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**On the Timing of Vertical Relationships**

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# On the Timing of Vertical Relationships\*

Etienne Billette de Villemeur<sup>†</sup> Richard Ruble<sup>‡</sup> Bruno Versaevel<sup>§</sup>

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## Abstract

In a real options model, we show that the standard analysis of vertical relationships transposes directly to investment timing. Thus, when a firm undertaking a project requires an outside supplier (e.g., an equipment manufacturer) to provide it with a discrete input to serve a growing but uncertain demand, and if the supplier has market power, investment occurs too late from an industry standpoint. The distortion in firm decisions is characterized by a Lerner-type index, and we show how market growth rate and volatility affect the extent of the distortion. If the initial market demand is high, greater volatility increases the effective investment cost, and results in lower value for both firms. Vertical restraints can restore efficiency. For instance, the upstream firm can induce entry at the correct investment threshold by selling a call option on the input. Otherwise, if two downstream firms are engaged in a preemption race, the upstream firm sells the input to the first investor at a discount which is chosen in such a way that the race to preempt exactly offsets the vertical distortion, and this leader invests at the optimal time.

JEL classification: C73; D43; D92; L13

Keywords: Investment timing; Preemption; Real options; Vertical relations

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# 1 Introduction

In real option models of investment, the cost of the investment (strike price) is often tacitly taken to reflect economic fundamentals closely. This assumption seems reasonable when the investment consists of R&D that is performed largely in-house, or in industries such as real estate development that may rely on competitive outside contractors. However, there are many other cases in which a firm wishing to exercise an investment option depends on an outside firm with market power to provide it with a discrete input (e.g., a key equipment) it needs to start producing and selling. Thus, an electricity producer may buy a nuclear plant from an outside firm, an oil company that decides to drill offshore must acquire a platform from a specialized supplier, or an aeronautics firm will coordinate aircraft development with an engine manufacturer. In addition, strategic issues may arise if several firms seek to exercise related investment options, and call upon the same supplier. To illustrate, in the next section we outline the case of a market for a new vaccine, where demand is related to the diffusion of an emerging pathogen, and firms must invest in a factory constructed to exact specifications before starting operations.

This paper uses advances in real options games to build a model of vertical relationships in which the cost of a firm's investment is endogenous.<sup>1</sup> We adopt similar specifications to models by Boyer, Lasserre and Moreaux [1], Mason and Weeds [16], and Smit and Trigeorgis [20], and incorporate an upstream equipment supplier that prices with market power. There are thus two ways to approach the contributions of this paper. On the one hand, it extends the real options literature, including strategic real options game, to encompass a richer industrial structure, and on the other, the paper shows how insights drawn from the study of vertical relationships apply in stochastic dynamic settings.

Specifically, we show that the standard analysis of vertical relationships translates directly to investment timing, with investment trigger replacing price as the decision variable of the downstream firm. When the upstream supplier exercises market power, a vertical effect akin to double marginalization causes the downstream firm to unduly delay its investment decision relative to the optimal

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<sup>1</sup>For recent surveys of game theoretic real options models, see Boyer, Gravel, and Lasserre [2], and Huisman, Kort, Pawlina, and Thijssen [12]. Among economic extensions of real option models, Grenadier and Wang [11] comes closest to our work here, as it studies the effect of agency issues on option exercise (albeit, in a corporate governance framework). Moreover, Lambrecht, Pawlina, and Teixeira [14] and Patel and Zavadov [17] develop alternative approaches to investment options in vertical structures.

exercise threshold for the industry. This distortion increases with both market growth and volatility. Thus, the industry earns lower profits under separation than under integration. In contrast with the standard real option framework, greater volatility decreases upstream and downstream firm value near the exercise threshold, because of the simultaneous presence of two effects: the option value of delay is balanced by a greater markup choice by the upstream firm.

If feasible (for example, because the upstream firm has information regarding the stochastic final demand), vertical restraints that take the form of an option or down payment restore the industry optimum. Without vertical restraints, the upstream firm benefits from the presence of a second downstream firm, although this possibly occurs at the expense of aggregate industry profits. We find that the race between downstream firms to preempt one another exactly balances the incentive to delay caused by the upstream firm’s mark-up, so the leader invests at the optimal integrated threshold (as in the integrated case with a single buyer), whereas the follower invests at the separation threshold (for duopoly profits), a type of “no distortion at the top” result. The leader receives a discounted price, and this discount decreases when the volatility rises.

The remainder of the paper is organized as follows. In Section 2, we motivate the analysis, and our model specifications, by examining the case of an emerging market for a new vaccine. In Section 3 we describe the model, with one upstream supplier and one downstream firm, and investigate the basic vertical externality. This is done by comparing the equilibrium outcomes in the integrated case, which we use as a benchmark, with the outcomes of the separated case. In Section 4, we discuss the introduction of vertical restraints that restore the industry optimum. In Section 5, we introduce a second downstream firm and compute the preemption equilibrium, before comparing the investment threshold and pricing outcomes with the single-firm case. Final remarks appear in Section 6. All the proofs and derivations are in the appendix.

## **2 An Example: Investments in the Vaccine Industry**

A recent example of investment in production facilities for a new vaccine against the dengue fever motivates our model. Dengue is a disease caused by any of four closely related virus serotypes transmitted by mosquitoes. It strikes people with low levels of immunity, with symptoms that include intense joint and muscle pain, headache, nausea, and fever. The most severe form of the disease is the dengue hemorrhagic fever. Although it occurs mostly in Asian and Latin American countries, where it is a leading cause of hospitalization, the disease spreads to new parts of the globe

each year, including countries such as Australia and the United States (Puerto Rico, the U.S. Virgin Islands, the Texas-Mexico border, Pacific islands, and most recently Florida).<sup>2</sup>

To address this problem, one or more vaccines are due to arrive over the next years. With clinical studies reaching their final phase, Sanofi Pasteur, the vaccines division of Sanofi Aventis SA, was the first to launch the construction of a new plant north of Lyon (France) in 2009, with an annual capacity of 100 million doses per year. The total investment amounts to \$477 million (see Carroll [3]). The main facility, which concentrates the firm's production technology, has been specifically designed for the processing of the novel vaccine. Most components of the investment, including fees and wages, capital amortization, and the costs for the safety and quality qualification procedure, are unrecoverable. The equipment is sourced on an intermediate market from specialized input providers, and represents on the order of 35-40% of the plant construction cost. The customized lyophilisators, which use liquid nitrogen refrigeration for freeze drying operations, constitute a central piece of equipment in the mass production process. The suppliers of pharma freeze drying technology are highly concentrated, suggesting the possibility of market power. In Europe, lyophilizers are supplied by firms such as Usifroid, a subsidiary of Telstar SA which recently claimed a French market share of 80% in freeze drying equipment solutions for the pharmaceutical industry. In 2009, GlaxoSmithKline Plc (GSK), another leading vaccine producer, announced that it would develop and manufacture another dengue vaccine with a Brazilian partner. GSK is thus likely to also invest in additional production capacities in the foreseeable future, at some point in time that will depend on demand forecasts.<sup>3</sup>

The potential demand for the future vaccines is clearly growing, though future levels are uncertain. The number of reported cases, as measured annually, is a simple indicator of the magnitude of future demand.<sup>4</sup> According to the World Health Organization (WHO) “[a]n estimated 2.5 billion people live in over 100 endemic countries and areas where dengue viruses can be transmitted. Up to 50 million infections occur annually with 500,000 cases of dengue hemorrhagic fever and 22,000 deaths mainly among children”. The number of countries reporting cases is another demand indicator. The WHO indicates that “[p]rior to 1970, only 9 countries had experienced cases of dengue hemorrhagic fever; since then the number has increased more than 4-fold and continues to rise”.<sup>5</sup>

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<sup>2</sup>See World Health Organization [22] and Center for Disease Control [4].

<sup>3</sup>Sources on Telstar's market shares in Europe and GSK's project to manufacture a vaccine for dengue fever are <http://www.telstar-lifesciences.com/en/> and <http://www.gsk.com/media/pressreleases/2009>, respectively.

<sup>4</sup>The issue of demand forecasts for new vaccines is discussed thoroughly in Center for Global Development [5].

<sup>5</sup>Source: <http://www.who.int/csr/disease/dengue/impact/en/index.html>.

This real-world situation, where firms' choice of investment timing depends on the cost of a key equipment, which is necessary in order to serve a growing though uncertain demand, and is delivered by an upstream supplier with market power, is thus emblematic of the many market cases captured by our model specifications, which are described in the next section.

### 3 The Basic Vertical Externality

Investment in a discrete input is necessary to operate on a final market. It can be produced and used by the same firm (integration), or produced by an upstream supplier and used by one or several other firms (separation). The flow profit resulting from investment is  $Y_t \pi_M$  where  $\pi_M$  is an instantaneous monopoly profit, and  $Y_t > 0$  is a random shock assumed to follow a geometric Brownian motion with drift  $dY_t = \alpha Y_t dt + \sigma Y_t dZ_t$ . The non-negative parameters  $\alpha$  and  $\sigma$  represent the market's expected growth rate (or "drift") and volatility, respectively, and  $Z_t$  is a standard Wiener process.<sup>6</sup> A lowercase  $y = Y_t$  is used to denote the current level of the state variable. The threshold  $y_i$  is a decision variable. It triggers the investment (whence the subscript  $i$ ) in the discrete input when attained by  $Y_t$  for the first time and from below, at a future date which is stochastic. The cost  $I$  of the input is positive. The discount rate  $r > \alpha$  is common to all firms.<sup>7</sup>

#### 3.1 Integrated Case

Suppose that a single firm produces the discrete input, decides at what threshold  $y_i$  to invest, for a current market size  $y \leq y_i$ , and earns the subsequent flow profit. The value of a firm that decides to invest when the market reaches size  $y_i$ , given the investment cost  $I$  and the current market size  $y$ , is:

$$V(y, y_i, I) = \left(\frac{y}{y_i}\right)^\beta \left(\frac{\pi_M}{r - \alpha} y_i - I\right), \quad (1)$$

where  $\beta \equiv \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$ , which is referred to in what follows as a discounting term. The expressions of  $V(y, y_i, I)$  in (1), and of  $\beta$ , are standard in real option models (see Dixit and

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<sup>6</sup>The geometric brownian motion is derived from  $Y_t = Y_0 \exp\left[\left(\alpha - \frac{1}{2}\sigma^2\right)t + \sigma Z_t\right]$  by using Itô's lemma.

<sup>7</sup>It can be proved easily that a firm increases value by waiting to invest forever if  $r \leq \alpha$ .



