{n}}$ if and only if $\beta \gtreqless \theta/2$.

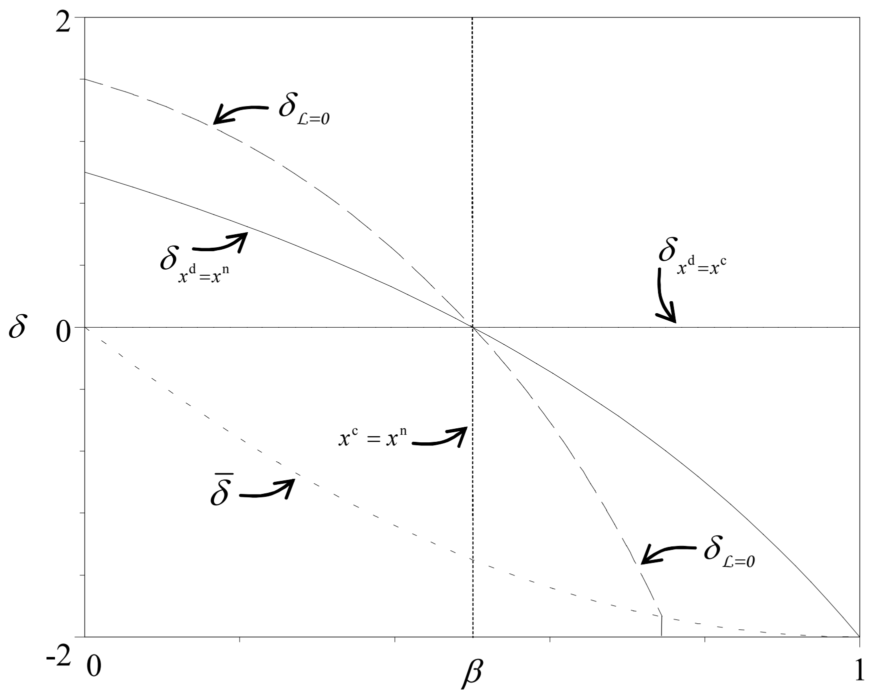

Figure 1: **(R&D outcomes):** $x^{\mathrm{d}} \gtreqless x^{\mathrm{n}}$ if and only if $\delta \gtreqless \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, and $x^{\mathrm{d}} \gtreqless x^{\mathrm{c}}$ if and only if $\delta \gtreqless \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0$.

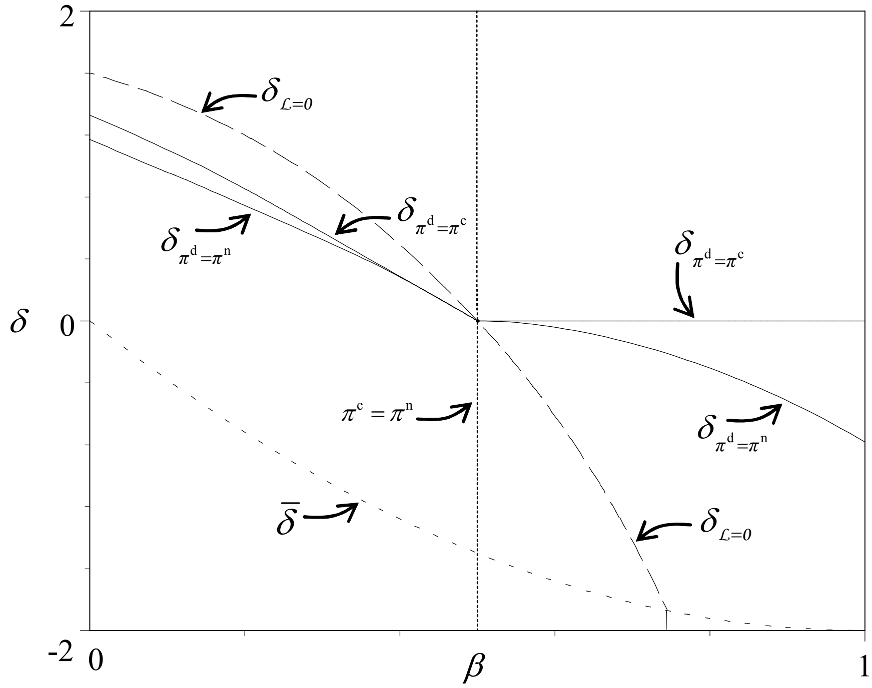

Figure 2: **(Firms' profits):** $\pi^{\mathrm{d}} \gtreqless \pi^{\mathrm{n}}$ if and only if $\delta \gtreqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$, and $\pi^{\mathrm{d}} \gtreqless \pi^{\mathrm{c}}$ if and only if $\delta \gtreqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$.

