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The optimal short-term management of flexible nuclear plants in a competitive electricity system as a case of competition with reservoir.

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Abstract

In many countries, the electricity systems are quitting the vertically integrated monopoly organization for an operation framed by competitive markets. It therefore questions how flexible nuclear plants capable of load-following should be operated in an open market framework. A number of technico-economical features of the operation of flexible nuclear plants drive our modelling complex which makes difficult to determine the optimal management of the nuclear production within our model. In order to examine the existence of an equilibrium and calculate it, we focus on a short-term (monthly) management horizon of the fuel of nuclear reactors. The marginal cost of nuclear production being (significantly) lower than that of thermal production induces a discontinuity of producer's short-term profit. The problem of discontinuity makes the resolution of the optimal short-term production problem extremely complicated and even leads to a lack of solutions. That is why it is necessary to study an approximate problem (continuous problem) that constitutes a "regularization" of our economical problem (discontinuous problem). Its resolution provides us with an equilibrium which proves the existence of an optimal production trajectory.

Key words: Electricity market, nuclear generation, competition with reservoir, optimal short-term production problem, price discontinuity, quadratic programming.

JEL code numbers: C61, C63, D24, D41, L11.

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1 Introduction

Nuclear generation differentiates itself from other technologies by its important fixed cost and low variable cost (Bertel and Naudet (2004), DGEMP & DIDEME (2003, 2008), MIT (2003, 2009), Cour des Comptes (2012)). This is the main reason why nuclear is deemed to serve the baseload demand. The nuclear plants being operated as baseload plants, produce energy at a constant rate using their full capacity in order to cover their fixed costs. From a market perspective, nuclear is "economically suitable" to operate at baseload because of its relatively low variable costs, which imply a low marginal cost. Nevertheless, an important participation of nuclear in the generation of a country can lead to a different operation. A typical example of this is France which is distinct from other countries (like UK or Sweden) because nuclear generation accounts for 80% of generation and 53% of installed generating capacity. The high share of nuclear in the national mix asks nuclear plants to be flexible[2] operating occasionally at semi-base load that corresponds to less than 5000 hours of operation per year and responds to a part of the variable demand (Regulatory Commission of Energy (2007), Pouret and Nuttall (2007), Bruynooghe et al. (2010)). That type of relative importance of the nuclear production vis-à-vis other generation technologies is an exception and the literature on this subject is extremely reduced.

However, in numerous countries, electricity systems are quitting the vertically integrated monopoly organization for an operation framed by competitive markets (e.g. European Union). This reopens -both empirically and theoretically- the question of nuclear operation. Economic reasoning supports that in a changing environment, the choice and operation of generation may also change. In the previously monopolistic and vertically integrated markets, the optimal management of this production technology was mainly a technical issue, as there was a guarantee of selling the whole nuclear production, adjustments of production levels could be done by other even more flexible technologies (e.g. hydro, gas). In a competitive setting, this question is not only of a technological nature (Chevalier (2004)). Given the differences in production costs of different technologies and the variations in market prices, the maximization of the economic value of nuclear production becomes a crucial issue for producers. Consequently, a question arises: what could be the optimal management of a flexible nuclear set in a competitive setting? Within this competitive framework, we address the medium-term horizon (1 to 3 years) of management to take into account the fluctuations of demand according to the seasons of a year. In the medium-term, the managers of a large nuclear set (like the French) have to set their seasonal variation of output in order to satisfy the seasonal demand. We emphasize two stylized seasons: a season of high demand and a season of low demand. In continental Europe, it corresponds respectively to winter and summer. In this medium-term horizon, a core feature of market based nuclear is that the nuclear fuel works as a "reservoir" of energy - partly similar to a water reservoir of hydro energy. This feature of nuclear is based on the discontinuous reloading of nuclear reactors. Nuclear units stop only periodically (from 12 to 18 months) to reload their fuel. Then a new period of production ("campaign" of production) starts. A producer allocates a limited and exhaustible amount of nuclear fuel between winter and summer to respond to demand's variations and to maximize its profit. Therefore, we will look at this question as a rational economic analysis of the operation of a nuclear fuel "reservoir". While such an analytical frame obviously makes sense from a theoretical point of view, it is both highly unconventional and entirely unexplored in the public literature. This is mainly

[2]The european pressurized reactor (EPR), which is an evolution of the pressurized water reactor (PWR), is an example of a III+ generation nuclear reactor which is designed to accommodate load-following operation (AREVA (2005), Goldberg and Rosner (2012)).

due to the existence of a number of difficulties that one faces on a theoretical plan in order to build a model of optimal management of flexible nuclear plants in a competitive market.

There exist difficulties that result from the relatively high technological complexity that characterizes nuclear as an electricity generation technology. This technological complexity can be seen through the potentially higher investment and exploitation costs than those identified for other energy sources using conventional techniques (e.g. fossil fuel generation technologies). Several technico-economical constraints have to be considered when a nuclear producer searches for an equilibrium of the problem of optimal allocation of the nuclear fuel stock during a campaign of production. These constraints have to do mostly with the flexible operation of nuclear plants (minimum/maximum production constraints) and the operation of the nuclear fuel reservoir which is based on the mode of fuel reloading of nuclear reactors (nuclear fuel constraints).

Another difficult aspect of our problem concerns the implication of competition in the optimal management of a flexible nuclear set. The question of optimal operation of such a nuclear set has not been raised so far for a competitive market given that France has not fully opened till now its electricity market to competition like other countries e.g. UK. Indeed, the French historical operator (EDF) holds the total capacity of the nuclear set which offers the majority of the total electricity generation, and thus has a dominant position in its historical national electricity market. Therefore, the creation of a benchmark of optimal operation in a competitive framework has not been done yet. We build a microeconomic deterministic dynamic model of optimal management of flexible nuclear plants by taking into account the feature of the reservoir in a market where producers disposing a certain amount of nuclear capacity compete with each other to maximize their profits and at the same time to even meet energy demand. This benchmark could give some insight with respect to the optimal production behaviour of nuclear producers in a market based electricity system.

The constraints imposed by the equality between supply and demand (called supply-demand equilibrium constraints) play a decisive role in the determination of the optimal management of flexible market based nuclear. In view of the large proportion of nuclear in the electricity system, the global balance between supply and demand depends mainly on the nuclear production. This makes the nuclear set responsible to a large extent for ensuring this balance and preventing potential disruptions of supply which could lead to a "blackout". As a consequence, each nuclear producer has to take account of constraints intrinsic to the public interest and social welfare such that of the equality between supply and demand. Consequently, in the medium-term horizon, the nuclear fuel reservoir has to be managed so that imbalances between production and consumption are avoided during a campaign of production.

We study the optimal management of the fuel reservoir of a flexible nuclear unit given the decentralization of the nuclear generation. We assume the existence of two flexible types of generation: nuclear and thermal (e.g. coal, gas, etc.). In view of the complexity of our model, the producers starting with the optimization of their nuclear production do not immediately know how to manage all the factors affecting the market equilibrium in the medium-term (all over a fuel campaign) within our model. For this reason, they reduce their management horizon to that portion of the market being easier to foresee: the monthly horizon[3]. Each generation units manager playing on a market base aims at the determination of a production profile that: (*i*) respects the constraints imposed by the flexible operation of a nuclear unit (e.g. load-following operation of an EPR) and the thermal generation capacity for each month (minimum/maximum production constraints), (*ii*) respects the constraints imposed by the inter-temporal management of the nuclear fuel stock over the entire time horizon of production

[3]This will be called short-term management of the production (during the medium-term horizon).

(nuclear fuel constraints), (*iii*) respects the constraints induced by the overall equality between supply and demand (i.e. total supply equals total demand) at every moment over each month, (*iv*) maximizes the value of profit for each month.

The interest of a short-term horizon of management of the nuclear fuel reservoir is to set up a model that will take into account several complex technico-economical characteristics of the operation of flexible nuclear units in a competitive framework and then to examine the existence of a production path that verifies the above conditions ((*i*), (*ii*), (*iii*), (*iv*)) and determine it within this model. This will serve as a reference for the existence of an equilibrium before the producers proceed with a more complex problem based on a multi-annual optimization implying an augmentation of their management horizon to one or more campaigns of production ("non-myopic" case). It will also permit to analyze the outcome of such approach consisting of a partially "myopic" (short-sighted) optimization of the nuclear production and identify its limits before the producers turn their interest into the "non-myopic" case.

In section 2, we build a general deterministic dynamic model to study the optimal short-term operation of flexible nuclear plants in a perfect competitive setting. In section 3, we study the optimal short-term production behaviour. We introduce the notion of the merit order equilibrium in the case of $N \geqslant 2$ producers. Then, we present different approaches to calculate a merit order equilibrium. Finally, in section 4, we collect some basic data to feed our model. In order to get a complete data set suitable for the numerical modelling, we proceed with an interpolation of the missing data. Then, we analyze the production and nuclear fuel decisions of our last approach within a simple numerical model. The section 6 concludes.

2 Model: Perfect competitive case

In this section, we describe our general deterministic dynamic model of a perfectly competitive electricity market where there exist two types of generation: nuclear and thermal. Nuclear is a complex generation technology given its technical and economical characteristics. Therefore, we deal with a demanding modelling with respect to nuclear technology in order to be realistic and at the same time we make assumptions which permit to our model to be manageable. In our model, the price in the electricity market is determined by the merit order price rule which implies perfect competition according to which firms treat price as a parameter and not as a choice variable. Price taking firms guarantee that when firms maximize their profits (by choosing the quantity they wish to produce and the technology of generation to produce it with) the market price will be equal to marginal cost. First, our modelling aims at determining the optimal short-term management of a flexible nuclear generation set in that competitive regime. We want to look out to the medium-term horizon which is characterized by the seasonal variation of demand between winter and summer. Second, there are constraints imposed by the flexible operation of nuclear units, production and nuclear fuel storage and the supply-demand equilibrium that play a central role in determining the equilibrium outcomes in this wholesale electricity market. In view of our framework of perfect competition, we focus on the wholesale spot market[4] assuming that there is no bilateral contracting regime between retailers/consumers and producers within our model. The wholesale spot prices are paid by the retailers/consumers directly to the producers.

For simplicity reasons and in the absence of access to detailed data the electricity importa-

[4]Statistics presented by the French energy regulator (CRE) showed that the volumes traded on the French wholesale market (intra-day and day-ahead) represented about 12% of the total volumes in 2011 and around 14% in 2012 (Sources: EPEX Spot, EPD, courtiers).

tions/exportations are not taken into account within our model. However, in the hypothetical case that electricity importations/exportations were part of our modelling, they could be considered either exogenous or endogenous to our model. If they were exogenous then the demand would be translated by the production that is imported/exported. This would modify the value of the demand but it would not affect our modelling. On the contrary, if they were endogenous, the complexity of the modelling would increase since several new parameters have to be taken into account in our model e.g. technical constraints imposed by the transmission power lines, the price elasticity of foreign demand, etc.

Our work centers only on the nuclear fuel storage and the optimal management of the nuclear fuel reservoir without considering the production coming from hydro units with possibility of storage (peaking[5] power plants) because of the additional capacity and storage constraints which would increase the complexity of the model. There exists an extensive literature that studies the optimal management of hydro-reservoirs in mixed hydro-non-nuclear thermal competitive markets and where one can see several modellings of the optimal production problem and notice the increased level of difficulty from a theoretical and numerical point of view (Ambec, Doucet (2003), Arellano (2004), Bushnell (2003)).

Similarly, the stochastic nature of the renewable electricity production, from sources such as wind power and solar power would complicate our model which is the main reason why we do not take them into consideration. The electricity production coming from renewable energy plants is variable or intermittent[6] because of the stochastic nature of weather patterns. This means that the renewable energy production should be a stochastic endogenous variable in our model. Therefore, its consideration would impose to realize a radically different modelling, a stochastic modelling, whose nature is not consistent with the deterministic character of our model. Note that mathematical proofs and numerical data can be found in the second chapter and the annexes of the Ph.D. thesis (Lykidi (2014)).

2.1 Modelling the demand

The demand, being exogenous, is considered perfectly inelastic. It is obviously a simplification. It can nevertheless be motivated by some arguments. In the short-term to medium-term the demand is less sensitive to price because it is already determined by previous investments in electrical devices and ways of life whose evolutions require time. We can also consider that, in this time scale, the consumers are not able to observe and respond in price evolutions in real time. Consequently, the sensitivity of demand to price is extremely low. In the case that we add a price elasticity of demand in our model, it would have an arbitrary value since there are no specific elements that permit to estimate its value. In the French case, most of the consumers and a significant part of the firms have a fixed price contract, being a regulated price contract set by the government and precisely by the French energy regulator (CRE) and the Ministry in charge of economy and energy. Regulated tariffs do not give information regarding the evolution of the spot price and therefore, the evolution of the production costs in the wholesale market which justifies the absence of elasticity of demand within our model. The case of tariffs which take different values for different hours during a day (hours when demand is high and low, respectively) gives access to a price relatively close to the spot price but if one aggregates e.g.

[5]Peaking power plants are power plants that generally run only when there is a high demand, known as peak demand, for electricity.