Pindyck [7], Chapter 5, or Chevalier-Roignant and Trigeorgis [6], Chapters 11-12).<sup>8</sup> We will use the property that  $\beta$  is decreasing in  $\alpha$  and in  $\sigma$  throughout the paper.

The integrated firm's decision problem is  $\max_{y_i} V(y, y_i, I)$ . Since the objective is quasiconcave, differentiating (1) gives the value-maximizing investment trigger,  $y^* = \frac{\beta}{\beta-1} \frac{r-\alpha}{\pi_M} I$ , which serves as a benchmark throughout the analysis. Then the value of the firm that invests at  $y^*$  is:

$$V(y, y^*, I) = \left(\frac{y}{y^*}\right)^\beta \frac{I}{\beta-1}. \quad (2)$$

It is assumed throughout that the current market size  $y$  at  $t = 0$  is positive and sufficiently small relative to  $I$  so that it is not profitable to invest immediately.

### 3.2 Separated Case

Suppose that the input production and investment decisions are made by distinct firms. We suppose that the upstream firm, as an input producer on the intermediate market, does not observe  $Y_t$  at (almost) any date  $t$ .<sup>9</sup> As a separate entity, it chooses an input price  $p_S \geq I$  that is independent of the random shock. The downstream firm is assumed to be a price-taker in the intermediate market.<sup>10</sup> Given  $p_S$ , it observes the final market shock, and decides when to invest, at  $y_i$ . To establish the equilibrium in  $(y_i, p_S)$  we proceed by backwards induction.

The value of a downstream firm that decides to invest when the market reaches size  $y_i$ , given the investment cost  $p_S$  and the current market size  $y$ , is:

$$V(y, y_i, p_S) = \left(\frac{y}{y_i}\right)^\beta \left(\frac{\pi_M}{r-\alpha} y_i - p_S\right), \quad (3)$$

all  $y \leq y_i$ . The separated firm's decision problem is  $\max_{y_i} V(y, y_i, p_S)$ , and the associated value-maximizing investment trigger is  $y_S(p_S) = \frac{\beta}{\beta-1} \frac{r-\alpha}{\pi_M} p_S$ , which is increasing in  $p_S$ , with  $y_S(I) = y^*$ .

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<sup>8</sup>The term  $\left(\frac{y}{y_i}\right)^\beta$  in (1) reads as the expected discounted value, measured when  $Y_t = y$ , of receiving one monetary unit when  $Y_t$  reaches  $y_i$  for the first time. In the certainty case  $\sigma = 0$ , we have  $\beta = \frac{r}{\alpha}$  and  $\left(\frac{y}{y_i}\right)^\beta = e^{-r(t_i-t)}$ , which is the standard continuous time discounting term.

<sup>9</sup>We relax this assumption only in Section 4 which discusses possible vertical restraints.

<sup>10</sup>As in Tirole [21], this is “for simplicity” only that we “assume that the manufacturer chooses the contract” (p. 173), and the outside option of the downstream firm is normalized to zero.

At the current market size  $y$ , the upstream firm's value is:

$$W(y, p_S) = \left( \frac{y}{y_S(p_S)} \right)^\beta (p_S - I), \quad (4)$$

all  $y \leq y_S$ . Given  $y_S(p_S)$ , the upstream firm's decision problem is  $\max_{p_S} W(y, p_S)$ , leading to the optimal price which is to set  $p_S^* = \frac{\beta}{\beta-1}I$ . In what follows, for compactness denote  $y_S(p_S^*)$  by  $y_S^*$ . We find:

**Proposition 1** *In the separated case, there is a unique equilibrium characterized by:*

$$y_S^* = \left( \frac{\beta}{\beta-1} \right)^2 \frac{r-\alpha}{\pi_M} I \text{ and } p_S^* = \frac{\beta}{\beta-1} I. \quad (5)$$

Substituting back (5) into (3-4), we obtain the firm values in the separated equilibrium case:

$$V(y, y_S^*, p_S^*) = \frac{\beta}{\beta-1} \left( \frac{y}{y_S^*} \right)^\beta \frac{I}{\beta-1} \text{ and } W(y, p_S^*) = \left( \frac{y}{y_S^*} \right)^\beta \frac{I}{\beta-1}. \quad (6)$$

From (6) we obtain that  $\frac{V(y, y_S^*, p_S^*)}{W(y, p_S^*)} = \frac{\beta}{\beta-1}$ , implying that  $V(y, y_S^*, p_S^*) > W(y, p_S^*)$ , all  $\beta > 1$ . Using (2) we find that  $\frac{V(y, y_S^*, p_S^*) + W(y, p_S^*)}{V(y, y^*, I)} = \left( 1 + \frac{\beta}{\beta-1} \right) \left( \frac{\beta}{\beta-1} \right)^{-\beta} \in \left( \frac{2}{e}, 1 \right)$ . Hence the industry value is lower under separation than under integration, as is to be expected.

### 3.3 Comparative Statics

In the separated case, the upstream firm introduces a distortion by charging a price  $p_S$  above the cost  $I$ . This is analogous to the baseline model of vertical externality<sup>11</sup>, the investment trigger substituting for the final price as the downstream decision variable.

In fact, this model is formally identical to the baseline model with price choices, a final demand  $Q = aP^{-b}$ , a constant marginal cost of production  $c$ , and a wholesale price  $w$ , taking  $P \equiv y_i$ ,  $a \equiv \frac{\pi_M}{r-\alpha} y^\beta$ ,  $b \equiv \beta$ ,  $c \equiv \frac{r-\alpha}{\pi_M} I$ ,  $w \equiv p_S \frac{r-\alpha}{\pi_M}$ .

The vertical externality in the model may be gauged as follows. The trigger in the separated case,  $y_S^* = \frac{\beta}{\beta-1} y^*$ , is greater than in the integrated case. In static models of oligopoly, the Lerner index is often used as a measure of market power. For the upstream firm, we have:

$$L_p \equiv \frac{p_S^* - I}{p_S^*} = \frac{1}{\beta}. \quad (7)$$

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<sup>11</sup>See Tirole [21] for a description.

Formally,  $\beta$  plays the same role as the (absolute value of) the elasticity of demand in a monopoly model. By analogy, although it does not represent a price-cost margin, one may define a measure of the dynamic dimension of the downstream firm's market power as:

$$L_y \equiv \frac{y_S^* - y^*}{y_S^*} = \frac{1}{\beta}. \quad (8)$$

Note from (7) and (8) that  $L_p$  and  $L_y$  are fully characterized by  $\beta$ , and are impacted in the same proportions by a higher growth rate or a greater volatility.<sup>12</sup> Note also that  $\frac{V(y, y_S^*, p_S^*)}{W(y, p_S^*)} = \frac{p_S^*}{I} = \frac{y_S^*}{y^*} = \frac{\beta}{\beta-1}$  which implies that, in the separated case, the relative firm value is decreasing in  $\beta$ , as is the distortion in the two firms' decisions vis-à-vis the integrated case. The two measures  $L_p$  and  $L_y$  behave similarly and converge to 0 when  $\beta$  tends to infinity.

Another significant magnitude is the relative joint value under separation and integration, that is  $\frac{V(y, y_S^*, p_S^*) + W(y, p_S^*)}{V(y, y^*, I)} = \left(1 + \frac{\beta}{\beta-1}\right) \left(\frac{\beta}{\beta-1}\right)^{-\beta}$ . In contrast with the distortion in upstream and downstream choices, the distortion in the separated and integrated payoffs *decreases* when  $\alpha$  and  $\sigma$  increase, since  $\frac{d\left(\left(1 + \frac{\beta}{\beta-1}\right) \left(\frac{\beta}{\beta-1}\right)^{-\beta}\right)}{d\beta} < 0$ , with  $\lim_{\beta \rightarrow 1} \left(\left(1 + \frac{\beta}{\beta-1}\right) \left(\frac{\beta}{\beta-1}\right)^{-\beta}\right) = 1$  (recall that  $\beta$  is decreasing in  $\alpha$  and  $\sigma$ ).<sup>13</sup> This is because factors other than the vertical externality also affect firm values, including the denominator  $V(y, y^*, I)$ . When a rising  $\alpha$  or  $\sigma$  make  $\beta$  relatively low, the impact of the vertical distortion is small relative to the real option effect, and the joint value under separation converges to the value under integration. To summarize:

**Proposition 2** *The industry value is lower under separation than under integration. The distortion in firm decisions, as measured by  $L_p$  and  $L_y$ , is increasing in market growth rate and volatility, whereas the distortion in separated and integrated payoffs is decreasing in market growth rate and volatility.*

For the sensitivity analysis of firm choices and values we consider changes in the growth rate and in the volatility parameter separately.

□ *Growth.* The effect on  $y_S^*$  and  $p_S^*$  of a change in the growth rate is univocal:

$$\frac{dy_S^*}{d\alpha} > 0, \quad \frac{dp_S^*}{d\alpha} > 0.$$

<sup>12</sup>These expressions are comparable to those in Dixit, Pindyck, and Sodal [8].

<sup>13</sup>In Appendix 7.3 we fully characterize  $f(\beta) \equiv \left(1 + \frac{\beta}{\beta-1}\right) \left(\frac{\beta}{\beta-1}\right)^{-\beta}$ .

A change in these parameters does not impact only the magnitude of the vertical externality, as there is also a real option effect. To see that, focus first on the upstream firm's value. For notational simplicity, let  $V^* \equiv V(y, y_S^*, p_S^*)$ . By the envelope theorem,  $\frac{\partial V^*}{\partial y_S^*} = 0$ , and we find that:

$$\frac{dV^*}{d\alpha} = \frac{\partial V^*}{\partial \alpha} + \left( \frac{\partial V^*}{\partial \beta} + \frac{\partial V}{\partial p_S^*} \frac{dp_S^*}{d\beta} \right) \frac{d\beta}{d\alpha} = V^* \left[ \frac{\beta}{r - \alpha} + \left( \ln \frac{y}{y_S^*} + \frac{1}{\beta} \right) \frac{d\beta}{d\alpha} \right], \quad (9)$$

which is positive for all  $y \leq y_S^*$ . The direct effect in (9) is positive. The two terms between brackets, which describe the indirect effect, have opposite signs because a higher growth rate increases the investment option's value, but simultaneously raises the input price. However, the magnitude of the latter term is limited, so that the option value effect dominates the vertical effect. The sensitivity analysis is similar for the upstream firm, whose value in equilibrium  $W^* \equiv W(y, p_S^*)$  has an analogous form. Specifically, the effect of greater market growth on upstream value is univocal, as we find  $\frac{dW^*}{d\alpha} = W^* \left( \frac{\beta}{r - \alpha} + \left( \ln \frac{y}{y_S^*} + \frac{1}{\beta - 1} \right) \frac{d\beta}{d\alpha} \right) > 0$  (see Appendix 7.4), so in elasticity terms  $0 < \varepsilon_{W^*/\alpha} < \varepsilon_{V^*/\alpha}$ .