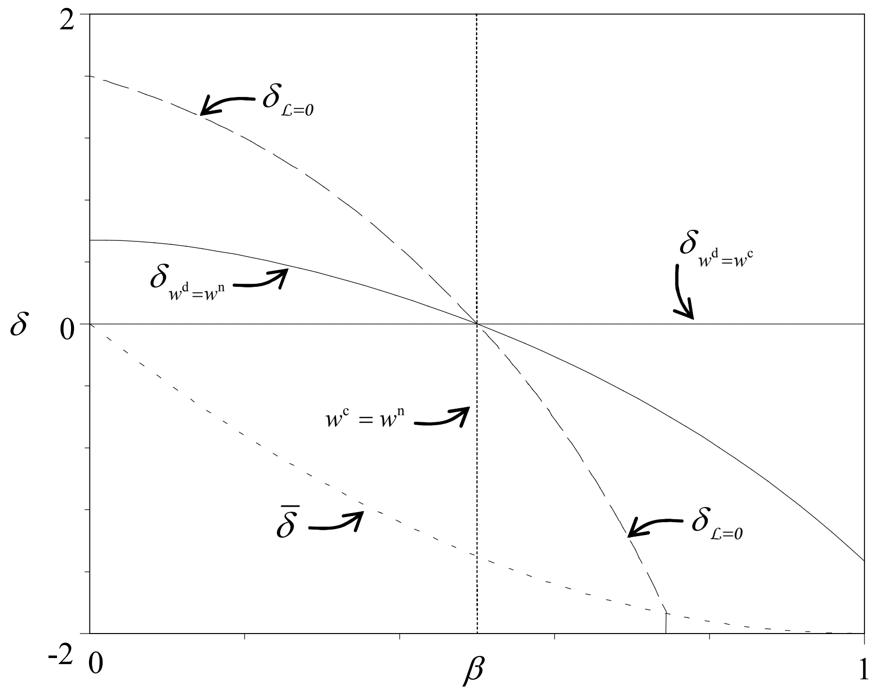

Figure 3: **(Social welfare):** $w^{\mathrm{d}} \gtreqless w^{\mathrm{n}}$ if and only if $\delta \gtreqless \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$, and $w^{\mathrm{d}} \gtreqless w^{\mathrm{c}}$ if and only if $\delta \gtreqless \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$.

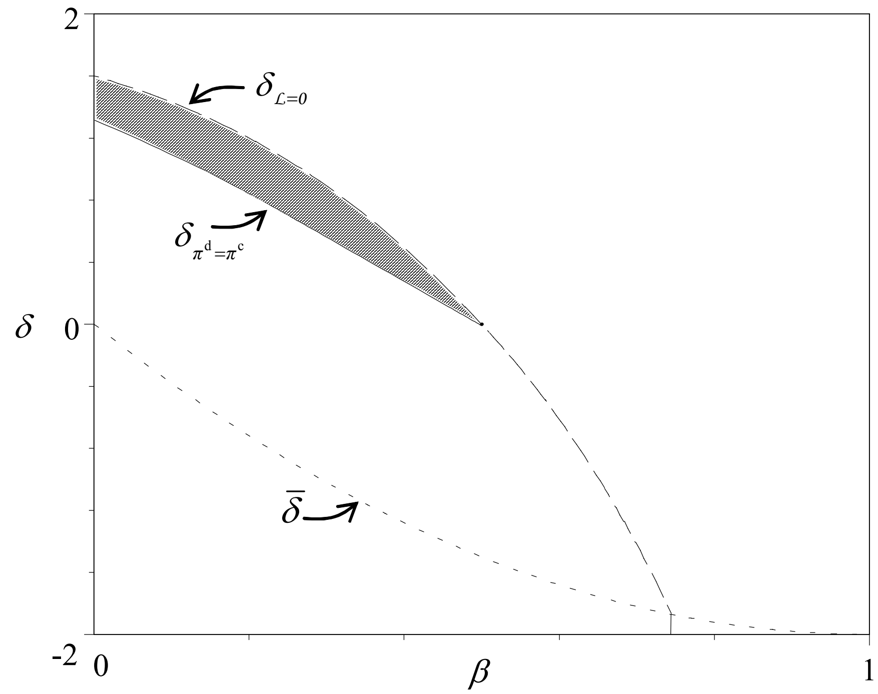

Figure 4: **(Delegation dominance):** the shaded area represents the set of points (β, δ) for which the delegated R&D game is a Pareto optimal organizational form of R&D, and the laboratory earns positive benefits.

