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Modeling Commercial Processes and Customer Behaviors to Estimate the Diffusion Rate of New Products *

Alain BLOCH Daniel KROB Ada Suk-Fung NG

Chaire "Action commerciale et distribution", Conservatoire National des Arts et Métiers
40, rue des Jeûneurs, 75002 Paris, France

Laboratoire d'Informatique de l'Ecole Polytechnique, École Polytechnique and CNRS
91128 Palaiseau Cedex, France

Departement of Transportation & Logistics, Malaysia University of Science and Technology
Kelana Square, Kelana Jaya, 47301 Petaling Jaya, Selangor, Malaysia

bloch@cnam.fr *dk@lix.polytechnique.fr* *adang@must.edu.my*

Abstract

This paper presents a generic mathematical model for depicting the diffusion of an innovative product on a given market. Our approach relies on a probabilistic modeling of each customer behavior with respect to the commercial process which is used to promote such a product. We introduce in particular the concept of coherent market that corresponds to a market which can be analyzed in a uniform way within our model. This last notion allows us to recover the classical empirical results that were discovered and widely studied by E.M. Rogers and his school (Rogers 1995). We explain finally how to use our approach as a support for analytic predictive marketing.

Keywords: Analytic marketing, coherent market, commercial process, customer behavior, diffusion of innovations, Markovian model, probabilistic modeling, waiting time.

1 Introduction

In his famous seminal book (Rogers 1962), Rogers introduced and popularized a generic framework for analyzing and describing how the mechanism of *diffusion of innovations* works inside a given population. In this general approach, "innovation" can of course be understood in a large way: it may indeed refer to various contexts such as the learning of a new practice, the use of a new tool, the launch of a new product, etc. The strength of Rogers' point of view comes from the fact that it can be applied to all these different types of situations, which also reflects in the huge amount of relied academic and professional literature (we refer to (Rogers 1995) for an up-to-date and comprehensive bibliography on this subject – see also (Gatignon and Robertson 1985) and (Cestre 1996) for interesting complements).

In the marketing sphere, Rogers' approach was especially applied in *strategic marketing* since it provides a rather good qualitative understanding of both the structure and the rate of the diffusion of a

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new product on a market, helping therefore managers or analysts – who are facing problems of innovation management – to take strategic business decisions (we refer to (Foster 1986) or (Moore 1999) for different applications of Rogers' theory to such contexts). However, it is still difficult to use Rogers' paradigm further in the marketing process (considered in its whole) – in particular if one wants to do *analytic or predictive marketing* – due to the fact that his approach is fundamentally empirical and non quantitative. The key problem is here that there still does not exist (up to our knowledge) any precise mathematical description of the nature of the cornerstone of Rogers' model, i.e. the characteristic "Gaussian type" curve that describes the *rate of diffusion of an innovation* inside a given population, that is to say the function $N(t)$ that measures the number of *new* people of the considered population that is accepting such an innovation at time t (see Figure 1) !

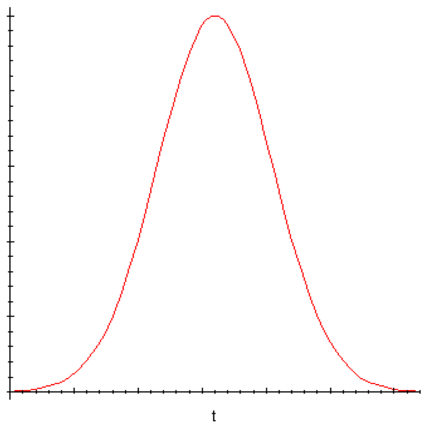

Figure 1: *The characteristic Gaussian type diffusion rate in Rogers' model.*

In this paper, we try to cross the gap that exist between the remarkable qualitative results provided by Rogers' work and a more mathematical point of view. We indeed propose a simple probabilistic model – which is intended to capture globally both the diffusion rate of the knowledge of a given innovation among a given population, the structure of the process which is designed to bring this innovation to the considered population and finally the individual behavior (within this last process) of the members of such a population with respect to the acceptance of the innovation – from which we can derive Rogers' qualitative results (i.e the Gaussian type curves that measure the rates of diffusion of an innovation in a population, as predicted by Rogers' works). In other words, we provide a mathematical framework (with a large number of parameters and dimensions for the purpose of fine operational modeling) which models the mechanism of diffusion of innovations in a given population in a way which is compatible with Rogers' predictions ! Observe in particular than the usual probabilistic interpretation of Rogers' model – which claims that the duration times for the acceptation of a given innovation in an arbitrary population are distributed according to a Gaussian normal law – can now be seen as a special (and quite trivial) case of our own parametric model (see Section 3 for more details).

Even if our modeling approach can capture rather general types of diffusion of innovation phenomena, we will however only concentrate in this paper – in order to be more specific – on the special case of diffusion of innovations in marketing contexts, which corresponds to the particular situation where one wants to understand the penetration rate of a new product on a market (see for instance (Goldsmith and Hofacker 1991) and (Béji-Bécheur and Pras 2000) for different classical approaches to this last problem). This special context (which is studied in Section 2) is indeed both especially interesting due to its business interest and its potential applications, and enough generic for illustrating the different dimensions of our model (the reader will easily apply our modeling framework to other types of situations). As a matter of fact, our model allowed us to recover (in this special context) the usual results predicted by Rogers' theory, but also to find several new sorts of marketing patterns that were not captured in Rogers' initial model (see Section 3 for more details). Moreover it can also be used – in a numerically predictive way – as a mathematical framework for an analytic marketing process (we give a few insights on this last and difficult topic in Section 4).

