A Dynamic Explanation of the Crisis of the Welfare State

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Abstract

Although the crisis of the Welfare State has been evoked for quite a long time, figures show that such a phenomenon has arisen only recently. Furthermore, it is not a common feature in all developed countries. This paper aims at explaining these two empirical facts. We use an overlapping generations model in which agents decide to educate themselves or not endogenously. Furthermore, at each date, the working population vote on the size of a redistributive policy. Firstly, we show that the share of the educated population can be the engine of the crisis of the Welfare State. Moreover, our paper emphasizes that the expectations of agents about the size of redistributive policies, can explain the timing differential in the crisis of the Welfare State between developed countries.

Keywords: Welfare State, Indeterminacy, Education, Redistribution.

JEL Codes: D72, H55, I2

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1 Introduction

Since the end of the second World War, the role of governments has dramatically expended. Indeed, they set up an objective solidarity which mainly aims at insuring agents against life risks, and at reducing wealth inequalities (Merrien 2007, Barr 2002). However, for a few years, a growing literature consider that the Welfare State is in "crisis" or that it will be soon. The Welfare State would suffer from a questionable efficiency, from a legitimacy crisis, and from financial difficulties (Rosanvallon 1992). Concerning the first point, the literature argues that the private sector provides a better alternative to public expenditures because of the improvement of its efficiency (Bergh 2008). Moreover, the size of transfers has a significant distorsive impact on economic decisions. Secondly, the opacity of the management of public funds would imply that citizens do not necessarily consider any further expenditure as useful. Finally, the aging of the population, the increase in the unemployment (notably in Europe), or the improvement of the quality of health services have increased the burden of taxation. Although the "crisis" of the Welfare State is detailed in a fair number of studies, empirical data do not confirm their assertions. Indeed, the changes in the size and in the structure of the Welfare State seem to be incremental (Bergh 2008). However, it is possible to distinguish some trends about the size of the public sector in developed countries[1] (see figure 1). More specifically, countries can be classified in two groups. In the first one, the fiscal burden has increased or is remained stable. France, Germany, the United-Kingdom and the United-States among others, belong to this group. In the second one, the fiscal burden has decreased for a few years. Canada and the Netherlands describe such a trend. Consequently, even if the term "crisis" does not seem to be the most appropriate to describe the situation, it seems that some countries have questioned the size of their Welfare State. Let us now consider the case of the European Union. In figure 2, it is shown that the size of the public sector has begun to decrease for a few years. Consequently, in average, countries have begun to question the size of their government.

In this paper we propose a model which is compatible with the previous facts. We assume that tax revenue is only used to redistribute resources between socio-economic groups. Our main argument is that the evolution of the political support for redistribution can explain the dynamics of the Welfare State. Moreover, in our model, the period from which the size of public spending decreases depends on the expectations of agents. This

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[1]In this paper the size of the public sector is represented by the share of the tax revenue in the GDP.
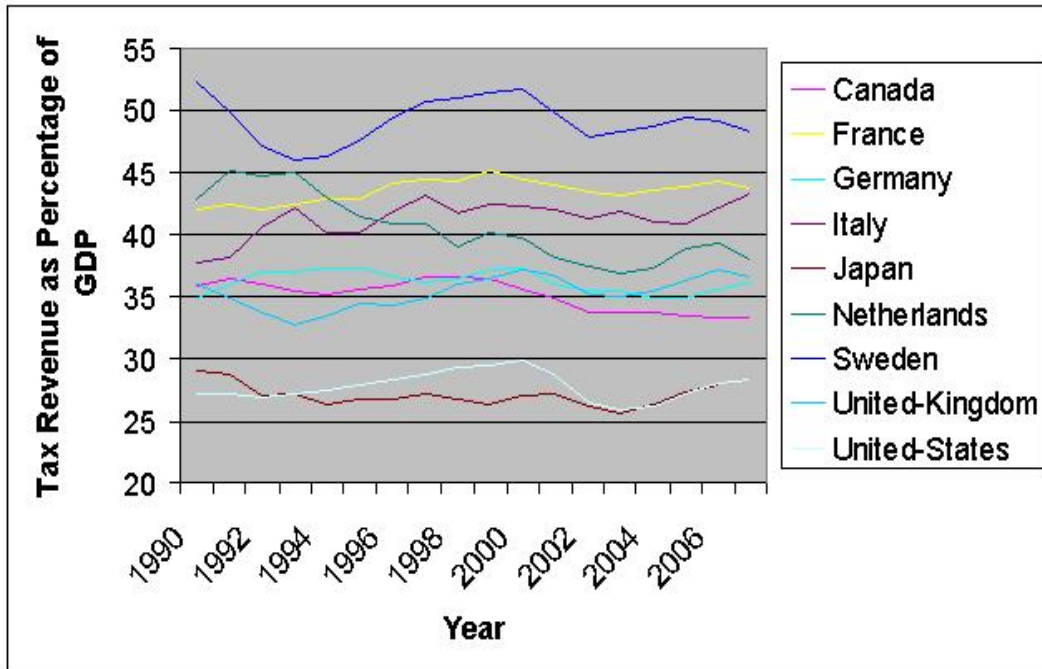
Figure 1: Tax Revenue as percentage of GDP for some developed countries. *Source*: OECD.

can explain the timing differential in the crisis of the Welfare State between near countries.

Some papers have previously studied the endogenous determination of the size of public spending (identified with the expenditures of redistribution) using political equilibrium models (Meltzer and Richard 1981, Persson and Tabellini 2000). In these static models, agents supply their labor endogenously, given their productivity endowment and given the size of public transfers. Moreover, it is assumed that agents vote on the size of public spending knowing their distorsive impact on labor supply. Public transfers are financed through a tax rate on wages, and are used to finance a flat benefit. The equilibrium tax rate is determined by the agent endowed with the median wage. Belletini and Ceroni (2007) use the same kind of model, but they assume (i) an exogenous labor supply, and (ii) that agents can face a liquidity constraint. In that case, the median voter is not necessarily the agent endowed with the median wage. Compared to these models, we add a dynamic environment. Furthermore, as in Belletini and Ceroni (2007), we do not consider the case of an endogenous labor supply to keep the model tractable.

