Antiquity in the shape of a Canon.

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Views on antiquity from the outlook of mathematics »

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In the ancient extant Chinese writings in which practitioners evoke the genesis and history of mathematics, Antiquity plays a key part. Features of mathematical knowledge on which these texts place emphasis and which are in fact, as we now know, specific to China (such as a conception of the overall architecture of mathematics, its organization in nine sections), properties attached to it (such as its power), are referred to the way in which it developed in Antiquity. In addition to this, representations of Antiquity play an essential role in shaping practices of mathematics to which these writings attest.

These remarks lead me to the two main issues I shall address in this paper. In a first part, I shall focus on analyzing the perception of Antiquity disclosed by how the discourse about mathematics put it into play. In the following two parts, I shall describe how representations of Antiquity shaped some of the mathematical practices to which these texts bear witness: respectively, editing scriptures from Antiquity and investigating them.

The earliest of such writings turns out to be the commentary Liu Hui 劉徽 completed in 263 on the Canon *The Nine Chapters on Mathematical Procedures* (*Jiuzhang suanshu* 九章算術)—hereafter abbreviated to *The Nine Chapters*—, which is today believed to have been composed during the Han dynasty, about two thousand years ago. It is in Liu Hui's preface that we find the development about Antiquity in which we are interested. Incidentally, Liu Hui's commentary is also the earliest one composed on this Han Canon to have survived. However, in the subsequent centuries, *The Nine chapters* remained the subject of an active interest, as is evidenced by the successive editions and commentaries that came down to us and to which we come back below.

Liu Hui's preface bears witness, within the sphere of mathematical activities, to representations and uses of Antiquity that regularly recurred in the mathematical writings referring to history in subsequent centuries. I shall hence take it as a reference point in the discussion, analyzing how it brings Antiquity into play and the operations through which Antiquity is made present in the form of artifacts and practices. Moreover, it is by reference to this preface that I shall point out continuities and discontinuities in later representations and

uses of Antiquity. These are to be found in specific *loci* for the development of historical discourse within ancient mathematical literature: the chapters relating to mathematics in the dynastic histories as well as the prefaces to the successive editions of, and commentaries on, the books that were progressively selected as Canons —above all, the one that became perceived as the most important of them all: *The Nine Chapters*.

It is not surprising that prefaces turn out to be, with dynastic histories, the main places where such developments about the genesis of mathematical knowledge occur. However, it is interesting that, more specifically, these developments lay emphasis on Antiquity especially at times when the canonical literature received a specific attention —a point for which we shall account below. For mathematics as well as more generally, in addition to the Wei-Jin period during which Liu Hui composed his commentary, this designates the Tang and the Song dynasties, during which a stronger interest in canonical writings is clearly perceptible. It was in 656 that Li Chunfeng 李淳風 presented to the throne a commentary on *The Nine chapters* and Liu Hui's commentary, composed by a group of scholars under his supervision —for the sake of simplicity, below, let us designate it as "Li Chunfeng's commentary". Today, there is no extant edition of the Han Canon that would not include Liu Hui's and Li Chunfeng's commentaries. Moreover, during the Song dynasty, *The Nine Chapters* and the two commentaries became the focus of a renewed interest. They were printed in 1084, in an edition to be reprinted by Bao Huanzhi 鮑瀣之 in 1213. New commentaries were composed on this set of texts, like the one by Jia Xian 賈憲, in the 11th century, which was printed by Rong Qi in 1148 榮啟 and was to be in its turn commented upon by Yang Hui 楊輝 in 1261. All these actors expressed views that related the emergence of mathematics to Antiquity in a way that will be examined below.

Now that the questions we shall investigate and the corpus of texts on the basis of which we shall consider them were introduced, let us turn to the perception of Antiquity as evidenced by the 3rd century commentator, Liu Hui.

Representations, and modes of presence, of Antiquity within the domain of mathematics

According to Liu Hui's preface, Antiquity constituted the time period during which mathematics took shape and developed. And he relates how this process took place, by reference to the account of the genesis of culture as found in the "Great commentary", or "Commentary on the attached verbalizations" of the *Classic of changes*.[1] The discourse about Antiquity thus takes a passage in the canonical literature as its reference point, and it accounts for an emergence of mathematics in Antiquity by modifying this passage accordingly.

Liu Hui transforms this account, when quoting it, in the first place by inserting the creation of the fundamentals of mathematics —i.e.: the multiplication table ("procedure of nine (times) nine" *jiujiu zhi shu* 九九之術)— quite early in the narrative.[2] The opening section of Liu Hui's preface reads as follows:

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[1] *Xici zhuan*, last chapter, paragraph 2, in *Zhouyi dachuan jinzhu*, pp. 558-568. For a critical edition and a translation of the whole preface, I refer the reader to [Chemla & Guo 2004], pp. 126-129, and, for more details on the interpretation of the preface, see my footnotes to the translation in *ibid.*, pp. 747-759.

