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Julien Chevallier. Modelling the convenience yield in carbon prices using daily and realized measures. 2010. halshs-00463921v2

**HAL Id: halshs-00463921**

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Preprint submitted on 25 May 2010

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# Modelling the convenience yield in carbon prices using daily and realized measures

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December 2, 2009

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**Abstract:** This article investigates the modelling of the convenience yield in the European carbon market by using daily and intraday measures of volatility. The convenience yield stems from differences in spot and futures prices, and can explain why firms hold inventories. The main findings are that (i) a simple AR(4) process best describes the 2008 convenience yield, and (ii) there exists a non-linear relation between spot and futures prices. The approach developed in this article captures 74% of the explanatory power for the 2008 convenience yield variable in an autoregressive framework, with carbon spot price levels, moving averages and carbon futures realized volatility measures as exogenous regressors. These results are of interest for energy utilities, risk-managers, and traders exposed to the variation of carbon prices.

*JEL Classification:* C5; G1; Q4.

*Keywords:* Convenience Yield; Carbon Price; EU ETS; High-frequency Data; Realized Volatility.

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# 1 Introduction

On commodity markets, the investigation of the convenience yield appears as a central empirical issue, since it allows practioners, hedgers, brokers and other market operators in the field to buffer themselves against unanticipated changes in market conditions (see Pindyck (2001) for a through discussion on this issue). The basic intuition is that differences in spot and futures prices occur due to the cost of holding inventory.

In many economic and financial applications, the theory of storage aims at explaining the differences between spot and futures prices by analyzing the reasons why agents hold inventories. According to Geman (2005), inventories have a productive value since they allow to meet unexpected demand, avoid the cost of frequent revisions in the production schedule and eliminate manufacturing disruption. Working (1949) defined the notion of convenience yield as a benefit that accrues to the owner of the physical commodity. Brennan (1958) further defined the convenience yield as an embedded timing option attached to the commodity, since inventory allows to put the commodity on the market when prices are high and hold it when prices are low. Recent applications of the theory of storage to the modelling of commodity prices include Considine and Larson (2001), Wei and Zhu (2006), Geman and Ohana (2009) and Stronzik et al. (2009) for crude oil or natural gas markets.

This article focuses on the modelling of the convenience yield for carbon spot and futures prices, which are exchanged since 2005 on the European Union Emissions Trading Scheme (EU ETS). The EU emissions trading system has been created by the Directive 2003/87/CE. Across 27 Member States, the EU ETS covers large plants from CO<sub>2</sub>-intensive emitting industrial sectors with a rated thermal input exceeding 20 MWh. One allowance exchanged on the EU ETS corresponds to one ton of CO<sub>2</sub> released in the atmosphere, and is called a European Union Allowance (EUA) (see Alberola et al. (2008) for more details.).

This issue is of particular importance for risk-managers and traders in energy utilities regulated by the scheme, as they need to cover themselves against financial, political, and economic risks specific to this market (see Chevallier et al. (2009) for more details). Besides, carbon prices convey some interesting characteristics in terms of commodity modelling, since the costs of storage are null. Carbon permits indeed only exist in the balance sheets of the companies regulated by the scheme, and the costs of storing them physically are insignifiant.

There is very limited literature on the investigation of the convenience yield in the European carbon

market. To our best knowledge, only Borak et al. (2006) address this issue. They show that the market has changed from initial backwardation to contango with significant convenience yields in future contracts for the Kyoto commitment period starting in 2008. Their main result features that a high fraction of the yields can be explained by the price level and volatility of the spot prices. The authors conclude that the yields can be interpreted as market expectation on the price risk of CO<sub>2</sub> emissions allowance prices and the uncertainty of EU allocation plans for the Kyoto period.

Based on recent developments in financial econometrics (Andersen, Bollerslev, Diebold and Labys, henceforth ABDL (2003)), we use in this article both daily and intradaily data for risk measures to model the convenience yield. High-frequency data is indeed superior in estimation if some biases (linked to microstructure noise) are correctly accounted for<sup>2</sup>. This methodology, joint with standard ARMA filtering, yields some interesting results for the modelling of the convenience yield in carbon spot and futures prices, which is predictable (with a R-squared of 74%) using autoregressive processes. Using realized volatility measures shows two effects: 1) the explanatory power is reduced to 35%, and 2) realized volatility significantly impacts the convenience yield in carbon prices. Our results extend Borak et al. (2006), who used daily data only and whose study period covers the early years of the EU ETS (2005-2006).

The remainder of the article is composed as follows. Section 2 describes the data used. Section 3 details the modelling of the convenience yield. Section 4 develops realized volatility estimation techniques. Section 5 presents the empirical results. Section 6 concludes.

## 2 Data

We detail below the data used for carbon prices at both daily and intra-daily frequencies, as well as for the risk-free rate.

First, concerning daily spot and futures carbon prices, *Bluenext* is the market place dedicated to CO<sub>2</sub> allowances based in Paris. It has been created on June 24, 2005 and has become the most liquid platform for spot trading: 72% of the volume of spot contracts are traded on Bluenext according to Reuters. The *European Climate Exchange* (ECX) is the market place based in London. It has been created on April 22,

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<sup>2</sup>Note differences between implied volatility (extracted from option prices) and realized volatility (computed from intraday data) in the context of convenience yield estimates for the carbon market are left for further research.