Some other papers have studied the linkages between growth and redistribution (Alesina and Rodrik 1994, Persson and Tabellini 1994). In these second kind of papers, the tax rate
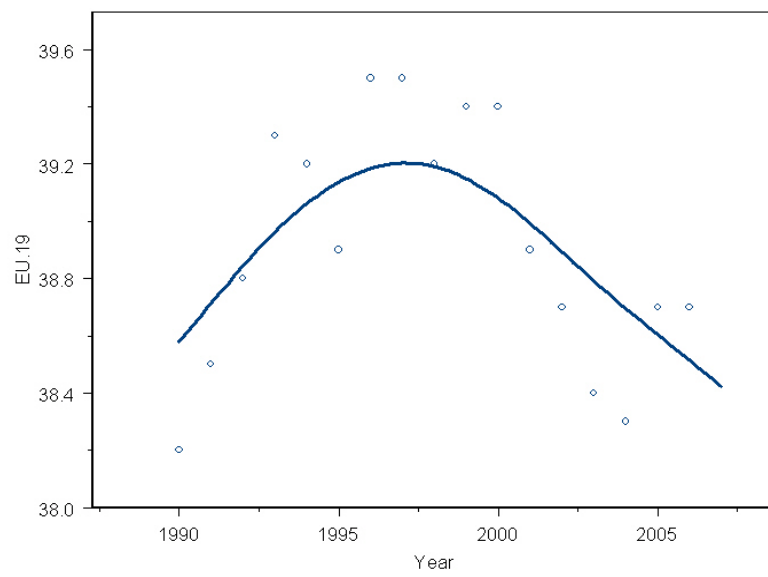
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Figure 2: Tax Revenue as percentage of GDP in the European Union (19 countries) between 1990 and 2007. _Source_: OECD. The line is get using the command Smoothing-Spline plot in S-Plus.

bears on the return of savings. Consequently, the fiscal system has a distorsive impact on capital accumulation. These transfers are used to finance flat benefits for Persson and Tabellini (1994), and to finance Government spending on productive activities for Alesina and Rodrik (1994). The common feature of these two papers is to show that more inequalities lead to a higher demand for redistribution, which increases the tax rate on capital accumulation. It implies that the growth rate of the economy decreases. These models can be considered as the first models which include both a dynamic environment and a demand for redistribution. However, in their models, the structure of the population is constant over time and they only consider the dynamics of the income distribution. Perotti (1993) incorporates an endogenous determination of the structure of the population *à la* Galor and Zeira (1993), into a model in which the size of the social security system is endogenously determined. However, he does not study the dynamics of the structure of the population[2].

Our paper completes this literature on two significant points. (1) These papers use static models. It implies that all changes in the size of the Welfare State come from modifications of some relevant parameters such as wage inequalities. Consequently, their models cannot provide an explanation of the dynamics of the size of Government. (2) Because the structure of the population is exogenous, these models do not study the impact of the increase in the share of the educated population on the demand for redistribution. However, the dynamics of modern economies also has an impact on the structure of the population, which changes the political support for intra-generational transfers.

Acemoglu (2002) reported two stylized facts concerning the evolution of the structure of the population. (1) There has been a large increase in the supply of skills in the US economy over the past sixty years. (2) There has been an increase in the relative return (the wage differential) of education. Firstly, an increase in the skill premium has a positive impact on the share of the educated population. Moreover, following an increase in the share of the educated population, the stock of knowledge in the economy raises, which increases the marginal productivity of skilled agents more than that of unskilled agents. We explore these two causalities.

In this paper, we assume an overlapping generations model in which agents live for two periods. In the first period of their life, agents decide to educate themselves or not, while in the second one agents work and supply one unit of labor inelastically. At each

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[2]Only Bisin and Verdier (2000, 2002) consider a vote on public spending and an endogenous structure of the population in a cultural transmission model.

period t, the working population vote on the size of the Welfare State which can take two values to simplify the model: a high and a low tax rate. Tax revenue is redistributed through flat benefits to workers. It implies that educated agents (who receive high wages) vote for the low tax rate, while uneducated agents (who receive low wages) vote for the high tax rate. As long as uneducated (educated) agents have the majority, the high (low) tax rate is chosen. Consequently, the structure of the population determines the size of the Welfare State in our model. Our accumulation variable is knowledge capital. The share of the educated population has a positive impact on the stock of knowledge. *Ceteris paribus*, a higher knowledge capital level has a positive impact on the share of the educated population because of the increase in the wage differential between the educated and the uneducated population. Conversely, the higher the tax rate (redistribution) is, the less agents decide to educate themselves. Finally, as the tax rate of period t has an impact on the educational choices of agents born at period $t-1$, there can be indeterminacy. Indeed, for a reasonably high value of the wage differential[3], if agents expect a low tax rate, then only a few of them decide to remain uneducated, and the low tax rate will actually be chosen. Conversely, if agents expect that the high tax rate will be chosen, then only a few of them decide to educate themselves, and the high tax rate is actually chosen.

This theoretical structure enables us to distinguish four relevant *scenarii* about the dynamics of the Welfare State. The first one is the following. Assuming that the initial wage differential is low, then at the first period, only a few agents decide to educate themselves and the majority of people (uneducated agents) vote for high vertical transfers. Then, because of the knowledge accumulation, the wage differential increases (Acemoglu 2002) and more agents decide to educate themselves. However, the educational decision also depends on the size of vertical transfers. Consequently, for a reasonably high value of the wage differential, if agents expect high vertical transfers, most of them remain uneducated, and actually, uneducated agents keep the majority. But if agents expect small vertical transfers, then most of them decide to educate themselves and educated agents actually get the majority, and they vote for a small Welfare State. Finally, once the wage differential is very high, whatever the size of vertical transfers, the share of the educated population is so high that agents know that a small Welfare State will be chosen in the next period. This scenario could describe the empirical evidence which we mentioned above. Indeed, in a first time the size of the Welfare Sate is significant because of a political support in favor of vertical redistribution. Then, the increase in

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[3]We define more precisely in the model what the term "reasonnably high" means.

the share of the educated population decreases the demand for vertical redistribution. Consequently, agents vote for smaller public spending. In the second scenario, the cost of education is so high that the economy reaches the low steady state before educated agents get the majority. Then, in the long run, the high tax rate is chosen, and no crisis of the Welfare State takes place. In the third scenario, we show that an economy uses a high tax rate as long as agents expect that the same tax rate will be chosen at the next period. However, once agents expect that the Welfare State will be less generous, then a larger share of population decides to educate itself and the low tax rate is chosen at steady state. Finally, in the fourth scenario, we show that there can appear cycles in the size of the Welfare State. It means that an economy can switch from high public spending to smaller one, according to the beliefs of agents.

This model allows us to study the dynamics of the Welfare State and to emphasize some properties related to indeterminacy. Moreover, our model suggests that the evolution of the Welfare State, and notably the "crisis of the Welfare State", is the result of new political pressures because of the endogenous dynamics of the structure of the population. To our knowledge, the literature on the welfare state has still not considered this problem previously.

This paper is organized as follows. Section 2 presents the basic structure of the model. In section 3, we study the equilibrium and the dynamics of the economy. Section 4 provides some concluding remarks.

