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The Economics of Performance Appraisals

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Abstract

Performance appraisals have become a widespread practice in OECD member countries. However, whereas the problem of constructing an optimal contract with subjective evaluation receives a lot of attention, firm-level performance appraisals are strikingly left outside of economic theory. The purpose of this paper is threefold: first, to theoretically define what performance appraisals are; second, to analyze the effects of incentive contracts on effort and wage using performance appraisals; and third, to theoretically quantify the selection effects driven by the implementation of performance appraisals.

Keywords: Performance appraisals, Super-modularity, Selection, Work intensification.

JEL Classification: M50, M54.

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> Performance ratings were given on the DOGNUT scale: Distinguished, Outstanding, Good, Needs Improvement, Unsatisfactory, and Too New to Evaluate. Receiving a Distinguished rating increased the raise about four to five percentage points compared to receiving a Good rating; it increased the bonus even more for Grade and Hay employees, though it had a smaller effect for PAQ employees.
>
> Gibbs and Hendricks (2004, pages 82/83).

1 Introduction

In the classical Principal-Agent model, if the Agent's level of effort is unobservable by the Principal, then the optimal contract depends on a verifiable measure of the Agent's performance. Most papers use the output of the task performed by the Agent as a verifiable measure. These papers construct the optimal wage as an increasing function of output. However, according to Macleod and Parent (1999) and Prendergast (1999), few firms use such a mechanism in the real world and employ instead a mechanism in which the employees' bonus depends on a subjective evaluation of their performance by the Principal. Such mechanisms (in which the Agent's reward does not depend on a verifiable measure of performance but instead on a subjective one) are called Principal-Agent models with subjective evaluation. In these models, the subjective performance measure is usually modeled as a signal.

We analyze in this paper the case where subjective evaluations are achieved through so-called performance appraisals, which are widespread in most OECD countries. For instance, in the US over 90 percent of large organizations employ some performance appraisal system (Murphy and Cleveland, 1991), and over 75 percent of state employment agencies require annual performance appraisals (Seldon et al., 2001).

While performance appraisals are one of the most widely researched areas in industrial psychology (see for instance Catano et al., 2007), they remain so limited in economic literature that, to the best of our knowledge, there is no specific economic definition of performance appraisals.

This is quite surprising since a significant number of empirical papers in economic literature deal directly or indirectly with performance appraisals (see

for instance Medoff and Abraham, 1980; Baker et al., 1994; Gibbs and Hendricks, 2004; Brown and Heywood, 2005; Diaye et al., 2008; Addison and Belfield, 2008; Gibbs et al., 2009; Engellandt and Riphahn, 2011).

Of course the existing models of subjective evaluation, especially the models of MacLeod (2003) and Levin (2003), provide keys to understanding the conditions under which incentive mechanisms with performance appraisals exist.

But these models will simply define performance appraisals as a way to produce two signals, one from the Principal and the other from the Agent. MacLeod (2003) denoted them respectively t and s. The Principal may condition the Agent's wage to the signal t depending on the correlation between t and s (for a perfect correlation for instance, the Agent's wage depends on t).

However it seems in real world that the opinion of the Principal always prevails and that the only thing the Agent can do, if he does not agree with the Principal's assessment about his performance, is not to sign the performance appraisals form.

Another difference with MacLeod (2003) is that in his analysis the Principal can cheat in his announcement of t. In our paper, we do not explicitly take into account the fact that the Principal can cheat. We simply assume that he can make an error in evaluating the performance of the Agent. We show (see claim 1) that a necessary condition for the existence of a compensation system based on performance appraisals is what we call efficiency in detecting the required level of effort. Suppose that the contract is designed in order to induce the Agent to make an effort of level $\bar{k}$. The performance appraisals system is efficient in detecting this level of effort $\bar{k}$ if the probability that the Agent's level of effort is evaluated (by the Principal) as being $\bar{k}$ when his true level is $\bar{k}$ is strictly higher than the probability that the Agent's level of effort is evaluated as being $\bar{k}$ when his true level is $k' < \bar{k}$. Our claim 1 requires the performance appraisals system to be consistent whatever the conditional probabilities for the Agent as being evaluated with effort $\bar{k}$ when the true effort is k. If this is not the case then the Agent's incentive constraints will not be fulfilled. Hence our analysis implicitly allows the Principal to manipulate the conditional probabilities for the Agent as being evaluated with effort $\bar{k}$ when the true effort is k, as long as the announced conditional probabilities remain consistent.

Let $Pr(I = i|K = k)$ be the probability (which is common knowledge) that the Agent's level of effort is evaluated as being i when his true level of effort is k;

I being the Agent's overall score that is provided by the performance appraisals system. This score is the signal announced by the Principal and we propose in this paper a way of constructing this signal. More precisely I is defined as the result of an aggregation of scores, one for each criterion entering in the process of the Agent's effort assessment. A score associated with a criterion s is found by the following way: (1) the true level v^s of s is a private information of the Agent; (2) however the Principal can use a direct mechanism in order to derive $rf(v^s)$ the score associated with the criterion s; (3) $rf(v^s)$ corresponds therefore to the subjective assessment by the Principal of the Agent's level of s.

In sum, we define performance appraisals as an institution, in the sense of the theory of mechanism design. Moreover, we suggest to use the Choquet integral in order to aggregate the scores.

We show also (claims 3 and 4) that performance appraisals lead to work intensification in the sense that some Agents provide efforts above the maximal effort 2 designated by the Principal. This intensification of work is profitable to these Agents because it increases their probability to have a good evaluation; and also profitable (when the revenue derived from the production process is sufficiently high) to the Principal because it increases his expected profit.

Let us define the optimal contract with performance appraisals as the contract derived from the Principal's expected profit program fulfilling the Agent's incentive and participation constraints. We deduce (claim 2) from this contract that in incentive schemes using performance appraisals, not only is the probability that the Agent gets his wage independent from the result of the task he performs, but also this wage itself is independent from the result of the task he performs. Both depend on the employee's evaluation and not (like in classical incentive schemes) on the outcome of the task he has performed. In other words, performance appraisals permit the managers to take into account the context, since the success of the task does not depend only on the effort provided by the employees (Baker et al., 1994).

Suppose that a concern of firms implementing performance appraisals is to be better off when compared to the case of classical incentive schemes. This could be the case if a firm moves from a classical incentive scheme to an incentive scheme using performance appraisals. Such firms could implement a contract we call FNWO (Firms Not Worse-Off) contract defined as the contract derived from the Principal's expected profit program fulfilling the Agent's incentive and

participation constraints and the constraint that the Principal's expected profit is at least equal to his expected profit in the classical incentive scheme. We show that a FNWO contract does not always exist and, when it does exist, we show that (for a wide class of utility functions including CARA or DARA functions) firms design performance appraisal schemes in such a way that the employees get a higher wage (compared to the case of a classical incentive scheme) but the probability of getting this wage is smaller. Hence workers who are able to provide an effort above the required effort in order to increase the probability of getting their wage, are attracted by such firms. In other words there is a high selection of workers in FNWO firms.

Let us for instance take a firm moving from a classical incentive scheme to a scheme using performance appraisals. Let us assume that this firm decides to implement a FNWO contract. Hence the ex-post wage of the employees will be higher compared to the starting situation. However, the probability of getting this wage will be weaker. As a consequence, some workers may be worse-off compared to the starting situation, and therefore may leave this firm. Only workers whose effort disutility is weak enough to allow them to make an effort above the required effort will be better-off.

However firms can use another type of contract that we call Pareto-Optimal contract (PO), defined as the contract derived from the Principal's expected profit program fulfilling the Agent's incentive and participation constraints, the constraint that the Principal's expected profit is at least equal to his expected utility in the classical incentive scheme, and the constraint that the Agent's expected utility is at least equal to his expected utility in the classical incentive scheme.

In the PO contract, firms design the performance appraisal contracts in such a way that both firms and their employees are not worse-off compared with the classical incentive scheme. A PO contract does not always exist and when it does exist, we show that (for a wide class of utility functions including CARA or DARA functions) firms design the performance appraisals scheme in such a way that the employees' wage is smaller than the wages of employees working in classical incentive schemes; however, the probability of getting this wage is higher. As a consequence, compared to the case of FNWO contracts, in PO contracts, the selection of low effort disutility workers is weaker.

Finally our model can be extended in several directions. For instance in order

to catch the "dynamic" aspects of performance appraisals (like their effects on employees' careers), we can consider the evaluation process as a repeated game instead of a static one. Another extension of the model could be to allow the situations in which the Agent considers that the performance appraisals system is not fair.

This paper includes five sections. The second section is devoted to the presentation of the characteristics of the production technology (submodular or supermodular), the Principal and the Agent. In section three we present our model of performance appraisals in the case of individual production. The fourth section is devoted to the comparison of the classical incentive contract and the performance appraisals contract. Finally the fifth section concludes. All proofs are relegated to appendix A.

2 Basic setting

We use (see Che and Yoo, 2001) a Principal-Agent framework in which production requires only one task. This task is performed by the Agent, who makes an effort decision unobservable by the Principal. Production, that is, the outcome of the task, is a random variable X that can either succeed ($X = 1$) or fail ($X = 0$) giving respectively R or 0 payoffs to the Principal. The Agent's individual effort, denoted K (K is a random variable from the Principal's stand point), belongs to the set $\Theta = \{0, 1, 2\}$ which is the set of *reasonable* levels of effort. In other words, the maximal level of effort that the Principal can reasonably incite the Agent to supply is $K = 2$. However the general set of levels of effort is $\Theta_g = \{0, 1, 2, 3, 4..., \overline{m}\}$. This level, $K = 2$, may be stated by the law (for instance the number of working hours per week), by a negotiation with the unions or by a direct negotiation with the employees. In any case, it is assumed to be known (and accepted) by the Agent.

Let $Pr(X = 1|K = k) = q_k$ and $Pr(X = 0|K = k) = 1 - q_k$, or the conditional probability of success of the task given the Agent's level of effort k, and the conditional probability of failure of the task given the Agent's level of effort k, respectively.