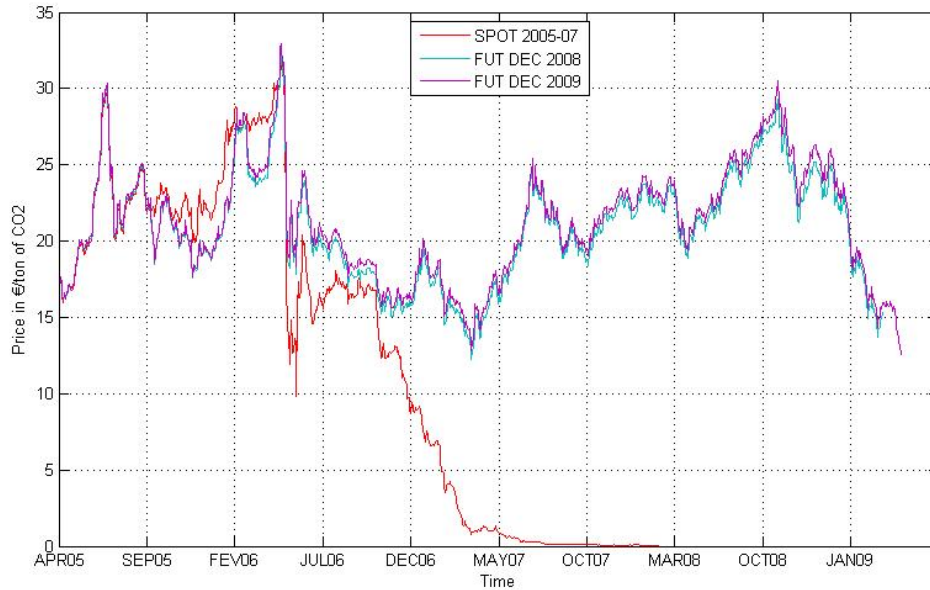


Figure 1: Carbon Spot and Futures Prices from April 22, 2005 to January 16, 2009  
 Source: Bluenext, European Climate Exchange

2005 and is the most liquid platform for futures and options trading: 96% of the volume of futures contracts are traded on ECX according to Reuters. Figure 1 shows the price path for daily carbon spot and futures prices.

The trading of Bluenext EUA spot prices started on June 24, 2005. However, from October 2006 until December 2007, CO<sub>2</sub> spot prices have been decreasing towards zero due to the banking restrictions implemented between 2007 and 2008 (Alberola and Chevallier (2009)). Due to this erratic and non-reliable behavior of spot prices during Phase I, we choose to work only with Phase II CO<sub>2</sub> spot prices in this article. The trading of CO<sub>2</sub> spot prices valid for Phase II started on Bluenext on February 26, 2008. Thus, the start of the second trading period of the EU ETS corresponds to the start of the dataset for spot prices used in this article. The minimum volume for trading is 1,000 tons of CO<sub>2</sub> equivalent. From February 26, 2008 to April 15, 2009, Bluenext Phase II spot prices reached an upper bound of €28/ton of CO<sub>2</sub> in May 2008, and a lower bound of €8/ton of CO<sub>2</sub> in February 2009, thereby probably capturing with some delay the depressing effect of the “credit crunch” crisis on global commodity markets. During our study period, the total volume of Bluenext Phase II spot prices exchanged is equal to 847 million tons.

The trading of ECX futures started on April 22, 2005 with varying delivery dates going from December

2005 to December 2012. Futures contracts with vintages December 2013 and 2014 were introduced on April 8, 2008. For the December 2009 futures contract, futures trade at €13.32/ton of CO<sub>2</sub> as of January 15, 2009, and have reached a maximum price of €32.90/ton of CO<sub>2</sub> in 2008. In the longer term, analysts forecast EUA prices of €20-25/ton of CO<sub>2</sub> over Phase II and €25-30/ton of CO<sub>2</sub> over Phase III, according to Reuters. From April 2005 to January 2009, the total volume of ECX futures exchanged for all vintages is equal to 40.67 billion.

As shown in Figure 1, given the non-reliable behavior of carbon spot prices during 2005-2007 (EU ETS Phase I), we choose to work only with carbon prices valid during 2008 (EU ETS Phase II). Indeed, as shown by Alberola and Chevallerier (2009), banking restrictions between 2007 and 2008 caused the disconnection between spot and futures prices between the two Phases. Besides, a structural break due to information revelation occurred in April 2006 for carbon prices of all maturities (Alberola et al. (2008)).

Second, concerning intraday carbon futures prices, our sample contains one year of tick-by-tick transactions for the ECX futures contract of maturity December 2008, going from January 2 to December 15, 2008. This is equivalent to 240 days of trading after cleaning the data for outliers, and until the expiration of the contract. The average amount of transactions for the ECX carbon futures tick-data is equal to 700 trades per day. This corresponds to an average of 50 seconds between each transaction.