Note finally that our work takes naturally place in one of the main research stream which is related to diffusion of innovations [1], that is to say to the *study of adoption of innovations in a social system*, even if our approach relies on a very different trend than Rogers's one (who followed a socio-cognitive learning intuition [2] when we followed a commercial/selling intuition to model the laws of diffusion of innovations). Our work can in particular be seen as a "discrete" revisitation of the early work of Bass (Bass 1969) who was the first to propose a diffusion model where one clearly distinguishes different modes of communication (interpersonal mode and mass media in his case) for bringing the knowledge of a given innovation to the end user [3]. Our approach can eventually also be connected to the study of diffusion networks where several interesting mathematical models have already been developed in the literature (see for instance (Granovetter and Soong 1983) or (Valente 1991)).

Modeling being always a difficult exercise, we would like to end this introduction by stressing the fact that our approach does not give any closed description of the mechanisms of diffusion of innovations, but provides rather a tool box that could be used to model lots of different contexts. One should also always keep in mind that we just intended to present in this paper a mathematical model that "save the phenomena" of diffusion of innovations. Our model should in particular clearly never be considered as a speculation about the layer of matter underlying these last phenomena ...

2 Modeling commercial processes and customer behaviors

2.1 The key elements of our model

As explained in the introduction, we will focus through all this paper on the question of modeling - in a mathematical way - the diffusion mechanism of a new product P on a given market M (which is here defined as a given set M of potential clients for P). More precisely, we want to be able to compute, estimate and/or simulate – under certain hypotheses that are provided below – the function $N_M(t)$ that gives the expected number of new clients (within M) of the product P at time t.

Let us therefore first make totally explicit the key elements of the mathematical model with which we will work. As already mentioned, we will consider from now on that we are dealing with a given *market* M, which is here just defined as a fixed set of individuals. We also suppose that there exists a mechanism – which is called a *commercial process* (see sub-section 2.2 for more details) – that allows first to bring the existence of a given product P to the knowledge of *each* people m belonging to M during

[1] We recall that the research works on diffusion of innovations can be structured in eight main research streams according to Rogers (see (Rogers 1995) chapter 2 pp. 88-95): earliness of knowing about innovation, rate of adoption of different innovations in a social system, innovativeness, opinion leadership, diffusion networks, rate of adoption (of an innovation) in different social systems, communication channel use, consequences of innovation.

[2] Rogers defines the diffusion of innovation mechanism (see (Rogers 1995) chapter 5 pp. 162-191) as a learning process that consists in five main steps: knowledge (initial exposition and understanding of the innovation), persuasion (forming a favourable or unfavourable attitude towards the innovation), decision (deciding whether to accept or to reject the innovation), implementation (putting the innovation into use), confirmation (i.e. reinforcement of an innovation-acquisition decision already made, but that may be reversed in case of exposition to conflicting messages about the innovation).

[3] Our approach is however very different from Bass' one since it is only based on discrete and probabilistic modeling when Bass makes use of differential equations for describing the laws of diffusion of innovations.

a certain period of time and that is also designed to always lead each member of M either to buy the product P (after some delay) or definitively to never buy P. Our last hypothesis is that all the members of the market M will have a given *customer behavior* with respect to the considered commercial process (see sub-section 2.3 for more details). In other words, the individual buying behavior is modeled here as a reaction to a given commercial process.

Note finally that our approach is based on giving – as this will be done in the two next sub-sections – precise probabilistic interpretations to the two main notions that we just informally introduced, that is to say commercial processes and customer behaviors. We indeed believe that diffusion of innovations can only be mathematically understood from a probabilistic point of view. This means in particular that we will not try to look for a deterministic formulation and/or expression for $N_M(t)$. This last function will indeed be considered in a probabilistic way and will be defined, up to now, as the following sum of individual probability densities over all members of the market M:

$$N_M(t) = \sum_{m \in M} d_{T_P(m)}(t) \ , \tag{1}$$

where $d_{T_P(m)}$ denotes the probability density of the real random variable $T_P(m)$ which represents the moment when a given member m of M eventually decides to buy the product P (the value $d_{T_P(m)}(t)$ is indeed the classical continuous representation of the "probability" that $T_P(m)$ equals t). Note that Equation (1) shows in particular that one should be able to compute for any given member m of M the probability density associated with $T_P(m)$ in order to approximate or simulate the function $N_M(t)$.

2.2 Modeling of a commercial process

Let us now fix the mathematical definition of the notion of commercial process that we will use in the sequel of this paper.

Definition 2.1 *A* commercial process *C is defined as a 6-uple $C = (D, G, I, B, N, \chi)$ where :*

- *$D = (D_i)_{i=1...n}$ is a finite family of independent waiting time processes, which are called the* entry channels *of C, i.e. a family of n real random variables D_i, each of them evolving in an interval of the form $[t_i, +\infty[\subset \mathbb{R}$ (where $t_i \geq 0$ denotes the* beginning time *of the i-th entry channel) and following an exponential law of parameter $\alpha_i > 0$ defined by*

$$\forall\ t \geq t_i,\ \mathbb{P}(D_i \geq t) = \exp(-\alpha_i\,(t - t_i))\,, \tag{2}$$

- *$G = (Q, T)$ is a directed acyclic graph, called the* core *of the commercial process, with Q as set of vertices and T as set of oriented edges (the elements of Q and T are respectively called the* states *and the* transitions *of the commercial process C),*
- *I, B and N are distinguished non-intersecting subsets of the set Q that are respectively called the* entry states*, the* buying states *and the* non-buying *states of C,*
- *χ is a function, called the* correspondence function*, that associates with each number $i \in [1, n]$ (indexing an entry channel) a unique entry state $\chi(i) \in I$ in the core of C.*

This last definition wants here to express that a commercial process is the superposition of a communication process and of a core commercial process as explained below.