The Principal is risk-neutral and the Agent is risk-averse with a utility function $U_\theta(r, k) = u(r) - v_\theta(k)$, where u is an increasing and concave function such that $u(0) = 0$, $u(r) \geq 0$ $\forall r \geq 0$, and the Agent's disutility function of effort

writes:

$$v_\theta(k) = \begin{cases} ke & \text{if } k \in \Theta = \{0,1,2\} \\ \\ (2+\theta k)e \quad \text{with} \quad 0 \leq \theta \leq 1 \quad \text{if} \quad k \in \{3,4,...\overline{m}\} \end{cases} \tag{2.1}$$

where the unit of effort noted e is strictly positive and θ is the Agent's index of effort disutility. θ is unobservable by the Principal, however, he or she knows that θ follows a uniform law with support $[0,1]$.

Hence the probability that the Agent has a type θ inferior to δ is δ.

The interpretation of the disutility function $v_\theta(k)$ is the following. Over $\Theta = \{0,1,2\}$ the disutility increased with effort by e unit. However between an effort level $k=2$ and effort level $k=3$, there is a jump $3\theta e$ in the cost of effort for the Agent. However this jump decreases with θ, the Agent's index of effort disutility. For instance for $\theta = 1$, the cost of effort jumps by 3e between effort 2 and effort 3; for $\theta = 0.5$ the cost of effort jumps only by 1.5e and for $\theta = 0$ the disutility of effort remains at level 2. The shape of $v_\theta(k)$ allows us to take into account the fact that it is costly for the agent to move from effort 2 to effort 3 and that not all agents are able to do it.

Table 1: Some values of $v_\theta(k)$ according to θ and k

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$\theta = 1$	e	$2e$	$5e$	$6e$	$7e$
$\theta = 0.5$	e	$2e$	$3.5e$	$4e$	$4.5e$
$\theta = 0.25$	e	$2e$	$2.75e$	$3e$	$3.25e$
$\theta = 0$	e	$2e$	$2e$	$2e$	$2e$

Of course the Principal cannot construct a contract $(w_k, k \geq 3)$ because a court of law may find from w_k the implicit level of effort (which is here unreasonable since it is higher than 2) that the Principal wants the Agent to provide. Likewise the Principal cannot construct a θ-dependent contract because it induces the Principal to offer a θ-dependent wage.

Moreover, for the sake of simplicity, we normalize the Agent's reservation utility to be equal to zero. Finally we state[1] that $1 > q_2 > q_1 > q_0 \geq 0$ and that

[1] Over $\Theta_g = \{0,1,2,3,4...,\overline{m}\}$, we have $1 > q_{\bar{m}} > ... > q_3 > q_2 > q_1 > q_0 \geq 0$.

$2q_1 \geq q_2$ (this condition guarantees that the participation constraint is fulfilled).

The relationship between effort and production is an important feature of production technology. As usual among the literature, we consider two alternatives: production is either super-modular over Θ ($q_2 - q_1 \geq q_1 - q_0$), or sub-modular over Θ ($q_2 - q_1 \leq q_1 - q_0$).

Broadly speaking when production is super-modular (respectively sub-modular), the marginal return on effort is increasing (respectively decreasing) with the level of effort.

Through the paper we assume that the income R is high enough so that the Principal wants the Agent to provide the effort $k = 2$.

It is straightforward to see (Che and Yoo, 2001) that if the production technology is super-modular then the Principal will implement the contract $(w_2^* = u^{-1}(2e/(q_2 - q_0));\ k = 2)$, which is designed in such a way that the Agent provides the maximal level of effort $k = 2$. Likewise if the production technology is sub-modular, then the Principal implements the contract $(w_2^* = u^{-1}(e/(q_2 - q_1));\ k = 2)$.

3 Incentive mechanism with performance appraisal

3.1 Definition of Performance appraisal

Let us first define what a performance appraisal is. This definition is important because it allows us to embed the analysis of performance appraisals into the more general framework of implementation theory framework.

Let us assume that the Principal determines, after seeking advice from the Agent, a finite set $S = \{1, \cdots, \bar{s}\}$ of criteria that he considers important for estimating the Agent's effort. This assumption of an agreement between the Principal and the Agent concerning the set of criteria is important because it avoids any fairness perception problems from the Agent's standpoint. Indeed, according to organizational psychologists (see for instance Bretz et al., 1992), the perceived fairness of the performance appraisal system is the main reason why performance appraisals fail within firms. Our model is therefore compatible with empirical findings (see Addison and Belfield, 2008) that the existence of unions in a firm has a positive impact on the likelihood of the implementation of a performance appraisal system in this firm.

Let us call $v^s \in \Xi^s$ the true level of the Agent's criterion s ($s = 1$ to $\bar{s}$ the cardinality of S). We assume that it is private information namely; that v^s is a random variable which, realized, is only observable by the Agent (ie. the Principal does not observe it). According to the revelation principle, we can restrict ourselves to the following direct mechanism $\Sigma^s = (\Xi^s; rf)$ where rf is a result function:

$$rf : \Xi^s \to \Xi^s$$

$$v^s \mapsto rf(v^s)$$

We call performance appraisal, the indirect mechanism $\Sigma = (\Sigma^1, ..., \Sigma^s, ...\Sigma^{\bar{s}}; ag)$ where the Σ^s ($s = 1, ..., \bar{s}$) are direct mechanisms and ag is an aggregation function of marks resulting from the assessment of each criterion:

$$ag : \prod_{s=1}^{\bar{s}} \Xi^s \to M$$

$$(rf(v^1), ..., rf(v^{\bar{s}})) \mapsto ag(rf(v^1), ..., rf(v^{\bar{s}}))$$

where M is a finite marks set (i.e, a set of overall performance rating).

In practice, firms take $\Xi^s = M$, $\forall s = 1 \cdots \bar{s}$. For instance $\Xi^s = M = \{U, N, G, O, D\}$, where the rating U means that the work performance is unacceptable, the rating N means that the work performance needs improvement (i.e., serious effort is needed to improve performance), the rating G means that the work performance is good (i.e., the work performance consistently meets the standards of performance for the position), the rating O means that the work performance is outstanding (i.e., is consistently above the standard of performance for the position), and rating D means that the work performance is distinguished.

From a theoretical standpoint, let us note that since the purpose of the performance appraisal is to get a subjective evaluation of effort[2], then the Principal can directly take $M = \Theta = \{0, 1, 2\}$ or $M = \Theta_g = \{0, 1, 2, 3, \cdots, \overline{m}\}$.

[2]We have stated in our definition that the purpose of the performance appraisal is to measure the Agent's effort; however, this definition is still valid, with a slight change, if the Principal uses performance appraisals in order to measure ex-ante (i.e, before the production takes place) some qualitative criteria like team spirit, innate ability etc.

3.2 How to aggregate ?

Our definition of performance appraisals includes an aggregation function denoted ag. The question is how to aggregate ?

From a practical standpoint there usually exists in firms no formal way to aggregate the scores and the managers follow their own judgment. Of course this method is not transparent and implies that an Agent may have two different overall scores if he is evaluated by two different persons, even if the two evaluators agree on the score per criterion. As a consequence, firms sometimes provide some guidelines to their managers about the distribution of overall scores they have to reach[3](for instance the distribution of overall scores among the employees has to be α_0 percent of score U, α_1 percent of score N, α_2 percent of score G, α_3 percent of score O, α_4 percent of score D, with $\sum_{i=0}^{4} \alpha_i = 1$) or firms recalculate all the overall scores of the employees in such a way that the associated distribution follows a normal law.

This question of the aggregation function is not trivial since it shows that given a set of criteria, saying that evaluation is subjective means two things. Firstly it means that evaluation has to reflect the preference of the Principal, and secondly that the result function $rf(v^s)$ is at last resort the Principal's subjective evaluation of v^s.

The weight of one criterion in the aggregation process will be completely subjective and will reflect the Principal own opinion concerning the weight of this criterion (that is, it will reflect his preference). For instance, suppose that $\#S = 2$ and $M = \{U, N, G, O, D\}$, where the rating U means that the work performance is unacceptable, the rating N means that the work performance needs improvement, the rating G means that the work performance consistently meets the standards of performance for the position, the rating O means that the work performance is consistently above the standard of performance for the position, and rating D means that the work performance is distinguished. Then if the Principal prefers (U, O) to (O, U) it means that from the Principal's standpoint, criterion 2 weighs more than criterion 1.

Let $\succcurlyeq$ be the Principal's preference over $\prod_{s=1}^{\bar{s}} M$. We will denote by $\succ$ the asymmetric component (strict preference) of $\succcurlyeq$ and by $\sim$ the symmetric part

[3]Another reason could be the alleged propensity of managers to give an average rating to employees.

(indifference). We require the aggregation function ag to fulfill the preference $\succcurlyeq$ of the Principal:

$$\forall\, x,\, y \in M^{\#S}\,,\; x \succcurlyeq y \Leftrightarrow ag(x) \geq ag(y) \tag{3.1}$$

The marks could be quantitative (M the set of marks is included in $\mathbb{R}_+$) or qualitative (M the set of marks is an ordinal scale).

If the marks are elements of $\mathbb{R}_+$ then a first obvious aggregation function is the weighted sum: $ag(x) = \sum_{i=1}^{i=\bar{s}} pr_i x_i$ with $pr_i \geq 0$ and $\sum_{i=1}^{\bar{s}} pr_i = 1$.

However the weighted sum is known not to be able to describe some situations in which some alternatives have a totally satisfactory score on one criterion ($pr_i = 1$) and not acceptable on the others ($pr_i = 0\,,\ \forall i \neq i_0$) and $M = \{0, 1, 2, 3, 4\}$. Let us consider the following three alternatives $x = (2, 2)$, $y = (0, 4)$ and $z = (4, 0)$. Suppose that $x \succ y \sim z$. $ag(x) = 2pr_1 + 2pr_2$, $ag(y) = 0pr_1 + 4pr_2$ and $ag(z) = 4pr_1 + 0pr_2$. Since $y \sim z$ then $ag(y) = ag(z)$; leading to $pr_1 = pr_2$. However $x \succ y$ then it must be the case that $ag(x) > ag(y)$. That is $2pr_1 + 2pr_2 > 4pr_1$. Since $pr_1 = pr_2$ then we must have $4pr_1 > 4pr_1$, which is of course impossible.

