



HAL
open science

Frequency of trade and the deterrminacy of equilibrium in economies of overlapping generations

Hippolyte d'Albis, Emmanuelle Augeraud-Véron

► **To cite this version:**

Hippolyte d'Albis, Emmanuelle Augeraud-Véron. Frequency of trade and the deterrminacy of equilibrium in economies of overlapping generations. 2012. halshs-00717158

HAL Id: halshs-00717158

<https://shs.hal.science/halshs-00717158>

Submitted on 12 Jul 2012

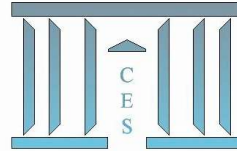
HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Documents de Travail du Centre d'Économie de la Sorbonne

C
E
S
W
o
r
k
i
n
g
P
a
p
e
r
s



Frequency of trade and the determinacy of equilibrium in economies of overlapping generations

Hippolyte d'ALBIS, Emmanuelle AUGERAUD-VÉRON

2012.42



Frequency of trade and the determinacy of equilibrium in economies of overlapping generations

Hippolyte d'ALBIS*

Paris School of Economics (University Paris 1)

Emmanuelle AUGERAUD-VÉRON†

University of La Rochelle (MIA)

June 19, 2012

Abstract

In a recent article, Demichelis and Polemarchalis (2007) highlighted the role played by the frequency of trade on the degree of indeterminacy of equilibrium in economies of overlapping generations. Assuming that time has a finite starting point and extends into the infinite future, they prove that the degree of indeterminacy increases with the number of periods in the life-span of individuals, which is assumed to be certain. We show that this result does not hold when individual longevity is uncertain. We build a discrete time model that uniformly converges to a standard continuous time overlapping-generation model when the frequency of trade is infinite. Deriving the equilibrium prices, we demonstrate that the degree of indeterminacy is independent of the frequency of trade and is always equal to one.

Keywords: Overlapping generations · Perpetual youth model · Determinacy · Continuous time · Discrete time

JEL Classification Numbers D50 · D90

*Correspondence: Centre d'Economie de la Sorbonne, 106 boulevard de l'Hôpital, 75013 Paris, France. E-mail: dalbis@univ-paris1.fr

†Correspondence: Université de La Rochelle - MIA, Avenue Michel Crépeau, 17042 La Rochelle, France. E-mail: eaugerau@univ-lr.fr

1 Introduction

The indeterminacy of the competitive equilibrium in economies of overlapping generations is a crucial issue for the design of monetary and fiscal policies. In an economy populated with individuals living for two periods and trading one commodity, it is well known that indeterminacy is of degree one: only the level of prices is indeterminate while the inflation rate is determinate. Increasing the number of traded goods increases the degree of indeterminacy in a truncated economy (see the nice exposition proposed by Geanakoplos and Polemarchakis, 1991). However, as claimed by Kehoe and Levine (1984), this property does not generalize to a model with many individuals and general preferences. Then, Kehoe et al. (1991) prove the uniqueness of the equilibrium near the steady state if dated consumption goods are gross substitutes at all price ratios close to the steady state. Uniqueness means that the number of stable eigenvalues exactly equals the number of initial conditions describing the distribution of financial assets among generations at the initial date of the economy.

In recent articles, Demichelis (2002) and Demichelis and Polemarchakis (2007) reassessed the role played by the number of generations in the determinacy issue by focussing on the number of periods in the life-span, or equivalently on the frequency of trade among generations. In the interesting case such that time has a finite starting point and extends into the infinite future, they prove its influence on the degree of indeterminacy. Indeed, the algebra shows that the number of eigenvalues whose modulus is lower than one monotonously increases with the frequency of trade. However, according

to the authors, despite the fact that the equilibrium equation in prices is linear, this counting of equations and unknowns does not necessarily imply an increase in the degree of indeterminacy. Using the convergence property of their discrete time model to its continuous time counterpart, they claim that the indeterminacies that appear in discrete time vanish in continuous time as solutions are time-shifts of a single path. Indeterminacies are hence a by-product of the discretization of the economy.