2   The Model

We assume an overlapping generations model in which agents live for two periods[4]. During their first period of life, agents decide to educate themselves or not. It is a binary choice. If an agent decides not to educate himself, then he spends the time of his first period of life on leisure[5]. The utility level of leisure is normalized to 0. Conversely, if an agent decides to educate himself, then he has to bear a psychological cost related to the learning process. In this paper, we assume that it represents the time which cannot be spent for leisure. When an agent is born, he is randomly endowed with an educational cost (θ). This cost reduces the utility level of agents who decide to educate themselves $(-v(\theta))$.

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[4]The size of generations does not matter in our model because there are no inter-generational transfers. Consequently, the size of each generation is assumed to be constant over time, and is denoted by N.

[5]The length of each period is normalized to 1. We assume that education only has a leisure cost.

θ necessarily belongs to the interval $\Omega_\theta = [\underline{\theta}, \bar{\theta}]$. $f(\theta)$ and $F(\theta)$ denote the density function and the cumulative distribution function of θ respectively. They are such that $f(\theta) > 0$, $\forall \theta \in \Omega_\theta$, $F(\underline{\theta}) = 0$ and $F(\bar{\theta}) = 1$. $f(\theta)$ also denotes the fraction of the population endowed with a cost θ.

We make the following assumption concerning the function $v(\theta)$:

Assumption 1: $v(\bar{\theta}) > 0$, $v(\theta)$ is of class C^1 and $v'(\theta) > 0$, $\forall \theta \in \Omega_\theta$.

The first part of this assumption only means that there exists a fraction of the population for whom the time spent on education is a cost. Otherwise, every agent would choose to be educated. It also implies that our model does not exclude the case in which education provides more utility than leisure for a part of the population. For these people, education is a way to blossom out. It is the case iff[6] $v(\underline{\theta}) < 0$.

The last part of this assumption implies that a longer time spent on the learning process reduces leisure time, and thus the utility level.

During their second period of life, every agent works and supplies one unit of labor inelastically. At period t, the wage level of an educated agent is denoted by w_t^e, while that of an uneducated agent is denoted by w_t^u. At each period t, each worker pays a proportional tax (τ_t) on his wage to finance the Welfare State. The Government uses this fiscal revenue to finance flat transfers (b_t). All agents benefit from these transfers.

Moreover, we assume that there only exists one good in the economy which is used as a *numeraire*.

2.1 Consumers

At period t, the budget constraint of an agent born at period $t-1$ can be written:

$$W_t^i = w_t^i(1 - \tau_t) + b_t \tag{1}$$

with $i \in \{e, u\}$. At period $t-1$, an agent expects the wealth which he will receive when he works. The expectation bears on the wage level $(w_t^{i,a})$ and on the size of the welfare state (τ_t^a, b_t^a). Indeed, our model includes a dynamics of wages, and at each period, the

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[6]In this paper "iff" means "if and only if".

size of the welfare state is endogenously chosen by the population.

Then, at period $t - 1$ we have:

$$W_t^{i,a} = w_t^{i,a}(1 - \tau_t^a) + b_t^a \tag{2}$$

Each agent is assumed to consume his wealth when he works. The utility level of an agent born at period $t - 1$ can be written[7]:

$$U_t^i = -(1 - I)v(\theta) + \ln\left(W_t^i\right) \tag{3}$$

with $I = 0$ if an agent decides to educate himself, and $I = 1$ if he decides to remain uneducated. At period $t - 1$ an agent decides to educate himself iff: $U_t^{u,a} < U_t^{e,a}$. This is equivalent to:

$$\ln\left(\frac{W_t^{e,a}}{W_t^{u,a}}\right) > v(\theta) \tag{4}$$

The use of the log-utility function implies that educational choices depend on the wealth inequality ratio. An increase in this ratio corresponds to an increase in the opportunity cost of remaining uneducated.

Equation (4) will enable us to define a threshold value $\tilde{\theta}_t$ such that, given the expectation of τ_t^a, agents for whom θ is smaller than $\tilde{\theta}_t$ decide to educate themselves, while agents endowed with a θ higher than $\tilde{\theta}_t$ decide to remain uneducated. Consequently, $F(\tilde{\theta}_t)$ denotes the share of the educated population.

2.2 Knowledge Capital

In our economy, the accumulation variable is knowledge capital. h_t denotes the knowledge capital, i.e. the available stock of knowledge at period t for each agent. In doing so, we assume that agents (educated and uneducated) only differ by their ability to transform knowledge capital into productivity, i.e. educated and uneducated agents differ by their ability to transfrom the stock of knowledge into a useful information for the production process. The dynamics of the knowledge capital is assumed to have the following form:

$$h_{t+1} = \Psi(h_t, \tilde{\theta}_t) \tag{5}$$

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[7]We use this utility function because of the tractability of our model.

with[8] $\Psi_1() > 0$ and $\Psi_2() > 0$. It implies that the current stock of knowledge has a positive impact on the future stock of knowledge[9]. Moreover, the share of the educated population at period t has a positive impact on the knowledge capital of period $t + 1$ because these people do research.

The timing of the model is the following. Agents decide at period $t - 1$ to educate themselves or not. Education consists in the acquisition of methods to transform and to improve the stock of knowledge into useful information for the production process. As for uneducated agents, their ability level is not sufficient to use the stock of knowledge efficiently. At period t, agents use the methods which they have acquired when they were young, given the current knowledge capital level. Moreover, educated agents develop new knowledge by research activities, which will only be available at period $t + 1$. It implies that there need time before this new knowledge can be used.

The previous assumptions imply that when an agent is born at period $t - 1$, he observes the current knowledge capital h_{t-1} and the share of the educated population $\tilde{\theta}_{t-1}$. Consequently, he rightly expects the value h_t. His only problem concerns the expectation of the size of the Welfare State of period t.

2.3 Firms

We assume that the technology of firms has the following form:

$$Y_t = f(h_t, L_t^e, L_t^u) \tag{6}$$

with $f_i() > 0$, $\forall i \in \{1, 2, 3\}$. It means that the stock of knowledge and both kinds of labor have a positive impact on the production level. More specifically, firms take the available stock of knowledge of the current period (h_t) as given. h_t is a positive externality for each firm. However, firms choose the quantity of labor which they decide to use in the production process. We make two further assumptions about the properties of the production function: $f_{ii}() < 0$, and $f_{ij}() > 0$ if $i \neq j$. These assumptions notably imply a decreasing marginal productivity of both kinds of labor. Assuming a perfect competition on the final good market and on the inputs markets, we have:

$$w_t^e = f_2(h_t, L_t^e, L_t^u) > 0 \tag{7}$$

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[8]g_i denotes the derivative of $g()$ with respect to its *ith* argument. g_{ij} denotes the derivative of g_i with respect to its *jth* argument.