Hence when M the set of marks is included in $\mathbb{R}_+$, the so-called Choquet integral (Choquet, 1953) is more appropriate. Loosely speaking, the Choquet integral permits us to define weights not only on each criterion, but also on groups of criteria.

Before defining the Choquet integral, let us state the following definition of a Choquet capacity. To simplify the exposition of our paper, this definition is based on the set S of criteria and on the set M of marks. Of course the original definition of a capacity and of a Choquet integral are based over much more general sets.

Let $P(S)$ be the power set of S the set of criteria. A function μ defined from $P(S)$ to $\mathbb{R}$ is a Choquet capacity if $\mu(\varnothing) = 0$ and $\mu(A) \leq \mu(B)$, $\forall A \subseteq B$. We will work here with normalized capacity, that is a capacity μ such that $\mu(S) = 1$.

A Choquet capacity could be additive (i.e. $\forall A, B,\, A \cap B = \varnothing,\, A, B \subseteq S$ then $\mu(A \cup B) = \mu(A) + \mu(B)$) or non additive. Hence a Choquet capacity is a kind of non necessarily additive probability.

Let x be a vector of marks belonging to $M^{\bar{s}}$. We suggest to define the aggregation function $ag(x)$ as the Choquet integral of x relative to a capacity μ:

$$ag(x) = \sum_{s=1}^{\bar{s}} [x_{\sigma(s)} - x_{\sigma(s-1)}] \mu(\{\sigma(s), \cdots, \sigma(\bar{s})\}) \tag{3.2}$$

where σ is a permutation on S such that $x_{\sigma(s)} \leq \cdots \leq x_{\sigma(\bar{s})}$ and $x_{\sigma(0)}$ is arbitrarily stated at 0.

The notion of the Choquet integral is more general than the one of the weighted sum in the sense that if the capacity is additive then the Choquet integral coincides with the weighted sum $ag(x) = \sum_{s=1}^{\bar{s}} \mu(\{s\}) x_s$.

If we apply the formula to the above example with $x = (2, 2)$, $y = (0, 4)$ and $z = (4, 0)$, then we get $ag(x) = 2\mu(\{1, 2\})$, $ag(y) = 4\mu(\{2\})$ and $ag(z) = 4\mu(\{1\})$. Since the Principal's preference is $x \succ y \sim z$, we obtain $2\mu(\{1, 2\}) > 4\mu(\{1\}) = 4\mu(\{2\})$, that is $\mu(\{1, 2\}) > 2\mu(\{1\}) = 2\mu(\{2\})$. Since the capacity is normalized and $S = \{1, 2\}$ then we have $\mu(\{1,\ 2\}) = 1$ and $\mu(\{1\}) = \mu(\{2\}) < \frac{1}{2}$. The capacity here is over-additive, pointing to the fact that the Principal prefers to have a balanced mark over the two criteria.

The use of the Choquet integral requires that the Principal examines all the interactions between all criteria, and this can be costly as the number of criteria increases. Fortunately there exists some ways to reduce the algorithmic complexity of computing the Choquet capacities, for instance by using a particular Choquet capacity introduced by Grabisch (1997). This capacity is called 2-additive capacity and is defined as a capacity μ whose Möbius transform[4] m^μ satisfies $m^\mu(A) = 0$ for all $A \subseteq S$ with $\#A > 2$ and there exists $A \subseteq S$ with $\#A = 2$ and $m^\mu(A) \neq 0$.

Suppose now that the marks are qualitative (for instance $M = \{U, N, G, O, D\}$). An obvious way to deal with this issue is to transform the ordinal problem into a cardinal one, and then use Choquet integral. For instance we can convert the qualitative marks into quantitative ones (in our example, by stating that U corresponds to 0, N to 1, G to 2, O to 3 and D to 4).

[4]The Möbius transform of a capacity μ is the solution m^μ of the equation $\mu(A) = \sum_{B \subseteq A} m^\mu(B)\ \forall A \subseteq S$. This (unique) solution writes $m^\mu(A) = \sum_{B \subseteq A} (-1)^{\#A \setminus B} \mu(B)$.

3.3 The Program of the Principal

Given the performance appraisal $\Sigma = (\Sigma^1, ..., \Sigma^{\bar{s}}; ag)$, let $ag(rf(v^1), \cdots, rf(v^{\bar{s}}))$ be the rating I obtained by the Agent after the production process. Of course $I \in \{0, 1, 2\} = \Theta$. If $I = 2$, then the Agent receives a wage $\overline{p}$ and gets nothing otherwise (remembering that we have stated the Agent's reservation utility to be 0). Let us call p the wage variable. We thus have:

$$p = \begin{cases} \overline{p} & \text{if } I = 2 \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

Of course the participation constraint (equation 3.4) and the incentive constraints (equations 3.5 and 3.6) must be fulfilled:

$$E[u(\overline{p})|k = 2] - 2e \geq 0 \tag{3.4}$$

$$E[u(\overline{p})|k = 2] - 2e \geq E[u(\overline{p})|k = 1] - e \tag{3.5}$$

$$E[u(\overline{p})|k = 2] - 2e \geq E[u(\overline{p})|k = 0] \tag{3.6}$$

Let us state $\gamma_k^i = Pr(I = i|K = k)$, the probability that the Agent's level of effort is evaluated as being i when his true level of effort is k. We assume this probability to be common knowledge and his choice (by the Principal) to take place before the Principal constructs the optimal contract. The probabilities γ_k^i represent in some sense the technology of evaluation.

The probability $\gamma_k^i = Pr(I = i|K = k)$ is influenced by whether the mechanism Σ permits us to reveal the true level of effort of the Agent, and this depends on the truth revealing property of the mechanisms $\Sigma^1, ..., \Sigma^{\bar{s}}$ and on the aggregation function that is used by the Principal. If the mechanism Σ permits us to reveal the true level of effort of the Agent then $\gamma_k^k = 1$ and $\gamma_k^{k'} = 0\,, \forall\, k' \neq k$. As a consequence the Principal is in the trivial case of perfect information.

In order to avoid triviality, let us assume that the mechanism Σ does not reveal the true level of effort of the Agent and that the Principal makes some errors in his evaluation of the Agent's level of effort.

The inequalities (3.4) to (3.6) lead to the following program Pmax:

$$
\left|
\begin{array}{ll}
\underset{\overline{p}}{Max}\ q_2 R - \gamma_2^2 \overline{p} & \\
\text{under the constraints:} & \\
(3.4) \quad \gamma_2^2 u(\overline{p}) - 2e \geq 0 & \\
(3.5) \quad (\gamma_2^2 - \gamma_1^2) u(\overline{p}) - e \geq 0 & \\
(3.6) \quad (\gamma_2^2 - \gamma_0^2) u(\overline{p}) - 2e \geq 0 &
\end{array}
\right. \qquad \text{(Pmax)}
$$

The information included in the performance appraisal Σ is summarized into the concept of an evaluation system, denoted $\mathbb{E}$, and defined as a profile $\mathbb{E} = \left(\Theta = \{0, 1, 2\}, \{\gamma_k^i\}_{k,i}\right)$.

Like we said above, the probabilities γ_k^i represent in some sense the technology of evaluation. Of course it may be the case that some technologies are more efficient in detecting a given level of effort than others.

We say that an evaluation system $\mathbb{E}$ is efficient in detecting a level of effort $k \in \Theta$ if the probability that the Agent's level of effort is evaluated as being k when his true level of effort is strictly higher than the same probability when his true level of effort is $k^{'} < k : \gamma_k^k > \gamma_{k^{'}}^k$, $\forall k^{'} < k$, $k^{'} \in \Theta$.

The first claim of this paper is that the efficiency of the Evaluation System for the level of effort 2 is a necessary condition for the existence of a solution for the program Pmax.

Hence even when there is no fairness perception problem, an performance appraisal system may fail if the associated technology of detection is not efficient and destroys incentives.

As a consequence, we are going to restrict ourselves through the paper to the class of Evaluation Systems which are efficient for the level of effort 2.

What is the optimal contract ? In order to reduce the number of solutions (see appendix A: proof of the claim 2) from the maximizing program, let us state the following hypothesis that the probability of evaluating the Agent's effort equal at 2 when, it is in fact lower than 2, is independent from the true level of effort: $\gamma_{k^{'}}^2 = \gamma^{'}$, $\forall k^{'} \in \{0, 1\}$.

Our second claim states that the optimal contract in incentive mechanisms using performance appraisal is $\{\overline{p}, \gamma_2^2, \gamma_0^2\}$ with:

$$\overline{p} = u^{-1}\Big(\frac{2e}{\gamma_2^2 - \gamma_0^2}\Big) \tag{3.7}$$

Hence in incentive schemes using performance appraisals, not only is the probability γ_2^2 for the Agent to get his wage independent from the result of the task he performs, but also this wage $\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ itself is independent from the result of the task he performs (see Appendix B for a numerical example). In other words, performance appraisals permit the managers to take into account the context, since the success of the task does not depend only on the effort provided by the employees (Baker et al., 1994).

3.4 θ-efficient performance appraisals systems and work intensification

If performance appraisals permit managers to take into account the context of work (usually in favor of the Agent), they could induce the Agent to work beyond the reasonable maximal level $k = 2$.

Why might an Agent of type θ increase his level of effort beyond the reasonable maximal level $k = 2$? Indeed one can note that the Agent's wage $\overline{p} = u^{-1}\big[\frac{2e}{\gamma_2^2 - \gamma_0^2}\big]$ does not change even if the Agent increases his effort beyond the required level $k = 2$.

In order to answer this question, let us state the following definition.