[2] In fact, in the ancient extant mentions to be found in Chinese sources, what we designate by the expression of "multiplication table" was called the "procedure of nine (times) nine". This reflects the fact that it is at the basis of both multiplication and division. This is made explicit by the 3rd century commentator Zhao Shuang, in his commentary on another Han Canon

> "In times past there was Baoxi, who, first, drew the eight trigrams to enter into communication with the capacities of clairvoyance and illumination, to classify the essentials of the myriad things, and then created the procedure of the multiplication table so that it be in concordance with the mutations of the six lines (of the hexagrams).
>
> "This arrived to Huangdi, who metamorphosed them [by working at the level of] the unfathomable, increased [their extension] by elongating them, and who, hence, established the structure of the calendar, tuned the musical tubes, and used them to inquire into the source of the Way (*dao*). Hence the essential and minute *qi* of the two exemplars and of the four models could model themselves on them."[3]
> 昔在庖犧氏始畫八卦，以通神明之德，以類萬物之情，作九九之數，以合六爻之變。暨于黃帝，神而化之，引而伸之，於是建曆紀、協律呂、用稽道原，然後兩儀四象精微之氣可得而效焉。

As is clear from this quotation, the emergence of the fundamentals of mathematics is placed quite early —and thus quite high— in the process of the development of culture that took place in Antiquity. The creation of the multiplication table, from which it will proceed, is attributed to Baoxi (Fuxi), just after he created the trigrams. Later on, the paragraph of the "Great commentary" explains how, on the basis of the trigrams and the hexagrams that they yield by duplication, the fundamental cultural artifacts could be invented. As for Liu Hui, on the one hand, he gives the fundamentals of mathematics to present an intimate relationship to the hexagrams. On the other hand, his preface recounts how mathematical knowledge developed from that time onwards. This opening section of his preface reveals a belief in the fact that mathematics is intrinsically linked to other dimensions of culture and is even essential in the shaping of its most fundamental elements.

In relation to these facts, Liu Hui endows mathematics with vast powers: its fundamentals belong to the set of elements on the basis of which Huangdi could shape the structure of the calendar, tune pitch-pipes and study the source of the *dao*. The theme of the power of mathematics impregnates more generally the whole preface, transferring to that particular body of knowledge qualities attached to Antiquity in the representation provided by the "Great commentary". This theme also pervades the prefaces of mathematical books to be written in the subsequent centuries in China: mathematics is given to enable tackling the problems relating to the cosmos, the earth as well as the heavens and beyond. However, these prefaces sometimes detach this thesis from the one of the emergence of mathematics in Antiquity, as the preface to the *Mathematical Canon by Sunzi* 孫子算經 (*ca.* 400).[4]

Moreover, also as a consequence of the representation of Antiquity that it endorses, the story of the development of mathematics that Liu Hui's preface recounts is that of a

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dealing with mathematical astronomy and cosmography: the *Gnomon of the Zhou* (*Zhoubi*), also known as *Mathematical canon of the Gnomon of the Zhou* (*Zhoubi suanjing*). Zhao Shuang writes: "The multiplication table is the origin of multiplication and division." ([Qian Baocong 1963], p. 14; [Guo Shuchun and Liu Dun 1998], p. 1.)

[3] The emphasis is mine.

[4] This date is taken from Qian Baocong. For this preface, see, respectively, [Qian Baocong 1963], volume 2, p. 279; [Guo Shuchun and Liu Dun 1998], volume 2, p. 1. The books of the subsequent centuries to have survived belonged for the most part —or at least were thought at one point of history to belong— to the collection of ten mathematical Canons prepared under the supervision of Li Chunfeng during the Tang dynasty. See their prefaces in the critical editions of the collection by [Qian 1963] or [Guo & Liu 1998].

creation, followed by a *structuring* and an *unfolding*. This is clear from the next paragraph, which reads as follows:

> "Writings tell that Lishou 隸首created mathematics (*shu* 數); nothing more precise has been heard about it. It is only when the Duke of Zhou established the Rites that [we know that] the nine parts of mathematics (*jiushu* 九數) existed. The development (*liu*) of these nine parts hence produced *The nine chapters*.=
> 記稱隸首作數，其詳未之聞也。按周公制禮而有九數。九數之流則九章是矣

If we recapitulate, first the fundamentals of mathematics were created. Thereafter, during Huangdi's time, mathematics was in its turn created, probably in the same way, on the basis of the fundamentals, as cultural artifacts took shape by relying on the hexagrams —this is at least what the movement of the text inclines to believe. The further step was that of a structuring in nine branches, the development of which yielded the Canon *The Nine Chapters*.