[6]An intermittent energy source is any source of energy that is not continuously available due to some factor outside direct control. The intermittent source may be quite predictable, for example, tidal power, but cannot be dispatched to meet the demand of a power system.

on a monthly level the result is not pertinent.

In order to obtain a more clear vision of the demand served by the nuclear and thermal units, we remove the part of the base load demand served by generation units whose electricity production is "fatal" and they are the first called to meet demand according to the merit order[7] (run-of-river[8] hydro plants, renewable energy plants). More precisely, the level of demand observed during a month is translated by the monthly production coming from the run-of-river hydro plants. Since the hydro technology with no reservoir (run-of-river) is a base load generation technology which is presumably never marginal, it is necessary to call up nuclear to cover the different levels of demand. As we explained before, we do not consider the production coming from renewable energy plants because of the variability or intermittency that characterizes their production level.

In our model, we do not take into consideration the seasonal variations of hydro production due to precipitation and snow melting because the corresponding data is not available. Therefore, we assume that the monthly run-of-river hydro-production is constant through the entire time horizon of our model. This assumption is also based on the relatively low volatility of the monthly run-of-river hydro-production due to a relatively low standard deviation[9] which results in a smooth evolution of its monthly value close to the mean over a year. This is not the case for the monthly production coming from renewable energy plants since it is significantly volatile (high levels of standard deviation) and is spread out over a large number of values during a year. This does not permit to assume that it is constant over the whole time horizon of the model and take it into account in the modelling of the demand as we did with the run-of-river hydro-production.

2.2 Modelling the time horizon

The time horizon of the model is T= 36 months[10] beginning by the month of January. We choose this time horizon because we need a sufficiently long time horizon to follow up the evolution of the optimal levels of production and nuclear fuel stock as well as the variations of price and profit. In order to keep our model simple, we assume that the value of profit is not discounted during the period T. We do not choose a longer time horizon in order to be consistent with this absence of the discount rate. We can certainly consider longer periods but the model will be less pertinent.

Let us now proceed with the modelling of the time horizon of the campaign of production. A French nuclear producer has two main options with respect to the scheduling of fuel reloading (Source: EDF (2008), CEA (2008)):

- per third (1/3) of fuel reservoir (representing a reloading of reactor's core per third of its

[7]The merit order is a way of ranking the available technologies of electricity generation in the same order as their marginal costs of production. This ranking results in a combination of different generation technologies to reach the level of demand at a minimum cost. The price in the market is therefore determined by the marginal cost of the "last technology" used to equilibrate supply and demand (perfect competitive case). This technology is also called marginal technology.

[8]The run-of-river hydro plants have little or no capacity for energy storage, hence they can not co-ordinate the output of electricity generation to match consumer demand. Consequently, they serve as base load power plants.

[9]In statistics and probability theory, standard deviation shows how much variation or "dispersion" exists from the average (mean, or expected value). A low standard deviation indicates that the data points tend to be very close to the mean; high standard deviation indicates that the data points are spread out over a large range of values.

[10]The time horizon of the model is a multiplicative of twelve, being expressed in months. Therefore it could be modified.

full capacity) that corresponds to 18 months of campaign and 396 days equivalent to full capacity for a unit of 1300 MW (cf. Subsection 2.5),
- per quarter (1/4) of fuel reservoir (representing a reloading of reactor's core per quarter of its full capacity) that corresponds to 12 months of campaign and 258 days equivalent to full capacity for a unit of 1500 MW.

We exclude the case of having both a campaign of 12 and 18 months to avoid complicate our model and because: (*i*) any changes on the choice of duration of the campaign have to be authorized by the Nuclear Safety Authority (NSA) which is tasked, on behalf of the state, with regulating nuclear safety in order to protect workers, patients, the public and the environment in France, (*ii*) of the design of the nuclear fuel rods intended for this specific reactor, (*iii*) the optimal allocation of the shutdowns of all 58 nuclear reactors for reloading is decided in advance according to safety rules imposed by NSA. In order to get a tractable model for our numerical simulations, we need a cyclic model for the modelling of the campaign. We do not retain the first modelling, hence a campaign of 18 months because it is not consistent with the "good" seasonal allocation of shutdowns of the nuclear units. Indeed, if the nuclear producer reloads fuel in summer when the demand is low the date of the next reloading will be then in winter when the demand is high. Consequently, we retain a modelling close to the second modelling, thus a duration of campaign equivalent to 12 months to get a cyclic model with a periodicity of one year. The one year period can be then broken down into 11 months being the period of production and 1 month corresponding to the month of reloading of the fuel.

Note that both options[11] of fuel reloading result from the operational schema of EDF (Electricité de France) that is strategically chosen in order to optimize the allocation of shutdowns of nuclear reactors for reloading. We do not deal with the question of the optimal allocation of shutdowns in this thesis for several reasons: (i) lack of operational data for confidentiality reasons, (ii) lack of information with regard to the periodical inspections of nuclear reactors and the inspections imposed by the Nuclear Safety Authority, (iii) it is already determined by the French nuclear operator (EDF) via a high level computational programming (model ORION). For all these reasons which are difficult to control in order to endogenously determine the optimal point of reloading of nuclear fuel and thus, the duration of the campaign within our model, the scheduling of fuel reloading is entirely exogenous. In our model, our focus is on optimizing the allocation of the nuclear fuel stored in the reservoir during the different campaigns of production for a reloading pattern provided by the French nuclear operator via the model ORION.

2.3 Modelling the generating units

We study a competitive electricity market with $N \geqslant 2$ producers who manage both nuclear and thermal generating units. A producer $n = 1, \cdots, N$ can operate with all types of nuclear generating units. In addition, each producer disposes of a certain amount of thermal capacity.

2.3.1 Concept of type

Among the nuclear generating units, we distinguish several essential intrinsic characteristics:
- available nuclear capacity,
- minimum capacity when in use,

[11]In the case of a unit of 900 MW, the scheduling of fuel reloading is the following: (*i*) 1/3 of fuel reservoir that corresponds to 18 months of campaign and 385 days equivalent to full capacity, (*ii*) 1/4 of fuel reservoir that corresponds to 12 months of campaign and 280 days equivalent to full capacity.

- month of their fuel reloading.

In our model, the minimum capacity is proportional to the available capacity, and this proportion is the same for all "physical" nuclear reactors. Therefore, for each "physical" nuclear reactor, we will focus on the month of fuel reloading, which permits us to define twelve "types" of nuclear units. Each type indexed by $j = 1, \cdots, 12$ corresponds to a different month of reloading of the nuclear unit. Then, a unit which belongs to the type of unit $j = 1$ (respectively $j = 2, \cdots, j = 12$) shuts down in January (respectively February, $\cdots$, December).

A nuclear plant[12] may contain several "physical" nuclear reactors, which (for operational reasons) do not reload on the same month. The characteristic "type" for the nuclear case is not related to the plant but to the reactor. Each producer $n = 1, \cdots, N$ owns a precise number of "physical" nuclear reactors that are grouped according to the month of reloading (independently of the locations) in order to constitute units. Therefore, it can hold a certain level of capacity from each type of nuclear unit.

For the thermal units, the modelling is the same except that the minimum capacity is equal to zero and that there is no month of reloading. There is a unique type of thermal units.

2.3.2 Notations

The level of the nuclear production during the month $t = 1, \cdots, T$ for the unit j of producer n will be denoted by q^{nuc}_{njt}. Moreover, the maximum nuclear production that can be realized by the unit j of producer n during a month is given by the parameter $Q^{n,j,nuc}_{max}$, while the minimum nuclear production is equal to $Q^{n,j,nuc}_{min}$.

The level of the thermal production during the month $t = 1, \cdots, T$ for the producer n will be denoted by q^{th}_{nt}. Furthermore, the maximum thermal production during a month for the producer n is given by the parameter $Q^{n,th}_{max}$ and corresponds to the nominal thermal capacity of producer n, while there is no minimum for thermal production $Q^{n,th}_{min} = 0$.

2.4 Modelling the production costs

The cost functions of both nuclear and thermal production are common among the different producers. The nuclear cost function is made of a fixed part determined by the cost of investment, the fixed cost of exploitation and taxes and a variable part which corresponds to the variable cost of exploitation and fuel cost (see page 18, Subsection 4.1). We assume that the cost function of the nuclear production is affine and defined as

$$C^{nuc}_{n,j}(q^{nuc}_{njt}) = a^{n,j}_{nuc} + b_{nuc} q^{nuc}_{njt}.$$

The thermal cost function is also made of a fixed part which corresponds to the cost of investment, the fixed cost of exploitation and taxes and a variable part covering the variable cost of exploitation, the fuel cost, the cost of CO_2 as well as the taxes on the gas fuel (see page 18, Subsection 4.1). We assume that the thermal production has a quadratic cost function $C^{th}_n(.)$ which is the following:

$$C^{th}_n(q^{th}_{nt}) = a^n_{th} + b_{th} q^{th}_{nt} + c^n_{th} {q^{th}_{nt}}^2.$$

Proposition 2.1 *The coefficients involved in the cost functions are determined by the capacity.*

- *The fixed part a^n_{th} of the thermal cost function is proportional to the capacity $Q^{n,th}_{max}$ while the coefficients b_{th}, c^n_{th} of the variable part of the thermal cost function are such that: (i)*

[12] A nuclear power plant is a thermal power station in which the heat source arises from nuclear reactions. A nuclear unit is the set that consists of two parts: the reactor which produces heat to boil water and make steam and the electricity generation system in which one associates: the turbine and the generator. The steam drives the turbine which turns the shaft of the generator to produce electricity (Source: SFEN).

b_{th} does not depend on the capacity $Q^{n,th}_{max}$, (ii) c^n_{th} is inversely proportional to the capacity $Q^{n,th}_{max}$.

- *The coefficient $a^{n,j}_{nuc}$ is proportional to the capacity $Q^{n,j,nuc}_{max}$ since it corresponds to the fixed part of the nuclear cost function.*

Proof
A proof of this proposition is given in the Ph.D. thesis on pages $92 - 93$ (Lykidi (2014)). □

The nuclear and thermal cost functions are monotone increasing and convex functions of q^{nuc}_{njt} and q^{th}_{nt} respectively. We choose a quadratic cost function for thermal because the marginal cost of the thermal production is increasing since it results from different fossil fuel generation technologies (e.g. coal, gas). Furthermore, the thermal production needed a non constant (increasing) marginal cost function to recover its fixed costs.

2.5 Modelling the nuclear fuel stock

Following the modelling of the production constraints, we will model the nuclear fuel stock and the associated constraints. We denote $S^{n,j}_{reload}$, the nuclear fuel stock of reloading available to the unit j of producer n. Rather than expressing this stock in kilograms of uranium or number of nuclear fuel rods, we will express it thanks to the conversion between the quantity of energy and the corresponding number of days of operation at full capacity. The number of days of operation equivalent to full capacity is constant for all j, n and does not exceed the 11 months which permits and obliges at the same time the modulation of the nuclear production. The nuclear fuel stock of reloading $S^{n,j}_{reload}$ is equal to the corresponding capacity of the units of type j of producer n (Capacity^{n,j,nuc}) multiplied by the number of hours equivalent to full capacity during a campaign. More precisely, one has:

$$S^{n,j}_{reload} = 1\times \text{ Capacity}^{n,j,nuc} \times \text{ Number of days equivalent to full capacity}\times 24$$

where the number 1 makes clear that we reason over a campaign of production.

The dynamic variable which constitutes the nuclear fuel stock of a unit over time is $S^{n,j}_t$ which by convention represents the quantity of fuel stored in the nuclear reservoir and available to the unit j of producer n at the beginning of the month t. Obviously, we have $S^{n,j}_t \geqslant 0$.

The evolution of the nuclear fuel stock is classic and is determined by the following rules:

$$S^{n,j}_1 \text{ given, } S^{n,j}_{t+1} = \begin{cases} S^{n,j}_t - q^{nuc}_{njt}, & \text{if no reload during month } t \text{ for unit } j \\ S^{n,j}_{reload}, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (1)$$

We have only to take into account the question of reloading. In the case that t is the month during which the producer n reloads the fuel of the reactor, the stock at the beginning of the following month (beginning of the campaign) is equal to $S^{n,j}_{reload}$.

We also impose

$$S^{n,j}_{T+1} \geqslant S^{n,j}_1 \quad (2)$$

The constraint (2) implies that a producer must keep its nuclear units at the end of the game at the same storage level as the initial one. A producer has to finish the period T at least with the same quantity of nuclear fuel as the initial one. In this way the producer has to "spare" its nuclear fuel during the production period. The consideration of this constraint is motivated by some arguments:

- The absence of this constraint could lead to an "over-consumption" of the nuclear fuel stock in order to reach the maximum nuclear production level; this could generate some negative effects (e.g. insufficient level of stock to reach at least the minimum nuclear production level during some months $t = 37, 38$, etc. (excluding the month of reloading)).

- The constraint (2) guarantees that the producer will start a new cycle of simulations of 36 months with a quantity of stock at least equal to $S_1^{n,j}$ at the beginning of the game.

Such a constraint is implicit for unit j if the end of period T coincides with the end of the campaign of unit j.