Third, the risk-free rate used below to compute the convenience yield between carbon spot and futures prices is the Euribor, as commonly used by market agents. The Euribor rates were accessed from Thomson Financial DataStream. Depending on the time until maturity, we use the Euribor contract with the relevant maturity. Descriptive statistics for all variables used in our econometric specification are given in Table 1. We observe that CO<sub>2</sub> spot and futures price series are characterized by a negative skewness, and the kurtosis coefficient is close to three, which is the value for the normal distribution. Taken together, these descriptive statistics suggest that CO<sub>2</sub> spot and futures price series exhibit some leptokurticity, which can be better fitted by the used of GARCH( $p, q$ ) modeling (Bollerslev (1986)).

Next, we provide some elements on the theory of storage and develop the computational steps in order to obtain the convenience yield from carbon spot and futures prices.

	Futures	Spot	Euribor	Spot price level	RV vol proxy	MA vol proxy
Mean	22.88945	22.65393	4.895124	-0.000724	0.015279	0.010033
Median	23.63000	23.38000	5.050000	0.000199	0.006450	0.009269
Maximum	29.33000	28.73000	5.448000	0.023344	0.329379	0.018092
Minimum	13.72000	13.70000	3.334000	-0.039217	8.17E-06	0.003589
Std. Dev.	3.591024	3.507532	0.469375	0.010621	0.031766	0.003374
Skewness	-0.810727	-0.805231	-1.392180	-0.659208	6.733913	0.424142
Kurtosis	2.962105	2.903512	4.612382	3.960755	59.51318	2.138923
Observations	240	240	240	240	240	240

Table 1: Descriptive statistics for the daily data used from January 2 to December 15, 2008  
Source: ECX, Bluenext, Thomson Financial Datastream

Note: *Futures* denotes the ECX December 2008 carbon futures price, *Spot* the Bluenext carbon spot price, *Euribor* the risk-free rate, *Spot price level* the regressor of the carbon spot price against a constant, *RV vol proxy* is the measure of realized volatility for the ECX December 2008 carbon futures contract as computed by Chevallier and Sévi (2009a), *MA vol proxy* is another proxy for volatility using moving averages of the carbon price, and *Std.Dev* is the standard deviation.

### 3 Modelling the convenience yield

This section describes how the convenience yield can be measured.

By the cost-of-carry relationship, and without storage costs for EUA allowances, the futures and spot prices are linked through  $S_t = F_T e^{-r(T-t)}$  with  $S_t$  the spot price at time  $t$ ,  $F_T$  the futures prices of a contract with delivery in  $T$  and  $r$  the interest rate (Working (1949), Brennan (1958)). Equivalently, we may write:

$$F_{t,T} = S_t e^{(r-y)(T-t)} \quad (1)$$

with  $F_{t,T}$  the futures price for maturity  $T$  at time  $t$ ,  $r$  the continuously compounded risk-free interest rate used by market agents at time  $t$  for maturity  $T$ , and  $y$  is the convenience yield on the commodity.

As Pindyck (2001) put it, this no-arbitrage condition states that the only cost of buying a commodity at time  $t$  and delivering it at maturity  $T$  is the foregone interest. Agents incur the opportunity cost of purchasing the asset, but in return they benefit from possessing the commodity and being able to trade it until maturity.

Hence, the convenience yield at time  $t$  for maturity  $T$  may be modelled directly as:

$$y_{t,T} = S_t e^{r(T-t)} - F_{t,T} \quad (2)$$

Figure 2 shows the carbon spot and futures prices during 2008, as well as the corresponding convenience

yield<sup>3</sup>.

In the bottom panel of Figure 2, we may observe that the convenience yield is strongly time-varying during 2008, going from -0.2 in June to 1.2 in April. These variations may be explained by (i) the delayed effect of the “credit crunch” crisis on the carbon market (Chevallier (2009)), and (ii) the 2007 compliance event which occurred in April 2008 (see Chevallier et al. (2009) for a detailed analysis of the effects of compliance events on investors’ expectation changes).

To test this relationship empirically, we adopt the following econometric specification<sup>4</sup>:

$$y_{t,T} = \alpha + \beta S_t + \gamma V_t + \varepsilon_t \quad (3)$$

with  $V_t$  a proxy for volatility,  $\varepsilon_t$  the error term.  $y_{t,T}$  is filtered through an ARMA process, which does not appear in the specification below, following the Box-Jenkins methodology.  $V_t$  may be either an autoregressive or a moving average process for a measure of volatility, which may be composed of realized volatility measures (as in ABDL (2003)). Thus, we will explicitly compare in Tables 7 and 8 several volatility proxies, which may be based on daily data (through the use of moving averages) or based on intraday data (through the use of realized volatility estimates). Since realized volatility estimates are computed over the time interval of one day, our econometric model may be carried out with a daily frequency.

In the next section, we detail how to compute realized volatility measures for carbon prices.

## 4 Estimation of realized volatility

This section presents different computation methods for realized volatility measures using the most recent developments in financial econometrics techniques.

Let  $p(t)$  denote a logarithmic asset price at time  $t$ . With no jump, the continuous-time diffusion process generally employed in asset and derivatives pricing may be expressed by a stochastic differential equation as:

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<sup>3</sup>Note we do not consider here breakpoint analysis in the time-series of carbon prices. This aspect is left for further research.

<sup>4</sup>Note that, given the interaction between carbon prices and other energy markets (Alberola et al. (2008)), we may also include other factors such as oil and gas prices in order to model the convenience yield for carbon prices. This area is left for further research.