[9]This is a standard assumption in the human capital literature.

$$w_t^u = f_3(h_t, L_t^e, L_t^u) > 0 \tag{8}$$

Finally, we make the following assumption about the wages of agents:

Assumption 2: $w_t^e > w_t^u$ and $\dfrac{f_{21}}{f_2}h > \dfrac{f_{31}}{f_3}h$.

The first part of this assumption implies that the wage level of educated agents is higher than the wage level of uneducated agents. Thus, it is implicitly assumed that the marginal productivity of unskilled workers is always finite, even if the quantity of unskilled workers is small. Formally, it means that $f_2(h, N, 0) > f_3(h, N, 0)$, $\forall h$, i.e. even if all agents decide to educate themselves and if firms employ them, then the scarcity of unskilled workers is not sufficient for their wage level to become higher than that of skilled workers.

The second part of this assumption implies that the elasticity of the wage of skilled agents with respect to knowledge capital is higher than the elasticity of the wage of unskilled agents with respect to knowledge capital. *Ceteris paribus*, it means that the increase in the stock of knowledge increases the wage inequality ratio w^e/w^u (Acemoglu 2002). In the Acemoglu's paper (2002), the stock of knowledge is represented by a technological bias in favor of skilled workers.

2.4 Government

The Government budget constraint can be written:

$$N\tau_t \bar{w}_t = N b_t \tag{9}$$

knowing that τ_t is endogenously determined at each period by a voting procedure. The political equilibrium is detailed below.

3 The Dynamic Equilibrium

The dynamic equilibrium of this economy is the sequence $\{\theta_t, \tau_t, h_t\}_{t=0,..,+\infty}$ which satisfies the first order conditions of firms ((7) and (8)), the educational choices of agents (4), and the government budget constraint (9). This sequence is such that the input markets and the output one are in equilibrium at each period. Thus, at each period we have:

$$L_t^e = NF(\tilde{\theta}_t) \tag{10}$$

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$$L_t^u = N(1 - F(\tilde{\theta}_t)) \tag{11}$$

Before studying the dynamics of the economy, we have to specify some properties of wages in equilibrium.

3.1 Some Properties of Wages in Equilibrium

Given equations (10) and (11) the wages of educated and uneducated agents can be written:

$$w^e(h_t, \tilde{\theta}_t) = f_2(h_t, NF(\tilde{\theta}_t), N(1 - F(\tilde{\theta}_t))) \tag{12}$$

$$w^u(h_t, \tilde{\theta}_t) = f_3(h_t, NF(\tilde{\theta}_t), N(1 - F(\tilde{\theta}_t))) \tag{13}$$

Our assumptions on the production function imply that that the knowledge capital (h_t) has a positive impact on both w^e and w^u ($w_1^e() > 0$ and $w_1^u() > 0$[10]). However, it can also be shown that the share of the educated population ($F(\tilde{\theta}_t)$) has a negative impact on the wage level of educated agents ($w_2^e() < 0$), and a positive one on the wages of uneducated agents ($w_2^u() > 0$). This result comes from the decreasing marginal productivity of labor.

The average wage of the economy can be written:

$$\bar{w}_t = F(\tilde{\theta}_t)w^e(h_t, \tilde{\theta}_t) + (1 - F(\tilde{\theta}_t))w^u(h_t, \tilde{\theta}_t) \equiv \bar{w}(h_t, \tilde{\theta}_t) \tag{14}$$

The function $\bar{w}(h_t, \tilde{\theta}_t)$ is such that $\bar{w}_1() > 0$. An increase in the knowledge capital level has a positive impact on the average wage of the economy because of the increase in the wages of educated and uneducated agents (with $\tilde{\theta}_t$ constant). However, the share of the educated population has an ambiguous impact on the average wage of the economy. Differentiating equation (14) with respect to $\tilde{\theta}_t$ we get:

$$\frac{\partial \bar{w}(h_t, \tilde{\theta}_t)}{\partial \tilde{\theta}_t} = \underbrace{f(\tilde{\theta}_t)(w^e - w^u)}_{A>0} + \underbrace{F(\tilde{\theta}_t)\frac{\partial w^e(h_t, \tilde{\theta}_t)}{\partial \tilde{\theta}_t}}_{B<0} + \underbrace{(1 - F(\tilde{\theta}_t))\frac{\partial w^u(h_t, \tilde{\theta}_t)}{\partial \tilde{\theta}_t}}_{C>0} \tag{15}$$

Elements A and C are both positive. Element A denotes the increase in the average wage because of the increase in the share of the population having the higher wage level. Element C denotes the increase in the wage level of uneducated agents because of the bigger scarcity of this input. Finally, element B is negative because of the smaller scarcity

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[10]Some simple calculations imply the result.

of the educated population.

Using this preliminary analysis and equation (5), we can determine the share of the educated population at each period. However, this choice depends on the result of the voting procedure. Consequently, we first study the determination of the share of the educated population and the size of transfers from the Government. Finally, we consider the dynamics of our economy, i.e. the sequence $\{\theta_t, \tau_t, h_t\}_{t=0,\ldots,+\infty}$.

3.2 The Share of the Educated Population

At period $t-1$, every agent for whom the following inequality is (not) satisfied, decides to educate himself (to remain uneducated):

$$\ln\left(\frac{w_t^e(1-\tau_t^a) + \tau_t^a \bar{w}_t}{w_t^u(1-\tau_t^a) + \tau_t^a \bar{w}_t}\right) > v(\theta) \tag{16}$$

If it exists, the threshold value $\tilde{\theta}_t$ is defined by:

$$LHS(h_t, \tilde{\theta}_t, \tau_t^a) \equiv \ln\left(\frac{w^e(h_t, \tilde{\theta}_t)(1-\tau_t^a) + \tau_t^a \bar{w}(h_t, \tilde{\theta}_t)}{w^u(h_t, \tilde{\theta}_t)(1-\tau_t^a) + \tau_t^a \bar{w}(h_t, \tilde{\theta}_t)}\right) = v(\tilde{\theta}_t) \tag{17}$$

Lemma 1 *(i) There exists a unique threshold value $\tilde{\theta}_t$ such that $\underline{\theta} < \tilde{\theta}_t < \bar{c}$ iff $LHS(h_t, \underline{\theta}, \tau_t^a) > v(\underline{\theta})$ and $LHS(h_t, \bar{\theta}, \tau_t^a) < v(\bar{\theta})$. (ii) This threshold is such that $\partial\tilde{\theta}_t/\partial\tau_t^a < 0$ and $\partial\tilde{\theta}_t/\partial h_t > 0$.*

Proof: See the appendix of this paper.□

The first element of the second part of lemma 2.1 shows that the size of the welfare state has a negative impact on the share of the educated population. Indeed, because the Government uses flat transfers, they redistribute resources in favor of unskilled agents. Consequently, an increase in the size of the Welfare State increases the opportunity cost to educate oneself.