1. The *communication process* (that by convention begins globally at the time $t = 0$) is first used to bring the knowledge of a given product P to all the people that are belonging to the market

M. It is based on the use of n distinct communication channels (modeled by D) that are typically dedicated to different segments of markets. In other words, if M_i denotes the subset of M that consists of all the persons that are (first) reached by the i-th channel, $(M_i)_{i=1...n}$ forms a partition of M. One can only begin to use the i-th communication channel at a specific beginning time t_i. Moreover the average time which is required to inform a given member m of M_i of the existence of the product P through the i-th channel, does only depend on this channel – and not on m – and is given by an exponential probabilistic law with parameter $\alpha_i > 0$ and beginning time $t = t_i$ (a classical way of modeling waiting times).

2. In a second time, the *core commercial process* tries to transform any potential customer – which is defined as a member of the market M who knows the existence of the product P – into a real effective consumer of P ! It is organized in such a way that every potential customer will pass by different states (the vertices Q of G) which correspond to different stages of this second process. A potential customer m arrives initially in the entry state $\chi(i) \in I$ if he was "captured" in the core commercial process by means of the i-th communication channel. Different commercial actions are then made in order to change the state of m up to transforming him (if possible) into an effective buyer of P, the allowed changes of states being modeled by the transitions T. The acyclicity hypothesis on G means that a potential customer can never come back to a previous stage of the core commercial process (which always "pushes" further or eliminates a potential client). The core commercial process finally ends when m enters either in a buying state (an element of B) or in a non buying state (an element of N): the first case corresponds to the situation where m became a real buyer of P, when the second one models that one knows that m will definitively never buy P.

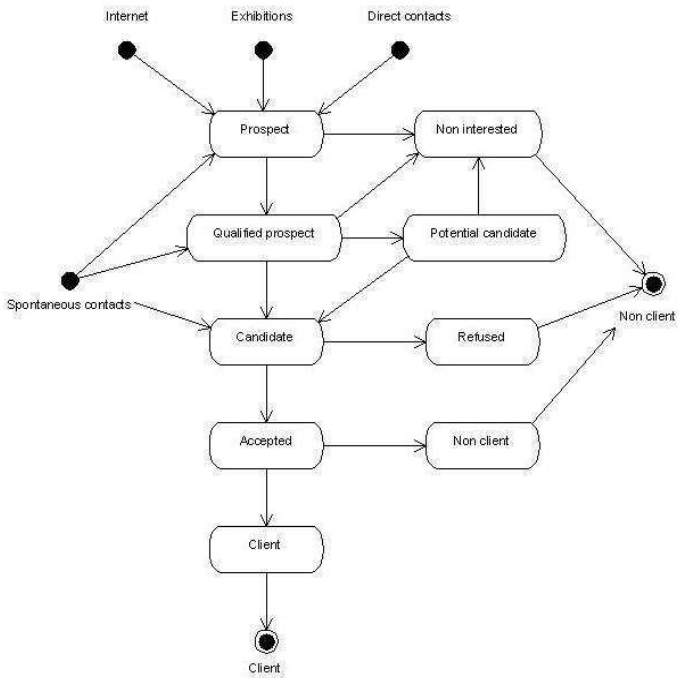

Figure 2: *Modeling of a (real) commercial process.*

To understand better our approach, let us now show how Definition 2.1 can be used for analyzing a real commercial process (which is graphically described by means of an UML activity diagram - see (Rumbach, Jacobson and Booch 1999) – in Figure 2) that deals with the selling of the MBA programme of a French engineering school (that allowed us gratefully to study its commercial mechanism). In this effective example, that we shall now present in full details, a new "product" is proposed once a year to the market. The associated *communication process*, that begins each February for ending in September, relies then on the following four types of independent communication channels (that are denoted below with the same names than in Figure 2 where these last channels are represented by black bullets):

- *Internet:* this channel refers to the Web site of the MBA programme (where information demands can be made),
- *Exhibitions:* this channel groups two entry channels (in the sense of Definition 2.1) which corresponds to the two presentations of the MBA programme that are organized each year in two major commercial exhibitions (they are denoted by Exhibition 1 and 2 in Table 1),
- *Direct contacts:* this channel models direct permanent commercial actions that take place on several university campuses in order to identify potential clients,
- *Spontaneous contacts:* this last channel corresponds to all spontaneous phoning, mailing or e-mailing actions that are made by potential clients; it groups in fact three different entry channels (in the meaning of Definition 2.1) depending whether a spontaneous contact will transform immediately into a prospect, into a qualified prospect or even into a candidate (see below for more details).

Observe that this analysis shows that these four communication channels decompose into seven entry channels in the sense of Definition 2.1 (see Table 1 for their exhaustive list). In order to be complete, we finally also provide in the same Table 1 the values (experimentally estimated) of the two parameters [4] that define precisely these different entry channels according to Definition 2.1.