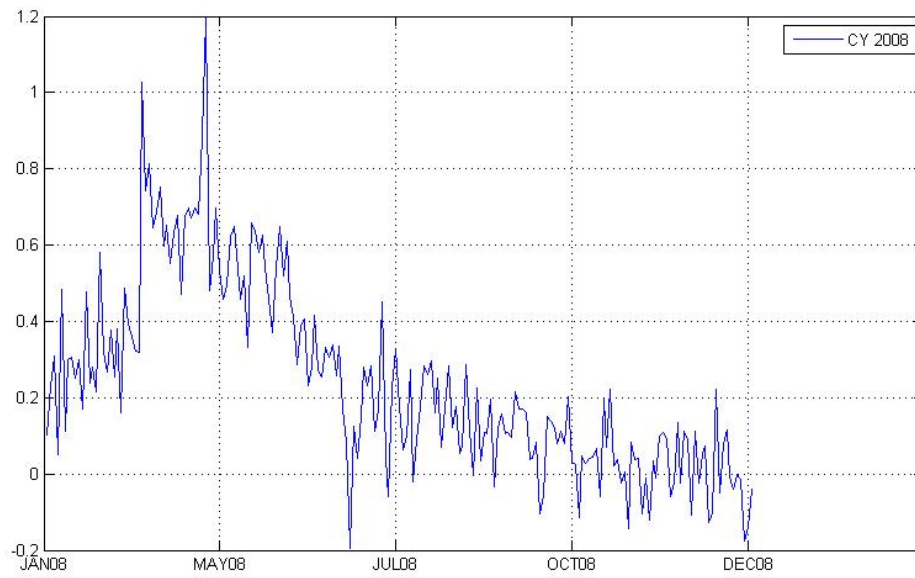
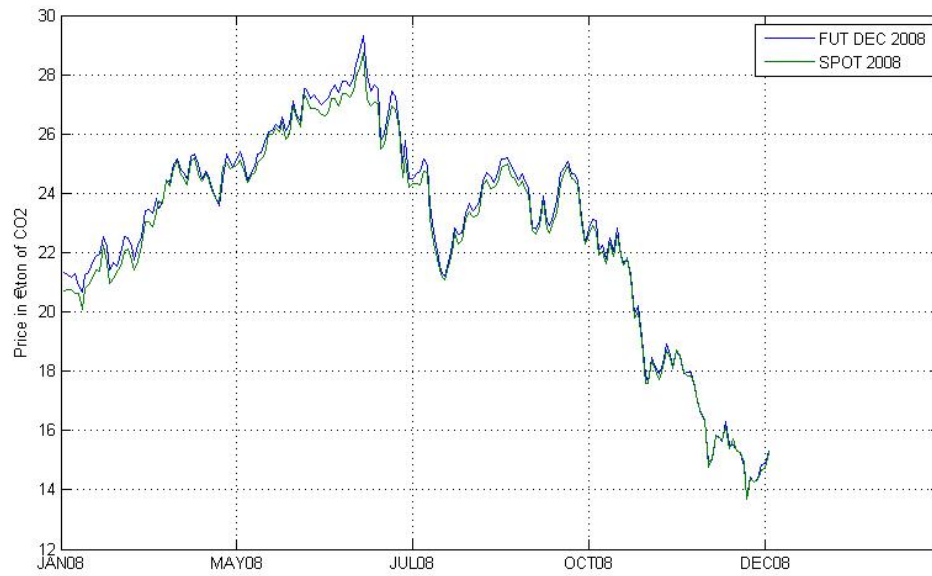


Figure 2: Carbon Spot and Futures Prices (top) and convenience yield (bottom) from January 2 to December 15, 2008

Source: Bluenext, European Climate Exchange, Euribor

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) \quad \text{with} \quad 0 \leq t \leq T \quad (4)$$

with  $\mu(t)$  a continuous and locally bounded variation process,  $\sigma(t)$  a strictly positive càdlàg (right continuous with left limits) stochastic volatility process, and  $W(t)$  a standard Brownian motion.

Next, let us consider the quadratic variation (QV) for the cumulative return process  $r(t) \equiv p(t) - p(0)$ :

$$[r, r]_t = \int_0^t \sigma^2(s)ds \quad (5)$$

The QV simply equals the integrated volatility of the process described in Eq. (4). The *realized volatility* (RV) is defined as the sum of returns at a frequency  $1/\Delta$ , or:

$$RV_{t+1}(\Delta) \equiv \sum_{j=1}^{1/\Delta} r_{t+j.\Delta, \Delta}^2 \quad (6)$$

When  $\Delta \rightarrow 0$ , using theory of quadratic variations, it can be shown (see Andersen, Bollerslev, Diebold and Ebens (2001), Barndorff-Nielsen and Shephard (2002)) that:

$$RV_{t+1}(\Delta) \rightarrow \int_0^t \sigma^2(s)ds \quad (7)$$

Theory suggests that optimal sampling corresponds to sampling at the highest possible frequency. However, this is not true in practice due to microstructure effects (bid-ask spread, rounding, non-synchronicity, etc.) which introduce noise in the price process. To mitigate the impact of microstructure noise, we examine the volatility signature plot (as in ABDL (2003)) for the 2008 carbon futures contract. In volatility signature plots, the realized volatility measure described in Eq. (6) is computed and plotted at different sampling frequencies.