The second element of the part (ii) of the lemma shows that, for a given tax rate, more agents decide to educate themselves if the knowledge capital level grows. Indeed, as the knowledge capital raises, the wage inequality ratio (w^e/w^u) increases because of the larger marginal impact of the knowledge capital on the wages of educated agents than on the wages of uneducated agents. Thus, the stock of knowledge increases the cost of remaining uneducated, and more agents decide to educate themselves.

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Thanks to lemma 2.1, we can define a function $\tilde{\theta}()$ such that:

$$\tilde{\theta}_t = \tilde{\theta}(h_t, \tau_t^a) \tag{18}$$

with $\tilde{\theta}_1() > 0$ and $\tilde{\theta}_2() < 0$.

3.3   The Political Equilibrium

In our economy, the government is only an institution which applies the decision taken by the majority of the population. Among the working population, each group of agents (educated and uneducated agents) is homogenous, thus all educated agents vote for the same tax rate. As for uneducated agents, they vote for the same tax rate which is different from that of educated agents.

We assume that agents have to choose between two tax rates, $\bar{\tau}$ and $\underline{\tau}$, such that: $1 > \bar{\tau} > \underline{\tau} > 0$. Agents choose the tax rate which maximizes their wealth level. The wealth level of an agent of type i can be written:

$$W_t^i = w_t^i(1 - \tau_t) + \tau_t \bar{w}_t \tag{19}$$

The wealth of educated agents is a decreasing function of τ_t (assumption 2), while the wealth of uneducated agents is an increasing function of τ_t. It implies that educated agents vote for the lower tax rate $\underline{\tau}$, while uneducated agents vote for the higher tax rate $\bar{\tau}$.

The political equilibrium is:

$$\tau_t = \begin{cases} \underline{\tau} & \text{if } F(\tilde{\theta}_t) > 1/2 \\ \bar{\tau} & \text{if } F(\tilde{\theta}_t) < 1/2 \end{cases}$$

We conclude from this analysis that there exists a threshold $\hat{\theta}_t$ such that:

$$F(\hat{\theta}_t) = 1/2 \tag{20}$$

As long as $\tilde{\theta}(h_t, \tau_t^a) < \hat{\theta}_t$, uneducated agents have the majority and the higher tax rate ($\bar{\tau}$) is chosen. Conversely, once $\tilde{\theta}(h_t, \tau_t^a) > \hat{\theta}_t$, educated agents have the majority and the lower tax rate $\underline{\tau}$ is chosen. It implies that the expectations of agents concerning the tax rate has an influence on the result of the vote. We define two threshold values for the knowledge capital level: h_a and h_b, such that:

$$F(\tilde{\theta}(h_a, \underline{\tau})) = 1/2 \tag{21}$$

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and

$$F(\tilde{\theta}(h_b, \bar{\tau})) = 1/2 \qquad\qquad (22)$$

Given that $\tilde{\theta}_1() > 0$ and that $\tilde{\theta}_2() < 0$, it is straightforward to show that $h_b > h_a$. More specifically, differentiating equation (20) with respect to h_t and τ_t^a we obtain:

$$\frac{dh}{d\tau^a} = \frac{\tilde{\theta}_2(h, \tau^a)}{\tilde{\theta}_1(h, \tau^a)}$$

This equation means that the gap between h_a and h_b is all the more small as the ratio between the marginal impact of the tax rate and the marginal impact of the stock of knowledge is low. It means that the differential between the elasticity of the wages of educated agents and the elasticity of the wages of uneducated agents with respect to the stock of knowledge plays a significant role. If this differential is high, then a small increase in h provides a strong incentive to educate oneself. Consequently, if τ highly increases, then it could be sufficient that h slightly increases for educated agents to get the majority.

Moreover, if the distance between h_a and h_b is small, then h_a and h_b take low (high) values if a large fraction of the population has small (big) educational costs. It could represent some other specificities of education which are not explicitly introduced in this paper such as low monetary or cultural costs of education. It means that the educated population gets the majority for low (high) values of the stock of knowledge.

Lemma 2 sums up the result of the vote for different values of h_t.

Lemma 2 *If $h_t < h_a$, then no agent born at period $t-1$ expects that $\underline{\tau}$ can be adopted and $\tau_t = \bar{\tau}$. For $h_t > h_b$, no agent expects the tax rate $\bar{\tau}$ can be adopted and thus $\tau_t = \underline{\tau}$. For $h_a < h_t < h_b$, then we have self-fulfilling prophecies.*

For $h_t < h_a$, the knowledge capital level is so small that agents know at period $t-1$ that uneducated agents will have the majority at period t. Conversely, for $h_t > h_b$, the knowledge capital level is so high that agents know at period $t-1$ that educated agents will have the majority at period t. For the intermediate case ($h_a < h_t < h_b$), if agents expect that the low (high) tax rate will be chosen, then a large (small) share of the population will educate itself and educated agents will (not) actually have the majority.

3.4 The Dynamics

The stock of knowledge of period t only depends on the share of the educated population of period $t-1$ and on the knowledge capital level h_{t-1} (equation (5)). Assuming that

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both can be observed without cost, agents know at period $t-1$ the value of the knowledge capital of period t (h_t). However, agents born at period $t-1$ expect the result of the vote of period t. This expectation has an impact on the share of the educated population at period t, and thus, it has an impact on the knowledge capital of period $t+1$.

With equation (18), equation (5) can be rewritten:

$$h_{t+1} = \Psi(h_t, \tilde{\theta}(h_t, \tau_t^a)) \equiv \Phi(h_t, \tau_t^a) \tag{23}$$

Using the properties of the function $\Psi()$, we have $\Phi_1() > 0$, i.e. a higher knowledge capital level at period t has a direct positive impact on h_{t+1}, and an indirect impact through the share of the educated population of period t. Given Lemma 2.2, the dynamics of our economy is described by:

$$h_{t+1} = \begin{cases} \Phi(h_t, \bar{\tau}) \text{ if } h_t < h_a, \text{ or if } h_a < h_t < h_b \text{ and } \tau_t^a = \bar{\tau} \\ \Phi(h_t, \underline{\tau}) \text{ if } h_t > h_b, \text{ or if } h_a < h_t < h_b \text{ and } \tau_t^a = \underline{\tau} \end{cases}$$

h_{t+1} is directly influenced by choices made at period $t-1$. Indeed, the share of the educated population of period t is determined at period $t-1$. However, this decision depends on the expected tax rate of period t. Consequently, for low (high) values of knowledge capital, agents know that the high (low) tax rate will be chosen at period t, and only a few (most) people decide to educate themselves, which has a negative (positive) impact on h_{t+1}. For $h_a < h_t < h_b$, if agents expect the tax rate $\bar{\tau}$ to be chosen at period t, then only a few people decide to educate themselves, which implies that uneducated agents will actually have the majority. At period $t-1$, the expectation of a high tax rate at period t has a negative impact on knowledge capital of period $t+1$.