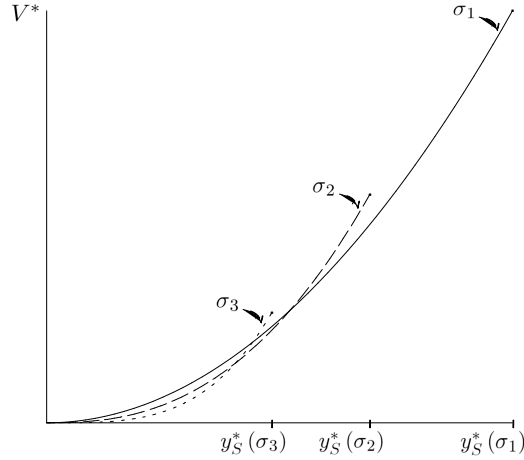


Figure 1: Downstream value  $V^* = V(y, y_S^*, p_S^*)$ , for  $y \leq y_S^*$ , with  $\frac{r - \alpha}{\pi_M} = I = 1$ ,  $r = 0.2$ ,  $\alpha = 0.05$  and  $\sigma_1 > \sigma_2 > \sigma_3$  such that  $\beta(\sigma_1) = 2$  (solid),  $\beta(\sigma_2) = 2.5$  (dash),  $\beta(\sigma_3) = 3.5$  (dots). For large initial market sizes, greater uncertainty (i.e., a lower  $\beta$ ) reduces firm value.

□ *Volatility.* The effect on  $y_S^*$  and  $p_S^*$  of a change in volatility rate is also univocal:

$$\frac{dy_S^*}{d\sigma} > 0, \quad \frac{dp_S^*}{d\sigma} > 0.$$

However, the effect of volatility on firm values is *not* univocal. Taking first the case of upstream value, and noting again that  $\frac{\partial V}{\partial y_S^*} = 0$ , we find that:

$$\frac{dV^*}{d\sigma} = \left( \frac{\partial V^*}{\partial \beta} + \frac{\partial V}{\partial p_S^*} \frac{dp_S^*}{d\beta} \right) \frac{d\beta}{d\sigma} = V^* \left( \ln \frac{y}{y_S^*} + \frac{1}{\beta} \right) \frac{d\beta}{d\sigma}, \quad (10)$$

which is positive (zero) if and only if  $y < (=) y_S^* \exp\left(-\frac{1}{\beta}\right) \equiv \hat{y}$  (note that  $\hat{y}$  is lower than  $y_S^*$  for all  $\beta > 1$ ). A change in  $\sigma$  has two opposite indirect effects on  $V^*$ . The first term between parentheses in (10) is the real option effect: greater volatility (hence a lower  $\beta$ ) has a positive impact on the downstream value. The second term is the vertical effect: greater volatility raises the upstream firm's optimal price  $p_S^*$ , lowering the downstream value. The net effect depends, in particular, on the current market size  $y$ . At low market sizes, the real option effect dominates and the downstream firm benefits from greater volatility, whereas at higher market sizes, which are closer to the investment trigger  $y_S^*$ , the real option effect is less important and the vertical effect tends to dominate. For large initial market sizes, in this model greater uncertainty thus reduces firm value, which stands in contrast with many real option models. Figure 1 illustrates the behavior of  $V(y, y_S^*, p_S^*)$  over  $[0, y_S^*]$  for several levels of  $\beta$ .

Similarly, the effect of greater volatility on the value of the upstream firm is also ambiguous. As  $\frac{dW^*}{d\sigma} = W^* \left( \ln \frac{y}{y_S^*} + \frac{1}{\beta-1} \right) \frac{d\beta}{d\sigma}$ , the crossover occurs at a lower threshold than for the upstream firm, that is at  $y_S^* \exp\left(-\frac{1}{\beta-1}\right) \equiv \check{y} < \hat{y}$ . Thus, both firms benefit from greater volatility at low enough market sizes, and both are harmed by volatility at high enough market sizes, and there exists a range of market sizes ( $\check{y}, \hat{y}$ ) over which the two firms have divergent preferences with respect to volatility.

The following proposition summarizes these results, making use of the inherent elasticity form in expressions such as (9) and (10).

**Proposition 3** *In the separated case, a higher growth rate or more volatility increase the upstream price and the downstream trigger. A higher growth rate increases upstream and downstream values, with  $0 < \varepsilon_{W^*/\alpha} < \varepsilon_{V^*/\alpha}$ . The effect of higher volatility on firm values depends on the market size:*

$$\begin{cases} 0 \leq \varepsilon_{W^*/\sigma} < \varepsilon_{V^*/\sigma} & \text{if } y \leq \check{y}; \\ \varepsilon_{W^*/\sigma} < 0 < \varepsilon_{V^*/\sigma} & \text{if } \check{y} < y < \hat{y}; \\ \varepsilon_{W^*/\sigma} < \varepsilon_{V^*/\sigma} \leq 0 & \text{if } \check{y} \leq y. \end{cases}$$

## 4 Vertical Restraints

In the baseline model of vertical externality, various contracting options or vertical restraints allow the separated structure to realize the integrated profit. Similar mechanisms apply here, although the interpretation is different because of the underlying dynamic nature of the model. We illustrate them by means of two examples.<sup>14</sup>

In Figure 2, the dashed line is the locus of the downstream firm's optimal responses to given upstream prices,  $y_S(p_S)$ . With the chosen parameters (that is,  $\beta = 2$  and  $I = \frac{\pi_M}{r-\alpha} = 1$ ), the separation outcome of Section 3.2 is  $(y_S^*, p_S^*) = (4, 2)$ . For a given  $y$  below the benchmark trigger  $y^* = 2$ , we may graph the isovalue curves of both firms in the plane  $(y_i, p_S)$ . The convex curves are the upstream isovalues, whereas the downstream isovalue curves are concave. The ordering of the curves follows from the monotonicity of the value functions  $V$  and  $W$  in  $p_S$ . Because  $p_S^*$  maximizes  $W(y, p_S)$ , the point  $(y_S^*, p_S^*)$  lies at a tangency of an upstream isovalue with the locus  $y_S(p_S)$ . To illustrate, when  $y = 1$ , say, the firm values in the separated case are  $V(1, y_S^*, p_S^*) = \frac{1}{8}$  and  $W(1, p_S^*) = \frac{1}{16}$ , whereas the integrated value is  $V(1, y^*, I) = \frac{1}{4}$ .

The two firms can be made better off by reaching a contractual agreement that yields a greater total value than in (1), the equilibrium of the separated case. The most direct value-maximizing contract specifies both the investment trigger and the price. This is analogous to resale price maintenance in the standard vertical framework. For simplicity, assume that the upstream manufacturer chooses the contract, and proposes it to the other firm, at some given  $y \leq y^*$ .

The contract proposal must satisfy the constraints that the downstream buyer earns no less than in the separation outcome with no vertical restraint. Should the upstream firm's current offer be rejected by the downstream firm, the upstream firm could not credibly commit *not* to sell the specific input at  $p_S^*$  at a future date when the trigger  $y_S^*$  is reached. It follows that the downstream firm's reservation value is  $V(y, y_S^*, p_S^*)$ . The upstream firm can appropriate all benefits on top of the latter downstream reservation level by dictating the trigger  $y^*$ , so that the total industry value is maximized, before charging the price for which the downstream participation constraint is exactly satisfied.

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<sup>14</sup>Note though that certain contractual arrangements, such as a maintenance contract, may not have bearing on the vertical externality. In the present model the distortion in investment timing arises because there is a mark-up on the overall cost of the input. It is independent of how this investment expense is allocated in time.

Formally, for any  $y \leq y^*$ , and by slightly abusing notation<sup>15</sup> to introduce  $y_i$  as an argument of the function  $W$ , the upstream firm's problem is:

$$\begin{aligned} & \max_{y_i, p_S} W(y, y_i, p_S) \\ \text{s.t.} \quad & p_S \leq \bar{p}_S(y_i), \end{aligned} \tag{11a}$$

$$p_S \geq \underline{p}_S(y_i). \tag{11b}$$

It is clear from Figure 2 that the first constraint is equivalent to  $V(y, y_i, p_S) \geq V(y, y_S^*, p_S^*)$ , and the second one to  $W(y, y_i, p_S) \geq W(y, y_S^*, p_S^*)$ , with  $\underline{p}_S(y_i)$  implicitly defined by  $W(y, y_S^*, p_S^*) = W(y, y_i, \underline{p}_S(y_i))$ , and  $\bar{p}_S(y_i)$  by  $V(y, y_S^*, p_S^*) = V(y, y_i, \bar{p}_S(y_i))$ . Total value maximization implies that  $y_i = y^*$ , and the upstream supplier maximizes its share of total value by charging  $p_S = \bar{p}_S(y^*)$ , so that (11a) is exactly satisfied. With the parameter values that we use in our example ( $\beta = 2$ ,  $y = I = \frac{\pi M}{r - \alpha} = 1$ ), the two participation constraints reduce to  $\frac{5}{4} \leq p_S \leq \frac{3}{2}$ , and the input supplier chooses  $y_i = y^* = 2$  and  $p_S = \bar{p}_S(2) = \frac{3}{2}$ .<sup>16</sup>

**Proposition 4** *Suppose that  $y \leq y^*$ . In a contract analogous to resale price maintenance, the upstream firm chooses the investment trigger  $y^*$  and charges the input price  $\bar{p}_S(y^*)$ , as defined by  $V(y, y_S^*, p_S^*) = V(y, y^*, \bar{p}_S(y^*))$ . The downstream value is the same as in the separation outcome, and the upstream value is  $W(y, y^*, \bar{p}_S(y^*)) > W(y, y_S^*, p_S^*)$ .*

One caveat is that the implementation of this contract requires the upstream firm to continuously monitor  $y$  until the market size reaches  $y^*$ , which may be costly. This was not needed in the separated case, with no vertical restraint. Nor is it clear that  $y^*$  is an easily verifiable contract provision, or that such contracts are used in practice.

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<sup>15</sup>In this example, we define  $W(y, y_i, p_S) = \left(\frac{y}{y_i}\right)^\beta (p_S - I)$ , all  $y \leq y_i$ .