Figure 3 shows that as frequency becomes higher, the realized variance includes an increasing noise component. Thus, the optimal sampling frequency for carbon prices may be determined at 15-min returns (see Chevallier and Sévi (2009a, 2009b) for a detailed analysis).

Figure 4 plots the three proxies of volatilities using realized measures. We use the last tick method which is superior to interpolation (ABDL (2003)). Subsampling (Zhang et al. (2005)) and kernel-based (Zhou

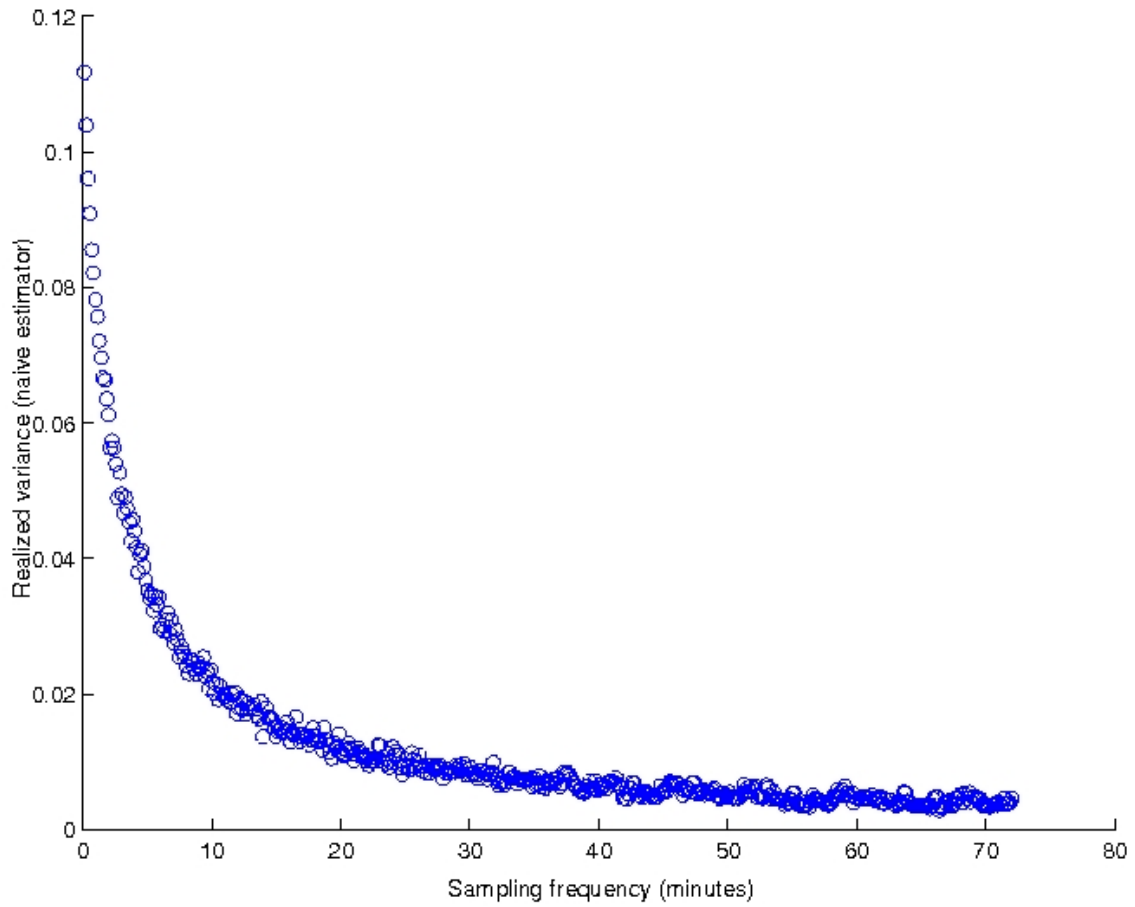


Figure 3: Volatility signature plot for 2008 carbon futures contract with sampling frequencies ranging from 0 to 80 minutes.