An (efficient) evaluation system $\mathbb{E} = (\Theta = \{0, 1, 2\}, \{\gamma_k^i\}_{k,i})$ can detect a given level of effort $k^{'} \geq 3$ if:

1. $\gamma_{k'}^2$ is well defined, and
2. $\gamma_{k'}^2$ respects the following pseudo-monotony condition:

$$\gamma_{k'}^2 \geq \gamma_k^2 \quad , \forall k \in \{2, 3, 4, ...k^{'} - 1\}.$$

For instance, the levels of effort associated with the ratings O and D can be considered as very high levels of effort.

If there exists a level of effort greater than 3 that an evaluation system can detect, then we will say that this evaluation system is θ-efficient.

It is easy to see that if the evaluation system is θ-efficient, then, when the Agent increases his level of effort beyond the required level $k = 2$, the probability of being detected $\Big($and thus of receiving the wage $\overline{p} = u^{-1}\big[\frac{2e}{\gamma_2^2-\gamma_0^2}\big]\Big)$ as having provided a level of effort $k = 2$, increases.

To illustrate, let us consider an Agent of type $\theta = 0$. If he provides an effort $k = \overline{m}$, he has the same effort disutility as when providing an effort $k = 2$ with almost the certainty of getting the wage $\overline{p} = u^{-1}\big(\frac{2e}{\gamma_2^2-\gamma_0^2}\big)$. Such an Agent is rational, since by increasing his level of effort beyond the maximal reasonable level ($k = 2$) he actually increases his expected utility.

Our third claim states that an Agent provides a level of effort $k^{'}$ strictly superior to the maximal reasonable level 2 only if the evaluation system $\mathbb{E} = (\Theta, \{\gamma_k^i\}_{k,i})$ can detect the level of effort $k^{'}$.

We state in claim 4 that given an evaluation system $\mathbb{E} = (\Theta, \{\gamma_k^i\}_{k,i})$ which can detect a level of effort $k^{'}$ strictly superior to 2, an Agent provides a level of effort $k^{'}$ if and only if his type is θ with $\theta < \delta(k^{'})$ where $\delta(k^{'}) = \frac{2}{\overline{k}}\frac{(\gamma_{k^{'}}^2-\gamma_k^2)}{(\gamma_2^2-\gamma_0^2)}$ and $\overline{k} = k^{'}1_{\{k=2\}} + (k^{'}-k)1_{\{k\geq 3\}}$, $\forall k \in \{2, ..., k^{'}-1, k^{'}+1, ...\overline{m}\}$.

The inequality $\theta < \delta(k^{'})$ in claim 4 can be rewritten in the following, readable, way:

$$\frac{\theta\overline{k}}{2} < \frac{\gamma_{k^{'}}^2 - \gamma_k^2}{\gamma_2^2 - \gamma_0^2} \tag{3.8}$$

An interesting interpretation of claim 4 is that two effects drive the Agent's decision to provide an effort higher than the maximal reasonable level 2.

The first effect is in relation with the quantity $\frac{\gamma_{k^{'}}^2-\gamma_k^2}{\gamma_2^2-\gamma_0^2}$ which expresses the marginal variation of the probability of getting the associated wage $\overline{p} = u^{-1}\big(\frac{2e}{\gamma_2^2-\gamma_0^2}\big)$. This marginal variation depends on the technology of evaluation and is therefore the same for all Agents whatever their types θ.

The second effect, which is a cost effect, is expressed by the quantity $\frac{\theta\overline{k}}{2}$. It measures the effort's marginal disutility when an Agent of type θ goes from effort 2 to a higher effort. This cost effect slows down the rate of increase of the Agent's level of effort. We can also note that for a given level of effort, the smaller the type θ, the weaker the cost effect.

To summarise, claim 4 states that an Agent decides to provide an effort beyond the maximal reasonable level 2 if the marginal variation of the probability of getting the associated wage $\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ is higher than the marginal cost.

A corollary of our fourth claim is that the Agent provides a level of effort at least equal to a given level $k^{'}$ (strictly greater than the maximal reasonable level 2) if and only if his type is $\theta < \delta(k^{'})$ with $\delta(k^{'}) = \frac{2}{k}\frac{(\gamma_{k^{'}}^2 - \gamma_k^2)}{(\gamma_2^2 - \gamma_0^2)}$ where $\overline{k} = k^{'}1_{\{k=2\}} + (k^{'} - k)1_{\{k \geq 3\}}$, $\forall k \in \{2, ..., k^{'} - 1\}$.

Let us illustrate this point with $k^{'} = 3$. An Agent provides an effort at least equal to 3 if and only if his type θ is strictly weaker than $\frac{2}{3}\left(\frac{\gamma_3^2 - \gamma_2^2}{\gamma_2^2 - \gamma_0^2}\right)$. Thus, the shape of the probability of detection γ_k^2 in the neighborhood of effort $k = 3$ plays a crucial role. For instance, if γ_k^2 is convex in the neighborhood of $k = 3$ in such a way that $\gamma_3^2 - \gamma_2^2 > \frac{3}{2}\left(\gamma_2^2 - \gamma_0^2\right)$, then all Agents, whatever their types, provide an effort at least equal to 3. If γ_k^2 is concave in the neighborhood of $k = 3$ in such a way that $\gamma_3^2 - \gamma_2^2 < \gamma_2^2 - \gamma_0^2$, then there are still some individuals who can provide an effort at least equal to $k = 3$. To illustrate, if $\gamma_0^2 = 0.1$, $\gamma_2^2 = 0.7$, $\gamma_3^2 = 0.9$, then Agents with type $\theta < \frac{2}{9}$ provide an effort at least equal to $k = 3$. Agents with type $\theta \geq \frac{2}{9}$ will provide an effort $k = 2$. The meaning is that "requiring" these Agents (through some external devices like harassment or the threat to be fired) to provide an effort greater than 3 will be counterproductive because it will be at the expense of the health of the Agents (indeed their expected utility will decrease).

Of course we do not say that the incentives mechanism with performance appraisals only attracts individuals who provide an effort higher than the maximal level 2 designated by the Principal (like we said above the probability that the Agent has a type $\theta \leq \delta$ is δ). Indeed, let us remember that the optimal contract is constructed by the Principal over the set of reasonable efforts $\Theta = \{0, 1, 2\}$. And over this set, all Agents have the same behavior with respect to disutilities of effort. We simply say that performance appraisals lead to work intensification in the sense that some Agents provide efforts above the maximal effort 2 designated by the Principal. This intensification of work is profitable to these Agents because it increases their probability of having a good evaluation. Hence their expected utilities increase from $\gamma_2^2 u(\overline{p}) - 2e$ to $\gamma_k^2 u(\overline{p}) - (2 + k\theta)e$. Concerning the Principal, let us remember that the probability q_k of success of the task is, over $\Theta_g = \{0, 1, 2, 3, 4..., \overline{m}\}$, a monotone

increasing function of the effort level k. Hence if the increase of the probability of success of the task when workers move from effort 2 to effort $k \geq 3$ is sufficiently high $\left(q_k - q_2 \geq \frac{\overline{p}}{R} \times (\gamma_k^2 - \gamma_2^2),\ k \geq 3\right)$[5], then this intensification of work is also profitable to the Principal because it increases his expected profit from $q_2 R - \gamma_2^2 \overline{p}$ to $q_k R - \gamma_k^2 \overline{p}$.

From an statistical standpoint, the claims 3 and 4 mean that it is likely, when measuring the effect of performance appraisals on firms' economic performance or on workers's effort, that the bulk of this effect is caused by selection effects.

4 Taking the classical incentive scheme as benchmark

In the same vein, let us compare the classical incentive scheme with the incentive scheme with performance appraisals. For instance a firm using a classical incentive scheme may want to move to an incentive scheme with performance appraisals.

Obviously firms which use performance appraisals may design their mechanism in order that:

• The Agent's expected utility incentive mechanisms using performance appraisals when he makes the effort $k = 2$ must be at least equal to his expected utility in the classical mechanisms when he makes the effort $k = 2$:

$$E[u(\overline{p})|k = 2] - 2e \geq E[u(w_2^*)|k = 2] - 2e \tag{4.1}$$

• and/or the Principal's expected profit in the incentive mechanisms using performance appraisals must be at least equal to his expected profit in the classical mechanisms:

$$q_2 R - \gamma_2^2 \overline{p} \geq q_2 (R - w_2^*) \tag{4.2}$$

We examine in this section two kinds of contracts which are derived from equations (4.1) and (4.2). The first kind of contract, because it fulfills equation (4.2), is called *Firms Not Worse Off Contract* (FNWO Contract). The second type of contract fulfills both equations (4.1) and (4.2); and is called *Pareto-*

[5]This is obviously the case when R is high enough.

Optimal Contract (PO Contract) in the sense that both firms and workers are not worse off (compared to a classical contract).

4.1 The FNWO Contract

The program of the Principal is:

$$\left|\begin{array}{ll} \underset{\overline{p}}{Max}\ q_2 R - \gamma_2^2 \overline{p} & \\ \text{under the constraints:} & \\ (3.4) \quad \gamma_2^2 u(\overline{p}) - 2e \geq 0 & \\ (3.5) \quad (\gamma_2^2 - \gamma_1^2) u(\overline{p}) - e \geq 0 & \\ (3.6) \quad (\gamma_2^2 - \gamma_0^2) u(\overline{p}) - 2e \geq 0 & \\ (4.2) \quad \gamma_2^2 \overline{p} \leq q_2 w_2^* & \end{array}\right. \tag{Pmax2}$$

It is easy to see from Pmax2 that at the optimum $\overline{p}$ must be such that:

$$\overline{p} \geq u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right) \tag{4.3}$$

and:

$$\overline{p} \leq \frac{q_2}{\gamma_2^2} \cdot w_2^* \tag{4.4}$$

The right term of equation (4.3) is the minimal wage necessary to incite workers playing effort $k = 2$; while the right term of equation (4.4) is the wage above which firms are worse off compared to the classical incentive scheme.

As a consequence, if $u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right) > \frac{q_2}{\gamma_2^2} w_2^*$ then Pmax2 does not admit a solution. Otherwise, if $u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right) \leq \frac{q_2}{\gamma_2^2} w_2^*$ then the optimal wage writes $\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$.