In this paper, we study the same economy except for the assumption on the individuals' longevity which is not anymore certain. We build a discrete time model similar to Farmer, Nourry and Venditti (2011) and that converges, when the frequency of trade is infinite, to the pure exchange economies' extensions of the Blanchard (1985) continuous time model. It is indeed known since Weil (1989) that the degree of indeterminacy is one in such a framework. The derivation of the equilibrium prices in the discrete time model allows us to claim that the degree of indeterminacy is independent of the frequency of trade. Moreover, the equilibrium is time-shifts of a single path. We are aware (see d'Albis and Augereau-Véron, 2011) that using a Poisson process to describe the survival function eases a lot the computation of the model and think that generalizing our results to realistic mortality patterns is a promising avenue of research.

The paper is organized as follows: in Section 2.1, the discrete time model is presented and the exact solution for the price dynamics is given. In section 2.2, we show that the equilibrium in discrete time uniformly converges to its continuous time counterpart. Proofs are gathered in Section 3.

2 The economy

The model closely follows Demichelis and Polemarchakis (2007) except for individual longevity that is uncertain. The economy is stationary, the distribution of the fundamentals being invariant with calendar time, and there is one commodity available at each date, which can not be stored or produced.

2.1 Discrete time

Discrete time has a finite starting point and extends into the infinite future:

$$0, (1/n), \dots, (t/n), \dots$$

where $t \in \mathbb{N}$ and $n \in [1, +\infty)$. The unit of time is given by $1/n$, which will be used for comparisons of equilibrium paths.

The duration of life is uncertain. As in Blanchard (1985), it is supposed to follow a Poisson process. Thus, as of time τ/n , an individual has a probability to be alive at time t/n equal to $(1 - \lambda/n)^{(t-\tau)}$, with $\lambda \in (0, 1)$. λ/n is the hazard rate of death which depends on the length of a period. Consequently, life expectancy at any age is a constant independent of n . It writes:

$$\sum_{s=1}^{\infty} \frac{s}{n} \frac{\lambda}{n} \left(1 - \frac{\lambda}{n}\right)^{s-1} = \frac{1}{\lambda}. \quad (1)$$

Despite life uncertainty, the model is similar to Demichelis and Polemarchakis (2007): increasing n increases the (expected) frequency of trade without modifying the (expected) duration of life.

At date $t/n = \tau/n, (\tau + 1)/n, \dots$, the consumption of the individual is $x(\tau, t; n)$ and his endowment is $e(t; n)$, a non-negative amount. Moreover,

across the life-span, the expected flow of endowments is:

$$\sum_{s=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^s e(\tau + s; n) = 1. \quad (2)$$

This normalization is necessary to keep the endowment distribution independent from changes in the unit of time. Following d'Albis and Augeraud-Véron (2009), a distinction is made between cohorts born before and after the initial date of the economy. For any individual belonging to a cohort born at time τ/n with $\tau = 1, 2, \dots$, the intertemporal utility function is:

$$u = \sum_{s=0}^{\infty} \left(1 - \frac{\delta}{n}\right)^s \left(1 - \frac{\lambda}{n}\right)^s \ln(x(\tau, \tau + s; n)), \quad (3)$$

where $\delta/n \in (0, 1)$ stands for the discount factor. Individuals have access to a competitive consumption-loan market and, as in Yaari (1965), to a competitive annuity market. The intertemporal budget constraint writes:

$$\sum_{s=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^s p(\tau + s; n) x(\tau, \tau + s; n) \leq w(\tau; n). \quad (4)$$

where $w(\tau; n)$ is the human wealth:

$$w(\tau; n) = \sum_{s=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^s p(\tau + s; n) e(\tau + s; n). \quad (5)$$

The optimal consumption path is (see Lemma 1 in Section 3):

$$x(\tau, \tau + s; n) = \frac{[1 - (1 - \frac{\lambda}{n})(1 - \frac{\delta}{n})] (1 - \frac{\delta}{n})^s}{p(\tau + s; n)} w(\tau; n). \quad (6)$$

Conversely, for individual belonging to cohorts born before the initial date of the economy and which is still alive, the utility function at time 0 is:

$$u = \sum_{s=0}^{\infty} \left(1 - \frac{\delta}{n}\right)^s \left(1 - \frac{\lambda}{n}\right)^s \ln(x(\tau, s; n)). \quad (7)$$

At the same date, the intertemporal budget constraint writes:

$$\sum_{s=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^s p(s; n) x(\tau, s; n) \leq p(0; n) a(\tau, 0; n) + w(0; n), \quad (8)$$

where $a(\tau, 0; n)$ denotes the initial amount of financial wealth, whose distribution among the population is given and whose aggregate value is zero.