Entry channel	Beginning time (t_i)	Value of α_i
Internet	0	1/70
Exhibition 1	15	1/10
Exhibition 2	45	1/10
Direct contacts	0	1/60
Spontaneous prospects	0	1/70
Spontaneous qualified prospects	0	1/70
Spontaneous candidates	0	1/70

Table 1: *Parameters of the entry channels of the commercial process described in Figure 2.*

Let us now pass to the description of the *core commercial process* of our example. This other process begins here when a potential future client – i.e. a student in our case – is identified by means of the previous different communication channels. Such a person is then considered as a commercial prospect, which is modeled by its entering into the *prospect* state (only a spontaneous contact can lead this person to another state if he wants spontaneously either to join an information meeting of the MBA programme or to become directly a candidate – see above): this means that one has enough information to begin on him a series of constructed commercial actions. The initial step of the core commercial process of our example consists then to enter into contact with all the commercial prospects in order to see whether they would like to come to an information meeting organized each week in our engineering school: if a prospect accepts this first proposal, he is then considered to arrive into the *qualified prospect* state (if not,

[4] All times are measured in days in this table.

he is pushed in the *non interested* state). After the information meeting, one proposes to each qualified prospect to apply to the MBA programme, which should lead to his venue to a series of application interviews. The fact that a qualified prospect accepts to come to these last interviews is then modeled by its entry in the *candidate* state (if he refuses, he is again put in the *non interested* state; if he wants to delay his decision, he is then first considered as a *potential candidate*, from which situation he can either become a real *candidate*, or definitively be *non interested*). A candidate can then either be *refused* or *accepted* to the MBA programme (this is modeled by his entry in two different states). If a candidate is accepted, he will however only became a *client* when he will have made a payment for reserving his place (if he never pays, he will be considered as a *non client* in our modeling).

We now totally presented both the correspondence function and the core graph of the commercial process of our example according to Definition 2.1. Note that all the states of the core graph that we just described, are represented by usual UML states in Figure 2. Observe also that the "client" state is the only buying state, when the "non interested", "refused" and "non client" states are the unique non buying states of this example in the meaning of Definition 2.1 (they are all connected to a single final state (denoted "non client") – in the UML meaning – in Figure 2).

Note finally that communication channels are modeled in Definition 2.1 with the most basic existing probabilistic model that takes into account a waiting time (which corresponds here to the time required to bring the knowledge of a product P to an arbitrary member of the market M). One can in particular obtain more accurate models of communication processes if waiting times are modeled differently (see for instance (Feller 1957), (Feller 1966) or (Brémaud 1988) for more subtle mathematical models).

2.3 Modeling of the customer behavior

We are now in position to propose a model for the behavior of the potential customers of the product P within the market M with respect to a given commercial process C (in the sense of Definition 2.1). Our approach is based on the interpretation of the directed graph $G = (Q, T)$ that models the core of C (see again Definition 2.1) as an homogeneous Markov chain (an idea that was already underlying in Miniard and Cohen's study of the early adoption mechanisms in innovation acquisitions – see (Miniard and Cohen 1983) and also (Fischer and Price 1992)) equipped with adapted probabilistic distributions for modeling the time required for passing a given transition of T.

Definition 2.2 *Let C be a commercial process (that works over a given market M), let $G = (Q, T)$ be the graph that models the core of C and let m be an arbitrary member of M. A* customer behavior *$\mathcal{B}(m, C)$ associated with m with respect to C is then defined as a couple $\mathcal{B}(m, C) = (p_m, \mathcal{N}_m)$ where:*

- *$p_m = (p_{m,t})_{t \in T}$ is a set of probabilities associated with all the transitions t of T in such a way that the following identity always holds*
$$\forall\ q \in Q,\ \sum_{t=(q,q')} p_{m,t} = 1\,, \tag{3}$$
- *$\mathcal{N}_m = (\mathcal{N}_{m,t})_{t \in T}$ is a set of independent normal distributions indexed by all the transitions t of T, each of them respectively being of mean $\mu_{m,t} > 0$ and of variance $\sigma_{m,t}$, i.e. defined by*
$$\forall\ a, b \in \mathbb{R},\ \mathbb{P}(a \leq \mathcal{N}_{m,t} \leq b) = \frac{1}{\sigma_{m,t}\,\sqrt{2\,\pi}} \int_a^b \exp(-\frac{(u - \mu_{m,t})^2}{2\,\sigma_{m,t}^2}\,du\,. \tag{4}$$

This definition intends just to express that a given potential customer m of our market has a probabilistic behavior within a given commercial process which is modeled by the fact that m will take the transition $t = (q, q')$ with probability $p_{m,t}$ when he lies in the state q of the core of such a commercial process and that m will need the time $\mathcal{N}_{m,t}$ for passing through this transition.

3 Diffusion of a new product on a market

3.1 Coherent markets

Definition 2.2 proposes a model for representing the behavior of a single member of a given market M with respect to a given commercial process C (in the sense of Definition 2.1). Hence we must consider all the different customer behaviors $\mathcal{B}(m, C)_{m \in M}$ for all members of M if we want to describe the global behavior of the market M in its whole. We are therefore naturally lead to introduce the notion of coherent market that corresponds to the situation where all probabilistic parameters of all considered customer behaviors $\mathcal{B}(m, C)$ only depend on the market M in its whole (and not on a single m).

