# Ambiguity, Agency Relationships and Adverse Selection

Gérard Mondello

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## Summary

*This paper applies to adverse selection theory the advances made in the field of ambiguity theory. It shows that i) a relevant second-best contract induces no production distortion considering the efficient agent as in the standard case. But the principal has to pay a higher information rent compared to the standard case; ii) This is due to the level of transfer paid to the inefficient agent which is higher than under the complete information system. The above results are reached when the agent has neither fully optimistic nor optimistic beliefs. When, he feels an extreme feeling then, the information rent and second best transfers are inside bounds similar to the SEU case; iv) as a consequence, the principal has to adopt a flexible behavior consisting in acquiring new information for becoming either entirely optimistic or pessimistic to minimize transfers and information rent in the proposed delegation contract.*

**Keywords:** Asymmetric information, agency theory, adverse selection, uncertainty, ambiguity theory, irreversibility, information arrival.

**JEL:**D80, D82, D86

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# I. INTRODUCTION

.

The present paper aims at applying to classic adverse selection theory recent progresses made by ambiguity theory[1]. Ambiguous choices appear when agents face situations characterized by “unmeasurable uncertainties”[2] following Frank Knight, i.e. uncertain events that no probability distribution can predict.

Adverse selection arises when a principal considers farming out a charge to an agent. Benefiting from increasing returns (division of work) or performing efficiently several tasks may motivate the delegation choice. This applies to all varieties of contractual relationships between two entities (corporate and workers, public utilities and firms, etc.). This process generates information asymmetries because the agents can easily hide private information from the principal (true ability, cost structure technologies, knowledge, etc.). However, an efficient use of economic resources involves restoring information symmetry between all parties and, the principal must design some incentive schemes to reach this goal. In classical adverse selection models, this involves allocating rent information to most proficient agents, and then inducing them to provide the first-best level of services (Laffont and Martimort (2002)).

To study the consequences of introducing “unmeasurable uncertainty” in the adverse selection scheme, we use the simplest formulation of Laffont and Martimort (2002), i.e. their one-shot adverse-selection model This is a contractual framework characterized by no-repetition, no-adjustment process. Hence, this paper’s main goal is characterizing the information rent extraction process under “true” uncertainty.

With the standard view, both principal and agent are rational and they maximize a Savage Expected Utility function (SEU). In its simplest version, the model considers that the agent is either efficient or inefficient and the principal ignores which kind of agent he meets. However, by assumption, the proportion between efficient and inefficient agent is common knowledge. Considering “real” uncertainty involves that a unique probability distribution cannot summarize the principal’s beliefs. This view comes from Ellsberg (1961) who showed that Savage’s axioms lead to paradoxical choices[3]. In our model, the principal forms beliefs

[1] The references about these advances are presented in the following of the introduction and in section 1 and appendix 1.
[2] Following the Frank Knight’s terminology (in Knight (1921)).
[3] See Teitelbaum (2007) for a review of the following literature.

about the probability distribution of the states of nature. The principal's objective function is a specific Choquet integral i.e. a"neo-additive capacity" as described by Chateauneuf Eichberger and Grant (2007), or still, Eichberger, Kelsey and Schipper (2005). This objective function is the weighted sum of the maximum value, the minimum one and the average value that the principal can expect from his transaction with the agent and this function replaces the standard SEU.

Under information asymmetry, the principal cannot directly enforce the first best contracts. To deter the efficient agent to mimic the inefficient one and to grasp some higher level of transfer against some low quality services, the principal accepts to pay him an informational rent. This is a second-best contract that induces the efficient agent to supply his first-best service. Substituting neo-capacity to standard SEU comes from the fact that the principal faces ambiguous beliefs and may feel either optimism or pessimism about the agent's efficiency. The paper shows then:

- First, considering agent's efficiency, a relevant second-best contract creates no production distortion likewise the standard case. However, unlike this case, the information rent to pay is much higher.
- Second, paradoxically, concerning the inefficient agent, the information rent due to transfers is higher than the first-best situation.
- Third, when the agent feels an extreme feeling (fully optimistic or pessimistic) then, the information rent and second best transfers lie inside bounds similar to the SEU case.
- Fourth, from the above results lead to consider that the principal has to adopt a flexible behavior that consists in acquiring of new information before choosing the contract terms. The aim is to become either fully optimistic or pessimistic and minimize both transfers (and information rent). Hence, introducing uncertainty and a neo-additive capacity as a utility function involves that the principal takes into account the opportunity costs associated with potentially irreversible choices.

Obviously, applying ambiguity for adverse selection is not that new. For instance, Wakker, Timmermans and Machielse (2007) analyze empirically insurance theory. They test how weary are agents about ambiguity. In a more theoretical dissertation, Martinez-Correa (2012) tests the impact of ambiguity on Borch-Arrow insurance markets. He shows that the level of coinsurance under ambiguity, compared to risk only, will depend on the relative importance of risk and ambiguity for each party. The differences in beliefs correspond to the ratios of the risk aversion on the ambiguity aversion parameters. These works are rather far from our purpose.

In a first part, we remind the theoretical background involved by the using of neoadditive capacities compared to standard capacities. In a second part, we describe the set of allocations that the principal can attain in spite of the lack of information he undergoes. He has to induce the agents to reveal their true nature by defining a set of incentive compatibility constraints and participation constraints. Then, in a third part, we introduce neo-additive capacities in the Principal's program. In a fourth part, we sketch the features of the informational belief system corresponding to the acquisition of new information. A fifth part concludes.

## II. Background

For the last sixty years, the subjective Savage expected utility theory (SEU) (Savage (1954)) ruled the field of uncertainty theory. SEU combines de Finetti's (1937) calculus of subjective probability and von Neumann and Morgenstern (1947)'s expected utility theory. Since its early formulation, this leadership empirical and theoretical works questioned this leadership (Allais (1952)). The critics focus on the violations of the SEU main rationality axioms. In SEU, the decision makers have personal probabilities defined on several uncertain states of nature. They express preferences on these states and the SEU axioms show under which conditions is represented a numerical expected utility. From their choices are inferred subjective probabilities.

In the early sixties, Ellsberg (1961) revitalized the distinction between risk and uncertainty (probabilities unknown) made by Keynes and Knight at the beginning of the XXth century. Ellsberg analyses the behavior of individuals who have to make similar choices facing two situations. In situation (A), an agent makes a choice among an alternative. Both events can occur under a known probability distribution. In situation B, the alternative is the same but the probability distribution is unknown. Most of the time, the agents prefer to make their choice in situation (A) where the probability distribution is given rather than in (B). This situation violates the sure-thing principle of the SEU. Indeed, implicitly, even ignoring the true probability of the alternative in B, by choosing in situation A, the agent does "as" if the chance of success in B were lesser than in A. Ellsberg explains this by the agents' aversion for ambiguity. The following example shows more precisely the point.

Let us assume that a manager (principal) has the opportunity to hire the services of an agent in a location A. A priori, he does not know whether the worker is skilled enough or not.

If this last one is qualified his payoff will be $\underline{S}$, if not, he will gain $\overline{S}$, ($\underline{S} > \overline{S}$ , ($\underline{S}$, $\overline{S} \in \mathbb{R}_+$)). The principal knows only the proportion of skilled and unskilled workers in the A area. Let respectively be $\underline{p}$ and $\overline{p} = 1 - \underline{p}$, $(1 \geq \underline{p} \geq 0)$ this proportion. In a different location (B), he has the opportunity to earn the same amounts $\{\overline{S}, \underline{S},\}$, but, the proportion $(\underline{p}_f)$ between skilled and inexperienced workers$(1 - \underline{p}_f)$ is unknown. Hence, the expectation of gain for location A is $E[S/A] = \underline{p}\ \underline{S} + (1 - \underline{p})\overline{S}$ and for location B:

$$\mathrm{E}[S/B] = \underline{p}_f\ \underline{S} + (1 - \underline{p}_f)\overline{S}\ .$$

If the principal chooses A rather than B, that means that he thinks that $\mathrm{E}[\mathrm{S/A}] > E[\mathrm{S/B}]$, or still:

$$\underline{p}\ \underline{S} + \left(1 - \underline{p}\right)\overline{S} > \underline{p}_f\ \underline{S} + (1 - \underline{p}_f)\overline{S}\ or,\ \overline{S} + \underline{p}(\underline{S} - \overline{S}) > \overline{S} + \underline{p}_f(\underline{S} - \overline{S})$$

And, naturally: $\underline{p} > \underline{p}_f$.