Assumption 3: $\Phi_{11}(h_t, \tau_t^a) < 0$ and $\Phi(0, \tau_t^a) \geq 0$.

The first part of this assumption means that the function $\Phi()$ is a strictly concave function of h_t, which is a sufficient condition for the stability of the steady state. The second part of this assumption implies that there exists a unique non-trivial steady state for knowledge capital. Let us call a *potential steady state*, a steady state which should be obtained if an economy kept the same tax rate τ at every period. $\underline{h}$ and $\bar{h}$ denote the steady state knowledge capital levels for the tax rates $\underline{\tau}$ and $\bar{\tau}$ respectively. Given assumption 4 and the properties of the functions $\Phi()$ and $\tilde{\theta}()$, it is straightforward to show
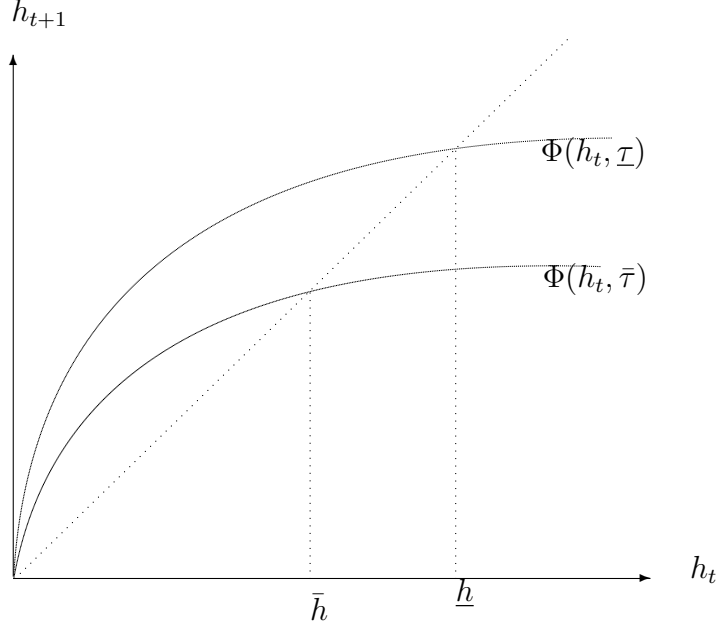
Figure 3: Potential steady states

that $\underline{h}$ and $\bar{h}$ are stable, and that $\underline{h} > \bar{h}$. This result comes from the negative impact of redistributive policies on the share of the educated population (see figure 3).

The dynamics of our economy is not trivial and we have to consider some cases according to the respective positions of $\bar{h}$, $\underline{h}$, h_a and h_b[11]. We assume that the initial value of knowledge capital h_0 is given. The first generation of agents born at period -1, decide to educate themselves or not, expecting the values taken by h_0 and by τ_0^a.

Case 1: $\bar{h} > h_b$ and $\underline{h} > h_b$

Figure 4 illustrates that case, which occurs as long as the gap between h_a and h_b is not too high and if a large share of the population has small educational costs. The gap between h_a and h_b is not too high if the knowledge capital has a dramatic impact on wage inequalities. Moreover, educational costs are small if there are no monetary or cultural barriers too significant concerning the access to education.

In this first case, the economy cannot converge towards the low steady state $\bar{h}$, because at that point, educated agents have the majority and the tax rate $\underline{\tau}$ is chosen. A large

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[11]We do not study all cases but only the ones which we consider to be the most relevant.
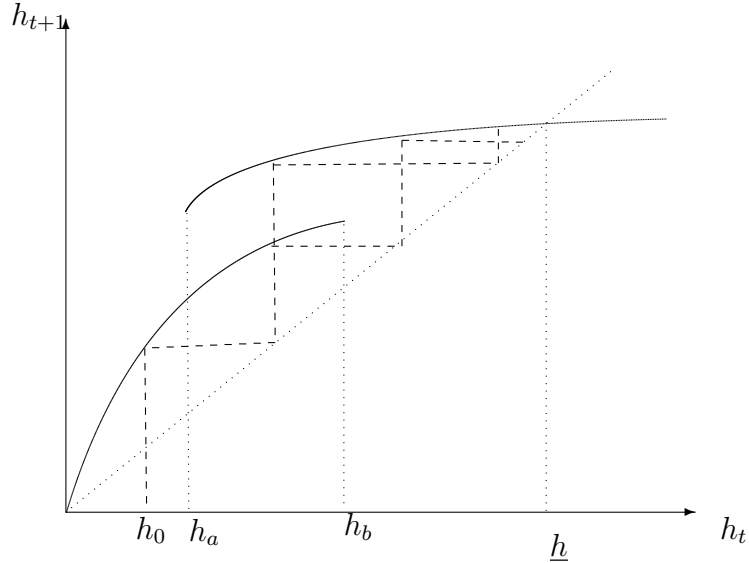
Figure 4: Illustration of case 1

Welfare State is only a temporary equilibrium. Let us consider that the initial value of knowledge capital is small ($h_0 < h_a$). Figure 4 gives two examples of trajectories. h_0 is so small that agents born at period -1 know that the high tax rate will be chosen at period 0. However, given the value h_1, agents born at period 0 can have two kinds of expectations. They can either expect that τ_1 will be high and in that case the knowledge capital follows the low trajectory; or they can expect that τ_1 will be low and the knowledge capital follows the high trajectory. In the long-run, whatever the initial value of h, the economy will have a small Welfare State $\underline{\tau}$ and knowledge capital will converge towards $\underline{h}$. Nevertheless, self-fulfilling prophecies can change the dynamics of the economy and the date from which a small Welfare State is adopted[12].

This case emphasizes that the size of the Government of near countries can follow different trajectories according to the beliefs of agents. Then, the "crisis of the Welfare State" can appear at different dates for countries having near initial characteristics.