Antiquity thus yields a time span on which to map this unfolding with respect to the progressive advent of other cultural artifacts. Simultaneously, this account provides a description of an architecture of mathematical knowledge, starting from the most basic elements. Both —the representation, in terms of unfolding, of the formation of mathematics and the correlated architecture of mathematical knowledge— can be evidenced until at least the Song-Yuan dynasties.

In the 13th century, the commentator Yang Hui also describes the development of mathematical knowledge as that of a structure that unfolds from the basis constituted of multiplication and division. However, for him, this pictures the architecture of mathematical knowledge, rather than a historical process. In relation to this, mathematics is no longer considered with respect to the history of culture, even though the theme of the immense extension of its power is still present.

In Liu Hui's preface, mathematics is given not only to have unfolded, but also to have unfolded throughout Antiquity in a specific way: starting from the "table of multiplication", mathematics was developed; subsequently, at the time of the duke of Zhou, a process of structuring took place, as a result of which it was organized into nine branches. *The nine chapters* is thus introduced as the embodiment, in book form, of the resulting structure of mathematical knowledge. In the subsequent centuries, its "chapters" constituted the "disciplines" within the framework of which mathematical practice was carried out in China, until as late as the 20th century. More precisely, there seems to be a correlation between the facts that a practitioner of mathematics develops a discourse on the Antiquity of mathematics, that he places emphasis on its organization in nine branches and that he presents his own contribution as developing one of these chapters. On the one hand, an author like Wang Xiaotong 王孝通, in the first half of the 7th century, in the preface to the *Jigu suanjing* 輯古算經(*Mathematical Canon continuing the Ancients*), endorses the themes of the immense powers of mathematics, of their structuring in nine branches as going back to the time of the Duke of Zhou. And, in correlation to this, he presents his book as a development of the fifth of *The Nine Chapters*. On the other hand, in contrast to this, the *Mathematical Canon by Sunzi* and the *Mathematical Canon by Zhang Qiujian* 張丘建算經 neither mention nor use this structuring of mathematical knowledge. Nor do these books evoke Antiquity in relation to mathematics.

By way of consequence, we may consider that this fundamental structure of mathematics in nine branches, which is considered to have taken shape in Antiquity,

constituted in fact *one mode of the social construction of the past in the present*. In the 7th century, Li Chunfeng accounted for how and why mathematics separated in nine branches, even though in a way different from Liu Hui's.[5] In the 13th century, in a separate chapter of his commentary, Yang Hui challenged this organization.[6] Alternative ways of structuring mathematical knowledge were introduced in China by the Jesuits from the 17th century onwards. However, this structuring in the traditional nine branches embodied by *The Nine Chapters* remained a reference point and a norm for the practice and development of mathematics. This feature allowed displaying, or restating, cultural continuity.

Furthermore, Liu Hui's preface stresses the contiguity between the structuring of mathematical knowledge in nine branches that took place in Antiquity and the book embodying it: *The Nine Chapters*. In correlation to this, quite early on, and certainly from Liu Hui's commentary onwards, *The Nine Chapters* itself, the composition of which is repeatedly given to have been carried out in Antiquity, was granted the status of a Canon (*jing* 經). This has important consequences for our topic. First, Antiquity was thus embodied in a book, purported to have specific properties. Secondly, throughout history, Antiquity was constantly made present in the form of this cultural artifact, or, to be more precise, by editions attempting to recover its original form. One could go even as far as to say that, as regards mathematics, Antiquity was equated with the Canon and the properties characterizing it, a strong emphasis on, and cultural restatement of the importance of, high Antiquity going along with launching endeavors to restore the Canon.

The book, as an ideal, and the successive attempts to restore it, all represent *another dimension of the social construction of Antiquity in the succeeding "presents"*. The shaping of the past in the form of canonical writings certainly deserves our attention, if we consider that, regarding mathematics, no other writing but the Canons survived in what was handed down through the written tradition. It is all the more meaningful for us because this is where the significance attached to Antiquity gets translated into actual mathematical practices. We shall now hence turn to analyzing the operations involved to ensure the presence of Antiquity in the form of such an artifact.

Editing as a key modality of the shaping of the past for the present

At different time periods, narratives were produced that came down to us, regarding *how* the book, as such, could be, and actually was, made available in the present. They display how different practitioners conceived of the relation one could establish to Antiquity, or, in other words, the operations they felt were necessary to shape Antiquity for a given present time. They also shed light on the mathematical activities that developed to yield editions of *The Nine Chapters* that were considered as adequate. Liu Hui's preface provides us again with the key elements entering into the composition of all these narratives.

The first such element consists of an explanation accounting for why the Canon was not directly available to the actors of a given present time in its original form. It thus

[5] See the "Monograph on the musical scale and the calendar *(Lülizhi* 律曆志)" of the *History of the Sui dynasty* (*Suishu* 隋書) that was prepared under Li Chunfeng's supervision, in [Yang Jialuo 楊家駱1977], volume 3, p. 1859.