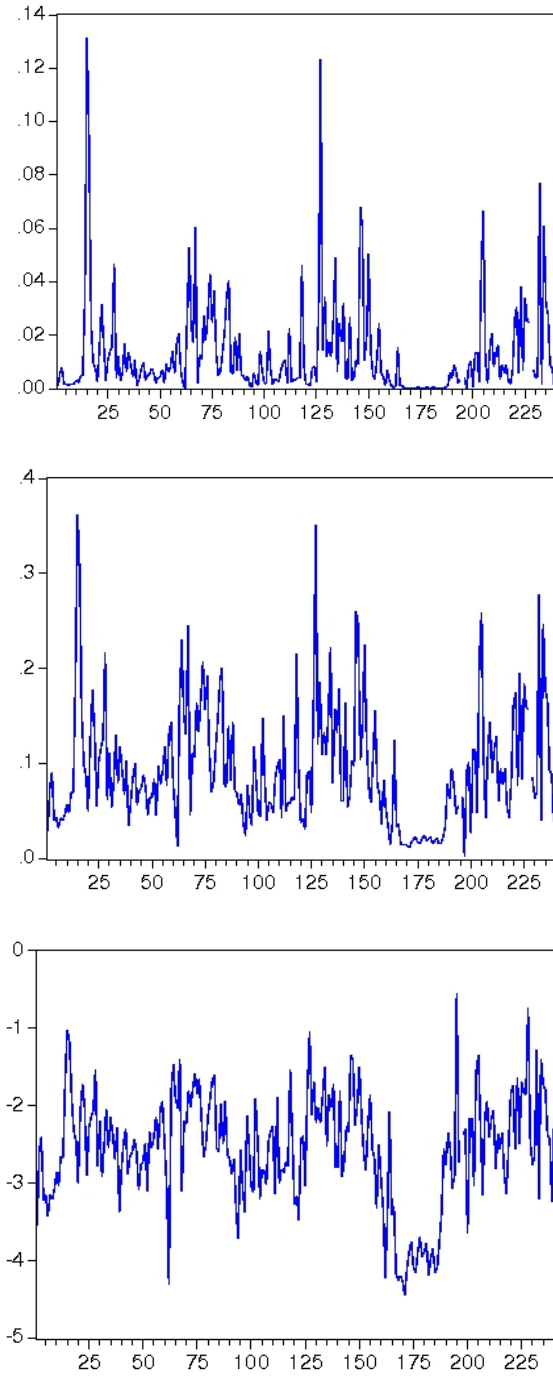


Figure 4: Daily realized variance (top panel), daily realized volatility in standard deviation form (middle panel), and daily realized volatility (bottom panel) for the 208 carbon futures contract

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.776088	0.0198
*McKinnon (1996) one-sided p-values.		
Test critical values:		
1% level	-4.005076	
5% level	-3.432682	
10% level	-3.140127	

Table 2: ADF Test Statistic for the 2008 convenience yield variable with carbon spot and futures prices

(1996), Hansen and Lunde (2006)) estimators are similar in nature. The time-series reveal the presence of jumps and structural breaks that may be taken into account using bipower and tripower variation measures (ABD (2007)). The extension of our work to jump-robust measures of realized volatility is also left for further research.

In the next section, we present the estimation results of Eq. (3) for the 2008 convenience yield between carbon spot and futures prices using daily and realized measures.

## 5 Empirical results

Our empirical approach is of interest for policy makers, since it will allow to derive informed conclusions on the modelling of the convenience yield in the carbon market, which can then be used for forecasting purposes. Thus, we estimate eq(3) with various specifications.

Using daily and intraday data as a risk measure, we aim at identifying here whether the convenience yield in carbon prices is highly time-variant. Within this framework, volatility can either be an autoregressive or moving average process which can possibly consist of realized volatility measures.

In this section, we present the modelling results of Eq. (3) following different specifications of the 2008 convenience yield variable through ARMA filtering, and the inclusion as exogenous regressors of carbon spot price levels, moving averages, and carbon futures realized volatility estimates.

We first run the Augmented Dickey-Fuller (1979) unit root test on the dependent variable.

From Table 2, we may reject the unit root hypothesis for the 2008 convenience yield variable. The time-series does not seem to be integrated of any order.

Table 3 presents estimation results with ARMA filtering. The best specification is obtained with autore-

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.368329	0.072681	5.067725	0.0000
AR(2)	0.189735	0.076136	2.492068	0.0136
AR(3)	0.214808	0.076070	2.823817	0.0053
AR(4)	0.190556	0.072244	2.637655	0.0091
Adj. R <sup>2</sup>	0.735475			
AIC	-1.179760			
SC	-1.110389			
LL	113.7177			
F-Stat.	0.000000			

Table 3: AR(4) Test Results for the 2008 convenience yield variable with carbon spot and futures prices

Note: The dependent variable is the 2008 convenience yield for carbon spot and futures prices. *Std.Error* is the standard error, *Prob.* is the probability value for statistical significance, *AR(p)* denotes the lags of the autoregressive components. The quality of the regression is verified through the following diagnostic tests: *Adj.R<sup>2</sup>* is the Adjusted R-squared, *AIC* is the Akaike Information Criterion, *SC* is the Schwartz information criterion, *LL* is the Log likelihood, and *F – Stat.* is the *p*-value of the F-Statistic.

gressive components up to order 4 (AR(4)): all lag orders are statistically significant at the 1% level. The R<sup>2</sup> is equal to 74%, which shows the high degree of predictability of the 2008 convenience yield variable for carbon spot and futures prices based on autoregressive processes only. All diagnostic tests are validated for this regression, and residuals are not autocorrelated<sup>5</sup>.

Variance estimation is a critical part of this article. Thus, we need to account for heteroskedasticity, as is often the case for financial time-series. To detect ARCH-effects, we re-estimate this regression with an ARCH term in the variance equation, as shown in Table 4.

The ARCH component is significant at the 5% level, while other estimated coefficients remain stable. This regression confirms the robustness of our previous results with an AR(4) process. This specification also allows to adjust for serial correlation in the data<sup>6</sup>.

Three other proxies of volatilities may be used: 1) daily realized variance, 2) daily realized volatility in standard deviation form, and 3) daily realized volatility. In what follows, we present the results of measuring the convenience yield in carbon markets using these three proxies.

Here, instead of AR components, we introduce the carbon spot price level and the realized volatility measure for the 2008 carbon futures contract as exogenous regressor of the convenience yield in Eq. (3).

<sup>5</sup>This comment applies in the remainder of the article.