The nuclear production q_{njt}^{nuc} during a campaign (11 months) for the unit j of producer n can not exceed the nuclear fuel stock of reloading $S_{reload}^{n,j}$ available to the unit j of producer n at the beginning of the campaign. For example, given the schema stock-production flow represented by (1) and the positivity of the nuclear fuel stock ($S_t^{n,j} \geqslant 0$), the nuclear production q_{n1t}^{nuc} realized by the unit 1 of producer n during its first campaign is such that:

$$\sum_{t=2}^{12} q_{n1t}^{nuc} \leqslant S_{reload}^{n,1} \tag{3}$$

We recall that the nuclear units of type $j = 1$ reload their nuclear fuel in January. From the constraint (3) and (1), we deduce that the producer n finishes the campaigns with a quantity of stock superior or equal to zero. However, a producer spends all its nuclear fuel stock of reloading $S_{reload}^{n,j}$ during a campaign. Several reasons lead us to this ascertainment:

- The length of a campaign is given by the maximum number of days during which a nuclear unit produces until **exhaustion** of its fuel of reloading.
- The evaluation of the variable part (b_{nuc}) of the nuclear cost function which partially corresponds to the fuel cost is based on the fact that a producer uses all the available nuclear fuel stock: if a producer keeps paying in order to obtain the fuel stock $S_{reload}^{n,j}$ even in the case that it does not consume all the stock during a campaign, then this cost should be regarded as a fixed cost which is paid at the beginning of each campaign. Consequently, the fuel cost should be integrated into the fixed part of the nuclear cost function, which means that the coefficient $a_{nuc}^{n,j}$ and thus the nuclear cost would be modified.
- The cost that a producer undergoes to get rid of the unused nuclear fuel at the end of the campaigns (cost related to the reprocessing of nuclear fuel).

Note also that there exists an obvious analogy with Walras' Law where the inequality budget constraint is represented as an equality.

For these reasons, the constraint (3) will now take the form:

$$\sum_{t=2}^{12} q_{n1t}^{nuc} = S_{reload}^{n,1} \tag{4}$$

Similarly, the constraint (2) can not hold as inequality constraint ($S_{T+1}^{n,j} > S_1^{n,j}$) which means that the surplus of stock at the end of the game is zero. Thus, the constraint (2) will become:

$$S_{T+1}^{n,j} = S_1^{n,j} \tag{5}$$

Note that if $j = 1$, then the condition (5) becomes $S_{T+1}^{n,1} = S_1^{n,1} = 0$ which is obviously true.

In view of the above ascertainment regarding the exhaustion of the nuclear fuel stock at the end of campaigns, the nuclear fuel constraints for the nuclear unit j of producer n are defined in the table that follows:

We observe that the nuclear units of type $\{2, \cdots, 11\}$ have two additional constraints than the nuclear units of type 1 and 12. This is due to the presence, at the beginning and end of the game, of campaigns that we will qualify as incomplete.

j=1	j ∈ {2, ⋯, 11}	j=12
$\sum_{t=2}^{12} q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=1}^{j-1} q_{njt}^{nuc} = S_1^{n,j}$	$\sum_{t=1}^{11} q_{n12t}^{nuc} = S_{reload}^{n,12}$
$\sum_{t=14}^{24} q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=j+1}^{j+12-1} q_{njt}^{nuc} = S_{reload}^{n,j}$	$\sum_{t=13}^{23} q_{n12t}^{nuc} = S_{reload}^{n,12}$
$\sum_{t=26}^{T} q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=j+12+1}^{j+2\cdot 12-1} q_{njt}^{nuc} = S_{reload}^{n,j}$	$\sum_{t=25}^{T-1} q_{n12t}^{nuc} = S_{reload}^{n,12}$
	$\sum_{t=j+2\cdot 12+1}^{T} q_{njt}^{nuc} = S_{reload}^{n,j} - S_1^{n,j}$	

Table 1

2.6 Number of optimization variables and of optimization constraints

In our model, the total number of optimization variables is equal to $N \cdot (J \cdot T + T) = N \cdot (12 \cdot 36 + 36) = N \cdot 468$. The number of constraints resulting from the equality between supply and demand is $T = 36$. In addition, the number of nuclear fuel constraints is $N \cdot ((2 \cdot K + 1) \cdot (J - 2) + (2 \cdot K) \cdot 2) = N \cdot ((2 \cdot 3 + 1) \cdot (12 - 2) + (2 \cdot 3) \cdot 2) = N \cdot 82$, where K represents the number of campaigns within our model. Lastly, the number of minimum and maximum nuclear and thermal production constraints is equal to $N \cdot (J \cdot T + T) = N \cdot (12 \cdot 36 + 36) = N \cdot 468$. Hence, the total number of optimization constraints is equal to $N \cdot 550 + 36$. Even in the case of a unique producer ($N = 1$), the number of variables (468) and of optimization constraints (586) are quite large which leads to computational difficulties. This is because, the level of difficulty of the numerical program to compute a solution of an optimization problem is increasing with respect to the size of the model (number of optimization variables, number of optimization constraints).

In general, computational difficulties can result from: (i) the difficulty of the numerical program in calculating a global optimum since it can stop running when it finds a first solution which could be a local optimum of the optimization problem and not proceeding until it finds a global optimum, (ii) the sensibility of calculations with regard to the initial point that one chooses so that the program start running (different initial points can lead to different results), (iii) the duration of calculations which is increasing with respect to the size of the model.

3 Equilibrium and approaches of calculation

In this section, we introduce the notion of a merit order equilibrium in the case of several producers ($N \geqslant 2$). Then, we present our different approaches to calculate a merit order equilibrium.

3.1 The notion of merit order equilibrium

Let us introduce the definition of a merit order equilibrium with respect to a system of prices $p \in \mathbb{R}_+^T$.

Definition 3.1 The production vector $(\overline{q}_n)_{n=1}^N = (((\overline{q}_{1jt}^{nuc})_{j=1}^J, \overline{q}_{1t}^{th})_{t=1}^T, \cdots, ((\overline{q}_{Njt}^{nuc})_{j=1}^J, \overline{q}_{Nt}^{th})_{t=1}^T)$ is a merit order equilibrium with respect to a system of prices $p \in \mathbb{R}_+^T$ if:

(i) for all n, $\overline{q}_n$ is a feasible production vector: (a) it respects the nuclear fuel constraints, for all j and (b) it respects the minimum/maximum production constraints, for all j, t.

(ii) the price, at each month t, is determined by the marginal cost of the marginal technology. It is called the merit order price associated with the production vector $(\overline{q}_n)_{n=1}^N$.

(iii) at each date t, it respects the equality between supply and demand

$$\sum_{n=1}^{N}(\sum_{j=1}^{J}\overline{q}_{njt}^{nuc}+\overline{q}_{nt}^{th})=D_t-Q_t^{hyd}. \tag{6}$$

Let us now give some precisions to each condition ((i), (ii), (iii)) that makes part of the definition of the merit order equilibrium in order to better understand it. More precisely, we specify that:

(i) The nuclear fuel constraints for the unit j are provided by subsection 2.5 of this paper. The minimum/maximum nuclear and thermal production constraints take the form

$$\begin{cases} Q_{min}^{n,j,nuc}\leqslant q_{njt}^{nuc}\leqslant Q_{max}^{n,j,nuc}, & \text{if no reload during month } t \text{ for unit } j \\ q_{njt}^{nuc}=0, & \text{if unit } j \text{ reloads during month } t \end{cases} \tag{7}$$

$$0\leqslant q_{nt}^{th}\leqslant Q_{max}^{n,th} \tag{8}$$

(ii) In view of the subsection 2.2.4 where we determine the nuclear and thermal production costs, the merit order price p associated with a feasible production vector $(q_n)_{n=1}^N$, $p=(p_t)_{t=1}^T=(\Phi_{nt}(q_{nt}))_{t=1}^T=(\Phi_{nt}(q_{nt}^{nuc},q_{nt}^{th}))_{t=1}^T=(\Phi_{nt}((q_{njt}^{nuc})_{j=1}^J,q_{nt}^{th}))_{t=1}^T=\Phi_n(q_n)$ is calculated in month t as follows:

$$p_t=\Phi_{nt}(q_{nt})=\begin{cases} mc_n^{th}(q_{nt}^{th}), & \text{if } q_{nt}^{th}>0 \\ mc^{nuc}(q_{njt}^{nuc}), & \text{if } q_{nt}^{th}=0 \end{cases}=\begin{cases} b_{th}+2c_{th}^n q_{nt}^{th}, & \text{if } q_{nt}^{th}>0 \\ b_{nuc}, & \text{if } q_{nt}^{th}=0 \end{cases} \tag{9}$$

The price is calculated independently of n because according to Proposition 2.1 on page 8, the variable part of the thermal cost c_{th}^n is inversely proportional to the capacity $Q_{max}^{n,th}$ which means that there exists $\lambda\in\mathrm{R}$ such that $c_{th}^n=\dfrac{\lambda}{Q_{max}^{n,th}}$. Hence, in the case that $q_{nt}^{th}>0$, we have

$$p_t=b_{th}+2c_{th}^n q_{nt}^{th}=b_{th}+2\frac{\lambda}{Q_{max}^{n,th}}q_{nt}^{th}=b_{th}+2\lambda\frac{q_{nt}^{th}}{Q_{max}^{n,th}}=b_{th}+2\lambda r_t$$

where $r_t=\dfrac{q_{nt}^{th}}{Q_{max}^{n,th}}$ is the rate of use of the thermal capacity at time t. It varies over time while it is constant from one producer to another. We emphasize that the merit order price p_t is discontinuous on production vectors whose thermal component q_{nt}^{th} is equal to zero (see Figure 1 on page 13).

3.2 Supply behaviour with respect to the merit order price

In this section, we describe the supply behaviour with respect to the merit order price in order to show why producers can not decide on operating the nuclear units given an externally determined intertemporal pattern of prices (e.g. from European Energy Exchange) even if one assumes they can perfectly foresee. From a theoretical and numerical perspective, we expose the barriers of computing a merit order equilibrium. This leads producers to make decisions based on the patterns of seasonal demand.

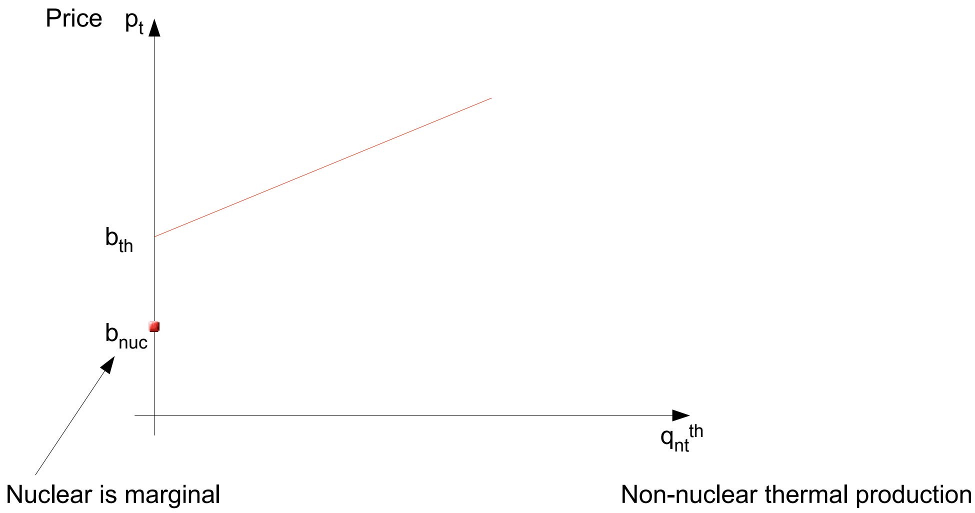

Figure 1: Price discontinuity

3.2.1 First theoretical difficulty of computing a merit order equilibrium

Let us consider a feasible production vector $(q_n)_{n=1}^N = ((q_{nt})_{t=1}^T)_{n=1}^N = ((q_{nt}^{nuc}, q_{nt}^{th})_{t=1}^T)_{n=1}^N = (((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th}))_{t=1}^T)_{n=1}^N$. We can associate to this production a price $p = \Phi_n(q_n)$ or equivalently $(q_n)_{n=1}^N = (\Phi_n^{-1}(p))_{n=1}^N = ((\Phi_{nt}^{-1}(p_t))_{t=1}^T)_{n=1}^N$. We indicate $(((\Phi_{nt}^{-1})^{nuc}(p_t))_{t=1}^T)\ _{n=1}^N$ the nuclear component and $(((\Phi_{nt}^{-1})^{th}(p_t))_{t=1}^T)_{n=1}^N$ the thermal component of the function $((\Phi_{nt}^{-1}(p_t))_{t=1}^T)\ _{n=1}^N$. From a theoretical point of view, supply does not behave well with respect to the merit order price within our model because $\Phi_n^{-1}(.)$ is not a function but a correspondence of the merit order price. This means that a set of different feasible production levels (and not a single feasible production level) are associated with the price.