<sup>16</sup>The constraints (11a-11b) are compatible whenever  $\underline{p}_S(y_i) \leq \bar{p}_S(y_i)$ , which always holds if  $y_i = y^* < y_D^*$ . This results from continuity of  $V(y, y_i, p_S)$  and  $W(y, y_i, p_S)$  in  $y_i$  and  $p_S$ , together with  $V(y, y_i, p_S)$  being monotone increasing in  $y_i$  on  $[y^*, y_D^*]$ , and decreasing in  $p_S$ , whereas  $W(y, y_i, p_S)$  is monotone decreasing in  $y_i$ , and increasing in  $p_S$ . Obviously, the participation constraint of the input supplier, who writes the contract, can be omitted. However, when the bargaining power is more evenly distributed among the parties, the price  $p_S$  can be chosen anywhere in the interval  $[\underline{p}_S(y^*), \bar{p}_S(y^*)]$ .

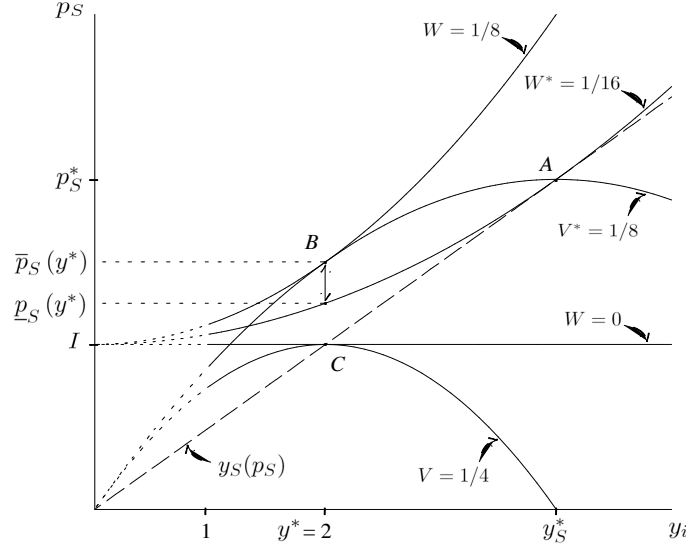


Figure 2: Upstream and downstream isovalues ( $\beta = 2$ ,  $y = I = \frac{\pi M}{r - \alpha} = 1$ ). Point  $A$  describes the separated equilibrium, the upstream firm charges  $p_S^* = 2$ , and the downstream firm enters at  $y_S^* = 4$ . Points  $B$  and  $C$  describe joint-value maximizing contracts, as chosen by the upstream firm under the constraint that the downstream firm earns no less than  $V^* = \frac{1}{8}$ . In both contracts the upstream firm chooses the investment level  $y^* = 2$ . In  $B$ , the input price is  $\frac{3}{2}$ , and in  $C$  it is  $I$ , so that the upstream supplier takes no margin. In the latter case the supplier can sell an up-front option with strike  $I$  and specify a transfer payment  $t_S^* = \frac{1}{8}$ , resulting in a downstream value  $V(1, y^*, I)$  equal to the reservation level  $V^* = \frac{1}{8}$ .

A contractual alternative is to set the equivalent of a two-part tariff. In this case, for any  $y \leq y^*$ , the integrated value is realized by means of an up-front option offered to the downstream firm on the specific input at an exercise price,  $p_S$ .<sup>17</sup> We know from Section 3.2 that the input buyer maximizes its private value by exercising the option when  $Y_t$  reaches the barrier  $y_S(p_S)$ . As in the previous contractual example, the objective of the upstream supplier is to induce the choice of the efficient investment trigger by the input buyer, and to appropriate the value in excess of the downstream reservation level  $V(y, y_S^*, p_S^*)$ . This can be done through a transfer payment,  $t_S$ , made at  $y$ , which we interpret here as the option premium. This contract also corresponds to a non-refundable deposit on the specific input.

<sup>17</sup>Airlines typically buy options to purchase planes conditioned on air traffic volumes.



The upstream problem is then:

$$\max_{p_S, t_S} W(y, p_S) + t_S$$

$$\text{s.t. } V(y, y_S(p_S), p_S) - t_S \geq V(y, y_S^*, p_S^*), \quad (12a)$$

$$W(y, p_S) + t_S \geq W(y, p_S^*). \quad (12b)$$

With the joint-value maximizing input price  $p_S^* = I$ , the downstream firm chooses to invest when  $Y_t = y_S(I) = y^*$ . The transfer payment is chosen under the condition that the downstream firm's participation constraint is exactly satisfied.<sup>18</sup> With the same parameter values as in Figure 2, (12a) and (12b) reduce to  $\frac{1}{16} \leq t_S \leq \frac{1}{8}$ , the downstream firm invests at  $y_S(1) = y^* = 2$ , hence the upstream firm chooses  $t_S = t_S^*(1) = \frac{1}{8}$ , as paid by the downstream firm when the market size is  $y = 1$ .

**Proposition 5** *Suppose that  $y \leq y^*$ . In a contract analogous to a two-part tariff, the upstream firm charges the price  $I$ , and chooses the transfer  $t_S^*(y) = V(y, y^*, I) - V(y, y_S^*, p_S^*)$ . The downstream value is the same as in the separation outcome, and the upstream value is  $W(y, I) + t_S^*(y) > W(y, p_S^*)$ .*

This kind of contract imposes a smaller informational requirement on the upstream firm than the previous one, although it does require it to have an estimate of the current market size  $y$  at the date at which the option is written.

If the contracting alternatives, as described in this section, are not available to the upstream firm, the presence of a downstream firm may act as a substitute. As it results in earlier investment, the race to preempt downstream counteracts the “double marginalization” distortion, at least for the first firm that invests.

## 5 Downstream Duopoly

In this section the structural assumptions are those of Section 3, except that on the intermediate market the upstream firm faces two downstream buyers, that also compete on the final market. We

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<sup>18</sup>The constraints (12a-12b) are compatible whenever  $V(y, y_S(p_S), p_S) + W(y, p_S) \geq V(y, y_S^*, p_S^*) + W(y_S^*, p_S^*)$ , which always holds if  $p_S = I$ . As in the case of the previous vertical restraint, the participation constraint of the upstream firm, which writes the contract, can be omitted. We keep it here in order to describe the range of possible transfer payments when the bargaining power is less asymmetrically distributed.

build on the analysis of Fudenberg and Tirole [9] (preemption), Grenadier [10], Mason and Weeds [16], and Boyer, Lasserre, and Moreaux [1] (preemption under uncertainty).<sup>19</sup> Now  $Y_t$  describes an industry-wide shock, so that the flow profits are  $Y_t\pi_M$  (monopoly) if a single firm has entered the final market, and  $Y_t\pi_D$  (duopoly profits) if both firms have invested, with  $\pi_D < \pi_M$ .

The upstream firm is constrained to a single instrument, the spot price of the specific input, but it may charge different prices at different dates (intertemporal price discrimination). It may thus condition the spot price on the information it receives regarding the demand of downstream firms (in particular, how many firms are present). In what follows  $p_L$  denotes the spot price charged to the first firm to invest (the “leader”), and  $p_F$  denotes the spot price for the second firm (the “follower”). We also assume that the upstream supplier cannot make commitments at one date regarding prices at some future date.

In the absence of strong positive technological externalities at the downstream stage (if total duopoly profits are lower than monopoly profits), the integrated optimum from the industry’s viewpoint is for a single downstream firm to be active. However, in the separated case, and in the absence of sufficient other instruments, we have seen that the downstream firm invests too late. Therefore the upstream firm may find it profitable to allow a second firm into the market. Then the race to preempt downstream, as it typically results in earlier investment, can counteract the “double marginalization” distortion, and thereby functions as a substitute for the vertical restraints examined in Section 4.<sup>20</sup>

## 5.1 Equilibrium

The underlying strategies of the downstream firms are the “simple” mixed strategies defined in Fudenberg and Tirole [9], which consist of (augmented) distributions of investment thresholds, conditional on the number of downstream firms to have already invested.<sup>21</sup> In order to determine the equilibrium, it suffices to determine two investment triggers,  $y_P$  and  $y_F$ , at which the leader and the follower invest, respectively. In equilibrium, the identity of the leader and follower are indeterminate,

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<sup>19</sup>A comprehensive discussion of these contributions can be found in Chevalier-Roignant and Trigeorgis [6].

<sup>20</sup>If an exclusive dealing clause is allowed, an upstream firm that is able to implement resale price maintenance or price the downstream option contract can potentially use the threat of downstream duopoly, altering the terms discussed in Section 4: it offers the downstream firm exclusivity but benefits from a lower reservation value which corresponds to the ex-ante downstream value in a preemption equilibrium, as displayed in (21).

<sup>21</sup>See also Huisman, Kort, and Thijssen [13].

in that either firm effectively invests first, with equal probability. The follower trigger results from standard arguments: once the leader has invested, the subgame between the upstream firm and the follower is identical to that in Section 3.2.

In what follows, when the current market size is  $y$ , the value of a follower that invests at a threshold  $y_F$  and pays a price  $p_F$  is:

$$F(y, y_F, p_F) = \left(\frac{y}{y_F}\right)^\beta \left(\frac{\pi_D}{r - \alpha} y_F - p_F\right). \quad (13)$$

By the same arguments as in Section 3.2, the optimal second spot price for the upstream firm is  $p_F^* = \frac{\beta}{\beta-1}I$ , and the optimal follower investment threshold is  $y_F^* = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_D}I$ . Compared with the case where the specific input is produced internally (Boyer, Lasserre, and Moreaux [1], Mason and Weeds [16]), the follower invests at a level of  $y$  that is  $\frac{\beta}{\beta-1}$  times higher, and has lower value by a factor of  $\left(\frac{\beta}{\beta-1}\right)^{1-\beta}$ .

**Remark 1**  $F(y, y_F^*, p_F^*)$  does not depend on  $(p_L, y_P)$ .

Indeed, what the firm takes into account when it chooses an investment trigger, as a follower, is the profit flow it may expect in the future. This flow is *not* impacted by the investment cost of the leader, nor by its exact investment date.