[6] See the last chapter "Organizing the categories of the *Detailed explanations of The nine chapters on mathematical methods (Xiangjie jiuzhang suanfa zuanlei* 纂類)" in Yang Hui's commentary of 1261.

designates which events occurred that, according to that narrative, are to be blamed for the difficulties they brought about for getting access to Antiquity. Liu Hui gives *The Nine Chapters* as having been damaged by the process of transmission. He writes —and this is the sequel of what was already quoted from his preface:

> "Formerly, the cruel Qin burnt the books. The procedures of the Canon got scattered and damaged. 往者，暴秦焚書，經術散壞。

Here, Qin shihuangdi's burning of the books accounts for the dispersion and damaging of its pieces, which make it impossible to have the scripture in its original form —Note that here Liu Hui uses the term *jing* 經 to refer to the book. Later, in other narratives, different historical events will play the same part in a story that is also shaped in terms of damaging and restoring. In any case, the conclusion is the same: the Canon is no longer available and the access to its text as it once was, at the time of the Duke of Zhou, requires editing. This leads to the second ingredient of such narratives: the operations needed to recover the genuine Canon, according to the actors.

Liu Hui's preface continues with a description of the operations that he believed the Han editors of *The Nine Chapters* carried out to yield the text available to him:

> "After that time, the Bei Ping Marquis Zhang Cang (*ca* 250-152 BCE) and the Assistant of the Grand Minister of Agriculture Geng Shouchang (active *ca* 57-44 BCE) both acquired a universal reputation for their excellence in mathematics. On the basis of scraps of the ancient text that had survived, Zhang Cang and others made both excisions and completions.
> 自時厥後，漢北平侯張蒼、大司農中丞耿壽昌皆以善算命世。
> 蒼等因舊文之遺殘，各稱刪補。

Thus, according to Liu Hui, the text of *The Nine Chapters* that he could read had been restored on the basis of fragments that could be gathered in Han times. Moreover, on this basis, the editorial operations included *excising and completing* them.

This description hence reveals that, in Liu Hui's perspective, the Canon had been damaged in two opposite ways. On the one hand, some of its pieces had been dispersed and lost, which led to losing its original content and organization. On the other hand, it had been disfigured by superfluous additions. This is why the Han scholars striving to restore it thus supplemented what was thought had been lost, and, in addition, also had to *delete* the superfluous. The latter operation relates to a value associated to the book as "Canon" —and more generally to scriptures ascribed to Antiquity: their being "made simple (*yue*)" by the Sages to whom their composition is ascribed. Commentators bear witness to that fact that this virtue of simplicity was constantly ascribed to such texts as Canons along history, especially to the Confucian scriptures. In correlation to this, the editorial operation *par excellence* carried out by Confucius himself to restore canonical texts bequeathed by the Sages of Antiquity and perceived as having been damaged by the process of transmission was precisely excising (*shan* 刪).[7] We thus see that, in Liu Hui's view, the lasting representation of a scripture of Antiquity as "having been made simple" had guided the work of restoring the text of *The Nine Chapters*. This point reveals how, according to his understanding, Antiquity had been shaped for the present in Han times. Moreover, it also discloses aspects of his own

[7] [Henderson 1991], p. 27; [Owen 1992], pp. 195-196.

approach to *The Nine Chapters* and shows how it adhered to collective perceptions of Antiquity.

However, for the 3rd century commentator Liu Hui, the Han editors failed to restore the book correctly. The reason he adduces to bring this problem to light reveals another value attached to, and hence an expectation towards, Antiquity: in Liu Hui's view, the Canon should be all-encompassing. In contrast to this anticipation, Liu Hui shows that it fails to cover a category of problems, thereby exhibiting that the book had not been recovered adequately. He writes:

> "But (in *The Nine Chapters* as restored by the Han editors) there is nothing of the category (*lei*) of (…the problem introduced...). Therefore the procedures made by Zhang Cang and the others do not yet suffice to exhaust extensively all mathematics (*bo jin qun shu*).
> 九章 (…) 無 (…) 之類。然則蒼等為術猶未足以博盡群數也

For Liu Hui, this property which participates in his representation of Antiquity provides a criterion for assessing the editorial work that, in the Han dynasty, aimed at restoring the Canon. Let us note that, here, a value —the completeness of the text— combines with mathematical work —testing the completeness— for evaluating whether the text was restored adequately. Their combination leads Liu Hui to conclude that the Han text of *The Nine Chapters* is problematic.[8] If we analyze the episode from our perspective, his reaction is quite revealing in two ways. First, he seems to attribute the same vision of Antiquity as his own to the Han editors. One can even assume, following Liu Hui, that the guiding value that oriented their way of *completing* the text was that it once was all-encompassing. Secondly, the way in which Liu Hui proceeds to carry on the editing again casts light on how the values associated to Antiquity, and hence expected from *The Nine Chapters*, were further put into play in shaping the Canon for the present.