Case 2: $h_a > \bar{h}$ and $h_a > \underline{h}$

Figure 5 illustrates that case, which occurs as long as the gap between h_a and h_b is not

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[12]As in Bisin and Verdier (2000, 2002), the beliefs of agents matter to study the dynamics of our economy. However, we assume that there is no coordination problem, i.e. agents share the same beliefs about the future policy.
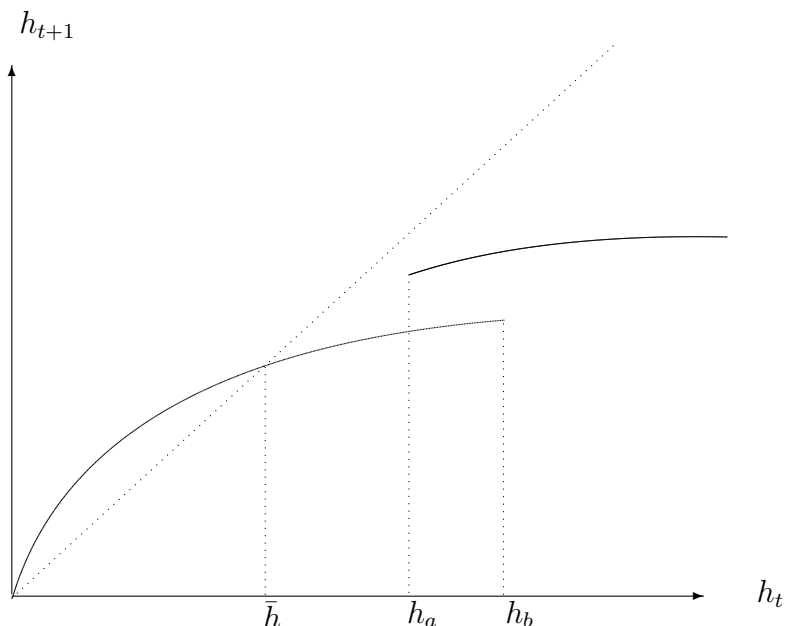
Figure 5: Illustration of case 2

too high[13] and if the educational costs remain high for a significant part of the population. The low access to education implies that the wage differential has to be very high for a large share of agents decide to educate themselves. Consequently, the economy reaches the low steady state $\bar{h}$ before the educated population gets the majority. A wide Welfare State is chosen at all periods. This case could explain why some countries do not seem to question the size of their Welfare State, and go on to use a high tax level (see figure 1).

In the two previous cases, the technology of the production process is such that the knowledge capital has a significant impact on the wage gap, and then on educational choices. Let us remove this assumption. It implies that the redistributive policy has henceforth a dramatic impact on educational choices. More formally, the distance between the thresholds h_a and h_b becomes high.

Case 3: $\underline{h} > h_b > \bar{h} > h_a$

Figure 6 illustrates that case. It occurs as long as the distance between h_a and h_b is

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[13]Let us recall that this is the case as long as the knowledge capital has a significant impact on the wage gap between educated and uneducated agents.
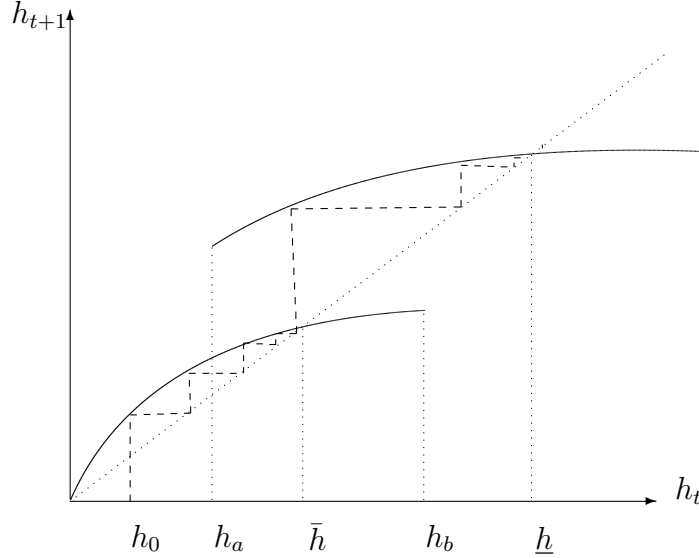
Figure 6: Illustration of case 3

significant but not too high. Moreover, the value h_b above which agents know that the low tax rate will be chosen at the next period, is smaller than the potential steady state of the knowledge capital. It means that the technology of the knowledge accumulation is very efficient.

In figure 6 we have represented the dynamics of an economy whose initial knowledge capital (h_0) is very low. In the long-run, the economy can converge towards the low steady state $\bar{h}$ such that a majority of the population votes for a high Welfare State. The economy can remain in this situation as long as expectations are stable. However, if a generation expects that a low tax rate will be chosen at the next period, then only a few agents decide not educate themselves, and the educated population actually gets the majority. Consequently, the crisis of the Welfare State occurs only if agents expect that it will do. Using this analysis, countries of figure 1 can differ in the long-run only because of different expectations. Compared to the first case, here the expectations affect the steady state of the economy.

Case 4: $h_b > \underline{h} > \bar{h} > h_a$

Figure 7 illustrates that case, which occurs as long as the gap between h_a and h_b is high, i.e. if the share of the educated population depends more on the redistributive
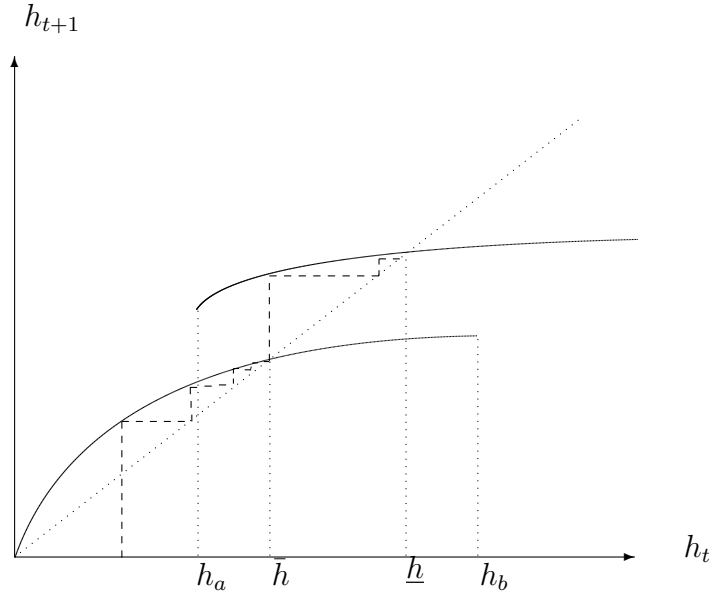
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Figure 7: Illustration of case 4

policies than on the elasticity of wages with respect to the stock of knowledge. Moreover, the technology of the accumulation of the knowledge capital is not very efficient because the highest steady state ($\underline{h}$) is smaller than h_b. In this specific context the economy can switch from one steady state to the other according to the beliefs of agents about the size of the Welfare State. For example, let us assume that h_0 is sufficiently small for agents to expect a large Welfare State to be adopted at period 0. If this belief endures for every following generation, knowledge capital will converge towards $\bar{h}$ which is associated with a large social security system. But, once at steady state, if the beliefs of agents change and if young agents expect that the Welfare State will be less generous, then a large part of the population decides to educate itself and $\underline{\tau}$ will actually be chosen. If this belief endures, the economy will converge towards $\underline{h}$ with a less redistributive Welfare State. In the same way, once the economy reaches this new steady state, if some generations of agents believe that a large welfare state will be chosen, the economy will come back to the steady state $\bar{h}$.