An interesting aspect of the FNWO contracts is about workers.

We state in claim 5 that when the production technology is sub-modular, then:

- (a.) If $\frac{q_2}{q_1} \geq \frac{2\gamma_2^2}{\gamma_2^2 + \gamma_0^2}$, then all individuals, whatever their type θ, prefer to work for firms which use FNWO contracts rather than for firms which use classical incentive schemes.

- (b.) If $\frac{q_2}{q_1} < \frac{2\gamma_2^2}{\gamma_2^2+\gamma_0^2}$, then only individuals of type

$$\theta \leq \frac{1}{k'}\Big(\frac{2\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_1}\Big) \tag{4.5}$$

with $k' \geq 3$, prefer to work for firms which use FNWO contracts rather than for firms which use classical incentive schemes.

Likewise, we state in claim 6 that when the production technology is supermodular, then:

- (a.) If $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$, then all individuals, whatever their type θ, prefer to work for firms which use FNWO contracts rather than for firms which use classical incentive schemes.
- (b.) If $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$, then only individuals of type

$$\theta \leq \frac{2}{k'}\Big(\frac{\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_0}\Big) \tag{4.6}$$

with $k' \geq 3$, prefer to work for firms which use FNWO contracts rather than for firms which use classical incentive schemes.

In order to understand our fifth and sixth claims, let us recall that the Principal receives the income R only when the task succeeds ($X = 1$). Concerning incentive mechanisms using performance appraisals, the Agent receives the wage $\overline{p}$ if he has been evaluated as having provided a level of effort equal to 2.

Hence the condition $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$ from our sixth claim states that the relative increase in the probability of receiving the wage $\overline{p}$, when the Agent switches from the level of effort 0 to the level of effort 2, has to be smaller than the relative increase in the probability of success of the task (which is for the Principal the probability of receiving the income R), when the Agent switches from the level of effort 0 to the level of effort 2.

In such a case, the FNWO contract designed by the Principal does not need to be selective.

Otherwise, if the relative increase in the probability of receiving the wage $\overline{p}$ when the Agent switches from the level of effort 0 to the level of effort 2

is strictly higher than the relative increase in the probability of success of the task when the Agent switches from the level of effort 0 to the level of effort 2, then the Principal designs the FNWO contract in order to recruit only the most productive individuals.

The interpretation of condition $\frac{q_2}{q_1} \geq \frac{2\gamma_2^2}{\gamma_2^2+\gamma_0^2}$ in claim 5 is the same. Indeed, let us note that this condition writes also as $\frac{q_2}{q_1} \geq \frac{\gamma_2^2}{\gamma_0^2}$.

An interesting corollary from our fifth and sixth claims is that if we restrict the analysis to the class of utility functions u whose inverse are such, that $u^{-1}(\lambda r) > \lambda u^{-1}(r)\,, \forall \lambda > 1$, then the only FNWO contracts which are implemented are those in which $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$.

We can see that the CARA or DARA utility functions belongs to this class of utility functions. This corollary is interesting because it says that if the Principal's main concern is not to be worse off, compared to the classical incentive scheme, then he designs the performance appraisals scheme in such a way that the employees get a higher wage but the probability of getting this wage is smaller. Hence workers who are able to provide an effort above the required effort $k = 2$, in order to increase the probability of getting the wage $\overline{p}$, will be attracted to such a kind of firm. In other words, there will be a high selectivity of workers in FNWO firms.

4.2 The Pareto-Optimal Contract

Let us recall here that the Principal designs the performance appraisals contract in such a way that neither him, nor the Agent, is worse off compared to the classical incentive scheme. Such Pareto-Optimal contracts seem suitable for public organizations or for firms in relatively protected sectors.

The program of the Principal is:

$$
\left|\begin{array}{ll}
\underset{\overline{p}}{Max}\ q_2 R - \gamma_2^2 \overline{p} & \\
\text{under the constraints:} & \\
(3.4) \quad \gamma_2^2 u(\overline{p}) - 2e \geq 0 & \\
(3.5) \quad (\gamma_2^2 - \gamma_1^2) u(\overline{p}) - e \geq 0 & \\
(3.6) \quad (\gamma_2^2 - \gamma_0^2) u(\overline{p}) - 2e \geq 0 & \\
(4.1) \quad \gamma_2^2 u(\overline{p}) \geq q_2 u(w_2^*) & \\
(4.2) \quad \gamma_2^2 \overline{p} \leq q_2 w_2^* &
\end{array}\right. \qquad \text{(Pmax3)}
$$

Our seventh claim states that if the production technology is sub-modular then the pareto-optimal contract in incentive mechanisms using performance appraisal is:

Either $\{\overline{p}, \gamma_2^2, \gamma^{'}\}$ with $\overline{p} = u^{-1}\Big(\frac{2e}{\gamma_2^2 - \gamma'}\Big)$ and

$$\begin{cases} \frac{q_2}{q_1} \geq \frac{2\gamma_2^2}{\gamma_2^2 + \gamma'} \\ \\ u^{-1}\Big[\frac{2e}{\gamma_2^2 - \gamma'}\Big] \leq \frac{q_2}{\gamma_2^2} u^{-1}\Big[\frac{e}{q_2 - q_1}\Big] \end{cases}$$

Or $\{\overline{p}, \gamma_2^2, \gamma^{'}\}$ with $\overline{p} = u^{-1}\Big(\frac{q_2}{\gamma_2^2} \times \frac{e}{q_2 - q_1}\Big)$ and

$$\begin{cases} \frac{q_2}{q_1} < \frac{2\gamma_2^2}{\gamma_2^2 + \gamma'} \\ \\ u^{-1}\Big[\frac{q_2}{\gamma_2^2} \times \frac{e}{q_2 - q_1}\Big] \leq \frac{q_2}{\gamma_2^2} u^{-1}\Big[\frac{e}{q_2 - q_1}\Big] \end{cases}$$

Likewise we state in claim 8 that if the production technology is super-modular then the pareto-optimal contract in incentive mechanisms using performance appraisal is:

Either $\{\overline{p}, \gamma_2^2, \gamma^{'}\}$ with: $\overline{p} = u^{-1}\Big(\frac{2e}{\gamma_2^2 - \gamma'}\Big)$ and

$$\begin{cases} \frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma'} \\ \\ u^{-1}\Big[\frac{2e}{\gamma_2^2 - \gamma'}\Big] \leq \frac{q_2}{\gamma_2^2} u^{-1}\Big[\frac{2e}{q_2 - q_0}\Big] \end{cases}$$

Or $\{\overline{p}, \gamma_2^2, \gamma^{'}\}$ with $\overline{p} = u^{-1}\Big(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\Big)$ and

$$\begin{cases} \frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma'} \\ \\ u^{-1}\Big[\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\Big] \leq \frac{q_2}{\gamma_2^2} u^{-1}\Big[\frac{2e}{q_2 - q_0}\Big] \end{cases}$$

In summary, according to claim 8 (the interpretation for claim 7 being the same) when the technology production is super-modular then the wage $\overline{p}$ is $u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ if the relative increase in the probability of receiving the wage

$\overline{p}$ when the Agent switches from the level of effort 0 to the level of effort 2 is smaller than the relative increase in the probability of success of the task (which is for the Principal the probability of receiving the income R) when the Agent switches from the level of effort 0 to the level of effort 2; and if the wage $\overline{p}$ is less than the wage above which firms are worse off compared to the classical incentive scheme.Otherwise if $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma'}$, then $\overline{p} = u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\right)$. In this latter case, only the probability that Agent gets his wage is independent from the result of the task he performs; however, his wage $\overline{p} = u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\right)$ depends partly on the result of the task he performs.

Let us conclude this subsection with a corollary derived from claims 7 and 8.

If we restrict ourselves to the class of utility functions u whose inverse are such that $u^{-1}(\lambda r) > \lambda u^{-1}(r)\,, \forall \lambda > 1$, then firms implement either PO contracts in which $\overline{p} < w_2^*$ and $q_2 < \gamma_2^2$, or degenerate PO contracts in which $\overline{p} = w_2^*$ and $q_2 = \gamma_2^2$.

This means that in PO contracts (in which both firms and employees are not worse off compared to the classical incentive scheme), employees' wages are smaller than the wages of employees working in classical incentive schemes; however, the probability of getting their wage is higher. As a consequence, compared to the case of FNWO contracts, in PO contracts the selection of low effort disutility workers will be weaker.

5 Conclusion

In this paper we proposed an economic model of performance appraisals. It allows us to understand the effects of performance appraisals over employees' levels of effort and over their wages. Moreover as in most papers in the literature concerning Principal-Agent models with subjective evaluation, we implicitly consider that the main difference between the classical contracts and those with performance appraisals is that in the latter, whatever the outcome of the production, the Agents receive their bonus if they have been evaluated as having played the appropriate level of effort. As a consequence, from the workers standpoint performance appraisals are associated with fair wages and work recognition (except, of course, in the cases where the evaluation is biased).

This explanation is attractive but is not exclusive. Other explanations exist in sociology, in theories of organizations, and in industrial relations literature. For instance, performance appraisals may be a "domination method" used by firms that intensify work by imposing both business-bureaucratic constraints and market constraints. Performance appraisals may also contribute to elaborating the formalization of work organization. Lastly, performance appraisals might deter social unrest within organizations in which the dispute potential is high; that is to say, faced with the possibility of expressing themselves during interviews, employees would be less encouraged to contest management.

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Appendix

A Proofs.