The optimal consumption path is:

$$x(\tau, s; n) = \frac{[1 - (1 - \frac{\lambda}{n})(1 - \frac{\delta}{n})] (1 - \frac{\delta}{n})^s}{p(s; n)} [p(0; n) a(\tau, 0; n) + w(0; n)]. \quad (9)$$

Each individual belongs to a large cohort of identical individuals. Therefore even though each individual's life-span is stochastic, there is no aggregate uncertainty. The law of large numbers applies, and thus, the size of each cohort is decreasing at rate λ/n . Then, at time t/n , the size of the cohort born at time τ/n is $(\gamma/n) N(\tau; n) (1 - \lambda/n)^{(t-\tau)}$ where $\gamma/n > 0$ is the birth rate for the unit of time $1/n$ and $N(\tau; n)$ is the size of the population at time τ/n . Since a new cohort is born at each date, $N(t; n)$ satisfies:

$$N(t; n) = \sum_{\tau=-\infty}^t \frac{\gamma}{n} N(\tau; n) \left(1 - \frac{\lambda}{n}\right)^{(t-\tau)}. \quad (10)$$

Assume, as in Demichelis and Polemarchakis (2007), that the population is stationary and normalized to 1 for computing the birth rate such that: $\gamma = \lambda$.

The aggregate endowment, $e(t; n)$, is supposed to be constant and using (2), is equal to:

$$e(t; n) = \frac{\lambda}{n}. \quad (11)$$

At the equilibrium, aggregate demand equals aggregate endowment if and only if

$$\sum_{\tau=-\infty}^t \left(1 - \frac{\lambda}{n}\right)^{(t-\tau)} x(\tau, t; n) = 1, \quad (12)$$

is satisfied. Substituting (6), (9) and (11) in (12) yields the equilibrium equation for prices:

$$p(t; n) = \left[\left(1 - \frac{\delta}{n}\right) \left(1 - \frac{\lambda}{n}\right) \right]^t \left[1 - \left(1 - \frac{\lambda}{n}\right) \left(1 - \frac{\delta}{n}\right) \right] \times \left[\sum_{i=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^i p(i; n) + \sum_{\tau=1}^t \frac{\frac{\lambda}{n} \sum_{i=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^i p(\tau + i; n)}{\left[\left(1 - \frac{\delta}{n}\right) \left(1 - \frac{\lambda}{n}\right)\right]^{\tau}} \right] \quad (13)$$

Note that initial conditions $a(\tau, 0; n)$ vanish at the equilibrium since aggregate assets equal zero. Changing the order of summation yields an equation similar to Demichelis and Polemarchakis (2007):

$$p(t; n) = p(0; n) + \sum_{i=1}^{\infty} c(i, t) p(i; n), \quad (14)$$

where:

$$c(i, t) = \frac{\left(1 - \frac{\lambda}{n}\right)^i \left[1 + \frac{\lambda}{n} \sum_{\tau=1}^{\min\{i, t\}} \left(1 - \frac{\lambda}{n}\right)^{-2\tau} \left(1 - \frac{\delta}{n}\right)^{-\tau} \right]}{\left[\left(1 - \frac{\delta}{n}\right) \left(1 - \frac{\lambda}{n}\right)\right]^{-t} \left[1 - \left(1 - \frac{\lambda}{n}\right) \left(1 - \frac{\delta}{n}\right) \right]^{-1}}. \quad (15)$$

Since the price dynamics is linear, it can be solved explicitly.

Proposition 1. *The equilibrium prices satisfy:*

$$p(t; n) = p(0; n) \left(1 - \frac{\delta}{n}\right)^t. \quad (16)$$

Proof. See Section 3.