To determine the preemption threshold  $y_P$ , given  $p_L$ , it is necessary to refer to the value of a firm that invests immediately at the current market size  $y$ , given that its rival invests optimally as a follower. Let  $L(y, p_L)$  denote this value, which has a different form from the  $V(\cdot)$  expressions is the rest of the paper:

$$L(y, p_L) = \frac{\pi_M}{r - \alpha} y - p_L - \left(\frac{y}{y_F^*}\right)^\beta \frac{\pi_M - \pi_D}{r - \alpha} y_F^*. \quad (14)$$

Although this function is commonly used in preemption models, it is also useful to consider a more general expression of (14), that is  $\tilde{L}(y, y_L, y_F^*, p_L) = \left(\frac{y}{y_L}\right)^\beta \left(\frac{\pi_M}{r-\alpha} y_L - p_L\right) - \left(\frac{y}{y_F^*}\right)^\beta \frac{\pi_M - \pi_D}{r-\alpha} y_F^*$ . The function  $\tilde{L}(y, y_L, y_F^*, p_L)$  measures the value, at the current market size  $y$ , of a firm that is free to invest at  $y_L$  as a leader.<sup>22</sup> We have  $L(y, p_L) = \tilde{L}(y, y_L, y_F, p_L)$  when the constraint  $y_L \equiv y$  is imposed, and  $y_F = y_F^*$ .

**Remark 2**  $\arg \max_{y_L} \tilde{L}(y, y_L, y_F^*, I) = \{y^*\}$ .

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<sup>22</sup>See Reinganum [18].

In other words, when it incurs the “true” cost of investment  $p_L = I$ , a firm that is free to choose  $y_L$  invests at the same date as in the integrated case (with a single firm). This is another illustration of the “myopic” behavior as coined by Leahy [15].

The analysis of the investment game based on the functions (13) and (14) closely follows that of existing models. The threshold  $y_P$ , which is defined by  $L(y_P(p_L), p_L) = F(y_P(p_L), y_F^*, p_F^*)$ , is a function of  $p_L$ . We define  $y_P^* \equiv y_P(p_L^*)$ , where  $p_L^*$  denotes the upstream supplier’s value-maximizing price. We find:

**Proposition 6** *In the separated case with two downstream firms, there is a unique equilibrium characterized by:*

$$(i) \text{ downstream triggers} : y_P^* = \frac{\beta}{\beta-1} \frac{r-\alpha}{\pi_M} I, \quad y_F^* = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_D} I, \quad (15)$$

$$(ii) \text{ upstream prices} : p_L^* = \left[1 - \Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right)\right] p_F^*, \quad p_F^* = \frac{\beta}{\beta-1} I, \quad (16)$$

with  $\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)^{1-\beta} - \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)^{-\beta} \in \left(0, \left(\frac{\beta-1}{\beta}\right)^{\beta-1} \frac{1}{\beta}\right)$ .<sup>23</sup>

The intuition for this result is that, in a preemption equilibrium, rent equalization implies that, for any investment cost chosen upstream, including  $p_L = I$ , the leader’s value is pegged on the follower payoff  $F(y, y_F^*, p_F^*)$ . The latter value does not depend on  $p_L$  (Remark 1). By raising the price  $p_L$  above  $I$ , the upstream firm increases the cost of leading the sequence of investments, and thereby raises the preemption equilibrium trigger  $y_P(p_L)$ . It also appropriates any additional monetary gain on top of the constant share  $\tilde{L}(y, y_P(p_L), y_F^*, p_L) = F(y, y_F^*, p_F^*)$  retained by the downstream leader. Therefore, the supplier’s value-maximizing strategy is to set  $y_P$  equal to the investment trigger  $y^*$ , as this trigger maximizes the joint value of the two vertically related units. This is the same investment trigger as the one chosen by the leader when it incurs the “true” cost  $I$  (Remark 2).

The comparison of investment thresholds and input prices, across the single downstream firm and two-firm scenarios, and the integrated case, follows directly from the expressions in (5) and (15-16).

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<sup>23</sup>For all  $\beta > 1$ , we have  $\left(\frac{\beta-1}{\beta}\right)^{\beta-1} \frac{1}{\beta} < \frac{1}{\beta}$ , implying that  $\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right) < 1$ .

**Proposition 7** *In a preemption equilibrium, downstream triggers and upstream prices satisfy the following rankings:*

$$y_P^* = y^* < y_S^* < y_F^* \text{ and } I < p_L^* < p_F^* = p_S^*. \quad (17)$$

In the downstream duopoly case, the upstream supplier induces an investment threshold for the first firm, via the price  $p_L$ , that is identical to the investment threshold in the integrated case (3.1), that is  $y^*$ , analogously to a “no discrimination at the top” result. The threat of preemption among downstream firms thus has the effect of a vertical restraint, insofar as it induces investment at the correct trigger for the first firm. The race to be first exactly counterbalances the incentive that the leader would otherwise have to delay, if its investment date resulted from the optimization of an investment threshold. This substitute for a vertical restraint does not represent an industry first-best however if, as it may be assumed, the presence of a second firm reduces industry profit ( $\pi_D < \pi_M/2$ ).

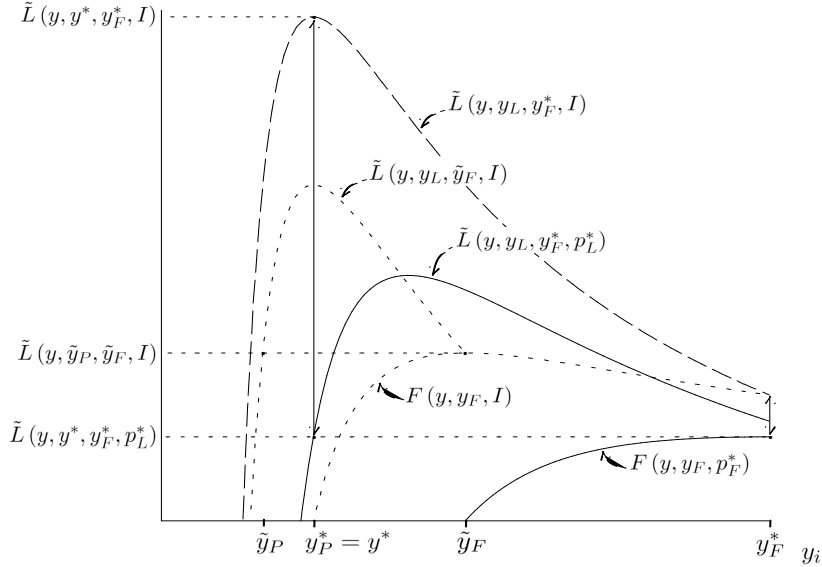


Figure 3: Leader and follower values at current market size  $y = 1$  as a function of investment trigger  $y_i$  (with  $\beta = 2$ ,  $I = r - \alpha = \pi_M = 2\pi_D = 1$ ). The preemption trigger  $y_P^* = y^*$  in the separated case maximizes the integrated leader value (that is, the leader value if  $p_L = I$ ), and is greater than the trigger under preemption when downstream firms face the true investment cost. By charging  $p_L^* > I$ , the upstream

firm appropriates the value differential  $\tilde{L}(y, y^*, y_F^*, I) - \tilde{L}(y, y^*, y_F^*, p_L^*)$ . By charging  $p_F^* > I$ , it also earns the difference  $F(y, y_F^*, I) - F(y, y_F^*, p_F^*)$ .

To illustrate, in Figure 3 the two solid curves refer to the separated case. The quasi concave one represents  $\tilde{L}(y, y_L, y_F^*, p_L^*)$ , that is the value of the leader as a function of  $y_L$ , and measured at a given  $y$  (specifically,  $y = 1$ ) provided that the follower invests at the optimal threshold  $y_F^*$ , and for an upstream value-maximizing price  $p_L^*$  (with  $\beta = 2$ ,  $I = r - \alpha = \pi_M = 2\pi_D = 1$ ). The other solid curve is a graph of  $F(y, y_F, p_F^*)$ , which has the same expression as in (13). Note that, when  $y = y_F^*$ , the leader value is higher since  $p_L^* = \frac{13}{8} < 2 = p_F^*$ . The preemption threshold  $y_P^*$  is determined by the condition that firms are indifferent at that point between investing as a leader or waiting to invest as a follower. In this figure, the dashed curve represents the upstream firm's optimization problem. It describes the reference (or "true") leader value, based on the actual investment cost  $I$  (i.e.,  $p_L = I$ ) for all possible  $y_L \leq y_F^*$ , and for  $y_F = y_F^*$  (i.e.,  $p_F = p_F^* > I$ ). This is the graph of  $\tilde{L}(y, y_L, y_F^*, I)$ , which reaches a maximum when  $y_L = y^*$ .

We also represent the case with two integrated downstream firms (i.e.,  $p_L = p_F = I$ ). We know from existing models with similar specifications (e.g., Boyer, Lasserre, and Moreaux [1], Mason and Weeds [16]) that, in that case, in a preemption equilibrium the leader invests first at  $\tilde{y}_P$ , which is strictly less than  $y^* = \frac{\beta}{\beta-1} \frac{r-\alpha}{\pi_M} I$ , as computed in Section 3.1. Then the follower invests at  $\tilde{y}_F = \frac{\beta}{\beta-1} \frac{r-\alpha}{\pi_D} I$ . In Figure 3, the two dotted curves represent  $\tilde{L}(y, y_L, \tilde{y}_F, I)$  and  $F(y, y_F, I)$ . It is straightforward to check that  $\tilde{y}_P < y_P^* < \tilde{y}_F < y_F^*$ .<sup>24</sup>

Consider now the upstream value in the preemption equilibrium of Proposition 6, that is

$$\tilde{W}(y, p_L^*, p_F^*) = \left( \frac{y}{y_P(p_L^*)} \right)^\beta (p_L^* - I) + \left( \frac{y}{y_F^*} \right)^\beta (p_F^* - I). \quad (18)$$

This value can be visualized in Figure 3 by reinterpreting each term on the right hand side of the equality sign in (18) as follows. On the one hand, the supplier chooses  $p_L$ , shifting the leader value function, to maximize the difference between the reservation value that must be given to the leader and the reference leader value at the preemption trigger  $y_P(p_L)$ . By charging exactly  $p_L^* > I$ , so that the leader invests at  $y_P^* = y_P(p_L^*) = y^*$ , the supplier appropriates the value differential

$$\tilde{L}(y, y^*, y_F^*, I) - \tilde{L}(y, y^*, y_F^*, p_L^*) = \left( \frac{y}{y_P(p_L^*)} \right)^\beta (p_L^* - I). \quad (19)$$

<sup>24</sup>The comparison of  $\tilde{y}_F$  with  $y_S^*$  is less straightforward in that it depends on the ratio  $\frac{\pi_D}{\pi_M}$ .

In addition, the upstream supplier earns the difference between the value that the follower would earn as an integrated firm, and the value it earns as a separate entity, with  $y_F = y_F^*$  in both cases. Formally, by charging  $p_F^* > I$ , the supplier appropriates

$$F(y, y_F^*, I) - F(y, y_F^*, p_F^*) = \left(\frac{y}{y_F^*}\right)^\beta (p_F^* - I). \quad (20)$$

The magnitudes (19) and (20) are represented by the vertical arrows in Figure 3.