Proof of Claim 1. The Evaluation System is not efficient for the level of effort 2 if $\gamma_2^2 \leq \gamma_1^2$ or $\gamma_2^2 \leq \gamma_0^2$. If $\gamma_2^2 \leq \gamma_1^2$, then there is no $\overline{p}$ which respects constraint (3.5). And if $\gamma_2^2 \leq \gamma_0^2$, then there is no $\overline{p}$ which respects constraint (3.6). ■

Proof of Claim 2. Let us first note that equations (3.4) to (3.6) in the program Pmax write respectively:

$$\overline{p} \geq u^{-1}\left(\frac{2e}{\gamma_2^2}\right) \tag{3.4}$$

$$\overline{p} \geq u^{-1}\left(\frac{e}{\gamma_2^2 - \gamma_1^2}\right) \tag{3.5}$$

$$\overline{p} \geq u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right) \tag{3.6}$$

It is easy to see that if (3.6) is satisfied, then the participation constraint (3.4) is also satisfied. Moreover if $\gamma_2^2 - \gamma_1^2 > \gamma_1^2 - \gamma_0^2$ then the optimal solution writes $\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$. Else if $\gamma_2^2 - \gamma_1^2 \leq \gamma_1^2 - \gamma_0^2$ then the optimal solution writes $\overline{p} = u^{-1}\left(\frac{e}{\gamma_2^2 - \gamma_1^2}\right)$. However since $\gamma_{k'}^2 = \gamma', \forall k' \in \{0,1\}$, then $\gamma_1^2 - \gamma_0^2 = 0$. Since the evaluation system is efficient for the level of effort 2 then $\gamma_2^2 > \gamma_1^2$. As a consequence, we have $\gamma_2^2 - \gamma_1^2 > \gamma_1^2 - \gamma_0^2$ and $\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$. ■

Proof of Claim 3. On the one hand, if an Agent provides a level of effort k' superior strictly to the maximal reasonable level 2 then $\gamma_{k'}^2 u(\overline{p}) - (\theta k' + 2)e > \gamma_2^2 u(\overline{p}) - 2e$. This inequality writes also $(\gamma_{k'}^2 - \gamma_2^2)u(\overline{p}) > \theta k' e$. Since $\theta k' e \geq 0$ it implies necessarily that $\gamma_{k'}^2 - \gamma_2^2 > 0$.

On the other hand, if an Agent provides a level of effort k' superior strictly to the maximal reasonable level 2 then for all $k \in \{3, ...\overline{m}\}$, $k \neq k'$: $\gamma_{k'}^2 u(\overline{p}) - (\theta k' + 2)e > \gamma_k^2 u(\overline{p}) - (\theta k + 2)e$. This inequality writes also $(\gamma_{k'}^2 - \gamma_k^2)u(\overline{p}) > \theta(k' - k)e$. If $k \in \{3, ..., k' - 1\}$, then $\theta(k' - k)e \geq 0$. Hence $\gamma_{k'}^2 - \gamma_k^2 > 0$. ■

Proof of Claim 4. Indeed, the Agent provides an effort $k' \geq 3$ if and only if:

$$\begin{cases} \gamma_{k'}^2 u(\overline{p}) - (2+\theta k')e > \gamma_2^2 u(\overline{p}) - 2e \\ \\ \gamma_{k'}^2 u(\overline{p}) - (2+\theta k')e > \gamma_k^2 u(\overline{p}) - (2+\theta k)e \ , \quad \forall\ k \in \{3,..,k'-1,k'+1,..\overline{m}\} \end{cases}$$

These two inequalities obviously lead to claim 4. ■

Proof of the corollary of Claim 4. If the level of effort played by the Agent is exactly k', then it must be the case that this level of effort maximizes his expected utility over the set $\{2,3,...,\overline{m}\}$. However, the corollary of claim 4 is simply about the minimal level of effort k' that could be played by the Agent. This level of effort, of course, must be the best one only over the set $\{2,3,...k'\}$. ■

Proof of Claim 5.

Since the production technology is sub-modular, then $w_2^* = u^{-1}\left(\frac{e}{q_2-q_1}\right)$. An individual, whatever his type θ, prefers FNWO contracts instead of classical contracts if and only if his expected utility is higher with FNWO contracts. That is to say, if and only if equation (4.1) is fulfilled when $\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2-\gamma_0^2}\right)$.

This equation (4.1) writes also: $\gamma_2^2 u(\overline{p}) \geq q_2 u(w_2^*)$.

That is:

$$\overline{p} \geq u^{-1}\left[\frac{q_2}{\gamma_2^2} u(w_2^*)\right] \tag{A.1}$$

Hence, in order to prove claim 5, we need to compare (4.3) to (A.1), that is to solve the following inequality $u^{-1}\left(\frac{2e}{\gamma_2^2-\gamma_0^2}\right) \geq u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{e}{q_2-q_1}\right)$. This inequality obviously leads to $\frac{q_2}{q_1} \geq \frac{2\gamma_2^2}{\gamma_2^2+\gamma_0^2}$.

Hence an individual, whatever his type θ, prefers FNWO contracts instead of classical contracts if and only if $\frac{q_2}{q_1} \geq \frac{2\gamma_2^2}{\gamma_2^2+\gamma_0^2}$.

As a consequence, $\frac{q_2}{q_1} < \frac{2\gamma_2^2}{\gamma_2^2+\gamma_0^2}$ means that the expected utility of individuals who would like to provide an effort $k=2$ is strictly weaker than their expected utility in the classical incentive scheme. Hence only individuals who are able to provide an effort k at least equal to 3 may prefer FNWO contracts instead of classical contracts. Such individuals are characterized by $\gamma_{k'}^2 u(\overline{p}) - (2+\theta k')e \leq q_2 u(w_2^*) - 2e$ with $k' \geq 3$. That is by $\theta \leq \frac{1}{k'}\left(\frac{2\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_1}\right)$ with $k' \geq 3$.

■

Proof of Claim 6.

Since the production technology is super-modular, then $w_2^* = u^{-1}(2e/(q_2 - q_0))$. Hence, in order to prove claim 6, we need to compare (4.3) to (A.1), that is to solve the following inequality $u^{-1}\left(\frac{2e}{\gamma_2^2-\gamma_0^2}\right) \geq u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2-q_0}\right)$. This inequality obviously leads to $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$.

Hence an individual, whatever his type θ, prefers FNWO contracts instead of classical contracts if and only if $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$. As a consequence, $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$ means that the expected utility of individuals who would like to provide an effort $k = 2$ is strictly weaker than their expected utility in the classical incentive scheme. Hence only individuals who are able to provide an effort k at least equal to 3 may prefer FNWO contracts instead of classical contracts. Such individuals are characterized by $\gamma_{k'}^2 u(\overline{p}) - (2 + \theta k^{'})e \leq q_2 u(w_2^*) - 2e$ with $k' \geq 3$. That is by $\theta \leq \frac{2}{k'}\left(\frac{\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_0}\right)$ with $k' \geq 3$. ■

In order to prove the corollary of claims 5 and 6, let us state the following lemma.

Lemma 1 *Let us restrict to the class of utility functions u whose inverse are such that $u^{-1}(\lambda r) > \lambda u^{-1}(r), \forall \lambda > 1$. If $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$, then the expected utility of individuals (when they provide an effort $k = 2$) is strictly weaker than their expected utility in the classical incentive scheme: $\gamma_2^2 u(\overline{p}) - 2e < q_2 u(w_2^*) - 2e$.*

Proof of Lemma 1. The inequality $\gamma_2^2 u(\overline{p}) - 2e < q_2 u(w_2^*) - 2e$ is equivalent to $\overline{p} < u^{-1}\left(\frac{q_2}{\gamma_2^2} u(w_2^*)\right)$. Since u belongs to the class of utility functions u whose inverse are such that $u^{-1}(\lambda r) > \lambda u^{-1}(r), \forall \lambda > 1$, and since $q_2 > \gamma_2^2$, then $u^{-1}\left(\frac{q_2}{\gamma_2^2} u(w_2^*)\right) > \frac{q_2}{\gamma_2^2} \times u^{-1}(u(w_2^*))$. Hence $u^{-1}\left(\frac{q_2}{\gamma_2^2} u(w_2^*)\right) > \frac{q_2}{\gamma_2^2} w_2^*$. According to the equation (4.2), $\gamma_2^2 \overline{p} \leq q_2 w_2^*$. Thus $u^{-1}\left(\frac{q_2}{\gamma_2^2} u(w_2^*)\right) > \overline{p}$. ■

Proof of the corollary of Claim 5 and Claim 6.

Let us first see that firms can a priori implement six kinds of FNWO contracts.

- FNWO contracts in which $\overline{p} = w_2^* \quad and \quad q_2 = \gamma_2^2$
- FNWO contracts in which $\overline{p} > w_2^* \quad and \quad q_2 > \gamma_2^2$

- FNWO contracts in which $\overline{p} = w_2^* \quad and \quad q_2 > \gamma_2^2$
- FNWO contracts in which $\overline{p} < w_2^* \quad and \quad q_2 = \gamma_2^2$
- FNWO contracts in which $\overline{p} < w_2^* \quad and \quad q_2 > \gamma_2^2$
- FNWO contracts in which $\overline{p} < w_2^* \quad and \quad q_2 < \gamma_2^2$

Our strategy of proof is to show that firms get their highest expected profit when they implement FNWO contracts with $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$.

Let us look at FNWO contracts in which $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$. In such a case, firms attract only the most productive individuals; and by the same token, increase their expected profits (to illustrate, see Appendix B).

Indeed, according to the above lemma 1, if $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$ then $\gamma_2^2 u(\overline{p}) - 2e < q_2 u(w_2^*) - 2e$. As a consequence, if the production technology is sub-modular (see claim 5) then only individuals of type $\theta \leq \frac{1}{k'}\left(\frac{2\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_1}\right)$ with $k' \geq 3$, prefer to work for firms which use FNWO contracts with $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$ (rather than for firms which use classical incentive schemes). And if the production technology is super-modular (see claim 6) then only individuals of type $\theta \leq \frac{2}{k'}\left(\frac{\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_0}\right)$ with $k' \geq 3$, prefer to work for firms which use FNWO contracts with $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$ (rather than for firms which use classical incentive schemes). As a consequence, the expected profit of firms increases from $q_2 R - \gamma_2^2 \overline{p}$ to $q_{k'} R - \gamma_{k'}^2 \overline{p}$.

Let us compare the FNWO contracts with $\overline{p} > w_2^*$ and $q_2 < \gamma_2^2$ to the five other type of contracts.