For the moment, the existence of the Welfare State is too recent to invalidate or to confirm the plausibility of such a case. So, it might be read as a theoretical result about the cycles in a political economy model with indeterminacy.

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4 Conclusion and Discussion

In this paper we have tried to provide some explanations about the differences observed in the dynamics of the size of the Welfare States of developed countries (figure 1)[14]. We notably explain why some countries keep a high level of public spending, whereas others have begun to reduce the fiscal burden of their Government. Our main argument is that the demand for redistribution can be engine of the dynamics of the Welfare Sate. It implies that the structure of the population, which influences the demand for redistribution, plays a main role to study the previous empirical facts. We have shown that some *scenarii* can be considered. In case 1, the beliefs of agents can change the period from which the size of public spending decreases. In case 2, the steady state value of the knowledge capital is too small for educated agents get the majority. Then, a wide Welfare State is chosen at all periods. In case 3, the beliefs of agents can change both the dynamics and the steady state of the economy. Indeed, an economy keeps a wide tax rate as long as agents expect that such a situation will remain the same at the next period. However, once a generation expect that a change will occur, then the economy adopts a low Welfare State. In case 4, we show that that there can appear cycles in the size of public spending.

This paper emphasizes that the dynamics of the size of a Government does not only depend on objective factors such as the wage inequality ratio,... but that it also depends on subjective factors such as the beliefs of agents. We have shown that it can have a significant impact on the result of the voting procedure. It also proves the importance of the public debate and of the diffusion of a belief. Some agents, whose educational costs are low, have an incentive to convince every others that the Welfare State will be less generous at the next period, for it actually to be the case. This role played by subjective factors has already been studied in a few papers. In Alesina and Angeletos (2005), the beliefs of agents about the fairness of the economic system determines the demand for redistribution. Bisin and Verdier (2000, 2002) also use the indeterminacy property in a cultural transmission model in which agents vote on the size of the Welfare State. In this model, there are two kinds of agents: those who like a wide Welfare State, and those who dislike it. Parents prefer that their children have the same preferences as they, and parents have to make an effort for this is the case. Parents also take into account the result of the vote which will occur when their children will be in the working population. Bisin and Verdier (2000, 2002) show that there can be indeterminacy in the initial periods, but that,

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[14]Our main objective was not to explain the initial differences in the level of taxes which has already been widely studied in the literature.

in the long-run, an economy converges either towards a low Welfare State, or towards a high Welfare State.

Finally, this paper has only considered one aspect of the Welfare State: the vertical redistribution. Nevertheless, it also provides insurance against life risks, public spending in infrastructure,.... Our story is only one way to consider the role of the Welfare State and its dynamics. These elements should be considered to tell the complete story of the dynamics of Welfare States.

5   APPENDIX

This appendix provides the proof of lemma 2.1.

(i) The RHS is a strictly increasing function of $\tilde{\theta}_t$ by assumption (see assumption 1). There only remains to proove that the LHS is a decreasing function of $\tilde{\theta}_t$.

Differentiating the ratio of the LHS of equation (17), we obtain an expression of the following form:

$$\frac{\frac{\partial w^e}{\partial \tilde{\theta}}(1-\tau^a)(w^u(1-\tau^a)+\tau^a\bar{w})-\frac{\partial w^u}{\partial \tilde{\theta}}(1-\tau^a)(w^e(1-\tau^a)+\tau^a\bar{w})+\tau^a(1-\tau^a)(w^u-w^e)\frac{\partial \bar{w}}{\partial \tilde{\theta}}}{\left[w^u(h,\tilde{\theta})(1-\tau^a)+\tau^a\bar{w}(h,\tilde{\theta})\right]^2}$$

Using equations (14) and (15) this expression becomes:

$$\underbrace{\frac{(1-\tau^a)w^u\frac{\partial w^e}{\partial \tilde{\theta}}-\frac{\partial w_u}{\partial \tilde{\theta}}(1-\tau^a)(w^e(1-\tau^a)+\tau^a\bar{w})}{\left[w^u(h,\tilde{\theta})(1-\tau^a)+\tau^a\bar{w}(h,\tilde{\theta})\right]^2}}_{<0}$$

$$\underbrace{\frac{+\tau^a(1-\tau^a)(w^u-w^e)[f(\tilde{\theta})(w^e-w^u)+(1-F(\tilde{\theta}))\frac{\partial w^u}{\partial \tilde{\theta}}]}{\left[w^u(h,\tilde{\theta})(1-\tau^a)+\tau^a\bar{w}(h,\tilde{\theta})\right]^2}}_{<0}$$

(ii) The LHS is a strictly decreasing function of τ^a_t.

Differentiating the ratio of the LHS of equation (17) with respect to h_t, we obtain an expression of the following form:

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$$\frac{\frac{\partial w^e}{\partial h}(1-\tau^a)(w^u(1-\tau^a)+\tau^a\bar{w})-\frac{\partial w^u}{\partial h}(1-\tau^a)(w^e(1-\tau^a)+\tau^a\bar{w})+\frac{\partial \bar{w}}{\partial h}\tau^a(1-\tau^a)(w^u-w^e)}{\left[w^u(h,\tilde{\theta})(1-\tau^a)+\tau^a\bar{w}(h,\tilde{\theta})\right]^2}$$

Using equations (14) we obtain:

$$\frac{(1-\tau^a)\left(\frac{\partial w^e}{\partial h}w^u-\frac{\partial w^u}{\partial h}w^e\right)}{\left[w^u(h,\tilde{\theta})(1-\tau^a)+\tau^a\bar{w}(h,\tilde{\theta})\right]^2} > 0$$

The numerator is positive under assumption 2 and if $\tau^a < 1$. As we assume that τ cannot be equal to 1 then this condition is satisfied.∎

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