Hence, by choosing A, implicitly, the principal allocates a lower probability to the meeting of a skilled worker in B and he has expressed his aversion for ambiguous situation.

Ellsberg (1961) shows that the usual distribution probabilities cannot represent beliefs: the agents do not use probability in a linear manner, as does it the expected utility model. Ellsberg's paradox highlights the non-additivity of "subjective probability" (or "beliefs") attributed to complementary events. The agents' preferences do not respect the principles of the Savage theory of uncertainty (pre-order principle and the sure-thing principle). Schmeidler considers this as the "first principle of bayesianism" i.e. the representation of uncertainty by a single measure of additive probability (Gilboa and Schmeidler (1989), Schmeidler (1989)). Schmeidler used Choquet expected utility (henceforth CEU), which involves representing individuals' beliefs with non-additive probabilities (or capacities). Following this approach, individuals maximize the expected value of a utility function with respect to a non-additive belief. Mathematically, this corresponds to a Choquet integral.

The Ellsberg's paradox also gave rise to empirical experiments. Laboratory tests showed no proportional relationships but distortions between losses and gains in the mind of people. In those years, the literature about uncertainty hugely grew. Gonzalez and Wu (1999) attempted to bring together theoretical and empirical studies to define the effective individuals' behavior facing uncertainty. Kahneman, and Tversky (1979), Tversky and Kahneman (1986) and 1992) or still Tversky and Fox (1995) or Tversky and Wakker (1995), Prelec (1998) deeply inspired their studies. They try to understand how individuals distort outcomes and probabilities. Hence, the agents systematically tend to overweight low probabilities and underweight high ones. A trend of theoretical works systematized these empirical approaches and simplified Schmeidler's advances. Hence, Eichberger, Kelsey and Schipper (2005) noted that the general CEU model applies hardly because of intrinsic mathematical complexity and a high number of free parameters. For instance, a capacity on a set with n elements involves $2^{n}$ parameters, while (n-1) parameters describe a probability distribution on the same set. Consequently, Chateauneuf, Eichberger and Grant (2007) and Eichberger, Kelsey and Schipper (2005) develop the concept of neoadditive capacity. Neoadditive capacity is a probability weighting function and "Neo" is the abbreviation for Non-Extremal Outcome additive. This approach helps to assimilate the contribution associated to the inverse-S-shaped probability weighting function (Gonzalez and Wu (1999). Indeed, under Choquet expected utility with a neo-additive capacity, a weighted sum of the minimum utility, the maximum utility, and the expected utility represent an agent's preferences. This relationship links together experimental analysis and theoretical approaches. Concretely, the choice of a concrete act $a$ that yields uncertain outcomes represents his preferences:

$$V_p(a) = \lambda\, M(a) + \gamma\, m(a) + (1 - \gamma - \lambda) E_p(a)$$

Where $M(a)$ represents the maximum value of the act $a$ and $m(a)$ its minimum, $E_p(a)$ the expected value. The parameters λ and γ express respectively the level of optimism and the degree of belief of the agent in the expected value of $a$. For instance consider that if $\gamma = 0$, the agent will assess his preferences by the expected value of $a$ and the maximum value:

$$V_p(a) = \lambda\, M(a) + (1 - \lambda) E_p(a)$$

Hence, $\lambda$ measures the decision-maker's level of optimism while γ expresses his pessimism degree[4]. So the agent will over-weight good and bad outcomes compared to expected utility theory. When considering neo-additive-capacities, only the best and worst outcomes are over-weighted. In appendix one, we provide a simplified version of these developments.

## III. The basic framework

Now, we consider a principal (regulator) that delegates to an agent the production of $q$ positive units of a good. Its value is $S(q)$ where:

- $S'(q) > 0$, (increasing in $q$), and negative or null in the second derivative $S''(q) \leq 0$. Thus, $S(q)$ is concave with $S(0) = 0$. As usual in model in this kind, there are no fixed

[4] See Teitelbaum (2007)

costs and for payment for the supply of $S(q)$, the agent receives an amount $t$ (transfer). The agent's production costs are assumed unobservable from the principal, but, it is common knowledge that the marginal costs can take values in the following interval:

- $\Psi = \{\underline{\theta}, \overline{\theta}\}$ with $\underline{\theta}, \overline{\theta} > 0$ and $\Delta\theta = \overline{\theta} - \underline{\theta} > 0$. The index $\underline{\theta}$ shows an efficient agent ($\underline{\theta}$-Agent) while the $\overline{\theta}$ index corresponds to an inefficient one ($\overline{\theta}$-Agent). It is assumed that the Principal meets the efficient $\underline{\theta}$-Agent with a probability of $\vartheta$ and the inefficient $\overline{\theta}$-Agent with a probability $1 - \vartheta$. The cost function that is assumed linear in $q$:
    - $C(q, \underline{\theta}) = \underline{\theta} q$ with a probability $\vartheta$ or, (1)
    - $C(q, \overline{\theta}) = \overline{\theta} q$ with a probability $1 - \vartheta$. (2)

The linearity of cost will be our working assumption.

## 1. The contracting sets under complete and incomplete information

Here, basically, a contract is an agreement by which the agent engages to supply a certain amount of goods or services to the principal in exchange for a payment (transfer). Let $\mathcal{B}$ be the set of feasible contractual allocations where:

$$\mathcal{B} = \{(q, t) : q \in \mathbb{R}_+, t \in \mathbb{R}\} \quad (3)$$

A third party can observe and verify these variables (generally, authors refer to a court of law or a specialized agency).

The usual asymmetric information framework assumes that the agent discovers his type, and then the Principal offers a contract that the agent can either accept or reject. Once accepted, the principal enforces the contract. This is the timing of the contractual relationship.

### *The complete Information Optimal Contract*

The complete information between the principal and the agent model constitutes the benchmark of this agency relationship. From the first order conditions of the social welfare function ($W^{CI}$), ($W^{CI}(q, \theta) = S(q) - C(q, \theta)$, $\theta \in \Psi$) the production levels define as:

- For an efficient agent: $S'\left(\underline{q}^*\right) = C'\left(\underline{q}^*, \underline{\theta}\right) = \underline{\theta}$ (4)
- For an inefficient one: $S'\left(\overline{q}^*\right) = C'\left(\overline{q}^*, \overline{\theta}\right) = \overline{\theta}$ (5)

Depending on the agent's nature (efficient or not), under a complete information regime, the production level will be either $\underline{q}^*$ or $\overline{q}^*$, for non negative social functions:

$$W^{CI}\left(\underline{q}^*, \underline{\theta}\right) = S\left(\underline{q}^*\right) - C\left(\underline{q}^*, \underline{\theta}\right) \text{ or } W^{CI}\left(\underline{q}^*, \underline{\theta}\right) = S\left(\overline{q}^*\right) - C\left(\overline{q}^*, \overline{\theta}\right)$$

The condition that the social value of production achieved with an efficient agent is always higher than an inefficient one, i.e. :

$$S\left(\underline{q}^{*}\right) - \underline{\theta}\,\underline{q}^{*} \geqq S(\overline{q}^{*}) - \underline{\theta}\overline{q}^{*} \geqq S(\overline{q}^{*}) - \overline{\theta}\overline{q}^{*} \quad (6)$$

This is a direct result of $\Delta\theta > 0$.

An important issue in what follows is that $\underline{q}^{*} > \overline{q}^{*}$, i.e. the optimal production of the efficient agent is greater than the one of the inefficient agent. This, because the principal's marginal value of output is decreasing. Implementing the first best production levels entails for the principal to make a take-it-or leave-it offer to the agent which involves a zero profit opportunity. Afterward, the efficient contracts under asymmetric information write as $\mathcal{B}^{\mathrm{AI}*} = \left\{(\underline{t}^{*}, \underline{q}^{*}) \text{ and } (\overline{t}^{*}, \overline{q}^{*})\right\} \sqsubseteq \mathcal{B},$ will have to meet the participation constraints:

$$\underline{t} - \underline{\theta}\,\underline{q} \geqq 0 \quad (7)$$

$$\overline{t} - \overline{\theta}\,\overline{q} \geqq 0 \quad (8)$$

(with equality respected for an optimal contract).

Constraints (7) and (8) are called the " agent's participation constraints" and mean that the principal has to supply that level of transfer which insures to the agent a utility level at least as high as the agent could obtain without doing anything. Hence an optimal contract under complete information involves that $\underline{t}^{*} = \underline{\theta}\,\underline{q}^{*}$ and $\overline{t}^{*} = \overline{\theta}\overline{q}^{*}$.