- The case $\overline{p} = w_2^*$ and $q_2 = \gamma_2^2$ is a trivial one. If firms implement FNWO contracts with $\overline{p} = w_2^*$ and $q_2 = \gamma_2^2$ then their expected profit $\delta(k')\left[q_{k'} R - \gamma_{k'}^2 \overline{p}\right] + (1 - \delta(k'))\left[q_2 R - \gamma_2^2 \overline{p}\right]$ is lower than their expected profit $\left(q_{k'} R - \gamma_{k'}^2 \overline{p}\right)$ in the case of FNWO contracts with $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$;
  where $\delta(k')$ writes $\frac{1}{k'}\left(\frac{2\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_1}\right)$ if the technology production is sub-modular and writes $\frac{2}{k'}\left(\frac{\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_0}\right)$ if the technology production is super-modular, $k' \geq 3$.

- Let us take the case of FNWO contracts with "$\overline{p} = w_2^*$ and $q_2 > \gamma_2^2$" or "$\overline{p} < w_2^*$ and $q_2 = \gamma_2^2$" or "$\overline{p} < w_2^*$ and $q_2 > \gamma_2^2$". Obviously firms do not implement such contracts because they need in order to work that the individuals who are recruited have a type θ weaker than $\frac{1}{k'}\left(\frac{2\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_1}\right)$ if the technology production is sub-modular and is weaker than $\frac{2}{k'}\left(\frac{\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_0}\right)$ if the technology production is super-modular, $k^{'} \geq 3$.
However these individuals prefer to work for firms which implement the FNWO contracts with $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$. To see this, suppose that a firm A implements a scheme with $\overline{p} \leq w_2^*$ and $q_2 > \gamma_2^2$. If another firm B keeps the same γ_2^2 but slightly increases $\overline{p}$ (by slightly increasing γ_0^2) then the individuals would prefer to work for this firm B.

- Finally, if the Principal implements the FNWO scheme with $\overline{p} < w_2^*$ and $q_2 < \gamma_2^2$ then two cases occur:
  - Either all individuals whatever their type θ, prefer to work for firms which use the FNWO contracts with $\overline{p} < w_2^*$ and $q_2 < \gamma_2^2$ rather than for firms using classical incentive schemes. In this case however, firms prefer the FNWO contracts with $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$ because their expected profit is higher.
  - Or are recruited only individuals with a type $\theta \leq \frac{1}{k'}\left(\frac{2\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_1}\right)$ or $\theta \leq \frac{2}{k'}\left(\frac{\gamma_{k'}^2}{\gamma_2^2-\gamma_0^2} - \frac{q_2}{q_2-q_0}\right)$. In this case, individuals of type θ prefer to work for firms which implement the FNWO contracts with $\overline{p} > w_2^*$ and $q_2 > \gamma_2^2$ because their expected utility is higher.

■

Proof of Claim 7.

Since the production technology is sub-modular, then $w_2^* = u^{-1}\left(\frac{e}{q_2-q_1}\right)$. Like in the proof of claim 5, the comparison of equation (4.3) and equation (A.1) leads to the following inequality: $\frac{q_2}{q_1} \geq \frac{2\gamma_2^2}{\gamma_2^2+\gamma_0^2}$. Hence if $\frac{q_2}{q_1} \geq \frac{2\gamma_2^2}{\gamma_2^2+\gamma_0^2}$, then $\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2-\gamma_0^2}\right)$; else $\overline{p} = u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{e}{q_2-q_1}\right)$. Moreover, the optimal contract must respect the constraint (4.4). Let us remember that this constraint (which guarantees that the Principal's expected benefit is at least equal to his expected benefit in the classical mechanism) writes $\overline{p} \leq \frac{q_2}{\gamma_2^2} \cdot w_2^*$. Finally we get our claim 7 by using the hypothesis which states that $\gamma_{k'}^2 = \gamma^{'}$, $\forall k^{'} \in \{0,1\}$.

■

Proof of Claim 8. Since the production technology is super-modular, then $w_2^* = u^{-1}(2e/(q_2 - q_0))$. Like in the proof of claim 6, the comparison of equation (4.3) and equation (A.1) leads to the following inequality: $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$. Hence if $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$, then $\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$; else $\overline{p} = u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\right)$. Finally, we get our eighth claim by using the hypothesis which states that $\gamma_{k'}^2 = \gamma^{'}$, $\forall k^{'} \in \{0, 1\}$; and by constraining the optimal contract to fulfill the inequality (4.4). ■

B Numerical examples with super-modular technology production (For Online Publication).

Let us take a logarithmic utility function $u(r) = ln(r+1)$ and let us suppose that:

- the revenue $R = 1000$ and the effort unit is $e = 1$;
- $q_2 = 0.7$, $q_1 = 0.2$ and $q_0 = 0.05$; the technology production is therefore super-modular over $\Theta = \{0, 1, 2\}$;
- $\Theta = \{0, 1, 2\}$ and $\Theta_g = \{0, 1, 2, 3\}$.

B.1 The case $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$

B.1.1 The case $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$ and $q_2 < \gamma_2^2$

Suppose that $\gamma_2^2 = 0.8$ and $\gamma_1^2 = \gamma_0^2 = \gamma' = 0.05$. Suppose also that $\gamma_3^2 = 0.9$ and $q_3 = 0.8$.

Table B.1: Results

	Classical Contract	Optimal PA Contract[1]	FNWO PA Contract[2]	PO PA Contract[3]
Parameters	$q_2 = 0.7$ $q_1 = 0.2$ $q_0 = 0.05$	$\gamma_2^2 = 0.8$ $\gamma' = 0.05$	$\gamma_2^2 = 0.8$ $\gamma' = 0.05$	$\gamma_2^2 = 0.8$ $\gamma' = 0.05$
Wage	$w_2^* = u^{-1}(\frac{2}{q_2 - q_0})$ $w_2^* = 20.69$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 13.39$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 13.39$	$\overline{p} = u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\right)$ $\overline{p} = 13.76$
Expected Utility	$0.7ln(20.69 + 1) - 2$ $= 0.153$	$0.8ln(13.39 + 1) - 2$ $= 0.133$	$0.9ln(13.39 + 1) - (2 + 3\theta)$ ≥ 0.153	$0.8ln(13.76 + 1) - 2$ $= 0.153$
Expected Profit	$0.7R - 0.7 \times 20.69$ $= 685.51$	$0.7R - 0.8 \times 13.39$ $= 689.28$	$0.8R - 0.9 \times 13.39$ $= 787.94$	$0.7R - 0.8 \times 13.76$ $= 688.98$

[1] = Optimal contract in the performance appraisals scheme.
[2] = Firms Not Worse Off Contract in the performance appraisals scheme.
[3] = Pareto-Optimal Contract in the performance appraisals scheme.

Since $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$, then, according to our sixth claim, the FNWO contract concerns only individuals with type $\theta \leq \frac{2}{3}\left(\frac{\gamma_3^2}{\gamma_2^2 - \gamma_0^2} - \frac{q_2}{q_2 - q_0}\right)$. That is, the individuals with type $\theta \leq 0.082$.

By providing an effort equal to 3, their wage (13.39) is still the same; however, their probability γ_3^2 of getting this wage is 0.9 (instead of the "official"

figure of $\gamma_2^2 = 0.8$). As a consequence, their expected utility $\big(0.9 \times ln(13.39 + 1) - (2 + \theta \times 3)\big)$ is higher than 0.153 (their expected utility in the classical incentive mechanism). For instance, the expected utility of individuals having $\theta = 0.082$ is 0.153, and the expected utility of individuals having $\theta = 0.072$ is 0.183. At the same time, the Principal is also better off since his expected profit $\big(0.8R - 0.9 \times 13.39 = 787.94\big)$ is higher than his expected's profit in the classical incentive scheme $\big(685.51\big)$.

B.1.2 The case $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$ and $q_2 = \gamma_2^2$

Suppose that $\gamma_2^2 = 0.7$ and $\gamma_1^2 = \gamma_0^2 = \gamma' = 0.01$. Suppose also that $\gamma_3^2 = 0.9$ and $q_3 = 0.8$.

Table B.2: Results

	Classical Contract	Optimal PA Contract[1]	FNWO PA Contract[2]	PO PA Contract[3]
Parameters	$q_2 = 0.7$ $q_1 = 0.2$ $q_0 = 0.05$	$\gamma_2^2 = 0.7$ $\gamma' = 0.01$	$\gamma_2^2 = 0.7$ $\gamma' = 0.01$	$\gamma_2^2 = 0.7$ $\gamma' = 0.01$
Wage	$w_2^* = u^{-1}(2/q_2 - q_0)$ $w_2^* = 20.69$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 17.14$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 17.14$	$\overline{p} = u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\right)$ $\overline{p} = 20.69$
Expected Utility	$0.7ln(20.69 + 1) - 2$ $= 0.153$	$0.7ln(17.14 + 1) - 2$ $= 0.028$	$0.9ln(17.14 + 1) - (2 + 3\theta)$ ≥ 0.153	$0.7ln(20.69 + 1) - 2$ $= 0.153$
Expected Profit	$0.7R - 0.7 \times 20.69$ $= 685.51$	$0.7R - 0.7 \times 17.14$ $= 688$	$0.8R - 0.9 \times 17.14$ $= 784.56$	$0.7R - 0.7 \times 20.69$ $= 685.51$

[1] = Optimal contract in the performance appraisals scheme.
[2] = Firms Not Worse Off Contract in the performance appraisals scheme.
[3] = Pareto-Optimal Contract in the performance appraisals scheme.

Let us see that since $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$, then, according to our sixth claim, the FNWO contract concerns only individuals with type $\theta \leq \frac{2}{3}\left(\frac{\gamma_3^2}{\gamma_2^2 - \gamma_0^2} - \frac{q_2}{q_2 - q_0}\right)$. That is, the individuals with type $\theta \leq 0.151$.

B.1.3 The case $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$ and $q_2 > \gamma_2^2$

Suppose that $\gamma_2^2 = 0.65$ and $\gamma_1^2 = \gamma_0^2 = \gamma' = 0.01$. Suppose also that $\gamma_3^2 = 0.9$ and $q_3 = 0.8$.