A problem was diagnosed in the composition of *The Nine Chapters*. The final part of Liu Hui's preface is devoted to explaining how the commentator, continuing the work of his predecessors, elaborated a solution for it. It thus highlights the operations at play in the "recovery" of the Canon. They are clearly in continuity with what was described of the work of the Han editors.

First, Liu Hui goes back to ancient writings, looking for textual evidence allowing retrieving the missing piece of the Canon. He writes:

> "Within the nine parts of mathematics, I investigated the one named "double difference". I examined its essential points so as to make them extend to/be efficient for this (problem).
> 徽尋九數有重差之名，原其指趣乃所以施于此也。
> (…Here follows the statement of the procedures defining this category of problems and able to provide a solution for the one raised)

Once a possible candidate —that is, a procedure that was not included in the received version of the Canon and was likely to cover the gap discovered— has been found in the available evidence, mathematical analysis is put into play. The point is not here whether the

[8] Let us stress the fact that what is "missing" is not a problem, but a "category (*lei*)" of problem. For an interpretation of this term, see the relevant entry in my glossary included in [Chemla & Guo 2004].

procedure named "double difference", which was dug out from ancient texts, can be applied directly to the situations not covered by *The Nine Chapters*. Rather, analysis inquires into its "essential points (*zhiqu*)". This concept needs to be elucidated. On the basis of an analysis of various occurrences in the commentaries, I suggest that it designates general operations brought to light by the commentator as underlying the procedure, when exploring the reasoning that allowed establishing it —in Liu Hui's words, when looking for its meaning (*yi* 意).[9] The procedure indicates in this way general operations that may prove to extend their efficiency to solving the missing category of problems. This description by Liu Hui of his mode of reading ancient procedures and of his interpretation of how they cover a range of problems is, I believe, a unique piece of evidence of the kind.

After the textual research and the mathematical work were carried out, the commentator finally expounds how, like his predecessors, he concludes by complementing the received version of *The Nine Chapters*:

> I elaborated the "double difference" and wrote a commentary on it so as to explore in depth the meaning (that was given to it by) the Ancients. I joined it after (the chapter) "Base and height " (*gougu,* i.e.: the last chapter)."
> 造重差，并為注解以究古人之意，綴於句股之下。

His description bears witness to the fact that the editorial work goes hand in hand with the production of the commentary, whereby the "meaning of the Ancients" is brought to light and, by the same stroke, the range of problems covered by the procedures determined. We shall come back to this issue below. We see how his description of his own activity nicely fits with the operations he attributed to the Han editors.

In conclusion, let us sum up what we so far found. Liu Hui associates two key values to the Canon bequeathed from Antiquity: it was made simple and yet all-encompassing. These are the two properties that were to be damaged by transmission and the accidents of history. These are also tests against which to assess the text recovered by the editors to serve as Canon for the present. At last, these are the guiding principles ruling the exercice of editing.

The episode described in Liu Hui's preface reveals two facets of the shaping of Antiquity at a given time. On the one hand, it shows the social and intellectual factors at play for reshaping the past. On the other hand, it highlights how values attached to Antiquity — that is, modes of representation of the past—, which are put into play through mathematical work, enter in the actual social construction of Antiquity.

The phenomenon is, I believe, quite general. Such a description also accounts for how Greek mathematical texts of Antiquity were shaped in the second half of the 19th century.[10] The editorial work was guided by values philologists attached to ancient Greece. As a consequence, they produced editions whereby these values were projected in the ancient texts. However, the values orienting them sharply differed from those disclosed by Liu Hui's preface.

Even though between the 7th and the 13th centuries, in China, editors and commentators manifested a range of slightly different attitudes towards the received text, they still envisioned the access to Antiquity as a story of damaging and restoring. Moreover, they bear witness to the fact that the two values of "being made simple", and, though, "all-encompassing" associated with *The Nine Chapters* remained key features in the approach to Canons from Antiquity. To take only one example, in 1261, Yang Hui composed an edition and a commentary of the Canon and successive layers of earlier commentaries on it. In his

[9] On all these terms, see my glossary, *op. cit.*.
[10] See [Chemla 1999].

preface, he stressed the property of *The Nine Chapters* to be all encompassing.[11] However, the text as he has it appears to him to have been damaged by unnecessary accretions. This reflects his expectation that the text should conform to the ideal of simplicity that was discussed above. To express this doubt in the edition, Yang Hui designates a part in it that, in his view, relates to the original meaning, and opposes it to a part he feels is superfluous, without, however, deleting it. We hence see how, in this case, the edition of the Canon of Antiquity as text takes on new forms, whereby the editor takes less liberty with the received documents.