More precisely, given the relationship (9), there is a distinction between the thermal production of producer n in the month t which is a function (and not a correspondence) of the form

$$(\Phi_{nt}^{-1})^{th}(p_t) = q_{nt}^{th}(p_t) = \begin{cases} \dfrac{p_t - b_{th}}{2c_{th}^n}, & \text{if } p_t \geqslant b_{th} \\ 0, & \text{if } p_t \leqslant b_{th} \end{cases} \tag{10}$$

and the nuclear production $(\Phi_{nt}^{-1})^{nuc}(p_t) = q_{nt}^{nuc}(p_t)$ (of producer n with respect to the price p_t in the month t) which is not a function of the price. From a mathematical point of view, if we analyze (9), one has $p_t = \Phi_{nt}(q_{nt}) = \Phi_{nt}(q_{nt}^{nuc}, q_{nt}^{th})$ but in fact, the function $\Phi_{nt}(.)$ is independent of the first argument i.e. the nuclear production. Consequently, the function $\Phi_{nt}(.)$ can not be one-to-one[13]. Therefore, we will never have unicity of the value of the nuclear production of producer n for a given value of the price p_t in the month t. In particular, when nuclear is the marginal technology i.e. $p_t = b_{nuc}$, the nuclear production can take any value for this value of price in order to maximize profit. Classically, the coordination of producers is realized by the "invisible hand", a mechanism which is based on the evolution of price and the choice of the optimal production levels according to it. However, it is not the case here considering the fact that nuclear production is a correspondence within our model and thus, price signals can not lead to an equilibrium of the optimal production problem. Inevitably, coordination

[13] A function $f(.)$ is called one-to-one or injective if $f(a) \neq f(b)$ for any two different elements a, b of the domain of the function.

issues between producers at a production level will come up. Each producer having a range of feasible nuclear production options would have to coordinate with all producers so that the overall equilibrium between demand and production (resulting from all producers) is respected each month and its profit is maximized at the same time.

3.2.2 Second theoretical difficulty of computing a merit order equilibrium

In figure 1, if $b_{nuc} < p_t < b_{th}$ then there is no correspondence $(((\Phi_{nt}^{-1}(p_t))_{t=1}^{T})_{n=1}^{N})$ since the nuclear production $(\Phi_{nt}^{-1})^{nuc}(p_t)$ of producer n in the month t is an empty set. Therefore, we can not compute the production $\Phi_{nt}^{-1}(p_t)$ of producer n by looking at the merit order price p_t during the month t.

3.2.3 Numerical difficulty of computing a merit order equilibrium

In the case that the nuclear production of producer n $((q_{njt}^{nuc}(p_t))_{j=1}^{J})_{t=1}^{T}$ was a function of the merit order price and not a correspondence, the problem would be different. More precisely, in view of the inter-temporal management of the nuclear fuel stock (see page 10, Subsection 2.5), the system operator has to look at the equality between supply and demand over the entire time horizon T of the model in order to determine the price within the merit order equilibrium. In this typical case of $T = 36$ months and by taking as example the simplest case of one aggregate producer ($N = 1$), we have to deal with a large non linear system

$$\textstyle\sum_{j=1}^{12} q_{jt}^{nuc}(p_t) + q_t^{th}(p_t) = D_t - Q_t^{hyd}, \quad \text{for all } t$$

of 36 equations involving 36 unknowns $((p_t)_{t=1}^{T})$ which is difficult to solve numerically.

Note that this system is based on the auxiliary variables q_{jt}^{nuc}, q_t^{th} whose number is $J \cdot T + T = 12 \cdot 36 + 36 = 468$.

In view of the above theoretical and numerical difficulties, we conclude that it is not possible to calculate a merit order equilibrium by looking at the inter-temporal pattern of prices in this first approach.

3.3 A second approach to calculate a merit order equilibrium

In this approach, each producer n operating with a certain level of nuclear capacity of unit j and an amount of thermal capacity determines its optimal level of supply $((q_{njt}^{nuc})_{j=1}^{J},\ q_{nt}^{th})$ during the month t via the maximization of its current profit given the optimal production levels realized in the previous months.

At time t, the producer n could try to solve the following optimal production problem:

$$\max_{(((q_{\nu j\tau}^{nuc})_{j=1}^{J}, q_{\nu\tau}^{th})_{\tau=1}^{T})_{\nu=1}^{N} \in G^t} p_t \cdot (\sum_{j=1}^{J} q_{njt}^{nuc} + q_{nt}^{th}) - \sum_{j=1}^{J} C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th}) \tag{11}$$

where p_t is a given parameter and G^t is the set of feasible solutions of the optimization problem (11) defined as

$$G^t = \left\{ (((q_{\nu j\tau}^{nuc})_{j=1}^{J}, q_{\nu\tau}^{th})_{\tau=1}^{T})_{\nu=1}^{N} \in K \ s.t. \begin{array}{ll} q_{\nu j\tau}^{nuc} = \widetilde{q}_{\nu j\tau}^{nuc}, & \text{for all } \nu, j \text{ and for all } \tau < t \\ q_{\nu\tau}^{th} = \widetilde{q}_{\nu\tau}^{th}, & \text{for all } \nu \text{ and for all } \tau < t \\ Q_{min}^{\nu,j,nuc} \leqslant q_{\nu j\tau}^{nuc} \leqslant Q_{max}^{\nu,j,nuc}, & \text{for all } \nu, j \text{ and for all } \tau \\ 0 \leqslant q_{\nu\tau}^{th} \leqslant Q_{max}^{\nu,th}, & \text{for all } \nu \text{ and for all } \tau \end{array} \right\}$$

The set K is defined by all the production vectors of the form $q = ((q^{nuc}_{\nu j1})^J_{j=1}, \cdots, (q^{nuc}_{\nu jT})^J_{j=1}, q^{th}_{\nu 1}, \cdots, q^{th}_{\nu T})^N_{\nu=1}$ that respect the nuclear fuel constraints for all ν as well as the supply-demand equilibrium constraint

$$\sum_{\nu=1}^{N}(\sum_{j=1}^{12} q^{nuc}_{\nu jt} + q^{th}_{\nu t}) = D_t - Q^{hyd}_t, \quad \text{in month } t.$$

For simplicity reasons, we used the notation G^t which is the reduced form of the notation $G^t((((\widetilde{q}^{nuc}_{\nu j\tau})^J_{j=1}, \widetilde{q}^{th}_{\nu\tau})^{t-1}_{\tau=1})^N_{\nu=1})$ where $(((\widetilde{q}^{nuc}_{\nu j\tau})^J_{j=1}, \widetilde{q}^{th}_{\nu\tau})^{t-1}_{\tau=1})^N_{\nu=1}$ is the optimal production vector of the months preceding the month t. In the set of feasible solutions G^t, we look at the production realized in all months until the month t because of the inter-temporal nature of nuclear fuel constraints. In view of the construction of the set of feasible solutions G^t in the month t, we deduce that $G^1 \subseteq G^2 \subseteq \cdots \subseteq G^T$.

A production vector $(((\overline{q}^{nuc}_{\nu j\tau})^J_{j=1}, \overline{q}^{th}_{\nu\tau})^T_{\tau=1})^N_{\nu=1}$ is an equilibrium of the optimal short-term optimization problem (11) if it is a merit order equilibrium and in addition to this it maximizes the profit of the producer ν during the month t on the set of feasible solutions G^t, for all ν, t.

However, this approach could be qualified as "short sighted" since the equality between supply and demand in future periods is not taken into consideration in the optimization problem (11). This may lead to a failure of the system to equilibrate supply and demand in future periods while at the same time respecting production and nuclear fuel constraints. In fact, in our numerical example, we find that there exists a month $t \in \{1, \cdots, T\}$ such that the set of feasible solutions G^t is empty. More precisely, we verify (through a numerical test) the nonexistence of feasible solutions within the set G^{16} in April ($t = 16$) of the second year of period T. We intentionally present here a mistaken approach in order to show that too high a level of "short-sightedness" with regards to future demand is not bearable. The equality between supply and demand has to be seen with a minimum anticipation in order to manage the current use of the resevoir. For this reason, we proceed with the next and final approach to calculate a merit order equilibrium.

3.4 Final approach to calculate a merit order equilibrium

In view of the second approach to calculate a merit order equilibrium, the nuclear set has to be managed so that the equality between supply and demand is respected over the whole period T. For this reason, we provide a third and final approach to calculate a merit order equilibrium in order to take into account the supply-demand equilibrium constraint in future periods within the set of feasible solutions of the optimization problem (11).

More precisely, at time t, the producer n may attempt to solve the following optimal production problem

$$\max_{(((q^{nuc}_{\nu j\tau})^J_{j=1}, q^{th}_{\nu\tau})^T_{\tau=1})^N_{\nu=1} \in H^t} p_t \cdot (\sum_{j=1}^{J} q^{nuc}_{njt} + q^{th}_{nt}) - \sum_{j=1}^{J} C^{nuc}_{nj}(q^{nuc}_{jt}) - C^{th}_n(q^{th}_t) \tag{12}$$

where H^t is the set of feasible solutions of the optimization problem (12) defined as follows:

$$H^t = \left\{ (((q^{nuc}_{\nu j\tau})^J_{j=1}, q^{th}_{\nu\tau})^T_{\tau=1})^N_{\nu=1} \in M \ s.t. \begin{array}{ll} q^{nuc}_{\nu j\tau} = \widetilde{q}^{nuc}_{\nu j\tau}, & \text{for all } \nu, j \text{ and for all } \tau < t \\ q^{th}_{\nu\tau} = \widetilde{q}^{th}_{\nu\tau}, & \text{for all } \nu \text{ and for all } \tau < t \\ Q^{\nu,j,nuc}_{min} \leqslant q^{nuc}_{\nu j\tau} \leqslant Q^{\nu,j,nuc}_{max}, & \text{for all } \nu, j \text{ and for all } \tau \\ 0 \leqslant q^{th}_{\nu\tau} \leqslant Q^{\nu,th}_{max}, & \text{for all } \nu \text{ and for all } \tau \end{array} \right\}$$

The set M is defined by all the production vectors of the form $q = ((q^{nuc}_{\nu j1})^J_{j=1}, \cdots, (q^{nuc}_{\nu jT})^J_{j=1}, q^{th}_{\nu 1}, \cdots, q^{th}_{\nu T})^N_{\nu=1}$ that respect the nuclear fuel constraints for all ν as well as the supply-demand equilibrium constraint

$$\sum_{\nu=1}^{N}(\sum_{j=1}^{12} q^{nuc}_{\nu jt} + q^{th}_{\nu t}) = D_t - Q^{hyd}_t, \quad \text{for all } t.$$

The set M differs from the set K defined in our previous approach to calculate a merit order equilibrium because, in this approach, the producer n ensures that supply will meet demand during the entire time horizon of the model. To simplify, the notation H^t is used for $H^t((((\tilde{q}^{nuc}_{\nu j\tau})^J_{j=1}, \tilde{q}^{th}_{\nu\tau})^{t-1}_{\tau=1})^N_{\nu=1})$ where $(((\tilde{q}^{nuc}_{\nu j\tau})^J_{j=1}, \tilde{q}^{th}_{\nu\tau})^{t-1}_{\tau=1})^N_{\nu=1}$ is the optimal production realized in the months preceding the month t. The set H^t has the same properties with those mentioned for the set G^t.

The optimal short-term production problem (12) determines the supply of the producer n $((q^{nuc}_{njt})^J_{j=1}, q^{th}_{nt})$ during the month t, given the optimal nuclear and thermal production $(((\tilde{q}^{nuc}_{nj\tau})^J_{j=1}, \tilde{q}^{th}_{n\tau})^{t-1}_{\tau=1})$ realized in the previous months. A production vector $(((\overline{q}^{nuc}_{\nu j\tau})^J_{j=1}, \overline{q}^{th}_{\nu\tau})^T_{\tau=1})^N_{\nu=1}$ is an equilibrium if it is a merit order equilibrium and it maximizes the profit of the producer ν during the month t on the set of feasible solutions H^t, for all ν, t.

3.4.1 The decrease of short-term profit in the absence of thermal production

Under some rather mild assumptions (satisfied by our numerical data), we show that the absence of thermal production during the month t induces a decrease of the profit during this month which results from a decrease of the price.

Let us focus on the set H^t_{th} defined as

$$H^t_{th} = \left\{ (((q^{nuc}_{\nu j\tau})^J_{j=1}, q^{th}_{\nu\tau})^T_{\tau=1})^N_{\nu=1} \in M \ s.t. \begin{array}{ll} q^{nuc}_{\nu j\tau} = \tilde{q}^{nuc}_{\nu j\tau}, & \text{for all } \nu, j \text{ and for all } \tau < t \\ q^{th}_{\nu\tau} = \tilde{q}^{th}_{\nu\tau}, & \text{for all } \nu \text{ and for all } \tau < t \\ Q^{\nu,j,nuc}_{min} \leqslant q^{nuc}_{\nu j\tau} \leqslant Q^{\nu,j,nuc}_{max}, & \text{for all } \nu, j \text{ and for all } \tau \\ 0 < q^{th}_{\nu t} \leqslant Q^{\nu,th}_{max}, & \text{for all } \nu \end{array} \right\}$$

The price is determined by the thermal production during the month t in the set H^t_{th}.

Remark 3.1 *For all $t \in \{1, \cdots, T\}$, H^t_{th} is contained in H^t and H^t is contained in M ($H^t_{th} \subset H^t \subset M$).*

We will now make use of Proposition 3.1 in order to prove the decrease of the profit at production vectors with zero levels of thermal production at date t.