With respect to firm values, intuition suggests that, in a preemption equilibrium, a downstream firm is worse off, and the upstream firm is better off, than under bilateral monopoly since there is competition downstream. The first of these comparisons is not immediate to verify, since the first investment occurs at a lower threshold  $y_P^* < y_S^*$ , and also faces a lower input price  $p_L^* < p_S^*$ , which is value enhancing. The closed-form expression of  $y_P^*$  is useful to resolve this ambiguity, as it allows us to exactly evaluate the downstream firm value, which we denote by  $\tilde{V}(y, p_L^*, p_F^*)$ , for all market sizes  $y < y_P^*$ . This value is given by:

$$\tilde{V}(y, p_L^*, p_F^*) = \frac{1}{2} \left(\frac{y}{y_P^*}\right)^\beta \left(\frac{\pi_M}{r - \alpha} y_P^* - p_L^*\right) + \left(\frac{y}{y_F^*}\right)^\beta \left(\frac{\pi_D - \frac{1}{2}\pi_M}{r - \alpha} y_F^* - \frac{1}{2}p_F^*\right). \quad (21)$$

The expression (21) reflects the fact that, ex-ante, a firm is equally likely to be a leader or a follower under preemption. The comparison of the equilibrium value of the upstream firm  $\tilde{W}(y, p_L^*, p_F^*)$  when there are two downstream buyers, with  $W(y, p_S^*)$ , is not immediate either. Although the upstream firm sells its input twice, the first sale is discounted and the second occurs at a more removed date.

Industry value increases under duopoly only conditionally, since although the investment threshold of the first firm is more efficient, industry flow profits decrease with the investment of the second firm. When  $\beta$  is high, which corresponds for example to low volatility of the demand process, the option component weighs less on firm value than the vertical distortion (see Section 3.3.). The benefit gained from correcting the vertical externality by introducing a second firm then outweighs the destruction of downstream profits that occurs under duopoly.

**Proposition 8** *For all  $y \leq y_P^*$ , for all  $\beta$  and all  $\pi_D < \pi_M$ , the downstream value is lower and the upstream value is higher in a preemption equilibrium than under bilateral monopoly:*

$$\tilde{V}(y, p_L^*, p_F^*) < V(y, y_S^*, p_S^*) \text{ and } W(y, p_S^*) < \tilde{W}(y, p_L^*, p_F^*). \quad (22)$$

*Moreover, for large enough  $\left\{\beta, \frac{\pi_M}{\pi_D}\right\}$ , total industry value is greater in a preemption equilibrium than under bilateral monopoly.*

## 5.2 Comparative Statics

A noteworthy feature of the specification with vertical separation and two downstream firms, compared with similar real option games, is that the solution in the preemption scenario is analytic. The closed-form expression of  $y_P^*$  facilitates the comparative statics, which are consistent with the interpretation of the model given in Section 3.3.

First, we know from Proposition 6 that in the downstream duopoly case the supplier sells the specific input to the first entrant at a discount as  $\frac{p_L^*}{p_F^*} = 1 - \Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right) < 1$ . We find that the effect on this discount of a higher growth rate  $\alpha$  or volatility  $\sigma$ , and also of a higher profit ratio  $\frac{\pi_M}{\pi_D}$ , is univocal.

**Proposition 9** *For all admissible parameter values:*

$$\frac{d\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right)}{d\beta} < 0, \quad \frac{d\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right)}{d\frac{\pi_M}{\pi_D}} < 0. \quad (23)$$

*Therefore, the markdown is decreasing in  $\beta$  and  $\frac{\pi_M}{\pi_D}$ .*

Next, recall from (7-8) that the parameter  $\beta$  is analogous to an elasticity of demand in the intermediate market, and lower elasticity (greater  $\beta$ ) results in a greater vertical distortion, with higher prices and triggers. Although these features are robust to the introduction of a second downstream firm, an additional remark is warranted. In fact  $p_L^* = \left[1 - \Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right)\right] p_F^*$ , with both  $\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right)$  and  $p_F^*$  increasing in  $\beta$ , which implies that the net effect of more growth or volatility on the first spot price, expressed here in terms of elasticities, is a priori ambiguous. By definition,

$$\varepsilon_{p_L^*/\beta} = \underbrace{\varepsilon_{(1-\Lambda(\beta, \frac{\pi_M}{\pi_D}))/\beta}}_{>0} + \underbrace{\varepsilon_{p_F^*/\beta}}_{<0}, \quad (24)$$

that is to say a greater  $\beta$  dampens the price discrimination effect by decreasing the spread in prices on the one hand, but also decreases the follower price on the other. The net effect can be shown to be negative, but the comparative static in the preemption game is thus not a direct corollary of the bilateral monopoly case. We find that changes in market conditions, as captured by the parameter  $\beta$ , have a greater impact on the follower input price than the leader input price.



**Proposition 10** *In a preemption equilibrium, a higher market growth rate and volatility result in higher triggers  $\{y_P^*, y_F^*\}$  and higher downstream prices  $\{p_L^*, p_F^*\}$  with:*

$$\varepsilon_{p_F^*/\beta} = \varepsilon_{p_S^*/\beta} < \varepsilon_{p_L^*/\beta} < 0 \text{ and } \varepsilon_{y_F^*/\beta} = \varepsilon_{y_S^*/\beta} < \varepsilon_{y_P^*/\beta} < 0. \quad (25)$$

Finally, the comparative statics of  $\tilde{V}(y, p_L^*, p_F^*)$  and  $\tilde{W}(y, p_L^*, p_F^*)$ , which involve additional effects from those of  $V(y, y_S^*, p_S^*)$  and  $W(y, p_S^*)$ , do not appear to have straightforward characterizations. We find in particular that  $\beta$  may have either a monotone, or an ambiguous effect on total industry value. (See Section 7.9 in the Appendix.)

## 6 Discussion

In this paper, we have studied investment timing when firms depend on an outside supplier to provide a discrete input (e.g., a key equipment), developing a dynamic version of a heretofore static model. The upstream firm's mark-up depends on the stochastic process followed by downstream flow profits. A vertical externality arises because the upstream firm's pricing induces the downstream firm to delay the exercise of its investment option. This distortion, akin to a Lerner index, increases with both market growth and volatility. Downstream firm values are more sensitive to the market growth rate than upstream values, and in contrast with the standard real option framework, greater volatility decreases firm value near the exercise threshold. If the input supplier has sufficient information regarding downstream demand, it can induce optimal investment timing by means of standard vertical restraints. Otherwise, the upstream supplier benefits from the presence of a second downstream firm, which results in a preemption race and acts as a partial substitute for vertical restraints. The input is then sold to the downstream leader at a discount which increases with volatility, the leader's price is less sensitive to market growth and volatility than the follower's, and the leader invests at the optimal threshold, resulting in greater industry profits when growth and volatility are sufficiently low.

In the first place, one may allow for upstream competition. If suppliers compete in prices and there is a single downstream firm, then the integrated optimum is restored. On the other hand, in an industry with two upstream and two downstream firms, upstream competition presumably results in a standard preemption race downstream, and the leader invests too early. Second, a strong qualitative prediction of the model is that, under duopoly, the first input is sold at a discount. In

practice, we would expect that learning effects which decrease the upstream firm's production cost for the second input supplied, should reduce the apparent discount that is offered to the leader. Third, the assumption of a geometric Brownian motion for the stochastic market process can be relaxed. For example, Poisson jumps may be included to allow for greater risk. Many of the results here are unchanged, with the exception of those comparative static results that rely on the analytic expression of  $\beta$ . Eventually, the analysis of this paper has adhered to the classical assumption that the contract terms are decided by the upstream firm. However, one could envisage that it is the downstream firms that have market power in the input market, and therefore write the contract, or alternatively that downstream firms may use some other device, such as the threat of reversion to a tacit collusion equilibrium if such exists.

As stated in the introduction, this paper may be viewed alternatively as an extension of the existing real options literature, or as the extension of the classic Industrial Organization analysis of vertical relationships to a stochastic dynamic setting. Although this latter extension involves strong formal analogies between the static model and the downstream duopoly case, it also uncovers some significant differences. This is clearest in the downstream duopoly case, where under preemption the upstream supplier treats the leader and follower differently, and only a partial correction of the vertical externality is feasible given the sequence of actions. Finally, besides shedding light on an incentive for dynamic price discrimination, the model presented here makes several predictions with respect to decision timing and firm values which only await empirical testing on a suitable set of data.

## 7 Appendix

### 7.1 Sensitivity of $\beta$ to $\alpha$ and $\sigma$

The derivatives of  $\beta$  with respect to the growth and volatility parameters  $\alpha$  and  $\sigma$  arise throughout the paper, and have the following expressions:

$$\frac{d\beta}{d\alpha} = \frac{-\beta}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\sigma^2\right)} < 0, \quad (26)$$

$$\frac{d\beta}{d\sigma} = \frac{-2(r - \alpha\beta)}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\sigma^2\right)\sigma^3} \leq 0. \quad (27)$$

With respect to the signs of these expressions, note that by assumption that  $\frac{1}{2} - \frac{\alpha}{\sigma^2} < 0$ , and  $r \geq \alpha\beta$ , with equality only if  $\sigma = 0$ .