Table B.3: Results

	Classical Contract	Optimal PA Contract[1]	FNWO PA Contract[2]	PO PA Contract[3]
Parameters	$q_2 = 0.7$ $q_1 = 0.2$ $q_0 = 0.05$	$\gamma_2^2 = 0.65$ $\gamma' = 0.01$	$\gamma_2^2 = 0.65$ $\gamma' = 0.01$	$\gamma_2^2 = 0.65$ $\gamma' = 0.01$
Wage	$w_2^* = u^{-1}(2/q_2 - q_0)$ $w_2^* = 20.69$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 21.76$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 21.76$	No PO contract exists
Expected Utility	$0.7ln(20.69+1) - 2$ $= 0.153$	$0.65ln(21.76+1) - 2$ $= 0.031$	$0.9ln(21.76+1) - (2+3\theta)$ ≥ 0.153	No PO contract exists
Expected Profit	$0.7R - 0.7 \times 20.69$ $= 685.51$	$0.7R - 0.65 \times 21.76$ $= 682.78$	$0.8R - 0.9 \times 21.76$ $= 780.41$	No PO contract exists

[1] = Optimal contract in the performance appraisals scheme.
[2] = Firms Not Worse Off Contract in the performance appraisals scheme.
[3] = Pareto-Optimal Contract in the performance appraisals scheme.

Since $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$, then, according to our sixth claim, the FNWO contract concerns only individuals with type $\theta \leq \frac{2}{3}\left(\frac{\gamma_3^2}{\gamma_2^2 - \gamma_0^2} - \frac{q_2}{q_2 - q_0}\right)$. That is, the individuals with type $\theta \leq 0.219$.

Moreover there exists no pareto-optimal contract using performance appraisal. The reason is that our function u^{-1} is an exponential one. Hence whatever γ_2^2 with $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$ and $q_2 > \gamma_2^2$, the following inequality is true:

$$\overline{p} = u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\right) > \frac{q_2}{\gamma_2^2} \times u^{-1}\left(\frac{2e}{q_2 - q_0}\right)$$

More precisely, the contract $\left(\overline{p} = u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\right) = 26.48,\, \gamma_2^2,\, \gamma'\right)$ respects the constraint (4.1) in the program Pmax3, but it does not respect the constraint (4.2). Remind that Pmax3 is the program which leads to the pareto-optimal contract in the performance appraisal scheme (PO PA contract).

The Agent is not worse off since his expected utility $\left(0.65 \times ln(26.48+1) - 2 = 0.153\right)$ is the same as his expected utility in the classical incentive mechanism $\left(0.7 \times ln(20.69+1) - 2 = 0.153\right)$. However, the Principal is worse off. Indeed, his expected profit $\left(0.7R - 0.65 \times 26.48 = 682.78\right)$, when he implements the contract $\left(\overline{p} = 26.48,\, \gamma_2^2,\, \gamma'\right)$, is weaker than his expected profit in the classical incentive scheme $\left(685.51\right)$.

To conclude, the Principal never implements a PO PA contract with $\frac{q_2}{q_0} < \frac{\gamma_2^2}{\gamma_0^2}$

and $q_2 > \gamma_2^2$ because his expected profit is lower.

B.1.4 Comments on the FNWO Contracts

The FNWO contracts in tables B.1 and B.2 are theoretically possible, but they are less likely to happen compared to the FNWO contract in table B.3. For instance, if we take an individual with type $\theta = 0.082$, who chooses to work for firms which use a FNWO contract like in table B.1, then his expected utility is 0.153. However, if this individual works for firms which use a FNWO contract like in table B.3, then his expected utility is 0.566. Likewise, if we take an individual with type $\theta = 0.151$, who chooses to work for firms which use a FNWO contract like in table B.2, then his expected utility is 0.153. However, if this individual works for firms which use a FNWO contract like in table B.3, then his expected utility is 0.357.

B.2 The case $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$

Remind that the condition $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$ means that the relative increase in the probability of receiving the wage $\overline{p}$, when the Agent switches from the level of effort 0 to the level of effort 2, is smaller than the relative increase in the probability of success of the task (which is for the Principal the probability of receiving the income R), when the Agent switches from the level of effort 0 to the level of effort 2. As a consequence, the FNWO performance appraisal contract, when it exits, does not need to be selective. For instance to the contrary of the expected utilities in tables B.1, B.2 and B.3, in table B.4 the workers selection parameter θ does not intervene in the calculus of the expected utility of workers.

B.2.1 The case $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$ and $q_2 < \gamma_2^2$

Suppose that $\gamma_2^2 = 0.8$ and $\gamma_1^2 = \gamma_0^2 = \gamma' = 0.06$. Suppose also that $\gamma_3^2 = 0.9$ and $q_3 = 0.8$. Since $q_2 < \gamma_2^2$, the FNWO and the PO contracts exist but there are degenerate in the sense that their characteristics coincide with the the characteristics of the optimal PA contract. The associated wage ($\overline{p} = 13.92$) is smaller than the wage in the classical contract $w_2^* = 20.69$, but the probability (0.8) to get the wage $\overline{p}$ is higher than the probability (0.7) to get w_2^*. Nevertheless both the workers' expected utility and the firm's expectted profit are higher than in the classical incentive scheme.

Table B.4: Results

	Classical Contract	Optimal PA Contract[1]	FNWO PA Contract[2]	PO PA Contract[3]
Parameters	$q_2 = 0.7$ $q_1 = 0.2$ $q_0 = 0.05$	$\gamma_2^2 = 0.8$ $\gamma^{'} = 0.06$	$\gamma_2^2 = 0.8$ $\gamma^{'} = 0.06$	$\gamma_2^2 = 0.8$ $\gamma^{'} = 0.06$
Wage	$w_2^* = u^{-1}(2/q_2 - q_0)$ $w_2^* = 20.69$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 13.92$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 13.92$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 13.92$
Expected Utility	$0.7ln(20.69 + 1) - 2$ $= 0.153$	$0.8ln(13.92 + 1) - 2$ $= 0.162$	$0.8ln(13.92 + 1) - 2$ $= 0.162$	$0.8ln(13.92 + 1) - 2$ $= 0.162$
Expected Profit	$0.7R - 0.7 \times 20.69$ $= 685.51$	$0.7R - 0.8 \times 13.92$ $= 688.86$	$0.7R - 0.8 \times 13.92$ $= 688.86$	$0.7R - 0.8 \times 13.92$ $= 688.86$

[1] = Optimal contract in the performance appraisals scheme.
[2] = Firms Not Worse Off Contract in the performance appraisals scheme.
[3] = Pareto-Optimal Contract in the performance appraisals scheme.

B.2.2 The case $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$ and $q_2 \geq \gamma_2^2$

Suppose that $\gamma_2^2 = 0.6$ and $\gamma_1^2 = \gamma_0^2 = \gamma' = 0.05$. Suppose also that $\gamma_3^2 = 0.9$ and $q_3 = 0.8$.

Table B.5: Results

	Classical Contract	Optimal PA Contract[1]	FNWO PA Contract[2]	PO PA Contract[3]
Parameters	$q_2 = 0.7$ $q_1 = 0.2$ $q_0 = 0.05$	$\gamma_2^2 = 0.6$ $\gamma^{'} = 0.05$	$\gamma_2^2 = 0.6$ $\gamma^{'} = 0.05$	$\gamma_2^2 = 0.6$ $\gamma^{'} = 0.05$
Wage	$w_2^* = u^{-1}(2/q_2 - q_0)$ $w_2^* = 20.69$	$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right)$ $\overline{p} = 36.95$	No FNWO contract exists	No PO contract exists
Expected Utility	$0.7 \times ln(20.69 + 1) - 2$ $= 0.153$	$0.6 \times ln(36.95 + 1) - 2$ $= 0.181$	No FNWO contract exists	No PO contract exists
Expected Profit	$0.7R - 0.7 \times 20.69$ $= 685.51$	$0.7R - 0.6 \times 36.95$ $= 677.82$	No FNWO contract exists	No PO contract exists

[1] = Optimal contract in the performance appraisals scheme.
[2] = Firms Not Worse Off Contract in the performance appraisals scheme.
[3] = Pareto-Optimal Contract in the performance appraisals scheme.

There exists no "pareto-optimal" or "firms not worse off" contract using performance appraisal. Indeed, the contract $\left(\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right) = 36.95,\ \gamma_2^2,\ \gamma^{'}\right)$

respects the constraint (4.1) in the program Pmax3, but it does not respect the constraint (4.2). The reason is that our function u^{-1} is an exponential one. Hence if we are not in the trivial case with $q_2 = \gamma_2^2$ and $q_0 = \gamma_0^2$, then whatever γ_2^2 with $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$ and $q_2 \geq \gamma_2^2$, the following inequality is true:

$$\overline{p} = u^{-1}\left(\frac{2e}{\gamma_2^2 - \gamma_0^2}\right) \geq u^{-1}\left(\frac{q_2}{\gamma_2^2} \times \frac{2e}{q_2 - q_0}\right) > \frac{q_2}{\gamma_2^2} \times u^{-1}\left(\frac{2e}{q_2 - q_0}\right)$$

The Agent is better off since his expected utility $\big(0.6 \times ln(36.95 + 1) - 2 = 0.181\big)$ is higher than his expected utility in the classical incentive mechanism $\big(0.7 \times ln(20.69 + 1) - 2 = 0.153\big)$. However, the Principal is worse off since his expected profit $\big(0.7R - 0.6 \times 36.95 = 677.82\big)$ is weaker than his expected profit in the classical incentive scheme $\big(0.7R - 0.7 \times 20.69 = 685.51\big)$.

To conclude, except in the degenerate case (i.e. $q_2 = \gamma_2^2$ and $q_0 = \gamma_0^2$), the Principal never implements a PO or a FNWO contract with $\frac{q_2}{q_0} \geq \frac{\gamma_2^2}{\gamma_0^2}$ and $q_2 \geq \gamma_2^2$ because his expected profit is lower.

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