Definition 3.1 *Let C be a commercial process that deals with a market M and let $\mathcal{B} = \mathcal{B}(m, C)_{m \in M}$ be the associated customer behaviors. The market M is then said to be a* coherent market *with respect to C if and only if the following conditions are satisfied:*

- **(C1)** *one has $p_m = p_{m'}$ for every m and m' in M,*
- **(C2)** *one has $\mathcal{N}_m = \mathcal{N}_{m'}$ for every m and m' in M,*

where we took here again the notations of Definition 2.2.

In other words, a given market M is coherent with respect to a given commercial process C when all its potential customers cross any transition $t = (q, q')$ belonging to the core of C with a probability p_t and in a time delay $\mathcal{N}_t$ that only depend on t.

Coherent markets are interesting since different parameters of commercial interest can be explicitly computed with them. One can for instance obtain a closed formula for estimating the number B_M of people that will eventually buy the product P for which was designed a commercial process C, if the underlying market is coherent with respect to C. To this purpose, let us consider the graph G that models the core of such a commercial process C on a coherent market M. We can construct a Markov chain – that only depends on M by definition of a coherent market – by equipping G with the transition probabilities given by Definition 2.2 (and that are involved in relations (3)), completed with probability 1 loops on each buying and non buying states of G. We shall then denote by $\mathcal{M}$ the associated stochastic matrix. Since G is here a directed acyclic graph, this last Markov chain is stationary, hence convergent. We can therefore consider the matrix $\mathcal{M}_\infty$ which is the limit of the sequence of all iterated powers of $\mathcal{M}$ (that can be obtained by simply solving a linear matrix system [5]). It is then straightforward to see that the following result holds (using here the notations we just introduced).

Proposition 3.2 *Let C be a commercial process for the product P on a coherent market M. For every $i \in I$, let m_i denote the number of persons that belong to the sub-market M_i of M captured by the i-th entry channel of C. The mathematical expectation B_M of the total number of buyers of the product P (considered as a real random variable) is then given by the following closed formula:*

$$B_M = \sum_{i \in I,\, f \in B} m_i \, (\mathcal{M}_\infty)_{i,f} \ . \tag{5}$$

Proof – Let q and q' be two arbitrary states of the core graph G of the commercial process C. Since the stochastic matrix $\mathcal{M}$ is associated with a stationary Markov chain [6], the classical theory of Markov chains (see (Feller 1957)) shows then that the (q, q')-th entry $(\mathcal{M}_\infty)_{q,q'}$ of $\mathcal{M}_\infty$ is exactly equal to the

[5] $\mathcal{M}_\infty$ is the unique stochastic matrix X that satisfies to the linear matrix equation $\mathcal{M}\, X = X$.

[6] Which means here that $\mathcal{M}_\infty = \mathcal{M}^N$ where N stands for the dimension of $\mathcal{M}$ (i.e. the number of states of G).

probability for an arbitrary member $m \in M$ (due to the fact that the market M is coherent with respect to C) to go from q to q' during the commercial process (i.e. from its beginning up to the moment when m has been definitively transformed either into a buyer or into a non buyer).

Let now $\mathcal{B}_M$ denote the real random variable that gives the total number of buyers of the product P when the commercial process C is fully accomplished. It is then easy to see that one has

$$\mathcal{B}_M = \sum_{m \in M} \mathcal{B}_m \tag{6}$$

where $\mathcal{B}_m$ stands for every member $m \in M$ for the real random variable which is equal to 1 or to 0 depending whether m becomes a buyer (or remains a non buyer) at the end of the commercial process C. Using the initially established result, we immediately deduce the desired Equation (5) as a direct consequence of this last relation and of the fact that $B_M = E(\mathcal{B}_M)$. ■

Note also that the usual interpretation of Roger's model can also be rather easily captured by our approach. One can indeed easily check that the function $N_M(t)$ that gives the expected number of new clients – within a coherent market M – of a new product P at time t, is given by a normal law if and only if the associated commercial process is described by the diagram of Figure 3. In other words, Roger's initial model is interpreted in this case in our own model as the diffusion of a new product on a coherent market through a commercial process which just has a unique entry channel without waiting time (i.e. the exponential law parameter takes the limit value 0) and whose core commercial process is trivial (it just separates in one single transition each potential client into a client or a non client).

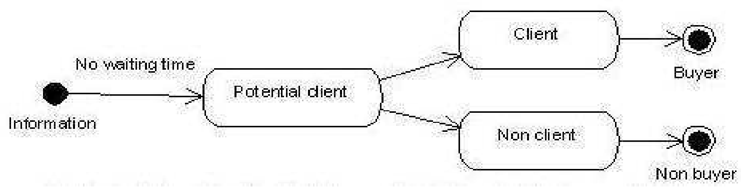

Figure 3: *Usual Rogers' model interpretation.*

3.2 Diffusion of a new product on a coherent market

One should first point out that our model is constructed in such a way that one can now very easily simulate the diffusion mechanism of a given new product on a coherent market. We can then recover in this general framework the behaviors that were predicted by Rogers in (Rogers 1995) (as described in Section 1), but also capture much more complex situations as we will see below when we will study the general behavior of our model. Before studying our model in full generality, let us however first consider – for the sake of simplicity – the example of the commercial process depicted by Figure 4 (which is moreover also one of the first examples with a bigger complexity than the interpretation – cf. the last subsection – of Roger's usual model !).