***The asymmetric information case***

We assume now that the efficiency type of the agent $\Psi = \{\underline{\theta}, \overline{\theta}\}$ is private information. It is well known that the set of efficient contracts $\mathcal{B}^{\mathrm{CI}*}$ defined under complete information cannot be implemented because the efficient type could simulate the behavior of the inefficient one and, when profitable, to get an informational rent $(\overline{t}^{*} - \underline{\theta}\overline{q}^{*})$. Consequently, the efficient allocation set under asymmetric information $\mathcal{B}^{\mathrm{AI}*}$ has to satisfy the following incentive compatibility constraints:

$$\underline{t} - \underline{\theta}\,\underline{q} \geqq \overline{t} - \underline{\theta}\overline{q} \quad (9)$$

$$\overline{t} - \overline{\theta}\,\overline{q} \geqq \underline{t} - \overline{\theta}\,\underline{q} \quad (10)$$

The condition for accepting a menu is to fulfill the following participation constraints:

$$\underline{t} - \underline{\theta}\,\underline{q} \geqq 0 \quad (11)$$

$$\overline{t} - \overline{\theta}\,\overline{q} \geqq 0 \quad (12)$$

Asymmetric information adds more constraints on the allocation of resources and a menu of contract is incentive feasible if it satisfies the incentive compatibility constraints (9) and (10) and the participation ones (11) and (12). When adding (9) and (10) we get

$\underline{t} - \underline{\theta}\, \underline{q} + \bar{t} - \bar{\theta}\, \bar{q} \geqq \bar{t} - \underline{\theta}\bar{q} + \underline{t} - \bar{\theta}\, \underline{q}$ and$-\underline{\theta}\, \underline{q} - \bar{\theta}\, \bar{q} \geqq -\underline{\theta}\bar{q} + -\bar{\theta}\underline{q}$ and $(\bar{\theta} - \underline{\theta})\, \underline{q} \geqq (\bar{\theta} - \underline{\theta})\bar{q}$ and, naturally:

$$\underline{q} \geqq \bar{q} \tag{13}$$

This well-known result will be useful in the following. Relationship (13) is an enforcement condition. Under complete information, the information rent level is null. Consequently, if respectively, $\underline{U}$ and $\bar{U}$ are the efficient and the inefficient agents' utility level, then, $\underline{U}^* = \underline{t}^* - \underline{\theta}\, \underline{q}^* = 0$ and $\bar{U}^* = \bar{t}^* - \bar{\theta}\bar{q}^* = 0$. Therefore, the agents cannot benefit from extra-rents extracted under asymmetric information. This is no longer the case under asymmetric information. The efficient agent will get the following utility level when he decides to mimic the inefficient one:

$$\underline{U} = \bar{t} - \underline{\theta}\bar{q} = \bar{t} - \bar{\theta}\, \bar{q} + \Delta\theta\bar{q} = \bar{U} + \Delta\theta\bar{q} \tag{14}$$

This relationship means that the information rent is carried out by the informational gain of the agent over the principal. Each category of agent can get the following information rent:

$$\underline{U} = \underline{t} - \underline{\theta}\, \underline{q} \tag{15}$$

$$\bar{U} = \bar{t} - \bar{\theta}\, \bar{q} \tag{16}$$

These writings (15) and (16) will be used in the following of the argument.

## 2. The Principal's choice under ambiguity

We consider now that the principal has beliefs represented by neo-capacities as defined above in the background of the study. Neo-capacities play a similar role to a subjective probability in the expected theory of utility.

***The Choquet utility expectation function of the principal***

Coming back to considerations about uncertainty, we define the utility function of the principal as a Choquet expected utility. This utility writes as the weighted sum of the maximum utility, the minimum utility, and the expected utility:

$$V_p(a) = \lambda\, M(a) + \gamma\, m(a) + (1 - \gamma - \lambda)E_p(a)$$

Appendix 1 recalls the conditions that lead to build neo-additive capacities and adapted Choquet integrals. To apply this to our model, we have to consider the space of states $\Psi = \{\underline{\theta}, \bar{\theta}\}$ to which is associated the skills of the agents Consequently, under asymmetric information the outcome space will be:

$\mathrm{X} = \{M(a), m(a)\}$. Then, in what follows (by abuse), $\mathrm{X} = \{M(q, \underline{\theta}), m(q, \overline{\theta})\}$

$$M(q, \underline{\theta}) = S(\underline{q}) - \underline{t} \tag{17}$$

$$m(\overline{\theta}) = S(\overline{q}) - \overline{t} \tag{18}$$

The regulator's beliefs about the nature of the agent are given by a neoadditive capacity μ based on the prior probability distribution: $p = \{\vartheta(\underline{\theta}), \vartheta(\overline{\theta})\}$, where $\vartheta(\overline{\theta}) = 1 - \vartheta(\underline{\theta})$ and in the following $\vartheta(\overline{\theta}) = 1 - \vartheta$ and $\vartheta(\underline{\theta}) = \vartheta$.

$$\mu(X) = \lambda M(\underline{q}) + \gamma m(\overline{q}) + (1 - \gamma - \lambda) p(X) \tag{19}$$

Where $\lambda, \gamma$ are real numbers such that $0 \leq \gamma \leq 1$ and $0 \leq \lambda \leq 1 - \gamma$. The regulator is optimistic when $\gamma = 0$ (and when $\lambda = 1$ he is extremely optimistic). Putting it otherwise, he gives lesser weight to the probability of revealing the inefficient agent and he is pessimistic whenever $\lambda = 0$ (extremely pessimistic with $\gamma = 1$). When $\lambda = \gamma = 0$, the capacity is additive and corresponds to the expected utility function. Then, the Choquet expected utility of the regulator is:

$$V_p(X) = \lambda M(\underline{q}) + \gamma m(\overline{q}) + (1 - \gamma - \lambda) E_p(X) \tag{20}$$

Or still,

$$V_p(X) = \lambda \left(S(\underline{q}) - \underline{t}\right) + \gamma\left(S(\overline{q}) - \overline{t}\right) + (1 - \gamma - \lambda)\left(\vartheta\left(S(\underline{q}) - \underline{t}\right) + (1 - \vartheta)\left(S(\overline{q}) - \overline{t}\right)\right) \tag{20'}$$

With a neo-additive capacity, the Choquet utility is the weighted average of the minimum, the maximum and the average payoff. For Eichberger, Kelsey and Schipper (2005, p. 6): "*Intuitively a neo-additive capacity describes a situation in which the individual believes the likelihood of events is described by the additive probability measure: However (s)he lacks confidence in this belief. In part (s)he reacts to this in an optimistic way measured by $\lambda$ and in part the reaction is pessimistic, measured by $\gamma$ .*"

***The principal's program***

For heuristic reasons, the principal's program writes as:

$$Max_{\{(\underline{q}, \underline{t}), (\overline{q}, \overline{t})\}} \left\{\lambda \left(S(\underline{q}) - \underline{t}\right) + \gamma\left(S(\overline{q}) - \overline{t}\right) + (1 - \gamma - \lambda)\left(\vartheta\left(S(\underline{q}) - \underline{t}\right) + (1 - \vartheta)\left(S(\overline{q}) - \overline{t}\right)\right)\right\} \tag{21}$$

Under the incentive and participation constraints (9) to (12).