<sup>6</sup>Results from the Ljung-Box-Pierce test statistic are not reproduced here to conserve space, but they may be obtained upon request to the authors.

	Coefficient	Std. Error	Prob.
AR(1)	0.337292	0.092377	0.0003
AR(2)	0.228832	0.084789	0.0070
AR(3)	0.210157	0.085976	0.0145
AR(4)	0.188615	0.073851	0.0106
Variance Equation			
C	0.014162	0.001197	0.0000
RESID(-1) <sup>2</sup>	0.177775	0.086428	0.0397
Adj. R <sup>2</sup>	0.732061		
AIC	-1.196933		
SC	-1.092877		
LL	117.3148		
F-stat.	0.000000		

Table 4: AR(4) and ARCH Test Results for the 2008 convenience yield variable with carbon spot and futures prices

Note: The dependent variable is the 2008 convenience yield for carbon spot and futures prices. *Std.Error* is the standard error, *Prob.* is the probability value for statistical significance,  $AR(p)$  denotes the lags of the autoregressive components. In the variance equation,  $RESID(-1)^2$  denotes the ARCH term. The quality of the regression is verified through the following diagnostic tests:  $Adj.R^2$  is the Adjusted R-squared, *AIC* is the Akaike Information Criterion, *SC* is the Schwartz information criterion, *LL* is the Log likelihood, and *F – Stat.* is the *p*-value of the F-Statistic.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.341998	0.106699	-3.205265	0.0016
SPOT	0.041766	0.004371	9.555048	0.0000
LOGSD(-1)	0.012412	0.024008	0.517001	0.6058
LOGSD(-2)	0.030391	0.025516	1.191031	0.2352
LOGSD(-3)	0.026523	0.024341	1.089625	0.2773
LOGSD(-4)	0.047349	0.025377	1.865828	0.0637
LOGSD(-5)	0.025009	0.024082	1.038525	0.3004
Adj. R <sup>2</sup>	0.348315			
AIC	-0.262496			
SC	-0.141097			
LL	31.41210			
F-stat.	0.000000			

Table 5: Realized Volatility with Lag 5 Results for the 2008 convenience yield variable with carbon spot and futures prices

Note: The dependent variable is the 2008 convenience yield for carbon spot and futures prices. *Std.Error* is the standard error, *Prob.* is the probability value for statistical significance, *SPOT* is the exogenous regressor for the level of carbon spot prices,  $LOGSD(p)$  denotes the lags of the realized volatility measure of the 2008 carbon futures contract in log-transformation. The quality of the regression is verified through the following diagnostic tests:  $Adj.R^2$  is the Adjusted R-squared, *AIC* is the Akaike Information Criterion, *SC* is the Schwartz information criterion, *LL* is the Log likelihood, and *F – Stat.* is the *p*-value of the F-Statistic.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.464655	0.106403	-4.366921	0.0000
SPOT	0.039174	0.004490	8.725252	0.0000
LOGSD(-1)	0.071591	0.019751	3.624685	0.0004
Adj. R <sup>2</sup>	0.296707			
AIC	-0.207194			
SC	-0.155166			
LL	22.26906			
F-stat.	0.000000			

Table 6: Realized Volatility with Lag 1 Results for the 2008 convenience yield variable with carbon spot and futures prices

Note: The dependent variable is the 2008 convenience yield for carbon spot and futures prices. *Std.Error* is the standard error, *Prob.* is the probability value for statistical significance, *SPOT* is the exogenous regressor for the level of carbon spot prices, *LOGSD(p)* denotes the lags of the realized volatility measure of the 2008 carbon futures contract in log-transformation. The quality of the regression is verified through the following diagnostic tests: *Adj.R<sup>2</sup>* is the Adjusted R-squared, *AIC* is the Akaike Information Criterion, *SC* is the Schwartz information criterion, *LL* is the Log likelihood, and *F – Stat.* is the *p*-value of the F-Statistic.

In Table 5, the specification with carbon spot price levels and realized volatility measures of carbon futures (up to lag 5) provides inferior results for the modelling of the 2008 convenience yield variable compared to AR processes. Indeed, the R<sup>2</sup> is merely equal to 35%.

As shown in Table 6, if we reduce the number of lags to 1 for the realized volatility component, the R<sup>2</sup> falls to 29%.

If we remove the realized volatility component as an exogenous regressor in Eq. (3), the specification with carbon spot price levels only has an explanatory power of 25% (Table 7). The latter result reinforces the belief that volatility is an important determinant in the modelling of the 2008 convenience yield variable for carbon spot and futures prices.

Finally, we may use moving averages as another proxy for volatility. In addition to the AR(4) configuration of the convenience yield variable, the inclusion as exogenous regressors of carbon spot price levels and moving averages for the volatility component yield to the best estimates.

In Table 8, the R<sup>2</sup> is indeed equal to 74% for such a regression. The inclusion of realized volatility measures (instead of moving averages) as a proxy for volatility does not yield to superior results (Table 9).