This commercial process has a unique entry channel – that is represented in Figure 4 by the black bullet (T) – modeled by an exponential law of parameter $\alpha > 0$. The corresponding core commercial process has four states – i.e. (P), (I), (N) and (C) – whose semantics can be directly read on this last figure. The other components of a commercial process can then be read on Figure 4. The market M on which this commercial process works is finally supposed to be coherent with respect to it. Let us finally also denote by a, b and c the means (which here only depend on the market) of the normal distributions that model – according to Definition 2.2 – the time delays that a potential customer requires respectively to go from (P) to (I), from (I) to (C) and from (P) to (C).

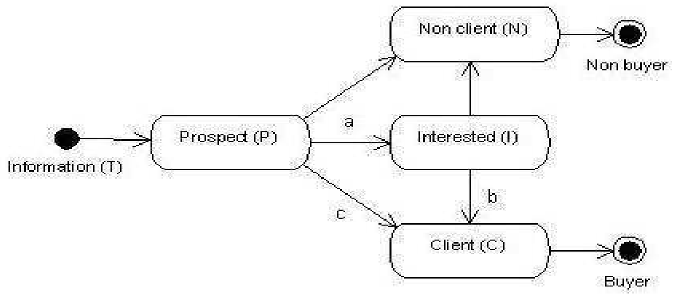

Figure 4: *Example of a theoretical commercial process.*

Under the previous hypotheses, it is possible to compute explicitly the probability density associated with the $\{0, 1\}$ random variable that measures whether a single member $m \in M$ will eventually become a buyer – i.e. enter in state (C) – (we will not give here the corresponding formula which is straightforward, but without any special interest in its own, and moreover contained in the general study we will do later). Using this last result, we were then easily able to study the exact buying behavior of M.

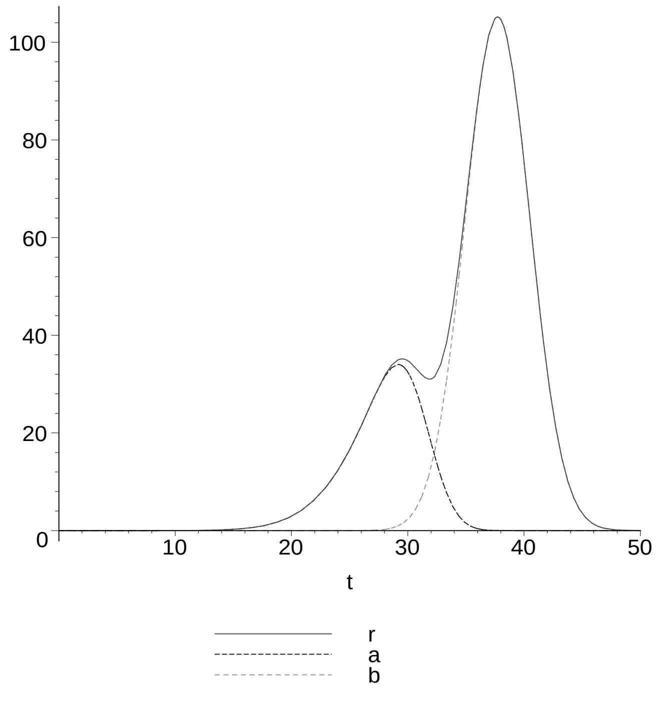

Figure 5: *Potential clients stay a long time in the interested state.*

Two key facts can now be pointed out. First of all, one can prove that Rogers' predicted result – i.e. the Gaussian shape of the function $N_M(t)$ that gives the expected number of new clients in our commercial process at the time t – occurs when one has either $a << c$ or $c << a$ (where $<<$ means here "much smaller"). In other words, one observes Gaussian shape curves for $N_M(t)$ as soon as one of the paths (in our commercial process) that may transform a given commercial prospect into an effective buyer has a mean time much bigger than all the other paths of the same nature.

On the other hand, new types of situations appear when a is comparable to c (which means that a prospect needs the same time on the average to be simply just interested or to become directly a buyer). In this case, one can prove that the function $N_M(t)$ has either a shape that consists of a small peak followed by a much bigger one as in Figure 5 (whose x and y axes represents respectively time and the number of buyers of P), that is obtained when $c > a$ and $c \approx a + b$, or conversely the "symmetric" shape, characterized by a high peak followed by a much smaller one, obtained when $c \approx a$ or $c > a + b$.

This last example illustrates in fact the general behavior of our model that we shall now present more precisely. To this purpose, let us first introduce a new definition. Let therefore C be an arbitrary commercial process (in the meaning of Definition 2.1) with G as core graph. A *buying path* of C is then by definition any path of G that goes from some input state of C to some buying state of C. Note that there exists always only a finite number of paths of G going from a state of C to another state of C due to the fact that G is supposed by definition to be a directed acyclic graph. This implies in particular that the number of buying paths is necessarily finite. We are now in position to state the following key result that gives the generic shape of the function $N_M(t)$ within our model (see Figure 7).

Proposition 3.3 *Let M be a coherent market with respect to a given commercial process C which has p buying paths. Then the function $N_M(t)$ – that gives the expected number of new clients in the commercial process C at each time t – is alternatively increasing and decreasing (the t-axis being two times its asymptote at $t = \pm\infty$) with a maximal number of p peaks* [7] .