Proposition 1 shows that, despite having a finite starting point for time, the equilibrium prices are featured by a balanced-growth path, which growth rate is determinate and given by $-\delta/n$. More importantly, since the initial prices $p(0; n)$ are unknown, the degree of indeterminacy is 1: there is a one

parameter family of solutions but all paths can be obtained from another one by a time-shift. The frequency of trade does not modify the degree of indeterminacy; it simply affects the constant rate of inflation.

2.2 Continuous time

The continuous time version of the model presented above is similar to Buiter (1988) and Weil (1989). This section derives the equilibrium prices and demonstrates the uniform convergence of the equilibrium computed in discrete time to its continuous time counterpart.

Time has a finite starting point:

$$0 \leq t < +\infty.$$

An individual born at time τ has a probability to be alive at time t equal to $e^{-\lambda(t-\tau)}$, where $\lambda > 0$ is the hazard rate. Consequently, life expectancy at any age is $1/\lambda$. Consumption at age $t - \tau$ is denoted $x(\tau, t)$ and the endowment received at date t is $e(t)$. An individual born at time $\tau > 0$ maximizes:

$$\int_{\tau}^{\infty} e^{-(\lambda+\delta)(s-\tau)} \ln x(\tau, s) ds, \quad (17)$$

subject to

$$\int_{\tau}^{\infty} e^{-\lambda(s-\tau)} p(s) x(\tau, s) ds \leq w(\tau), \quad (18)$$

where $p(s)$ denotes the price at time s , $\delta > 0$ denotes the discount rate and where

$$w(\tau) = \int_{\tau}^{\infty} e^{-\lambda(s-\tau)} p(s) e(s) ds. \quad (19)$$

The optimal consumption path satisfies:

$$p(t) x(\tau, t) = (\delta + \lambda) w(\tau) e^{-\delta(t-\tau)} \quad (20)$$

Conversely, an individual born at time $\tau \leq 0$ and still alive at time 0, maximizes:

$$\int_0^{\infty} e^{-(\delta+\lambda)s} \ln x(\tau, s) ds, \quad (21)$$

subject to

$$\int_0^{\infty} e^{-\lambda s} p(s) x(\tau, s) ds \leq p(0) a(s, 0) + w(0), \quad (22)$$

where $a(s, 0)$ is given. The optimal consumption path satisfies:

$$p(t) x(\tau, t) = (\delta + \lambda) [p(0) a(s, 0) + w(0)] e^{-\delta t}. \quad (23)$$

The population is stationary and the birth rate is λ . The aggregate endowment is constant, and satisfies: $e(t) = \lambda$. The condition for the aggregate demand to equal aggregate endowment is now

$$\int_{-\infty}^t e^{-\lambda(t-\tau)} x(\tau, t) d\tau = 1. \quad (24)$$

Using (20) and (23), this yields the equilibrium equation for prices:

$$\begin{aligned} p(t) = & (\delta + \lambda) e^{-(\delta+\lambda)t} \int_0^{\infty} e^{-\lambda s} p(s) ds \\ & + (\delta + \lambda) \lambda \int_0^t e^{-(\delta+\lambda)(t-\tau)} \left(\int_{\tau}^{\infty} e^{-\lambda(s-\tau)} p(s) ds \right) d\tau, \end{aligned} \quad (25)$$

or equivalently:

$$p(t) = \int_0^{\infty} p(s) \varphi(t, s) ds, \quad (26)$$

where

$$\varphi(t, s) = (\delta + \lambda) e^{-(\delta+\lambda)t} e^{-\lambda s} \left(1 + \lambda \frac{e^{(2\lambda+\delta) \min\{s,t\}} - 1}{(2\lambda + \delta)} \right).$$

Proposition 2. *The equilibrium prices satisfy*

$$p(t) = p(0) e^{-\delta t}. \quad (27)$$

Proof. See Section 3.

Proposition 3. *As $n \rightarrow \infty$, $p(t; n)$ converges uniformly to $p(t)$.*

Proof. See Section 3.

As in Demichelis and Polemarchakis (2007), the discrete time model converges to the continuous time one when the frequency of trade is infinite and the degree of indeterminacy of the equilibrium is still 1.