To sum up the results obtained, we find that (i) using an AR(4) process can already explain 74% of the convenience yield for CO<sub>2</sub> spot and futures prices; and (ii) using realized volatility significantly impacts



Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.565611	0.101884	-5.551507	0.0000
SPOT	0.035633	0.004434	8.035785	0.0000
Adj. R <sup>2</sup>	0.245863			
AIC	-0.188676			
SC	-0.155225			
LL	20.49021			
F-stat.	0.000000			

Table 7: Carbon Spot Price Level Results for the 2008 convenience yield variable with carbon spot and futures prices

Note: The dependent variable is the 2008 convenience yield for carbon spot and futures prices. *Std.Error* is the standard error, *Prob.* is the probability value for statistical significance, *SPOT* is the exogenous regressor for the level of carbon spot prices. The quality of the regression is verified through the following diagnostic tests: *Adj.R<sup>2</sup>* is the Adjusted R-squared, *AIC* is the Akaike Information Criterion, *SC* is the Schwartz information criterion, *LL* is the Log likelihood, and *F – Stat.* is the *p*-value of the F-Statistic.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPOT	0.012241	0.005817	2.104200	0.0368
VOL MA RET	-5.071107	8.302313	-0.610806	0.5421
AR(1)	0.381148	0.074571	5.111182	0.0000
AR(2)	0.183629	0.078429	2.341334	0.0203
AR(3)	0.198623	0.078691	2.524105	0.0125
AR(4)	0.149432	0.074178	2.014496	0.0455
Adj. R <sup>2</sup>	0.741826			
AIC	-1.182653			
SC	-1.077027			
LL	113.6214			
F-stat.	0.000000			

Table 8: Moving Averages as Volatility Proxy Results for the 2008 convenience yield variable with carbon spot and futures prices

Note: The dependent variable is the 2008 convenience yield for carbon spot and futures prices. *Std.Error* is the standard error, *Prob.* is the probability value for statistical significance, *SPOT* is the exogenous regressor for the level of carbon spot prices, *VOL MA RET* is the exogenous regressor for a proxy of volatility using moving averages. The quality of the regression is verified through the following diagnostic tests: *Adj.R<sup>2</sup>* is the Adjusted R-squared, *AIC* is the Akaike Information Criterion, *SC* is the Schwartz information criterion, *LL* is the Log likelihood, and *F – Stat.* is the *p*-value of the F-Statistic.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPOT	0.009428	0.006072	1.552626	0.1223
LOGSD	-0.011838	0.014946	-0.792022	0.4294
AR(1)	0.358909	0.073306	4.896050	0.0000
AR(2)	0.186561	0.076876	2.426772	0.0162
AR(3)	0.199642	0.076745	2.601353	0.0101
AR(4)	0.184547	0.073290	2.518040	0.0127
Adj. R <sup>2</sup>	0.736643			
AIC	-1.173727			
SC	-1.069671			
LL	115.1566			
F-stat.	0.000000			

Table 9: Counter-Factual Exercise with Realized Volatility Component (instead of Moving Averages) as Volatility Proxy Results for the 2008 convenience yield variable with carbon spot and futures prices

Note: The dependent variable is the 2008 convenience yield for carbon spot and futures prices. *Std.Error* is the standard error, *Prob.* is the probability value for statistical significance, *SPOT* is the exogenous regressor for the level of carbon spot prices, *LOGSD(p)* denotes the lags of the realized volatility measure of the 2008 carbon futures contract in log-transformation. The quality of the regression is verified through the following diagnostic tests: *Adj.R<sup>2</sup>* is the Adjusted R-squared, *AIC* is the Akaike Information Criterion, *SC* is the Schwartz information criterion, *LL* is the Log likelihood, and *F – Stat.* is the *p*-value of the F-Statistic.

the convenience yield, but the explanatory power is reduced to 35%. The methodology conducted here therefore provides useful information for market players in need to hedge against a potential carbon price risk. Besides, the analysis is based on most recent findings from financial econometrics.

## 6 Conclusion

The EU ETS has fostered the development of market place for CO<sub>2</sub> allowances in Europe. The most liquid spot trading place is the BlueNext in Paris, while futures and options trading takes place at the European Climate Exchange in London.

This article models the convenience yield in the European carbon market. The convenience yield stems from differences in spot and futures prices, and can explain why firms hold inventories. Hence, this analysis appears of particular importance for risk management and to traders in Europe.

Compared to previous literature, our approach builds on Borak et al. (2006) by taking explicitly into account various proxies for the volatility of carbon prices that may be of importance for the behaviour of the convenience yield term. More precisely, we focus on daily and realized measures to proxy for volatility of

carbon prices, and we assess their respective importance in the modelling of the convenience yield. High-frequency data for CO<sub>2</sub> allowances futures have been gathered from the European Climate Exchange.

We show that the 2008 convenience yield variable for carbon spot and futures prices may be modelled as a highly persistent variable. Indeed, after testing for various specifications with spot price levels, realized volatility measures or moving averages as exogenous regressors for volatility, the best results are achieved with a simple AR(4) specification.

Besides, we find evidence that daily and realized volatility measures are statistically significant in modelling the convenience yield variable. This result suggests that the relationship between carbon spot and futures prices is not linear.

That spot price levels, moving averages or intraday volatility estimates contribute to explain the convenience yield only at the margin (the explanatory power of regressions with autoregressive components alone is indeed very high) contains some useful information in terms of forecasting for utilities facing the need to hedge against carbon price changes.

## **Acknowledgements**

I wish to thank Benoît Sévi for fruitful discussions on this topic, as well as ECX for providing the tick-by-tick data. The usual disclaimer applies.

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