Using the following change of variable $\underline{U} = \underline{t} - \underline{\theta}\,\underline{q}$ and $\overline{U} = \overline{t} - \overline{\theta}\,\overline{q}$) in (15) and (16), (Laffont and Martimort(2002)), the program writes now:

$$\underset{\{(\underline{q},\underline{U}),(\overline{q},\overline{U})\}}{\text{Max}}\{V_p(X)\} =$$

$$\underset{\{(\underline{q},\underline{U}),(\overline{q},\overline{U})\}}{\text{Max}} \Big\{\lambda\left(S\left(\underline{q}\right) - \underline{\theta}\,\underline{q}\right) + \gamma\left(S(\overline{q}) - \overline{\theta}\,\overline{q}\right) + +(1-\gamma-\lambda)\left(\vartheta\left(S\left(\underline{q}\right) - \underline{\theta}\,\underline{q}\right) + (1-\vartheta)\left(S(\overline{q}) - \overline{\theta}\,\overline{q}\right)\right) - \overline{U}\left(\gamma + (1-\gamma-\lambda)\left((1-\vartheta)\right)\right) - \underline{U}(\lambda + (1-\gamma-\lambda)\vartheta)\Big\} \quad (22)$$

Under the modified set of constraints:

$$\underline{U} \geqq \overline{U} + \Delta\theta\overline{q} \quad (23)$$

$$\overline{U} \geqq \underline{U} - \Delta\theta\underline{q} \quad (24)$$

Under the participation constraints:

$$\underline{U} \geqq 0, \quad (25)$$

$$\overline{U} \geqq 0 \quad (26)$$

In order to solve this program, we note that the number of constraints reduces because of the ability of the $\underline{\theta}$-agent to mimic the inefficient one. This involves that the participation constraint (25) is always strictly satisfied. Then, (30) is fulfilled and combining it with (23) this involves that (25) is always strictly verified. Indeed, when a set of contracts allows an inefficient agent to reach his minimum utility level ($\overline{U} = 0$), consequently the efficient agent strictly reaches a higher level. The constraint (24) is irrelevant: the inefficient agent cannot claim that he is efficient when obviously he is not. Hence, only two constraints are relevant (23) and (26) and both are binding. Hence, we get:

$$\underline{U} = \Delta\theta\overline{q} \quad (27)$$

And,

$$\overline{U} = 0 \quad (28)$$

Replacing these values we get a reduced program:

$$\text{Max}_{\{\underline{q},\overline{q}\}}\Big\{\lambda\left(S\left(\underline{q}\right) - \underline{\theta}\,\underline{q}\right) + \gamma\left(S(\overline{q}) - \overline{\theta}\,\overline{q}\right) + (1-\gamma-\lambda)\left(\vartheta\left(S\left(\underline{q}\right) - \underline{\theta}\,\underline{q}\right) + (1-\vartheta)\left(S(\overline{q}) - \overline{\theta}\,\overline{q}\right)\right) - (\lambda + (1-\gamma-\lambda)\vartheta)\,\Delta\theta\overline{q}\Big\} \quad (29)$$

Hence, looking for the first order conditions considering $\underline{q}$ and $\overline{q}$ gives:

$$\frac{\partial V_p(X)}{\partial \underline{q}} = 0 \Rightarrow S'\left(\underline{q}^{SB}\right) = \underline{\theta} \quad (30)$$

And,

$$\frac{\partial V_p(X)}{\partial \overline{q}} = 0 \Rightarrow S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) \quad (31)$$

Consequently, if $\lambda = 0$ and $\gamma = 0$, then,

$$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta \frac{\vartheta}{1-\vartheta} \quad (31')$$

This last expression corresponds to the solution in which the principal is endowed with a Savage expected utility function (SEU-Principal in the following). Rewriting the above equation as : $(1-\vartheta)(S'(\overline{q}^{SB}) - \overline{\theta}) = \Delta\theta\,\vartheta$, as Laffont and Mortimort (2002) showed it, in this state of nature, increasing the inefficient agent's output by an infinitesimal amount *dq* raises allocative efficiency.

The SEU-principal's expected payoff improves by a term equal to the left-hand side of the equation times *dq*. Simultaneously, the extremely small change in output also increases the information rent of the efficient agent and this diminishes the principal's expected payoff by a term equal to the right-hand side above times *dq*. At this level the expected marginal efficiency cost is equal to the expected marginal cost of the rent brought about by an infinitesimal change of the inefficient type's output. Consequently, there is a tradeoff between rent extraction and efficiency. However, under true uncertainty, when the principal forms beliefs about the state of nature, the efficient rent increases at a higher level than expected in the standard case. This is the object of proposition 1 :

**Proposition 1:** *Under asymmetric information between principal and agent, when the principal is a Choquet Expected Utility maximizer, the optimal menu of contracts entails:*

*i)* *No output distortion for the efficient agent with respect to the first-best, $\underline{q}^{SB} = \underline{q}^*$, as for a SUE-Principal. However, concerning the inefficient type, there is an upward distortion, $\overline{q}^{SB} \geq \overline{q}^*$ with*

$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right)$ *where* $\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) < 0$,

*and $\underline{q}^* \geq \overline{q}^{SB} \geq \overline{q}^*$. This contrary to the SEU case,*

*ii)* *The efficient agent gets a positive information rent corresponding to:*

$$\overline{U}^{SB} = \Delta\theta\overline{q}^{SB} \quad (32)$$

*iii)* *The second-best transfers corresponds respectively to( for the efficient agent):*

$$\underline{t}^{SB} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{SB} \quad (33)$$

*And, for the inefficient one:*

$$\overline{t}^{SB} = \overline{\theta}\overline{q}^{SB} \quad (34)$$

(Proof in appendix 2).

The similarity of solutions between proposition 1 and the SEU case is only apparent. Indeed, both of them issue on no-distortion in production considering the efficient agent who supplies the first-best level. Indeed, the i) of proposition 1 shows that $\underline{q}^{SB} = \underline{q}^*$. This result was not a priori intuition. However, things change considerably with respect to the inefficient agent because the level of production supplied by this agent is higher than under the perfect information case: $\overline{q}^{SB} > \overline{q}^*$.

This situation has consequences on the level of transfers that the principal offers in each state of nature. Indeed, considering these transfers (in (33) and (34)), it appears clear that they are higher than under complete information, where from ($\overline{U}^* = \overline{t}^* - \overline{\theta}\overline{q}^* = 0$), we get $\overline{t}^* = \overline{\theta}\overline{q}^* < \overline{t}^{SB} = \overline{\theta}\overline{q}^{SB}$. From (33) we get $\underline{t}^{SB} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{SB}$ which is higher than under the SEU case because, under this regime the production supplied by the inefficient agent is below $\overline{q}^*$[5]. This argument establishes proposition 2:

**Proposition 2:** *Under asymmetric information and true uncertainty, with a Choquet Expected Utility maximizer principal, the level of transfers that the principal has to supply is higher than when the principal is a SEU maximizer.*

The proof of proposition 2 comes from the above argument.

Nevertheless, once accepted the methodological change concerning utility functions, linking the performances of both schemes is of little interest. Much more attractive is to check whether the principal could (should) not reduce the amount of the information rent to pay to the efficient agent. In order to answer to this question, we will examine the cases in which the principal feels pure or full optimism or pessimism.

**Proposition 3:** *Under asymmetric information, with a Choquet Expected Utility maximizer principal, when the principal feels either fully optimistic* ($\gamma = 0$) *or fully pessimistic* ($\lambda = 0$), *then:*

i) *The quantity supplied by the less efficient agent is less than under complete information.*

ii) *When fully optimistic, the second-best transfers corresponds respectively to( for the efficient agent):*

$$\underline{t}^{opt} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{opt} \quad (35)$$

*And, for the inefficient one:*

$$\overline{t}^{opt} = \overline{\theta}\overline{q}^{opt} \quad (36)$$

[5] See Laffont and Martimort (2002, chap.2).

(*Where* $\underline{q}^{op}$ *and* $\overline{q}^{op}$ *correspond to the second best solutions when the principal feels fully optimistic*),

*iii)* *When fully pessimistic, the second-best transfers corresponds respectively to( for the efficient agent):*

$$\underline{t}^{psm} = \underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{psm} \quad (37)$$

*And, for the inefficient one:*

$$\overline{t}^{psm} = \overline{\theta}\overline{q}^{psm} \quad (38)$$

(*Where* $\underline{q}^{psm}$ *and* $\overline{q}^{psm}$ *correspond to the second best solutions when the principal feels fully pessimistic*).

(Proof of proposition 3 in appendix 2).

Proposition 3 establishes that when the principal forms extreme beliefs such as full optimism ($\gamma = 0$) or full pessimism ($\lambda = 0$), then, as second-best, the production distortion will be of the same order than for the situation in which the Principal behaves like a SEU-Principal. Hence, this production level is lower than the first best level, which corresponds to the complete information case:

$$\overline{q}^{psm} < \overline{q}^{*} \text{ or } \overline{q}^{op} < \overline{q}^{*}.$$

For the principal this situation is highly preferable compared to the case where ($\gamma \neq 0$) and ($\lambda \neq 0$). Hence, with extreme beliefs, the transfers are lesser than with non extreme ones. The difference expresses the principal's gain if he could switch from moderated beliefs to the extreme ones. Consequently from (33) and (34) and from proposition 3 ii) and iii), we deduce (for the fully optimistic case) that for the efficient agent:

$$\underline{t}^{SB} - \underline{t}^{opt} = \underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{SB} - \left(\underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{opt}\right) = \Delta\theta\left(\overline{q}^{SB} - \overline{q}^{opt}\right) > 0 \quad (38)$$

And, for the inefficient one:

$$\overline{t}^{SB} - \overline{t}^{SB} = \overline{\theta}\,(\,\overline{q}^{SB} - \overline{q}^{\,opt}) \quad (39)$$

(The argument is the same for the fully pessimistic case).