## 7.2 Sensitivity of $y_S^*$ and $p_S^*$ to $\alpha$ and $\sigma$

Growth:

$$\begin{aligned}\frac{dy_S^*}{d\alpha} &= \frac{\partial y_S^*}{\partial \beta} \frac{d\beta}{d\alpha} + \frac{\partial y_S^*}{\partial \alpha} = \left(\frac{\beta}{\beta-1}\right)^2 \frac{(\sigma^2 + 2\alpha)I}{(\sigma^2(2\beta-1) + 2\alpha)} > 0, \\ \frac{dp_S^*}{d\alpha} &= \frac{\partial p_S^*}{\partial \beta} \frac{d\beta}{d\alpha} = -\frac{I}{(\beta-1)^2} \frac{d\beta}{d\alpha} > 0.\end{aligned}$$

Volatility:

$$\begin{aligned}\frac{dy_S^*}{d\sigma} &= \frac{\partial y_S^*}{\partial \beta} \frac{d\beta}{d\sigma} = -\frac{2y^*}{(\beta-1)^2} \frac{d\beta}{d\sigma} > 0, \\ \frac{dp_S^*}{d\sigma} &= \frac{\partial p_S^*}{\partial \beta} \frac{d\beta}{d\sigma} = -\frac{I}{(\beta-1)^2} \frac{d\beta}{d\sigma} > 0.\end{aligned}$$

## 7.3 Behavior of $f(\beta)$

Define  $f(\beta) \equiv \left(\frac{\beta}{\beta-1}\right)^{-\beta} \left(1 + \frac{\beta}{\beta-1}\right)$ . This expression arises several times throughout the paper. To establish that  $f(1) = 1$ , rewrite it as  $f(\beta) = \left(\frac{\beta}{\beta-1}\right)^{-\beta} + \left(\frac{\beta}{\beta-1}\right)^{1-\beta}$ . Since  $\lim_{\beta \rightarrow 1} (\beta-1)^{\beta-1} = 1$ ,  $f(1) = 1$ . To establish that  $\lim_{\beta \rightarrow \infty} f(\beta) = \frac{2}{e}$ , note that for the denominator,  $\left(\frac{\beta}{\beta-1}\right)^\beta = \left(1 + \frac{1}{\beta-1}\right)^\beta$ , and  $\lim_{\beta \rightarrow \infty} \left(1 + \frac{1}{\beta-1}\right)^\beta = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x \left(1 + \frac{1}{x}\right) = e$ . Finally,

$$\frac{df(\beta)}{d\beta} = \frac{(\beta-1)^{\beta-1}}{\beta^\beta} \left(2 - (2\beta-1) \ln \frac{\beta}{\beta-1}\right). \quad (28)$$

The last term in (28) is negative:  $\frac{d(2-(2\beta-1)\ln\frac{\beta}{\beta-1})}{d\beta} = -2\ln\frac{\beta}{\beta-1} + \frac{2\beta-1}{\beta(\beta-1)} > 0$ ,  $\frac{d^2(2-(2\beta-1)\ln\frac{\beta}{\beta-1})}{d\beta^2} = -\frac{1}{\beta^2(\beta-1)^2} < 0$  and  $\lim_{\beta \rightarrow \infty} \left(2 - (2\beta-1) \ln \frac{\beta}{\beta-1}\right) = 0$ . ■

## 7.4 Proof of Proposition 3

The effect of a change in  $\sigma$  on  $V^*$  and  $W^*$  follows directly from the expressions displayed in the main body of the paper, so that we need focusing on  $\alpha$  only. We first sign  $\frac{dV^*}{d\alpha}$ . After simplification, computations yield, for all  $y \leq y_S^*$ :

$$\frac{dV(y, y_S^*(p_S^*), p_S^*)}{d\alpha} = \left( \frac{\beta}{r - \alpha} + \left( \frac{1}{\beta} + \ln \frac{y}{y_S^*} \right) \frac{d\beta}{d\alpha} \right) V(y, y_S^*, p_S^*). \quad (29)$$

Since  $\left( \ln \frac{y}{y_S^*} \right) \frac{d\beta}{d\alpha} \geq 0$ ,

$$\frac{dV(y, y_S^*(p_S^*), p_S^*)}{d\alpha} \geq \left( \frac{\beta}{r - \alpha} + \frac{1}{\beta} \frac{d\beta}{d\alpha} \right) V(y, y_S^*, p_S^*). \quad (30)$$

Substituting for  $\frac{d\beta}{d\alpha}$ ,  $\frac{dV(y, y_S^*(p_S^*), p_S^*)}{d\alpha} > 0$  if  $g(\alpha, r, \sigma) \equiv \beta \left( \beta - \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \right) \sigma^2 - (r - \alpha) > 0$ . When  $\alpha = r \equiv z$ ,  $\beta = 1$ , and therefore  $g(z, z, \sigma) = \frac{1}{2} + \frac{z}{\sigma^2} > 0$ . Moreover,  $\frac{dg(\alpha, r, \sigma)}{dr} = \beta \left( \beta - \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \right)^{-1} > 0$ . Therefore,  $\frac{dV(y, y_S^*(p_S^*), p_S^*)}{d\alpha} > 0$  for all admissible parameter values.

Next, we sign  $\frac{dW^*}{d\alpha}$ . Similarly, after simplification, computations yield, for all  $y \leq y_S^*$ ,

$$\frac{dW(y, p_S^*)}{d\alpha} = \left( \frac{\beta}{r - \alpha} + \left( \frac{1}{\beta - 1} + \ln \frac{y}{y_S^*} \right) \frac{d\beta}{d\alpha} \right) W(y, p_S^*). \quad (31)$$

Again  $\left( \ln \frac{y}{y_S^*} \right) \frac{d\beta}{d\alpha} \geq 0$  implies that

$$\frac{dW(y, p_S^*)}{d\alpha} \geq \left( \frac{\beta}{r - \alpha} + \frac{1}{\beta - 1} \frac{d\beta}{d\alpha} \right) W(y, p_S^*). \quad (32)$$

Thus,  $\frac{dW(y, p_S^*)}{d\alpha} > 0$  if  $h(\alpha, r, \sigma) \equiv (\beta - 1) \left( \beta - \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \right) \sigma^2 - (r - \alpha) > 0$ . Taking  $\alpha = r \equiv z$ ,  $h(z, z, \sigma) = 0$ , and  $\frac{dh(\alpha, r, \sigma)}{dr} = (\beta - 1) \left( \beta - \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \right)^{-1} > 0$ . Therefore,  $\frac{dW(y, p_S^*)}{d\alpha} > 0$  for all admissible parameter values.

It remains to rank  $\varepsilon_{V^*/\alpha}$  and  $\varepsilon_{W^*/\alpha}$ . A simple reorganization of terms in (29) and (31), together with  $\frac{1}{\beta} < \frac{1}{\beta - 1}$ , directly leads to  $\varepsilon_{W^*/\alpha} < \varepsilon_{V^*/\alpha}$ . ■

## 7.5 Proof of Proposition 6

The optimal follower investment threshold  $y_F^*$  and second spot price  $p_F^*$  having been discussed in the text, only the first spot price  $p_L^*$  and the preemption threshold  $y_P^*$  remain to be established.

First, we determine the upstream firm's strategy space to be of the form  $[0, \bar{p}]$ . The value function  $F(y, y_F^*, p_F^*)$  is increasing and convex and  $L(y, p_L)$  is increasing and concave in  $y$ . The choice of  $p_L$  is bounded because the equation  $L(y, p_L) = F(y, y_F^*, p_F^*)$  must have at least one root in  $y$ , given  $p_L$ , for downstream firms to wish to invest first in the market. Setting  $\frac{dL(y, p_L)}{dy} = \frac{dF(y, y_F^*, p_F^*)}{dy}$ , we find that tangency occurs at  $\bar{y} = \left(\frac{\pi_M}{\beta\pi_M - (\beta-1)\pi_D}\right)^{\frac{1}{\beta-1}} y_F^*$ . Note that  $\bar{y} < y_F^*$  so long as  $\pi_D < \pi_M$ . Then,  $\bar{p}$  is defined implicitly by  $L(\bar{y}, \bar{p}) = F(\bar{y}, y_F^*, p_F^*)$ .

Second, for  $p_L \in [0, \bar{p}]$ , a preemption equilibrium exists at the threshold  $y_P(p_L)$  that verifies  $L(y_P(p_L), p_L) = F(y_P(p_L), y_F^*, p_F^*)$ . Specifically,  $y_P(p_L)$  is defined implicitly by:

$$\frac{\pi_M}{r - \alpha} y_P - p_L - \left(\frac{y_P}{y_F^*}\right)^\beta \frac{\beta}{\beta - 1} \left(\frac{\beta}{\beta - 1} \frac{\pi_M}{\pi_D} - 1\right) I = 0. \quad (33)$$

The decision problem of the upstream firm can then be examined. Its value when the current market size is  $y$  is:

$$\tilde{W}(y, p_L, p_F^*) = \left(\frac{y}{y_P(p_L)}\right)^\beta (p_L - I) + \left(\frac{y}{y_F^*}\right)^\beta (p_F^* - I). \quad (34)$$

From (33) we obtain an expression of  $p_L - I$ , which is plugged into (34). This leads to:

$$\tilde{W}(y, p_L, p_F^*) = y^\beta \left(-y_P^{-\beta} I + \frac{\pi_M}{r - \alpha} y_P^{1-\beta} - \frac{\beta}{\beta - 1} \left(\frac{\beta}{\beta - 1} \frac{\pi_M}{\pi_D} - 1\right) y_F^{*-\beta} I\right) + \left(\frac{y}{y_F^*}\right)^\beta (p_F^* - I). \quad (35)$$

Note that:

$$\tilde{W}(y, p_L, p_F^*) = V(y, y_P, I) + U(y, p_F^*), \quad (36)$$

where  $U(y, p_F^*)$  is independent of  $y_P$ , and  $V(y, y_P, I)$  is the integrated payoff (1) of Section 3.1. The upstream firm's decision problem is thus that of the integrated firm, and the first-order condition is satisfied at  $y_P^* = \frac{\beta}{\beta-1} \frac{r-\alpha}{\pi_M} I$ . Substituting into (33) gives the optimal first downstream spot price  $p_L^* = \left(1 - \Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right)\right) \frac{\beta}{\beta-1} I$ , with  $\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)^{1-\beta} - \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)^{-\beta}$ . Since  $\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D} > 1$ ,  $\left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D} - 1\right) \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)^{-\beta} = \Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right) > 0$ . Also,  $\Lambda(\beta, 1) = \left(\frac{\beta-1}{\beta}\right)^{\beta-1} \frac{1}{\beta} < \frac{1}{\beta}$ . We have  $\frac{d\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right)}{d\frac{\pi_M}{\pi_D}} = \beta \left(\frac{\pi_D}{\pi_M} - 1\right) \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)^{-\beta} < 0$  because  $\pi_M > \pi_D$ , so  $\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right) < \frac{1}{\beta}$ . In addition, it can be verified that  $y_P^* < \bar{y}$  if and only if  $\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right) < \frac{1}{\beta}$ , so the equilibrium preemption trigger is in the admissible range.