Proposition 3.1 *For all $t \in \{1, \cdots, T\}$, if H^t_{th} is a non-empty set, then the closure of H^t_{th} is equal to H^t ($\overline{H^t_{th}} = H^t$).*

Proof
A proof of this proposition appears in our Ph.D. thesis on page 112 (Lykidi (2014)).

□

From a geometrical point of view, it results from Proposition 3.1 that all the points of the set H^t and consequently those which belong to $H^t \setminus H^t_{th}$ and therefore characterized by zero levels of thermal production in the month t can be approached by points that belong to H^t_{th}. This result plays a central role in order to prove in the next proposition the discontinuity (decrease) of the profit at these particular points due to the discontinuity (decrease) of the price (discontinuous problems have been analyzed in an economic framework (cf. for example Bich and Laraki (2011)).

Proposition 3.2 *For all $t \in \{1, \cdots, T\}$, for all $n \in \{1, \cdots, N\}$, if H^t_{th} is a non-empty set, $b_{nuc} < b_{th}$ and $\overline{q} \in H^t \setminus H^t_{th}$, then there exists a sequence $(q_r)_{r \in \mathrm{N}} \in H^t_{th}$ with $\lim_{r\to\infty} q_r = \overline{q}$ such that $\lim_{r\to\infty} \pi^n_t(q_r) > \pi^n_t(\overline{q})$.*

Proof
A proof of this proposition is given in our Ph.D. thesis on pages $113 - 114$ (Lykidi (2014)).
□

Notice that the non-emptiness of the set H^t_{th} obviously depends on the values of the exogenous variables $(Q^{n,j,nuc}_{max}, Q^{n,j,nuc}_{min}, Q^{n,th}_{max}, S^{n,j}_{reload}, S^{n,j}_1, D_t, Q^{hyd}_t)$.

The inequality $b_{nuc} < b_{th}$ holds within our data; thus, according to Proposition 3.2, the profit at date t decreases for all the production vectors whose thermal component at this date is equal to zero. Consequently, it is not profitable for a producer who wants to maximize its profit to run only its nuclear units and be remunerated at a price $p_t = b_{nuc}$. Therefore, the producer will search for a solution that maximizes its profit among the production vectors of the set H^t_{th}.

The following corollary establishes the relation between the optimal short-term production problem on H^t and the optimal short-term production problem on H^t_{th}.

Corollary 3.1 *The current monthly profit maximization problem determined on H^t is equivalent to the current monthly profit maximization problem determined on H^t_{th} (same set of solutions and same value[14]), for all t.*

Proof
This corollary is an obvious consequence of Proposition 3.2.
□

It should be mentioned that, for all t, the value of both optimization problems exists (in the real line) since the profit function is polynomial and the set H^t along with the set H^t_{th} are bounded. We also notice that that if the problem (12) is determined on H^t which is a compact set, the objective function is not continuous according to Proposition 3.2 (see page 17). If the problem (12) is determined on H^t_{th}, the objective function is continuous in view of Proposition 3.2 while the set H^t_{th} is not compact. Hence, it is not possible to conclude on the existence of solutions of this problem (cf. for example Varian (1992)). If a solution of the optimal short-term production problem on H^t_{th} exists for all t, then it constitutes an equilibrium since all the conditions in order to be a merit order equilibrium are met and it maximizes the profit in the month t (see Corollary 3.1 and Definition 3.1 on page 11). In the next section, we provide a numerical illustration of this problem.

3.4.2 A property of the short-term profit function and its implications

The following lemma shows the concavity of the profit function of the optimal short-term production problem determined on H^t_{th}.

Lemma 3.1 *For all $t \in \{1, \cdots, T\}$, for all $n \in \{1, \cdots, N\}$ the profit function of the current monthly profit maximization problem on H^t_{th} is concave with respect to q_{nt}.*

Proof
A proof of this lemma is provided in our Ph.D. thesis on page 116 (Lykidi (2014)).
□

[14]The value of an optimization problem is defined as the upper bound of the set $\{f(x)|x \in C\}$, where f is the objective function and C is the set of feasible solutions. The value always exists even if the set of solutions is empty. When the set of solutions is nonempty, the value of an optimization problem is the common value $f(\overline{x})$ for any solution $\overline{x}$.

Remark 3.2 *The strict concavity of the profit function π_t^n with respect to the thermal production q_{nt}^{th} implies the unicity of solutions regarding the thermal component for all n, t. However, if we consider the other variables which do not affect the profit π_t^n and according to the proof of Lemma 3.1 the profit function π_t^n is concave with regard to q_{nt} for all n, t which does not imply necessarily the unicity of the entire solution.*

4 Numerical modelling

In this section, we provide an analytical description of our data set. Then, we study the nuclear and thermal production decisions coming from the resolution of the optimal short-term production problem analyzed in the previous section 3.4, within a simple numerical model solved with Scilab.

4.1 Data

The data used in our numerical dynamic model is French. It is of several years due to the difficulty of collection: ∗ level of French demand during the year 2007, ∗ generation capacity of hydro (run-of-river), nuclear and thermal, ∗ nuclear fuel stock of reloading and ∗ fixed and variable costs of nuclear, coal and gas generation.

Consumption data comes from the French Transmission & System Operator (named RTE). It gives the hourly consumption in MW for the entire year 2007 which we use to determine the monthly consumption. This data takes into account the electricity losses of the network, as estimated by RTE. RTE also provides the annual capacity of nuclear, gas and coal for the year 2009. The annual capacity and production of hydro and the nuclear fuel stock of reloading have been provided by EDF. The costs of production come from the official report "Reference Costs of Electricity Production" issued by the ministry of industry (DGEMP & DIDEME (2003, 2008)). The report gives the total cost for each technology as follows: investment cost, variable and fixed cost of exploitation, fuel cost, taxes, R&D costs and cost of CO_2 per ton for nuclear (EPR reactor), coal and gas given the level of operation (base load (8760h) and semi-base (3000h) operation). These costs are estimated for the year 2007 and 2015 with different discount rates (3%, 5%, 8%, 11%) taking into account the influence of exchange rate on the production cost.

4.1.1 Specific data assumptions for our numerical modelling

Our modelling is based on a scenario in which one dollar is equal to one euro, the discount rate is 8%, the cost of CO_2 per ton reaches 20 euros, the price of coal is 30 dollars per ton and the price of gas is 3.3 dollars per MBtu (1 MBtu/h = 293.1 kWh). The choice of this particular scenario is mainly based on the scenario considered by DGEMP & DIDEME in 2003 for the estimation of costs of the different types of generation technologies in 2007.

In view of the quadratic form of the thermal production cost, we need to specify that the value of the coefficient a_{th} corresponds to the fixed cost of coal and gas while the other coefficients have been determined by interpolation in order to meet the variable cost provided by our data. The consideration of the fixed costs of both technologies (nuclear, thermal) permits to obtain a more realistic vision of the profit within our medium-term horizon.

The capacity of each nuclear unit has been simulated[15] in order to approximate the graph

[15] Access to detailed nuclear capacity data for each short period of time is not possible due to the confidentiality of such data.

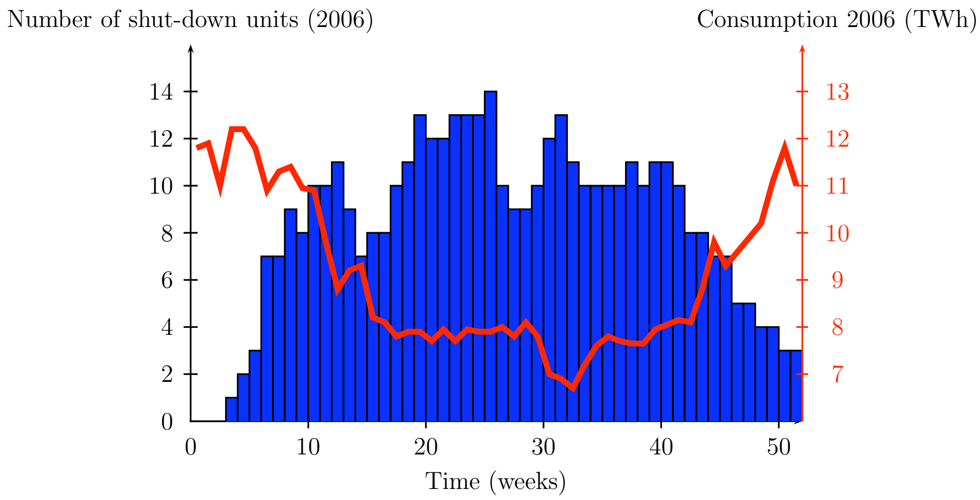

Figure 2: Availability of nuclear units, Source: EDF (2006)

of figure[16] 2. For example, the capacity of the nuclear unit $j = 1$ (respectively $j = 2, \cdots, j = 12$) corresponds to the average capacity of shut-down nuclear units in January (respectively February, $\cdots$, December) whose calculation has the difficulty that a month is not composed of an integer of weeks. This has to be considered in order to compute the average number of shut-down nuclear units and thus, the average nuclear capacity every month. The french nuclear set is composed of different types of nuclear reactors with different levels of capacity and we do not dispose a detailed data of the information for figure 2. The initial value of the nuclear fuel stock (S_1^j) has been set by simulating the nuclear fuel stock of each unit j available at the beginning of the time horizon of the model.

The number of days (or number of hours) equivalent to full capacity is calculated within our model as the product of the operating factor of a nuclear reactor (K_u percentage of time that the reactor is used at its maximum capacity during its availability period) and the number of days (or number of hours) of a given year: $\mathrm{K}_u \cdot 365 \approx 0.70 \cdot 365 \approx 256$. This number is almost identical with the number of days (or number of hours) during which a reactor of 1500 MW operates at full capacity in the case of a campaign of 12 months following the operational schema of EDF (see page 6, Subsection 2.2).

Finally, a nuclear unit can vary its capacity level between the nominal capacity and the technical minimum. In the case of an EPR, load follow[17] enables planned variations in energy demand to be followed and can be activated between 25% of nominal capacity (technical minimum) and 100% of nominal capacity (technical maximum) (NEA/AEN (2011)). In our numerical model, the minimum nuclear production is the 25% of the capacity of the units of type j i.e. $0.25\cdot$ Capacity^{j,nuc} and the maximum nuclear production is the 100% of the capacity of the units of type j i.e. $1\cdot$ Capacity^{j,nuc}.

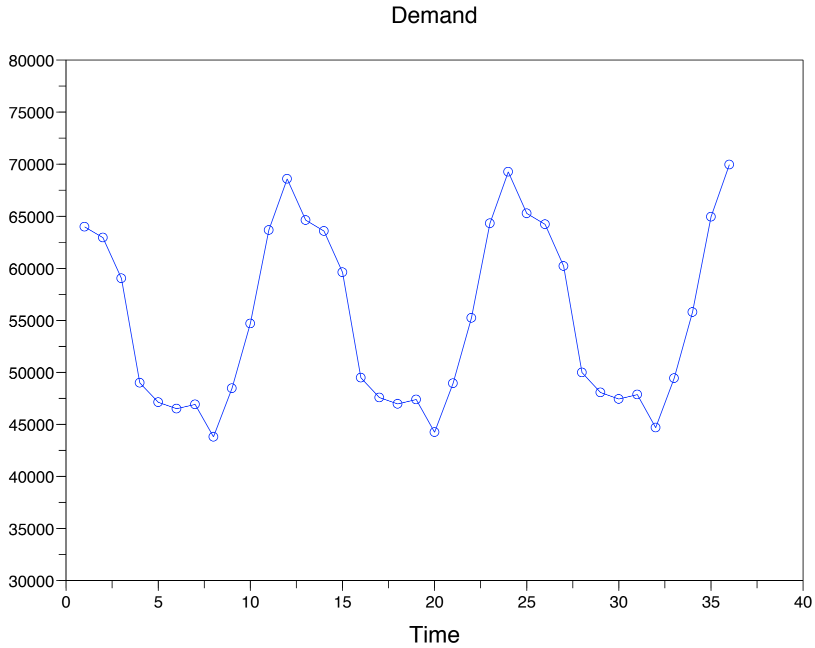

Figure 3: Simulated demand (in MW)

4.1.2 Numerical simulation of the levels of monthly demand.

The amounts of monthly demand[18] D_t obtained for the period January 2007 - December 2009 are presented in figure[19] 3. In particular, the values of monthly demand during the period January 2007 - December 2007 come from our historical data. Then, we reproduce these values by applying a positive rate on the monthly demand for the years that follow (2008 and 2009). We suppose an augmentation of the demand level by a rate of 1% per year to take into account the increasing trend of demand from one year to another. This implies a non periodic evolution of demand over time. One can see the seasonal variation of demand between winter (high demand in November-February) and summer (low demand in April - August). Part of the information is produced by the aggregation of the data since it takes into consideration the monthly levels of demand[20] and not the hourly levels of demand. Indeed, if we compare the peak monthly demand with the peak hourly demand for a given year, we could observe a difference between them. For the year 2007, the peak daily demand which is 80403 MW corresponds to an average hourly demand of 3350.1 MW that is inferior than 3352.5 MW corresponding to the peak hourly demand observed in our historical data.