Our analysis of editing as a mode of shaping Antiquity in the present has brought to light how mathematical work, in combination with values attached to Antiquity, was involved in the operations carried out for restoring *The Nine Chapters*. This, in turn, raises the more general issue of the scholarly and institutional practices that developed in relation to this presence of Antiquity in the form of a canonical text. Tackling this issue requires that we analyze further *which mode of presence* Antiquity had in the present, a question to which we now turn.

Approaching a text from Antiquity—looking for the "meaning/intention (*yi*)"

For each time period for which there is evidence that emphasis was being placed on Antiquity, as regards mathematics, Antiquity was present in the shape of a book. On the one hand, this book embodied the nine branches of mathematics. On the other hand, it was endowed with specific properties. To approach the issue just raised, we shall hence analyze uses to which practitioners put that book and mathematical practices that took shape with respect to it. This will be our method for dealing with how Antiquity was granted a certain form of presence within mathematical activity.

My remarks on these questions will be based on the evidence provided by the commentaries on *The Nine Chapters* composed, successively, by Liu Hui, Li Chunfeng and Yang Hui. In other words, I suggest that we concentrate on mathematical texts that were produced precisely with respect to the Canon. The modes of reading *The Nine Chapters*, as evidenced in the commentaries, betray the attitudes towards, and the uses of, Antiquity.

The key fact, in my view, is that commenting on the Canon was in itself one form of mathematical activity. This testifies to the fact that editing *The Nine Chapters* was not a mere statement of prestige for the discipline, but was meaningful as such for its practitioners. In other terms, the artifacts that ensured the presence of Antiquity were in that way significant objects for the present.

The questions considered can thus be translated into more specific ones: which kind of mathematical practice was required for commenting on *The Nine Chapters*? What was the purpose of exegesis? Again, Liu Hui's preface gives us clues that prove relevant. As we saw above and as it is repeatedly restated in the preface, the commentary aimed at "explor[ing] the meaning (*yi* 意) of the Ancients". Now, how are we to interpret this quest concretely?[12] It turns out, quite interestingly, that observing how Liu Hui actually comments on *The Nine Chapters* provides evidence to account for what that meant for him. Let us examine a passage of his commentary that will make this point clearer.

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[11] On these points, see [Chemla 2003 & forthcoming].

[12] I treated this question in Chapter A of [Chemla & Guo 2004]. The reader who would be interested in a more detailed interpretation of *yi* is referred to my glossary where the available evidence is gathered (*Ibid.*, pp. 1018-1022).

The Canon on which he comments is composed of problems and algorithms solving them. In this respect, the piece on which Liu Hui comments in the passage we are interested in is more or less representative of the whole book. It reads as follows:[13]

> "SUPPOSE AGAIN ONE HAS A NUMBER—PRODUCT OF 1644866437500 *CHI*. ONE ASKS HOW MUCH THE DIAMETER OF THE SPHERE MAKES.
> ANSWER: 14300 *CHI*.
> PROCEDURE FOR EXTRACTING THE SPHERICAL ROOT:
> ONE PUTS THE QUANTITY OF *CHI* OF THE NUMBER—PRODUCT, MULTIPLIES IT BY 16, AND DIVIDES BY 9. TO DIVIDE WHAT IS OBTAINED BY EXTRACTION OF THE CUBE ROOT GIVES THE DIAMETER OF THE SPHERE."

> 又有積一萬六千四百四十八億六千六百四十三萬七千五百尺，問爲立圓徑幾何。
> 荅曰：一萬四千三百尺。
> 開立圓術曰：置積尺數，以十六乘之，九而一。所得，開立方除之，即立圓徑。

In this problem, the Canon provides an amount that corresponds to the volume of a sphere, and the question is to determine the diameter of the corresponding sphere. The commentator brings to light that the procedure given amounts to considering that the sphere fills up $\frac{9}{16}$ of the circumscribed cube. If this is established, multiplying the volume of the sphere by 16 and dividing this by 9 yields the volume of the cube, the side of which is equal to the diameter of the sphere. This is why extracting the cube root of the value obtained yields the diameter of the sphere. Before he develops this last argument, Liu Hui explains how "those who made the procedure" may have come to the conclusion that the sphere fills up $\frac{9}{16}$ of the circumscribed cube. That is to say, he restores the reasoning that may have been followed. He writes:

> "Those who made this procedure probably relied on the *lü*'s[14] of 1 for the diameter and 3 for the circumference (of the circle). If one hence supposes that the surface

[13] This is problem 24 of chapter 4. For the critical edition, see [Chemla & Guo 2004], pp. 378-385. We oppose the text of the Canon to that of the commentary, by writing the former with capital letters and the latter with small letters.
[14] Here, this term designates numbers expressing the ratio between the geometrical entities. They can be multiplied or divided by the same factor, without losing their quality of expressing the ratio. For more detail on this concept, the reader is referred to the entry in my glossary, *op. cit.*

of the circle fills 3/4 of the surface of the square,[15] the circular cylinder thus also fills the 3/4 of the cube.