Notes

[1]The National Science Foundation (2006) uses the term "contract R&D" to denote a transaction with external parties involving R&D payments or income, regardless of the actual legal form of the transaction. The quoted figures do not include contract R&D expenses by U.S. companies that do not perform internal R&D, or that contract out R&D to companies located overseas.

[2]For details on these contracts, and other examples, see: www.recap.com/bday.nsf.

[3]For example, Kamien, Tauman and Zang (1988) examine the case of a superior product which is licensed to producers of an inferior substitute through a fixed fee. Kamien, Oren and Tauman (1992b) compare alternative licensing strategies, namely a fixed fee, a per unit royalty, and the auctioning of a fixed number of licenses.

[4]For instance, Kamien et al. (1992a) consider spillovers on the input side of the R&D stage, Suzumura (1992) introduces a second-best welfare criterion, Motta (1992) and Rosenkranz (1995) consider quality-improving R&D, Vonortas (1994) distinguishes between generic research and commercial development, Poyago-Theotoky (1999) and Kamien and Zang (2000) endogeneize spillovers, and Hinloopen (2000) introduces Bertrand competition on the final market with more than two firms.

[5]See Martin (2001) for a critical overview of that literature.

[6]In the business literature, the outsourcing of *new* technological knowledge – under conditions stipulated in a contract agreed beforehand – is clearly distinguished from situations in which firms license *existing* technological knowledge (Howells, James and Malik 2003, for example).

[7]This terminology is similar to what Laffont and Martimort (1997) call "type 1" (indirect) and "type 2" (direct) externalities.

[8]By contrast, in an *intrinsic* common agency model the agent is forced to contract with all principals, or none of them.

[9]By comparison, in a *private* common agency set-up a principal may contract only on the quantities it specifically receives from the agent.

[10]This is done as in the present paper, but for more general functional forms, by associating to the common agency setting a transferable utility cooperative game (see Appendix C, which builds on Laussel and Le Breton (2001)). The characteristic function of this cooperative game is defined as the maximum level of joint profits as obtained by the

laboratory and any subset of contracting firms (that is either none, or only one of the two firms, or both of them). Then it is proved that the laboratory appropriates some non-zero share of joint profits if the characteristic function reflects substitutabilities in R&D dimensions (which may result from negative externalities of the direct and/or indirect kinds). Alternatively, the laboratory breaks-even if the characteristic function reflects complementarities in R&D dimensions. The proposed sufficient conditions are shown to be satisfied by many specifications used in a sample of papers on the industrial organization of R&D.

[11] Aghion and Tirole (1994) and Ambec and Poitevin (2001) differ fundamentally in their theoretical use of the non-deterministic R&D assumption. While Aghion and Tirole (1994) emphasize the non-contractibility of the uncertain R&D outcome, Ambec and Poitevin (2001) stress an informational problem as attached to the risky nature of R&D tasks.

[12] In d'Aspremont and Jacquemin (1988), and in the cooperative R&D model of the present paper, the two firms are subject to "ex-ante equal treatment", in the terminology introduced by Leahy and Neary (2005). This means that the symmetric firms are assumed to invest equally in R&D when they cooperate, "a natural starting point" (p. 384) when side-payments between firms are ruled out.

[13] This alternative set-up constitutes an example in a class of principal-agent models surveyed by Gal-Or (1997, Section 3, pp. 241-248). In these models, contracts are observable, several principals compete in the market-place, and each of them delegates the production of an input to its own specific agent.

[14] Most papers refer in particular to the US National Cooperative Research Act (NCRA) of 1984, and its 1993 subsequent amendment, which is usually described as a means to reduce legal disincentives to participate in contractual R&D agreements. This act gives firms the possibility of limiting antitrust analysis to the rule of reason (in lieu of the *per se* illegality rule) on all "properly defined, relevant research and development markets". It also limits antitrust recoveries against registered agreements to lower damages if the terms of the submitted agreement are found to violate the law. According to Martin (1996, p. 270) it leads antitrust authorities to adopt a "*permissive* attitude" (added emphasis). More recently, Amir et al. (2003) extend this comment to other geographical areas by emphasizing that "a permissive antitrust attitude towards R&D cooperation has been the norm in Europe and Japan early on" (p. 184).

[15] This state of affairs leads Majewski (2004) to question the cooperative character of these cases: "the fact that a significant subset of observations organizes their collaboration as a nexus of arms-length contracts raises the question of what 'collaborative R&D' means" (p. 20). We see this as an encouragement to the examination of delegated R&D set-ups in complement to existing comparisons of cooperative *versus* non-cooperative R&D set-ups.