[16]Each blue bar shows the number of shut-down nuclear units during a week and the red line shows the evolution of the consumption over time. The different levels of consumption are measured on the right axis while the number of shut-down nuclear units is reflected on the left axis.

[17]The minimum requirements for the maneuverability capabilities of modern reactors are defined by the utilities requirements that are based on the requirements of the grid operators.

[18]Note that we did a rescaling on this data to take into account the diversity on the length of the months.

[19]In our numerical model, the unit of energy used for measuring demand, stock and production is the MWh.

[20]Note that the monthly demand in 2007 results from the aggregation of the hourly demand found within our historical data.

4.1.3 Numerical results deduced from our data set

Let us now provide a couple of numerical results obtained within our data set:

- In view of our modelling of the nuclear production cost and the value of its coefficients computed in this data (which is calculated for the year 2007), we determine the average nuclear cost evaluated here at 37.25 euros per MWh. This price of the nuclear MWh is very close to the range of nuclear electricity prices[21] (37.5 - 38.8 euros per MWh) appeared in the analysis of the Regulatory Commission of Energy (CRE) before Fukushima accident. It does not take a value significantly lower than 37.5 euros per MWh as it was asserted by alternative producers in order to compete EDF. However, it does not exceed the 38.8 euros per MWh (being eventually close to the price of 42 euros per MWh set by the French government to contribute to the security of supply of France and the work that EDF might have to start to enhance nuclear safety after Fukushima disaster in Mars 2011 (Les Echos (20/04/2011)) which is totally understandable since the cost evaluations of DGEMP & DIDEME do not take into account the extra costs resulting from measures intended to improve nuclear safety after Fukushima accident.

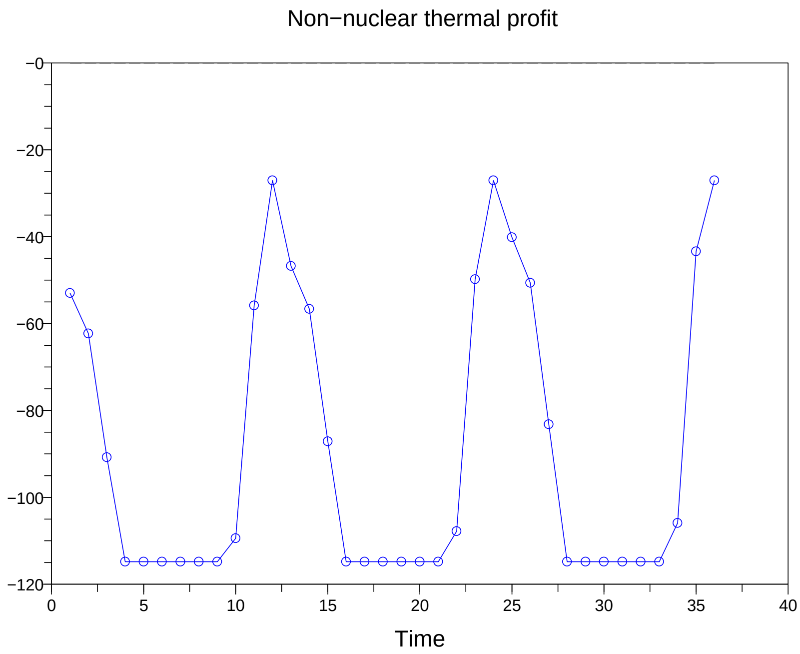

Figure 4: Thermal profit (in Euro (million))

- In order to examine whether the thermal production is profitable or not and the potential to pay for the cost coming from it every month, we determine the threshold of profitability of the thermal production realized by N producers (considered also as one aggregate producer) during the month t (θ_N). To do this, we take the profit resulting from the aggregate monthly thermal production equal to zero: $\theta_N = N\sqrt{\frac{a_{th}}{c_{th}}}$. If the monthly thermal production level realized by the N producers is higher (lower) than θ_N, then the profit is positive (negative). The value of the threshold of profitability[22] (which is

[21] In 2010, the French energy regulator (CRE) estimated that a price between 37.5 and 38.8 euros per MWh would permit the development of competition, at least in the market of professionals consumers, according to the regulator, without penalizing EDF (Le Monde (01/02/2011)).

[22] In figure 7, the red crossed line represents the threshold of profitability of the thermal production.

independent of the number of producers in the case of N identical producers) provided by our numerical model is $\theta \cong 18$ GW (or equivalently 13 TWh per month) which exceeds the level of the aggregate monthly thermal capacity Q^{th}_{max} and thus, it leads always to negative aggregate thermal profits (see Figure 4 on page 21). So, we conclude that the quadratic form of the thermal production cost in combination with the pricing of production at marginal cost (merit order price) causes important losses to the aggregate producer in our model. This could be explained by the numerous approximations (in particular the absence of mark-up rate) that we made within our model (see General results on page 27, Subsection 4.3). The condition $\theta > Q^{th}_{max}$ (observed numerically) is not incompatible with positive profits. Indeed, the producer could manage to cover the thermal production cost and even obtain a strictly positive profit if another generation technology with higher marginal cost becomes the marginal technology (hydro technology with storage, oil, etc.). In this case, the thermal production would be remunerated above its own marginal cost which would help to recuperate its cost. However, the marginal cost of the marginal technology (coming after thermal) needs to be non constant so that the thermal production becomes profitable for the producer.

4.2 "Equivalence" of merit order equilibrium between an economy with N producers and an economy with one aggregate producer

From a purely logical (mathematical) perspective, our numerical model is characterized by a high level of complexity because of the high number of optimization variables and the important amount of operational constraints even in the case of a unique producer (for $N = 1$, we obtain 586 constraints in total). To reduce the number of optimization variables and operational constraints of our numerical model in order to facilitate the calculation of a merit order equilibrium, we show through the following proposition that the merit order equilibrium in an economy with $N \geqslant 2$ producers is "equivalent" to the merit order equilibrium in an economy with one aggregate producer ($N = 1$). We use this mathematical proposition to calculate an equilibrium of the original economy by working in an auxiliary economy with one aggregate producer from now on. The aggregate producer holds the capacity of all the types of nuclear units as well as the total thermal capacity. In particular, this will allow to simplify the notations q^{nuc}_{njt} and q^{th}_{nt} by considering the notations q^{nuc}_{jt} and q^{th}_{t} that represent respectively the level of the nuclear production during the month t for the unit j and the level of the thermal production during the month t (and similarly for the stock).

Proposition 4.1 *Let us consider an economy E with several producers and let $\widetilde{E}$ be the auxiliary economy with a unique producer obtained by the aggregation of the N producers of E.*

- (α) *If q is a merit order equilibrium of $\widetilde{E}$ then it can be decentralized as a merit order equilibrium $(q_n)_{n=1}^{N}$ of E with regard to the same prices.*
- (β) *Conversely, if $(q_n)_{n=1}^{N}$ is a merit order equilibrium of E then its aggregation defined by $q = \sum_{n=1}^{N} q_n$ is a merit order equilibrium of $\widetilde{E}$ for the same prices.*

Proof

A proof of this proposition is provided in our Ph.D. thesis on pages 131 − 136 (Lykidi (2014)).
□

4.2.1 Economic implications of Proposition 4.1

In view of Proposition 4.1, we may say that the merit order equilibrium q of the centralized economy $\widetilde{E}$ (auxiliary economy) is "equivalent" to the merit order equilibrium $(q_n)_{n=1}^N$ of the decentralized economy E (original economy). This proposition permits to determine a merit order equilibrium of the decentralized economy E by working in the economy $\widetilde{E}$ with one aggregate producer which holds the capacity of all the types of nuclear units and the total thermal capacity and aims to satisfy the demand each month. From an economical point of view, we deduce that at the optimum, the decentralized economy is not "superior" than the economy with a unique producer (centralized economy), in terms of production, in the sense that a merit order equilibrium of the decentralized economy constitutes a merit order equilibrium of the economy with a unique producer and vice versa. Consequently, the decentralization of the nuclear production segment is "neutral" in comparison with the centralized management regarding the optimal production levels obtained in both organizational forms within our model.

4.3 Simulation results

In order to resolve the optimal short-term production problem numerically within our data set, we deal with the problem of discontinuity of the merit order price which leads to the discontinuity (decrease) of the current monthly profit. To do this, we resolve an approximate problem (continuous problem) that is a "regularization" of our economical problem (discontinuous problem). The proof of several mathematical propositions appeared in this section can be found in the Annex A of the Ph.D. thesis of Lykidi (2014).

4.3.1 "Regularization" of the optimal short-term production problem

The hypothesis of Proposition 3.2 that $b_{nuc} < b_{th}$ holds within our data, thus the discontinuity and more precisely the decrease of price at production vectors characterized by zero thermal production during a month induces a discontinuity and specifically a decrease of the value of profit during this month. A point that we need to stress is the nonexistence of an algorithm that maximizes a discontinuous function. Theoretically and numerically, we proceed with a "regularization" of the merit order price to treat the problem of discontinuity and resolve the optimal short-production problem. Theoretically, we dealt with this problem through an equivalent optimization problem being the optimal short-term production problem determined on the subset H_{th}^t of the set of feasible solutions H^t (see Corollary 3.1 on page 17). We recall that the set H_{th}^t is characterized by strictly positive thermal production levels and thus, the marginal cost of thermal determines the price for all t. Within this set, the current monthly profit is a continuous function. Numerically, we propose an alternative model, where the price is given by the thermal marginal cost ($mc^{th}(0) = b_{th}$) instead of the nuclear marginal cost (b_{nuc}) when nuclear is the marginal technology. Thus, at the date t, the price p_t will be

$$p_t = \begin{cases} mc^{th}(q_t^{th}), & \text{if } q_t^{th} > 0 \\ mc^{th}(0), & \text{if } q_t^{th} = 0 \end{cases} = \begin{cases} b_{th} + 2c_{th}q_t^{th}, & \text{if } q_t^{th} > 0 \\ b_{th}, & \text{if } q_t^{th} = 0 \end{cases} \tag{13}$$

This means that a producer receives at least b_{th} (Euros per MWh) when it runs only its nuclear units to cover the monthly demand. In view of this "regularization" of the merit order price, the current monthly profit being now a continuous function is maximized on the entire set of feasible solutions H^t within our numerical model resulting in a continuous optimization problem, the "regularized" problem. However, the "regularized" problem (continuous problem) and the economical problem (12) described in subsection 3.4 (discontinuous problem) differ with respect to the objective function which is the profit. More precisely, the profit of the economical problem is smaller than the profit of the "regularized" problem since the value of b_{nuc} (5.01 Euro/MWh) is less important than the value of b_{th} (26.24 Euro/MWh). Nevertheless, for all t, we show that the value of the "regularized" problem and the value of the economical problem are identical, meaning that the optimal value of the profit is the same for both optimization problems (see Annex A, Proposition A.1). Consequently, the "regularized" problem is a "good" approximation of our economical problem (Boyd and Vandenberghe (2004)).

From a theoretical perspective, in view of Proposition A.3 demonstrated in the Annex A, for all t, if a solution of the "regularized" problem does not belong to the set H^t_{th}, then the set of solutions of the economical problem is empty. Numerically, the solution of the "regularized" problem whose results are analyzed in this section does not belong to the set H^t_{th} since there are months during which the thermal production is zero. Therefore, the set of solutions of the economical problem is empty, hence the interest of focusing on the numerical solution coming from the "regularized" problem. This numerical solution constitutes only an "approximate" solution of our economical problem.

4.3.2 General results

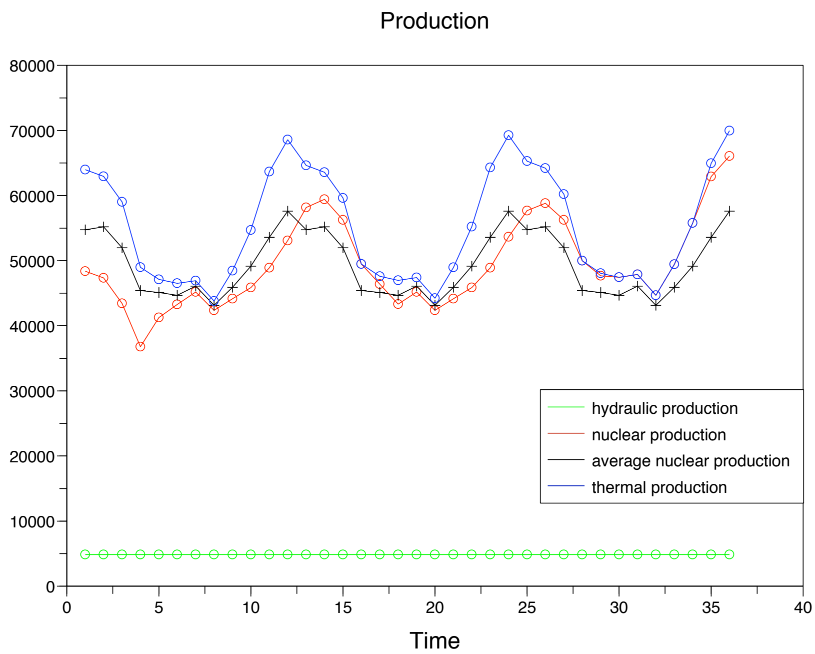

Figure 5: Simulated hydro(run-of-river)/nuclear/thermal production (in MW)

Nuclear follows the seasonal variations of demand by decreasing during summer and increasing during winter (see Figure[23] 5, Figure[24] 6). The monthly nuclear production almost never

[23]The average nuclear production in the month t given that some unit is inactive during this month (month of reloading) is represented by the black crossed line. Its evolution is periodic and is determined by the red line. For example, the average nuclear production in January is the average nuclear production of the months $t = 1, 13, 25$.