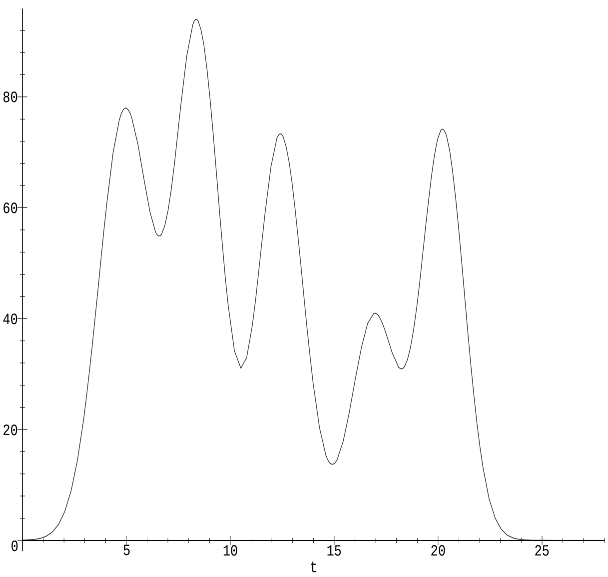

Figure 6: *Generic shape of the function $N_M(t)$ in our model.*

Proof – Let M be a coherent market (whose size will be denoted by n) with respect to a commercial process C. For every member m of M, let us then denote by T_m the real random variable which represents the moment when m will eventually become a buyer within the commercial process C. Note first that

[7] The term "peak" refers here to an arbitrary maximum of $N_M(t)$, i.e. any point that separates an increasing part from a decreasing part of this function.

the study of $N_M(t)$ on the coherent market M reduces to the study of the density of the random variable T_m for any single member m of M due to the obvious following relation

$$N_M(t) = n\, d_{T_m}(t) \ . \tag{7}$$

Let us therefore now fix a given member $m \in M$. Before going further, we shall however introduce first a number of new notations. We will denote by BP the (finite) set that consists of all the buying paths of the commercial process C (as defined above). For every entry state i of C, we will also denote by $\mathcal{E}_i$ the exponential distribution that models the communication time of the i-th entry channel in Definition 2.1. For every transition $t = (i, j)$ of the core graph G of C (where i and j stand for the states of C that form respectively the initial and final vertices of t), we will finally denote by $\mathcal{N}_{i,j}$ the normal distribution which models the crossing time of transition (i, j) for m according to Definition 2.2.

Observe at this point that m can only become a buyer in our model if his buying behavior corresponds to the following of one of the buying paths of C. This property implies then immediately that one has

$$d_{T_m} = \sum_{\pi=(i_1,\ldots,i_r)\in BP} p_\pi \, d_{\mathcal{E}_{i_1}+\mathcal{N}_{i_1,i_2}+\ldots+\mathcal{N}_{i_{r-1},i_r}} \tag{8}$$

where the sum is taken over all buying paths $\pi = (i_1, \ldots, i_r) \in BP$ (where the last list represents the sequence of states of C that forms the path π inside the core graph G of C) and where p_π denotes the discrete probability of taking this path within the Markov chain [8] naturally associated with G by means of Definition 2.2 (see Section 3.1).

Let us now recall (see for instance (Brémaud 1988)) that the sum of two independent normal distributions $\mathcal{N}_1$ and $\mathcal{N}_2$ (of respective means μ_1 and μ_2 and variances σ_1 and σ_2) is again a normal distribution (of mean $\mu_1 + \mu_2$ and of variance $\sqrt{\sigma_1^2 + \sigma_2^2}$). Due to this last result and to the fact that all the normal distributions given by Definition 2.2 are supposed to be independent, we can rewrite relation (8) as

$$d_{T_m} = \sum_{\pi=(i_1,\ldots,i_r)\in BP} p_\pi \, d_{\mathcal{E}_{i_1}+\mathcal{N}_\pi} \tag{9}$$

where $\mathcal{N}_\pi$ stands for the normal distribution that results from the addition of all the normal distributions $\mathcal{N}_{i_1,i_2}, \ldots, \mathcal{N}_{i_{r-1},i_r}$. We are therefore naturally lead to compute the density of the sum of an exponential distribution and of a normal distribution, which will be the purpose of the next lemma.

Lemma 3.4 *Let $\mathcal{E} = \mathcal{E}_{\alpha,t_0}$ be an exponential distribution of parameter α and beginning time t_0 [9] and let $\mathcal{N} = \mathcal{N}(\mu, \sigma)$ be a normal distribution of mean μ and variance σ. The sum $\mathcal{S}$ of these two probability distributions, i.e. $\mathcal{S} = \mathcal{E} + \mathcal{N}$, has then a density which is given by the following formula:*

$$\forall\ t \in \mathbb{R},\ d_{\mathcal{S}}(t) = \frac{\alpha\, \exp(\,\dfrac{\alpha^2\sigma^2}{2} + \mu\,\alpha)}{\sigma\,\sqrt{2\,\pi}}\ \exp(-\alpha\,(t-t_0)) \int_{-\infty}^{t-t_0} \exp(-\frac{(u-\mu-\alpha\,\sigma^2)^2}{2\,\sigma^2})\, du \ . \tag{10}$$

Proof of the lemma – The sum of two real random variables that have a density has always a density which is the convolution product of the two involved densities (see for instance (Brémaud 1988)). This classical result implies in our case that $\mathcal{S}$ has a density which is given by the following formula:

$$d_{\mathcal{S}}(t) = \int_{-\infty}^{+\infty} d_{\mathcal{E}}(t-u)\, d_{\mathcal{N}}(u)\, du = \int_{-\infty}^{t-t_0} \alpha\, \exp(-\alpha(t-u-t_0))\, \frac{1}{\sigma\,\sqrt{2\,\pi}} \exp(-\frac{(u-\mu)^2}{2\,\sigma^2})\, du$$

[8] If $\mathcal{M}$ stands for the stochastic matrix associated with this Markov chain and if $\mathcal{M}_\infty$ is the limit of its iterated powers (see Section 3.1), then one has $p_\pi = (\mathcal{M}_\infty)_{i_1,i_r}$ for every buying path $\pi = (i_1, \ldots, i_r)$ of BP.