Finally, as  $p_L^*$  and  $p_F^*$  are given under the assumption of price-taking by downstream firms, only the parameters of  $L(y, p_L)$  and  $F(y, y_F^*, p_F^*)$  are altered (specifically, the investment cost which is

asymmetric for the first and second firm to invest), so Fudenberg and Tirole [9]’s argument applies to establish that downstream firms seek to invest immediately off the equilibrium path if no firm has entered yet when the market size reaches  $y_P^*$ . ■

## 7.6 Proof of Proposition 8

The downstream value in equilibrium is:

$$\tilde{V}(y, p_L^*, p_F^*) = \frac{1}{2} \left( \frac{y}{y_P^*} \right)^\beta \frac{\beta}{\beta-1} \Lambda \left( \beta, \frac{\pi_M}{\pi_D} \right) I + \frac{1}{2} \left( \frac{y}{y_F^*} \right)^\beta \frac{\beta}{(\beta-1)^2} \left( 1 + \beta - \beta \frac{\pi_M}{\pi_D} \right) I. \quad (37)$$

Evaluating,

$$\tilde{V}(y, p_L^*, p_F^*) - V(y, y_S^*, p_S^*) = \frac{(\beta-1)^{\beta-2}}{\beta^{\beta-1}} \left( \frac{y}{y_P^*} \right)^\beta \left( \left( \frac{\pi_M}{\pi_D} \right)^{-\beta} - 1 \right) I < 0. \quad (38)$$

The upstream value in equilibrium is:

$$\tilde{W}(y, p_L^*, p_F^*) = \left( \frac{y}{y_P^*} \right)^\beta \left( 1 - \beta \Lambda \left( \beta, \frac{\pi_M}{\pi_D} \right) \right) \frac{I}{\beta-1} + \left( \frac{y}{y_F^*} \right)^\beta \frac{I}{\beta-1}. \quad (39)$$

Recall that  $\frac{\pi_M}{\pi_D} > 1$ . After simplification,  $\tilde{W}(y, p_L^*, p_F^*) > W(y, p_S^*)$  if and only if:

$$\Gamma \left( \beta, \frac{\pi_M}{\pi_D} \right) \equiv \left( \left( \frac{\beta}{\beta-1} \right)^\beta - 1 \right) \left( \frac{\pi_M}{\pi_D} \right)^\beta - \frac{\beta^2}{\beta-1} \frac{\pi_M}{\pi_D} + \beta + 1 > 0. \quad (40)$$

Note first that  $\Gamma(\beta, 1) = \left( \frac{\beta}{\beta-1} \right)^\beta - \frac{\beta}{\beta-1} > 0$ . Next, we compute:

$$\frac{d\Gamma \left( \beta, \frac{\pi_M}{\pi_D} \right)}{d\frac{\pi_M}{\pi_D}} = \beta \left( \left( \frac{\beta}{\beta-1} \right)^\beta - 1 \right) \left( \frac{\pi_M}{\pi_D} \right)^{\beta-1} - \frac{\beta^2}{\beta-1}. \quad (41)$$

Since  $\frac{\pi_M}{\pi_D} > 1$ ,

$$\frac{d\Gamma \left( \beta, \frac{\pi_M}{\pi_D} \right)}{d\frac{\pi_M}{\pi_D}} > \beta \left( \left( \frac{\beta}{\beta-1} \right)^\beta - 1 \right) - \frac{\beta^2}{\beta-1} = \frac{\beta(2\beta-1)}{\beta-1} \frac{1-f(\beta)}{f(\beta)} > 0,$$

where the last inequality follows because  $f(\beta) < 1$  (see section 7.3), and therefore  $\Gamma \left( \beta, \frac{\pi_M}{\pi_D} \right) > 0$ .

The industry value with two firms is:

$$2\tilde{V}(y, p_L^*, p_F^*) + \tilde{W}(y, p_L^*, p_F^*) = \left(\frac{y}{y_P^*}\right)^\beta \frac{I}{\beta-1} + \left(\frac{y}{y_F^*}\right)^\beta \frac{\beta}{(\beta-1)^2} \left(1 + \beta \left(1 - \frac{\pi_M}{\pi_D}\right)\right) I. \quad (42)$$

After simplification,  $2\tilde{V}(y, p_L^*, p_F^*) + \tilde{W}(y, p_L^*, p_F^*) > V(y, y_S^*, p_S^*) + W(y, p_S^*)$  if and only if:

$$\left(\frac{y_S^*}{y_P^*}\right)^\beta \frac{\beta-1}{2\beta-1} + \left(\frac{y_S^*}{y_F^*}\right)^\beta \frac{1+\beta \left(1 - \frac{\pi_M}{\pi_D}\right)}{2\beta-1} = \frac{1+\beta}{2\beta-1} \left(\frac{\pi_M}{\pi_D}\right)^{-\beta} - \frac{\beta}{2\beta-1} \left(\frac{\pi_M}{\pi_D}\right)^{1-\beta} + \frac{1}{f(\beta)} > 1. \quad (43)$$

Since  $\frac{1}{f(\beta)} > 1$  and  $\lim_{\beta \rightarrow \infty} \frac{1}{f(\beta)} = \frac{e}{2}$  (section 7.3), (43) holds for  $\beta$  and  $\frac{\pi_M}{\pi_D}$  large enough. Moreover, the condition (43) is indeed violated for admissible parameter values. Let  $\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \frac{1+\beta}{2\beta-1} \left(\frac{\pi_M}{\pi_D}\right)^{-\beta} - \frac{\beta}{2\beta-1} \left(\frac{\pi_M}{\pi_D}\right)^{1-\beta} + \frac{1}{f(\beta)} - 1$ . For a given  $\beta$ , this is minimized at  $\frac{\pi_M}{\pi_D} = \frac{1+\beta}{\beta-1}$ . Then,  $\Delta\left(\beta, \frac{1+\beta}{\beta-1}\right) = \frac{1}{2\beta-1} \left(\frac{\beta^\beta}{(\beta-1)^{\beta-1}} - \left(\frac{\beta-1}{\beta+1}\right)^{\beta-1}\right) - 1$ , and  $\lim_{\beta \rightarrow 1} \Delta\left(\beta, \frac{1+\beta}{\beta-1}\right) = -1 < 0$ . ■

## 7.7 Proof of Proposition 9

We have established that  $\frac{d\Lambda}{d\frac{\pi_M}{\pi_D}} = \beta \left(\frac{\pi_D}{\pi_M} - 1\right) \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)^{-\beta} < 0$  for all  $\pi_M > \pi_D$  in the proof of Proposition 6. The other derivative is  $\frac{d\Lambda}{d\beta} = \frac{1}{\beta-1} \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)^{-\beta} \left[\frac{\pi_M}{\pi_D} - 1 - \ln\left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right) \left(\beta \frac{\pi_M}{\pi_D} - (\beta-1)\right)\right]$ . The sign of  $\frac{d\Lambda}{d\beta}$  is that of the expression in square brackets. The derivative of this expression with respect to  $\frac{\pi_M}{\pi_D}$  is  $(\beta-1) \left(\frac{\pi_D}{\pi_M} - 1\right) - \beta \ln\left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)$ , which is negative for all  $\beta > 1$  and  $\pi_M \geq \pi_D$ . Then take  $\frac{\pi_M}{\pi_D} = 1$  for a lower bound, which is  $-\ln\frac{\beta}{\beta-1} < 0$ . It follows that  $\frac{d\Lambda}{d\beta} < 0$ . ■

## 7.8 Proof of Proposition 10

For the comparative statics, note that the functional form of the triggers  $y_P^*$  and  $y_F^*$  is similar to that of  $y_S^*$ , and that  $p_F^* = p_S^*$ . The calculations are similar to those of the bilateral monopoly case of Section 7.2. To show that  $\frac{dp_L^*}{d\beta} < 0$ , let  $z \equiv \frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}$  for compactness and first evaluate this expression:

$$\frac{dp_L^*}{d\beta} = -\frac{I}{(\beta-1)^2} \left[1 + z^{-\beta} (-\beta(\beta-1)(z-1) \ln z + (\beta-2)z - (\beta-1))\right]. \quad (44)$$

Consider  $\frac{dp_L^*}{d\beta}$  as a function of  $\frac{\pi_M}{\pi_D}$ . After simplification,

$$\frac{\partial^2 p_L^*}{\partial \beta \partial \frac{\pi_M}{\pi_D}} = -\frac{\beta z^{-\beta} (\pi_M - \pi_D) (\beta \ln z - 2) I}{(\beta-1) \pi_M}, \quad (45)$$

so  $\frac{dp_L^*}{d\beta}$  is quasiconcave in  $\frac{\pi_M}{\pi_D}$  and attains a global maximum at  $\frac{\beta-1}{\beta}e^{\frac{2}{\beta}}$ . Then, again after simplification,

$$\frac{\partial p_L^*}{\partial \beta} \Big|_{\frac{\pi_M}{\pi_D} = \frac{\beta-1}{\beta} e^{\frac{2}{\beta}}} = - \frac{\left(1 + (\beta-1)e^{-2} - \beta e^{-2\frac{\beta-1}{\beta}}\right) I}{(\beta-1)^2}. \quad (46)$$

The sign of the numerator in (46) depends on the sign of the expression  $\Upsilon(\beta) \equiv 1 + (\beta-1)e^{-2} - \beta e^{-2\frac{\beta-1}{\beta}}$ . We have  $\Upsilon(1) = 0$  and  $\Upsilon'(\beta) = e^{-2} - \frac{\beta-2}{\beta} e^{-2\frac{\beta-1}{\beta}} > 0$ . Therefore,  $\frac{\partial p_L^*}{\partial \beta} \Big|_{\frac{\pi_M}{\pi_D} = \frac{\beta-1}{\beta} e^{\frac{2}{\beta}}} < 0$ , from which it follows that  $\frac{dp_L^*}{d\beta} < 0$ . ■

## 7.9 Sensitivity of Firm Values to $\beta$ under Preemption

The simplest expression to consider is the total industry value, whose comparative static behavior is independent of  $\Lambda\left(\beta, \frac{\pi_M}{\pi_D}\right)$ . We have:

$$\frac{d(2\tilde{V} + \tilde{W})}{d\beta} = \left(\frac{y}{y_P^*}\right)^\beta \ln\left(\frac{y}{y_P^*}\right) \frac{I}{\beta-1} + \left(\frac{y}{y_F^*}\right)^\beta \left(2\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D} + \beta \ln\left(\frac{y}{y_F^*}\right)\right) \frac{I}{(\beta-1)^2}. \quad (47)$$

Consider the value of this expression at the preemption threshold  $y_P^*$ , to find:

$$\frac{d(2\tilde{V} + \tilde{W})}{d\beta} \Big|_{y=y_P^*} = \left(\frac{y_P^*}{y_F^*}\right)^\beta \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D} - \beta \ln\left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)\right) \frac{I}{(\beta-1)^2}, \quad (48)$$

which has the sign of  $\left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D} - \beta \ln\left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)\right)$ . For a given  $\frac{\pi_M}{\pi_D}$ ,  $\lim_{\beta \rightarrow 1} \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D} - \beta \ln\left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)\right) = +\infty$  by l'Hôpital's rule, while  $\lim_{\beta \rightarrow \infty} \left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D} - \beta \ln\left(\frac{\beta}{\beta-1} \frac{\pi_M}{\pi_D}\right)\right) = -\infty$ . ■



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