If, furthermore, one supposes that, the cylinder being represented by the *lü* of the square, 12, what represents the *lü* of the sphere is 9,[16] then, in addition to this, the sphere fills 3/4 of the circular cylinder.

(…computation on fractions …)

Therefore the sphere fills 9/16 of the cube. This is why, when one multiplies its volume by 16 and divides by 9, one obtains the volume of the cube.

The diameter of the sphere and the side of the cube are equal, hence, if one divides this by extraction of the cube root, one obtains the diameter."

爲術者蓋依周三徑一之率。令圓冪居方冪四分之三，圓囷居立方亦四分之三。更令圓囷爲方率十二，爲丸率九，丸居圓囷又四分之三也。(...) 故丸居立方十六分之九也。故以十六乘積，九而一，得立方之積。丸徑與立方等，故開立方而除，得徑也。"

The key point for us here is that he concludes this whole development by asserting: "But this meaning/reasoning (*yi*) is wrong. 然此意非也", thereby using the term *yi* in which we are interested in a most significant way. Let us analyze further what we can learn from the occurrence of the term here.

First of all, it is quite interesting that Liu Hui designates by this term the reasoning that he made explicit and that accounts for why the procedure could be thought of as yielding the correct result. More generally, in his commentary, he uses *yi* to designate the "meaning" of an operation or a sequence of operations, that is their "intention" or what they aim at computing within a given context. And, bringing together the "meanings" of all key steps of a procedure —what could be interpreted as its "essential points (*zhiqu*)"— leads to developing the proof of why it is correct and thus establishing the meaning of its result. So the *yi* as inquired into by the commentator relates to the proof of the correctness of the procedure. This fits perfectly well with the fact that this is indeed what we find in each of Liu Hui's or Li Chunfeng's commentaries that are placed after virtually each such piece of the Canon.

Secondly, what is even more interesting is that here, Liu Hui's formulation reveals that he attributes this *yi*, which he is making explicit, to the authors of the procedure. His commentary would thus bring to light their "meaning", which again fits quite well with the description of his commentarial activity in his preface as "exploring the meaning of the Ancients". In other words, his interpretation of the Canon would amount to reading in *The Nine Chapters* the reasons underlying the correctness of its procedures.

Thirdly, here, the commentator asserts that this reasoning he makes explicit is wrong, a statement that he then sets out to establishing. However, if he can prove that the previous

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[15] After the problem dealing with the area of the circle, Liu Hui highlighted how these data of 1 and 3 for expressing the diameter and the circumference of the circle entail having the area of the circle fill up ¾ of the area of the circumscribed square. In his words, the *lü* of the area of the circle is 3 when that of the area of the circumscribed square is 4. From this, he derives that, within the context of the same values, the cylinder occupies ¾ of the circumscribed cube.

[16] Liu Hui just recalled the values of 3 and 4 as *lü*'s for the volumes of, respectively, the cylinder and the circumscribed cube. Now, if the *lü* attached to the cube is 16, in relation to that of 9 for the *lü* of the sphere, the *lü* correlatively attached to the inscribed cylinder must be 12. Hence Liu Hui derives from these values the corresponding *lü*'s of 9 and 12 for the volumes of the sphere and the circumscribed cylinder. Dividing both numbers by 3 yields the ratio of 3 to 4 for that of the volume of the sphere to that of the circumscribed cylinder.

reasoning is incorrect and if he stresses that the procedure still yields good approximations, he cannot replace the proof with any other correct one. Nor can he provide any other procedure for solving the same problem. The question is thus open to determine whether the criticism addresses the way in which the Canon was restored, or the *yi* that he thinks to read in *The Nine Chapters*, or even the Canon itself. Whatever the answer would be, the essential fact remains here that the *yi* of the Canon, from the understanding of which Liu Hui states that his commentary derives, takes on here the form of a reasoning accounting for the correctness of the procedure.

So far, we delineated what the practice of searching for the *yi* of *The Nine Chapters* could be. We see, with the example examined, how it could lead to considering open problems. More generally, like editing the text of the Canon, or assessing previous attempts of recovering it, interpreting it clearly also requires carrying out mathematical activity. This naturally raises the question of the purpose that the commentators assigned to this search. I argued elsewhere[17] that they carried out the proofs of the correctness of the procedures in such a way as to, by comparison, bring to light the "fundamental procedures" to which the variety of all procedures could be reduced. This is how, in my view, the commentators understand that the procedures of the Canon, when their *yi* is adequately explored, point out the most general procedures that are underlying any of them, whether it be in the Canon or not. And this provides, I think, at least partly, an interpretation of the belief, shared by the commentators, that *The Nine Chapters* was complete.[18] The Canon would hence be a means for accessing to fundamental operations, which commentators would exploit to this end and *via* exploring its "meaning". Such a view is expressed quite explicitly by the Song editor of *The Nine Chapters* Bao Huanzhi, which shows that the practice of reading the Canon and its motivation remained stable for quite a long time. In his preface to *The Nine Chapters,* he writes:

> "Among the books of mathematical procedures, there are altogether ten schools.
> One can only take *The Nine Chapters* as the first of the Canons.
> With the methods of its nine parts of mathematics (*jiu shu*), there is nothing that is not complete.
> Although the procedures established by the various schools present variation, when one looks for the original meaning (*yi*), they all come from it.[19] Thus, in addition, one understands why there is nobody, among the successors, who took (their writings) to replace the old scripture of the Zhou and the Han.[20]
> 算數之書凡數十家，獨以"九章"為經之首，以其九數之法無所不備。諸家立術雖有變通，推其本意，皆自此出，而且知後人無以易周、漢之舊也。"[21]
> (My emphasis)

We saw in the first part of this paper that texts of Antiquity were endowed with specific properties: that of "having been made simple" and yet "all-encompassing". Here, we sketched the specific approach to them, in relation to a specific quest that the commentators pursue. We now see how both dimensions relate to each other. This constitutes a form of

[17] See [Chemla 2003 and forthcoming].
[18] To account for this belief, it is also important to see how the commentators interpreted a problem as standing for a category. This point is developed at length in [Chemla 2003a].
[19] We have here again the theme that the "essential points" are contained in the Canon.
[20] Note that this sentence seems to reflect that the time period related to Antiquity may have changed in Song times.
[21] [Bao Huanzhi 1213], p. 491.

continuity in the exegesis. However, within this common framework, the commentaries also display discontinuities. In fact, if they carry on the same kind of quest and read the scripture in the same way, the set of operations that they deem fundamental on the basis of their reading of *The Nine Chapters* differs.[22]

Conclusion

Throughout this paper, I hope to have shown convincingly some strong continuity, beyond changes, in the ways in which artifacts deemed to originate in Antiquity were approached, at the time periods during which Antiquity was a major focus of interest between the Han and the Song dynasty. First of all, we saw a stability in the way in which Antiquity has been made present, in the form of "Canons", and in the representation shared by the actors regarding how these scriptures became available for the present time. Moreover, the values attached to *The Nine Chapters* as a Canon and the correlative approaches to its text —looking for the meaning of the Ancients, assuming the completeness of the Canon— also displayed forms of continuity. It would certainly be worth pushing these issues further and examining in detail the approaches to, and uses of, these texts after China came in contact with Europe through the mediation of the Jesuits.

What about today? How are mathematical canons from Antiquity approached? If I may venture some hypotheses about the present time, I would suggest that present-day Antiquities seem to be quite different from the ancient ones, if we look at them from the point of view of mathematics. On the one hand, they became "globalized", "international", and did so in different ways in China and in the West. On the other hand, long after the contact of China with the West that begun at the end of the 16th century, a shift can be noted in the conception of, and approach to, *The Nine Chapters*, within the framework of what could be called the "Battles of Antiquities". It would be worth documenting this process of change in detail. For the time being, I shall only stress how, today, these "Antiquities" play a part in shaping representations of "peoples" or "societies", which are often used to grade them with respect to one another.

To sketch it coarsely, in the West, as regards mathematics, some social groups shaped —or endorsed— the idea that Antiquity was at large dominated by ancient Greece, geometry and the emergence of the axiomatic-deductive practice of mathematics. Such representations of worldwide Antiquity are probably correlated with what appears to me as a turn in the shaping of Chinese Antiquity. If we look at its contemporary forms —leaving a further analysis for a future publication—, the discourse about Chinese Antiquity has now split into two regimes, according to the main historiographic focus selected.

In one of these regimes, the event of "the emergence of mathematical proof" has become the most significant factor. That provoked a reshaping of Antiquity based on this new central element. As a consequence, the texts that mediate the perception of Antiquity changed: after roughly two thousand years of valuing, in terms of Antiquity, mainly the Canon, the 1970s experienced a major shift, with a partial transfer of the interest on the commentaries.

However, a second regime of discourse about Antiquity has recently appeared. In it, *The Nine Chapters* defines an "Oriental" mode of practicing mathematics, which constitutes an alternative with respect to the "Western" mode, represented by Euclid's *Elements of geometry*. In addition, the "Oriental" mode, shaped in Antiquity and granted a form of essence, is also perceived as that towards which world mathematics is now shifting. Indeed, in

[22] On this point —the varying set of fundamental operations according to the commentators—, see [Chemla forthcoming].

the way in which it is considered as having defined contemporary options, Antiquity remains a very contemporary concern.

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