When he must define a contract, the principal with "moderate" beliefs will incur more cost transfers compared to the fully optimistic or pessimistic situations. If, however, he could change his state of mind and move towards one of these "extreme cases", then he would restrain these costs. This could be done by waiting and/or acquiring more information. Before going further we deduce from proposition 3, the proposition 4 that states that:

**Proposition 4 :** *When* $\gamma > \lambda$, *then* $S'(\overline{q}^{op}) > S'(\overline{q}^{psm})$ and $\overline{q}^{op} < \overline{q}^{psm}$ *and the reverse for* $\gamma < \lambda$.

(Proof of proposition 4 in appendix 2).

This proposition means that the ratio of pessimism to optimism is fundamental to determine what situation will bring more advantage. This result is important for the following.

## IV. THE PRINCIPAL'S CHOICE UNDER IRREVERSIBILITY POSSIBILITY

As a result, the combination of propositions 1 to 3 issues on the theory of irreversible choices. Here irreversibility stems from the high transfer costs that the principal can incur if he proposes to agents an immediate contract rather than waiting for new information and better opportunities. Consequently, with ambiguity theory, the principal has the possibility to wait to get further information and then to propose a contract on better conditions at the right moment.

If, after acquiring new information, the principal becomes either fully optimistic or pessimistic, that means that he gets better information about the states of nature occurence. Then, he is able to minimize the efficient agent's information rent. The arrival of new information changes on a Bayesian basis the probability of states of nature (Epstein (1980). This process modifies the value of the neoadditive capacities. Then, the question is whether there exists a convergence process from the state of "normal" state of mind (little information) towards one of the two extreme states. Studying such a convergence process would deserve a full article but going further would require too much space.

Consequently, a simple illustration is given. It relies on the location choice previously developed in the first part of this article. We recall that the principal knows only the proportion of skilled and unskilled workers in the A area, i.e. respectively $\underline{p}$ and $\overline{p} = 1 - \underline{p}, (1 \geq \underline{p} \geq 0)$ this proportion. This ratio will be the prior probability on which he forms beliefs.

Let be $\lambda$ and $\gamma$ respectively the level of optimism and pessimism ( $1 \geq \gamma \geq 0$ and $1 - \gamma \geq \lambda \geq 0$). Therefore, the principal's Choquet expected utility is:

$$ChE(S) = \lambda\, \underline{S} + \gamma\, \overline{S} + (1 - \lambda - \gamma)\left(\underline{p}\underline{S} - \left(1 - \underline{p}\right)\overline{S}\right) \quad (40)$$

We make the following change variable:

$$\lambda = \alpha\beta, \gamma = \alpha(1 - \beta) \text{ and, naturally, } (1 - \lambda - \gamma) = 1 - \alpha \quad (41)$$

Then,

$$ChE(S) = \alpha\beta\, \underline{S} + \alpha(1 - \beta)\, \overline{S} + (1 - \alpha)\left(\underline{p}\underline{S} - \left(1 - \underline{p}\right)\overline{S}\right) \quad (42)$$

This writing is conforming with Chateauneuf and alii (2007) and it improves the understanding of pessimism and optimism which are represented respectively by $(1-\beta)$ and $\beta$, where $\alpha$ represents the preference for ambiguity. The higher $\alpha$, the lesser will be the weight of the linear expectation in the principal opinion. Then, $(1-\alpha)$ represents the aversion for ambiguity parameter.

We can consider that $ChE(S)$ is the expected gain for a given initial level of information in present time $k=0$, let be $\varphi_0$ this level we rewrite it as $ChE(S|\varphi_0)$.

Considering the second best optimum level we get the following level of transfers from (33) and (34), $\underline{t}^{SB} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{SB}$ and $\overline{t}^{SB} = \overline{\theta}\overline{q}^{SB}$.

Let us assume that the principal has the opportunity of waiting another period (or stage 2) in order to get new information. By assumption, with it, he can become either fully optimistic $\beta = 1$ or fully pessimistic $\beta = 0$. His level of aversion/preference for ambiguity can stay identical (fixed $\alpha$). At stage 2, the information $\varphi$ will be considered as either positive $\varphi^+$ if he becomes fully optimistic, or negative $\varphi^-$ (fully pessimistic). Then with the set of initial information$\varphi^1$ on stage 1, his expected gain is:

$$\alpha\beta(\varphi^0)\,\underline{S} + \alpha(1-\beta(\varphi^0))\,\overline{S} + (1-\alpha)\left(\underline{pS} - \left(1-\underline{p}\right)\overline{S}\right) \tag{43}$$

At stage 2 the set of information becomes $\varphi^2 = \{\varphi^-, \varphi^+\}$ and let be $\beta(\varphi)$ the level of the optimism ratio after the coming of new information with:

$$\beta(\varphi^2) = \begin{cases} 1 \text{ if } \varphi^2 = \varphi^+ \\ 0 \text{ if } \varphi^2 = \varphi^- \end{cases} \tag{44}$$

Then the expected Choquet utility for stage 1, after the arrival of new information is then:

$$ChE(S|\varphi^2) = \alpha\beta(\varphi^2)\,\underline{S} + \alpha(1-\beta(\varphi^2))\,\overline{S} + (1-\alpha)\left(\underline{pS} - \left(1-\underline{p}\right)\overline{S}\right) \tag{45}$$

For $\varphi^2 = \{\varphi^-, \varphi^+\}$

Obviously from proposition 3 ii), for the efficient agent $\underline{t}^{SB} > \underline{t}^{opt}$ and $\underline{t}^{SB} > \underline{t}^{psm}$ and, the inefficient one*:* $\overline{t}^{SB} > \overline{t}^{opt}$ and $\overline{t}^{SB} > \overline{t}^{psm}$. As a consequence, without any further assumption, the problem would be very simple, because, as we know it from proposition 2 and 3, here, it is more profitable to wait because in this case, by proposition 3, $ChE(S|\varphi) > ChE(\mathrm{S}|\varphi_0)$ for $\varphi^2 = \{\varphi^-, \varphi^+\}$. Hence, whatever the relationship between $\alpha$ and $\beta$ taking into account the change of variable (41), it is better for the principal to wait further information.

However, things change when the information becomes costly. Let $c, c > 0$, the cost of new information, this increases the transfer costs and makes the information waiting process more risky (with $\underline{S}'(c) < 0$ and $\overline{S}'(c) < 0$). For instance, by assumption we could have a situation in which:

$$ChE(S|\varphi^+) > ChE(S|\varphi_0) > ChE(S|\varphi^-) \quad (46)$$

Where :

$$ChE(S|\varphi) = \alpha\beta(\varphi)\,\underline{S}(c) + \alpha(1-\beta(\varphi))\,\overline{S}(c) + (1-\alpha)\left(\underline{p}\underline{S}(c) - \left(1-\underline{p}\right)\overline{S}(c)\right) \quad (47)$$

For $\varphi = \{\varphi^-, \varphi^+\}$

The problem transforms in a classic problem with irreversibility that can be dealt with quasi-option theory. As such, it will not be analyzed here. Indeed, we gave an over-simplified model drawn from Fisher and Arrow (1974), Fisher and Peterson (1976) or still Henry (1974) and Freeman (1985). The object is to know whether a positive quasi-option may be defined from the waiting of new information. Here, this information is supposed costly because we gave a sure alternative about the convergence of beliefs about a full optimistic situation or full pessimistic situation. In fact, the number of stages could be longer knowing that convergence towards these polar situations is not certain. The gathering of information could be organized through a Cremer and Khalil (1992) (also Crémer, Khalil and Rochet (1998)) process where the principal finds it profitable to organize competition between several agents, even though he has monopoly power and can push a single agent down to his reservation utility.

## V. CONCLUSION

Introducing "true uncertainty" and aversion/preference for ambiguity in the specific context of basic agency relationships leads to two kinds of issues. First, as expected, ambiguity theory changes somehow the standard results compared to situations that consider probability additive utility functions. Hence, regarding the second best solutions, there is no production distortion about the quantity supplied by the efficient agent as in the classic case. Nevertheless, the level of transfers and the information rent highly increases because the transfers due to the inefficient agent are higher than the one supplied under complete information. These transfers form the base of the information rent paid to the efficient agent. This change in results induces a change in the behavior of the principal and this induces the second kind of results.