[24]The maximum nuclear production during the month t ($\sum_{j=1}^{J} Q^{j,nuc}_{max}(t)$) given that some unit is inactive during this month (month of reloading) is represented by the purple dotted line. This periodic quantity is

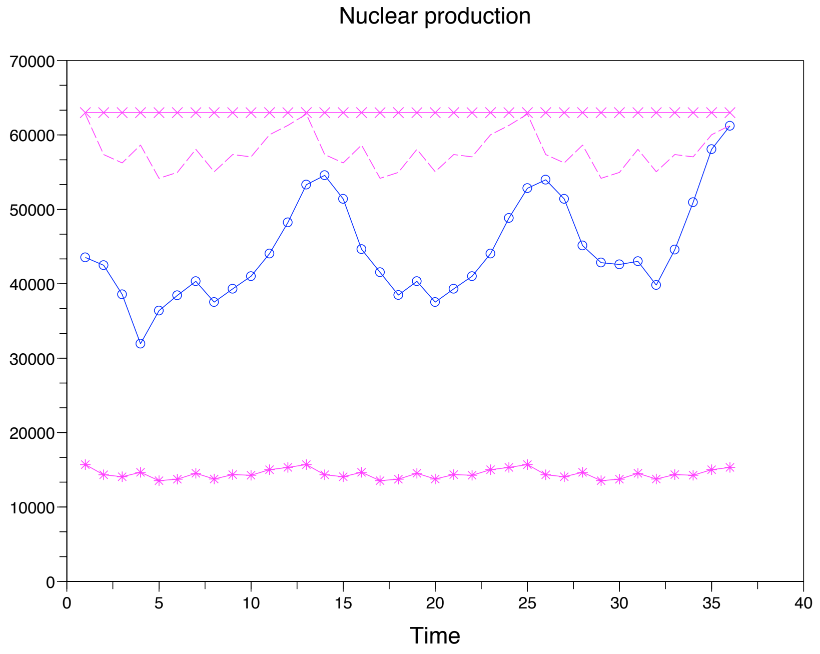

Figure 6: Simulated nuclear production (in MW)

reaches its maximum value (see Figure 6).

Similarly, the thermal production adjusts to demand's seasonal variations during the entire

obviously below the nominal capacity of the French nuclear set represented by the crossed purple line. The minimum nuclear production during the month t given that some unit is inactive during this month (month of reloading) is represented by the purple line of asterisks.

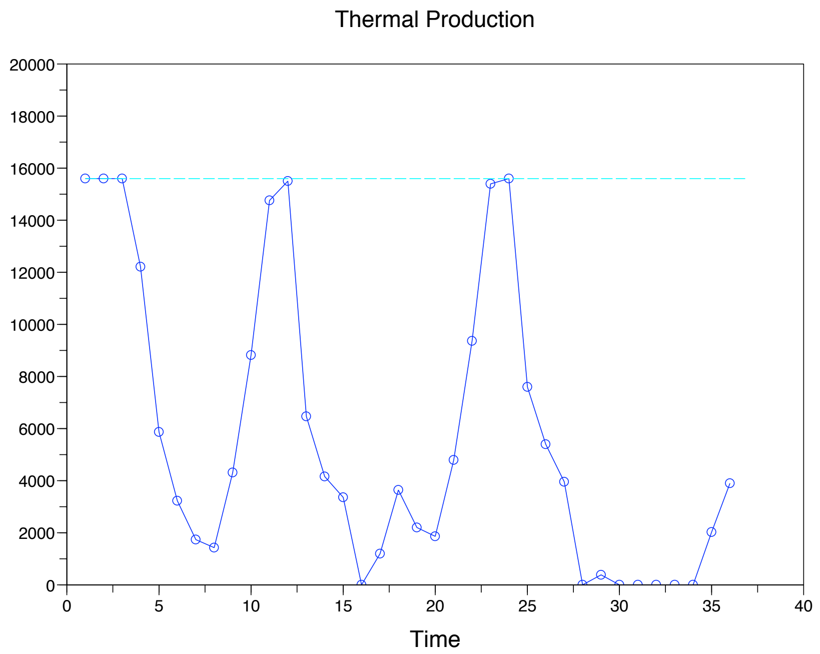

Figure 7: Simulated thermal production (in MW)

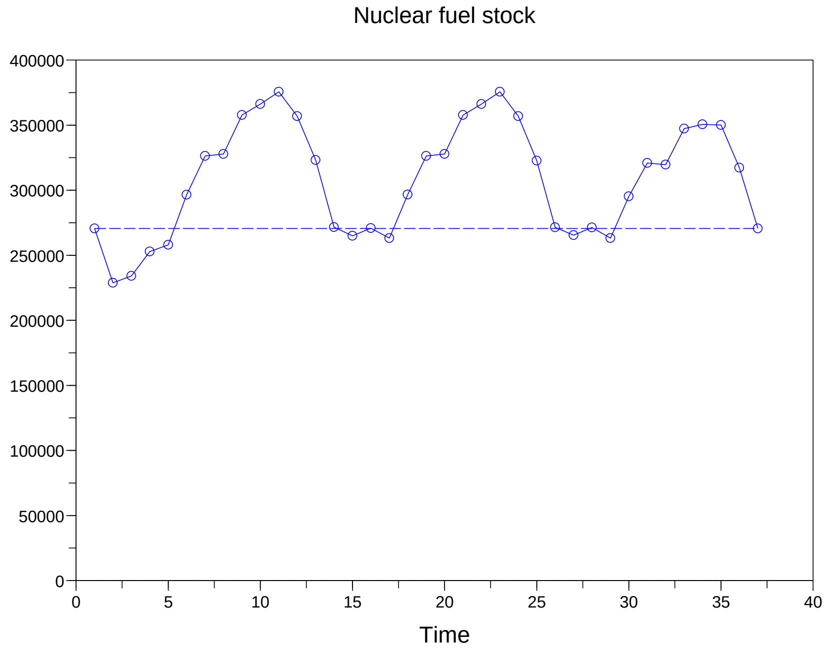

Figure 8: Simulated nuclear fuel stock (in MW)

time horizon T (see Figure 5, Figure 7). The monthly thermal production reaches its maximum value[25] at the beginning of period T when the nuclear production is significantly low and then during the month of December of 2008 and of 2009 to meet the peek levels of demand (see Figure 7). However, the thermal production becomes zero at the end of period T since the nuclear production is very important and covers the monthly demand.

We also observe that the nuclear fuel stock decreases during periods of high demand while increases during periods of low demand (see Figure 8). The trend of the stock appears fundamentally above the "stock of reference"[26] during the time horizon T of the model.

We should note, before we procced with a more detailed analysis of our simulation results, that the reader should not focus on the precise amount of profit since it depends on too many of the approximations we did (euro/dollar, oil prices, CO_2 cost, discount rate, no mark-up rate, absence of profits coming from the run-of-river hydro technology, etc.) and because our modelling does not consider the electricity importations/exportations or the production coming from renewable and hydro storage plants (see Figure 9).

4.3.3 Analytical results

We shall separate period T into three sub-periods to give some structure to the following discussion regarding the evolution of nuclear and thermal production, of stock, of price and of profit. According to figure 5, we distinguish first a sub-period during which the nuclear production is below its average value and the thermal is the marginal technology. A medium sub-period with a nuclear production that oscillates around its average value and a basically periodical evolution of the nuclear and thermal production. Finally, a third sub-period during

[25]The maximum thermal production during a month (Q^{th}_{max}) is represented by the white blue dotted line and corresponds to the nominal thermal capacity (including coal, gas, fuel, etc.) of the French set.

[26]The "stock of reference" is represented by the blue dotted line which shows the value of stock at the beginning, being also the value of stock at the end.

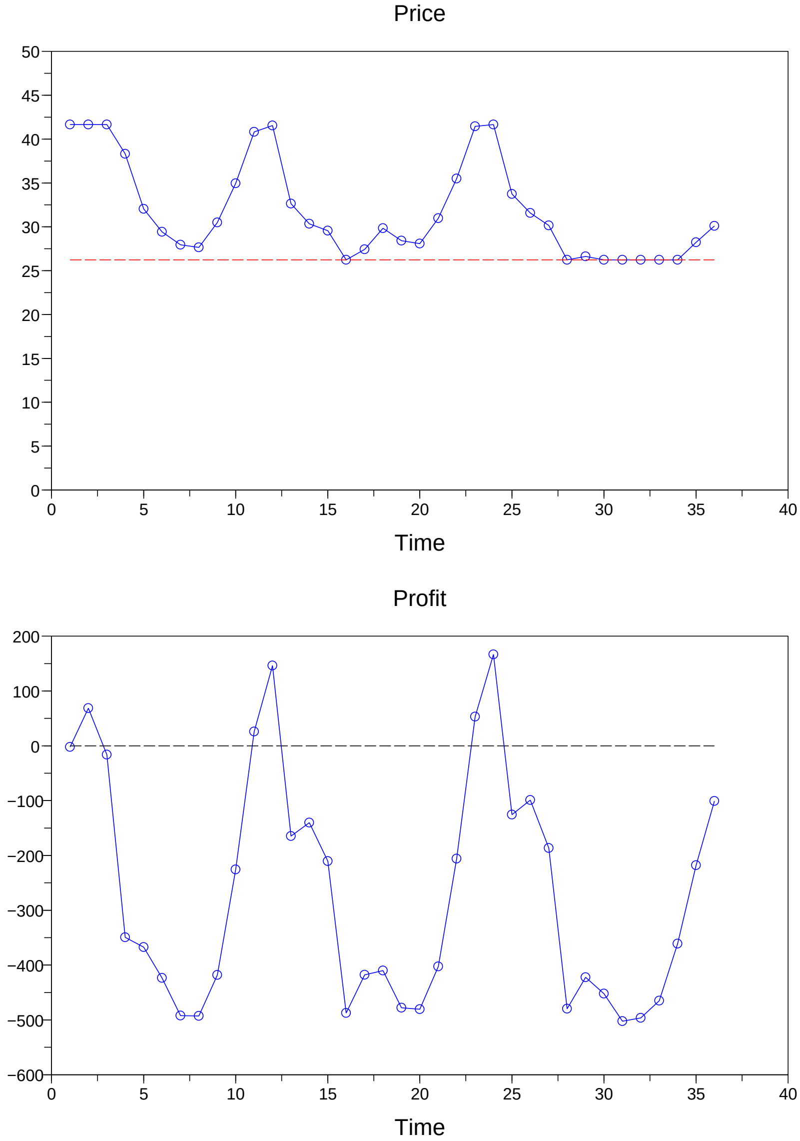

Figure 9: Simulated "regularized" price (in Euro/MWh)/Simulated "regularized" profit (excluding profit coming from hydro (run-of-river) generation) (in Euro (million))

which nuclear production is above its average value and it is mainly the marginal technology.

4.3.4 First sub-period (January 2007 - April 2007)

The nuclear production decreases from January to April due to the decrease of the demand during this period. We also observe that the nuclear production is significantly low during the first months of the simulation in comparison with the same months of the following years (see Figure 5, Figure 6).

On the contrary, the thermal production increases significantly because of the important levels of demand and the low levels of the nuclear production of this period (see Figure 5, Figure 7). In particular, the thermal production reaches its maximum value during the period January - March.

Paradoxically, the amounts of stock observed in this period are less important than those of the corresponding period of the next years. It seems that the nuclear fuel stock is "overused" during the first sub-period and this results in quantities of stock significantly lower than the "stock of reference" (see Figure 8). In particular, there is an impulsion to decrease the stock from January to February which drives the trend of the nuclear fuel stock below its reference value over the entire first sub-period. From February to April, the quantity of stock increases progressively following the decrease of the demand and thus the decrease of the nuclear production without however exceeding the "stock of reference".

The price[27] and the profit detected during the months of the first sub-period are higher than the price and the profit perceived during the same months of the next years because of the maximum levels of thermal production noticed in these months (see Figure 9). Consequently, if the nuclear units underproduce then the thermal units need to overproduce to meet the levels of demand of this period. Hence, significant gains[28] are generated if we compare them with the gains obtained during the same months of the years that follow.

4.3.5 Medium sub-period (May 2007 - May 2009)

The nuclear production follows the seasonal variations of demand which means high production during winter and low production during summer (see Figure 6). This implies "low" levels of nuclear fuel stock during winter and "high" levels of nuclear fuel stock during summer (see Figure 8). Clearly, the essentially periodical evolution of the nuclear production implies a periodical evolution for the nuclear fuel stock too. Note that the trend of the stock is above the "stock of reference" suggesting that it is not "overconsumed". However, during months of high demand when the nuclear production is important, the stock decreases significantly and reaches its "reference" value.