3 Proofs

Lemma 1. *For all $s = 0, 1, \dots$, the optimal individual consumption is (6) if $\tau \in \mathbb{N}_+$ and (9) if $\tau \in \mathbb{Z}_-$.*

Proof. For $\tau \in \mathbb{N}_+$, the individual maximizes (3) subject to (4). The Lagrangian writes:

$$L = \sum_{s=0}^{\infty} \left(1 - \frac{\delta}{n}\right)^s \left(1 - \frac{\lambda}{n}\right)^s \ln(x(\tau, \tau + s; n)) + \mu w(\tau; n) - \mu \sum_{s=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^s p(\tau + s; n) x(\tau, \tau + s; n). \quad (28)$$

For for all $s = 0, 1, \dots$, the first order conditions on $x(\tau, \tau + s; n)$ write:

$$\frac{\left(1 - \frac{\delta}{n}\right)^s}{x(\tau, \tau + s; n)} - \mu p(\tau + s; n) = 0. \quad (29)$$

Replace (29) in (4) to compute:

$$\mu = \frac{\sum_{s=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^s \left(1 - \frac{\delta}{n}\right)^s}{w(\tau; n)}, \quad (30)$$

and (30) in (29) to obtain (6). For $\tau \in \mathbb{Z}_-$, the individual maximizes (7)

subject to (8). The Lagrangian writes:

$$L = \sum_{s=0}^{\infty} \left(1 - \frac{\delta}{n}\right)^s \left(1 - \frac{\lambda}{n}\right)^s \ln(x(\tau, s; n)) + \mu [p(0; n) a(\tau, 0; n) + w(0; n)] - \mu \sum_{s=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^s p(s; n) x(\tau, s; n). \quad (31)$$

For for all $s = 0, 1, \dots$, the first order conditions on $x(\tau, \tau + s; n)$ write:

$$\frac{\left(1 - \frac{\delta}{n}\right)^s}{x(\tau, s; n)} - \mu p(s; n) = 0. \quad (32)$$

Replace (32) in (8) to compute:

$$\mu = \frac{\sum_{s=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^s \left(1 - \frac{\delta}{n}\right)^s}{p(0; n) a(\tau, 0; n) + w(0; n)}, \quad (33)$$

and (33) in (32) to obtain (9). \square

Proof of Proposition 1. Equation (13) can be rewritten as a second order difference equation that can be solved explicitly. From (13), one indeed has

$$p(t+1; n) = \left(1 - \frac{\delta}{n}\right) \left(1 - \frac{\lambda}{n}\right) p(t; n) + \left[1 - \left(1 - \frac{\lambda}{n}\right) \left(1 - \frac{\delta}{n}\right)\right] \frac{\lambda}{n} \sum_{i=0}^{\infty} \left(1 - \frac{\lambda}{n}\right)^i p(t+1+i; n), \quad (34)$$

and finally

$$p(t+2; n) = \left(2 - \frac{\delta}{n}\right) p(t+1; n) - \left(1 - \frac{\delta}{n}\right) p(t; n). \quad (35)$$

The solutions of the characteristic polynomial of (35) are: 1 and $\left(1 - \frac{\delta}{n}\right)$.

Denote the increase in prices with $q(t; n) = p(t+1; n) - p(t; n)$ and conclude

that $q(t; n) = \left(1 - \frac{\delta}{n}\right)^t q(0; n)$ and hence that:

$$p(t; n) = \frac{1 - \left(1 - \frac{\delta}{n}\right)^t}{\frac{\delta}{n}} q(0; n) + p(0; n). \quad (36)$$

Replace (36) in (13) to obtain after tedious computations:

$$q(0; n) + p(0; n) \frac{\delta}{n} = 0. \quad (37)$$

Replacing (37) in (36) yields (16). \square

Proof of Proposition 2. The proof is similar to the one of proposition 1. Differentiating (25) twice with respect to time yields to

$$\frac{d^2 p(t)}{dt^2} = -\delta \frac{dp(t)}{dt}. \quad (38)$$

Define $q(t) = dp(t)/dt$ and conclude that $q(t) = q(0) e^{-\delta t}$ and that

$$p(t) = q(0) \frac{1 - e^{-\delta t}}{\delta} + p(0). \quad (39)$$

Replacing (39) in (25) yields

$$q(0) + \delta p(0) = 0. \quad (40)$$

Use (39) and (40) to obtain (27). \square

Proof of Proposition 3. The proof proceed showing that (13) converges to (25) when $n \rightarrow +\infty$. Using $(1 - \frac{x}{n})^y \sim e^{-\frac{xy}{n}}$ rewrites (13) as follows:

$$p(t; n) \sim e^{-(\delta+\lambda)\frac{t}{n}} (\lambda + \delta) \times \left[\frac{1}{n} \sum_{i=0}^{\infty} e^{-\lambda \frac{i}{n}} p(i; n) + \frac{1}{n} \sum_{\tau=1}^t \frac{\frac{\lambda}{n} e^{\lambda \frac{\tau}{n}} \sum_{i=0}^{\infty} e^{-\lambda(\frac{\tau+i}{n})} p(\tau+i; n)}{e^{-(\delta+\lambda)\frac{\tau}{n}}} \right] \quad (41)$$

Now using $\frac{1}{n} \sum_{s=0}^{\infty} f(a + \frac{s}{n}) \sim \int_a^{\infty} f(s) ds$, it follows that:

$$p(t; n) \sim e^{-(\delta+\lambda)\frac{t}{n}} (\lambda + \delta) \times \left[\int_0^{\infty} e^{-\lambda i} p(i) di + \lambda \int_0^{\frac{t}{n}} e^{(\delta+\lambda)\tau} \int_{\tau}^{\infty} e^{-\lambda(i-\tau)} p(i) did\tau \right] \quad (42)$$

Let $t' = t/n$, and thus:

$$p(t; n) \sim e^{-(\delta+\lambda)t'} (\lambda + \delta) \times \left[\int_0^\infty e^{-\lambda i} p(i) di + \lambda \int_0^{t'} e^{(\delta+\lambda)\tau} \int_\tau^\infty e^{-\lambda(i-\tau)} p(i) did\tau \right] \quad (43)$$

Conclude that $\lim_{n \rightarrow +\infty} p(t; n) = p(t)$ where $p(t)$ is given by (25). \square

References

- [1] Albis (d'), H., Augeraud-Véron, E., 2009. Competitive growth in a life-cycle model: existence and dynamics. *International Economic Review* 50, 459-484.
- [2] Albis (d'), H., Augeraud-Véron, E., 2011. Continuous-time overlapping generations models. R. Boucekkine, Y. Yatsenko and N. Hritonenko (Eds.), *Optimal Control of Age-structured Populations in Economy, Demography, and the Environment*, Routledge, Taylor and Francis, p. 45-69.
- [3] Blanchard, O. J., 1985. Debt, deficits and finite horizons. *Journal of Political Economy* 93, 223-247.
- [4] Buiter, W., 1988. Death, birth, productivity growth and debt neutrality. *Economic Journal* 98, 279-293.
- [5] Demichelis, S., 2002. Solution of two equations arising in the overlapping generations model. *Mathematical Models and Methods in Applied Sciences* 12, 1269-1280.
- [6] Demichelis, S., Polemarchakis, H. M., 2007. The determinacy of equilibrium in economies of overlapping generations. *Economic Theory* 32, 461-475.
- [7] Farmer, R. E. A., Nourry, C., Venditti, A., 2011. Debt, deficits and finite horizons: The stochastic case. *Economics Letters* 111, 47-49.
- [8] Geanakoplos, J. D., Polemarchakis, H. M., 1991. Overlapping generations. In: Hildenbrand, W., Sonnenschein, H. (eds.) *Handbook of mathematical economics*, vol. IV. North Holland.
- [9] Kehoe, T. J., Levine, D. K., 1984. Intertemporal separability in overlapping-generations models. *Journal of Economic Theory* 34, 216-226.

- [10] Kehoe, T. J., Levine, D. K., Mas-Colell A., Woodford, M., 1991. Gross substitutability in large-square economies. *Journal of Economic Theory* 54, 1-25.
- [11] Weil, Ph., 1989. Overlapping families of infinitely-lived agents. *Journal of Public Economics* 38, 183-198.
- [12] Yaari, M., 1965. Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies* 32, 137-160.