Hence, second, when the principal feels "extremely" pessimistic or optimistic (respectively, the corresponding degree of optimism (or pessimism) is null) then the results are similar to the adverse selection model with a SEU-principal. Consequently, when possible, the principal could minimize the level of transfer if, some way, he could feel "extreme" beliefs. This involves that before designing a contract, it could be better to wait for new information. Therefore, a principal endowed with a Choquet expected utility has the possibility to propose an immediate contract to the agent as in the standard scheme, or to await news for a better knowledge about the future states of nature. This knowledge can drive him to become either fully optimistic (the probability of finding efficient agents increase) or pessimistic in the reverse case. If he decides to offer a delegation contract then, he knows that the level of transfer lies inside an acceptable range compared to the complete information situation. The postponement of the contract formation will depend upon the cost of information, its rate of arrival, the subjective probability of staying at the same level of uncertainty, etc.

As a conclusion, the introduction of ambiguity does not invalidate the relevancy of asymmetric information theory. It leads to consider the possibility of irreversible choices and writes it in the field of the literature dedicated to the theory of options and quasi-options because it becomes obvious that the principal has to take into account the opportunity costs of waiting. The above results apply for the simplest model with linear costs; further research will extend to situations that are more complicated.

# BIBLIOGRAPHY


Allais Maurice, " *Le comportement de l'Homme rationnel devant le risque : critique des postulats de l'École américaine* ", *Econometrica*, (1953)21: 503-546.

Arrow, Kenneth J., and Anthony C., Fisher, "Environmental Preservation, Uncertainty, and Irreversibility," *Quarterly Journal of Economics*, 88, (1974), May: 312-319.

Camerer, Colin, "Individual Decision Making'. pp. 587 – 703 in The Hand-book of Experimental Economics , edited by John. H. Hagel and Alvin. E.Roth (1995), Princeton, N.J.: Princeton University Press.

Camerer, Colin, and Martin Weber, "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity", *Journal of Risk and Uncertainty*, (1992), 5: pp.325 – 70.

Chateauneuf, Alain, Jürgen, Eichberger and Simon Grant S. "Choice under uncertainty with the best and worst in mind: Neo-additive capacities", *Journal of Economic Theory,* (2007), vol. 137, issue 1, pages 538-567.

Cicchetti, Charles J. and A. Myrick, Freeman III, "Option Demand and Consumer Surplus: Further Comment.1," *Quarterly Journal of Economics*, (1971), 85 (Aug): 528-539.

Crémer, Jacques, and Fahad Khalil, "Gathering Information before Signing a Contract," *European Economic Review*, (1992), 38: 675-682.

Crémer, J., F. Khalil and J.C. Rochet, (1998), \Strategic Information Gathering before a Contracts is Offered," *Journal of Economic Theory*, 81,163-200.

Ellsberg, Daniel, 1961. "Risk, Ambiguity, and the Savage Axioms". *Quarterly Journal of Economics* 75: 643 – 69.

Eichberger, Jürgen, David Kelsey, and Burkhart,C.Schipper,C. "Ambiguity and social interaction", (2005), Discussion paper n°59, *Department of Economics, University of California.*

Eichberger, Jürgen, David, Kelsey, Burkhart,C.Schipper, "Ambiguity and social interaction", *Oxford Economic Papers*, (2009), Volume 61, Issue 2, pp. 355-379.

Freeman, A.Myrick.III, "Supply Uncertainty, Option Price and Option Value," *Land Economics*, (1985), 6, 176-181.

Fisher, Anthony C. and Frederick, M. Peterson, "The Environment and Economics: A Survey", *Journal of Economic Literature*, (1976), 14(March): 1-33.

Fisher, Anthony C. and Krutilla, John V. "Valuing Long Run Ecological Consequences and Irreversibilities", *Journal of Environmental Economics and Management*, (1974), 1: 96-108.

Gilboa, Itzak., "Expected Utility with Purely Subjective Non-additive Probabilities", *Journal of Mathematical Economics*, (1987), 16:65 – 88.

Gilboa, Itzak, and Schmeidler David., 1989. "Maxmin Expected Utility with a Non-unique Prior", *Journal of Mathematical Economics*, 18:141 – 53

Gonzalez, Richard, and George Wu, "On the Shape of the Probability Weighting Function", *Cognitive Psychology*, (1999), 38:129–66.

Johansson, Per-O., The Economic Theory and Measurement of Environmental Benefits, (1987), Cambridge Press, Cambridge.

Henry, Claude. "Option-Values in the Economics of Irreplaceable Assets," *Review of Economic Studies:* Symposium on The Economics of Exhaustible Resources, (1974), pp.89-104.

Kahneman, Daniel, Amos Tversky. "Prospect Theory: An Analysis of Decision under Risk", *Econometrica*, (1979), 47: 263-91

Knight, Frank H., *Risk, Uncertainty and Profit*, (1921), Boston: Hougthon Mifflin.

Laffont, Jean-Jacques and David Martimort, *The Theory of Incentives: The Principal-Agent*, (2002),Model Princeton University Press.

Martinez-Correa, Jimmy, "Decisions under Risk, Uncertainty and Ambiguity: Theory and Experiments" (2012). Risk Management and Insurance Dissertations. Paper 29.

Prelec, Drazen "The Probability Weighting Function", *Econometrica*, (1998), Vol. 66, p. 497-527.

Savage, Leonard J. The Foundations of Statistics . 1954. New York: John Wiley.

Schmalensee, Richard, "Option Demand and Consumer's Surplus: Valuing Price Changes Under Uncertainty." *American Economic Review*, (1972), 62 (Dec): 813-824

Teitelbaum Joshua., "A unilateral accident model under ambiguity", *Journal of Legal Studies*, (2007), 36: 431-477.

Tversky, Amos, Craig, R. Fox,"Weighing risk and uncertainty*", Psychological Review*, (1995), 102: 269–283.

Tversky, Amos and Daniel Kahneman "Rational choice and the framing of decisions". *Journal of Business*, (1986), 59,S251–S278.

Tversky, Amos and Daniel Kahneman, D., "Advances in prospect theory: Cumulative representations of uncertainty", *Journal of Risk and Uncertainty*, (1992),,5,297–323.

Tversky, A., Derek. J., Koehler "Support theory: A non-extensional representation of subjective probability", *Psychological Review*, (1994),101: 547–567.

Tversky, A., Peter Wakker, "Risk attitudes and decision weights", Econometrica,63, (1995), 1255–1280.

von Neumann, John, and Oscar Morgenstern, *Theory of Games and Economic Behavior*. (1947), Princeton: Princeton University Press.

Wakker, Peter P. Daniëlle R. M. Timmermans Irma Machielse "The Effects of Statistical Information on Risk and Ambiguity Attitudes, and on Rational Insurance Decisions", *Management Science* (2007), 53(11), pp. 1770–1784.

Weisbrod, Burton, A. "Collective Consumption Services of Individual Consumption Goods," *Quarterly Journal of Economics*, (1964), 78 (3):471-477.

## Appendix 1

## NEO-CAPACITIES AND AMBIGUITY

The principal forms beliefs that are represented by neo-additive capacities. Before giving an understanding of this concept, it is necessary to define the notion of capacity that plays a similar role to subjective probabilities under Savage uncertainty.

A capacity is an extension of a probability, and as such is a real valued function $\tau(.)$ that assigns a real value to the set of events $\mathcal{E}$, $\tau: \mathcal{E} \rightarrow \mathbb{R}$, where $\mathcal{E}$ is the σ-algebra of events from the finite set of states of nature A (which could be assumed to be infinite too in the general case), (with $a \in A, \{a\} \in \mathcal{E}$). $\tau(.)$ to be considered as a capacity has to fulfill two conditions

i) For all $E, F \in \mathcal{E}$, and $E \subseteq F$, then $\tau(E) \subseteq \tau(F)$ as monotonicity condition and,

ii) As normalization conditions, $\tau(\emptyset) = 0$ and $\tau(A) = 1$.

Convexity of a capacity is verified by the following relationships:

$\tau(E \cup F) \geq \tau(E) + \tau(F) - \tau(E \cap F)$ (and concave in the opposite situation).

Choquet integrals are used to integrate capacities. To do that is considered a simple function f ( where simple means of finite range such that $f: A \rightarrow \mathbb{R}$, f is $\mathcal{E}$-mesurable) that takes values $\mu_1 \geq \mu_2 \dots \geq \mu_n$.