[16]Sinclair-Desgagné (2001) also mentions, in an example, that R&D contracts could be analyzed in a common agency set-up where principals condition their payments to the agent on the quantities received by other principals.

[17]Dobler and Burt (1996, p. 416) observe that "R&D services normally are purchased through one of the two methods of compensation: a fixed price for a level of effort (e.g., fifty days) or a cost plus fixed or award fee".

[18]A Nash equilibrium in a common agency game is coalition-proof if it is robust to credible threats of deviations by some subsets of the principals (see Bernheim, Peleg and Whinston (1987) for a formal definition). When there are only two principals, as in our delegated R&D model, a coalition-proof equilibrium is Pareto-efficient among principals (Bernheim and Whinston 1986b, p. 16 footnote 11).

[19]For more on this see Kirchsteiger and Prat (2001, footnote 7 on p. 354), and Martimort (2005, pp. 19-20).

[20]In this example, the numerical values are obtained from the algebraic expressions displayed in the proof of Proposition 2.

[21]It is straightforward to compare the asymmetric equilibrium R&D levels $\bar{x}_1 = 0$ and $\bar{x}_2 \equiv \bar{x}^{\mathrm{d}} > 0$ (say), as obtained for all $\delta < \bar{\delta}$, with the symmetric interior solution $x_1^{\mathrm{d}} = x_2^{\mathrm{d}} = x^{\mathrm{d}}$ to the joint-profit maximizing problem (*14*) when $\delta = \bar{\delta}$. From the equilibrium expressions computed in the appendix (see sections A.2 and C.1) one finds $\bar{x}^{\mathrm{d}} = 2x^{\mathrm{d}}$ on the frontier $\bar{\delta}$. Next, continuity of $\Lambda(\mathbf{x})$ in δ implies that $\bar{\pi}^{\mathrm{d}} = \pi^{\mathrm{d}}$ on the frontier $\bar{\delta}$. This implies that one may focus on the consumer surplus to compare total welfare with an interior solution or a corner solution on $\bar{\delta}$. For all β and θ less than 1, simple algebra leads to $\bar{w}^{\mathrm{d}} > w^{\mathrm{d}}$ (with an equality sign if $\beta = 1$ or $\theta = 1$), again on the frontier $\bar{\delta}$.

[22]More formally, $r(x_i) = \frac{\gamma}{2}(1+\beta)x_i^2$, in the cooperative and non-cooperative in-house R&D games. In this case, Amir (2000) demonstrates that d'Aspremont and Jacquemin (1988), in which $\mathbf{x}$ is a vector of R&D outcomes, and Kamien et al. (1992a), in which $\mathbf{x}$ describes R&D investments, yield the same effective cost-reducing outcomes. We let the laboratory's cost function be $s(\mathbf{x}) = \frac{\gamma}{2}(1+\beta)(x_1^2 + x_2^2) - \delta x_1 x_2$.

[23]A first step towards the generalization of the delegated R&D model to more than two firms is easily obtained by applying two recent results by Billete de Villemeur and Versaevel (2003). They establish sufficient conditions for $\Pi(S)$, as defined in (43), with $S \in 2^N$ and $\#N \geq 2$, to be either strictly subadditive or convex (and thus for the laboratory to earn positive benefits or to exactly break even, respectively). However, $\Pi(S)$ should be strongly subbadditive to obtain $\pi^{\mathrm{d}} = \Lambda - \Pi(N \backslash \{i\})$, for all i, in all TSPNE. The strong subadditivity property (see Laussel and Le Breton (2001, p. 104) for a formal definition) is stronger than the strict additivity property used in this paper. The two properties coincide for $\#N = 2$ only.

[24]The conjecture that an agent in charge of R&D activities may have no superior information about project returns before acting dates back from Holmstrom (1989), who sees it as a ”reasonable assumption if we are at the initial stages of a research undertaking" (p. 310). Interestingly, this falls in line with the specifications of a theoretical paper on common agency by Laussel and Le Breton (1998), who introduce a random parameter in the agent's cost function which is not realized at the contracting stage. In this setting, the laboratory's cost would become $\mu s(\mathbf{x})$, where μ is a positive random variable. The laboratory would not know the realization of μ before accepting or refusing the firms' contracts (that is, strategies $t_i(\mathbf{x})$ in our notation, $i = 1, 2$), but would learn it before producing R&D services (that is, $\mathbf{x}$).

[25]Lemma 1-(ii) implies that $\delta_{\check{x}_j=\hat{x}_j=0} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\mathcal{L}=0}$.

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