[9] I.e. defined by the fact that its density $d_{\mathcal{E}}$ has support in $[t_0, +\infty[$ and is defined by setting $d_{\mathcal{E}}(t) = \alpha\, \exp(-\alpha\,(t-t_0))$ or equivalently $\mathbb{P}(\mathcal{E} \geq t) = \exp(-\alpha\,(t-t_0))$ for every $t \geq t_0$.

from which a straightforward computation leads immediately to the desired relation (10). ■

Our proposition follows now immediately both from relations (7) and (9) and from the fact that each density $d_{\mathcal{S}}(t)$ (as defined by relation (10)) is always an alternatively increasing and decreasing function (the t-axis being its asymptote at $t = \pm\infty$) with a single peak (even it has nothing to do with a Gaussian distribution [10]). We give in Figure 7 a generic example of the shape of such a density function. ■

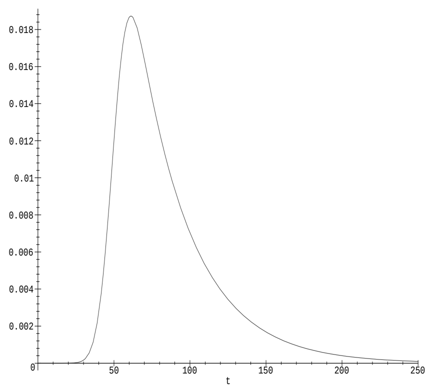

Figure 7: *Density of the sum of an exponential distribution and a normal distribution.*

3.3 Diffusion of a new product on a non coherent market

In practice, a market has of course no reason to be coherent in the meaning of Definition 3.1. This is however not really a limitation of our approach as soon as one is able to decompose a given market into several coherent independent markets with respect to a given commercial process (such a decomposition can clearly be seen as a natural marketing segmentation in our framework). We indeed easily get then a global model for such an incoherent market by superposing the different coherent models resulting of this process. Note that the curves for $N_M(t)$ that are generically obtained in this way will typically contain several consecutive peaks in the line of the final analysis of the last subsection (cf. Figure 7).

Observe finally that such curves can also be analyzed as the superposition of the buying behaviors of different types of people. To illustrate this mode of analysis, let us take again the example of Figure 5 that can indeed be interpreted as the superposition of the buying behaviors of two categories of people, that is to say early adopters who quickly became buyers (curve a) and more mature clients who became buyers only after a delay of thinking (curve b). The resulting gap in Figure 5 can then be interpreted as the fact that mature clients are much less reactive than early adopters (note that this type of gap was especially studied by Moore in (Moore 1999)).

4 Elements of conclusion

We propose in this paper an operational model for analyzing the mechanism of diffusion of a new product on a market. Our model was especially designed in order to take practically into account real marketing processes as shown in the example of Section 2. It can therefore also be naturally used as a support for analytical marketing. Adapted questionnaires may indeed be created in order to analyze a market as an union of coherent segments in our meaning. If these questionnaires can lead to the identification or the

[10] Its shape comes from the fact that this function results from the multiplication of a decreasing exponential with a sigmoid (a S shaped curve) as it immediately follows from relation (10).

approximation of all the parameters of our model, it is then easy to predict the level (see Equation (5)) and the rate of sales (see above) of a given new product on the studied market ! Hence we would get in this way a new method for doing *analytic marketing*.

Observe also that several new research trends can easily be imagined on the basis of our results. The most obvious new research direction is of course the *improvement of our modeling framework*, the main objective being here the obtention of more realistic models that would take diffusion of innovation more precisely into account. To achieve such an objective, one must probably try to apply our ideas in new effective application contexts (with respect to the ones where the model was already applied and from which it emerged): the generic patterns that will naturally arise from the study of new applications could clearly help a lot to improve our modeling approach.

In a totally different direction, an interesting, challenging and open research subject would also consist in introducing *pricing* within our model. Our approach does indeed not directly integrate neither the customer behavior with respect to the price of a given new product which is proposed to him (this last point can only be modeled indirectly by playing with the transition probabilities of our model), nor the way of establishing such a price in order to obtain the best possible customer reactions (within a given commercial process). It would therefore clearly be of main importance to be able to incorporate these different price-oriented aspects within our framework.

Note finally that our model does absolutely not pretend to be better than other models. In some sense, we even not really presented a model in the usual way, but rather a generic framework – based here on Markovian modeling – for dealing with diffusion of innovation, that should of course always be adapted in each practical situation. As stated in the introduction, one should always keep in mind that we just wanted to present a simple mathematical approach that "save the phenomena" (that is to say that took into account the realness of diffusion of innovation). We hope that we succeeded both in this direction and in showing that the mechanism of diffusion of innovation can be much more complicated than Rogers' initial approach could let it think (which was also one of our motivations).

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