Hence, Choquet integral of a simple function f with respect to a capacity $\tau(.)$ is defined as:

$$V(f/\tau) = \int f\, d\tau = \sum_{\mu \in f(A)} \mu[\tau(\{s/f \geq \mu\}) - \tau(\{s/f > \mu\})] \qquad (16)$$

The neo-additive capacity is a special kind of capacity and it is called as such because it is additive on “non-extreme” outcomes (Neo) as defined by Chateauneuf Grant …( 2007). These authors consider that a neo-capacity and its Choquet integral are a particular parametrization of the Choquet Expected Utility. “Neo-additive capacities may be viewed as a convex combination of an additive capacity and a special capacity that only distinguishes between whether an event is impossible, possible or certain” (CEG p.540.

To build the neo-additive capacity they consider that the set of events $\mathcal{E}$ is partitioned into three subsets :

i) The set of null events $\mathcal{N}$, where $\emptyset \in \mathcal{N}$ and for $G \subset H$, and $G \in \mathcal{N}$ if $H \in \mathcal{N}$.

ii) The set of “universal events” $\mathcal{W}$, in which an event is certain to occur, (complement of each member of the set $\mathcal{N}$), $\mathcal{W} = \{E \in \mathcal{E}: A - E \in \mathcal{N}\}$

iii) The set of essential events, $\mathcal{E}^*$, in which events are neither impossible nor certain. This set is composed of the following: , $\mathcal{E}^* = \mathcal{E} - (\mathcal{N} \cup \mathcal{W})$.

We can define now the neo-additive capacity:

**Definition** (Chateauneuf and alii (2007)

*A neo additive capacity $\rho(.)$ is defined as a linear combination of*

*i) An additive belief $p(.)$, that corresponds to a finite additive probability where $p(E) = 0$ if $E \in \mathcal{N}$ and $p(B) = 1$ if $B \in \mathcal{W}$,*

*ii) A non-additive belief $\tau^1$ where $\tau^1(E) = 0$ if $E \in \mathcal{N}$ and 1 otherwise,*

*iii) A non-additive belief $\tau^0$ where $\tau^0(E) = 1$ if $E \in \mathcal{W}$ and 0 otherwise,*

*Then, for $\gamma \geq 0$ and $\lambda \geq 0$ such that $\gamma + \lambda \leq 1$, a neo-capacity writes as:*

$$\rho(E) = \gamma\, \tau^0(E) + \lambda\, \tau^1(E) + (1 - \gamma - \lambda) p(E)$$

*This for all $E \in \mathcal{E}$.*

Then, we can define the Choquet integral of a neo-capacity as a weighted sum of the minimum, the maximum and the expectation of a simple function $f: \mathcal{E} \rightarrow \mathbb{R}$ such that

$z = inf\,(f)$ if $z = f^{-1}(x: x \geq z) \in \mathcal{W}$ and for $y > z, z = f^{-1}(x: x > y) \notin \mathcal{W}$ and a similar argument is developed for $z = \sup(f)$ if $z = f^{-1}(x: x \geq z) \in \mathcal{N}$ and for $y > z, z = f^{-1}(x: x > y) \notin \mathcal{N}$. Hence, we draw the following relationship:

$$V(\, f \,/\, p, \lambda, \gamma) = \lambda.\sup(f) + \gamma \inf(f) + (1 - \gamma - \lambda) E_p(f) \qquad (.)$$

Where $\mathrm{E_p(f)}$ is the expected value of the welfare function taking into account the type of agents, and from the linearity of the Choquet integral with respect to the capacity, $\mathrm{V}\big(\,\mathrm{f}\,/\,\nu_0(.)\big) = \inf(\mathrm{f})$ and $\mathrm{V}\big(\,\mathrm{f}\,/\,\nu_1(.)\big) = \sup(\mathrm{f})$, (proof see CFG(2002, 3).

From the definition of the Choquet integral, we can derive the following results:

i) If $\gamma, \lambda = 0$, then $\rho_{\gamma,\lambda=0}(E) = E_p(f)$ (the expected utility),

ii) $\lambda = 0$, then $\rho_{\lambda=0}(E) = \gamma \inf(f) + (1 - \gamma) E_p(f)$ (pure pessimism),

iii) $\gamma = 0$, then $\rho_{\gamma=0}(E) = \lambda \sup(f) + (1 - \lambda) E_p(f)$ (pure optimism),

iv) $\gamma + \lambda = 1$, $\rho_{\gamma+\lambda=1}(E) = \lambda.\sup(f) + \gamma \inf(f)$ (Hurwitz criteria).

All that means that the Choquet integral of a neo-additive utility may be represented as a weighted average of the expected utility, the maximum utility and the minimum one. That means that neo-additive capacities allow to take into consideration situatios in which the probability of events is described by the additive probability distribution $p(E)$, however the agent lacks confidence in this belief and he may feel either optimistic, (measured by $\lambda$), or pessimistic (measured by $\gamma$).

## Appendix 2:

## PROOFS OF THE PROPOSITIONS

**Proposition 1:** *Under asymmetric information between principal and agent, when the principal is a Choquet Expected Utility maximizer, the optimal menu of contracts entails:*

*i)* *No output distortion for the efficient agent with respect to the first-best, $\underline{q}^{SB} = \underline{q}^*$, as for a SUE-Principal. However, concerning the inefficient type, there is an upward distortion, $\overline{q}^{SB} \geq \overline{q}^*$ with*
$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right)$ *where* $\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) < 0$,
*and* $\underline{q}^* \geq \overline{q}^{SB} \geq \overline{q}^*$. *This Contrary to the SEU case,*

*ii)* *The efficient agent gets a positive information rent corresponding to:*

$$\overline{U}^{SB} = \Delta\theta\overline{q}^{SB} \tag{A2.1}$$

*iii)* *The second-best transfers corresponds respectively to( for the efficient agent):*

$$\underline{t}^{SB} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{SB} \tag{A2.2}$$

*And, for the inefficient one:*

$$\overline{t}^{SB} = \overline{\theta}\overline{q}^{SB} \tag{A2.3}$$

**Proof:**

This proof requires several steps:

**Proof of i)**

- *Concerning the efficient agent*, considering $S'\left(\underline{q}^{SB}\right) = \underline{\theta}$, we verify immediately that $S'\left(\underline{q}^{SB}\right) = S'\left(\underline{q}^*\right) = \underline{\theta}$ then $\underline{q}^{SB} = \underline{q}^*$.
- *Concerning the inefficient type*, from, $S(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right)$ we can verify that that:

$$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) \leq \overline{\theta}.$$

To show this point, let us assume that: $\overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) > \overline{\theta}$. Developing this expression, recalling that: $\overline{\theta} - \underline{\theta} = \Delta\theta$, we get: $\frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta} > 1$ or still, $\vartheta < \frac{\lambda}{(-1+\gamma+\lambda)}$.

Let us notice that $\frac{\lambda}{(-1+\gamma+\lambda)} < 0$ because $(\gamma + \lambda) \leq 1$ then $(-1 + \gamma + \lambda) < 0$. However, $\vartheta$ is a probability and $\vartheta \in [0,1]$ and the above relationship is impossible. Then,

$$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) \leq \overline{\theta}$$

That means, because by assumption that $S'(q)$ is a decreasing function ($S''(q) \leq 0$), that $S'(\overline{q}^{SB}) \leq S'(\overline{q}^*)$, then $\overline{q}^{SB} \geq \overline{q}^*$.

- It remains to show that $\underline{q}^* \geq \overline{q}^{SB}$

We apply the same argument than previously and we examine the conditions for which here

$$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\ \lambda + (-1+\gamma+\lambda)\vartheta}\right) < \underline{\theta}$$

Developing this expression, recalling that: $\overline{\theta} - \underline{\theta} = \Delta\theta$, leads to study:

$\Delta\theta + \Delta\theta\left(-1 + \frac{1}{1-\ \lambda+(-1+\gamma+\lambda)\vartheta}\right) < 0$, or,

$$\left(\frac{\Delta\theta}{1-\ \lambda + (-1+\gamma+\lambda)\vartheta}\right) < 0$$

As the numerator is always positive, we have to study the conditions for a negative denominator: $1 -\ \lambda + (-1+\gamma+\lambda)\vartheta < 0$ . Putting it otherwise, the condition becomes:

$$\frac{(1-\ \lambda)}{(1-(\gamma+\lambda)} < \vartheta$$

Obviously this cannot hold, because $(1-(\gamma+\lambda) < (1-\ \lambda)$ that involves that

$$\frac{(1-\ \lambda)}{(1-(\gamma+\lambda)} > 1$$

This is contradictory with the fact that $\vartheta$ is a probability then,

$$\frac{(1-\ \lambda)}{(1-(\gamma+\lambda)} \geq \vartheta$$

That means that $S'(\overline{q}^{SB}) \geq \underline{\theta}$, and consequently, $\underline{q}^* \geq \overline{q}^{SB}$, QED.