Similarly, the thermal production is high during winter (respectively low during summer) because of the high (respectively low) demand. In particular, the thermal production is increasing during winter (beginning from September) until it reaches its peak value November and December. It becomes decreasing in summer without however reaching its minimum value because of the very low levels of nuclear production (see Figure 7).

The price is high during months of high demand (winter) by taking its highest value through the period November - December and relatively low during months of low demand (summer). The profit is high during winter and at the beginning of spring while lower profits are realized during summer. Moreover, we can see that its value can be decomposed in a cyclical component and a linear trend which is slightly increasing.

4.3.6 Last sub-period (June 2009 - December 2009)

In figures 5 and 6, we observe that the nuclear production increases significantly during the last sub-period, especially from September to December. Particularly, during the last two months of this period (November, December), nuclear production approaches its maximum level. Inevitably, the nuclear fuel stock decreases without however being lower than the "stock of reference" (see Figure 8).

In view of the "overproduction" of the nuclear units, the demand is totally covered by the nuclear production with only exception the month of November, December when demand increases significantly and the participation of the thermal production is necessary. Therefore, the thermal production is very low becoming zero during the majority of the months of the last sub-period except of the period November - December (see Figure 5, Figure 7).

[27]The red dotted line indicates the "regularized" price level when nuclear is the marginal technology.

[28]Recall that the mark-up rate is taken equal to zero.

Nuclear is mainly the marginal technology and determines the price at the last sub-period. For this reason, we see that the price and the profit reach their lowest levels during this period. This is noticed even at the end of the last sub-period since the low participation of the thermal production leads to a low price and hence to a low profit in November, December (see Figure 9).

4.3.7 The duration of marginality of nuclear and thermal generation technology

We deduce that thermal is the marginal technology during most of the months of period T while nuclear is marginal only at the end of period T. The thermal production is used almost always to satisfy global demand and maximize the instantaneous monthly profit despite the fact that the nuclear production is remunerated above its "real" marginal cost (b_{nuc}) when it is marginal due to the "regularization" of the merit order price. The valorization of the nuclear production could imply a more important period of marginality for nuclear since producers are no more penalized by low prices (e.g. run only the nuclear units during periods of low demand). Nevertheless, this is not the case when the producers do not know how to manage optimally the resevoir in a market-based electricity system and thus, they choose a short-term horizon of operation which however makes them "short-sighted" with respect to future profits. Therefore, the numerical results regarding the duration of marginality of nuclear provide only an indication of this duration without certainty of how it will evolve in the case of a yearly or multi-annual optimization. This constitutes a limit of the short-term profit maximization problem.

4.3.8 General remarks

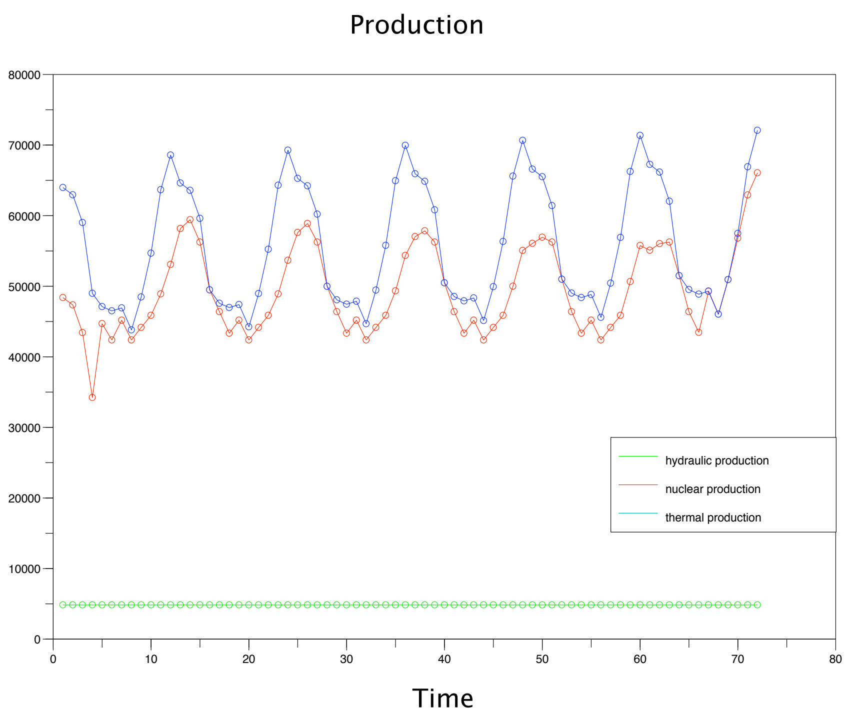

Figure 10: Simulated hydro(run-of-river)/nuclear/thermal production (in MW) (T=72)

In view of Remark 3.2, we obtain unicity of solutions regarding the thermal component but considering the other variables which do not influence the current monthly profit the complete solution is not automatically unique.

To end, if we modify the length of the time horizon of the model (i.e. $T \geqslant 36$ months), the behaviour of the producer does not change. The evolution of the nuclear and thermal production during the first and the last sub-period as well as the fundamentally periodical evolution of the production during the medium sub-period are the same (e.g. for $T = 72$, see Figure 10).

5 Conclusion

In this paper, we looked at the question of the optimal management of a flexible nuclear set (like the French set) in a competitive electricity system on a short-term basis. Flexible nuclear plants respond to variations in demand as a result of the design of modern nuclear reactors and of the significant share of nuclear in the national energy mix of a country. The originality of our research consists of the assumption that the nuclear fuel functions as a "reservoir" of energy in the medium-term horizon. This key feature of nuclear fuel as a "reservoir" is based on the periodical shutdown of nuclear reactors for fuel reloading. The consideration of the periodical interruptions of production to reload nuclear reactors with fuel made immediately our model complex. Their timing and frequency determined the modelling of the generating units by specifying their essential inherent characteristics (available nuclear capacity, month of reloading) that play a crucial role in the choice of the optimal production.

In view of the complexity of the nuclear generation technology and the introduction of competition in a rather monopolistic electricity market, the simultaneous consideration of all these components (abundance of nuclear, flexibility, competitiveness, nuclear fuel reservoir) within our model is not straightforward on a technical, mathematical and economical level. Therefore, an enormous work concerning the modelling of the parameters and the numerical calibration of our model has been realized in this paper. A number of assumptions and simplifications needed to be made in order to overcome the potential difficulties concerning the construction of the model and the resolution of the optimization problems resulted in it. Obviously, these assumptions limited our modelling to some extent, however they were necessary in order to reach to the existence of solutions. The choice of reasoning in months[29] rather than weeks is a compromise between refinement of the model and computational capacity. The obtained theoretical and numerical results are intrinsic to our model and any deviations from reality are observed mainly because nuclear managers and in particular the French nuclear operator (EDF) does not take into account our considerations. For example, as stated in the report of CRE in 2007, nuclear has been the marginal technology during periods of low demand meaning that thermal has not been the marginal technology over the entire year as it is suggested by the numerical results of the optimal short-term production problem. Note that a finer management horizon of the reservoir e.g. a week instead of a month could show periods during which nuclear is the marginal technology. Economically, these results coming from the different approaches proposed in our model and in which the system operator may be interested provide insights in order to supply conclusions for policy and industry.

Three different approaches were distinguished in order to determine an equilibrium of the optimal short-term production problem. In our first approach, we showed that there are theoretical and numerical difficulties of calculating an equilibrium by looking at the seasonal pattern

[29] We also find this reasoning in articles which study the optimal management of hydro-reservoirs in mixed hydro-thermal electricity systems (e.g. Arellano (2004), Bushnell (1998)).

of price as a result of the behaviour of supply vis-à-vis the merit order price and of the inter-temporel management of the nuclear fuel stock. We do not claim that this behaviour will persist if we choose marginal cost functions different than those of our model. However, our choice of marginal cost functions (nuclear and thermal) being a satisfactory approximation of the marginal costs of generation technologies as they appear in the merit order could give some insights with respect to the potential barriers of determining an equilibrium. Thus, we considered the inter-temporal behaviour of demand in the computation of an equilibrium. In our second approach, we observed (via a numerical simulation) that when the equality between supply and demand is taken into account only for the month t in the optimal short-term production problem, there exists a month during which the production constraints and the supply-demand equilibrium constraint can not be respected simultaneously. Therefore, a high level of "short-sightedness" with respect to future demand is intolerable in a market where nuclear has a dominant position and it leads to a failure to compute an equilibrium. This also implies that any level of "short-sightedness" regarding nuclear capacity investments in future months is not acceptable in order to avoid future disruptions on supply. For this reason, in our last approach, the equality between supply and demand is sufficiently anticipated in future months in order to manage the current use of the resevoir and an equilibrium is calculated within a numerical model.

From a mathematical perspective, we showed that the discontinuity (decrease) of the merit order price induced by the high marginal cost of thermal and the low marginal cost of nuclear results in a discontinuity (decrease) of producer's current monthly profit. The discontinuity of the merit order price is well-known and does not result in an absence of equilibrium in the case of a static model. Here, the dynamic character of our model resulting from the modelling of the nuclear fuel stock as a reservoir brings up the difficulty of finding an equilibrium theoretically and numerically in view of the discontinuity of the price and hence of the profit.

To treat the problem of discontinuity of the merit order price within our numerical model, we "regularized" our economical problem (discontinuous problem) through the "regularization" of the merit order price. We showed that this difficulty to find theoretically and numerically an equilibrium has economic consequences since it leads to overpay electricity during months of nuclear marginality. The "regularization" of the merit order price refers to an "opportunity" cost which is added to the marginal cost of nuclear. The "opportunity" cost then represents a compensation of producers for using only their nuclear capacities to meet demand instead of calling thermal generation technologies (coal, gas). Therefore, they benefit from a higher market price which helps to amortize their high fixed costs. This can be seen in the monitoring report of the French energy regulator (CRE) in 2007 which validates the use of nuclear capacities given that the use is based on a calculation of "opportunity" cost (which is not accessible/verifiable). According to this report, on the day-ahead market, the price reflected the valorization, decided by the historical operator (EDF), of nuclear production when nuclear is the marginal technology. The level of this valorization is generally higher than the marginal cost of nuclear production. Thus, a producer, even dominant, may legitimately seek the optimization of its income given that it does not constitute an abuse of dominant position or price manipulation. Within our numerical example, we concluded that the set of solutions of the economical problem (discontinuous problem) is empty. Hence, we concentrated our analysis on the solution of an approximate problem (continuous problem), the "regularized" problem, by proving that the value of both optimization problems (the "regularized" problem and the economical problem) is the same. This numerical solution is only an "approximate" solution of our economical problem.

The "regularization" of the merit order price erased "irregularities" i.e. the discontinuity of the price and of the profit and led to a “regularized” optimal production problem, in which we found an equilibrium. Thereby, we reached a satisfying situation which permits to obtain an equilibrium. The fact that if an economic phenomenon is not “regular”, it is not intellectually satisfying can be found for example in Balasko’s work regarding the theory of general equilibrium (Balasko (1988)). More precisely, Balasko mentions that, within a “regular” economy, we can find several properties of an equilibrium (e.g. continuity, existence and stability of equilibrium with respect to the parameters that define the economy) that we hope to verify in all economies since they are desirable for an equilibrium (insofar as they are not verified, infinitely more complex and also difficult to manage phenomena occur).

In view of the results coming from the resolution of the “regularized” problem (continuous problem), the period during which nuclear is marginal is shorter than one could expect given the “regularization” of the merit order price and the abundance of nuclear in the energy mix. Basically, we observed that producers use thermal capacities to maximize their current monthly profit and meet demand during almost the whole time horizon of the model. Hence, market price is determined almost always, even during seasons of low demand (summer), by the thermal marginal cost. The partial “short-sightedness” that characterizes producers with regard to future profit optimization does not permit them to use capacities (nuclear and thermal) in a more efficient way (e.g. using only nuclear capacities during summer).

To conclude, in this paper, we determined the optimal production behaviour by looking at the short-term (monthly) horizon of operation of the nuclear fuel reservoir. This mode of operation is not based on the direct optimization of the production over the entire period T. It could, however, correspond to the prudent behaviour of a nuclear set quitting the monopoly era and discovering step by step how flexible nuclear plants are operated in a competitive electricity market. Practically, this work permitted to model the nuclear fuel reservoir in presence of fossil fuel technologies (coal, gas) and then to identify and deal with a large number of mathematical, technical and computational difficulties regarding the modelling of the optimal production problem and its theoretical and numerical resolution in order to find an equilibrium (e.g. number of optimization constraints, number of optimization variables, price discontinuity, data set, computational algorithms, etc.). Despite this complex work, we verified the existence of an optimal production trajectory and we calculated it. We also became aware of the consequences of a partially “myopic” (short-sighted) approach regarding the optimal production behaviour and hence the interest of proceeding with the “non-myopic” case. Consequently, we conclude that this work is fundamental for the future analysis of the inter-temporal optimization as it would result from the full optimization of production during the whole time horizon of the model. No doubt that market based management of flexible nuclear plants would then like to look at determining a global optimum of the optimal production problem. This further analysis is studied in a companion paper.

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