**Proofs of ii) and iii):**

Calculus are made on the definition of information rents and the level of constraints in a similar mode achieved by Laffont Martimort (2002) chap.2 section 2.

**Proposition 3:** *Under asymmetric information, with a Choquet Expected Utility maximizer principal, when the principal feels either fully optimistic* ($\gamma = 0$) *or fully pessimistic* ($\lambda = 0$), *then:*

*i)* *The quantity supplied by the less efficient agent is less than under complete information.*

*ii)* *When fully optimistic, the second-best transfers corresponds respectively to( for the efficient agent):*

$$\underline{t}^{opt} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{opt} \quad \text{(B 2.1)}$$

*And, for the inefficient one:*

$$\overline{t}^{opt} = \overline{\theta}\overline{q}^{opt} \quad \text{(B 2.2)}$$

(*Where* $\underline{q}^{op}$ *and* $\overline{q}^{op}$ *correspond to the second best solutions when the principal feels fully optimistic*),

*iii)* *When fully pessimistic, the second-best transfers corresponds respectively to( for the efficient agent):*

$$\underline{t}^{psm} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{psm} \quad \text{(B 2.3)}$$

*And, for the inefficient one:*

$$\overline{t}^{psm} = \overline{\theta}\overline{q}^{psm} \quad \text{(B 2.4)}$$

(*Where* $\underline{q}^{psm}$ *and* $\overline{q}^{psm}$ *correspond to the second best solutions when the principal feels fully pessimistic*).

**Proof of proposition 3:**

Concerning point i) we proceed in two steps. The first one analyzes the situation in which the principal feels fully or purely optimistic and the second one in which he is pessimistic.

***a) Pure optimism***

A Principal purely optimistic means that $\gamma = 0$ (then, because $\lambda \leq 1-\gamma$, $\lambda$ can take values between 0 and 1). We keep the same program, constraints and simplification over constraints as before and we get:

$$\underset{\{\underline{q},\overline{q}\}}{Max}\Big\{\lambda\,\left(S\left(\underline{q}\right)-\underline{\theta}\,\underline{q}\right)+(1-\lambda)\left(\vartheta\left(S\left(\underline{q}\right)-\,\underline{\theta}\,\underline{q}\,\right)+(1-\vartheta)\left(S(\overline{q})-\overline{\theta}\,\overline{q}\right)\right)$$
$$-\left(\lambda+(1-\lambda)\vartheta\right)\Delta\theta\overline{q}\Big\}$$

Looking for second best solutions under the assumption of a pure optimistic principal leads to the following results. Hence, the first order conditions considering $\underline{q}$ and $\overline{q}$ give:

$$\frac{\partial V_p(X)}{\partial\underline{q}}=0\Rightarrow S'\left(\underline{q}^{op}\right)=\underline{\theta}$$

And,

$$\frac{\partial V_p(X)}{\partial\overline{q}}=0\Rightarrow S'\left(\overline{q}^{op}\right)=\overline{\theta}+\frac{v}{(1-v)(1-\lambda)}\Delta\theta+\frac{\lambda}{1-\lambda}\Delta\theta.$$

Where $\underline{q}^{op}$ and $\overline{q}^{op}$ correspond to the second best solutions when the principal feels fully optimistic. Naturally concerning the efficient agent we can see that, as in the general case there is no distortion in production and$S'\left(\underline{q}^{op}\right)=\underline{\theta}$, and as a consequence, $\underline{q}^{op}=\underline{q}^{*}$.

We can check immediately that, $\frac{v\Delta\theta}{(1-v)(1-\lambda)}\Delta\theta>0$ and $\frac{\lambda}{1-\lambda}\Delta\theta$ and, as a consequence $\overline{q}^{op}<\overline{q}^{*}$ because $S'\left(\overline{q}^{op}\right)>S'\left(\overline{q}^{*}\right)=\overline{\theta}+\frac{v\Delta\theta}{1-v}\Delta\theta$. The first term of the proposition 3 is proved. This result means that under ambiguity and pure optimism the level of informational distortion considering the inefficient agent is higher than under the lack of ambiguity of the standard result. The above relationship means that when the principal feels fully optimistic, he expects that the inefficient agent is more inefficient than under SEU.

***b) Pure pessimism***

We proceed alike as above and we consider that the principal is now absolutely pessimistic ($\lambda = 0$).The objective function becomes then:

$$\underset{\{\underline{q},\overline{q}\}}{Max}\Big\{\gamma\left(S(\overline{q})-\overline{\theta}\,\overline{q}\right)+(1-\gamma)\left(\underline{\vartheta}\left(S\left(\underline{q}\right)-\,\underline{\theta}\,\underline{q}\,\right)+\left(1-\underline{\vartheta}\right)\left(S(\overline{q})-\overline{\theta}\,\overline{q}\right)\right)$$
$$-\left((1-\gamma)\underline{\vartheta}\right)\Delta\theta\overline{q}\Big\}$$

Looking for second best solutions under the assumption of a pure optimistic principal leads to the following results. The first order conditions considering $\underline{q}$ and $\overline{q}$ give:

$$\frac{\partial V_p(X)}{\partial \underline{q}} = 0 \Rightarrow S'\left(\underline{q}^{psm}\right) = \underline{\theta}$$

And ,

$$\frac{\partial V_p(X)}{\partial \overline{q}} = 0 \Rightarrow S'\left(\overline{q}^{psm}\right) = \overline{\theta} + \frac{\vartheta(1-\gamma)\Delta\theta}{1+\vartheta(-1+\gamma)}$$

Where $\underline{q}^{psm}$ and $\overline{q}^{psm}$ correspond to the second best solutions when the principal feels fully pessimistic. As previously and the general case, concerning the efficient agent there is no distorsion in production and $S'\left(\underline{q}^{psm}\right) = \underline{\theta}$, and as a consequence, $\underline{q}^{psm} = \underline{q}^{*}$. However, concerning the inefficient agent, distorsion exists and we have to check if $\frac{\vartheta(1-\gamma)\Delta\theta}{1+\vartheta(-1+\gamma)} > 0$. It appears immediately that naturally, $\vartheta(1-\gamma)\Delta\theta > 0$, then then to verify the relationships, we must have $1 + \underline{\vartheta}(-1+\gamma) > 0$. Let us assume that $1 + \underline{\vartheta}(-1+\gamma) \leq 0$. That means that $1 \leq (1-\gamma)\,\vartheta$ which is a contradiction because $0 < \gamma < 1$ and $0 < \vartheta < 1$. Hence, $\frac{\underline{\vartheta}(1-\gamma)\Delta\theta}{1+\underline{\vartheta}(-1+\gamma)} > 0$ and, consequently

$S'\left(\overline{q}^{psm}\right) > S'\left(\overline{q}^{*}\right)$ and $\overline{q}^{psm} < \overline{q}^{*}$. The second term of the proposition is then proved.

ii) Concerning the proof of ii) and iii) we proceed in a similar way as for point iii) of proposition 1.

**Proposition 4 :** *When $\gamma > \lambda$, then $S'\left(\overline{q}^{op}\right) > S'\left(\overline{q}^{psm}\right)$ and $\overline{q}^{op} < \overline{q}^{psm}$ and the reverse for $\gamma < \lambda$.*

**Proof in appendix 2:**

Let us assume that $S'\left(\overline{q}^{op}\right) > S'\left(\overline{q}^{psm}\right)$ or

$$\overline{\theta} + \frac{v}{(1-v)(1-\lambda)}\Delta\theta + \frac{\lambda}{1-\lambda}\Delta\theta > \overline{\theta} + \frac{\vartheta(1-\gamma)\Delta\theta}{1+\vartheta(-1+\gamma)}$$

After simplification we get:

$$v > -\frac{\lambda}{\gamma-\lambda}$$

This is true for $\gamma > \lambda$. When, $\gamma < \lambda$, then $S'\left(\overline{q}^{op}\right) < S'\left(\overline{q}^{psm}\right)$ QED.